Foreign Currency Pricing

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Abstract

A special case of dollarization is analyzed: quotation of prices in dollars. The proposed explanation is price stickiness: when price adjustment is costly, firms can prefer to fix their prices in a stable foreign currency rather than in an unstable domestic one in order to avoid frequent price changes.

The proposed model shows how the choice of price-setting currency made by a firm depends on the inflation rate, exchange rate volatility, the pricing currency of competitors and input suppliers, and the shape of the demand function. The model predicts that there are two Nash equilibria in the economy populated by symmetric firms: an equilibrium with uniform ruble pricing and an equilibrium with uniform dollar pricing.

It is shown that in economy with less competition a smaller increase in inflation is needed to make an individual firm deviate from the equilibrium with uniform ruble pricing and turn to pricing in dollars.

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Q: What is the difference between one dollar and one ruble?

A: One dollar.

(A Russian joke of early 1990s)

1 Introduction

This paper analyzes a particular form of dollarization - quotation of prices of goods and services in dollars instead of local currency. Such practice became widespread in Russia during the early years of transition when inflation was persistently high; it is still popular today, although inflation became much lower. It should be noted that the practice of quotation of prices in a foreign currency does not require any actual use of that currency: in most cases transactions are still carried out in the domestic currency, and the foreign money is used solely as a unit of account.

Previous theoretical research on dollarization was mainly focused on the relative money demand for domestic and foreign money. Such approach is useful to capture the use of foreign money as a store of value: to make savings, individuals choose assets denominated in the currency, which yields the higher expected return. The store of value function of money is, to some extent, linked to its means of exchange function: people can often find it more convenient to conduct the big-ticket transactions in the currency in which they hold their savings. Hence, the standard approach can, at least partly, explain the use of foreign currency as a means of exchange. However, it hardly helps to understand the use of foreign currency as a unit of account (see Calvo and Vegh (1996) for the survey of the theoretical research on dollarization). In this sense, our paper complements the existing research on dollarization by analyzing its relevance to this last remaining function of money - unit of account. However, since this form of dollarization is quite different in its nature from what is usually studied in this literature, we label the phenomenon of quoting prices in foreign currency as "foreign currency pricing," further denoted FCP.

We explain the firms' decision to denominate prices in dollars as a case of price stickiness. At the times of high inflation, quoting prices in the domestic currency would require frequent price adjustments. If price adjustment is costly due to some sort of menu costs, sellers can prefer to denominate prices for their products in a stable foreign currency, which allows keeping prices unchanged for a much longer period. However, the strategy of switching individually to a different price-setting currency has certain drawbacks, making a firm vulnerable to fluctuations of the exchange rate, which can throw the firm's price rather far from the prices of others.

The model presented in the paper shows how the choice of the price-setting currency made by an individual imperfectly competitive firm is determined by the following features of the environment: the relation between the inflation rate and the exchange rate volatility, the currency in which competitors set prices, and the currency in which inputs are priced. The pricing strategy of competitors is important for the firm's decision in the case of a high degree of real price rigidity in the sense of Ball and Romer (1990). This real rigidity is introduced in the paper as a smoothed-out kink in the demand curve, following Kimball (1995). Such a demand curve makes firms desire to keep their prices close to those of the competitors.

It is pointed in Calvo and Vegh (1996) that dollarization in the standard understanding of the word typically appears to exhibit "hysteresis," in the sense that the degree of dollarization (measured as the proportion of foreign currency deposits in broad money) does not fall immediately in response to a reduction in inflation. Although there is no consensus in explaining hysteresis for this type of dollarization [1], hysteresis in the foreign currency pricing is easy to explain. Our model predicts that the firms, which turn to dollar pricing during high inflation period, can continue denominating prices in dollars long after the stabilization of inflation. The source of hysteresis in the model is multiple equilibria, which arise when firms try to avoid large deviations of their price from the prices of competitors: no firm would decide to switch to a different price-setting currency individually even if the inflation environment changes.

Finally, it is shown in the model that the level of competition has an important influence on the choice of price-setting currency made by the firms within the economy. In a less competitive economy a lower inflation rate is needed to make firms switch to dollar pricing. This finding is consistent with an informal observations that the more expensive luxury items, such as fancy restaurants, have been practicing FCP most vigorously. These services could be thought as being less competitive.

In a way, the paper is related not so much to the literature on dollarization, but rather to the

[1] Among the most popular explanations attempts are those based on the role of financial adaptation (Dornbusch and Reynoso 1989, Dornbusch, Sturzenegger and Wolf 1990)), costs of switching to a different currency (Guidotti and Rodriguez 1992, Sturzenegger 1993), optimal portfolio considerations under the assumption of perfect capital mobility (Calvo and Vegh 1996)).

debate on local currency pricing (LCP) versus producer currency pricing (PCP) in the modern "New open economy macroeconomics" paradigm (see Lane (1991) for a survey). Thus, foreign currency pricing can be thought of as an extreme case of producer currency pricing in that literature, in the sense that FCP implies perfect exchange rate pass-through not only for imports, but potentially for all goods in the economy, even nontradable ones. Hence, FCP may have strong implications for monetary policy and exchange rate volatility, which should be low in this case to allow consumption smoothing. Within the LCP-PCP debate, a paper analogous to ours is Friberg (1998). The difference, however, is that Friberg studies the choice of currency, in which to price imports, based purely on exchange rate uncertainty with price being predetermined but not necessarily rigid. In contrast, we concentrate on the role of inflation and exchange rate volatility in sticky-price environment.

The rest of the paper is organized as follows. Section 3 demonstrates existence of multiple equilibria in a simple reduced-form example, where firms suffer quadratic losses from deviating from the optimal price. In the example, we ignore the issue of input pricing, and concentrate solely on inflation rate, exchange rate volatility, and the pricing currency of competitors. In this example, the result is that a dollar-pricing economy would never revert back to local currency, even if inflation is brought to zero. In the section 4 we extend the model to include prices of inputs, real rigidity, and endogenous rate of price adjustment. In this section we demonstrate that the most important factor determining the pricing currency is the denomination of input prices, especially in the case of constant elasticity of demand, which makes the optimal price be a constant mark-up over the input price, independently of the pricing strategy of the competitors. However, this tendency can be countered by a sufficiently strong real rigidity in the form of a smoothed out kink in the demand curve, which makes the prices of competitors matter. Section 5 concludes.

2 Some facts

Although this paper is a theoretical exercise, we motivate our analysis with a snapshot of several product groups in Moscow. Table 1 demonstrates, in which currencies sellers in Moscow denominate their prices among 20 different product groups. These numbers were obtained by making phone calls to 20 sellers among each of these product groups, picking them at random from the Yellow

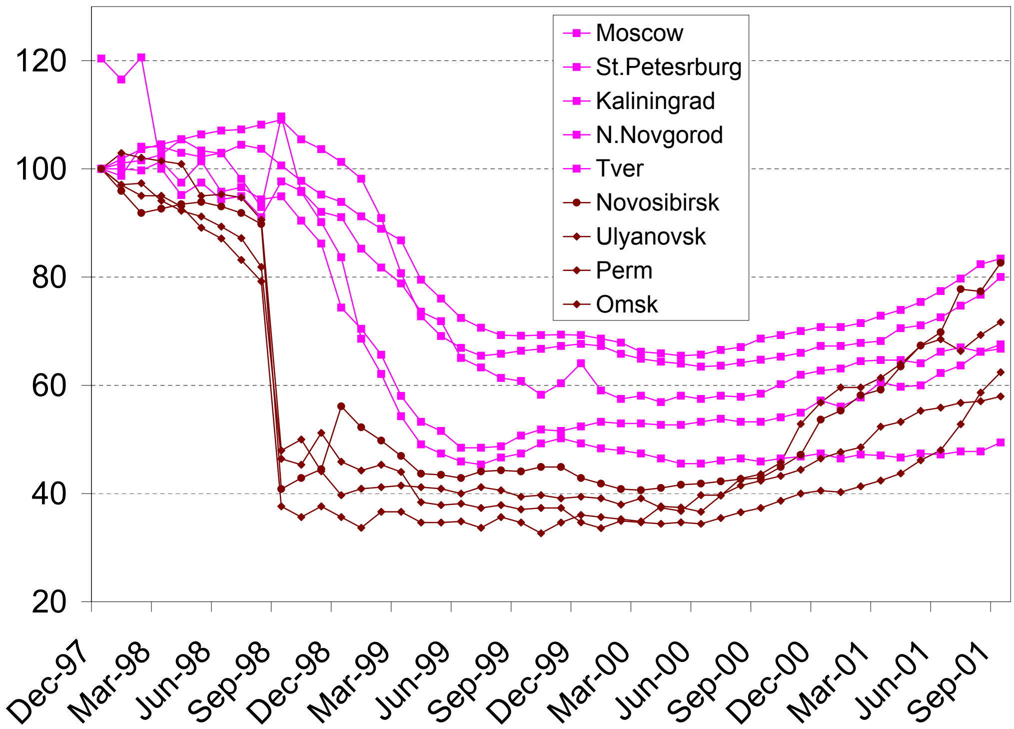
Table 1: Pricing currencies for some product groups

Number	Product	% in rubles	% in dollars
1	Advertising	0%	100%
2	Bank equipment	5%	95%
3	Ward robes	15%	85%
4	Keyboards	20%	80%
5	Auto body repair	25%	75%
6	Translation	30%	70%
7	Lamps	43%	57%
8	Home improvement	55%	45%
9	Aerobics classes	56%	44%
10	Plumbing supplies	57%	43%
11	English classes	58%	42%
12	Restaurants	60%	40%
13	Office furniture	67%	33%
14	Glass installation	75%	25%
15	Wash machines	77%	23%
16	Freezers	85%	15%
17	Refrigerators	95%	5%
18	Lunch deliveries	100%	0%
19	Notary services	100%	0%
20	Garages	100%	0%
The numbers were obtained by phone calls to 20 sellers within each of these groups in Moscow in the fall of 2001. The sellers were picked at random from the Yellow Pages.			

Pages. Each of the sellers was asked how much a certain product costs, and then we noted, in which currency the seller announced the price. From the table we see that pricing in dollars is widespread but not dominant. In advertising, everything is priced in dollars, while lunch deliveries, notary services, and garages, were all priced in rubles. The other product groups had mixed representation of both types of pricing. This table may seem to contradict our theoretical result that in equilibrium everyone should turn to one currency. This is likely to be explained by low homogeneity within these product groups - if we decomposed restaurants into cheap and expensive ones, we would see that the latter category would be almost uniformly dollar pricing.

An example of markets which are clearly stuck in different equilibria is apartment markets in different cities. Some Russian cities, such as Moscow, St.Petersburg, Kaliningrad, Tver, and Nizhniy Novgorod, have been pricing their apartments almost exclusively in dollars. Other cities, such as Novosibirsk, Omsk, Perm, and Ulyanovsk have been pricing in rubles. Figure 2 demonstrates that

Figure 1: Russian cities: behavior of apartment prices



Source: The Russian Guild of Realtors.

such pricing lead to drastically different behavior of prices, determined by their denomination, following the large devaluation of ruble and output collapse in August 1998.

It is easy to think of other groups of goods, which are priced predominantly in dollars. For example, Russian internet stores have their price lists predominantly in dollars. Many stores put dollar prices on the internet, while at the same time quoting ruble prices in the stores themselves. This type of behavior suggests that stores perhaps would like to price in dollars, but prefer not to do so, because they need a special permission from the city of Moscow. Yet, their ruble prices may be directly tied to a certain fixed dollar value, which they count for themselves and quote on the internet.

3 Expected losses from price stickiness in a simple framework

Here we investigate the relative losses of a firm that sets the price of its product in either rubles or dollars in either dollarized environment (when all other firms price in dollars) or in an environment when everyone else sets prices in local currency (hereafter rubles). These losses will be assessed and compared along the steady-state path with a constant rate of inflation. The theoretical framework is that of Ball, Mankiw and Romer (1988), where a continuum of small monopolisticaly competitive firms produce differentiated products and set prices in a staggered fashion. The deviation from that model is that money supply grows at a constant predictable rate μ, so that in case all firms price in local currency, the economy follows a steady-state path, even though the prices are sticky. That is, in a staggered price setting environment, the aggregation across firms guarantees a smooth rate of inflation and constant output according to the quantity equation

$$y_t = m_t - p_t, \tag{1}$$

where y is log output, m is log money supply and p is the log aggregate price level, and constant velocity is normalized to unity. Normalizing the initial price level and the output to unity as well, and hence their logarithm to zero, we get

$$p_t = m_t = \mu t.$$

An important assumption is made about the path of the exchange rate e. Although the rate is expected to follow the price level according to the PPP hypothesis, it is allowed to fluctuate randomly around that trend. So in every period, the log exchange rate is distributed according to

$$e_t \sim (\mu t, \sigma^2).$$

Note that if all of the firms in the economy set their prices in dollars, then the general price level fluctuates as well, and so does the output by (1). If all prices are set in rubles, however, the price level is smooth and output is constant.

The form of price stickiness is assumed to be as in Calvo (1983). That is, each firm gets a signal to adjust its price at a stochastic rate α. Then, at each moment of time, the firm's losses are quadratic in the deviation of the current price from the optimal one and these losses are represented by the formula

$$\frac{K}{2}(p_{i,t} - p_t^{\#})^2, \tag{2}$$

where $p_{i,t}$ is the price charged by the firm i, and $p_t^{\#}$ is the instantaneously optimal "desired" price (identical for all firms), given by the standard expression

$$p_t^{\#} - p_t = \phi(y_t - \bar{y}), \tag{3}$$

where $\bar{y}$ is the full employment output, here equal to zero, and ϕ is a measure of "real rigidity."

Let us now turn to the examination of the four distinct cases: pricing in dollars and rubles in the dollar and ruble pricing environment.

3.1 Pricing in rubles with everyone else pricing in rubles.

First of all, when everyone prices in rubles, the loss function can be expressed in terms of deviation from the aggregate price level, as it is simultaneously the desired price. This can be seen from (3) and the fact that with pricing in rubles output is constant at the full employment level.

Then, a representative firm chooses its reset price at time zero by solving

$$\min_{p_i} E_0 \frac{K}{2} \sum_{t=0}^{\infty} (1-\alpha)^t (p_i - p_t)^2. \tag{4}$$

Thus, the firm chooses a constant price p_i to minimize losses incurred while this price is in effect. We call this the "reset" price, as this is the price, which the firms chooses once given a stochastic signal to adjust. In principle, the subscript i is not needed, because any firm adjusting its price at time t would choose the same reset price. Yet, since we concentrate on profit losses of an individual firm, we keep the subscript for tractability reasons. The future losses are discounted by the probability of the price still being in effect at t, equal to $(1-\alpha)^t$. E_0 denotes expectation at time zero, which in this case in unnecessary because the problem is entirely deterministic.

The solution to the minimization problem is obtained remembering that $p_t = \mu t$ and observing that $\sum_{t=0}^{\infty}(1-\alpha)^t = \frac{1}{\alpha}$ and $\sum_{t=0}^{\infty} t(1-\alpha)^t = \frac{1-\alpha}{\alpha^2}$. The resultant optimal reset price at time zero (throughout the paper denoted by a star) is then given by

$$p_i^* = \frac{\mu(1-\alpha)}{\alpha}.$$

Thus, we see that the price depends positively on the inflation rate, which is quite intuitive: the optimal price is a weighted average of future optimal prices, which are expected to be higher the higher the inflation rate. Likewise, a higher α implies lower price because the expected length of time for this price to remain in effect is smaller, and hence, future high aggregate price level is discounted more heavily.

Plugging the expression for p^* into (4), and observing that $\sum_{t=0}^{\infty} t^2(1-\alpha)^t = \frac{(2-\alpha)(1-\alpha)}{\alpha^3}$, we obtain the following expression for expected losses:

$$E_0 L_{rr} = \frac{K}{2}\frac{\mu^2(1-\alpha)}{\alpha^3}.$$

Here, L_{rr} stands for "losses with pricing in rubles when others price in rubles." Again, it is quite intuitive that these losses depend positively on the rate of inflation, and negatively on the rate of price adjustment.

3.2 Pricing in dollars with everyone else pricing in rubles

Once the dollar pricing is in the picture, uncertainty is introduced. The problem now becomes

$$\min_{p_i^f} E_0 \frac{K}{2}\sum_{t=0}^{\infty}(1-\alpha)^t(e_t + p_i^f - p_t)^2. \tag{5}$$

Here, the firm i sets a constant price p_i^f, and the ruble price is then $e_t + p_i^f$, which on average grows in line with the optimal price, but with deviations of the size determined by the variance of the exchange rate. Maximizing, we get

$$p_i^{f*} = 0.$$

Thus, there is perfect certainty-equivalence here, which, of course, comes from the assumption of quadratic loss: the firm sets the price at the current optimum, as the ruble price is expected to grow with that optimum. The variability of the ruble price does not affect the decision. Plugging this zero into (5) and noting that $E_0 e_t^2 = \sigma^2 + \mu^2 t^2$ we get

$$L_{dr} = \frac{K}{2}\frac{\sigma^2}{\alpha}.$$

As expected, the losses depend on the variability of the exchange rate, but not on the inflation rate.

3.3 Pricing in rubles with everyone else pricing in dollars

When general pricing is in dollars, output is no longer constant, and the aggregate price level is no longer equivalent to the desired price. Instead, since $p_t = e_t$, $\bar{y}_t = m_t - \mu t = 0$, while $y_t = m_t - e_t$, the desired price is obtained from (3) to be

$$p_t^{\#} = e_t + \phi(\mu t - e_t) = \phi\mu t + (1-\phi)e_t.$$

Then, the minimization problem is

$$\min_{p_i} E_0 \frac{K}{2} \sum_{t=0}^{\infty} (1-\alpha)^t (p_i - \phi\mu t - (1-\phi)e_t)^2. \tag{6}$$

The optimal reset price is

$$p_i^* = \frac{\mu(1-\alpha)}{\alpha},$$

which is exact same as the price quoted by a firm pricing in rubles in ruble environment (Section 3.1). Again, with certainty equivalence only expectations matter.

Expected losses, on the other hand, have an additional term in comparison to the previous case:

$$E_0 L_{rd} = \frac{K}{2}\left(\frac{\mu^2(1-\alpha)}{\alpha^3} + \frac{(1-\phi)^2\sigma^2}{\alpha}\right).$$

The first term in the brackets is the same as before and represents the losses from not keeping up with inflation. The second term is additional losses associated with being away from the group. That is, in a competitive environment, the firm incurs losses not only because the firm is away from the full-employment output but also because the firm is away from everyone else. Here, the aggregate price level fluctuates with the exchange rate, but the firm i does not adjust its price, and hence its relative price is highly variable. Losses thus caused are especially big for low values of ϕ, which makes sense: low ϕ implies strong real rigidity, that is, each firm's optimal price is more dictated by everyone's prices rather than by the aggregate demand. Such a situation is likely to occur in a more competitive system, as stressed in Calvo (2000).

3.4 Pricing in dollars with everyone else pricing in dollars

This last possibility is quite straight-forward as all of the relevant issues have been discussed already. The minimization problem is

$$\min_{p_i^f} E_0 \frac{K}{2} \sum_{t=0}^{\infty} (1-\alpha)^t (p_i^f - \phi\mu t + \phi e_t)^2, \tag{7}$$

the optimal price is $p_i^{f*} = 0$ as before, and the expected losses are

$$L_{dd} = \frac{K}{2}\frac{\phi^2\sigma^2}{\alpha}.$$

Here, once again, the losses depend on the value of ϕ, this time positively. Again, a low ϕ means that firms would not want to deviate from each other much, and this is precisely what is achieved when everyone prices in dollars: even though all are away from the full employment price, all are together, and hence the losses are small. If ϕ is large, on the other hand, aggregate demand is a bigger consideration than the relative price, and so the losses from fluctuating far from the steady-state are large.

3.5 Comparing the losses

Summarizing the above findings, we get the following table of the expected losses:

Table 2: Comparison of expected losses

	All prices in	
Firm i prices in	rubles	dollars
rubles	$\frac{K}{2}\frac{\mu^2(1-\alpha)}{\alpha^3}$	$\frac{K}{2}\left(\frac{\mu^2(1-\alpha)}{\alpha^3}+\frac{(1-\phi)^2\sigma^2}{\alpha}\right)$
dollars	$\frac{K}{2}\frac{\sigma^2}{\alpha}$	$\frac{K}{2}\frac{\phi^2\sigma^2}{\alpha}$

One important result is that it is impossible to say whether uniform pricing in dollars is better or worse than uniform pricing in rubles. Comparison of E_0L_{rr} and E_0L_{dd} depends on the values of the inflation rate, exchange rate volatility, and the degree of real rigidity. Of course, an argument can be made that such comparison is difficult because the volatility of the exchange rate should be influenced by the choice of the economy to price in dollars.

The most sticking result, however, is that $E_0L_{rd} > E_0L_{dd}$ for reasonable values of ϕ, that is, whenever $\phi < 1/2$ with $\mu = 0$. This implies that if everyone in the economy prices in dollars, then no firm will choose to switch to rubles *even if inflation is brought to zero.* This is once again caused by the fact that in a competitive environment firms lose more from being away from others rather than being away from full employment output. At the same time, if inflation is brought to a low enough level, all firms would benefit from an organized switch to ruble pricing. Thus, we face a situation of multiple equilibria, where the economy can be stuck at a dominated dollarized equilibrium indefinitely. It would take a coordinated action to jump to a dominant one.

Note that the same is true in the opposite direction, but not to the same extent. In a ruble environment, a firm would choose to switch to dollar pricing unilaterally as soon as $\mu > \alpha\sigma\sqrt{1-\alpha}$ (this is the condition for $E_0 L_{dr} < E_0 L_{rr}$). Thus, with high enough inflation, the economy will switch to dollar pricing. However, all firms would benefit from a coordinated switch at yet a lower value of inflation as $E_0 L_{dd} < E_0 L_{rr}$ whenever $\mu > \phi\alpha\sigma\sqrt{1-\alpha}$. Between these two levels of inflation, the economy would once again be stuck at a dominated equilibrium.

4 Introducing input costs

The drawback of the model proposed in Section 3 is that it does not take into account the denomination of input prices, and is hardly in line with the empirical facts. Thus, reasonable parameter values would suggest that all prices in Russia should be now denominated in dollars; yet, we observe such practice only with respect to a fraction of goods and services, as shown in Table 1. Besides, it is clear that a significant portion of inputs (say, electricity or transportation) is priced according to ruble-denominated state-controlled tariffs. Hence, in this section we develop a model, which allows explicit modelling of input pricing. In this model we also endogenize the rate of price adjustment.

A principle methodological difference in this section is that we turn to continuous time, which makes the optimization easier and allows analytical solutions. As in Section 3, we allow the exchange rate to fluctuate randomly around the inflation trend. Formally, we assume that log exchange rate e_t follows the continuous-time mean reverting process:

$$d(e_t - \mu t) = -\rho(e_t - \mu t)dt + \sigma dz\,,$$
$$e_0 = 0\,,$$

where dz is an increment of a Wiener process. Thus, the exchange rate follows a standard Ornstein-Uhlenbeck process. If $\rho = 1$ then the process looks very much like the white noise around the trend assumed in Section 3. It will be shown, however, that the more realistic assumption of slower mean-reversion with $\rho > 1$ will not make a qualitative difference.

If $e_0 = 0$, then the log exchange rate the conditional distribution of log rate e_t is normal (see, for example, Dixit and Pindyck (1994)):

$$e_t - e_0 \sim N\left(\mu t\,,\, \frac{\sigma^2}{2\rho}\left(1 - e^{-2\rho t}\right)\right)\,. \tag{8}$$

As t increases the variance of the log rate converges to $\frac{\sigma^2}{2\rho}$, which is the unconditional variance of the process.

Beside the aggregate price level P we introduce the aggregate price of inputs used in the production process P^I. Both prices can be denominated either in rubles or in dollars, the logs of ruble values of P and P^I at every moment t are given by

$$p_t = \begin{cases} \mu t & \text{if firm's competitors quote prices in rubles} \\ e_t & \text{if they quote prices in dollars, and} \end{cases}$$

$$p_t^I = \begin{cases} \mu t & \text{if firm's inputs are denominated in rubles} \\ e_t & \text{if they are denominated in dollars.} \end{cases}$$

Thus, we assume that the input price grows with the general inflation, the only question is whether they grow monotonically or fluctuate with the exchange rate. An alternative would be to allow stickiness in the input price as well, by letting P^I be fixed in a certain currency, during the period when the output price is fixed. Such a specification would arguably be more realistic if the input were a single intermediate good. More likely, however, the single input is a composite of many goods, and hence its price should grow with inflation. The volatility of the input price then depends on the fraction of these inputs priced in dollars.

We also introduce an explicit "menu cost" F of changing the price, which will allow us to determine the rate of price adjustment endogenously. This endogeneity is especially valuable, because the ruble-pricing firms are likely to change their prices more frequently facing a positive rate of inflation. The last difference from Section 3 is that the time interval δ, during which the price is fixed, remains constant, not stochastic, once chosen by a firm. This interval here is the analog of $1/\alpha$, the expected length of time during which a price remained fixed in the stochastic case. Thus, this fixed interval specification is a special case of the stochastic rate with zero variance, provided that the timing of price adjustments by individual firms is spread evenly over time.[2]

To capture the effect of the input price on firm's pricing strategy we introduce the real profit function:

$$\pi\left(\frac{P_i}{P}\right) = \frac{P_i}{P} \cdot Q\left(\frac{P_i}{P}\right) - c\left(Q\left(\frac{P_i}{P}\right)\right).$$

Instead of choosing an exact analytical expression for $Q(p_i)$, we describe the function implicitly through the elasticity of demand for firm's output with respect to its relative price, $\epsilon(P_i/P) = -\frac{\partial \ln Q(P_i/P)}{\partial \ln(P_i/P)}$, around the steady state $\frac{P_i}{P} = 1$. Furthermore, following Kimball (1995), we allow the

[2]The problem with such a specification is that in equilibrium all firms would prefer to adjust prices simultaneously, so an equilibrium with a uniform distribution of price adjustment in time is not stable. However, such an assumption makes further calculations simpler.

individual demand curves to have a smoothed-out kink at the steady state firm's relative price. This particular form of "real rigidity" implies that it is easier for a firm to lose customers by raising its relative price above unity than to attract new customers by lowering its relative price below unity. In terms of the elasticity of demand this means that around the steady state elasticity $\epsilon(P_i/P)$ is an increasing function of the relative price. Hence, we characterize the demand function by two parameters: the elasticity of demand at the unity relative price $\epsilon(1)$, and the (non-negative) rate of change in elasticity with the deviations of relative price from unity $\frac{\epsilon'(1)}{\epsilon(1)} \geq 0$. When $\frac{\epsilon'(1)}{\epsilon(1)}$ is big, firms do not want to deviate from others.

We assume that real costs faced by a firm are given by a linear cost function: $c(Q) = a\,\frac{P^I}{P} \cdot Q$. Then, the real profit function takes the form:

$$\pi\left(\frac{P_{it}}{P}\right) = \left(\frac{P_{it}}{P} - a\frac{P_t^I}{P}\right) \cdot Q\left(\frac{P_{it}}{P}\right). \tag{9}$$

Hence, the instantaneously desired price $\frac{P_{it}^{\#}}{P}$, that at any moment t maximizes firm's profits, satisfies the standard expression:

$$\frac{P_{it}^{\#}}{P} = a\,\frac{P_t^I}{P} \cdot \mathcal{M}(P_{it}^{\#}/P), \tag{10}$$

where $\mathcal{M}(P_i^{\#}/P) = \frac{\epsilon(P_i^{\#}/P)}{\epsilon(P_i^{\#}/P)-1}$ is the optimal mark-up over the marginal cost. We assume that the desired mark-up $\mathcal{M}(P_i^{\#}/P)$ is a non-increasing function of the desired relative price. Then equation (10) always has a unique solution. We normalize the marginal cost coefficient a so that $\frac{P_i^{\#}}{P} = 1$ when relative price of inputs is equal to unity:

$$a = \frac{1}{\mathcal{M}(1)}.$$

Finally, we assume that the fixed price does not deviate far from the instantaneously optimal trajectory, which means that $(p_i^{\#} - p_i)$ is small, and that volatility of the exchange rate is small enough, so that relative prices do not fluctuate too much. Then, as it is shown in Appendix A, the corresponding fluctuations of firm's instantaneously desired price around unity $(p_i^{\#} - p)$ are also small, which allows us to employ the standard second order approximations.

4.1 The Desired Price Trajectory

Log-linearizing equation (10) around unity we can approximate the log firm's instantaneously desired price as a weighted average of logs of the competitors' and input prices:

$$p_i^{\#} = (1-\eta)p + \eta p^I + \underline{o}\left(p_i^{\#} - p\right) . \tag{11}$$

The weight coefficient η is related to the elasticity of firm's desired mark-up with respect to its desired relative price:

$$\eta = \frac{1}{1 - \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}} . \tag{12}$$

The elasticity of the desired mark-up at the unity desired relative price reflects the curvature of the smoothed-out kink of the demand function. It can be expressed in terms of elasticity of demand at the unity relative price and its rate of change around unity:

$$\frac{\mathcal{M}'(1)}{\mathcal{M}(1)} = -\frac{\epsilon'(1)}{\epsilon(1)[\epsilon(1)-1]} . \tag{13}$$

Formulas (11) - (13) are derived in Appendix B. The interpretation of the parameter η is once again the degree of real rigidity, similarly to ϕ in Section 3. The parameter η shows how much the desired price depends on the prices of competitors relative to the marginal cost.

We see from (13) that $\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}$ is non-positive: when a firm's relative price grows, its desired mark-up declines or stays the same. Therefore, $0 < \eta \leq 1$, which implies that the firm's instantaneously desired price depends positively on the input price and non-negatively on the price of competitors. The result that the price of competitors can influence the firm's desired price stems from the specific assumption about the shape of the demand curve. Under the standard assumption of constant elasticity of demand ($\frac{\epsilon'(1)}{\epsilon(1)} = 0$ and, hence, $\eta = 1$) the firm's desired price is equal to a constant markup over the marginal cost, thus, the optimal price trajectory of an individual firm is not affected by the pricing strategies of the competitors.

However, if the demand curve has a sufficiently steep smoothed-out kink at the firm's optimal relative price, a small increase in relative price can lead to a sensible reduction in the market share while a similar decline in relative price would lead only to a small increase in the market share. Then, random fluctuations of the firm's relative price around unity have on average a negative effect on the firm's profits. Thus, calculating the desired price trajectory, an individual firm is heeding not only the time-path of its marginal cost, but also tries to avoid large random deviations from the price trajectory of the competitors. The willingness of an individual firm to keep in line with others is the source of the multiplicity of equilibria, which was obtained in Section 3, and will be obtained here as well. High sensitivity of the markup here corresponds to a low value of β in Section 3.

4.2 Losses from Costly Price Adjustment

When price adjustment is costly, a profit-maximizing firm fixes its price for a certain period of time, instead of changing it at every instant. Thus, the firm incurs profit losses from two sources: deviation of price from the desired trajectory during the period the price is kept fixed in either currency, and the costly price change.

In the described setup the second-order approximation for the instantaneous profit losses from price non-optimality is the following:

$$\pi\left(\frac{P_{it}^{\#}}{P}\right) - \pi\left(\frac{P_{it}}{P}\right) \approx K \cdot \left(p_i - (1-\eta)\, p_{it} - \eta\, p_t^I\right)^2, \tag{14}$$

where K is related to the firm's desired mark-up at unity relative price and its rate of change:

$$K = \frac{1 - \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}}{2\left[\mathcal{M}(1) - 1\right]}. \tag{15}$$

Expressions (14)-(15) are derived in Appendix C.

With a continuous stochastic process for the exchange rate, the expectation as of period 0 of firm's accumulated profit losses from price non-optimality during each episode of fixed price depends on the expectation as of period 0 of distribution of the exchange rate at the beginning of this episode. Furthermore, these episodes are not independent, since the firm's expectation about the exchange rate at the beginning of the next period of fixed price depends on the current value of the exchange rate. Therefore, an optimizing firm, when choosing the pricing currency, at period 0 should make expectations about its profit losses during each episode of fixed price.

If a firm denominates its price in rubles, its expected, at the time of kth price adjustment, accumulated losses from price non-optimality during the period between the k-th and $(k+1)$-th price changes are equal to

$$L_k^{Rub}(p_{ik}) = \mathbf{E}_k \int_{k\delta}^{(k+1)\delta} K \cdot \left(p_{ik} - (1-\eta)p_t - \eta p_t^I\right)^2 dt, \tag{16}$$

where p_{ik} is log firm's fixed ruble price during this period. At time 0, the firm chooses the sequence of optimal ruble prices $\{p_{ik}\}|_{k=1}^{\infty}$ and the optimal length of time between price changes δ that minimize expected total profit losses per unit of time:

$$Loss^{Rub} = \min_{\{p_{ik}\},\,\delta} \lim_{n\to\infty} E_0 \left[\frac{1}{n\delta} \left(\sum_{k=0}^{n} L_k^{Rub}(p_{ik}) + nF \right) \right] \tag{17}$$

Similarly, if a firm denominates its price in dollars, its expected, as of period 0, losses from price non-optimality during the period between the k-th and $(k+1)$-th price changes are given by

$$L_k^{Doll}(p_{ik}^f) = \mathbf{E}_0 \int_{k\delta^f}^{(k+1)\delta^f} K \cdot \left(e_t + p_{ik}^f - (1-\eta)p_t - \eta p_t^I \right)^2 dt, \tag{18}$$

where p_{ik}^f is log firm's dollar price between the k-th and $(k+1)$-th price changes. The firm chooses the sequence of optimal dollar prices $\{p_{ik}^f\}\Big|_{k=1}^{\infty}$ and the optimal length of time between price changes δ^f that minimize expected total profit losses per unit of time:

$$Loss^{Doll} = \min_{\{p_{ik}^f\},\,\delta^f} \lim_{n\to\infty} \left[\frac{1}{n\delta^f} \left(\sum_{k=0}^{n} L_k^{Doll}(p_{ik}^f) + nF \right) \right] \tag{19}$$

The firm chooses the currency to quote prices in by comparing expected losses per unit of time associated with either pricing strategy.

4.3 Analyzing the Results

The solution to the optimization problems (17) and (19) is shown in Appendix D. We summarize the results below.

i. If a firm quotes its price in rubles the optimal length of time between the price changes δ^* and optimal set of the reset prices $\{p_{ik}^*\}$ are given by

$$\delta^* = \left(\frac{6F}{K\mu^2} \right)^{\frac{1}{3}}; \qquad p_{ik}^* = \left(k - \frac{1}{2} \right) \mu\delta^* .$$

Naturally, the optimal length of time between the price adjustments δ^* declines with inflation: with a higher inflation a firm has to revise its price more frequently in order to keep in line with the growing prices of competitors and growing input prices. Substituting expression for δ^* into the expression for the optimal reset price p_{ik}^* we see that the optimal reset price depends positively on inflation as before.

Table 3: Losses associated with ruble and dollar pricing

	Competitors price in rubles	Competitors price in dollars
Inputs prices in rubles	$Loss^{Rub} = \Lambda\left(K\mu^2\right)^{\frac{1}{3}}$ $Loss^{Doll} = K\frac{\sigma^2}{2\rho}$	$Loss^{Rub} = \Lambda\left(K\mu^2\right)^{\frac{1}{3}} + K(1-\eta)^2\frac{\sigma^2}{2\rho}$ $Loss^{Doll} = K\eta^2\frac{\sigma^2}{2\rho}$
Inputs prices in dollars	$Loss^{Rub} = \Lambda\left(K\mu^2\right)^{\frac{1}{3}} + K\eta^2\frac{\sigma^2}{2\rho}$ $Loss^{Doll} = K(1-\eta)^2\frac{\sigma^2}{2\rho}$	$Loss^{Rub} = \Lambda\left(K\mu^2\right)^{\frac{1}{3}} + K\frac{\sigma^2}{2\rho}$ $Loss^{Doll} = 0$

Note that neither the optimal length of time between the price changes nor the reset price depends on the pricing strategies of competitors and input suppliers. Only the inflation rate and the parameters of the demand function influence the firm's choice of the reset ruble price and frequency of its adjustment. Again, due to certainty equivalence, variance does not matter.

ii. If a firm denominates its price in dollars, the optimal length of time between the price changes δ^{f*} and the optimal sequence of the reset dollar prices $\{p_{ik}^{f*}\}$ are given by

$$\delta^{f*} = \infty \; ; \qquad p_{ik}^{f*} = 0 \quad \text{for all } k\,.$$

Under the assumption of stable zero dollar inflation a firm does not need to adjust its dollar price, instead it fixes the price at the optimal unity level once and for all. This replicates the result obtained in Section 3. Note that in the case of dollar pricing, precisely as in the case of pricing in rubles, the patterns of price adjustment by an individual firm are not affected by the choice of price-setting currencies by the firm's competitors and input suppliers.

iii. The individual firm's expected profit losses per unit of time associated with either pricing-in-rubles or pricing-in-dollars strategy under the different pricing strategies of the competitors and input suppliers are summarized in Table 3. $\Lambda \equiv \frac{1}{4}\left(6F\right)^{\frac{2}{3}}$ is a constant coefficient.

We see that losses associated with ruble pricing increase with inflation: under the higher rate of inflation an individual firm has to revise its price more frequently to keep it in line with other prices. Higher volatility of the exchange rate (higher values of the marginal variance of log rate $\frac{\sigma^2}{2\rho}$) also increases losses from ruble pricing if either firm's competitors or its input suppliers, or both of them denominate prices in dollars. Then, the instantaneously desired price, determined by the input and competitors' prices, fluctuates around the inflation trend together with the exchange rate, and the ruble pricing strategy does not allow a firm to adjust its price to these fluctuations.

Losses from dollar pricing do not depend on inflation since the expected exchange rate is assumed to follow the inflation trend. They increase in the exchange rate volatility in the case when either firm's competitors, or input suppliers, or both of them denominate their prices in rubles. Then, fluctuations of the firm's fixed dollar price exceed in size the fluctuations of the instantaneously desired price, the average difference between the two prices depends on the volatility of the exchange rate. Hence, the higher the volatility, the larger are the deviations of the actual dollar price from the desired level due to the fluctuations of the exchange rate, the bigger the profit losses.

Note that the individual firm's losses associated with dollar pricing strategy are zero if everyone in the economy denominates price in dollars. In this case the desired price, determined by the price of competitors and input price, stays constant in dollars. Then, having once fixed its dollar price at the optimal level a firm does not have to adjust this price any more: the time-path of this price exactly follows the trajectory of the desired price. Therefore, the firm does not incur any losses from price non-optimality nor from the costly price change.

4.4 Choice of the Price-setting Currency

An individual firm chooses the pricing currency by comparing expected profit losses per unit of time associated with each strategy. Since losses from ruble pricing increase in inflation and losses from dollar pricing do not depend on the inflation rate, there always exists a unique threshold value of inflation, which corresponds to the switch of an individual firm from pricing in rubles to pricing in dollars.

The expressions for the threshold values of inflation $\hat{\mu}$ under the different pricing strategies of the firm's competitors and input suppliers are presented in Table 4. The first and second subscripts of the threshold values $\hat{\mu}$ correspond to the currencies of denomination of the input and competitors' prices respectively (R - rubles, D - dollars). Zero threshold values indicate that an individual firm will prefer to quote its price in dollars even if inflation is reduced to zero.

Table 4 also demonstrates that exchange rate volatility makes firms desire to price in rubles, that is, threshold values of inflation increase in σ. This is always the case whenever the threshold is above zero. The threshold is zero, on the other hand, when losses from ruble pricing are always higher due to pricing strategy of competitors and suppliers.

Table 4: Threshold values of inflation

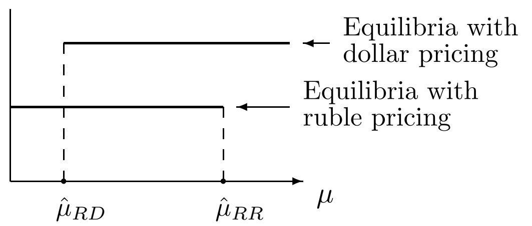
	Competitors price in rubles	Competitors price in dollars
Inputs priced in rubles	$\hat{\mu}_{RR} = \left(\frac{1}{2\Lambda\rho}\right)^{\frac{3}{2}} K\sigma^3$	$\hat{\mu}_{RD} = \begin{cases} 0 & \text{if } \eta < \frac{1}{2} \\ \left(\frac{2\eta-1}{2\Lambda\rho}\right)^{\frac{3}{2}} K\sigma^3 & \text{if } \eta \geq \frac{1}{2} \end{cases}$
Inputs priced in dollars	$\hat{\mu}_{DR} = \begin{cases} \left(\frac{1-2\eta}{2\Lambda\rho}\right)^{\frac{3}{2}} K\sigma^3 & \text{if } \eta < \frac{1}{2} \\ 0 & \text{if } \eta \geq \frac{1}{2} \end{cases}$	$\hat{\mu}_{DD} = 0$

For any currency of denomination of the input prices two equilibria exist:

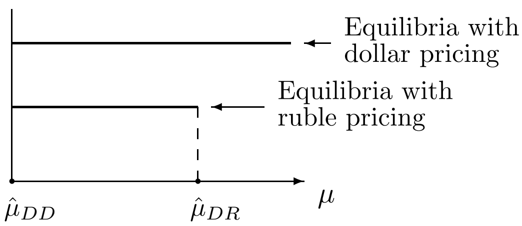
- *equilibrium with the uniform ruble pricing* exists when $\mu < \hat{\mu}_{\cdot R}$
- *equilibrium with the uniform dollar pricing* exists when $\mu > \hat{\mu}_{\cdot D}$

The areas of existence of the equilibria with the uniform ruble pricing and uniform dollar pricing can be illustrated by the following diagrams:

Inputs priced in rubles



Inputs priced in dollars



We see that under the increasing inflation an economy, sooner or later, switches from ruble pricing to the uniform pricing in dollars. As it is seen from Table 4, if the input price is denominated in dollars, the lower rate of inflation is enough to push the economy into the dollar pricing equilibrium, which is quite reasonable, as the firm's desired price trajectory is affected by the input price.

An important result is that exit from the equilibrium with uniform dollar pricing when inflation drops is possible only if the input price is denominated in rubles. If the input price is denominated in dollars firms incur zero profit losses quoting uniformly their prices in dollars. Then, even reduction of inflation to zero will not make individual firm deviate from the group and turn back to pricing in rubles.

4.5 The Hysteresis Effect

Informal observations in Russia and other countries (for example, Israel) suggest that FCP exhibits hysteresis: during high inflation firms turn to quoting prices in dollars but reduction of inflation does not immediately push firms back to the local currency. Firms can continue denominating prices in dollars after the stabilization of inflation.

The presented model captures this effect. With the assumption of non-constant elasticity of demand ($\eta < 1$) the model predicts hysteresis: as we can see from Table 4, the threshold value of inflation which corresponds to the fall of the economy into the equilibrium with uniform dollar pricing $\hat{\mu}_{\cdot R}$, unless it is zero, exceeds the threshold value which corresponds to the exit from dollar-pricing equilibrium $\hat{\mu}_{\cdot D}$:

$$\hat{\mu}_{\cdot R} > \hat{\mu}_{\cdot D} \quad \text{when } \hat{\mu}_{\cdot R} > 0\,.$$

The source of hysteresis in the model is again the desire of an individual firm to keep in line with the group, which stems from the assumption about the shape of the demand curve. When the demand curve has a smoothed-out kink at the firm's optimal relative price, random deviations of an individual price from the aggregate price of competitors are associated with profit losses for a firm.

Note, that under the traditional assumption of constant elasticity of demand ($\eta = 1$) no hysteresis is predicted by the model: there is one and the same threshold value of inflation which is associated with both, the fall into and the exit from the equilibrium with uniform dollar pricing: $\hat{\mu}_{\cdot R} = \hat{\mu}_{\cdot D}$. With constant elasticity of demand the firm's optimal price at every moment is equal to a constant markup over its marginal cost and, therefore, the pricing strategy of an individual firm is determined in full by the pricing strategy of its input suppliers and is not affected at all by the pricing strategy of competitors. Hence, an individual firm quotes its price in dollars whenever its costs are denominated in dollars and chooses between the ruble and dollar pricing, comparing the inflation rate and the exchange rate volatility, when its costs are denominated in rubles.

Thus, only introduction of a demand function with a smoothed-out kink at the firm's optimal relative price, as in Kimball (1995), allows to capture the hysteresis effect in the imperfectly competitive framework.

4.6 The Influence of Market Power

The relation between the areas of existence of the equilibria with uniform ruble and dollar pricing, and, respectively, the degree of hysteresis, depend on the coefficients η and K. By equation (11), η relates the firm's instantaneously desired relative price to the prices of competitors and input suppliers; according to approximation (14), K shows the magnitude of profit losses from price non-optimality. These two coefficients are determined by the shape of the demand function around the steady state relative price $p_i = 1$, they are expressed in terms of the firm's desired mark-up at the unity desired relative price $\mathcal{M}(1)$, and the rate of change in the desired mark-up with the deviations of desired relative price from unity $\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}$.

Both, the desired mark-up at the unity relative price $\mathcal{M}(1)$ and the rate of change in the mark-up around unity $\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}$, are related to the firm's market power ξ. With higher market power firms are less sensitive to the deviations from others, this implies higher desired mark-up and lower (in the absolute value) rate of change in the mark-up with the deviations of relative price from unity:

$$\frac{\partial \mathcal{M}(1)}{\partial \xi} > 0\,, \quad \frac{\partial \left[-\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}\right]}{\partial \xi} \leq 0\,.$$

Then, from expression (12) for the coefficient η we see that η is a non-declining function of market power: $\frac{\partial \eta}{\partial \xi} \geq 0$. This suggests that when market power is sufficiently high the desired price trajectory of an individual firm is determined primarily by the price of inputs, the price of competitors has only a moderate influence on the desired pricing strategy. However, if the market power is low a firm incurs sensible losses from the deviations from the group, therefore, choosing the desired price trajectory, it pays attention not only to the input price but also to the price of competitors. From expression (15) for the coefficient K we have $\frac{\partial K}{\partial \xi} < 0$. Thus, with the higher market power the deviations of an individual price from its instantaneously optimal level become less costly for a firm.

Therefore, from Table 4 we obtain:

$$\frac{\partial \hat{\mu}_{RR}}{\partial \xi} < 0\,; \quad \frac{\partial \hat{\mu}_{DR}}{\partial \xi} \leq 0\,;$$

$$\frac{\partial \hat{\mu}_{DR}}{\partial \xi} < 0 \quad \text{if} \quad \hat{\mu}_{DR} > 0\,.$$

The signs of the derivatives imply that the fall into the equilibrium with uniform dollar pricing is easier in a more monopolized economy: lower values of inflation are enough to make firms turn from pricing in rubles to pricing in dollars. This result holds disregarding of the currency in which the input price is denominated. The intuition for the result is straightforward: with the higher market power it is less costly for an individual firm to deviate from the group. Thus, a more moderate rate of inflation is sufficient for a firm to find it beneficial to turn, even individually, to pricing in dollars, which allows to avoid frequent price adjustment. Hence, identical firms start pricing in dollars, and the economy falls into the dollar-pricing equilibrium at a lower rate of inflation.

Anecdotal evidence suggests that in Moscow during the early years of transition, prices for a number of goods, including clothing, sport equipment, furniture, etc., were being denominated almost uniformly in dollars in the expensive shops, and mainly in rubles in the cheaper shops and markets. Furthermore, prices for the domestic products of high quality in the expensive shops were often denominated in dollars, while prices for the low quality imports in the cheap shops and markets were usually set in rubles. These informal observations are consistent with the predictions of the model, as expensive goods are generally less homogeneous and less competitive.

It is also seen from Table 4 that, in the framework of this model, we cannot make any predictions concerning the influence of the market power on the exit from equilibrium with uniform dollar pricing without making more concrete assumptions about the shape of the demand function. The sign of $\frac{\partial \hat{\mu}_{RD}}{\partial \xi}$ can be determined only after choosing some form of explicit relation between the mark-up at the unity relative price $\mathcal{M}(1)$ and its rate of change around unity $\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}$.

5 Conclusion

The model predicts that not only the relation between the rate of inflation and the exchange rate volatility is important in determining the optimal choice of the price-setting currency made by an individual firm, but so are the pricing strategies of the firm's competitors and its input suppliers.

It is shown, that in the partly dollarized environment, where firms can choose between pricing in rubles and pricing in dollars two equilibria exist: an equilibrium with uniform ruble pricing in the industry, and an equilibrium with uniform dollar pricing. The relation between the ranges of these two equilibria is determined by the pricing strategy of the input suppliers and by the assumption about the curvature of the demand function.

The model is capable to capture the hysteresis effect: it is shown, that under the non-constant elasticity of demand, the inflation rate which is needed to make firms switch from the equilibrium with ruble pricing to the equilibrium with dollar pricing exceeds the rate of inflation under which firms can exit from the dollar pricing equilibrium and turn back to pricing in rubles. It is also shown that exit from the equilibrium with uniform dollar pricing is possible only when the input price is denominated in rubles; when input price is denominated in dollars no firm will turn individually from pricing in dollars back to pricing in rubles even if the inflation rate is reduced to zero.

Finally, the model predicts that in the industries with lower competition a lower inflation rate is needed to make firms fall into the equilibrium with uniform dollar pricing.

Appendices

A Small fluctuations of desired relative price

The firm's desired relative price $\frac{P_i^{\#}}{P}$ and the relative input price $\frac{P^I}{P}$ are related by

$$\frac{P_i^{\#}/P}{\mathcal{M}(P_i^{\#}/P)} = \frac{P^I/P}{\mathcal{M}(1)} \,. \tag{20}$$

Differentiating equation (20) w.r.t. relative input price $\frac{P^I}{P}$ we obtain:

$$\frac{d\left(\frac{P_i^{\#}/P}{\mathcal{M}\left(P_i^{\#}/P\right)}\right)}{d\left(P^I/P\right)} = \frac{1}{\mathcal{M}(1)} \,,$$

or

$$\frac{d(P_i^{\#}/P)}{d(P^I/P)} \cdot \left(\frac{1}{\mathcal{M}(P_i^{\#}/P)} - \frac{\mathcal{M}'(P_i^{\#}/P)}{\mathcal{M}(P_i^{\#}/P)} \cdot \frac{(P_i^{\#}/P)}{\mathcal{M}(P_i^{\#}/P)}\right) = \frac{1}{\mathcal{M}(1)} \,. \tag{21}$$

At $\frac{P^I}{P} = 1$ we have $\frac{P_i^{\#}}{P} = 1$ and, hence, from (21) we get:

$$\left.\frac{d(P_i^{\#}/P)}{d(P^I/P)}\right|_{\frac{P^I}{P}=1} = \frac{1}{1 - \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}} \,.$$

With a non-increasing mark-up function, the rate of change in mark-up at unity, $\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}$, is non-positive, which implies that $0 < \left.\frac{d(P_i^{\#}/P)}{d(P^I/P)}\right|_{\frac{P^I}{P}=1} \leq 1$. Therefore, small deviations of the relative input price $\frac{P^I}{P}$ from unity are associated with only small fluctuations of the desired relative price $\frac{P_i^{\#}}{P}$.

B Derivation of formulas (11)-(13)

Rewriting equation (11) for the instantaneously desired relative price $\frac{P_i^{\#}}{P}$ in logs, and taking into account that $a = \frac{1}{\mathcal{M}(1)}$ we get:

$$p_i^{\#} - p = -\ln \mathcal{M}(1) + \ln \mathcal{M}(P_i^{\#}/P) + p^I - p\,. \tag{22}$$

We then approximate $\ln \mathcal{M}(P_i^{\#}/P)$ by Taylor with respect to log desired relative price $p_i^{\#} - p_i$ in the neighborhood of unity desired relative price:

$$\ln \mathcal{M}(P_i^{\#}/P) = \ln \mathcal{M}(1) + \left.\frac{d \ln \mathcal{M}(P_i/P)}{d(p_i - p)}\right|_{\frac{P_i}{P}=1} \cdot \left(p_i^{\#} - p\right) + \underline{o}\left(p_i^{\#} - p\right),$$

which gives

$$\ln \mathcal{M}(P_i^{\#}/P) = \ln \mathcal{M}(1) + \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}\left(p_i^{\#} - p\right) + \underline{o}\left(p_i^{\#} - p\right).$$

Substituting this approximation into the equation (22) we obtain:

$$p_i^{\#} - p = \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}\left(p_i^{\#} - p\right) + p^I - p + \underline{o}\left(p_i^{\#} - p\right),$$

or

$$\left(p_i^{\#} - p\right)\left[1 - \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}\right] = p^I - p + \underline{o}\left(p_i^{\#} - p\right).$$

Finally, we define $\eta =\equiv \frac{1}{1 - \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}}$, and, hence, get the following equation for the instantaneous desired relative price $p_i^{\#}$:

$$p_i^{\#} = (1-\eta)\cdot p + \eta \cdot p^I + \underline{o}\left(p_i^{\#} - p\right). \tag{23}$$

The relation between the rate of change in the desired mark-up at the unity desired relative price $\frac{\mathcal{M}'(1)}{\mathcal{M}(1)}$ and the elasticity of demand is obtained by differentiating the expression for the desired mark-up $\mathcal{M}(P_i^{\#}/P) = \frac{\epsilon(P_i^{\#}/P)}{\epsilon(P_i^{\#}/P)-1}$. This relation is given by

$$\frac{\mathcal{M}'(1)}{\mathcal{M}(1)} = -\frac{\epsilon'(1)}{\epsilon(1)(\epsilon - 1)}\,. \tag{24}$$

C Approximation for profit losses, formulas (14) - (15)

To derive an approximation for the profit losses from price non-optimality we expand the profit function $\pi(P_i/P)$ by Taylor w.r.t. log relative price $p_i - p$ around the log desired relative price

$p_i^{\#} - p$:

$$\pi(P_i/P) = \pi(P_i^{\#}/P) + \left.\frac{d\pi(P_i/P)}{d(p_i - p)}\right|_{p_i = p_i^{\#}} \cdot \left(p_i - p_i^{\#}\right) + \frac{1}{2}\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}} \cdot \left(p_i - p_i^{\#}\right)^2 + \underline{o}\left(p_i - p_i^{\#}\right)^2.$$

Since $\frac{P_i^{\#}}{P}$ is the instantaneously optimal price we have: $\left.\frac{d\pi(P_i/P)}{d(p_i - p)}\right|_{p_i = p_i^{\#}} = \left.\frac{d\pi(P_i/P)}{d(P_i/P)} \cdot \frac{P_i}{P}\right|_{\frac{P_i}{P} = \frac{P_i^{\#}}{P}} = 0.$ Therefore, the instantaneous profit losses are equal to

$$\pi(P_i^{\#}/P) - \pi(P_i/P) = -\frac{1}{2}\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}} \cdot \left(p_i - p_i^{\#}\right)^2 + \underline{o}\left(p_i - p_i^{\#}\right)^2. \tag{25}$$

Here the log instantaneously desired relative price $p_i^{\#} - p$ is determined by the relation (22), and the profit function is given by

$$\pi\left(\frac{P_{it}}{P}\right) = \left(\frac{P_{it}}{P} - a\frac{P_t^I}{P}\right) \cdot Q\left(\frac{P_{it}}{P}\right).$$

Hence, the coefficient in the Taylor expansion (25) is equal to

$$-\frac{1}{2}\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}} = \frac{1}{2}\frac{P_i^{\#}}{P} Q\left(\frac{P_i^{\#}}{P}\right)\left[\frac{P_i^{\#}}{P} \cdot \frac{\epsilon'(P_i^{\#}/P)}{\epsilon(P_i^{\#}/P)} + \epsilon(P_i^{\#}/P) - 1\right]. \tag{26}$$

Again, remaining agnostic about the exact functional forms of $\epsilon(P_i/P)$ and $\mathcal{M}(P_i/P)$ which are needed to calculate the exact values of $p_i^{\#}$ and $\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}}$, we approximate these values by Taylor. For the purposes of getting a second order Taylor approximation for the profit losses the zero order approximation for $\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}}$ and the first order approximation for $p_i^{\#}$ are sufficient. The zero order approximation for $\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}}$ is obtained from (26), where, for convenience, we normalize the demand at the unity relative price $Q(1)$ to unity:

$$\left.\frac{d^2\pi(P_i/P)}{d(p_i - p)^2}\right|_{p_i = p_i^{\#}} = -\left[\frac{\epsilon'(1)}{\epsilon(1)} + \epsilon(1) - 1\right] + \underline{o}(1). \tag{27}$$

The first order approximation for $p_i^{\#}$ is given by (23):

$$p_i^{\#} = (1 - \eta) \cdot p + \eta \cdot p^I + \underline{o}\left(p_i^{\#} - p\right). \tag{28}$$

Substituting relations (27) and (28) into the expression (25) for the instantaneous profit losses from price non-optimality we get the following second order approximation for the losses:

$$\pi(P_i^{\#}/P) - \pi(P_i/P) = K \cdot \left(p_i - (1 - \eta)\, p - \eta\, p^I\right)^2 + \underline{o}^2, \tag{29}$$

where K is the constant coefficient,

$$K = \frac{1}{2}\left(\frac{\epsilon'(1)}{\epsilon(1)} + \epsilon(1) - 1\right),$$

and the residual term $\underline{o}^2$ is given by

$$\begin{aligned}\underline{o}^2 &= \underline{o}\left[(p_i^{\#} - p)\cdot(p_i - (1-\eta)\,p - \eta\,p^I)\right] + \underline{o}\left(p_i^{\#} - p\right)^2 + \underline{o}\left(p_i^{\#} - p_i\right)^2 = \\ &= \underline{o}\left[(p_i^{\#} - p)\cdot\left((p_i^{\#} - p) - (p_i^{\#} - p_i) - \eta(p^I - p)\right)\right] + \underline{o}\left(p_i^{\#} - p\right)^2 + \underline{o}\left(p_i^{\#} - p_i\right)^2.\end{aligned}$$

Under the assumption of small values of $(p_i^{\#} - p_i)$ and $(p^I - p)$, and the resultant closeness to zero of $(p_i^{\#} - p)$, the residual term $\underline{o}^2$ is negligibly small in comparison with the main term $K\cdot\left(p_i - (1-\eta)p - \eta p^I\right)^2$.

Finally, using the expression for the desired mark-up $\mathcal{M}(P_i^{\#}/P) = \frac{\epsilon(P_i^{\#}/P)}{\epsilon(P_i^{\#}/P)-1}$, and the equation (24), which relates the rate of change in the desired mark-up around unity to the elasticity of demand, we express the coefficient K in terms of the mark-up function:

$$K = \frac{1 - \frac{\mathcal{M}'(1)}{\mathcal{M}(1)}}{2[\mathcal{M}(1) - 1]}.$$

D Solution to the optimization problems (17) and (19)

Here we investigate in detail the case when both, the price of the competitors and the input price are being set in rubles. The other cases, when either firm's competitors, or input suppliers, or both of them denominate prices in dollars are treated in a similar way.

When both the firm's competitors and input suppliers quote their prices in rubles we have $p_t = p_t^I = \mu t$. Hence, a firm which denominates its price in rubles solves the following optimization problem:

$$Loss^{Rub} = \lim_{n\to\infty}\left[\frac{1}{n\delta}\left(\sum_{k=0}^{n} L_k^{Rub}(p_{ik}) + nF\right)\right] \to \min_{\{p_{ik}\},\,\delta}, \tag{30}$$

where

$$L_k^{Rub}(p_{ik}) = \mathbf{E}_0 \int_{k\delta}^{(k+1)\delta} K\cdot(p_{ik} - \mu t)^2\,dt. \tag{31}$$

The optimal reset price for the period between the $k-th$ and $(k+1)-th$ price changes p_{ik} is found by solving

$$L_k^{Rub}(p_{ik}) = \mathbf{E}_0 \int_{k\delta}^{(k+1)\delta} K\cdot(p_{ik} - \mu t)^2\,dt \to \min_{p_{ik}}.$$

Solving the first order condition $\int\limits_{k\delta}^{(k+1)\delta} (p_{ik} - \mu t)dt = 0$ we get an expression for the optimal reset price:

$$p_{ik}^{*} = \left(k - \frac{1}{2}\right)\mu\delta.$$

It is easy then to calculate the firm's profit losses from price non-optimality during the period between the $k-th$ and $(k+1)-th$ price changes:

$$L_k^{Rub}(p_{ik}^{*}) = \frac{1}{12}K\mu^2\delta^3.$$

Note that losses in the latter expression do not depend on the number of period k. This is because the exchange rate does not enter firm's optimization problem in the particular case we are considering, and the inflation rate is steady. Substituting this expression into (30) for the total profit losses per unit of time we obtain:

$$Loss^{Rub}(p_{ik}^{*}) = \left[\frac{1}{12}K\mu^2\delta^2 + \frac{F}{\delta}\right] \rightarrow \min_{\delta}.$$

Hence,

$$\delta^{*} = \left(\frac{6F}{K\mu^2}\right)^{\frac{1}{3}},$$

and

$$\left(Loss^{Rub}\right)^{*} = \frac{1}{4}(6F)^{\frac{2}{3}}(K\mu^2)^{\frac{1}{3}}.$$

Similarly, a firm which denominates its price in dollars solves the following optimization problem:

$$Loss^{Doll} = \lim_{n\rightarrow\infty}\left[\frac{1}{n\delta^f}\left(\sum_{k=0}^{n} L_k^{Doll}(p_{ik}^f) + nF\right)\right] \rightarrow \min_{\{p_{ik}^f\},\, \delta^f}, \tag{32}$$

where

$$L_k^{Doll}(p_{ik}^f) = \mathbf{E}_0 \int\limits_{k\delta^f}^{(k+1)\delta^f} K\cdot(e_t + p_{ik}^f - \mu t)^2\, dt. \tag{33}$$

In this problem distribution of the exchange rate matters. We need the first and second moments of the log exchange rate to solve the problem. From (8), using $e_0 = 0$, we have:

$$\begin{aligned} \mathbf{E}_0(e_t) &= \mu t\,, \\ \mathbf{E}_0(e_t^2) &= \mu^2t^2 + \tfrac{\sigma^2}{2\rho}\left(1 - e^{-2\rho t}\right)\,. \end{aligned} \tag{34}$$

The optimal reset dollar price is found by minimizing $L_k^{Doll}(p_{ik}^f)$. The first order condition for this problem is

$$\mathbf{E}_0 \int\limits_{k\delta^f}^{(k+1)\delta^f} (e_t + p_{ik}^f - \mu t)\, dt = 0\,,$$

and hence,

$$p_{ik}^{f*} = 0 \text{ for any } k.$$

Substituting the zero log optimal reset price into expression (33) for the expected profit losses from price non-optimality during the period between the k-th and $(k+1)$-th price changes, and using formula (34) for the moments of log exchange rate we obtain:

$$L_k^{Doll}(p_{ik}^{f*}) = \frac{K\sigma^2}{2\rho}\left[\delta^f - \frac{e^{-2\rho(k-1)\delta^f} - e^{-2\rho k\delta^f}}{2\rho}\right]. \tag{35}$$

Unlike the previous case, here the expected losses from price non-optimality between the k-th and $(k+1)$-th price changes depend on k. This is because the uncertainty about future values of the exchange rate, as of moment $t = 0$, increases with time, and, respectively, expected profit losses are bigger for the later periods.

Using (35) we can calculate firm's total expected profit losses per unit of time:

$$Loss^{Doll}(p_{ik}^{f*}) = \lim_{n\to\infty}\left[\frac{1}{n\delta^f}\left(\sum_{k=0}^{n} L_k^{Doll}(p_{ik}^{f*}) + nF\right)\right] =$$

$$= \lim_{n\to\infty}\left[\frac{1}{n\delta^f}\cdot\frac{K\sigma^2}{2\rho}\left(n\delta^f - \frac{1-e^{-2\rho n\delta^f}}{2\rho}\right) + \frac{F}{\delta^f}\right] = \frac{K\sigma^2}{2\rho} + \frac{F}{\delta^f}\,.$$

Minimizing the latter losses with respect to the length of time between price adjustments δ^f we finally obtain:

$$\delta^{f*} = \infty,$$

and hence,

$$\left(Loss^{Doll}\right)^* = \frac{K\sigma^2}{2\rho}\,.$$

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