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Abstract

This paper studies the use of discretionary rewards in a finitely repeated principal-agent relationship with moral hazard. We show that the principal, when she obtains a private subjective signal about the agent's performance, may pay discretionary bonuses to provide credible feedback to the agent. Consistent with the often observed compression of ratings, we show that in equilibrium the principal communicates the agent's interim performance imperfectly, i.e. she does not fully differentiate good and bad performance. Furthermore, we show that small rewards can have a large impact on the agent's effort provided that the principal's stake in the project is small. Our analysis further reveals that, also in accordance with the empirical findings, the principal may ex ante prefer to choose a 'smoky', rather than a fully transparent performance monitoring system, thereby acquiring an implicit commitment device to reward the agent through discretionary bonuses.

JEL codes: D82, J33, M50

Keywords: discretionary rewards, feedback, self confidence, subjective performance, moral hazard, monitoring system

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1 Introduction

Incentive problems are part of most interpersonal relationships. Much of the literature on incentive contracts has focused on how verifiable performance measures can be used to mitigate these problems. But most people do not work in jobs where verifiable performance measures are available for all important dimensions of performance (Prendergast, 1999), and many aspects of their work are not described by explicit contracts. Instead, many firms use extensively discretionary rewards that are based on subjective, non-contractible performance measures.

In this paper we study how the principal can use a subjective performance measure to motivate the agent. Despite the non-contractibility, we show that the principal may have an incentive to offer discretionary rewards in a finitely repeated game. This happens because the principal has private information on the worker's performance, and, if the information is favorable, wants to communicate this credibly to the agent to boost his incentives. Since the principal is tempted to overstate the agent's performance, cheap talk would be ineffective in this situation.¹ Thus, the rewards are used to ensure the credibility of the interim feedback given to the agent.

We show that a bonus gives positive feedback about past performance to the agent, and increases his motivation to exert effort in the future. This result, establishing the informational content of a reward, is in line with the literature on motivation both in psychology and economics (see e.g. Deci and Ryan (1985) and Bénabou and Tirole (2003)). One of our main results, saying that larger bonuses correspond to higher performance levels comes, probably, with little surprise. However, we also show that in equilibrium the bonus is proportional to the principal's payoff from the project's success. An interesting implication of this is that even a small, insignificant bonus can have a substantial effect on the agent's subsequent motivation.

We also explore whether *ex ante* the principal prefers to choose a performance measurement system that is purely transparent to the agent, or "smoky", assuming that in both cases performance cannot be observed by a third party and thus cannot be part of a formal contract. We show that in some circumstances the principal may choose the smoky performance measurement system, thus gaining an indirect commitment to reward the agent for successful performance. This finding can explain why firms often use monitoring systems that provide little information to the agent about performance, even when more informative performance measures are available

¹One can check that only the babbling, uninformative equilibrium would be possible if costly signalling were replaced by a cheap talk game in our model.

(Gibbs, 1991). The reasoning behind it is that private information of the principal ensures a positive interim bonus in equilibrium. By contrast, if the agent could perfectly observe performance, there would be no reason to provide feedback, and the principal could not credibly commit to paying a bonus.

Our model furthermore predicts that good performance is not fully separated from bad performance by the principal. This result provides a new insight into the reluctance of managers to rate workers differently. That is, unsuccessful agents are sometimes given the same bonus as successful ones. The resulting compression of rating of performance is well documented in the literature (Prendergast (1999)). Intuitively, if the agent takes a bonus as providing positive feedback on performance, the principal is tempted to offer the bonus even after bad performance in order not to demotivate the agent.

The driving assumption behind these results is that the principal has private information about the agent's performance. The widespread use of performance feedback systems shows the relevance of this assumption (cf. Gibbs (1991)). It seems especially relevant when workers are in their learning phase, produce a complex good, or contribute to a project that involves many individual tasks. In such cases, the experience and overview might enable the manager to form a better judgement of an employee's performance.

Our model is closely related to that of Bénabou and Tirole (2003). Unlike our model, they assume that, at the *ex ante* stage, the principal has private information about the task (or about the agent himself), and show that rewards have informational content. More precisely, they find that a promise of a higher bonus by the principal sends a negative signal to the agent, whereas in our set-up a high bonus provides positive feedback. One crucial difference is that Bénabou and Tirole focus on objective performance measures, rather than subjective, which implies that bonuses are contractible. By offering a high bonus, the principal reveals her low confidence in the agent, hence the negative feedback result. In our case, the principal cannot commit to a contract and, *ex post*, has higher incentives to give a reward if she can motivate the agent for the next period, that is, if it gives positive feedback.

The existing literature on subjective performance measures is mostly focussed on infinitely repeated games (e.g. MacLeod and Malcomson (1989); Levin (2003)).² In that setup, noncontractible bonuses occur in equilibrium through, for instance, the threat of termination of the relationship. Discretionary bonuses then become self-enforcing. Given that such reputation mechanisms are relatively well understood, we focus on a finitely repeated game. In this case, no self-enforcing discretionary bonus based on reputation

²See also Bull (1987), Baker et al. (1994), Pearce and Stacchetti (1998).

can exist in equilibrium.

Several other papers study subjective performance measures in finitely repeated games. MacLeod (2003) studies a one-shot game where there is no balanced budget between what the principal gives as a bonus and what the agent receives. This 'no balanced budget' condition allows for a positive equilibrium bonus. For instance, the principal could commit to a fixed bonus that may be paid to the agent or a third party. The principal can promise to reward the agent after good performance, and to give the bonus away to a third party after bad performance ('burn the money'). Given the commitment to paying a bonus, after a success the principal has no incentive to renege on giving it to the agent. Actual examples of such burning of money do exist (see Fuchs (2005)), but it is unclear how important this practice is in reality. We study the situation in which the budget is balanced.

The role of bonus as a feedback mechanism is also studied in Lizzeri, Meyer and Persico (2003) and Fuchs (2005). In both papers it is assumed that the principal has private information on performance. However, both papers draw no link between the bonus and the information about ability. Lizzeri et. al. (2003) assume that there is no link between ability and performance. Interestingly, they also find that the principal may want to hide information but for other reasons. The trade-off in their paper is between giving better incentives by providing better information and a higher wage bill. Also, they assume that output is verifiable at the end of the last period so that ultimately performance is objectively measurable. In the model by Fuchs (2005) there is no differentiation in abilities among agents, so the bonus gives no feedback on the ability of the agent. This excludes a positive equilibrium bonus in the finitely repeated version of his model with a balanced budget.

Finally, there is a strand of literature that explains the use of discretionary bonuses by virtue of social preferences. For instance, Fehr, Gächter, and Kirchsteiger (1997) argue that, where explicit contracts are absent, reciprocal behavior by agents can serve as a contract enforcement device. If the principal is motivated by reciprocity, she might reward the agent for good performance even if there are no enforceable contractual obligations to do so. Note that this approach is quite different from ours, in that it assumes that agents do not have purely selfish preferences, whereas we focus on the role of information. In our view, the two approaches are complementary.

The rest of the paper is organized as follows. Section 2 presents the model and discusses the main assumptions. In section 3 the equilibria are derived. Section 4 explores the principal's incentives to choose a transparent or a "smoky" monitoring system. Section 5 compares our results to the literature on "hidden costs of rewards". In section 6 we briefly discuss the modification of our results to more general variants of our framework. Finally, section 7

concludes and suggests possible extensions.

2 The model

2.1 Preliminaries

We consider a two-period principal-agent relationship, in which both parties are risk-neutral. The agent is facing a sequence of two identical projects and, at the beginning of each period $t = 1, 2$, he decides whether to exert effort or not: $e_t \in \{0, 1\}$. The agent's cost $c(e)$ of effort is increasing; we normalize $c(0) = 0$ and $c(1) = c$.

The agent is characterized by his ability to perform the task, θ , that can be either high, $\theta = \theta_H$, or low, $\theta = \theta_L < \theta_H$. The outcome of each project y_t can be a success or a failure $y_t \in \{S, F\}$. The probability of success is jointly determined by the agent's ability θ (which is assumed to be constant across periods) and his choice of effort:

$$\Pr\{y_t = S \mid e_t\} = e_t\theta. \quad (1)$$

Thus, ability and effort are complements. Note also that not exerting effort induces failure with certainty. For simplicity, we assume that the effort decision is observed by the principal; however, it is not observed by the third parties and therefore is non-contractible.³

In case of success, the agent receives a payoff of $V > 0$ and the principal gets $W > 0$. Failure yields nothing to both. The agent's reservation utility is normalized to 0.

At the end of each period, the principal can compensate the agent by giving a discretionary bonus $b \geq 0$. We assume that the budget is balanced, so that the reward paid by the principal is equal to the reward obtained by the agent. The discount factor is normalized to 1 for simplicity.

2.2 The main assumptions

We assume that neither the agent himself, nor the principal precisely know the agent's ability. This imperfect self-knowledge of the agent can arise whenever he is facing a new task or when his retrospective evaluation of past experiences is distorted (see e.g. Kahneman (1994)). Note that the assumption that the agent does not know his ability is quite common in the

³See Section 6 for a brief analysis of more realistic cases where the principal does not observe the agent's effort, and failure to exert effort does not lead to failure with certainty.

literature on labor contracts (e.g. as in Holmstrom's (1999) career concerns model).

Both the principal and the agent share the same prior beliefs about the agent's ability (which are common knowledge): they believe that with probability ρ the agent is talented ($\theta = \theta_H$) and with probability $1 - \rho$ his ability is low ($\theta = \theta_L$). Based on these prior beliefs, the agent forms an estimate of his chances to succeed if he decides to exert effort. This estimate, akin to self-confidence, is simply

$$E[\theta] = \rho\theta_H + (1 - \rho)\theta_L.$$

Clearly, self-confidence is increasing in the prior probability of being talented, ρ . Sometimes, with some abuse of language, we shall equate ρ with the agent's initial self-confidence.

Although unable to observe the agent's ability directly, the principal observes his performance y_t at the end of period t . By contrast, we assume that performance cannot be perfectly observed by the agent himself or by any third party. There are many circumstances in which the principal may be in a better position to estimate the agent's performance than the agent himself, a fact that is illustrated by the ubiquity of performance feedback systems in organizations (Gibbs (1991)). A manager may often know better how useful an employee's work was for the firm, a professor may understand better the quality of a student's paper, etc. This is particularly true in situations where agents are in their learning phase: at school or at new jobs. Other applications concern complex jobs, where each agent is responsible for a tiny part of the final product and objective performance measures are fuzzy measures of ability or effort.

The agent does not directly observe his first-period performance y_1 , but receives instead an (imperfectly) informative private signal $\sigma \in (0, 1)$. This signal has a conditional distribution depending on the first-period outcome $G(\sigma | y) = G_y(\sigma)$ and a continuous positive density $g(\sigma | y) = g_y(\sigma)$. The realized signal σ is private information to the agent, but the conditional distribution functions are common knowledge. We assume that a high signal brings good news about the outcome. Formally,

Assumption 1 *The likelihood ratio*

$$l(\sigma) \equiv \frac{g_S(\sigma)}{g_F(\sigma)} \tag{2}$$

is continuous in σ and assumes all values in $[0, +\infty)$. Furthermore, the monotone likelihood ratio property (MLRP) is satisfied: $l(\sigma)$ is everywhere increasing in σ .

Neither the first- nor the second-period outcomes are observed by any third party. It follows that a performance-contingent reward cannot be specified in a contract, because the agent or a third party would not be able to verify the truthfulness of the principal's claim. The non-contractibility is one of the main departures from the model of Bénabou and Tirole (2003): in their model an outcome-contingent contract can be written. In practice, this feature of non-contractibility is present in most jobs (Prendergast (1999)).

We also assume that the payoff V that the agent receives in case of success is sufficiently high to motivate him to work even if he expects no bonus to be paid, provided that the agent's self-confidence (or feeling of competence) is high enough. The determination of the agent's payoff V is not modelled explicitly, but can be thought of as the future discounted reward from completing the task successfully. For instance, being successful increases the probability of finding a new job in the future, getting a promotion, obtaining useful skills or receiving peer recognition. On the other hand, it might also represent the intrinsic motivation of an agent from performing well on a task⁴. To make things interesting, we make the following assumption:

Assumption 2 *Were the agent perfectly informed about his type, he would undertake the task without a bonus if and only if he had high ability: $\theta_L V < c < \theta_H V$.*

2.3 Timing

Each period is divided into three stages. In the first stage of the first period, the agent decides whether or not to exert effort. In the second stage, the outcome is realized and observed by the principal, while the agent receives his private signal σ . In the third stage, the principal pays a bonus b . The second period is identical, save for we omit the agent's signal about his second-period performance since it has no impact on the analysis.

3 Equilibrium analysis

In this section we analyze Perfect Bayesian equilibria (PBE) of the principal-agent game. However, for a wide range of parameters there is a continuum

⁴In the words of Deci and Ryan [1985, 43] "Intrinsic motivation is the innate, natural propensity to engage one's interests and exercise one's capacities, and in so doing, to seek and conquer optimal challenges. Such motivation emerges spontaneously from internal tendencies and can motivate behavior even without the aid of extrinsic rewards or environmental controls".

of perfect Bayesian equilibria in this game. This is a common feature of signaling games. Rather than characterize the whole equilibrium set, we shall impose some further restrictions. Indeed, some of the equilibria are less reasonable than others, because out-of-equilibrium beliefs they stipulate are less plausible. In characterizing the equilibrium we therefore apply a standard refinement, *Never a Weak Best Response* (NWBR) refinement for signaling games⁵. We also restrict attention to those equilibria, in which the agent chooses to work if indifferent.

3.1 Continuation equilibria

In this section we analyze equilibria of continuation "subgames" starting after the second stage of period 1, that is, after the agent has chosen his first-period effort.

The first thing to note is that no bonus is ever offered to the agent in period 2: paying a bonus is costly to the principal, and, since it can have no impact on decisions made in the past, the principal should never give a bonus at the final stage of the game. The agent, of course, foresees that no bonus is given in period 2. Whether he exerts effort in this period depends on his posterior belief of being the high type, ρ' : the agent works in period 2 if and only if:

$$[\rho'\theta_H + (1 - \rho')\theta_L]V \geq c. \quad (3)$$

We have made two simplifying assumptions: first, the principal observes the agent's effort and second, the low effort leads to failure with certainty. These assumptions are not crucial for the results (see Section 6), but make the analysis more transparent.

Lemma 1 *Assume first that the agent exerts low effort at the first stage, $e_1 = 0$. In this case the principal does not pay a bonus, $b_1 = 0$, and the agent chooses to exert effort in the second period if $\rho \geq \bar{\rho}$, where $\bar{\rho}$ is determined by*

$$[\bar{\rho}\theta_H + (1 - \bar{\rho})\theta_L]V = c. \quad (4)$$

Proof. In the subgame that follows $e_1 = 0$ the agent gets no new information and chooses to exert effort in the second period if his payoff from

⁵See Cho and Kreps (1987) and Fudenberg and Tirole (1991) for a definition. The refinement is somewhat stronger than the *Intuitive Criterion* and, in the current context, is equivalent to the Banks and Sobel's (1987) *universal divinity* criterion.

working on the second project is non-negative, i.e. $\rho \geq \bar{\rho}$, where $\bar{\rho}$ is determined by (4). The principal does not pay a bonus in the first period: she has no private information to be communicated to the agent through costly signaling, and a bonus has no impact on the agent's incentives in the second period. ■

We now proceed with the analysis of the more interesting case, the subgame that follows the agent's choice to exert high effort in the first period, $e_1 = 1$. This time, the agent's posterior beliefs depend on the new information he gets: the signal σ about the first-period outcome and the bonus b_1 paid by the principal. Of course, what matters is not the bonus per se, but its informational content. Suppose that the agent gets signal σ and bonus b_1 that, he believes, is paid by the principal with probability x_S after success and with probability x_F after failure. Then, the agent updates his belief on being the high ability type from ρ to ρ' with:

$$\frac{\rho'}{1 - \rho'} = \frac{\rho}{1 - \rho} \frac{\theta_H g_S(\sigma) x_S + (1 - \theta_H) g_F(\sigma) x_F}{\theta_L g_S(\sigma) x_S + (1 - \theta_L) g_F(\sigma) x_F}. \quad (5)$$

The following lemma characterizes the agent's optimal choice of second-period effort:

Lemma 2 *Let $e_1 = 1$. There exist threshold levels of the initial self-confidence $\tilde{\rho}_S$ and $\tilde{\rho}_F \geq \tilde{\rho}_S$ such that in any continuation equilibrium:*

- (i) *If $\rho < \tilde{\rho}_S$, the agent chooses $e_2 = 0$ after any signal σ and bonus b .*
- (ii) *If $\rho \geq \tilde{\rho}_F$, the agent chooses $e_2 = 1$ after any signal σ and bonus b .*
- (iii) *If $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F)$, works if and only if he gets sufficiently good news (i.e. signal σ exceeds a threshold level $\tilde{\sigma}$ that depends on ρ and bonus b_1 received in the first period).*

Proof. See the Appendix. ■

Remark 1 *A similar statement can be formulated in terms of the agent's cost of effort c : there exist \tilde{c}_S and $\tilde{c}_F \leq \tilde{c}_S$ such that the agent never works if $c > \tilde{c}_S$, always works if $c \leq \tilde{c}_F$ and for $c \in (\tilde{c}_F, \tilde{c}_S]$ works if and only if he gets sufficiently good news (σ high enough).*

Indeed, if the agent has a sufficiently low initial self-confidence $\rho < \tilde{\rho}_S$ (or his cost of effort c is high), it is optimal for him to shirk in the second period even if he were sure that the first-period project had been successful: this information would still be insufficient to compensate for his initial pessimism. The signal σ about the outcome, as well as the bonus b_1 paid by the principal, are irrelevant for this range of parameters (this is part (i) of

Lemma 2). Similarly, if the agent has sufficiently high initial self-confidence (or low disutility of effort), the agent chooses to work in period 2 even if he is sure that he has suffered a failure in the first task (part (ii) of Lemma 2).

The thresholds $\tilde{\rho}_S$ and $\tilde{\rho}_F$ are derived in the appendix. Their characterization is straightforward: $\tilde{\rho}_S$ is the level of the initial self-confidence that makes the agent indifferent between choosing $e_2 = 1$ and $e_2 = 0$ if he knows that the first project has been successful, $y_1 = S$. Similarly, $\tilde{\rho}_F$ is the level of the initial self-confidence that makes him indifferent if he learns that $y_1 = F$. For intermediate levels of initial self-confidence $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F)$ (or cost of effort $c \in (\tilde{c}_F, \tilde{c}_S]$), were the agent to know the first-period outcome, it would be optimal for him to work after success and to shirk after failure. Therefore, within this range of parameters the agent's reaction is sensitive to the news he gets: if sufficiently good news arrives (that is, high enough σ for a given principal's policy), he works in period 2. If he gets sufficiently bad news, he does not work in period 2. More precisely, in the appendix we show that the agent chooses to work if signal σ exceeds $\tilde{\sigma}$ that satisfies

$$l(\tilde{\sigma}) = \frac{x_F}{x_S} A, \quad (6)$$

where A is a compound parameter measuring task unattractiveness.⁶ This parameter is decreasing in initial self-confidence and increasing in the cost of effort. Thus, the threshold signal $\tilde{\sigma}$ is decreasing in initial self-confidence (for a given principal's policy). Note, furthermore, that the threshold signal is decreasing in the probability that the bonus is paid after a success (x_S), which makes it more likely that the outcome was a success, and increasing in the probability that the bonus is paid after a failure (x_F), which makes it more likely that the outcome was a failure.

Consider now the the principal's behavior in period 1. For $\rho \notin [\tilde{\rho}_S, \tilde{\rho}_F)$, the principal gives no bonus in period 1, since she is not able to influence the agent's behavior in period 2. From here on, we focus on the intermediate range of initial self-confidence $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F)$. For these values of ρ , the agent's second-period behavior is sensitive to the information about the outcome of the first period, and, therefore, the principal may try to signal through a bonus that the agent was successful in period 1.

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$$A := \frac{(1 - \rho)(1 - \theta_L)(c - \theta_L V) - \rho(1 - \theta_H)(\theta_H V - c)}{\rho\theta_H(\theta_H V - c) - (1 - \rho)\theta_L(c - \theta_L V)}.$$

For the values of ρ in the interval from $\tilde{\rho}_S$ to $\tilde{\rho}_F$, A is equal to the ratio of the expected loss from working in case of a failure to the expected gain from working in case of a success. In the absence of any intermediate information the agent would work if and only if $A(\rho) \leq 1$, that is $\rho \geq \bar{\rho}$, where $\bar{\rho}$ is defined in (4).

The following lemma, that shows that in equilibrium a higher bonus increases the likelihood of effort, will prove to be helpful in deriving the equilibrium bonus. Define $\tilde{\sigma}(b)$ as the threshold signal such that the agent works for all $\sigma \geq \tilde{\sigma}(b)$ after getting bonus b . Then:

Lemma 3 *For any bonuses $b_1 > b_2$ offered in equilibrium with positive probability it must be that $\tilde{\sigma}(b_1) < \tilde{\sigma}(b_2)$.*

Proof. The proof is straightforward. Suppose $b_1 > b_2$ are equilibrium bonuses but $\tilde{\sigma}(b_1) \geq \tilde{\sigma}(b_2)$. Then a lower bonus b_2 (weakly) increases the likelihood of effort. Clearly, this makes the principal unambiguously better off giving b_2 so that b_1 can not be an equilibrium bonus. ■

We shall show that under our assumptions the continuation equilibrium is (generically) unique: it is either pooling or semi-separating. In the pooling equilibrium, the principal always offers the same reward. In the semi-separating equilibrium, she always offers one reward after success, but randomizes between this reward and another one after failure. To gain intuition, we show the derivation of these two equilibria in detail in Subsections 3.1.1 and 3.1.2 before stating Proposition 1 that summarizes the analysis of continuation equilibria for the case $e_1 = 1$.

3.1.1 Pooling equilibrium

We keep assuming throughout that the agent exerts effort in period 1 and that $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F]$. We are now looking for a *pooling* equilibrium, in which the principal gives the same bonus \tilde{b} independent of the intermediate outcome. Given this bonus, the agent only works for signals exceeding $\tilde{\sigma} > 0$, where $l(\tilde{\sigma}) = A$ (see equation (6) with $x_S = x_F = 1$). Denote by $\hat{\theta}_F$ and $\hat{\theta}_S$ the estimates of the agent's ability by the principal, conditional on failure and success in the first period respectively. Thus:

$$\hat{\theta}_y = E[\theta \mid e_1 = 1, y_1 = y]. \quad (7)$$

Formulas for these probabilities are given in the Appendix in the proof of Proposition 1. The expected second-period equilibrium payoff for the principal is then given by:

$$E[U_2^P \mid e_1 = 1, y_1 = y] = \hat{\theta}_y(1 - G_y(\tilde{\sigma}))W - \tilde{b}. \quad (8)$$

Assume that the principal deviates from the equilibrium strategy, and offers a bonus $\hat{b} = \tilde{b} + \varepsilon$ with $\varepsilon > 0$. For \tilde{b} to be the equilibrium bonus such a

deviation should not be profitable for the principal. A necessary condition for this is that the agent does not believe that \hat{b} is given only after success if ε is small enough (otherwise, the principal could achieve an upward jump in the probability of effort at an infinitesimal cost by giving \hat{b} instead of \tilde{b}). Let $\hat{\sigma}$ be the agent's reaction to bonus \hat{b} which makes the principal indifferent between deviating or not after failure:

$$\hat{\theta}_F(1 - G_F(\tilde{\sigma}))W - \tilde{b} = \hat{\theta}_F(1 - G_F(\hat{\sigma}))W - \hat{b}. \quad (9)$$

We shall now invoke the NWBR restriction on the agent's out-of-equilibrium beliefs, which stipulates that for the agent not to believe that \hat{b} is given only after success, it must be the case that:

$$\hat{\theta}_S(1 - G_S(\tilde{\sigma}))W - \tilde{b} \geq \hat{\theta}_S(1 - G_S(\hat{\sigma}))W - \hat{b}. \quad (10)$$

In other words, if the principal is indifferent between a bonus \hat{b} and \tilde{b} after a failure, he should not be better off with a bonus \hat{b} after a success. Combining (9) and (10) yields:

$$\hat{\theta}_S(G_S(\tilde{\sigma}) - G_S(\hat{\sigma})) \leq \hat{\theta}_F(G_F(\tilde{\sigma}) - G_F(\hat{\sigma})). \quad (11)$$

Dividing both sides by $\tilde{\sigma} - \hat{\sigma}$ (note that by Lemma 3 $\hat{\sigma}$ is necessarily smaller than $\tilde{\sigma}$ for a positive ε) and taking the limit $\varepsilon \rightarrow +0$ one gets $\hat{\theta}_S g_S(\tilde{\sigma}) \leq \hat{\theta}_F g_F(\tilde{\sigma})$ or:

$$l(\tilde{\sigma}) \leq \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (12)$$

Conversely, assume that (12) is satisfied and consider a possible deviation $\hat{b} = \tilde{b} + \varepsilon$. Since the MLRP implies that⁷:

$$\frac{G_S(\tilde{\sigma}) - G_S(\hat{\sigma})}{G_F(\tilde{\sigma}) - G_F(\hat{\sigma})} < l(\tilde{\sigma}), \quad (13)$$

for any $\hat{\sigma} < \tilde{\sigma}$, condition (12) implies a strict version of inequality (10). Then, according to NWBR, the agent has to believe in failure after receiving out-of-equilibrium bonus \hat{b} and the principal has no incentive to deviate to

⁷To prove this (following Milgrom (1981)): suppose $l(x) < l(z) \forall z \in [x, y]$. Then, since $l(z) \equiv g_S(z)/g_F(z)$, $g_F(z)l(x) < g_S(z)$. This implies $\int_x^y g_F(z)l(x)dz < \int_x^y g_S(z)dz$. Integration yields: $l(x) < [G_S(y) - G_S(x)] / [G_F(y) - G_F(x)]$. Similarly, for $l(z) < l(y) \forall z \in [x, y]$, it follows that $[G_S(y) - G_S(x)] / [G_F(y) - G_F(x)] < l(y)$.

a higher bonus. Similar reasoning shows that the principal does not want to deviate to a slightly lower bonus if and only if

$$l(\tilde{\sigma}) \geq \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (14)$$

There are two cases now. Either both inequalities (12) and (14) are satisfied simultaneously, which implies a restriction on parameters

$$A = \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (15)$$

In this case for any $\tilde{b} \in [0, \tilde{b}_S]$ with $\tilde{b}_S = \hat{\theta}_F(1 - G_F(\tilde{\sigma}_S))W$ there exists a pooling continuation equilibrium in which the principal gives \tilde{b} , the agent works if $\sigma \geq \tilde{\sigma}$ (with $l(\tilde{\sigma}) = A$) after the equilibrium bonus and never after an out-of-equilibrium bonus.

In the generic case, if (15) is not satisfied, the only candidate for the pooling equilibrium is the one with $\tilde{b} = 0$. In this case the agent's limited liability prevents the principal from downward deviations. For the upward deviations to be unprofitable, (12) must be satisfied, which implies a restriction on the parameters:

$$A \leq \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (16)$$

Note that this condition is both necessary and sufficient for the existence of some pooling equilibrium.

3.1.2 Semi-separating equilibrium

Consider now a *semi-separating* equilibrium where the principal offers \tilde{b}_S after success and randomizes between \tilde{b}_S and \tilde{b}_F after failure (with probabilities \tilde{x}_S and \tilde{x}_F). In this case, bonus \tilde{b}_F is given only in case of failure and therefore perfectly reveals it. It follows that $\tilde{b}_F = 0$, since there is no reason for the principal to incur a cost for conveying a negative signal.

In equilibrium the principal must be indifferent between \tilde{b}_S and \tilde{b}_F after a failure (otherwise she would not be willing to mix):

$$\hat{\theta}_F(1 - G_F(\tilde{\sigma}_S))W - \tilde{b}_S = 0, \quad (17)$$

where $\tilde{\sigma}_S$ is the threshold signal for which an agent works after a bonus \tilde{b}_S . Note that after $\tilde{b}_F = 0$ the agent does not work (recall that $\hat{\theta}_F V < c$ for

$\rho \in (\rho_S, \rho_F)$ so the payoff for the principal is zero. Condition (17) determines the bonus \hat{b}_S .

Moreover, the principal should not want to deviate to any bonus above or below \tilde{b}_S . Following the same logic as in the pooling case above, the principal indeed will not deviate if the agent believes a failure occurred after observing a deviation from the equilibrium bonus. Suppose first that the principal deviates to $\hat{b}_S = \tilde{b}_S - \varepsilon$. NWBR assumption implies that if

$$\hat{\theta}_F(1 - G_F(\hat{\sigma}_S))W - \hat{b}_S = 0, \quad (18)$$

it must be that

$$\hat{\theta}_S(1 - G_S(\hat{\sigma}_S))W - \hat{b}_S \leq \hat{\theta}_S(1 - G_S(\tilde{\sigma}_S))W - \tilde{b}_S \quad (19)$$

Again, if this inequality did not hold, the agent should believe that a success has occurred by the NWBR assumption, such beliefs being incompatible with the equilibrium.

Together, these conditions imply:

$$l(\tilde{\sigma}_S) \geq \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (20)$$

(Recall that a deviation to a smaller bonus is considered, hence $\hat{\sigma} > \tilde{\sigma}$).

Condition (20) implies that $\tilde{\sigma}_S > 0$. Indeed, suppose $\tilde{\sigma}_S = 0$. Then, $l(0) \geq \hat{\theta}_F/\hat{\theta}_S > 0$ – a contradiction to Assumption 2 which stipulates that the likelihood ratio $l(\sigma)$ is monotone and assumes all values in $[0, +\infty)$. Hence, $\tilde{\sigma}_S > 0$, so the agent does not always work after bonus \tilde{b}_S . Then, the principal should also not be willing to deviate to a (slightly) higher bonus $\hat{b}_S = \tilde{b}_S + \varepsilon$ to separate the success outcome. Like in the case of pooling, this implies:

$$l(\tilde{\sigma}_S) \leq \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (21)$$

Hence, combining (20) and (21) gives:

$$l(\tilde{\sigma}_S) = \frac{\hat{\theta}_F}{\hat{\theta}_S} \quad (22)$$

as the only possibility. According to (6), the agent's reaction is given by

$$l(\tilde{\sigma}_S) = \tilde{x}_F A. \quad (23)$$

Condition (22) determines $\tilde{\sigma}_S$, (23) defines \tilde{x}_F and (17) determines \tilde{b}_S . Finally note that conditions (22) and (23) imply that $A \geq \hat{\theta}_F/\hat{\theta}_S$ as $\tilde{x}_F \leq 1$.

3.1.3 Rewards, self-confidence, and motivation

The following proposition summarizes the analysis of continuation equilibria after $e_1 = 1$:⁸

Proposition 1 *For $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F)$ there always exists a (generically) unique⁹ continuation equilibrium after the agent has chosen $e_1 = 1$, satisfying the NWBR criterion. Furthermore, there exists a unique value of initial self-confidence ρ^* , defined as the (unique) solution to $A = \hat{\theta}_F/\hat{\theta}_S$, such that:*

- (i) *If $\rho \in [\tilde{\rho}_S, \rho^*)$, the unique continuation equilibrium is semi-separating: the principal always gives a bonus $\tilde{b}_S = \hat{\theta}_F(1 - G_F(\tilde{\sigma}))W$ after success ($\tilde{x}_S = 1$), and randomizes between \tilde{b}_S and $\tilde{b}_F = 0$ after failure with probabilities $\tilde{x}_F = \frac{l(\tilde{\sigma})}{A}$ and $1 - \tilde{x}_F$ respectively. After receiving bonus \tilde{b}_S the agent works if his signal σ exceeds the threshold $\tilde{\sigma}_S$, determined by $l(\tilde{\sigma}_S) = \frac{\hat{\theta}_F}{\hat{\theta}_S}$; after getting no bonus the agent does not work: $\tilde{\sigma}_F = 1$.*
- (ii) *If $\rho \in (\rho^*, \tilde{\rho}_F)$, the unique continuation equilibrium is pooling and no bonus is ever given by the principal. The agent works if his signal exceeds threshold $\tilde{\sigma}$ determined by $l(\tilde{\sigma}) = A$.*

Remark 2 *Proposition 1 can be re-stated in terms of disutility of effort: there exists a threshold value c^* defined as the unique solution to $A = \hat{\theta}_F/\hat{\theta}_S$ such that the unique continuation equilibrium is pooling for $c \in (\tilde{c}_F, c^*)$ and semi-separating for $c \in (c^*, \tilde{c}_S]$, with equilibrium strategies being the same as those described in parts (ii) and (i) of Proposition 1 respectively.*

That the unique continuation equilibrium for sufficiently high initial self-confidence or low disutility of effort is a pooling equilibrium with no bonus is quite intuitive. Indeed, for such parameters the threshold signal $\tilde{\sigma}$, which makes the agent indifferent between working or not in the second period in the pooling equilibrium, is relatively low. Low signals are more likely after failure (due to the MLRP assumption), so it is after failure that the principal gains more from a marginal decrease in $\tilde{\sigma}$. According to the logic of the NWBR refinement, this makes the agent interpret an (out-of-equilibrium) increase in the bonus as coming from the principal who observed a failure,

⁸The proposition states the equilibrium conditions for values of ρ such that $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F)$. Recall that it was already established in Lemma 2 that for values of $\rho \notin [\tilde{\rho}_S, \tilde{\rho}_F)$ no bonus is ever offered. For $\rho < \tilde{\rho}_S$, $\tilde{\sigma} = 1$ (the agent never works) and for $\rho \geq \tilde{\rho}_F$, $\tilde{\sigma} = 0$ (the agent always works).

⁹The equilibrium is unique unless $\hat{\theta}_F/\hat{\theta}_S = A$, in which case there is a continuum of pooling equilibria.

thus undermining the principal's incentives to increase the bonus. In fact, a *decrease* in bonus would signal to the agent that the principal observed a success, but the agent's limited liability prevents the principal from paying negative bonuses.

A more interesting result of the proposition is, however, that there is a region where the principal does give a positive bonus in (continuation) equilibrium, and this bonus increases the agent's self-confidence. In this region, the agent is relatively unlikely to make efforts in the second period, so, were the principal to play a pooling strategy, the threshold $\tilde{\sigma}$ would be high. High signals are more likely after success, and in this case it is a principal who observed a success who would gain more from a marginal decrease in $\tilde{\sigma}$. This time the agent would interpret an (out-of-equilibrium) increase in bonus as a signal of success, thus destroying the pooling equilibrium. In this case, by paying a positive bonus, the principal sends a costly credible signal to the agent that a success has occurred.

Besides establishing the use of bonuses in equilibrium, this proposition also rules out equilibria where good performance is completely separated from bad performance.¹⁰ It may seem counterintuitive that even bad performance would be rewarded, but there is considerable evidence that this often occurs in practice (Prendergast (1999)). Supervisors are reluctant to differentiate good from bad performance, resulting in a well documented compression of ratings. Prendergast (1999) conjectured that a possible reason could be that the supervisor avoids to discourage the agent by revealing poor performance to him. A similar view is shared by Beer (1990) who notes that many people are rated on the high side because managers "...do not want to damage an employee's self esteem, thereby demotivating the employee..."¹¹. This interpretation fits well our result¹².

Another important point is that for levels of initial self-confidence below the threshold level ρ^* , the size of the reward \tilde{b}_S is proportional to W . Intuitively, if the principal derives higher benefits from a success, a bonus of a given size becomes relatively less costly, and the principal has to increase the size of the bonus to keep it credible in equilibrium. This means that the agent can be very glad to get even a seemingly negligible reward provided that it is given by a principal that does not have too much interest in the agent's performance. An almost costless praise by a disinterested, but slightly al-

¹⁰This result relies on the assumption that "really bad" signals exist, i.e. $l(0) = 0$. There is an exception: the continuation equilibrium is separating if $\rho = \tilde{\rho}_S$, i.e. on the set of parameters of measure 0.

¹¹Quoted in Gibbs (1991, p. 7).

¹²Another reason mentioned by Prendergast (1999, p. 30) is that it is simply an unpleasant task to offer poor ratings to workers.

truistic division manager may still be very pleasant and convincing. In such a case it is not a reward per se that makes the agent happy but mostly its informational content.

3.1.4 Comparative statics

Comparative statics with respect to ρ Proposition 1 identifies a unique threshold ρ^* such that when the initial self-confidence ρ crosses the threshold value ρ^* , the equilibrium switches from the semi-separating to the pooling regime. In the pooling regime the only relevant equilibrium parameter – the probability that the agent works in the second period – increases with ρ .

Other comparative static results with respect to ρ are less clear-cut. For example, the impact of ρ on the size and frequency of the positive bonus is ambiguous. This is not surprising: the agent's equilibrium strategy is determined by the ratio $r(\rho) = \hat{\theta}_F/\hat{\theta}_S$ which itself varies nonmonotonically in ρ . At $\rho = 0$ and $\rho = 1$ this ratio equals unity (there is no uncertainty about the agent's ability and the intermediate outcome does not bring new information). For intermediate values of ρ the principal's estimate of the agent's ability is lower after observing a failure than after observing a success, and consequently $\hat{\theta}_F/\hat{\theta}_S$ is smaller than one. Its derivative

$$r'(\rho) = \frac{(\theta_H - \theta_L)^2(\rho^2\theta_H^2(1 - \theta_H) - (1 - \rho)^2\theta_L^2(1 - \theta_L))}{(\rho(1 - \theta_H) + (1 - \rho)(1 - \theta_L))^2(\rho\theta_H^2 + (1 - \rho)\theta_L^2)^2}.$$

is negative if $\rho \in (0, \rho_r)$ and positive if $\rho \in (\rho_r, 1)$, where ρ_r is determined by

$$\frac{\rho_r}{1 - \rho_r} = \frac{\theta_L}{\theta_H} \sqrt{\frac{1 - \theta_L}{1 - \theta_H}}.$$

Whether $r(\rho) = \hat{\theta}_F/\hat{\theta}_S$ is decreasing or increasing (or both patterns take place) for values of $\rho \in [\tilde{\rho}_S, \rho^*)$ depends on the specific parametrization of the model.

If $r'(\rho) > 0$, the threshold $\tilde{\sigma}$ is increasing in initial self confidence, ρ , as well as the probability of a bonus after a failure, x_F . The impact on the size of the bonus after a success, \tilde{b}_S , is ambiguous. If $r'(\rho) < 0$, $\tilde{\sigma}$ is decreasing, \tilde{b}_S is increasing, but the impact on x_F is ambiguous.

Comparative statics with respect to c The ratio of the agent's ability estimates $\hat{\theta}_F/\hat{\theta}_S$, does not depend on c . On the other hand, A , the measure of task unattractiveness, is increasing in c . First, this implies that in the pooling equilibrium the probability that the agent works in the second period decreases in c . Second, an increase in the cost of effort causes a switch

in equilibrium regimes when c crosses the threshold c^* . In this case the equilibrium bonus rises discontinuously from 0 to $b_S = \hat{\theta}_F(1 - G_F(\tilde{\sigma}_S))W$. Third, in the semi-separating regime the equilibrium value of the positive bonus, b_S , and the agent's strategy, $\tilde{\sigma}$, do not depend on c , but the probability that the principal gives a positive bonus b_S after a failure, x_F , decreases with c . The agent is less likely to work in that case; therefore, the bonus must provide a stronger positive feedback.

3.2 The first-period choice of effort

We now proceed by characterizing the equilibria of the whole game, in particular, specifying the agent's choice of effort in the first period; continuation equilibria following different effort choices have been described in the previous sections. Fix all parameters of the model but the initial self-confidence ρ and denote $\mathcal{R}_1 \subset [0, 1]$ the set of values of ρ for which there exists a PBE (satisfying the NWBR refinement) in which the agent chooses $e_1 = 1$. Similarly, let $\mathcal{R}_0 \subset [0, 1]$ be the set of values of ρ for which there exists a PBE (satisfying the NWBR refinement) in which the agent chooses $e_1 = 0$.

Proposition 2 *i) An equilibrium exists and is generically unique; in particular, $\mathcal{R}_1 \Delta \mathcal{R}_0 = (\mathcal{R}_1 \setminus \mathcal{R}_0) \cup (\mathcal{R}_0 \setminus \mathcal{R}_1)$ is an open set everywhere dense on $[0, 1]$.*

ii) $[\bar{\rho}, 1] \subset \mathcal{R}_1$, where $\bar{\rho}$ is defined as the (unique) solution of $A(\rho) = 1$.

iii) $[0, \tilde{\rho}_S) \subset \mathcal{R}_0$.

iv) $\mathcal{R}_1 \cap [\tilde{\rho}_S, \rho^) \neq \emptyset$.*

Proposition 2 shows that there exists equilibrium in which the agent works in the first period if he is sufficiently optimistic. Since, as argued before, the expected bonus may be nonmonotonic in ρ , the agent's equilibrium choice of effort also need not be monotonic with respect to ρ (or, similarly, with respect to c , the cost of effort¹³); part iv) of Proposition 2 thus cannot be strengthened, but at least it shows that the intersection of the two sets is nonempty. These parameters not only directly affect the agent's intrinsic benefit from working, but also affect the probability that a bonus is given after the first period and the size of the bonus.

We can now emphasize the dual role that rewards play in our model (for those values of parameters under which the continuation equilibrium is semi-separating). First of all, a reward paid by the principal at the intermediate stage provides the agent with positive feedback and encourages him

¹³A statement, similar to Proposition 2, could be formulated in terms of cost of effort c ; see also previous remarks concerning the classification of continuation equilibria.

to continue. This effect is purely informational in nature. It's existence also affects the agent's behaviour in period 1 because this information acquisition is valuable: it allows the agent to make a better, more informed choice of effort in the second period. By exerting effort in the first period, the agent receives valuable information which he otherwise would not get. Secondly, the agent foresees that a bonus is paid with positive probability provided he chooses to exert effort, which gives him an additional stimulus to work hard in period 1. This is a traditional incentive effect.

This second effect provides a rationale for giving feedback through the use of rewards that are useful to the agent. Indeed, in the model we have been assuming that discretionary rewards (bonuses) are simply monetary transfers from the principal to the agent. However, the principal could give feedback to the agent in many other ways, including burning money or wasting time, as long as such actions were costly to her. But, since actions that have direct benefits for the agent (i.e. various kinds of rewards) create an additional incentive effect, their use for providing feedback is more attractive.

4 Optimal performance measures

In the previous section we showed that even noncontractible bonuses may occur in equilibrium, by their virtue of providing feedback. In addition, the use of rewards created an additional, direct incentive effect. We can stress the point even further by showing that the desire to exploit the incentive effect may be a reason for the principal to obscure performance results in the first place. That is, we show that the principal may prefer subjective performance measures over performance measures that the principal and the agent can agree on¹⁴. This result corresponds well to the common practice of using monitoring systems that provide little information even though more informative performance measures are available (Gibbs, (1991)).

Let us now assume that the principal can influence the precision of the performance measure. She could, for instance, make the whole process more transparent to the agent, employ co-workers as peer reviewers, or specify the goals in more detail. To make things extreme, we assume that at the beginning of the game the principal can commit herself to either one of the following two situations: (i) a situation as before, in which the agent only gets an imperfect signal about performance, or (ii) a situation in which the performance is perfectly observable to the agent. We denote these two

¹⁴We still assume in this section that a third party cannot verify performance, even if the principal and the agent can agree on the performance (cf. for instance Levin (2003), section III).

regimes by smoky performance measure (SPM) and transparent performance measure (TPM) respectively. The choice of the performance measurement system is observed by the agent before he decides on his first-period effort.

It is fairly easy to show that in both situations the agent exerts effort in period 1 for lower levels of initial self-confidence (or higher levels of disutility of effort) than if he would receive no information at all. In both situations he receives some information on his performance by exerting effort in period 1. This information is valuable, as it enables him to form a better judgement about his ability, allowing for a better effort decision in period 2. On top of this informational acquisition, by exerting effort the agent may increase the probability of receiving a bonus.

From the viewpoint of the principal, a comparison between the two situations turns out to be ambiguous. The prospect of receiving valuable information gives incentives to the agent to exert effort in period 1. This favours making performance as transparent as possible. The downside of providing the precise information is that it negatively affects the equilibrium bonus: the private information of the principal is precisely the reason that a bonus occurs in equilibrium. In the extreme case when the principal makes performance completely transparent (TPM), the agent rightly foresees that no bonus will be given in equilibrium.

Note that the principal is always better off in the agent exerts effort in equilibrium, so that whenever an agent works in only one of the two regimes the principal always prefers the regime in which the agent works. If the agent exerts effort in both regimes, it is easily seen that the principal prefers to have a transparent performance measure available, as this saves her any signalling costs. Which performance measurement system the principal chooses turns out to depend on her payoff from a success, W . The trade-off she makes is between providing better information and giving direct incentives through a bonus. Since the equilibrium bonus increases linearly in W , the direct incentive effect dominates the incentive effect for high enough values of W and is dominated for low values of W . The following proposition and corollary summarize this.

Let \mathcal{R}^P be the set of values of ρ such that given performance measure $P \in \{T, S\}$ (transparent and smoky, respectively) there exist a NWBR equilibrium in which the agent exerts effort in period 1; thus, $\mathcal{R}^S = \mathcal{R}_1$.

Proposition 3 *There exist $\underline{W} \leq \overline{W}$ such that*

- i) If $W > \overline{W}$, $\mathcal{R}^T \subset \mathcal{R}^S$ (and $\mathcal{R}^T \neq \mathcal{R}^S$).*
- ii) If $W < \underline{W}$, $\mathcal{R}^S \subset \mathcal{R}^T$.*

Corollary 1 *For sufficiently high values of W , there exist values of initial*

self-confidence for which the principal strictly prefers a smoky performance measure to a transparent performance measure.

For $W > \bar{W}$ there exists a range of parameters¹⁵ such that the agent does not exert effort with a transparent performance measure but does exert effort with a smoky performance measure. The implication of this is that hiding the intermediate performance from the agent may be a way to deal with the otherwise non-contractible externality. Ex post rewards paid by the principal for signaling purposes serve as an ex ante mechanism to motivate the agent to work hard from the beginning.

5 The hidden costs of rewards

So far the focus has been on how noncontractible rewards stimulate motivation. By contrast, the papers by Bénabou and Tirole (2003) and Suvorov (2003) study how rewards can *decrease* motivation. As argued earlier, an important difference with their approach is that they consider contractually specified performance-contingent bonuses. To sketch the argument: if a principal observes that the agent has low ability, she also expects him to have low self-confidence. She therefore proposes a high-powered contract which specifies a high reward in case of success to motivate the agent anyway. This makes the agent realize that he must be of low ability, which lowers his self-confidence even further. Nevertheless, the external reward induces the agent to exert effort, but once rewards are withdrawn, the agent is no longer motivated to work hard.

Those papers agree with the experimental evidence on this effect. Deci et al. (1999) survey the literature and find that such crowding-out of motivation is confirmed by a meta-analysis of more than one hundred earlier studies. In sum, there is a rich body of experiments showing that there are hidden costs of rewards¹⁶.

How much motivation is crowded out depends to a great extent on the nature of the reward (Deci and Ryan (1985)). For example, rewards contingent on performance have an effect on motivation, but the effect of rewards contingent on fulfilling the task are less profound. In line with the model in this paper, no such effect has been found in experiments where rewards were not explicitly made contingent on performance in advance.

¹⁵An interval of initial self-confidence parameters ρ other parameters being fixed or an interval of the cost parameters c .

¹⁶See also Kohn (1993), Deci and Ryan (1985), Frey (1997), and Frey and Jegen (2002).

It may be possible to replicate the negative correlation between rewards and self-confidence in a version of our model. In the model of section 2, the principal has more to gain from rewarding an agent after a success. Thus, after success she gives rewards more often making them good news for the agent that raises his self-confidence. However, so far it has been assumed that effort and ability are complements, so that the principal indeed wants to increase the agent's self-confidence. When effort and ability are substitutes, a higher self-confidence becomes a bad thing from the principal's viewpoint, and the results of the model are reversed: the principal now has an incentive to spend resources to signal a failure, hence reducing self-confidence and encouraging more effort. The reward upsets the agent, but urges him to work harder in the second period. However, in some cases it may be unnatural to assume that the principal gives rewards to the agent after failures – this would have perverse ex ante effects. It may be more likely that the principal will exert costly effort to explain (prove) to the agent that he failed in the first period, or even undertake costly actions to punish the agent.

Consider the following stylized model: the agent has to perform a task that has again only two possible outcomes, failure and success. The agent's payoff is $\lambda V - ec$, where $\lambda = 1$ in case of success and $\lambda = 0$ in case of failure. As before, the agent's ability can be θ_H with probability ρ or θ_L with probability $1 - \rho$; effort $e \in \{0, 1\}$. We assume now that ability and effort are substitutes rather than complements: if the agent is very smart, he may master the task without much effort. If he is less gifted, he can compensate the lack of ability by working harder.

Formally, the agent succeeds if $\theta + e \geq \psi$, where $\theta_L < \psi < \theta_H < 1$. As before, we suppose that the agent does not know his ability. Thus, effort is sufficient for success, as is the high ability. Assume also that $c < V$ so that the low-ability agent would exert effort were he to know that he cannot succeed otherwise. The agent faces two identical tasks, but does not observe the outcome of the first until both are finished, and again there is a principal that observes the first outcome immediately and gets payoff W after each success of the agent.

If $c > (1 - \rho)V$, the agent will choose not to exert effort in the first period. In this case, if the principal observes the agent's failure in the first period, she will choose to signal this and encourage the agent to exert effort. Any action with some positive cost to the principal will do – were the agent to succeed in the first period, the principal would not want to spend any resources since she would learn that the agent is strong and would be sure to get W in any case.¹⁷ If, for some reason, the principal decides to signal a failure by giving

¹⁷The NWBR refinement does not allow to pin down a unique equilibrium due to the

a positive reward to the agent, this bonus brings bad news to the agent but persuades him to work hard. As in Bénabou and Tirole (2003) and Suvorov (2003), the bonus works as a short-term reinforcer but in the long run it decreases the agent's self-confidence.

6 Modifications of the basic model

In this section we briefly discuss what happens when two of the simplifying assumptions we made are lifted, one in turn.

6.1 Unobservable effort

First, we shall show that the main results do not change much if we make a more standard assumption that the agent's first-period effort is not observed by the principal. Indeed, the results for the continuation equilibria obtained in Section 3.1 remain valid, with a qualification that they apply to the continuation equilibria that occur on the equilibrium path, where the principal holds correct beliefs about the agent's first period choice of effort. However, now the analysis must be also extended to out-of-equilibrium situations, in which the agent deviates and chooses the first period effort that is different from what is expected by the principal.

When parameters are such that the continuation equilibrium is semi-separating, shirking becomes (weakly) more attractive than in the model with observable effort. Indeed, if the principal believes that the agent exerted effort, the agent may still count on getting a positive bonus with some probability even if he shirks since the principal does not observe the deviation. Similarly, if the principal believes that the agent shirks in the first period, but the agent deviates and chooses to supply effort, such a deviation will not be detected and rewarded by the principal unless the project succeeds (in which case the principal updates her beliefs about the agent's strategy). As a result, in the model with unobservable effort there is a set of parameters of a positive measure for which there are multiple equilibria, i.e. statement (i) of Proposition 2 is no longer valid. That is, for a set of values of $\rho \in (\tilde{\rho}_S, \rho^*)$ of a positive measure, there exist equilibria with $e_1 = 0$ and $e_1 = 1$, as well as a mixed-strategy equilibrium in which the agent randomizes between the two effort levels. Other statements of Proposition 2, as well as those of Proposition 3 remain valid.

open set problem: any positive bonus convinces the agent that he failed and thus is weak.

6.2 Positive probability of success under shirking

Another important assumption we made was that the low effort leads to failure with certainty. We now discuss what changes when this assumption is lifted. Suppose that the model is as described in Section 2, save for now the probability of success of the agent of ability θ that chooses $e = 0$ is given by θk with some $k \in (0, 1)$; we also assume that after he chooses $e_1 = 0$, the agent now gets signal σ that has the same distribution conditional on success/failure as that of the signal arriving after he chooses $e_1 = 1$. To check the robustness of our findings, we will be particularly interested in the limiting case of $k \approx 0$. We keep assuming, as in the main model, that the first period effort is observed by the principal.

Now the continuation subgame that follows the choice of the low effort by the agent is qualitatively similar to the subgame that follows the choice of high effort. More specifically, Lemma 2 is now true for the case $e_1 = 0$ with thresholds $\tilde{\rho}_S$ and $\tilde{\rho}_F$ replaced by appropriate $\tilde{\rho}_{S0}$ and $\tilde{\rho}_{F0}$; one can check that $\tilde{\rho}_{S0} = \tilde{\rho}_S$ and $\tilde{\rho}_{F0} \in (\tilde{\rho}_S, \tilde{\rho}_F)$. Proposition 1 now applies to the continuation subgame after $e_1 = 0$, with $A, \hat{\theta}_F, \hat{\theta}_S$ and ρ^* replaced by $A_0, \hat{\theta}_{F0}, \hat{\theta}_{S0}$ and ρ_0^* . Once again we see that the role of bonuses in our framework is to give credibility to communication, rather than to reward the agent's effort. Even if the agent shirks, but is lucky in the first period, the principal wants to give a reward: then, the agent learns that he has been lucky and, therefore, probably has high ability, which makes it worthwhile to exert effort on the second stage.

It can be checked that in the limit, for k small enough, all the statements of Propositions 2 and 3 are valid. Indeed, $\tilde{\rho}_{F0} \rightarrow \bar{\rho}$ when $k \rightarrow 0$; hence, for all $\rho > \bar{\rho}$, in the limit when $k \rightarrow 0$ the continuation equilibria after the agent chooses $e_1 = 0$ are pooling with no bonus offered. In contrast, for all $\rho \in (\tilde{\rho}_S, \bar{\rho})$ the continuation equilibria after the agent chooses $e_1 = 0$ are almost separating (i.e. $\tilde{x}_{F0} \rightarrow 0$ when $k \rightarrow 0$). Hence, for $\rho \in (\tilde{\rho}_S, \bar{\rho})$ the shirking option may be quite attractive for the agent: he gets almost perfect information from the principal. Thus, \mathcal{R}_1 , the set of values of ρ for which in equilibrium $e_1 = 1$, contracts, while \mathcal{R}_0 , the set of values of ρ for which in equilibrium $e_1 = 0$, expands, compared to the basic model with $k = 0$.

However, if k is small, the probability that the agent gets a bonus after shirking is negligible. Thus, when $\rho \in (\tilde{\rho}_S, \bar{\rho})$, he faces the following tradeoff: to get (almost) perfect information by shirking or to get a bonus from the principal with non-negligible probability by working. Clearly, the optimal behavior depends on the equilibrium size of the reward: if W is small enough, the shirking option becomes more attractive, if W is large, the agent prefers to work in the equilibrium.

7 Conclusions

Studies by psychologists have shown that rewards can undermine motivation, which stimulated economists to examine the effects of rewards in more detail. As Bénabou and Tirole (2003), we focus on the role of self-confidence and give a new explanation for the use of discretionary rewards and emphasize their role in stimulating motivation.

Although we have framed our model as a principal-agent relationship where the agent is uncertain about his performance, other settings are possible. For example, the agent could be unsure about his own payoff or cost of effort rather than his ability. Another possibility is that the agent cares about the principal's payoff (e.g. through altruism), but is unaware of how much utility the principal derives from his effort. Or the principal could be more or less altruistic towards the agent and his generosity in the first period could suggest that nice gifts are on the way if the agent also succeeds in the second.

This paper is only one of the first few attempts to study formally the interaction between rewards and self-confidence. Future work could generalize some of the assumptions. One possible extension is to consider more than two periods. This would shed light on the dynamics of rewards and self-confidence like in Suvorov (2003). Will the rewards have to increase to keep the agent motivated? Will the absence of a reward in a given period discourage the agent more if some rewards were given before than if no rewards were ever given?

Another extension would be to study an environment where both contractually specified and discretionary rewards might be used and to see what would be the optimal mix of them. For this, both subjective and objective measures of performance should be available, as in Baker et. al. (1994) and Pearce and Stacchetti (1998).

It would be also interesting to consider the model with several agents. For example, two agents may simultaneously perform a similar task in the same environment: their chances to succeed are perfectly correlated conditional on the abilities and efforts, abilities themselves being unknown and independent. What will an agent infer when he is not rewarded but sees the colleague to get a bonus? What will be the principal's policy? We hope to give answers to these questions in the future work.

8 Appendix

Proof of Lemma 2. The agent chooses to work in the second period if and only if inequality (3) is satisfied. It can be written as:

$$\frac{\rho'}{1 - \rho'} \geq \frac{c - \theta_L V}{\theta_H V - c} =: \phi, \quad (24)$$

where ϕ is the ratio of the expected loss of working for the low ability agent to the expected gain of the high-ability agent. Suppose the agent gets signal σ and bonus b_1 that, he believes, is paid by the principal with probability x_S after success and with probability x_F after failure. Then, the agent updates his belief on being the high ability type from ρ to ρ' given by (5). Substituting (5) into (24), one finds that the agent works in the second period if:

$$(\rho\theta_H - (1 - \rho)\theta_L\phi) l(\sigma)x_S \geq ((1 - \rho)(1 - \theta_L)\phi - \rho(1 - \theta_H)) x_F. \quad (25)$$

If ρ is smaller than $\tilde{\rho}_S$ determined by

$$\frac{\tilde{\rho}_S}{1 - \tilde{\rho}_S} = \frac{\theta_L}{\theta_H} \phi, \quad (26)$$

the LHS of (25) is non-positive, while the RHS is non-negative: the agent chooses $e_2 = 0$ irrespective of signal σ and the principal's strategy x_F, x_S (since at least one of x_F, x_S must be positive, the LHS is strictly smaller than the RHS).

If ρ is larger than $\tilde{\rho}_F$ determined by

$$\frac{\tilde{\rho}_F}{1 - \tilde{\rho}_F} = \left(\frac{1 - \theta_L}{1 - \theta_H} \right) \phi,$$

the LHS of (25) is non-negative, while the RHS is non-positive: the agent now chooses $e_2 = 1$ irrespective of signal σ and the principal's strategy x_F, x_S .

It is easy to see that $\tilde{\rho}_S < \tilde{\rho}_F$. For $\rho \in (\tilde{\rho}_S, \tilde{\rho}_F)$, the inequality (25) can be rewritten as

$$\frac{g_S(\sigma)}{g_F(\sigma)} \geq \frac{x_F}{x_S} A, \quad (27)$$

where

$$A = \frac{(1 - \rho)(1 - \theta_L)\phi - \rho(1 - \theta_H)}{\rho\theta_H - (1 - \rho)\theta_L\phi}. \quad (28)$$

For $\rho \in (\tilde{\rho}_S, \tilde{\rho}_F)$ the agent chooses $e_2 = 1$ for any signal $\sigma > \tilde{\sigma}$ with:

$$\frac{g_S(\tilde{\sigma})}{g_F(\tilde{\sigma})} = \frac{x_F}{x_S} A(\rho). \quad (29)$$

Since by assumption $g_S(\sigma)/g_F(\sigma)$ assumes all values in $[0, +\infty)$, the threshold signal $\tilde{\sigma} \in [0, 1]$ always exists (it may be equal to 0 or 1). When $\rho = \tilde{\rho}_S$, the agent works only if he is sure that success has occurred, that is if $x_F = 0$. ■

Proof of Proposition 1.

The estimates of the agent's ability conditional on observing failure and success in the first period are respectively:

$$\hat{\theta}_F = \frac{\rho(1 - \theta_H)\theta_H + (1 - \rho)(1 - \theta_L)\theta_L}{\rho(1 - \theta_H) + (1 - \rho)(1 - \theta_L)}, \quad (30)$$

$$\hat{\theta}_S = \frac{\rho\theta_H^2 + (1 - \rho)\theta_L^2}{\rho\theta_H + (1 - \rho)\theta_L}. \quad (31)$$

Consider the task unattractiveness parameter A as a function of ρ , $A(\rho)$. Denote $r(\rho) = \hat{\theta}_F/\hat{\theta}_S$ the ratio of the agent's expected abilities after failure and after success, again considered as function of ρ .

Lemma 4 *There exists a unique ρ^* such that $A(\rho) = r(\rho)$. If $\rho \in (\tilde{\rho}_S, \rho^*)$, then $A(\rho) > r(\rho)$; if $\rho \in (\rho^*, \tilde{\rho}_F)$, then $A(\rho) < r(\rho)$.*

Proof. Equation $A(\rho) = r(\rho)$ is equivalent to

$$\begin{aligned} & ((1 - \rho)(1 - \theta_L)\phi - \rho(1 - \theta_H)) (\rho(1 - \theta_H) + (1 - \rho)(1 - \theta_L)) (\rho\theta_H^2 + (1 - \rho)\theta_L^2) \\ &= (\rho\theta_H - (1 - \rho)\theta_L\phi) (\rho(1 - \theta_H)\theta_H + (1 - \rho)(1 - \theta_L)\theta_L) (\rho\theta_H + (1 - \rho)\theta_L), \end{aligned}$$

or, dividing both sides by $(1 - \rho)^3$ and denoting $\alpha = \frac{\rho}{1 - \rho}$,

$$\begin{aligned} & ((1 - \theta_L)\phi - \alpha(1 - \theta_H)) (\alpha(1 - \theta_H) + (1 - \theta_L)) (\alpha\theta_H^2 + \theta_L^2) \\ &= (\alpha\theta_H - \theta_L\phi) (\alpha(1 - \theta_H)\theta_H + (1 - \theta_L)\theta_L) (\alpha\theta_H + \theta_L). \end{aligned} \quad (32)$$

Denote by $Q_1(\alpha)$ the LHS of this equation which is a cubic polynomial in α with three real roots $\alpha_1^L = -\frac{1 - \theta_L}{1 - \theta_H}$, $\alpha_2^L = -\frac{\theta_L^2}{\theta_H^2}$, $\alpha_3^L = \frac{1 - \theta_L}{1 - \theta_H}\phi$, it tends to $-\infty$ when $\alpha \rightarrow +\infty$ and tends to $+\infty$ when $\alpha \rightarrow -\infty$. The RHS (which we denote by $Q_2(\alpha)$) is a also cubic polynomial in α with three real roots $\alpha_1^R = -\left(\frac{1 - \theta_L}{1 - \theta_H}\right)\left(\frac{\theta_L}{\theta_H}\right)$, $\alpha_2^R = -\frac{\theta_L}{\theta_H}$, $\alpha_3^R = \frac{\theta_L}{\theta_H}\phi$; it tends to $+\infty$ when $\alpha \rightarrow +\infty$ and tends to $-\infty$ when $\alpha \rightarrow -\infty$. Looking at the order of the roots ($\alpha_1^L < \alpha_1^R < \alpha_2^R < \alpha_2^L < \alpha_3^R < \alpha_3^L$), it can be easily seen that $Q_1(\alpha) - Q_2(\alpha)$

changes sign on each of the intervals (α_1^L, α_1^R) , (α_2^R, α_2^L) and (α_3^R, α_3^L) , and thus equation (32) has a solution on each of the intervals (in particular, we have proven the existence of ρ^* because $\alpha_3^R = \frac{\tilde{\rho}_S}{1-\tilde{\rho}_S}$ and $\alpha_3^L = \frac{\tilde{\rho}_F}{1-\tilde{\rho}_F}$). Since (32), as a cubic polynomial in α has at most three real roots, we have also proven the uniqueness. ■

It only remains to prove now the (generic) uniqueness of the continuation equilibrium. To prove it, we first get some intermediate results.

Lemma 5 *If bonus \tilde{b} is given in equilibrium with probability $\tilde{x}_S > 0$ after success and with $\tilde{x}_F > 0$ after failure, and $\tilde{\sigma}$ is the agent's reaction to the bonus (i.e. the agent works if and only if he received a signal above $\tilde{\sigma}$), then $l(\tilde{\sigma}) = \frac{\tilde{x}_F}{\tilde{x}_S}A$ and either*

- $\tilde{b} > 0$ and $l(\tilde{\sigma}) = \hat{\theta}_F/\hat{\theta}_S$ or
- $\tilde{b} = 0$ and $l(\tilde{\sigma}) \leq \hat{\theta}_F/\hat{\theta}_S$.

Proof. The agent's optimal reaction $\tilde{\sigma}$ to the principal's policy is determined by $l(\tilde{\sigma}) = \frac{\tilde{x}_F}{\tilde{x}_S}A$ unless the agent would find it worthwhile to always work when offered \tilde{b} , i.e. $\tilde{\sigma} = 0$. In the latter case we should have $l(0) \geq \frac{\tilde{x}_F}{\tilde{x}_S}A$ in contradiction with Assumption 1 which states $l(0) = 0$.

When $\tilde{b} > 0$, for the principal not to be able (and *a fortiori* willing) to signal that the agent has succeeded in period 1 by deviating to $\tilde{b} \pm \varepsilon$ for a small $\varepsilon > 0$, it must be the case that $l(\tilde{\sigma}) = r(\rho)$ (see the analysis of the pooling equilibrium in section 3.1.1). When $\tilde{b} = 0$, only deviations to $\tilde{b} + \varepsilon$ are relevant so the requirement reduces to $l(\tilde{\sigma}) \leq \hat{\theta}_F/\hat{\theta}_S$. ■

Lemma 6 *In equilibrium only one bonus is offered after success.*

Proof. Assume that b_1 and $b_2 > b_1$ are offered after success with positive probability, and $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ are the corresponding agent's reactions ($\tilde{\sigma}_1 > \tilde{\sigma}_2$ by Lemma 3). The smaller bonus, b_1 , must be offered after failure with a positive probability (otherwise the agent would always work after b_1 and the principal would never give the larger one, b_2). For the principal not to be willing to separate the successful outcome by offering $b_1 + \varepsilon$, it must be that $l(\tilde{\sigma}_1) \leq \hat{\theta}_F/\hat{\theta}_S$. Then,

$$b_2 - b_1 = \left[\hat{\theta}_S(G_S(\tilde{\sigma}_1) - G_S(\tilde{\sigma}_2)) \right] W < \left[\hat{\theta}_F(G_F(\tilde{\sigma}_1) - G_F(\tilde{\sigma}_2)) \right] W. \quad (33)$$

The equality in (33) comes from the principal's indifference between b_1 and b_2 , and the inequality follows from $l(\tilde{\sigma}_1) \leq \hat{\theta}_F/\hat{\theta}_S$ and MLRP¹⁸ and implies that the principal strictly prefers to give b_2 rather than b_1 after a failure – a contradiction. ■

Corollary 2 *At most two different bonuses are offered with positive probability in equilibrium. There are three potential types of equilibrium:*

- A. *pooling* – the same bonus offered to both types;
- B. *semi-separating* – the principal always gives \tilde{b}_S after success and randomizes between \tilde{b}_S and $\tilde{b}_F \neq \tilde{b}_S$ after failure;
- C. *separating* – the principal always gives \tilde{b}_S after success and $\tilde{b}_F \neq \tilde{b}_S$ after failure.

Lemma 7 *There are no separating continuation equilibria.*

Proof. In a separating equilibrium the principal gives $\tilde{b}_F = 0$ after a failure (there is no sense to incur any cost to send a negative signal) and $\tilde{b}_S > 0$ after success. The agent always works after \tilde{b}_S and never works after \tilde{b}_F . For this pair of bonuses to be an equilibrium, the principal should not strictly prefer to give \tilde{b}_S after failure:

$$\hat{\theta}_F W - \tilde{b}_S \leq 0. \quad (34)$$

If (34) were a strict inequality, then the principal could reduce \tilde{b}_S by a small ε so that (34) would still be satisfied and according to the NWBR criterion the agent should believe success has occurred: the set of the agent's reactions that make the principal indifferent between giving 0 and $\tilde{b}_S - \varepsilon$ after failure – the empty set – is strictly included in the set of reactions that make her indifferent between \tilde{b}_S and $\tilde{b}_S - \varepsilon$ after success. Hence, \tilde{b}_S is uniquely determined:

$$\tilde{b}_S = \hat{\theta}_F W. \quad (35)$$

It must also be the case that the principal cannot separate the success outcome by a bonus lower than \tilde{b}_S . The necessary and sufficient condition for this is

$$l(0) \geq \frac{\hat{\theta}_F}{\hat{\theta}_S}. \quad (36)$$

¹⁸MLRP implies

$$\frac{G_S(\tilde{\sigma}_1) - G_S(\tilde{\sigma}_2)}{G_F(\tilde{\sigma}_1) - G_F(\tilde{\sigma}_2)} < l(\tilde{\sigma}_1).$$

To see this, assume that the agent's reaction $\hat{\sigma}$ to an out-of-equilibrium bonus \hat{b} is such that the principal is indifferent between deviating to \hat{b} after a failure or not:

$$\hat{\theta}_F(1 - G_F(\hat{\sigma}))W - \hat{b} = 0. \quad (37)$$

Then we need the principal not to be willing to deviate after success:

$$\hat{\theta}_S(1 - G_S(\hat{\sigma}))W - \hat{b} < \hat{\theta}_S W - \tilde{b}_S, \quad (38)$$

or, using $\tilde{b}_S = \hat{\theta}_F W$

$$\hat{\theta}_S G_S(\hat{\sigma}) > \hat{\theta}_F G_F(\hat{\sigma}). \quad (39)$$

For (39) to be satisfied for all $\hat{\sigma}$, a necessary and sufficient condition is (36). Since $l(0) = 0$, this can be ruled out. ■

Thus, we see that for $\rho \in (\tilde{\rho}_S, \rho^*)$ there exists a unique continuation that is semi-separating; for $\rho \in (\rho^*, \tilde{\rho}_F)$ there exists a unique continuation equilibrium that is pooling with no bonus being paid after both outcomes. The only exception occurs when $\rho = \rho^*$, i.e. $\hat{\theta}_F/\hat{\theta}_S = A(\rho)$: in this non-generic case there is a continuum of pooling equilibria with the range of possible bonuses $[0, \tilde{b}_S]$ with $\tilde{b}_S = \hat{\theta}_F(1 - G_F(\tilde{\sigma}_S))W$. ■

Proof of Proposition 2.

Let the continuation equilibrium (satisfying NWBR) be characterized by bonus \tilde{b}_S that the principal pays after success and, with probability \tilde{x}_F , after failure, and by the agent's reaction $\tilde{\sigma}$ to this bonus. Note that both pooling equilibria (with $\tilde{x}_F = 1$) and semi-separating equilibria (with $\tilde{x}_F < 1$) fit this characterization. Equilibrium values of these parameters are given in Proposition 1 for the case $\rho \in (\tilde{\rho}_S, \tilde{\rho}_F)$; for $\rho \notin (\tilde{\rho}_S, \tilde{\rho}_F)$ bonus $\tilde{b}_S = 0$, $\tilde{x}_F = 1$ and $\tilde{\sigma}$ is irrelevant. The agent weakly prefers to work in the first period if

$$\begin{aligned} & \rho[(\theta_H V - c)(1 + \theta_H(1 - G_S(\tilde{\sigma})) + (1 - \theta_H)x_F(1 - G_F(\tilde{\sigma}))) \\ & \quad + b_S(\theta_H + x_F(1 - \theta_H))] \\ \geq & (1 - \rho)[(c - \theta_L V)(1 + \theta_L(1 - G_S(\tilde{\sigma})) + (1 - \theta_L)x_F(1 - G_F(\tilde{\sigma}))) \\ & \quad - b_S(\theta_L + x_F(1 - \theta_L))], \end{aligned} \quad (40)$$

and is indifferent between working and shirking if (40) is satisfied as an equality. Indeed, the left-hand side gives the expected gain from working for the high ability agent whereas the right-hand side gives the loss from working for the low ability one.

If $\rho > \rho^*$, the unique continuation equilibrium satisfying NWBR is pooling with no bonus ever offered: $\tilde{b}_S = 0, \tilde{x}_F = 1$. The agent's reaction $\tilde{\sigma}$ is characterized by $l(\tilde{\sigma}) = A$ when $\rho \in (\rho^*, \tilde{\rho}_F)$ and $\tilde{\sigma} = 0$ for $\rho \in [\tilde{\rho}_F, 1]$.

Let $\bar{\rho}$ be such that

$$\frac{\bar{\rho}}{1 - \bar{\rho}} = \phi,$$

or, equivalently, $A(\bar{\rho}) = 1$. Then,

$$\frac{\rho}{1 - \rho} > \phi \tag{41}$$

for all $\rho \geq \bar{\rho}$. Note that $A(\rho^*) = r(\rho^*) < 1$, therefore $\rho^* > \bar{\rho}$, and equilibria for $\rho \in [\bar{\rho}, \rho^*)$ are semi-separating. Besides, one can easily check that MLRP implies that

$$\frac{1 + \theta_H(1 - G_S(\tilde{\sigma})) + (1 - \theta_H)\tilde{x}_F(1 - G_F(\tilde{\sigma}))}{1 + \theta_L(1 - G_S(\tilde{\sigma})) + (1 - \theta_L)\tilde{x}_F(1 - G_F(\tilde{\sigma}))} > 1. \tag{42}$$

From (41) and (42) it follows that (40) is satisfied for all $\rho \geq \bar{\rho}$, which proves part ii) of the Proposition¹⁹, and, also, part iv) since $\bar{\rho} < \rho^*$.

If $\rho < \tilde{\rho}_S$, the unique continuation equilibrium is pooling with no bonus offered and the agent exerting no effort in the second period. Then, (40) reduces to

$$\frac{\rho}{1 - \rho} \geq \phi$$

which is not satisfied for any $\rho < \tilde{\rho}_S$ since, as can be easily seen, $\tilde{\rho}_S < \bar{\rho}$. This proves part iii) of the Proposition.

Existence of equilibrium has been proved by construction: when (40) is satisfied as a strict inequality, there exists an equilibrium with $e_1 = 1$; if (40) is not satisfied at all, there exists an equilibrium with $e_1 = 0$; finally, if (40) is satisfied as equality, both equilibria exist: one with $e_1 = 0$ and the other with $e_1 = 1$. Also note that the equilibrium (satisfying NWBR) will be unique, unless $\rho \in \{\tilde{\rho}_S, \rho^*, \tilde{\rho}_F\}$ or (40) is satisfied as equality. Clearly, parameter values for which none of these exceptions takes place, constitute an open and everywhere dense set in the manifold of admissible parameter values, so part i) is proven. ■

Proof of Proposition 3.

¹⁹When $\rho = \rho^*$, there is a continuum of pooling continuation equilibria, but the proof goes through for each of them.

We shall first characterize the set \mathcal{R}^T . First, note that $(0, \tilde{\rho}_S) \cap \mathcal{R}^T = \emptyset$ and $[\tilde{\rho}_F, 1) \subset \mathcal{R}^T$ – in these regions the agent does not care about information about the first-period outcome. When $\rho \in [\tilde{\rho}_S, \tilde{\rho}_F)$, if the agent decides to work in the first period in the TPM regime, the optimal strategy is to continue working in the second if the first project is successful and to switch to shirking if the project fails. Therefore, for these intermediate values of ρ it is optimal to work in the first period if $\rho \geq \bar{\rho}^T$, where

$$\frac{\bar{\rho}^T}{1 - \bar{\rho}^T} = \frac{1 + \theta_L}{1 + \theta_H} \phi.$$

Thus, $[\tilde{\rho}_S, \tilde{\rho}_F) \cap \mathcal{R}^T = [\bar{\rho}^T, \tilde{\rho}_F)$. Note that $\tilde{\rho}_S < \bar{\rho}^T < \bar{\rho} < \rho^*$.

For any $\rho \in (\tilde{\rho}_S, \rho^*)$, (40) is satisfied if W is large enough (since \tilde{b}_S is a linear increasing function of W and the other terms in (40) do not depend on W), which proves part (i) of the Proposition.

To prove part (ii), assume that $\rho < \bar{\rho}^T$. Then it can be easily checked that

$$\begin{aligned} & \rho(\theta_H V - c)(1 + \theta_H(1 - G_S(\tilde{\sigma})) + (1 - \theta_H)x_F(1 - G_F(\tilde{\sigma}))) \\ < & (1 - \rho)(c - \theta_L V)(1 + \theta_L(1 - G_S(\tilde{\sigma})) + (1 - \theta_L)x_F(1 - G_F(\tilde{\sigma}))), \end{aligned}$$

which implies that (40) is not satisfied for W small enough and $\rho \notin \mathcal{R}_1$ for these values of W . ■

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