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# Illegal Immigration and Migrant Networks: Is There an Optimal Immigration Quota Policy?

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# Illegal Immigration and Migrant Networks: Is There an Optimal Immigration-Quota Policy?

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*Abstract:* High-skill immigrants form networks that facilitate subsequent low-skill immigration. To minimize the effects of low-skill immigration on unemployment, a strict lean-against-the-wind strategy or a flexible lean-against-the-wind strategy that is supported by, and takes into account the effect of, border control, are considered. None of these policies is necessarily better than the other as regards leniency toward low-skill immigration as well as the number of illegal immigrants. High-skill immigration increases illegal immigration when the effect of migrant networks on the supply of low-skill immigrants is greater than the job-creation effect of these networks.

*Key words:* Migrant networks, immigration dynamics, low-skill immigration, illegal immigration, unemployment, lean-against-the-wind policy rules.

JEL Classification Numbers: F22, J61

# I Introduction

The recent literature on immigration has emphasized the "family and friends effect" as one of the main factors explaining migration (e.g., Stark and Wang, 2002). That is, the first wave of migrants forms a network that provides information and support for prospective migrants. Lucas (1997) points out that these migrant networks are related to high-skill veteran immigrants providing support to low-skill relatives and friends to follow their steps. The support provided by earlier migrants can take different forms: economic, cultural and emotional.

Immigration policies generally take the form of quotas. Immigration quotas exist due to excess supply of immigrants. The excess supply of immigrants is the source of illegal immigration. Therefore it is natural to inquire what are, if any, the relationship among immigrant policy quotas, illegal immigration and migrant networks.

This paper deals with this issue within a framework of supply of and demand for immigrants in which there are two types of immigrants, high-skill and low-skill workers. The immigration can be either legal or illegal. In line with recent literature, migrant networks facilitate the provision of support by earlier, high-skill immigrants, to low-skill immigrants. In the spirit of Hicks (1932) we assume that high-skill immigration is triggered by positive income differential between the country of destination and the country of origin. Although high-skill immigrants may be perceived to be the "scouts" who pave the immigration path for their less-skilful relatives and friends, we assume that their immigration is not restricted due to large excess demand for their types of skills in the host country. In contrast, we assume that low-skill immigrants face immigration quotas, and the excess supply of low-skill immigrants feeds the number of illegal immigrants<sup>1</sup>.

The flow of immigrants increases the supply of workers, which affects the unemployment rate in the host country. As a consequence, the government may set an immigration quota policy aiming at minimizing a loss function that depends on the unemployment rate. We analyze two different immigration quota policies – a strict lean against the wind and a flexible lean against the wind that takes into account the moderating effect of border enforcement on illegal immigration - and investigate which is the most efficient in increasing the number of legal immigrants and/or reducing the number of illegal immigrants. We find that there is no clear reason to expect that any of the policies is more efficient than the other. The choice between the policies on these criteria critically depends on factors such as the probability of success in entering illegally the destination-country, the excess supply of low-skill immigrants, the relationship among immigration quota, border-control investments, job creation and illegal immigration.

In addition, we analyze the role of immigration networks in affecting the number of low-skill and illegal immigrants. Of key importance is the identification of the support provided by high-skill immigrants to low-skill immigrants. In particular, we assume two types of "family and friends" effects. The first increases the demand for low-skill immigrants through the arrangement and creation of jobs for the low-skilled immigrants by their relatives and friends, high-skill, earlier immigrants (see Meng, 2000). The second effect increases the supply of low-skill immigrants through positive financial transfers

<sup>&</sup>lt;sup>1</sup> As in Ethier (1986) we ignore the illegal immigration of skilled workers, since most countries are willing to admit such immigrants.

made by earlier high-skill immigrants in favor of their low-skill relatives and friends in the country of origin so as to reduce the costs of, and induce, their immigration. When the effect of the immigration networks on the supply of low-skill immigrants dominates their effect on the demand for low-skill immigrants, the current number of illegal immigrants increases with the number of earlier high-skill immigrants.

The paper is structured as follows. Section II modifies Levy's (2002) supply of and demand for immigrant by introducing high-skill and low-skill immigrants, restricting the entry of low-skill immigrants by a strict lean-against-the-wind policy, and incorporating the effectiveness of border control into the strict lean-against-the-wind policy to form a flexible lean-against-the-wind policy. Section III derives the expected-employment-lossminimizing feedback coefficients and presents the conditions for leaning against the wind of low-skill immigration. Section IV derives the maximum quota and presents the demand for low-skill immigrants under strict and flexible leaning against the wind. Section V presents the condition under which the flexible strategy is more lenient than the strict strategy. Section VI analyzes the number of illegal immigrants resulting from the excess supply of low-skill immigrants and presents the condition under which the flexible strategy leads to a smaller number of illegal immigrants than the strict strategy. This section also shows that it is possible that none of these strategies is superior to the other as regards leniency and least number of illegal immigrants simultaneously. Section VII discusses the role of migrant networks and analyzes the impact of earlier high-skill immigration on current legal and illegal low-skill immigration. Section VIII concludes.

#### II Supply and demand determinants of low-skill immigrants

We assume that the immigration of low-skill people is facilitated by capable "family-and-friends" in the host country and hence take the current supply of low-skill immigrants  $(L_t^s)$  to be positively related to the number of high-skill veteran immigrants  $(H_{t-1})$ :

$$L_t^s = R(H_{t-1}), \ R' > 0.$$
 (1)

We assume that there is an excess demand for high-skill workers and hence there are no restrictions on high-skill immigration. In contrast, and similar to Ethier (1986), we assume that the market of low-skill labor has a rigid wage, presumably at a level above the market-clearing one, generating unemployment. Thus, the admittance of low-skill immigrants might aggravate the host-country's unemployment problem<sup>2</sup>. The current change in host-country's unemployment level (*U*) is given by:

$$U_t = L_t - J_t + V_t \tag{2}$$

where  $L_t$  is the number of low-skill immigrants who legally entered the host country at t,  $J_t$  is the number of vacant jobs at t, and  $V_t$  is the number of illegal immigrants who successfully entered the host country at t.

We assume that the host-country's government is aware of the adverse effect of low-skill immigration on the domestic level of employment as presented in equation (2) and postulate that the government sets the number of low-skill immigrants to be admitted so as to minimize the expected loss from an increase in unemployment above a desired

<sup>&</sup>lt;sup>2</sup> Agiomirgianakis and Zervoyianni (2001) focus on the impact of illegal immigration on social welfare. They show that illegal immigration reduces the inflationary bias associated with expansionary policies and thus has a positive overall impact on `social welfare' in the economy.

level. For tractability, the loss function (Å) is taken to be quadratic and the desired increase in the unemployment level to be zero:

$$\ddot{\mathbf{A}} = E[U_t]^2. \tag{3}$$

In setting the expected-loss-minimizing quota of low-skill immigrants, two alternative Lean Against the Wind (LAW) rules are considered. The first is a *Strict* LAW (S-LAW, hereafter):

$$L_t = g_0 - g_1 L_{t-1} \ge 0 \tag{4}$$

where  $g_0$  is the maximum quota of low-skill immigrants per period,  $g_1$  is a feedback coefficient reflecting the intensity of the policy-maker's reaction to the number of low-skill immigrants admitted in the previous period and thereby divergence from the maximum quota.

The second is a *Flexible* LAW (F-LAW, hereafter). That is, a LAW alleviated by the moderating effect of border enforcement (B) on illegal immigration<sup>3</sup>. Here, the effectiveness of recent past border enforcement positively affects the current quota of low-skill immigrants:

$$L_{t} = G_{0} - G_{1} [L_{t-1} - B_{t-1}] \ge 0$$
(5)

where  $G_0$  indicates the maximum periodical number of immigrants admitted,  $B_{t-1}$  is the number of illegal immigrants apprehended and detained in the previous period, and  $G_1$  is a feedback coefficient.

<sup>&</sup>lt;sup>3</sup> See Karlson and Katz (2003) for a policy mix involving border control.

#### III Feedback-coefficients and the conditions for leaning against the wind

The parameters of the S-LAW and F-LAW are found by minimizing the quadratic expected loss function, which is equivalent to minimizing the stationary variance of the unemployment level. By considering equations (4) and (2)

$$g_1^* = \arg\min\{g_1^2 \operatorname{var}(L) + \operatorname{var}(J) + \operatorname{var}(V) - 2g_1[\operatorname{cov}(L_{t-1}, V_t) - \operatorname{cov}(L_{t-1}, J_t) + \operatorname{cov}(V_t, J_t)]\}$$
(6)

and by considering equations (5) and (2)

$$G_{1}^{*} = \arg\min\{G_{1}^{2}[\operatorname{var}(L) + \operatorname{var}(B)] + \operatorname{var}(J) + \operatorname{var}(V) -2G_{1}[\operatorname{cov}(L_{t-1}, V_{t}) - \operatorname{cov}(L_{t-1}, J_{t}) + \operatorname{cov}(V_{t}, J_{t}) + \operatorname{cov}(B_{t-1}, J_{t}) - \operatorname{cov}(B_{t-1}, V_{t})]\}$$
(7)

Since var(L) > 0 and var(B) > 0 the second-order condition for minimum is satisfied in both cases and the expected-loss-minimizing feedback coefficients of S-LAW and F-LAW are given, respectively, by

$$g_{1}^{*} = \frac{\operatorname{cov}(L_{t-1}, V_{t}) - \operatorname{cov}(L_{t-1}, J_{t})}{\operatorname{var}(L)}$$
(8)

$$G_{1}^{*} = \frac{\operatorname{cov}(L_{t-1}, V_{t}) - \operatorname{cov}(L_{t-1}, J_{t}) + \operatorname{cov}(B_{t-1}, J_{t}) - \operatorname{cov}(B_{t-1}, V_{t})}{\operatorname{var}(L) + \operatorname{var}(B)} .$$
(9)

Equations (8) and (9) reveal that the feasibility of both the S-LAW and the F-LAW in regulating the number of low-skill immigrants depends crucially on the existence of differences between the stationary covariances. In the case of S-LAW, the feedback-coefficient  $g_1^*$  depends on a difference between the stationary covariances of the lagged number of legal low-skill immigrants with the current numbers of vacant jobs and illegal immigrants. It is reasonable to assume that the more restrictive the immigration quota in the past, the greater the number of vacant jobs in the host country in the present, that is, negative correlation between  $L_{t-1}$  and  $J_t$ . In the same vein, the more restrictive the immigration quota in the past, the larger the illegal immigration in the present, that is, negative correlation between  $L_{t-1}$  and  $V_t$ . Leaning against the wind is feasible if  $g_1^* > 0$ , which in turn requires that  $|\operatorname{cov}(L_{t-1}, J_t)| > |\operatorname{cov}(L_{t-1}, V_t)|$  or equivalently  $|\operatorname{cor}(L_{t-1}, J_t)sd(J)| > |\operatorname{cor}(L_{t-1}, V_t)sd(V)|$ .

In the case of F-LAW, it is sensible to assume that the larger the investment in border enforcement in the past, the lower the illegal immigration in the present; that is, a negative correlation between  $B_{t-1}$  and  $V_t$ . We assume that there is no correlation between investment in border enforcement in the past  $B_{t-1}$  and the present number of vacant jobs,  $J_t$ . Again, leaning against the wind is feasible if  $G_1^* > 0$ . This, in turn, requires that  $|\operatorname{cov}(L_{t-1}, J_t) + \operatorname{cov}(B_{t-1}, V_t)| > |\operatorname{cov}(L_{t-1}, V_t)|$ .

## IV Maximum quota and the demand for low-skill immigrants

The expected-loss-minimizing maximum quota of low-skill immigrants under S-LAW ( $g_0^*$ ) and under F-LAW ( $G_0^*$ ) are found by computing the stationary expectation of U from equation (2) and setting it to be equal to the desired increase in the level of unemployment, which was assumed to be equal to zero:

$$g_0^* = g_1^* E(L) + E(J) - E(V)$$
(10)

$$G_0^* = G_1^* [E(L) - E(B)] + E(J) - E(V) .$$
(11)

Using equations (4), (8) and (10) and assuming that low-skill immigrants admitted in earlier periods cannot be dumped, the derived demand for low-skill immigrants under S-LAW is

$$L_{St}^{d} = E(J) - E(V) - \left[\frac{\operatorname{cov}(L_{t-1}, V_{t}) - \operatorname{cov}(L_{t-1}, J_{t})}{\operatorname{var}(L)}\right] [L_{t-1} - E(L)]$$
(12)

or zero when the right-hand side of equation (12) is negative. That is, if the host country adopts the S-LAW policy, its demand for low-skill immigrants is equal to the greater number between zero and the expected number of vacant jobs minus the expected number of illegal immigrants and the product of the feedback-coefficient and the deviation of the number of immigrants admitted in the previous period from the stationary number.

Similarly, equations (5), (9) and (11) imply that the derived-demand equation for low-skill immigrants under F-LAW is:

$$L_{Ft}^{d} = E(J) - E(V) - \left[\frac{\operatorname{cov}(L_{t-1}, V_{t}) - \operatorname{cov}(L_{t-1}, J_{t}) - \operatorname{cov}(B_{t-1}, V_{t})}{\operatorname{var}(L) + \operatorname{var}(B)}\right] \left[L_{t-1} - E(L) + B_{t-1} - E(B)\right]$$
(13)

or zero if the right-hand side of equation (13) is negative. When F-LAW is adopted, the demand for legal low-skill immigrants is equal to the expected number of vacant jobs minus the expected number of illegal immigrants and the product of the optimal feedback coefficient and the deviation of the number of low-skill immigrants admitted in the previous period from the stationary number and the deviation of the border-control performance in the previous period from its stationary level.

In the next two sections we investigate whether the F-LAW strategy is better than the S-LAW strategy as regards leniency toward low-skill immigration and as regards the number of illegal immigrants.

## V Is the F-LAW more lenient than the S-LAW toward low-skill immigration?

An immigration policy is said to be more lenient than another if it admits a larger number of low-skill immigrants than the other. From liberal, egalitarian and global perspectives an immigration policy reflecting a high degree of leniency is preferred to that reflecting a lower degree. In that sense, leniency may be considered as a criterion for choosing between immigration policies.

Proposition 1: If  $G_1^*[B_{t-1} - E(B)] \le [g_1^* - G_1^*][L_{t-1} - E(L)]$ , then the F-LAW is more lenient than the S-LAW.

Proposition 1 suggests that despite of its inherent moderation of the quotaadjustment of low-skill immigration by the border-control's performance in limiting illegal, low-skill immigrants, the F-LAW policy on low-skill immigration is not necessarily more lenient than the S-LAW policy. This is due to the fact that the inequality  $G_1^*[B_{t-1} - E(B)] \le [g_1^* - G_1^*][L_{t-1} - E(L)]$  depends on a variety of conditions. For instance, if  $[B_{t-1} - E(B)] > 0$  and  $[L_{t-1} - E(L)] > 0$ , then three separate inequalities should be simultaneously satisfied for the inequality indicated in the proposition to hold: i)  $[g_1^* - G_1^*] > 0$ ; ii)  $G_1^* \le [g_1^* - G_1^*]$ , and iii) $[B_{t-1} - E(B)] \le [L_{t-1} - E(L)]$ . In view of these conditions and the conditions associated in other scenarios, it is safe to say that there is no clear reason to expect the F-LAW to be more lenient than the S-LAW toward low-skill immigration.

#### VI Does the F-LAW lead to a smaller illegal immigration than the S-LAW?

Illegal immigration reflects, in the context of the proposed model, the excess supply of low-skill immigrants. The supply of low-skill immigrants is given by equation (1) and the demand for low-skill migrants varies in accordance with the destination-country's low-skill immigration policy.

When the S-LAW policy on low-skill immigration is pursued the number of illegal immigrants is given by

$$V_{St} = q \left[ R(H_{t-1}) - L_{St}^{d} \right]$$
(14)

where q is the probability of being successful in entering the destination-country, which in the absence of investment in border enforcement is high.

When the F-LAW policy on low-skill immigration is adopted, the number of illegal immigrants is given by

$$V_{Ft} = I \left[ R(H_{t-1}) - L_{Ft}^{d} \right]$$
(15)

where l is the probability of being successful in entering the destination-country. As it is harder to enter in the destination-country when the government invests in border control, l < q.

A possible criterion for a destination-country for preferring one policy to the other is the least number of illegal immigrants, who are, in the context of our model, also endowed with low skills. Proposition 2: The F-LAW policy leads to a smaller number of illegal immigrants than

the S-LAW policy if 
$$\frac{\boldsymbol{q}}{\boldsymbol{l}} > \frac{[R(H_{t-1}) - L_{St}^d]}{[R(H_{t-1}) - L_{Ft}^d]}$$

Proposition 2 states that only if the ratio of the probability of entering illegally the destination-country under the S-LAW to that under the F-LAW (which is greater than one) exceeds the ratio of the excess supply of low-skill immigrants under the S-LAW to that under the F-LAW (which by Proposition 1 may or may not be greater than one), the F-LAW policy on low-skill immigration leads to a smaller number of illegal immigrants than the S-LAW rule.

It is interesting to note that even when the condition indicated in Proposition 1 is satisfied and the number of legal low-skill immigrants under the border-enforcement augmented F-LAW rule is larger than that under the S-LAW rule, the implementation of the former, which includes investment in border enforcement, may not necessarily generates a smaller number of illegal immigrants than the implementation of the latter, which does not includes investment in border enforcement. The probabilities of being a successful illegal immigrant under either policy have to be taken into account and their ratio has to be contrasted with the ratio of excess supply of low-skill immigrants, as stated in proposition 2.

Furthermore, leniency, as defined in the previous section, may be perceived as desirable aspect of an immigration policy, at least from liberal, egalitarian and global perspectives. Yet the aforementioned sort of independency between proposition 1 and proposition 2 leads us to conclude that there is not necessarily a consensus between the

leniency-criterion and the least-number-of-illegal-immigrants criterion in favor of the F-LAW rule or the S-LAW rule. Hence, we cannot claim that any of the investigated lowskill-immigration-quota policy is conclusively superior to the other as regards these two criterions simultaneously. The choice between the policies depends on factors such as the probability of success in entering illegally the host country, the excess supply of low-skill immigrants, and the relationship among immigration quota, border control investments, job creation and illegal immigration.

#### VII The role of migrant networks

A related important issue is to identify the role of migrant networks in low- skill migration. It was argued in the introduction that high-skill immigrants play the role of earlier immigrants and form networks that support low-skill workers to immigrate. Here Two types of support are examined in this section. The first one is the direct help in covering migration costs. This type of help increases the supply of low skill immigrants as stated in equation (1).

The second type of support is job creation. We assume that the number of jobs created for current low-skill immigrants increase with the number of earlier immigration of high skill workers. That is,  $J_t = J(H_{t-1}), J' > 0$ . In terms of the demand for current low-skill immigrants under any of the LAW rules, depicted by equations (12) and (13), the expected value of the job created increases with the number of early high-skill migrants: namely,  $\frac{dE[J(H_{t-1})]}{dH_{t-1}} > 0$ . Therefore, the impact of recent-past immigration of high skill workers on the current number of legal low-skill immigrants is positive and the same under each of the LAW immigration rules:

$$\frac{dL_{S_{t}}^{d}}{dH_{t-1}} = \frac{dL_{F_{t}}^{d}}{dH_{t-1}} = \frac{dE[J(H_{t-1})]}{dH_{t-1}} > 0$$
(16)

Equation (16) states that the demand for legal low-skill immigrants increases with the size of the past-period number of high-skill immigrants due to the effect of immigration networks in job creation.

It is also possible to assess the impact of high-skill immigration on the number of illegal immigrants. When the government implements the S-LAW low-skill immigration

policy, 
$$\frac{dV_{St}}{dH_{t-1}} = q \left[ \frac{dR(H_{t-1})}{dH_{t-1}} - \frac{dE[J(H_{t-1})]}{dH_{t-1}} \right] > 0$$
 if  $\frac{dR(H_{t-1})}{dH_{t-1}} > \frac{dE[J(H_{t-1})]}{dH_{t-1}}$ . That is,

the current number of illegal immigrants rises with the past-period number of high-skill immigrants if the impact of the migrant networks on the supply of low-skill immigrants is greater than their effect on job creation. When the border-enforcement augmented F-LAW low-skill immigration policy is adopted,  $\frac{dV_{Ft}}{dH_{t-1}} = I \left[ \frac{dR(H_{t-1})}{dH_{t-1}} - \frac{dE[J(H_{t-1})]}{dH_{t-1}} \right] > 0 \text{ if } \frac{dR(H_{t-1})}{dH_{t-1}} > \frac{dE[J(H_{t-1})]}{dH_{t-1}} \text{ and, as in the}$ 

case of the S-LAW, the current number of illegal immigrants rises with the past-period number of high-skill immigrants if the impact of the migrant networks on the supply of low-skill immigrants is greater than their effect on job creation. Thus, it is essential to assess the impact of migrant networks on the supply of and the demand for low-skill immigrants for explaining the impact of high-skill immigration on illegal immigration. When the effect of migrant networks in pushing the supply of immigrants, through direct financial help to prospective low-skill immigrants, is greater than their effect on the demand for low-skill immigrants through job creation, the number of illegal immigrants is bound to rise.

### VIII Concluding Remarks

This paper analyzes the relationship among immigration policy quotas, illegal immigrants and migrant networks. Immigrants are classified as high-skill and low-skill workers. High-skill immigration precedes and facilitates low-skill immigration. Migrant networks are related to the support given by high-skill immigrants to low-skill immigrants. High-skill immigration is free of barriers on entry, whereas low-skill immigration is subjected to quotas. The excess supply of low-skill immigrants generates illegal immigration.

The host country's government sets an immigration quota policy in order to minimize the effects of low-skill immigration on unemployment. Two policies were examined: a strict lean against the wind and a flexible one, which incorporates the government's investment in border control. Two criterions for identifying the best policy were considered. Under the first criterion, an immigration policy is said to be better than the other if it admits a greater number of low-skill immigrants, whereas under the second criterion, if it leads to a smaller number of illegal immigrants. It was found that these criteria do not necessarily agree that any of the lean-against-the-wind immigration policy is better than the other. The choice between the policies depends on factors such as the probability of success in entering illegally the host country, the excess supply of low-skill immigrants, and the relationship among immigration quota, border control investments, job creation and illegal immigration.

In addition, we analyze the impact of high-skill immigration on the number of legal and illegal low-skill immigrants. Due to the contribution of migrant networks to job creation the demand for legal low-skill immigrants increases when the past period number of high-skill immigrants rises. We find that when the effect of migration networks in expanding the supply of immigrants, through direct financial help to prospective low-skill immigrants, dominates the job-creation effect, the current number of illegal immigrants increased by past high-skill immigration.

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# APPENDIX

 $\begin{aligned} &Proof \ of \ Proposition \ l: \ G_{1}^{*}[ \ B_{t-1} - E(B)] \leq [g_{1}^{*} - G_{1}^{*}][L_{t-1} - E(L)] \Rightarrow \\ & [G_{1}^{*} - g_{1}^{*}][L_{t-1} - E(L)] + G_{1}^{*}[ \ B_{t-1} - E(B)] \leq 0 \Rightarrow \\ & -G_{1}^{*}[L_{t-1} - E(L) + B_{t-1} - E(B)] \geq -g_{1}^{*}[L_{t-1} - E(L)] \Rightarrow L_{Ft}^{d} \geq L_{St}^{d}. \ QED \\ & Proof \ of \ Proposition \ 2: \ \text{Recalling equations} \ (14) \ \text{and} \ (15), \\ & \frac{q}{l} > \frac{[R(H_{t-1}) - L_{St}^{d}]}{[R(H_{t-1}) - L_{Ft}^{d}]} \Rightarrow q[R(H_{t-1}) - L_{St}^{d}] > I[R(H_{t-1}) - L_{Ft}^{d}] \Rightarrow V_{St} > V_{Ft}. \ QED \end{aligned}$