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ABSTRACT

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We develop an urban-search model in which firms post wages. When all workers are identical, the Diamond paradox holds, i.e. there is a unique wage in equilibrium even in the presence of search and spatial frictions. This wage is affected by spatial and labor costs. When workers differ according to the value imputed to leisure, we show that, under some conditions, two wages emerge in equilibrium. The commuting cost affects the land market but also the labor market through wages. Workers' productivity also affects housing prices and this impact can be positive or negative depending on the location in the city. One important aspect of our model is that, even with positive search costs, wage dispersion prevails in equilibrium, a feature not possible in the non-spatial model.

JEL Classification: D83, J64, R14

Keywords: diamond paradox, urban land-use, spatial compensation, search frictions, wage dispersion

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1 Introduction

It is widely documented that unemployment varies between the regions of a country (Isserman et al., 1986, Gordon, 1987, Blanchflower and Oswald, 1994), between cities of different sizes and functions (Marston, 1985), between the inner and outer areas of cities and between the urban and rural areas. There are also stark spatial differences in incomes. For example, in the United States, the median income of central city residents is 40 percent lower than that of suburban residents. Despite these features, very few theoretical attempts have been made to better understand the working of the urban labor market and, in particular, urban unemployment and spatial wage dispersion. Indeed, labor economists and macroeconomists traditionally do not incorporate space directly into their studies (see e.g. Layard et al., 1991; Pissarides, 2000; Cahuc and Zylberberg, 2004), even though there are some well-known empirical studies of local labor markets (see e.g. Holzer, 1989; Eberts and Stone, 1992). Similarly, in urban economics, despite numerous empirical studies, the theory of urban labor economics has been relatively neglected. In most advanced urban textbooks (see, in particular, Fujita, 1989; Fujita et al. 1999; Fujita and Thisse, 2002) it is mainly assumed throughout perfect competition in the labor market and the issue of urban unemployment is not even discussed.

It seems, in particular, quite natural to introduce space in a search-matching model (Mortensen and Pissarides, 1999, Pissarides, 2000) because distance interacts with the diffusion of information. In his seminal contribution to search, Stigler (1961) puts geographical dispersion as one of the four immediate determinants of price ignorance. In most search models, say for example Diamond (1982), distance between agents or units implies a fixed cost of making another draw in the distribution. In other words, a spatial dispersion of agents creates more search frictions.

There is by now a small literature on urban search models (Zenou, 2008). In all these models (Simpson, 1992; Coulson et al., 2001; Sato, 2001; 2004; Wasmer and Zenou, 2002; 2006; Smith and Zenou, 2003), the wage is determined by a bilateral bargaining between the firm and the worker so that all workers are paid the same wage and no spatial wage distribution emerges in equilibrium. There is however an important literature in search (Mortensen, 2003) focussing on wage dispersion where firms post wages instead of bargaining them with workers. The starting point is the Diamond paradox (Diamond, 1971), which says that, when all workers are identical, then, even in the presence of search frictions, the only equilibrium is for all firms to post the reservation wage of workers. In order to obtain a wage dispersion and to avoid the Diamond paradox, researchers have introduced multiple job offers (Burdett and Judd, 1983), workers' heterogeneity (Albrecht and Axell, 1984), and on-the-job search (Burdett and Mortensen, 1998).

The aim of this paper is to develop an urban-search model in which firms post wages and derive the implications in the land and labor markets. To the best of our knowledge, this is the first paper that does so.

To be more precise, we first consider a model where all workers are homogenous and locate in a monocentric city. We characterize the steady-state equilibrium, which requires solving simultaneously an urban land use equilibrium and a labor market equilibrium. We show that the Diamond paradox holds. We also show that higher unemployment rate increases the employed workers' utility but decreases the equilibrium housing price in the employment area. We then extend this model by considering two types of workers who differ according to the value imputed to leisure. We show that, under some conditions, there is a spatial wage dispersion so that the Diamond paradox does not hold anymore. We show that the commuting cost affects the land market but also the labor market through wages. We also find that workers' productivity affects housing prices and that this impact can be positive or negative depending on the location in the city. One important aspect of our model is that, even with positive search costs, wage dispersion prevails in equilibrium, a feature not possible in the non-spatial model.

2 Ex ante identical workers

There is a continuum of ex ante identical workers whose mass is N and a continuum of ex ante identical firms whose mass is 1. Among the N workers, there are L employed and U unemployed so that $N = L + U$. The workers are uniformly distributed along a *linear*, *closed* and *monocentric* city. Their density at each location is taken to be 1. There is no vacant land in the city and all land is owned by absentee landlords. All firms are exogenously located in the Business District (BD hereafter) and consume no space. The BD is a unique employment center located at one end of the linear city. In a centralized city, it corresponds to the Central Business District (CBD), whereas, in a completely decentralized city, it represents the Suburban Business District (SBD). Workers are assumed to be infinitely lived, risk neutral and decide their optimal place of residence between the BD and the city fringe.

There is no on-the-job search and thus only the unemployed workers search for a job and receive information about job openings. We denote by a_U the offer arrival rate faced by an unemployed worker.[1] Workers respond to offers as soon as they arrive. There is no recall. Jobs are destroyed at exogenous rate δ. It is assumed that there exists a wage cumulative distribution function $F(w_L)$ that is known by everybody, i.e. workers know $F(w_L)$ but do not know which firm offers which wage. The support of $F(w_L)$ is $[0, \overline{w}_L]$, where $\overline{w}_L$ is very large.

A steady-state equilibrium requires solving *simultaneously* an urban land use equilibrium and a labor market equilibrium. It is convenient to present first the former and then the latter.

2.1 Urban land-use equilibrium

Each individual is identified with one unit of labor. Each employed worker goes to the BD to work and incurs a fixed monetary commuting cost τ per unit of distance. When living at a distance x from the BD, he/she also pays a land rent $R(x)$, consumes $h_L = 1$ unity of land and earns a wage w_L (that will be determined at the labor market equilibrium). The *instantaneous* (indirect) utility of an employed worker located at a distance x from the BD is equal to:

$$W_L(x) = w_L - \tau\, x - R(x) \tag{1}$$

and the bid rent is:[2]

$$\Psi_L(x, W_L) = w_L - \tau\, x - W_L \tag{2}$$

where W_L is the common utility level obtained by all employed workers in the city. Concerning the unemployed, they commute less often to the BD since they mainly go there to search for jobs. So, we assume that they incur a commuting cost $s\,\tau$ per unit of distance,

[1] The subscripts U and L stand for "unemployed" and "employed" respectively.

[2] The bid rent is a standard concept in urban economics. It indicates the maximum land rent that a worker located at a distance x from the BD is ready to pay in order to achieve a utility level (Fujita, 1989).

where $0 < s \leq 1$ is a measure of search intensity or search efficiency; s is assumed to be exogenous. For example $s = 1$ would mean that the unemployed workers go everyday to the BD (as often as the employed workers) to search for jobs. Thus, here, the cost of searching is captured through the increase in commuting costs since higher s implies higher commuting costs $s\,\tau\,x$. This is mainly because it is assumed that information about jobs is only gathered in the employment center (BD).[3] Unemployed workers consume $h_U = 1$ unity of land and thus their *instantaneous* (indirect) utility when residing at a distance x from the BD is given by:

$$W_U(x) = w_U - s\,\tau\,x - R(x) \tag{3}$$

where w_U indicates the unemployment insurance payment. The bid rent is thus given by:

$$\Psi_U(x, W_U) = w_U - s\tau\,x - W_U \tag{4}$$

where W_U is the common utility level obtained by all unemployed workers in the city. Because the bid rent of the employed workers is steeper than that of the unemployed workers, the former live close to jobs while the latter reside farther away. This pattern can capture both the European and American situations. Indeed, if the BD is interpreted as the Central Business District, then we have the European structure where the rich/employed workers live in the city-center and the poor/unemployed at the outskirts of the city. If the BD is the Suburban Business District, then the rich/employed workers live at the periphery while the poor reside in the city-center. What is important here is that in both situations the rich live close to jobs, which is the case in Paris and London and in New York or Los Angeles (Brueckner et al., 1999; Glaeser et al., 2008).

Definition 1 *An urban-land use equilibrium with ex ante identical workers is a 3-tuple* $(W_L^*, W_U^*, R^*(x))$ *such that:*

$$\Psi_U(N, W_U^*) = R_A = 0 \tag{5}$$

$$\Psi_U(L, W_U^*) = \Psi_L(L, W_L^*) \tag{6}$$

$$R^*(x) = \max\{\Psi_U(x, W_U^*), \Psi_L(x, W_L^*), 0\} \quad \textit{at each } x \in (0, N] \tag{7}$$

[3] We could also have introduced other search costs that are not-distance related. This would have complicated the model without altering any of our results.

Equations (5), (6) and (7) reflect the equilibrium conditions in the land market. Equation (5) says that, at the city fringe N, the bid rent of the unemployed workers must be equal to the agricultural land rent R_A, which is normalized to zero without loss of generality. Equation (6) states that, at L, the border between the employed and unemployed workers, the bid rent offered by the employed is equal to the bid rent offered by the unemployed workers. These two equations guarantee that the equilibrium land rent is everywhere continuous in the city. Finally, equation (7) defines the equilibrium land rent as the upper envelope of the equilibrium bid rent curves of all workers and the agricultural rent line. Observe that since all N workers consume 1 unit of housing each, and since there will be no vacant land inside the city, the distance from the BD to the urban fringe must be given by N and the border by L. As a result, the employed reside between 0 and L whereas the unemployed reside between L and N. Solving these equations leads to:

$$W_U^* = w_U - s\tau\, N \tag{8}$$

$$\begin{aligned} W_L^* &= w_L - (1-s)\,\tau\, L - s\tau\, N \\ &= w_L - (1-s)\,\tau\, N\,(1-u) - s\tau\, N \end{aligned} \tag{9}$$

$$R^*(x) = \begin{cases} \tau\,(sN - x) + (1-s)\,\tau N\,(1-u) & \text{for} \quad 0 \leq x \leq L \\ s\tau\,(N-x) & \text{for} \quad L < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{10}$$

Observe that the labor market affects the land market through both the unemployment rate and the wage. In particular, higher wages increases workers' utility while higher unemployment rate increases the employed workers' utility but decreases the equilibrium housing price in the employment area. Indeed, when u increases, $L = (1-u)\,N$, which is both the size of the employment area and the employment level, decreases. As a result, on average, the employed workers are closer to jobs and thus spend less in commuting costs, which increases their utility. This, in turn, decreases their bid rent (see (2)) and thus the housing price within the employment area also decreases at each x.

2.2 Labor-market equilibrium

We can now solve the labor-market equilibrium. We follow here the wage posting literature (Mortensen, 2000, 2003) where the total mass of firms is fixed to 1, so that there is no a freeentry condition and thus no endogenous job creation. Also, the contact rates for both firms and workers are exogenous and not determined using a matching function (as in Pissarides, 2000). Of course, as shown by Mortensen (2000) and Gaumont et al. (2006), including these two aspects in a wage posting model is straightforward and does not generally change the results.

Employed workers The Bellman equation for the employed workers is given by:

$$rI_L(w_L) = w_L - (1-s)\,\tau\,N\,(1-u) - s\tau\,N - \delta\,[I_L(w_L) - I_U] \tag{11}$$

where r is the discount factor. Indeed, employed workers obtain today $W_L^* = w_L - (1-s)\,\tau\,N\,(1-u) - s\tau\,N$, but can lose their job at rate δ, and then obtain a negative surplus of $-\,[I_L(w_L) - I_U]$. Equation (11) implies that:

$$I_L(w_L) - I_U = \frac{w_L - (1-s)\,\tau\,N\,(1-u) - s\tau\,N - rI_U}{r+\delta} \tag{12}$$

There is thus a reservation wage w_L^r, i.e. the wage below which unemployed workers refuse to accept a job offer, which is defined as follows:

$$I_L(w_L^r) - I_U = 0 \Leftrightarrow w_L^r = rI_U + (1-s)\,\tau\,N\,(1-u) + s\tau\,N \tag{13}$$

Unemployed workers The Bellman equation for the unemployed workers is given by:

$$rI_U = w_U - s\tau\,N + s\,a_U \int_{w_L^r}^{+\infty} [I_L(w_L) - I_U]\,dF(w_L) \tag{14}$$

where a_U is the exogenous job acquisition rate. Indeed, unemployed workers obtain today $W_L^* = w_U - s\tau\,N$, but can have a contact with a firm at rate $s\,a_U$, and transform this contact into a match if the offer is greater or equal than the reservation wage w_L^r. In that case, they obtain a positive surplus of $I_L(w_L) - I_U$. As stated above, there is a cost of searching s, which is captured by the total commuting costs $s\tau\,N$, and a reward since higher job search increases the contact rate $s\,a_U$ with a firm. Using (14), the wage reservation rule (13) can be written as:

$$w_L^r = w_U + (1-s)\,\tau\,N\,(1-u) + s\,a_U \int_{w_L^r}^{+\infty} [I_L(w_L) - I_U]\,dF(w_L)$$

which, using (12), is equivalent to:

$$w_L^r = w_U + (1-s)\,\tau\,N\,(1-u) + \frac{s\,a_U}{r+\delta}\int_{w_L^r}^{+\infty}\left[w_L - (1-s)\,\tau\,N\,(1-u) - s\tau\,N - rI_U\right]dF(w_L)$$

Using (13), we finally obtain:

$$w_L^r = w_U + (1-s)\,\tau\,N\,(1-u) + \frac{s\,a_U}{r+\delta}\int_{w_L^r}^{+\infty}(w_L - w_L^r)\,dF(w_L) \tag{15}$$

Unemployment rate The dynamics of the unemployment level is equal to:

$$\frac{d\,[u(t)N]}{dt} = \delta\,[1-u(t)]\,N - s\,a_U\,u(t)\,[1-F(w_L^r)]\;N$$

where $u(t)$ is the unemployment rate at time t. Indeed, at each time t, $[1-u(t)]\,N$ employed workers lose their jobs at rate δ while $u(t)\,N$ unemployed workers find a job at rate $s\,a_U\,u(t)\,[1-F(w_L^r)]$, which is the product of the contact rate $s\,a_U\,u(t)$ and the acceptation rate (workers only accept job offers with wages at least equal to their reservation wage w_L^r). In steady-state, $\frac{d[u(t)N]}{dt} = 0$ and thus, the unemployment rate u^*, is given by:

$$u^* = \frac{\delta}{\delta + s\,a_U\;[1-F(w_L^r)]} \tag{16}$$

Employment size in a firm Denote by $l(w_L)$ the employment level of a firm that offers a wage w_L to its employees. Denote also by $G(w_L)$ the proportion of employed workers in the economy receiving a wage no greater than w_L. The dynamics of $G(w_L)$ is given by

$$\frac{d\,[G(w_L,t)(1-u(t))N]}{dt} = s\,a_U\,[F(w_L) - F(w_L^r)]\,u(t)\,N - \delta G(w_L,t)\,[1-u(t)]\,N$$

where $d\,[G(w_L,t)(1-u(t))N]\,/dt$ is the variation of employed workers receiving a wage no greater than w_L, $s\,a_U\,[F(w_L) - F(w_L^r)]\,u(t)\,N$ is the flow at time t of unemployed workers into firms offering a wage no greater than w_L, $\delta G(w_L,t)\,[1-u(t)]\,N$ is the flow at time t of employed workers out of firms offering a wage no greater than w_L. In steady-state, $d\,[G(w_L,t)(1-u(t))N]\,/dt = 0$, and, using (16), we easily obtain the following steady-state value:

$$G(w_L) = \frac{F(w_L) - F(w_L^r)}{1 - F(w_L^r)} \tag{17}$$

We can now determine the employment size in a firm. The employment size $l(w_L)$ (the measure of workers) per firm earning a wage w_L can be expressed as

$$l(w_L) = \lim_{\varepsilon \to 0} \frac{G(w_L) - G(w_L - \varepsilon)}{F(w_L) - F(w_L - \varepsilon)}(1-u)N \tag{18}$$

where $[G(w_L) - G(w_L - \varepsilon)]\,(1-u)\,N$ represents the steady-state number of workers earning a wage in the interval $[w_L - \varepsilon, w_L]$ and $F(w_L) - F(w_L - \varepsilon)$ is the measure of firms offering a wage in the interval $[w_L - \varepsilon, w_L]$.

Lemma 1 *Equation (18) is equivalent to*

$$l(w_L) = \frac{s\, a_U\, N}{\delta + s\, a_U \left[1 - F(w_L^r)\right]} \tag{19}$$

Proof. See the Appendix.

Equation (19) specifies the steady-state number of workers available to a firm offering any particular wage, conditional on the wages offered by other firms, represented by the distribution $F(.)$, and the workers' reservation wage w_L^r. Thus, we have shown that, in steady-state:

$$l(w_L) = \begin{cases} \frac{s\, a_U\, N}{\delta + s\, a_U \left[1 - F(w_L^r)\right]} & \text{iff} \quad w_L \geq w_L^r \\ 0 & \text{iff} \quad w_L < w_L^r \end{cases} \tag{20}$$

Wage posting Firms post wages. As in Burdett and Mortensen (1998), firms are interested in maximizing steady-state profit, and will hire as many workers as are willing to accept. The profit of a firm that sets a wage w_L is given by:

$$\Pi = \max_{w_L} (y - w_L)\, l(w_L)$$

where y is the productivity of a worker. We have:

$$\Pi = \begin{cases} \max_{w_L} \left[\frac{s\, a_U\, N (y - w_L)}{\delta + s\, a_U \left[1 - F(w_L^r)\right]} \right] & \text{iff} \quad w_L \geq w_L^r \\ 0 & \text{iff} \quad w_L < w_L^r \end{cases} \tag{21}$$

Proposition 1 (Diamond's Paradox) *At the Nash equilibrium, all firms set the following wage:*

$$w_L^* = w_L^r$$

and thus $F(w_L)$ is degenerated to one point $w_L^* = w_L^r$.

Proof. See the Appendix.

This means that $w_L^* = w_L^r$ is a mass point and the wage distribution is degenerated to one point $w_L^* = w_L^r$. This result is due to the fact that $l(w_L)$ is independent of w_L. This is the so-called Diamond's paradox (Diamond, 1971).

2.3 Steady-state equilibrium

In equilibrium, since all firms set $w_L^* = w_L^r$, we have:

$$\int_{w_L^r}^{+\infty} (w_L - w_L^r)\, dF(w_L) = 0$$

and thus the reservation rule (15) is equivalent to:

$$w_L^* = w_L^r = w_U + (1-s)\,\tau\, N\,(1-u) \tag{22}$$

By using (16), we have:

$$w_L^* = w_L^r = w_U + (1-s)\,\tau\, N \left(1 - \frac{\delta}{\delta + s\, a_U\, [1 - F(w_L^r)]}\right)$$

and since $1 - F(w_L^r) = 1$, we finally obtain:

$$w_L^* = w_L^r = w_U + (1-s)\,\tau\, N \left(\frac{s\, a_U}{\delta + s\, a_U}\right) \tag{23}$$

This is the equilibrium wage obtained by all workers. The unemployment benefit w_U, is the only labor-market part of the wage. It increases with w_L^* because rI_U increases and thus workers are more demanding and increase their reservation wage. This is what is obtained in the non-spatial search model. The spatial part of the wage, $(1-s)\,\tau\, N \left(\frac{s\, a_U}{\delta + s\, a_U}\right)$, is what firms must give to workers to compensate for the spatial cost difference between employed and unemployed workers. This spatial compensation is calculated at $x = L = N \left(\frac{s\, a_U}{\delta + s\, a_U}\right)$, i.e. when the land rent of employed and unemployed workers is the same. In particular, if a_U increases or δ decreases, then wages increase because the spatial cost difference between employed and unemployed workers increases since employed workers are on average further away from jobs. Observe that

$$\frac{\partial w_L^*}{\partial s} \gtreqless 0 \Leftrightarrow s \lesseqgtr \frac{1}{2}$$

Indeed, there are two opposite effects of an increase of s on the wage w_L^*. On the one hand, increasing s reduces the spatial compensation since the spatial cost difference between employed and unemployed workers is smaller. On the other hand, it increases the chance to obtain a job and thus the employment rate, which, in turn, increases the distance to jobs for the employed worker located at $x = L$. This raises the spatial compensation and thus the wage.

For the model to make sense, we assume that $y > w_L^*$ so that firms do not make negative profits. This is equivalent to:

$$y - w_U > (1-s)\,\tau\,N\left(\frac{s\,a_U}{\delta + s\,a_U}\right) \tag{24}$$

Proposition 2 *Assume (24). Then, there is a unique steady-state equilibrium* $(w_L^{r*}, w_L^*,\ F^*(w_L), \Pi^*, u^*, W_U^*, W_L^*, R^*(x))$, *where* $w_L^{r*} = w_L^*$ *is defined by (23),* $F^*(w_L)$ *is degenerated to one point* w_L^*,

$$\Pi^* = \frac{s\,a_U\,N}{\delta + s\,a_U}\left[y - w_U - (1-s)\,\tau\,N\left(\frac{s\,a_U}{\delta + s\,a_U}\right)\right] \tag{25}$$

$$u^* = \frac{\delta}{\delta + s\,a_U} \tag{26}$$

$$W_U^* = w_U - s\tau\,N = W_L^* \tag{27}$$

and

$$R^*(x) = \begin{cases} \tau\,(sN - x) + (1-s)\,\tau N\left(\frac{s\,a_U}{\delta + s\,a_U}\right) & \text{for} \quad 0 \le x \le L \\ s\tau\,(N - x) & \text{for} \quad L < x \le N \\ 0 & \text{for} \quad x > N \end{cases} \tag{28}$$

Observe that all workers participate to the labor market because all workers search for a job and all accept a job if offered. Also, not surprisingly, $W_U^* = W_L^*$ and $I_L^* = I_U^*$.

2.4 Interaction between land and labor markets

Let us derive some comparative statics results. First, by differentiating (25), we have:

$$\Pi^* = \Pi\left(\underset{+}{y}, \underset{-}{w_U}, \underset{+}{a_U}, \underset{-}{\delta}, \underset{?}{s}, \underset{-}{\tau}, \underset{-}{N}\right)$$

Not surprisingly, when y, the productivity of workers, increases, firms' profits increase. The effects of w_U, s, τ and N only go through the wage w_L^* and thus when they increase w_L^*, firms' profits are reduced. The ambiguity of s stems from the ambiguity of the effect of s on w_L^* mentioned above. On the other hand, a_U and δ affect both the employment in the firm $l(w_L)$ and the wage w_L^*. As a result, when a_U increases or δ decreases, then both the employment $l(w_L)$ and the wage w_L^* increase, and thus the effect on profits is ambiguous. However, if the productivity y is high enough, then the first effect dominates the second one, and the net impact is positive.

Second, by differentiating the equilibrium land rent (28), for the employed workers, i.e. for $x \in [0, L]$, we obtain:

$$R_L^* = R\left(\underset{-}{x}, \underset{+}{a_U}, \underset{-}{\delta}, \underset{+}{s}, \underset{+}{\tau}, \underset{+}{N}\right)$$

These results are mainly due to effects on the competition on the land market. Indeed, when a_U increases or δ decreases, then the employment level $N(1-u^*)$ in the economy increases, which means that employed workers are on average further away from jobs. The access to the job center becomes more valuable, which increases the competition in the land market since employed workers bear higher commuting costs than the unemployed workers. As a result, housing prices increase everywhere in the city between $x = 0$ and $x = L$ but not in the unemployment area, i.e. when $x \in]L, N]$. Figure 1 illustrates this effect. Before the shock (i.e. increase in a_U or decrease in δ), the land rent is given by the normal line while after, it is described by the thick line. The equilibrium values with one and two stars correspond respectively to before and after the shock. Finally, an increase in τ, s or N, increases the competition in the land market because it becomes more costly to travel to the job center and therefore housing prices increase.

[*Insert Figure* 1 *here*]

3 Ex ante heterogenous workers

Following Albrecht and Axell (1984) and Gaumont et al. (2006), we now assume that there are two types of individuals in the economy who differ according to the value imputed to leisure. This assumption ensures that there can be at most two wages offered in equilibrium. Individuals are denoted by superscript $i = 0, 1$. Because the first individual is assumed to value more leisure than the other individual, we have:

$$s^1 > s^0 \tag{29}$$

The mass of type 0 individuals is N^0 and the mass of type 1 individuals is N^1, with

$$N = N^0 + N^1$$

and $N^i = U^i + L^i$. Thus:

$$L^i = N^i - U^i = \left(1 - u^i\right) N^i \tag{30}$$

where $u^i = U^i/N^i$ is the unemployment rate of type$-i$ workers.

3.1 Urban land-use equilibrium

In equilibrium, there will be four types of workers: the unemployed workers of types 0 and 1, with search intensities s^0 and s^1, and the employed workers earning a wage w_L^1 and w_L^0, with $w_L^1 > w_L^0$ (this will be shown below). As we will also see below, in equilibrium, workers of both types 0 and 1 can earn the high wage w_L^1 while only workers of types 0 can earn the low wage w_L^0. As the result, for employed workers, types do not always correspond to wages. We now relax the assumption of housing consumption equal to 1 for all workers and assume on the contrary that

$$h_L^1 > h_L^0 > h_U = 1 \tag{31}$$

where h_L^i is the housing consumption of an employed worker earning a wage w_L^i and h_U is the housing consumption of an unemployed worker. Even though it can be confusing to use the same notation i for workers' types and workers' wages, we keep it to avoid too many notations. Assumption (31) reflects the fact that richer workers consume more land, which is a well-documented fact (see e.g. Glaeser et al., 2007). Observe that, because the unemployed have the same revenue w_U, then they all consume the same amount of land h_U. As above, we can write the *instantaneous* (indirect) utility functions of an employed worker earning a wage w_L^i and a type$-i$ unemployed worker located at a distance x from the BD as:

$$W_L^i(x) = w_L^i - \tau\, x - h_L^i\, R(x)$$

$$W_U^i(x) = w_U - s^i\,\tau\,x - R(x)$$

Observe that the type $i = 0,1$ of a worker plays a role only when they are unemployed since it determines s^i. The type i is however irrelevant when they are employed since what matters is only the wage. As a result, in $W_U^i(x)$, the superscript i indicates the type of workers while, in $W_L^i(x)$, it represents the type of wage a worker earns. As we will see below, this will not be true for the *intertemporal* utilities since someone employed has to take into account the fact that he/she may be unemployed in the future and thus his/her type will matter even when employed. This is why there are four different instantaneous utilities but five different intertemporal utilities. Let us now determine the bid rents of the employed and unemployed workers. They are respectively given by:

$$\Psi_L^i(x, W_L) = \frac{w_L^i - \tau\,x - W_L^i}{h_L^i}$$

$$\Psi_U^i(x, W_U) = w_U - s^i \tau\,x - W_U^i$$

Depending on the assumptions we make, different types of urban equilibria can emerge. Because we want to be consistent with the previous section, we would like to focus on an equilibrium where the employed workers reside closer to jobs than the unemployed workers. For that, we assume

$$h_L^1 < \frac{1}{s^1} \tag{32}$$

which guarantees that, starting from the BD, we first locate the type-0 employed, then the type-1 employed, then the type-1 unemployed and, finally, the type-0 unemployed.[4]

Definition 2 *Assume (31) and (32). Then, an urban-land use equilibrium with ex ante heterogenous workers is a 5-tuple* $(W_L^{0*}, W_L^{1*}, W_U^{0*}, W_U^{1*}, R^*(x))$ *such that:*

$$\Psi_U^0(N, W_U^{0*}) = R_A = 0$$

$$\Psi_U^0(N - U^0, W_U^{0*}) = \Psi_U^1(N - U^0, W_U^{1*})$$

[4] If, on the contrary, we had assumed

$$h_L^0 > \frac{1}{s^0}$$

then we would have had an urban configuration where all the unemployed workers reside close to jobs while the employed workers live farther away from the BD.

$$\Psi_U^1(L, W_U^{1*}) = \Psi_L^1(L, W_L^{1*})$$
$$\Psi_L^1(L^0, W_L^{1*}) = \Psi_L^0(L^0, W_L^{0*})$$
$$R^*(x) = \max\left\{\Psi_U^0(x, W_U^{0*}), \Psi_U^1(x, W_U^{1*}), \Psi_L(x, W_L^*), 0\right\} \quad \textit{at each } x \in (0, N]$$

The interpretation of the equilibrium conditions are similar to the ones given in Definition 1, the only difference being that there are now three borders to be considered. Since $U^0 = u^0 N^0$, $L^0 = (1-u^0)\,N^0$, and $L = N - u^0 N^0 - u^1 N^1$, solving these equations leads to:

$$W_U^{0*} = w_U - s^0 \tau\, N \tag{33}$$

$$W_U^{1*} = w_U - s^1 \tau\, N + \left(s^1 - s^0\right) \tau\, u^0 N^0 \tag{34}$$

$$W_L^{1*} = w_L^1 - \tau N + \left(1 - h_L^1 s^0\right) \tau\, u^0 N^0 + \left(1 - h_L^1 s^1\right) \tau\, u^1 N^1 \tag{35}$$

$$W_L^0 = w_L^0 - \tau \left(N^0 + \frac{h_L^0}{h_L^1} N^1\right) + \tau\, u^0 N^0 \left(1 - s^0 h_L^0\right) + \left(1 - h_L^1 s^1\right) \frac{h_L^0}{h_L^1} \tau\, u^1 N^1 \tag{36}$$

$$R^*(x) = \begin{cases} \tau \left(\frac{N^1}{h_L^1} + \frac{N^0 - x}{h_L^0}\right) - \tau\, u^0 N^0 \left(\frac{1}{h_L^0} - s^0\right) & \\ \quad - \left(\frac{1}{h_L^1} - s^1\right) \tau\, u^1 N^1 & \text{for} \quad 0 \le x \le L^0 \\ \tau \left(\frac{N - x}{h_L^1}\right) - \left(\frac{1}{h_L^1} - s^0\right) \tau\, u^0 N^0 & \\ \quad - \left(\frac{1}{h_L^1} - s^1\right) \tau\, u^1 N^1 & \text{for} \quad L^0 < x \le L \\ s^1 \tau\, (N - x) - (s^1 - s^0)\, \tau\, u^0 N^0 & \text{for} \quad L < x \le N - U^0 \\ s^0 \tau\, (N - x) & \text{for} \quad N - U^0 < x \le N \\ 0 & \text{for} \quad x > N \end{cases} \tag{37}$$

The effects are here more complicated than for the homogenous case but the intuition remains the same. Indeed, the interaction between the land and the labor market is done through the wages w_L^0 and w_L^1 and the unemployment rates u^0 and u^1. Here also, an increase in u^0 and/or u^1 increase the workers' utility but decrease the equilibrium land rent.

3.2 Labor-market equilibrium

Firms post wages. Let $\theta \in [0,1]$ be the fraction of firms posting the high wage w_L^1 and thus $1-\theta$ the fraction posting the low wage w_L^0. As in the previous section, given any distribution of posted wages $F(w_L)$, each worker of type i will have a reservation wage w_L^{ri} such that he/she accepts a job if $w_L \geq w_L^{ri}$ and rejects it if $w_L < w_L^{ri}$, with $w_L^{r1} > w_L^{r0}$. It should also be clear that, in equilibrium, no firm will post anything other than the reservation wage of workers, as a firm posting $w_L \in (w_L^{r0}, w_L^{r1})$ could reduce w_L down to w_L^{r0} and make more profit per worker without changing the set of workers who accept. This was the same argument made in the proof of Proposition 1.

Unemployed workers Since we already know that the only two posted wages are w_L^{r1} and w_L^{r0}, the relevant steady-state Bellman equations for the unemployed workers are given by:

$$rI_U^0 = W_U^{0*} + s^0 a_U (1-\theta)\left(I_L^{0,0} - I_U^0\right) + s^0 a_U\, \theta \left(I_L^{0,1} - I_U^0\right)$$
$$rI_U^1 = W_U^{1*} + s^1 a_U\, \theta \left(I_L^{1,1} - I_U^1\right)$$

where I_U^i is the value function of an unemployed worker of type $i=0,1$ while $I_L^{i,j}$ is the value function of an employed worker of type $i=0,1$ and earning a wage $j=0,1$, where the superscript j corresponds to a wage w_L^{rj}. In this formulation, a value function $I_L^{1,0}$ cannot exist since a type-1 worker will always refuse a job offer with a wage w_L^{r0}. Indeed, type-1 workers accept the high wage w_L^{r1} but not the low wage w_L^{r0} while type-0 workers accept both wage offers. Since the reservation rule property implies that

$$I_L^{0,0} = I_U^0 \tag{38}$$

and

$$I_L^{1,1} = I_U^1 \tag{39}$$

the Bellman equations can be written as

$$rI_U^0 = W_U^{0*} + s^0 a_U\, \theta \left(I_L^{0,1} - I_U^0\right) \tag{40}$$

$$rI_U^1 = W_U^{1*} \tag{41}$$

Employed workers Similarly, the relevant steady-state Bellman equations for the employed workers are equal to:

$$rI_L^{0,0} = W_L^{0*} - \delta \left(I_L^{0,0} - I_U^0 \right)$$

$$rI_L^{0,1} = W_L^{1*} - \delta \left(I_L^{0,1} - I_U^0 \right)$$

$$rI_L^{1,1} = W_L^{1*} - \delta \left(I_L^{1,1} - I_U^1 \right)$$

Using the reservation rules (38) and (39), these equations simplify to:

$$rI_L^{0,0} = W_L^{0*} \tag{42}$$

$$rI_L^{0,1} = W_L^{1*} - \delta \left(I_L^{0,1} - I_U^0 \right) \tag{43}$$

$$rI_L^{1,1} = W_L^{1*} \tag{44}$$

Unemployment rate At the steady-state, the unemployment rates $u^i = U^i/N^i$ are equal to:

$$u^0 = \frac{\delta}{\delta + s^0\, a_U} \tag{45}$$

$$u^1 = \frac{\delta}{\delta + s^1\, a_U\, \theta} \tag{46}$$

Indeed, workers of type 0 accept any job offer (w_L^{r0*} or w_L^{r1*}) while workers of type 1 only accept high-wage jobs, which arrive at rate $s^1\, a_U\, \theta$. As a result, the higher the fraction of firms posting the high wage, the lower the unemployment rate for type-1 workers.

Wages We have the following result:

Proposition 3 *The firms post the following wages:*

$$w_L^{r1*} = w_U + \left(1 - s^1\right) \tau\, N + \left[s^1 - s^0 - \left(1 - h_L^1 s^0\right)\right] \tau\, u^0 N^0 - \left(1 - h_L^1 s^1\right) \tau\, u^1 N^1 \tag{47}$$

$$w_L^{r0*} = w_U + \tau \left(N^0 + \frac{h_L^0}{h_L^1} N^1 \right) - \left(\frac{r + \delta + s^1 a_U \, \theta}{r + \delta + s^0 a_U \, \theta} \right) s^0 \tau \, N - \left(1 - h_L^1 s^1 \right) \frac{h_L^0}{h_L^1} \tau \, u^1 N^1$$
$$+ \left[\frac{s^0 a_U \, \theta \, (s^1 - s^0 - (1 - s^0 h_L^0)) - (1 - s^0 h_L^0) \, (r + \delta)}{r + \delta + s^0 a_U \, \theta} \right] \tau \, u^0 N^0 \qquad (48)$$

Moreover, if

$$1 + \frac{s^0 \, (h_L^1 - h_L^0)}{(s^1 - s^0)} > \frac{N}{N^0} \frac{(\delta + s^0 a_U)}{\delta} \qquad (49)$$

holds, then $w_L^{r1*} > w_L^{r0*}$.

Proof. See the Appendix.

This proposition confirms that $w_L^{r1*} > w_L^{r0*}$, which is not always true since there is a short-run *cost* (higher commuting costs) and a long-run *gain* (higher contact rate with firms) of providing search effort. Inequality (49) is a sufficient condition that involves only parameters and guarantees that $w_L^{r1*} > w_L^{r0*}$. As can be seen in the Appendix, the high wage w_L^{r1*} is determined by (55), i.e. $W_L^{1*} = W_U^{1*}$ while the low wage w_L^{r0*} by (56), i.e.

$$W_L^{0*} = \frac{(r + \delta) \, W_U^{0*} + s^0 a_U \, \theta W_L^{1*}}{r + \delta + s^0 a_U \, \theta}$$

These two conditions are roughly equivalent to the ones obtained in the non-spatial case where wages and unemployment benefits are involved instead of utilities (see Gaumont et al., 2006, page 834). What is crucial here is the fact that the competition in the land market (through e.g. the commuting costs) affect the wage determination. Furthermore, we have:

$$\frac{\partial w_L^{r1*}}{\partial \theta} = - \left(1 - h_L^1 s^1 \right) \tau \, N^1 \frac{\partial u^1}{\partial \theta} > 0$$

Contrary to the non-spatial model, the high wage w_L^{r1*} depends on θ because an increase in θ affects negatively u^1, which affects the location of workers in city (the employed are closer to jobs), thus the competition in the land market and ultimately the wage. Furthermore, we have:

$$\frac{\partial w_L^{r0*}}{\partial \theta} = - \frac{(r + \delta) \, (s^1 - s^0) \, s^0 a_U \tau}{(r + \delta + s^0 a_U \theta)^2} \left(N - u^0 N^0 \right) - \left(1 - h_L^1 s^1 \right) \tau \, N^1 \frac{h_L^0}{h_L^1} \frac{\partial u^1}{\partial \theta} \gtreqless 0$$

since $\frac{\partial u^1}{\partial \theta} < 0$. A similar effect was present in the non-spatial model, but it was always positive. Here the mechanism is quite different since it goes through u^1 and thus the competition in the land market while, in the non-spatial model, it was through the job contact rate $s^0 a_U \, \theta$.

Firms Instead of following the approach of Albrecht and Axell (1984) as we did in the previous section, we now follow that of Gaumont et al. (2006) because it is simpler. Of course the two approaches are equivalent. Firms maximize steady-state profits. There are two types of firms $i = 0, 1$; those offering the high wage w_L^{r1*} (type-1 firms) and those offering the low wage w_L^{r0*} (type-0 firms). The profit of a firm of type i is given by:

$$\Pi^i = \frac{a_F \rho^i}{r+\delta} \left(y - w_L^{ri*} \right) \tag{50}$$

where ρ^i is the probability a random unemployed worker accepts a job offer at wage w_L^{ri*} and a_F is the exogenous rate at which a firm meets a worker. A job-match is when these two events are realized, which occurs at rate $a_F \rho^i$.

For a type-1 firm posting the high wage w_L^{r1*}, $\rho^1 = 1$ since a job offer is never turned down. On the contrary, for a type-0 firm posting the high wage w_L^{r0*},

$$\rho^0 = \frac{u^0 N^0}{u^0 N^0 + u^1 N^1}$$

since a job offer is only accepted by unemployed workers of type 0. Using (45) and (46), this can be written as:

$$\rho^0 = \frac{\left(\delta + s^1\, a_U\, \theta\right) N^0}{\left(\delta + s^1\, a_U\, \theta\right) N^0 + \left(\delta + s^0\, a_U\right) N^1} \tag{51}$$

In order to avoid the Diamond's paradox (Proposition 1), i.e. only the lowest wage is posted in equilibrium, one needs to write a condition that guarantees that both wages w_L^{r0*} and w_L^{r1*} coexist in equilibrium. For that, it has to be that, in equilibrium, firms are indifferent between posting w_L^{r0*} and w_L^{r1*}, otherwise the two wages cannot coexist together. This is an iso-profit condition. Let us thus calculate the profits Π^0 and Π^1.

Plugging $\rho^1 = 1$ and (51) into (50), we obtain:

$$\Pi^0 = \frac{a_F}{r+\delta} \frac{\left(\delta + s^1\, a_U\, \theta\right) N^0}{\left(\delta + s^1\, a_U\, \theta\right) N^0 + \left(\delta + s^0\, a_U\right) N^1} \left(y - w_L^{r0*} \right) \tag{52}$$

$$\Pi^1 = \frac{a_F}{r+\delta} \left(y - w_L^{r1*} \right) \tag{53}$$

where the wages w_L^{r1*} and w_L^{r0*} are given by (47) and (48), respectively. The iso-profit condition is thus $\Pi^1 = \Pi^0$, which is equivalent to:

$$\theta^* = \left(\frac{\delta + s^0\, a_U}{s^1\, a_U}\right) \frac{N^1}{N^0} \left(\frac{y - w_L^{r1*}}{w_L^{r1*} - w_L^{r0*}}\right) - \frac{\delta}{s^1\, a_U} \tag{54}$$

Observe that θ enters in w_L^{r0*} and w_L^{r1*} through u^1. We have the following result:

Proposition 4 *The sufficient conditions for a non degenerated labor-market equilibrium (i.e.* $0 < \theta^* < 1$*) to exist and to be unique are* $\underline{y} < y < \overline{y}$*, where* $\underline{y}$ *and* $\overline{y}$ *are respectively defined by (60) and (61) in the Appendix.*

Proof. See the Appendix.

Even if the conditions are much more complicated than for the non-spatial model, the intuition remains the same. The productivity y has to be large enough to prevent that all firms pay the lowest wage w_L^{r0*} and low enough to prevent that all firms pay the highest wage w_L^{r1*}. In other words, to obtain wage dispersion, the productivity y has to have intermediate values that belong to $\left(\underline{y}, \overline{y}\right)$.

The steady-state general equilibrium is then easy to calculate. We assume (31), (32), (49), and $\underline{y} < y < \overline{y}$. The value of θ^* is given by (58) in the Appendix. Then, plugging this value in (51) and (46), we obtain respectively ρ^{0*}, the equilibrium probability a random unemployed worker accepts a job offer at wage w_L^{r0*} and the equilibrium unemployment rate u^{1*} (the other unemployment rate u^{0*} is only function of parameters and determined by (45)). By plugging these values of unemployment rates u^{0*} and u^{1*} and the value of θ^* in (48) and (47), we obtain the wages w_L^{r0*} and w_L^{r1*}. Furthermore, by plugging these values of the wages w_L^{r0*} and w_L^{r1*} and the value of θ^* in (52) and (53), we obtain firms' equilibrium profits Π^{0*} and Π^{1*}. Finally, using the values of the wages and the unemployment rates in (33)$-$(37), we obtain the equilibrium utilities W_U^{0*}, W_U^{1*}, W_L^{0*}, W_L^{1*}, and the equilibrium land rent $R^*(x)$.

3.3 Numerical simulations

We run some numerical simulations in order to obtain reasonable values of unemployment rates. The values of the parameters (in monthly terms) are the following: There is 70 percent of workers with high value of leisure. The output y is normalized to 1.15 while the unemployment benefit has a value of 0.32. Pecuniary commuting costs τ are equal to 0.1. The discount rate is $r = 0.01$, whereas the job destruction rate is $\delta = 0.01$, which means that, on average, workers lose their job every eight years and four months. The contact rate of firms a_F is 1.5 while for workers it is $a_U = 1.3$ so that they have on average roughly a contact every 20 days. Table 1 summarizes these different values and the ones for search efforts and housing consumptions.

Table 1. Parameter values

y =1.15 Productivity	r =0.01 Pure discount rate
w_U =0.32 Unemployment benefit	δ =0.01 Job-destruction rate
a_F =1.5 Firms' job contact rate	a_U =1.3 Workers' job contact rate
N =10 Total population	N^0/N =70% Percentage of type−0 workers
τ =0.1 Pecuniary commuting cost	N^1/N =30% Percentage of type−1 workers
s^0=0.08 Search effort of type−0 workers	s^1=0.1 Search effort of type−1 workers
h_L^0=1.1 Housing consumption of type−0 workers	h_L^1=1.2 Housing consumption of type−1 workers

3.3.1 Steady-state equilibrium

Let us calculate the steady-state equilibrium for these parameters values. The numerical results of the equilibrium are displayed in Table 2.

Table 2. Steady-state equilibrium

$\theta^*(\%)$	44.61
$u^{0*}(\%)$	8.77
$u^{1*}(\%)$	14.71
w_L^{0r*}	1.11
w_L^{1r*}	1.13
$\rho^{0*}(\%)$	58.19
$W_U^{0*}(I_U^{0*})$	0.24 (22.69)
$W_U^{1*}(I_U^{1*})$	0.2212 (22.12)
$W_L^{0*}(I_L^{0,0*};\, I_L^{0,1*})$	0.2269 (22.69 ; 22.41)
$W_L^{1*}(I_L^{1,1*})$	0.2212 (22.12)
$\Pi^* = \Pi^0 = \Pi^1$	1.73291

In equilibrium, 44.61 percent of firms post the high wage w_L^{1r*}, which is slightly higher than w_L^{0r*}. Since the difference in search intensity between the two types is not very high, $\theta^* = 0.4461$ implies that u^{0*}, the unemployment rate of workers of type 0, is much lower than u^{1*}, the unemployment rate of type-1 workers (8.77 versus 14.71%). Indeed, the arrival rates for type-0 and type-1 workers are respectively given by $s^0 a_U = 0.104$ and $s^1 a_U\,\theta = 0.058$, which means that their average duration of unemployment is nine and half months and seventeen months, respectively. Furthermore, ρ^{0*}, the probability a random unemployed worker accepts a job offer at wage w_L^{r0*}, is equal to 58.19%. This means that the firms that post the high wage will transform a contact into a match with probability 1 while this will be true only in 58.19 percent of the time for firms posting the low wage since type-1 workers will always refuse such an offer. Since each firm has a contact with a worker every 20 days (i.e. $a_F = 1.5$), this also means that, on average, a match occurs every month for firms posting the low-wage. Table 2 also gives the different utilities (both instantaneous and intertemporal) and one can see that, because of a fiercer competition in the land market for employed workers, their utilities are not always higher than that of the unemployed workers. Finally, Figure 2 illustrates Proposition 4 by showing that, for low values of the productivity y (i.e. $\underline{y} \simeq 0.9$), $\theta^* \leq 0$ while for high values of y (i.e. $\overline{y} \simeq 1.2$), $\theta^* \geq 1$.

[*Insert Figure* 2 *here*]

3.3.2 Interaction between land and labor markets

We would like to pursue our analysis by investigating the interaction between land and labor markets. For that, we first examine the impact of a key spatial variable, the commuting cost τ, on the equilibrium labor market variables, w_L^{0r*}, w_L^{1r*}, θ^*, u^{1*}, and Π^*. The effects are complex since τ directly affects the land market through the land rent and the instantaneous utilities but also indirectly affects the labor market through the wages. Let us better understand these effects. By differentiating equations (33) to (36), one can see that an increase in τ, decreases the utilities of the unemployed workers of both types (i.e. W_U^0 and W_U^1) but has an ambiguous effect on the utilities of the employed workers. Indeed, when τ increases, the competition in the land market increases, so all workers pay higher housing prices and thus their utilities decrease. This is the direct effect. There is, however, an indirect effect that goes through u^1, since the latter is negatively affected by θ, which itself is affected by τ. So when τ increases, u^1 changes, which affects the location workers in the city, which, in turn, affects the competition in the land market and thus the utilities. The latter indirect effect is only true for the employed workers as can be seen in equations (35) and (36).

Figure 3a displays the negative impact of an increase in τ on wages. Take for example $w_L^1 = w_L^{1r*}$. By differentiating (47) with respect to τ, one can see that, holding u^1, constant, the relationship is positive. Indeed, as stated above, when τ increases, at a given u^1, the competition in the land market becomes fiercer so that bid rents increase and thus all utilities decrease. Since w_L^{1r*} is determined by $W_L^{1*} = W_U^{1*}$, then because these two utilities decrease and only the first one is a function of w_L^{1r*}, then, following a raise in τ, this wage has to increase for this equality to be true. Now, when we also take into account the fact that u^1 is a positive function of τ (this effect is indirect and goes through θ), the net effect is ambiguous. In the numerical example, the indirect negative effect is greater that the positive direct effect and thus the net effect is negative. The same intuition runs for the low-wage $w_L^0 = w^{0r*}$.

$[Insert\ Figure\ 3a\ here]$

Figures 3b, 3c, and 3d display the other comparative statics results. Not surprisingly, an increasing in the commuting cost τ decreases θ^*, the fraction of firms offering a high wage, but increases u^{1*}, the unemployment rate of type-1 workers, and Π^*, firms' profit. The intuition of these results is similar to that of the wages since the effect goes through the land market. The crucial aspect here is the fact that the land market amplifies the effect of the labor market.

$[Insert\ Figures\ 3b, 3c, 3d\ here]$

In order to further analyze the interaction between the two markets, let us now study the impact of a key labor-market variable, y, on the equilibrium land price $R^*(x)$. Figure 4 displays the result (the variables with one and two stars are respectively the equilibrium values before and after a change in y; the normal and thick lines correspond respectively to before and after the increase of y). Remember that L^i is the area in the city where the employed workers earning w_L^{ir*} reside while U^i is the area in the city where type$-i$ unemployed workers live. Looking at (37), an increase in y affects the bid rents and thus the competition in the land market only through u^1. In particular, y affects negatively u^1 since the latter is a negative function of θ, which is itself a positive function of y. So when y increases, the areas $L^0 = (1 - u^0)\, N^0$ and $U^0 = u^0 N^0$ are not affected while $L^1 = (1 - u^1)\, N^1$ expands and $U^1 = u^1 N^1$ shrinks (Figure 4). This is due to the fact that only the bid rents of the employed workers are affected by a change in y, and this effect is positive. Indeed, by differentiating (37), at a given x, one obtain:

$$\frac{\partial R^*_{L^0}(x)}{\partial y} = \frac{\partial R^*_{L^1}(x)}{\partial y} = -\left(\frac{1}{h^1_L} - s^1\right) \tau\, N^1 \frac{\partial u^1}{\partial \theta} \frac{\partial \theta}{\partial y} > 0$$

and

$$\frac{\partial R^*_{U^0}(x)}{\partial y} = \frac{\partial R^*_{U^1}(x)}{\partial y} = 0$$

where $R^*_{L^i}(x)$ and $R^*_{U^i}(x)$ are the equilibrium land rents at a distance x for the employed workers earning w_L^{ir*} and type$-i$ unemployed workers, respectively.

$[Insert\ Figure\ 4\ here]$

To better understand this result, Figures 5a, 5b, and 5c display the impact of y on the land rent at $x = 0$, $x = L^0$, and $x = L$, respectively. In these figures, one can see that the relationship is positive for $R^*(0)$ and $R^*(L^0)$ but negative for $R^*(L)$. Indeed, as stated above, when y increases, the employed's bid rents increase because the competition in the land market is fiercer due to the fact the unemployment rate u^1 decreases. So at $x = 0$ and at $x = L^0$ land rents increase because the bid rents of workers earning both w_L^{0r*} and w_L^{1r*} increase and these locations are not affected by a change in y (see Figure 4). Now, when y

increases, $\Psi_L^1(x, W_L)$, the bid rent of workers with high wages, increases while $\Psi_L^0(x, W_L)$, the bid rent of type-0 unemployed workers, is not affected. As a result, the location $x = L$ shifts rightward (from L^* to L^{**}), which makes the competition in the land market less fierce and thus the land price decreases. This is an interesting effect of workers' productivity on housing prices. Similar results can be obtained with other labor-market variables such as, for example, the job-destruction rate δ, which affects the equilibrium land rent only indirectly through u^1.

$[Insert\ Figures\ 5a, 5b, 5c\ here]$

4 Concluding remarks

In this paper, we propose a search-urban model where firms post wages. We first develop a model where all workers are identical and show that the Diamond paradox holds, i.e. there is a unique equilibrium wage even in the presence of search frictions. We investigate the interaction between land and labor markets and show, in particular, that higher unemployment rate increases the employed workers' utility but decreases the equilibrium housing price in the employment area. We then develop a model where there are two types of workers who differ according to the value imputed to leisure. We show that, under some conditions, two wages will emerge in equilibrium so that the Diamond paradox does not hold anymore. One interesting aspect of the results is to analyze how the two markets (land and labor) interact with each other. We show that the commuting cost τ directly affects the land market through the land rent and the instantaneous utilities but also indirectly affects the labor market through the wages. Another interesting and testable result is the impact of workers' productivity on housing prices. The impact can be positive or negative depending on the location in the city.

This model can easily be generalized to $K > 2$ types of workers where there will be K reservation wages $w_L^{1*}, ..., w_L^{K*}$, and in equilibrium these are posted with probabilities $\theta_1, ..., \theta_K$ with $\sum_{i=1}^{i=K} \theta_i = 1$ (see Gaumont et al., 2006). In our spatial model, this model will be very cumbersome to analyze since we will have to locate K types of workers in the city but it is clearly possible. However, this will not add very much in terms of intuition of the results than in the case of $K = 2$.

One the main critics made in the non-spatial literature is that these types of models are not robust to the introduction of any positive search cost. In that case, one typically ends up with the Diamond paradox and thus with the law of one wage, i.e. no wage dispersion in equilibrium. The interesting aspect of our spatial model is that this critic does not hold anymore. Indeed, in our model, there were positive search costs (introduced in the commuting costs) and we showed that the law of two wages still hold in equilibrium. This is because firms have to compensate workers for the search cost difference between the employed and unemployed workers due to the competition in the land market.

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APPENDIX

Proof of Lemma 1. Plugging (16) and (17) into (18) leads to

$$l(w_L) = \lim_{\varepsilon \to 0} \frac{\frac{F(w_L)-F(w_L^r)}{1-F(w_L^r)} - \frac{F(w_L-\varepsilon)-F(w_L^r)}{1-F(w_L^r)}}{F(w_L) - F(w_L - \varepsilon)} \frac{s\, a_U\, [1 - F(w_L^r)]}{\delta + s\, a_U\, [1 - F(w_L^r)]} N$$

Denote $\lim_{\varepsilon \to 0} F(w_L - \varepsilon) = F(w_L^-)$, then we have

$$\begin{aligned} l(w_L) &= \frac{\frac{F(w_L)-F(w_L^r)}{1-F(w_L^r)} - \frac{F(w_L^-)-F(w_L^r)}{1-F(w_L^r)}}{F(w_L) - F(w_L^-)} \frac{s\, a_U\, [1 - F(w_L^r)]}{\delta + s\, a_U\, [1 - F(w_L^r)]} N \\ &= \frac{\frac{F(w_L)-F(w_L^-)}{1-F(w_L^r)}}{F(w_L) - F(w_L^-)} \frac{s\, a_U\, [1 - F(w_L^r)]}{\delta + s\, a_U\, [1 - F(w_L^r)]} N \\ &= \frac{s\, a_U\, N}{\delta + s\, a_U\, [1 - F(w_L^r)]} \end{aligned}$$

which is (19). ■

Proof of Proposition 1. The first order condition of (21) is:

$$\frac{\partial \Pi}{\partial w_L} = \frac{-s\, a_U\, N}{\delta + s\, a_U\, [1 - F(w_L^r)]} < 0$$

Thus, since the profit is decreasing in wages when $w_L \geq w_L^r$, firms will set the lowest possible wage, which is $w_L^* = w_L^r$. No deviation is profitable since a lower wage than w_L^r leads to a zero profit and a higher wage does not increase neither productivity nor $l(w_L)$ but increase the cost of labor and thus leads to a lower profit. ■

Proof of Proposition 3. Solving (39) using (41) and (44) yields:

$$W_L^{1*} = W_U^{1*} \tag{55}$$

which using (34) and (35) leads to (47).

Observe now that using (43), we have:

$$I_L^{0,1} = \frac{W_L^{1*} + \delta I_U^0}{r + \delta}$$

Thus (40) can be written as:

$$\begin{aligned} rI_U^0 &= W_U^{0*} + s^0 a_U\, \theta \left[I_L^{0,1} - I_U^0 \right] \\ &= W_U^{0*} + s^0 a_U\, \theta \left[\frac{W_L^{1*} - rI_U^0}{r + \delta} \right] \end{aligned}$$

By solving this last equation in I_U^0, we obtain:

$$rI_U^0 = \frac{(r+\delta)\,W_U^{0*} + s^0 a_U\,\theta W_L^{1*}}{r+\delta+s^0 a_U\,\theta}$$

Using this last value, it is easy to verify that the reservation rule (38) is now given by:

$$W_L^{0*} = \frac{(r+\delta)\,W_U^{0*} + s^0 a_U\,\theta W_L^{1*}}{r+\delta+s^0 a_U\,\theta} \tag{56}$$

By solving (56) using (33), (35), (36) and (47), we obtain (48).

Let us now show that $w_L^{r1*} > w_L^{r0*}$. Using (47) and (48), this is equivalent to:

$$\left(1-s^1\right)\tau N + \left(\frac{r+\delta+s^1 a_U\,\theta}{r+\delta+s^0 a_U\,\theta}\right) s^0 \tau N - \tau\left(N^0 + \frac{h_L^0}{h_L^1} N^1\right)$$
$$+\left[s^0\left(h_L^1 - h_L^0\right) + \frac{\left(s^1-s^0\right)(r+\delta)}{r+\delta+s^0 a_U\,\theta}\right]\tau\, u^0 N^0$$
$$-\left(\frac{h_L^1-h_L^0}{h_L^1}\right)\left(1-h_L^1 s^1\right)\tau\, u^1 N^1 > 0$$

which can be written as

$$\tau N^1\left(\frac{h_L^1-h_L^0}{h_L^1}\right) - \tau N \frac{(r+\delta)\left(s^1-s^0\right)}{r+\delta+s^0 a_U\,\theta} + \left[s^0\left(h_L^1-h_L^0\right) + \frac{\left(s^1-s^0\right)(r+\delta)}{r+\delta+s^0 a_U\,\theta}\right]\tau\, u^0 N^0$$
$$-\left(\frac{h_L^1-h_L^0}{h_L^1}\right)\left(1-h_L^1 s^1\right)\tau\, u^1 N^1 > 0$$

After some rearrangements, we obtain:

$$\frac{N^1\left(\frac{h_L^1-h_L^0}{h_L^1}\right)\left[1-\left(1-h_L^1 s^1\right)u^1\right]\left(r+\delta+s^0 a_U\,\theta\right)}{(r+\delta)\left(s^1-s^0\right)} + \frac{s^0\left(h_L^1-h_L^0\right)u^0 N^0 s^0 a_U\,\theta}{(r+\delta)\left(s^1-s^0\right)}$$
$$+\frac{s^0\left(h_L^1-h_L^0\right)u^0 N^0}{\left(s^1-s^0\right)} + u^0 N^0 > N$$

Because $1-\left(1-h_L^1 s^1\right)u^1 > 0$ (since the unemployment rate $u^1 < 1$), a sufficient condition (that involves only parameters and thus no endogenous variables) for this inequality to be true is:

$$u^0 N^0\left[1 + \frac{s^0\left(h_L^1-h_L^0\right)}{\left(s^1-s^0\right)}\right] > N$$

Using (45), this can be written as:

$$\frac{\delta}{\delta+s^0 a_U}\left[1 + \frac{s^0\left(h_L^1-h_L^0\right)}{\left(s^1-s^0\right)}\right] > \frac{N}{N^0}$$

which is equivalent to (49). ■

Proof of Proposition 4. Using the wages w_L^{r1*} and w_L^{r0*} defined by (47) and (48), respectively, and the unemployment rate u^1, defined by (46), we have:

$$\frac{y - w_L^{r1*}}{w_L^{r1*} - w_L^{r0*}} = \frac{X + y + \frac{\left(1-h_L^1 s^1\right)\tau N^1\delta}{\delta+s^1 a_U \theta}}{Z - \frac{\delta\left(1-h_L^1 s^1\right)\left(h_L^1-h_L^0\right)\tau N^1}{h_L^1(\delta+s^1 a_U \theta)}} \tag{57}$$

where

$$X \equiv \left[s^0\left(1-h_L^1\right) + \left(1-s^1\right)\right]\tau u^0 N^0 - \left(1-s^1\right)\tau N - w_U$$

$$\begin{aligned} Z \equiv & \left[\left(s^1 - s^0\right)\left(\frac{r+\delta}{r+\delta+s^0 a_U \theta}\right) + \left(1 - \frac{h_L^0}{h_L^1}\right)\left(s^0 h_L^1 - 1\right)\right]\tau u^0 N^0 \\ & + \left[\frac{r+\delta+a_U s^1\theta}{r+\delta+a_U s^0\theta}s^0 - \left(1 - s^1 + \frac{h_L^0}{h_L^1}\right)\right]\tau N - \left(1 - \frac{h_L^0}{h_L^1}\right)\left(1-u^0\right)\tau N^0 \end{aligned}$$

Plugging this value (57) into (54), we obtain:

$$\theta^* = \left(\frac{\delta + s^0 a_U}{s^1 a_U}\right)\frac{N^1}{N^0}\left(\frac{X + y + \frac{\left(1-h_L^1 s^1\right)\tau N^1\delta}{\delta+s^1 a_U \theta^*}}{Z - \frac{\delta\left(1-h_L^1 s^1\right)\left(h_L^1-h_L^0\right)\tau N^1}{h_L^1(\delta+s^1 a_U \theta^*)}}\right) - \frac{\delta}{s^1 a_U} \tag{58}$$

which is equivalent to:

$$\frac{\delta N^0}{(\delta + s^0 a_U)\, h_L^1 N^1} + \frac{s^1 a_U N^0}{(\delta + s^0 a_U)\, h_L^1 N^1}\theta^* = \frac{(X+y)\left(\delta + s^1 a_U \theta^*\right) + \left(1 - h_L^1 s^1\right)\tau N^1 \delta}{Z h_L^1\left(\delta + s^1 a_U \theta^*\right) - \delta\left(1 - h_L^1 s^1\right)\left(h_L^1 - h_L^0\right)\tau N^1}$$

Let us define the following functions:

$$f(\theta) = \frac{\delta N^0}{(\delta + s^0 a_U)\, h_L^1 N^1} + \frac{s^1 a_U N^0}{(\delta + s^0 a_U)\, h_L^1 N^1}\theta$$

$$g(\theta) = \frac{(X+y)\left(\delta + s^1 a_U \theta^*\right) + \left(1 - h_L^1 s^1\right)\tau N^1 \delta}{Z h_L^1\left(\delta + s^1 a_U \theta^*\right) - \delta\left(1 - h_L^1 s^1\right)\left(h_L^1 - h_L^0\right)\tau N^1}$$

Then θ^* is defined by $f(\theta) = g(\theta)$. Observe that

$$f(0) = \frac{\delta N^0}{(\delta + s^0 a_U)\, h_L^1 N^1} > 0$$

$$f'(\theta) = \frac{s^1 a_U N^0}{(\delta + s^0 a_U)\, h_L^1 N^1} > 0$$

$$g(0) = \frac{X + y + (1 - h_L^1 s^1)\,\tau\, N^1}{Zh_L^1 - (1 - h_L^1 s^1)\,(h_L^1 - h_L^0)\,\tau N^1} > 0$$

$$g'(\theta) = \left(1 - h_L^1 s^1\right) \tau\, N^1 \delta Z h_L^1 s^1\, a_U - \delta \left(1 - h_L^1 s^1\right) \left(h_L^1 - h_L^0\right) \tau N^1\, (X + y)\, s^1\, a_U$$

First, we want that: $f(0) < g(0)$. This is equivalent to:

$$\frac{\delta N^0}{(\delta + s^0\, a_U)\, h_L^1 N^1} < \frac{X + y + (1 - h_L^1 s^1)\,\tau\, N^1}{Zh_L^1 - (1 - h_L^1 s^1)\,(h_L^1 - h_L^0)\,\tau N^1}$$

which is equivalent to:

$$y > \underline{\underline{y}}$$

where

$$\underline{\underline{y}} \equiv \frac{\delta N^0 \left[Z\, h_L^1 - (1 - h_L^1 s^1)\,(h_L^1 - h_L^0)\,\tau N^1\right]}{(\delta + s^0\, a_U)\, h_L^1 N^1} - X - \left(1 - h_L^1 s^1\right) \tau\, N^1 \tag{59}$$

Second, we want that: $g'(\theta) < 0$, which is equivalent to

$$y > \underline{y}$$

where

$$\underline{y} \equiv \frac{h_L^1}{h_L^1 - h_L^0}\, Z - X \tag{60}$$

It is easy to verify that $\underline{y} > \underline{\underline{y}}$ so the two conditions $y > \underline{\underline{y}}$ and $y > \underline{y}$ reduces to $y > \underline{y}$.

So far we have shown that $f(0) < g(0)$, $f'(\theta) > 0$ and $g'(\theta) < 0$. This guarantees that there exists a unique and strictly positive θ^*. Let us now show that $\theta^* < 1$. We have:

$$f(1) = \frac{\delta + s^1\, a_U}{\delta + s^0\, a_U} \frac{N^0}{h_L^1 N^1}$$

$$g(1) = \frac{X\,(\delta + s^1\, a_U) + (1 - h_L^1 s^1)\,\tau\, N^1 \delta}{Y h_L^1\,(\delta + s^1\, a_U) - \delta\,(1 - h_L^1 s^1)\,(h_L^1 - h_L^0)\,\tau N^1}$$

So, if when $\theta^* = 1$, $f(1) > g(1)$, then we are certain that $\theta^* < 1$ since the intersection between $f(\theta)$ and $g(\theta)$ occurs before $f(\theta) > g(\theta)$. The condition $f(1) > g(1)$ is equivalent to:

$$\frac{(\delta + s^1\, a_U)\, N^0}{(\delta + s^0\, a_U)\, h_L^1 N^1} > \frac{X\,(\delta + s^1\, a_U) + (1 - h_L^1 s^1)\,\tau\, N^1 \delta}{Y h_L^1\,(\delta + s^1\, a_U) - \delta\,(1 - h_L^1 s^1)\,(h_L^1 - h_L^0)\,\tau N^1}$$

which after some calculations leads to

$$y < \overline{y}$$

where

$$\overline{y} \equiv \frac{N^0 \left[Z\, h_L^1 \left(\delta + s^1\, a_U\right) - \delta \left(1 - h_L^1 s^1\right) \left(h_L^1 - h_L^0\right) \tau N^1\right]}{\left(\delta + s^0\, a_U\right) h_L^1 N^1} - \frac{\delta \left(1 - h_L^1 s^1\right) \tau\, N^1}{\delta + s^1\, a_U} - X \tag{61}$$

The results then follow. ■

Figure 1. Impact of an increase in a_U or a decrease in δ on the equilibrium land rent

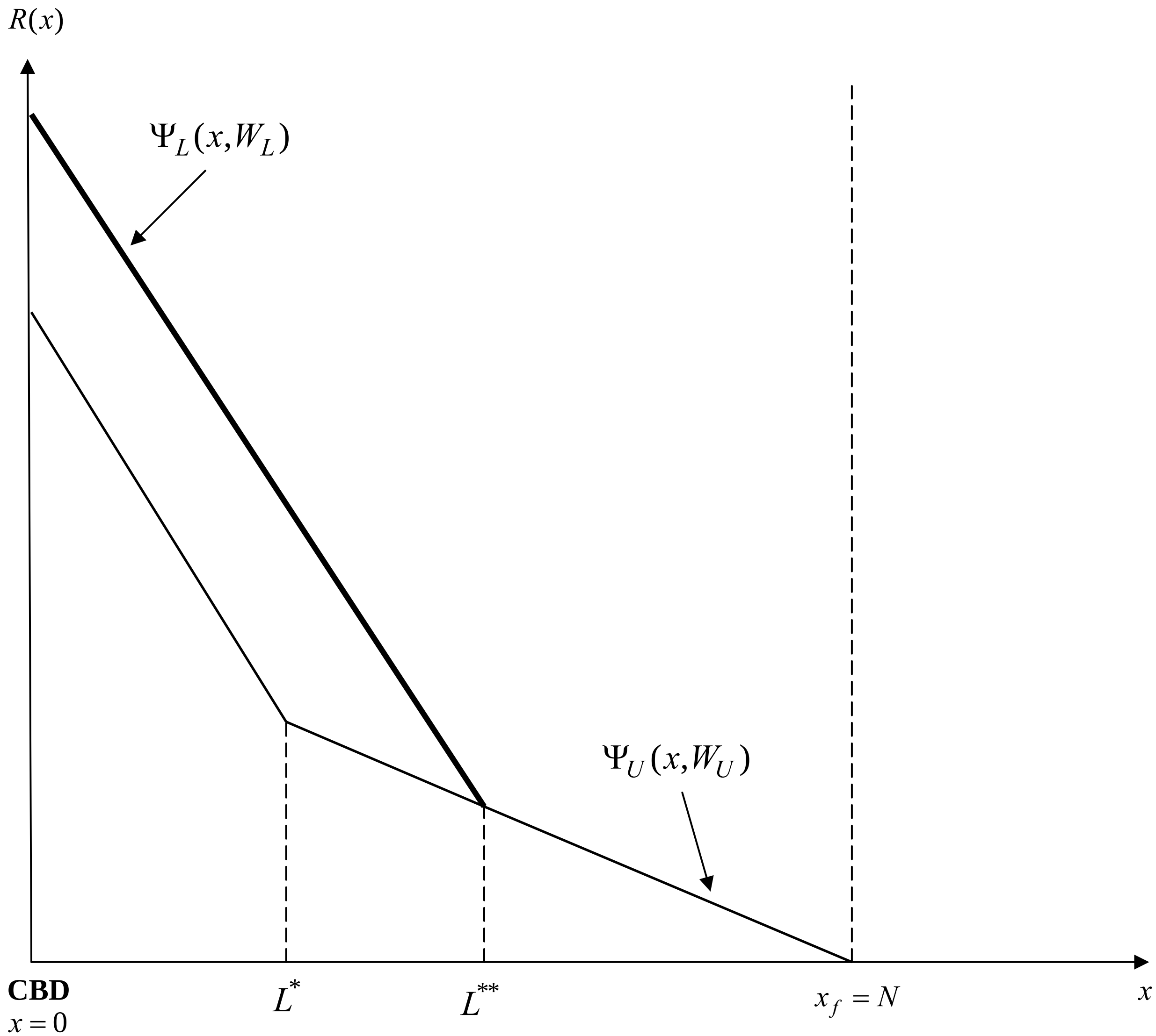

Figure 2. Impact of the productivity y on θ

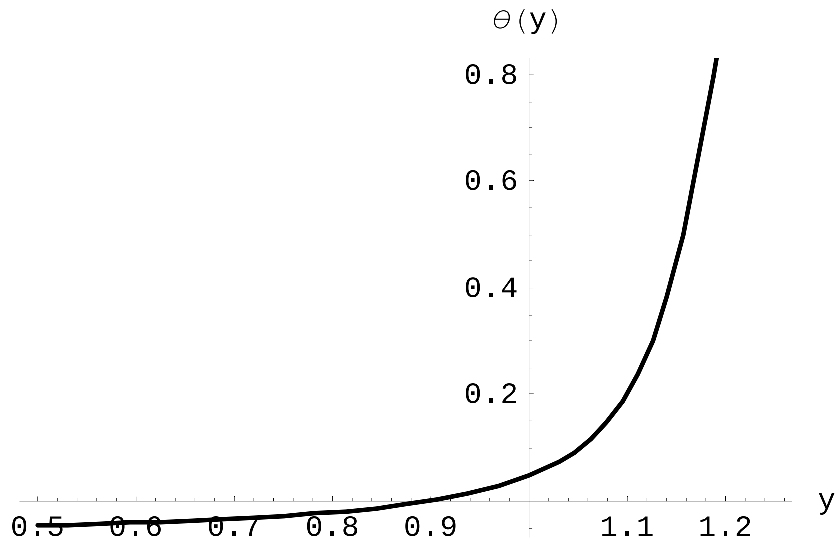

Figure 3a. Impact of the commuting cost τ on wages
(dash line w_L^0, solid line w_L^1)

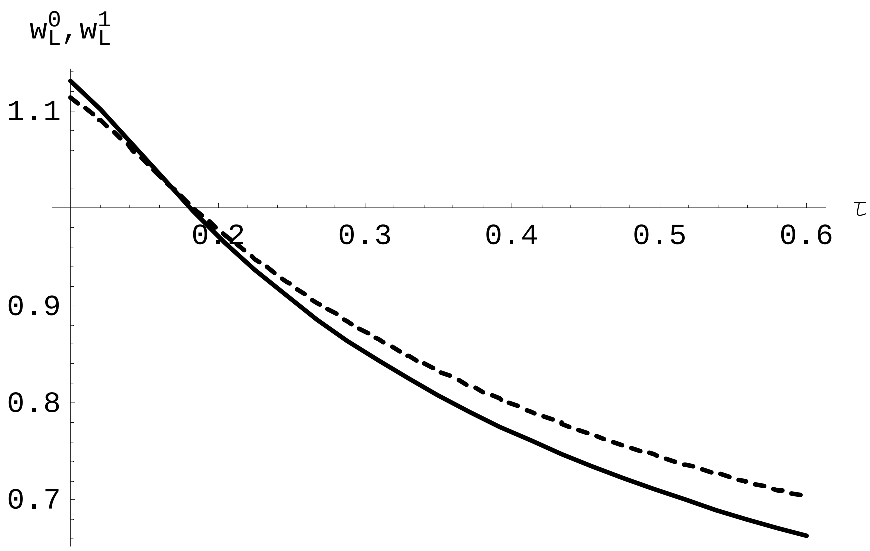

Figure 3b. Impact of the commuting cost τ on θ

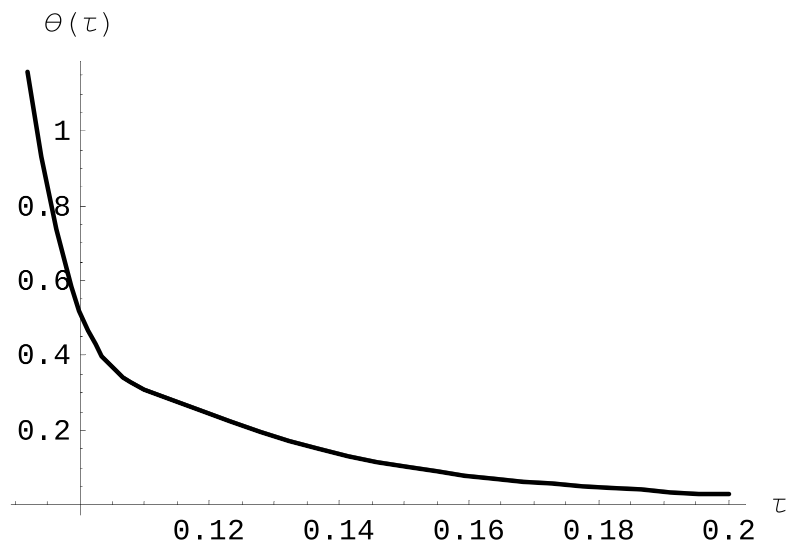

Figure 3c. Impact of the commuting cost τ on the unemployment rate u^1

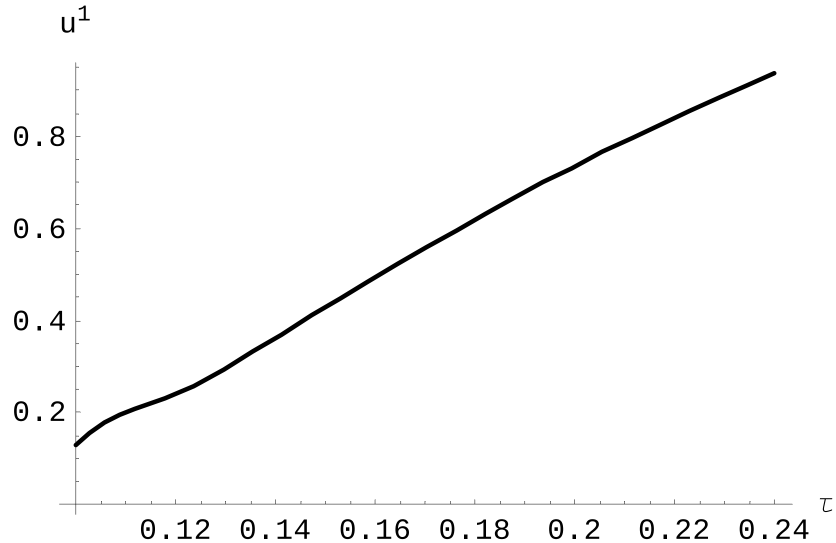

Figure 3d. Impact of the commuting cost τ on the profit Π^*

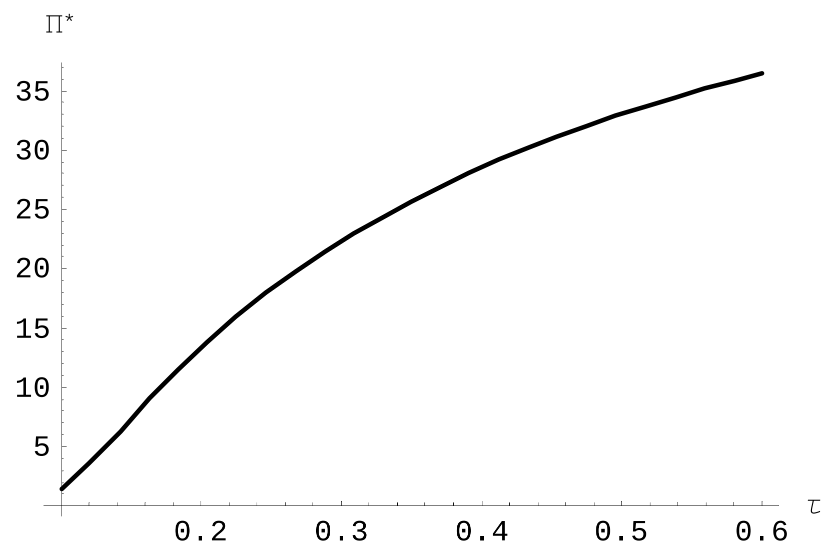

Figure 4: Impact of an increase in the productivity y on the equilibrium land rent

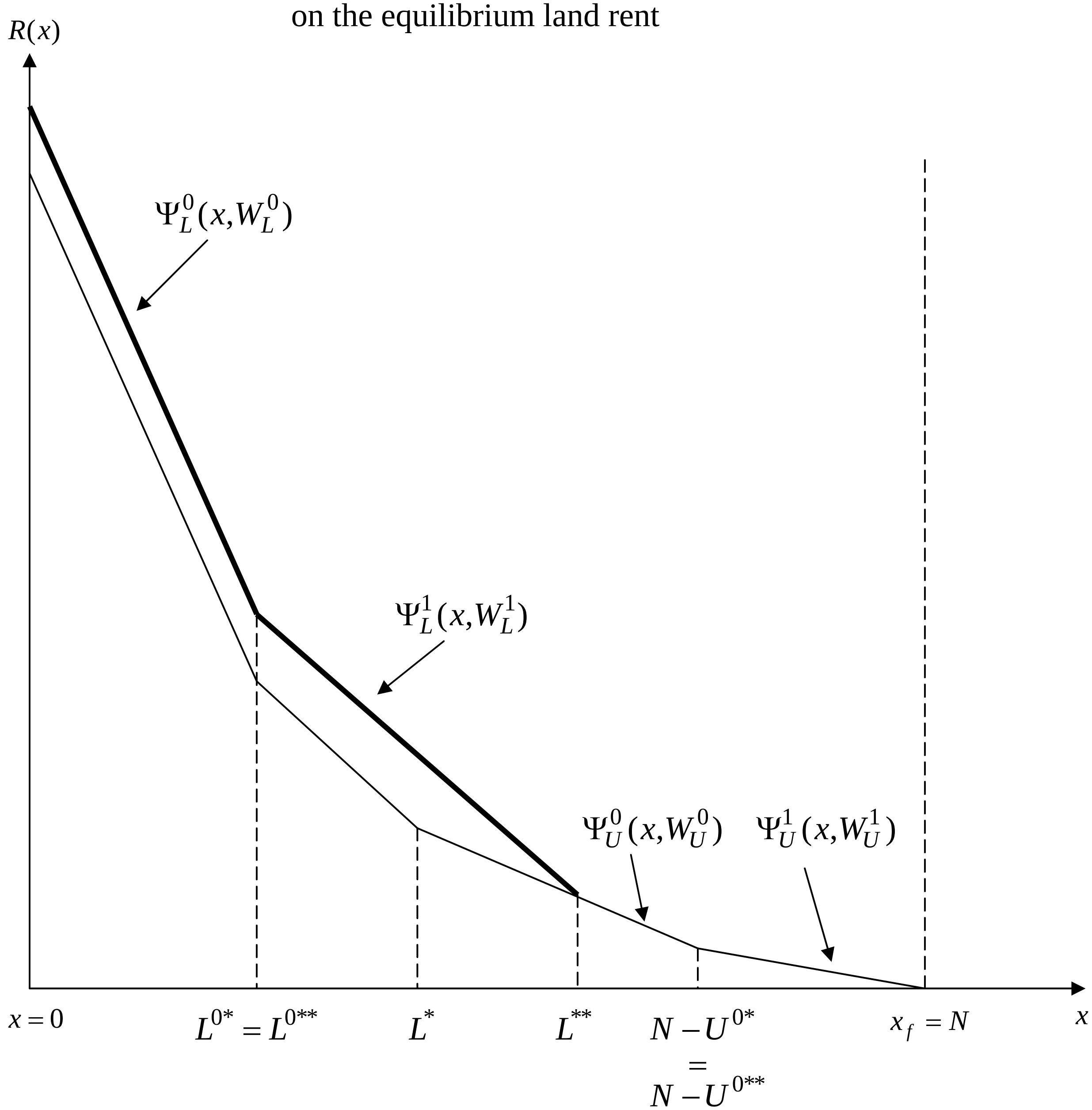

Figure 5a. Impact of the productivity *y* on the land rent at *x* = 0

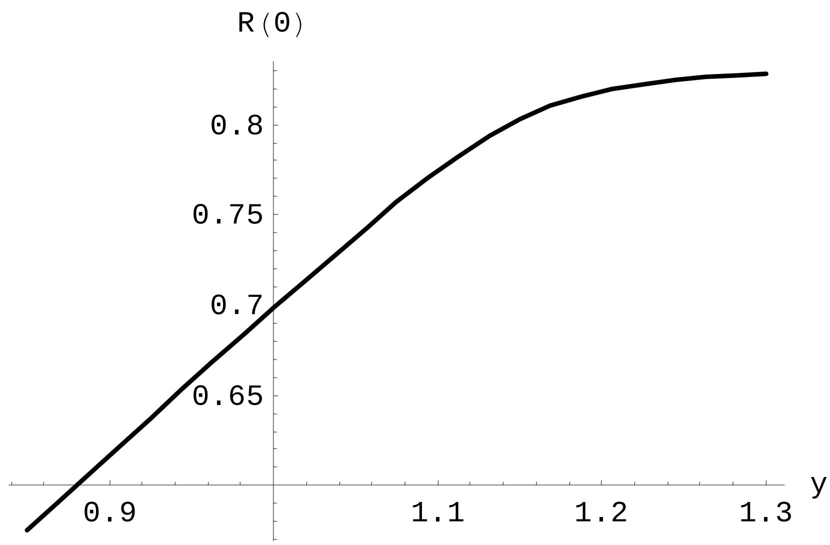

Figure 5b. Impact of the productivity *y* on the land rent at $x = L^0$

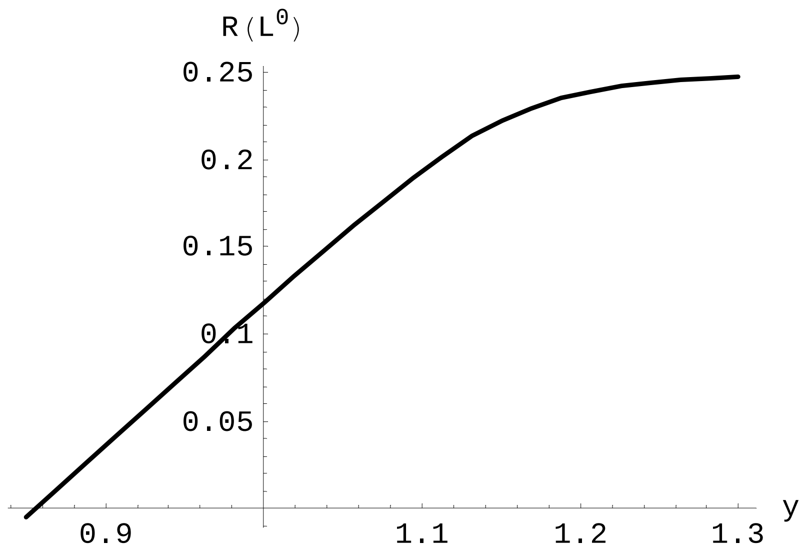

Figure 5c. Impact of the productivity *y* on the land rent at $x = L$