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On the Microeconomics of Diversification under Uncertainty and Learning

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Abstract: This paper investigates the microeconomics of diversification, based on a two-period model of an owner-managed firm facing uncertainty. The analysis utilizes a general state-contingent representation of uncertainty and learning. Economies of diversification are defined based on a certainty equivalent, which has three components: expected profit, the risk premium (measuring the cost of risk aversion), and the value of information associated with learning. The influence of scale effects, "trans-ray concavity" effects, and income effects on economies of diversification are examined in detail. We argue that, while scope economies and risk aversion can provide general incentives for diversification, information and learning can have the opposite effect. By integrating scope, risk, and the role of information, our analysis provides new insights on existing economic tradeoffs between firm diversification and specialization.

Key Words: diversification, risk, scope, learning, bounded rationality

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1. Introduction

This paper integrates three economic rationales for diversification by economic agents: scope, risk management, and learning. Scope economies and risk management have been distinct foci of extensive microeconomic research related to diversification. Starting with the seminal work of Baumol, Panzar, and Willig, economies of scope, which measure the cost reduction or profit gain associated with multi-output production or marketing processes,[1] have been broadly applied to understand the organization and performance of many sectors and industries, including higher education (e.g., Cohn et al.; De Groot et al., Foltz et al.), telecommunication (e.g., Evans and Heckman), banking (e.g., Berger et al.; Dietsch; Ferrier et al.; Lang and Welzel; Huang and Wang), R&D (e.g., Henderson and Cockburn; Klette), biotechnology (e.g., Arora and Gambardella), and health care (e.g., Prior).[2] These models examine the cost properties of multi-output production processes, looking for evidence of scope economies while abstracting from portfolio-risk considerations in the evaluation of diversification choice.

Risk management in portfolio theory is exemplified by the aphorism: "Don't put all your eggs in one basket". The analysis is typically based on risk aversion and the incentives it can provide to diversify among risky prospects (e.g., Markowitz; Tobin; Samuelson, 1967).[3] This approach has generated useful insights on the role of risk in many economic and financial decisions.[4] Yet, it is surprising that scope and risk rationales for diversification have not been integrated (as far as we know) in theoretical or applied microeconomics. If both scope and risk considerations are at play simultaneously in an economic agent's decision, it would seem crucial to identify properly their relative effects in microeconomic decision-making. For example, two activities in an agent's portfolio could have both scope economies and risk-reducing features, which would provide reinforcing motivations for pursuing them jointly to improve profits and reduce risk. Alternatively, diseconomies (economies) of scope and risk reduction (augmenting) features could cut against each other, making diversification decisions more complex in a way that a single rationale could not explain. At both theoretical and empirical levels, this suggests a need to incorporate both scope and risk rationales into the analysis of diversification choices in order to properly identify their individual roles and cumulative effects.

Integrating scope and risk in a microeconomic model is a central goal of this paper. Yet, the analysis of risk effects raises the question: how does a particular decision maker come to assess her uncertain environment? This typically takes place through learning. But can learning also play a role in diversification decisions? We argue that it can and does. As such, we add learning as a third major rationale for (or against) diversification in economic analysis. We show how learning can affect the incentive to diversify. To do so, we develop a model representing individual learning under bounded rationality. Bounded rationality means that, in complex environments, obtaining and processing information is difficult, making it infeasible for the decision maker to obtain perfect information about technology and market conditions (e.g., Simon; Conlisk; Gabaix et al.). This assumption is relevant for agents facing significant changes in their economic environment (e.g., technology, market conditions). And, it seems particularly important for entrepreneurs involved in innovations, i.e. in the discovery of knowledge leading to new technology, new products, and improved use of current resources. We argue that, under bounded rationality, learning plays an important role in diversification strategies: difficulties in information processing tend to have adverse effects on the incentive to diversify.[5] It means that, in addition to scope and risk management, learning also affects diversification decisions, and thus the primary focus of the article is to develop an integrated microeconomic model of diversification choices made by an owner-manager. The model provides new insights into diversification strategies, with implications for financial management, firm structure, household choices, and the economics of entrepreneurship.

Our analysis is also motivated by some of the difficulties economists face in explaining observed diversification choices. Indeed, discrepancies between theory and observed behavior have generated several “puzzles”. Here, we focus attention on two puzzles. One relates to the observed prevalence of investors holding poorly diversified portfolios (e.g., Blume and Friend; Calvet et al.; Campbell; Curcuru et al.; Goetzmann and Kumar; Kelly). Why are so many households willing to hold under-diversified portfolios? This is a challenge to portfolio theory which stresses the benefits of diversification. The empirical evidence also shows much variability in diversification strategies among households. For example, Goetzman and Kumar find that younger, less wealthy, and less sophisticated investors exhibit greater under-diversification. Explaining such heterogeneity remains challenging, as illustrated by our second puzzle: should a senior widow(er) accept the same level of risk exposure as a young entrepreneur?

Financial planners have long argued that very risky investments that appear suitable for young entrepreneurs should be avoided by widow(er)s, Yet, Samuelson (1969) showed that, under constant relative risk aversion, a young entrepreneur and a senior widow(er) with the same wealth should select exactly the same exposure to risk. We call this discrepancy between "conventional wisdom" on diversification and Samuelson's theoretical result the "Samuelson puzzle." We argue below that the Samuelson puzzle can be explained by introducing the role of learning in diversification strategies.[6] This means that traditional approaches to diversification focusing on scope and/or risk alone may be too narrow. An integrated approach also capturing the effects of learning is needed.

Our integrated approach also provides some new and useful insights into the economics of entrepreneurship. Entrepreneurs require refined knowledge and skills to learn about their environment and identify useful innovations related to technology, new products, and/or improved use of resources. This clearly involves risk management. But perhaps more importantly, this requires learning. Since good entrepreneurs can be distinguished by their ability to obtain and process information, entrepreneurial learning is crucial. Finally, entrepreneurs must try to integrate information about their economic environment in a useful way, which fits Lazear's 'jack of all trades' description of entrepreneurs. These lessons have two important implications. First, the economic functions of entrepreneurs are complex and cannot be reduced easily to simple roles. Second, all three components of our analysis (scope, risk and learning) appear to be important aspects of entrepreneurial activities. This suggests that our integrated approach to diversification will help provide new insights into the economics of entrepreneurship, or more generally into the economics of human capital (Schultz).

This article develops a dynamic model of a price-taking, owner-operated firm that explicitly incorporates all three diversification rationales: scope, risk and learning. The analysis is presented in the context of a two-period model under a state-contingent representation of uncertainty (Debreu; Chambers and Quiggin). The dynamic model provides a general representation of learning (including learning-by-doing; see Arrow, 1962). The focus on an owner-operated firm allows us to capture bounded rationality issues at the micro level, while setting aside the potential interactive and strategic effects that may arise in multiple agent environments. The microeconomic model is general and flexible.

Our analysis of diversification outcomes relies on a "certainty equivalent" representation which is introduced in section 3. The certainty equivalent approach is used in section 4 to propose a measure of economies of diversification and in section 5 to identify its three components: expected discounted profit (scope), a risk premium (capturing the role of risk aversion), and the value of information (capturing the benefit of learning process). In section 6, our analysis further decomposes each of these three components into scale effects, "trans-ray concavity" effects, and income effects. We show that, when applied to economies of scope, such a decomposition reduces to the analysis presented by Baumol and Baumol et al. However, the identification of scale effects and trans-ray concavity effects related to risk and learning are apparently new results. We then develop several conjectures about the nature and direction of these effects and explore in Section 7 their implications for diversification choices. In particular, we evaluate the implications of scale effects in risk management. We also discuss how the trans-ray concavity/convexity of the value of information affects diversification choices. Under bounded rationality, we conjecture that, through the trans-ray convexity of the value of information, learning tends to have adverse effects on diversification incentives. This is a key finding that helps to explain why entrepreneurs often hold highly specialized investments, and indicates that, while economies of scope and risk management may favor diversification, leaning effects will often favor specialization. These conjectures help to provide an answer to the "Samuelson puzzle." They also provide other new and useful insights into economic tradeoffs involved in diversification choices.

2. The Model

Consider a manager making decisions for a firm over time. For simplicity, we focus our attention on a two-period model. The firm is involved in a production process producing m outputs using n inputs at time t = 1 and time t = 2. The vector of m outputs chosen at time t is $\mathbf{y}_t = (y_{1t}, \ldots, y_{mt}) \in \Re^m_+$, and the vector of n inputs chosen at time t is $\mathbf{x}_t = (x_{1t}, \ldots, x_{nt}) \in \Re^n_+$, t = 1, 2. The manager faces uncertainty. The uncertainty comes from the production technology as well as market conditions. Production uncertainty, represented by R possible states, r = 1, …, R, reflects all uncertain factors related to the production process, ranging from imperfectly understood aspects of the technology to stochastic factors (such as unforeseen weather effects, possibility of strikes, or equipment breakdown), all of which affect production possibilities. Market uncertainty, represented by S possible states, s = 1, …, S, reflects all factors that generate uncertainty about market conditions and future market prices. Note that the number of states can be quite large. For example, if production uncertainty is generated by 10 random variables, each one taking one of 10 possible values, then $R = 10^{10}$, a very large number. Dealing with a large number of states can be quite difficult and problematic for the manager as well as for the economic analyst (e.g., Simon; Magill and Qinzii). Section 3 explores the implications of this "curse of dimensionality" for the assessment of diversification. We focus our attention on situations of incomplete risk markets, where risk exposure cannot be transferred entirely to other agents.

We consider the case where the manager is also the owner of the firm. While period-two decisions can depend on the information that becomes available about the states of nature, we assume that all period-one decisions are made *ex ante*. At time t, the owner-manager has a fixed amount of time T to allocate between leisure L_{et}, labor input in the firm L_{at}, and wage activities L_{wt} spent working outside the firm and earning a wage rate p_{Lt}. At time t, the manager's time constraint is

$$T = L_{at} + L_{wt} + L_{et}, \tag{1}$$

with $L_{at} \geq 0$, $L_{wt} \geq 0$, and $L_{et} \geq 0$, t = 1, 2.[7] At time t, the owner-manager also chooses a consumption good c_t, t = 1, 2. He/she faces price $p_{ct} > 0$ for consumption c_t, a wage rate $p_{Lt} > 0$ for wage labor L_{wt}, prices $\mathbf{p}_{yt}$

$\equiv (p_{y1t}, \ldots, p_{ymt}) \in \Re^{m}_{++}$ for outputs $\mathbf{y}_t$, prices $\mathbf{p}_{xt} \equiv (p_{x1t}, \ldots, p_{xnt}) \in \Re^{n}_{++}$ for inputs $\mathbf{x}_t$, t = 1, 2. Being the residual claimant, the owner-manager receives the period-one firm profit $(\mathbf{p}_{y1}{}^T \mathbf{y}_1 - \mathbf{p}_{x1}{}^T \mathbf{x}_1)$.[8] In period one, the owner-manager also chooses to invest an amount I into an asset yielding a unit return of $[1 + \rho(s)]$ in period two. It follows that the owner-manager's period-one budget constraint is

$$p_{c1}\, c_1 \leq w + p_{L1}\, L_{w1} + \mathbf{p}_{y1}{}^T \mathbf{y}_1 - \mathbf{p}_{x1}{}^T \mathbf{x}_1 - I, \tag{2a}$$

where w denotes initial wealth, and $(p_{L1}\, L_{w1})$ is wage income at time t = 1.

At time t = 2, the owner-manager chooses netputs $\mathbf{x}_2$, consumption good c_2, along with the time allocation L_{e2}, L_{a2} and L_{w2}. Under market condition s, the owner-manager faces market price $p_{c2}(s) > 0$ for c_2, a wage rate $p_{L2}(s)$ for L_{w2}, and prices $\mathbf{p}_{x2}(s)$ for netputs $\mathbf{x}_2$. Being the residual claimant, the owner-manager receives the period-two firm profit $(\mathbf{p}_{y2}{}^T \mathbf{y}_2 - \mathbf{p}_{x2}{}^T \mathbf{x}_2)$. Denote by $c_2(r, s)$, $\mathbf{x}_2(r, s)$ and $L_{w2}(r, s)$ the period-two decision for c_2, $\mathbf{x}_2$ and L_{w2}, respectively, under state (r, s). It follows that the owner-manager's period-two budget constraint is

$$p_{c2}(s)\, c_2(r, s) \leq p_{L2}(s)\, L_{w2}(r, s) + \mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s) + [1 + \rho(s)]\, I. \tag{2b}$$

Substituting (2a) into (2b) gives the manager's overall budget constraint

$$\begin{aligned} p_{c2}(s)\, c_2(r, s) \leq\; & [1 + \rho(s)][w + p_{L1}\, L_{w1} + \mathbf{p}_{y1}{}^T \mathbf{y}_1 - \mathbf{p}_{x1}{}^T \mathbf{x}_1 - p_{c1}\, c_1] \\ & + p_{L2}(s)\, L_{w2}(r, s) + \mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s). \end{aligned} \tag{3}$$

The period-two decisions for consumption and leisure under state (r, s) are respectively $c_2(r, s)$ and $L_{e2}(r, s)$. The associated decision rules under all possible states are $\tilde{c}_2 \equiv (c_2(1,1), \ldots, c_2(R, S))$ and $\tilde{L}_{e2} \equiv (L_{e2}(1,1), \ldots, L_{e2}(R, S))$. Using a state-contingent approach, the manager's preferences are represented by the *ex ante* utility function $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$. Note that this includes as a special case the expected utility (EU) model. Indeed, under the EU model, $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2}) = \sum_{r=1}^{R} \sum_{s=1}^{S} \Pr(r, s)\, U(c_1, L_{e1}, c_2(r, s), L_{e2}(r, s))$, where $\Pr(r, s)$ is the probability of facing the state (r, s) and $U(c_1, L_{e1}, c_2, L_{e2})$ is a von Neumann-Morgenstern utility function representing the manager's risk preferences. However, the state-contingent utility $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ applies under conditions much broader than the EU model. For example, it includes as special cases weighted utility (Chew), rank-dependent expected utility (Quiggin), prospect theory (Kahneman and Tversky), and general smooth preferences (Machina). Unlike the EU model, this allows for preferences that are not linear in the probabilities. And more generally, the state-contingent approach does not even require that the manager formulates a probability assessment of the states (Debreu). Throughout, we assume that $u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2})$ is strictly increasing in $(c_1, \tilde{c}_2)$. This implies that the owner-manager's preferences are non-satiated in the consumption goods $(c_1, \tilde{c}_2)$.

As noted above, the decisions made at time t = 1 (i.e., $\mathbf{y}_1$, $\mathbf{x}_1$, L_{a1}, L_{w1}, L_{e1}, c_1 and I) are chosen *ex ante*. This means that they do not depend on the states (r, s). However, the decisions made at time t = 2 can depend on the states. This includes the period-two consumption and leisure decisions $\tilde{c}_2 \equiv (c_2(1,1), \ldots, c_2(R, S))$ and $\tilde{L}_{e2} \equiv (L_{e2}(1,1), \ldots, L_{e2}(R, S))$. The nature of state-contingency reflects the amount of the manager's learning about his economic environment. Below, we assume that the period-two consumption/leisure decisions (c_2, L_{e2}) are made *ex post*. It means that $c_2(r, s)$ and $L_{e2}(r, s)$ can be different across <u>each</u> state (r, s). However, we want to capture the role of the learning process for other period-two decisions. This includes the output decisions $\tilde{\mathbf{y}}_2 \equiv (\mathbf{y}_2(1,1), \ldots, \mathbf{y}_2(R, S))$, the input decisions $\tilde{\mathbf{x}}_2 \equiv (\mathbf{x}_2(1,1), \ldots, \mathbf{x}_2(R, S))$, and the labor decisions $\tilde{L}_{a2} \equiv (L_{a2}(1,1), \ldots, L_{a2}(R, S))$ and $\tilde{L}_{w2} \equiv (L_{w2}(1,1), \ldots, L_{w2}(R, S))$. Let $\mathbf{z}_2 \equiv (\mathbf{y}_2, \mathbf{x}_2, L_{a2}, L_{w2}) = (z_{12}, \ldots, z_{m+n+2,2}) \in \Re^{m+n+2}$, with $\tilde{\mathbf{z}}_2 \equiv (\mathbf{z}_2(1,1), \ldots, \mathbf{z}_2(R, S)) \in \Re^{(m+n+2)RS}$. We allow the decisions $\mathbf{z}_2$ to reflect different amount of learning. This is done by considering different partitions of the state space $P \equiv \{1, \ldots, R\}\times\{1, \ldots, S\}$. Let P_i be a partition of P, i.e. a collection of disjoint subsets of P whose union is P. Assume that z_{i2} (the i-th decision variable in $\mathbf{z}_2 \equiv (\mathbf{y}_2, \mathbf{x}_2, L_{a2}, L_{w2})$) is chosen based on the information partition P_i such that

$$z_{i2}(r, s) = z_{i2}(r', s') \text{ if } (r, s) \text{ and } (r', s') \text{ are in the same element of } P_i, \tag{4}$$

$i = 1, \ldots, n+m+2$. Equation (4) means that, when choosing z_{i2}, the manager cannot distinguish between states that are in the same elements of the partition P_i. This can represent different amount of information available. At one extreme, <u>perfect information</u> corresponds to $P_i = P^+ \equiv \{(1, 1), \ldots, (R, S)\}$, where P^+ has

RS elements with each element corresponding to a state (r, s). Then, $P_i = P^+$ implies that the manager chooses z_{i2} *ex post*. At the other extreme, no information corresponds to $P_i = P^- \equiv \{P\}$, where P^- has only one element. Then, $P_i = P^-$ implies that the manager chooses z_{i2} *ex ante*. And partial learning corresponds to intermediate situations where the number of elements in P_i is greater than 1 but less than RS.

Denote by $\mathbf{P} = (P_1, \ldots, P_{n+m+2})$ the information structure supporting the second-period decisions $\mathbf{z}_2 \equiv (\mathbf{x}_2, L_{a2}, L_{w2}) = (z_{12}, \ldots, z_{n+m+2,2})$. To investigate the role of the learning process, we allow $\mathbf{P}$ to be endogenous. That is, we consider situations of active learning, where the manager uses the resources he/she controls to obtain information about his/her economic environment.

For a given information structure $\mathbf{P}$, let the feasible set $F(\mathbf{P}) \subset \Re^{n+2+(m+2)RS}$ represent the firm technology, where $(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})$ means that netputs $(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ are feasible under the information structure $\mathbf{P}$. Note the generality of this characterization. It guarantees feasibility for $(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ across all possible states. It allows for production as well as investment activities (where the first-period decisions generate uncertain second-period payoff). It allows for jointness between choosing $(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ and learning (the choice of $\mathbf{P}$) about both technology (represented by the states r = 1, …, R) and market conditions (represented by the states s = 1, …, S). As such, it can represent situations of active learning (including learning-by-doing; see Arrow, 1962). Under active learning, we assume that $F(\mathbf{P}) \subset F(\mathbf{P}')$ for any information structure $\mathbf{P}'$ that is at least as fine as $\mathbf{P}$. Then, $F(\mathbf{P}') - F(\mathbf{P})$ represents the set of resources required to learn so as to replace $\mathbf{P}$ by $\mathbf{P}'$. And the benefits obtained from the new information are associated with equation (4) (which becomes less restrictive). The feasible set $F(\mathbf{P})$ also allows for the possibility that labor activities outside the firm (L_{w1}, L_{w2}) can affect the productivity of labor within the firm (L_{a1}, L_{a2}). And it can reflect contractual and institutional restrictions imposed on labor choices both within and outside the firm. Finally, the characterization allows the amount of learning to be specific to each decision z_{i2}. This can represent situations where information processing requires the use of resources but with a learning process that varies across z_{i2}'s. [9]

Under economic rationality, the manager's decisions is represented by the optimization problem

$$W(w) = \text{Max } \{u(c_1, L_{e1}, \tilde{c}_2, \tilde{L}_{e2}): \text{equations (1), (2a), (2b) and (4)};$$

$$(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})\}. \tag{5}$$

Under non-satiation in $\tilde{c}_2$, note that the budget constraint (3) is always binding under each state (r, s). Below, we assume for simplicity that leisure is always positive, with $L_{e1} > 0$ and $L_{e2}(r, s) > 0$. Then, after substituting (1) and (3) into the utility function, the optimization problem (5) can be alternatively written as

$$W(w) = \text{Max } \{u[c_1, T - L_{a1} - L_{w1}, \ldots, [w + p_{L1} L_{w1} + \mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1 - p_{c1} c_1][1 + \rho(s)]/p_{c2}(s)$$

$$+ [p_{L2}(s) L_{w2}(r, s) + \mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/p_{c2}(s), \ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \ldots]:$$

$$L_{at} \geq 0, L_{wt} \geq 0, t = 1, 2; \text{ equation (4) evaluated at } \mathbf{P};$$

$$(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})\}. \tag{6}$$

Let $\mathbf{L}_t \equiv (L_{at}, L_{wt})$, t = 1, 2. Using backward induction, the optimization problem (6) can be decomposed into two stages: first choose ($\mathbf{y}_2$, $\mathbf{x}_2$, $\mathbf{L}_2$), conditional on ($\mathbf{y}_1$, $\mathbf{x}_1$, $\mathbf{L}_1$, c_1, $\mathbf{P}$); and second choose ($\mathbf{y}_1$, $\mathbf{x}_1$, $\mathbf{L}_1$, c_1, $\mathbf{P}$). The first stage decision is

$$u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P}) = \text{Max}_{y_2, x_2, L_2 \geq 0} \{u[c_1, T - L_{a1} - L_{w1}, \ldots,$$

$$[w + p_{L1} L_{w1} + \mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1 - p_{c1} c_1][1 + \rho(s)]/p_{c2}(s)$$

$$+ [p_{L2}(s) L_{w2}(r, s) + \mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/p_{c2}(s), \ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \ldots]:$$

$$\text{equation (4) evaluated at } \mathbf{P}; (\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})\}, \tag{7a}$$

with $\tilde{\mathbf{y}}_2^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})$, $\tilde{\mathbf{x}}_2^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})$ and $\tilde{\mathbf{L}}_2^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})$ being the corresponding optimal decision rules. The second stage decision is

$$W(w) = \text{Max}_{y_1, x_1, L_1 \geq 0, c_1, P} \{u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})\}, \tag{7b}$$

with ($\mathbf{y}_1^*$, $\mathbf{x}_1^*$, $\mathbf{L}_1^*$, c_1^*, $\mathbf{P}^*$) as corresponding optimal decisions.

How much learning typically takes place? When the economic environment of the firm is simple, assessing the uncertainty facing the owner-manager may be reasonably easy. Under such circumstances, obtaining perfect information may be attainable (provided that the decisions maker is willing to spend enough resources in the learning process). However, the economic environment of firms can be complex, especially during periods of significant market, technological, or institutional changes. Entrepreneurial activities seem fraught with this kind of complexity and characterized by opportunities for learning, where the number of states R and S is large. In this context, information acquisition and processing may prove difficult. When R and S are large, we define bounded rationality as any situation where $F(\mathbf{P}^+) = \emptyset$ where $\mathbf{P}^+$ represents perfect information. Under bounded rationality, this means that making all period-two decisions *ex post* is not feasible. Under such circumstances, while extensive learning remains feasible, perfect learning is impossible (Simon). In this context, our analysis provides a basis to investigate the economics of bounded rationality.

3. Certainty Equivalent under Uncertainty and Learning

Under incomplete risk markets, the owner-manager cannot transfer his/her risk exposure entirely to other agents. This means that risk exposure and information are expected to affect the welfare of the owner-manager. If so, how do risk and information affect production/investment decisions? This section explores under what conditions period-one netputs would be chosen in a way consistent with standard profit maximization. And if profit maximization does not apply, how can it be modified to account for risk and information effects?

First, we address the question: Does profit maximization apply to period-one inputs and outputs? Under non-satiation in (c_1, $\tilde{c}_2$), note that the optimization with respect to $\mathbf{x}_1$ in (6) implies the profit maximization problem

$$\pi(\mathbf{p}_{y1}, \mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P}) = \text{Max}_{y_1, x_1} \{\mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1:$$

$$(\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P})\}, \quad (8)$$

where $\mathbf{x}_1^{\pi}(\mathbf{p}_{y1}, \mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P})$ is the optimal solution for $\mathbf{x}_1$, and $\pi(\mathbf{p}_{y1}, \mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P})$ is a restricted profit function. The profit function $\pi(\mathbf{p}_{y1}, \mathbf{p}_{x1}, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}, \mathbf{P})$ is homogenous of degree one and convex in $(\mathbf{p}_{y1}, \mathbf{p}_{x1})$. Equation (8) is a standard profit maximization problem conditional on period-two state-contingent decisions $(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ and on the information structure $\mathbf{P}$. However, the conditionality on $(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ has important implications. The state-contingent choices $(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ control for the distribution of risk across all possible states. Controlling for risk exposure is the key reason why risk preferences do not play any role in (8). This can be seen as a significant advantage of (8): it applies irrespective of risk preferences. However, making equation (8) empirically tractable can be quite challenging. The reason is that it requires identifying the decisions $(\mathbf{y}_2, \mathbf{x}_2, L_{a2}, L_{w2})$ under all possible states. When the number of states is large, this is very demanding. This "curse of dimensionality" is the main reason why this approach has not been used much in the analysis of production/investment decisions under risk. This has two important implications. First, equation (8) shows that the maximization of profit remains a valid motivation for a firm under very broad conditions. Second, the problem with profit maximization under risk is not in its conceptual validity but rather in its empirical tractability.

When evaluating the state-contingent choices $(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2})$ proves difficult, equation (8) will not appear very attractive to support empirical analyses. Then, is there another way to proceed? The answer is yes, through the use of a "certainty equivalent." But this will come at a cost: if we no longer control for risk exposure, information and risk preferences will now play a role. To define a certainty equivalent, we focus our attention on profit. The discounted value of profit over the two periods is: $\mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1 + [\mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/[1+\rho(s)]$. While there is no uncertainty about period-one profit, $\mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1$, the discounted period-two profit, $[\mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/[1+\rho(s)]$, is subject to uncertainty. Define the expected value of period-two discounted state-contingent profit by $M(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2) \equiv \sum_{r=1}^{R} \sum_{s=1}^{S} \Pr(r, s)\, [\mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/[1 + \rho(s)]$, where Pr(r, s) is the subjective probability of facing the state (r, s).[10] To define a certainty equivalent, consider a situation characterized by: 1/ the replacement of discounted period-two profit by its expected value $M(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2)$; and 2/ no learning. Associate the absence of learning with the information structure $\mathbf{P}^0$, where $\mathbf{P}^0 \equiv (P^-, \ldots, P^-)$, with P^- having only one element. This means that, under the information structure $\mathbf{P}^0$, the period-two choice $\mathbf{x}_2$ and $\mathbf{L}_2$ is made *ex ante*, i.e. without any learning. Then, define "certainty equivalent" CE as the sure amount of income satisfying

$$\begin{aligned} &\text{Max}_{\mathbf{P}} \{u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})\} \\ &= \text{Max}_{y_2, x_2, L_2 \geq 0} \{u[c_1, T - L_{a1} - L_{w1}, \ldots, [w + CE + p_{L1} L_{w1} + p_{c1} c_1][1 + \rho(s)]/p_{c2}(s) \\ &\qquad + [p_{L2}(s) L_{w2}(r, s)]/p_{c2}(s), \ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \ldots]: \\ &\qquad \text{equation (4) evaluated at } \mathbf{P}^0;\ (\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P}^0)\}, \end{aligned} \tag{9}$$

where $u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})$ is defined in equation (7a). Note that the right-hand side of equation (9) is associated with no learning (as reflected by $\mathbf{P}^0$) and no uncertainty about $\mathbf{p}_{x2}(s)$ or $\mathbf{x}_2(r, s)$. As such, all uncertainty related to period-two production/investment decisions has been effectively eliminated. Solving equation (9) for CE gives the certainty equivalent $CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$, where $\boldsymbol{\alpha} = (w, \mathbf{L}_1, c_1)$.

From equation (7b), the period-one production/investment decisions $(\mathbf{y}_1, \mathbf{x}_1)$ involve the maximization of the left-hand side in equation (9). But the period-one decisions $(\mathbf{y}_1, \mathbf{x}_1)$ appear on the right-hand side in (9) only through the certainty equivalent CE. And under non-satiation in c_2, the right-hand side in (9) is an increasing function of CE. Then, equations (7b) and (9) yield the following result.

Proposition 1: The optimal period-one decisions $(\mathbf{y}_1, \mathbf{x}_1)$ satisfy

$$(\mathbf{y}_1, \mathbf{x}_1) \in \text{argmax}_{y_1, x_1} \{CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})\}. \tag{10}$$

Proposition 1 shows that the certainty equivalent $CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ has two attractive characteristics. First, being a sure amount of income, it provides a simple welfare measure of production/investment activities for the firm. Second, equation (10) shows that the certainty equivalent CE($\mathbf{y}_1$, $\mathbf{x}_1$, $\boldsymbol{\alpha}$) provides all the information necessary for the period-one decisions ($\mathbf{y}_1$, $\mathbf{x}_1$). Note that this representation is very general. It applies under any specification of risk preferences and learning process, even if the owner-manager decides to work only in the firm, i.e. if he/she chooses $L_{wt} = 0$, t = 1, 2. It also applies irrespective of the feasible set for $\mathbf{L}_t = (L_{at}, L_{wt})$, t = 1, 2. This allows for situations where labor contracts are not flexible and impose restrictions on the choice of (L_{at}, L_{wt}). As such, the certainty equivalent CE given in (9) provides a broad characterization of the factors affecting period-one netput decisions ($\mathbf{y}_1$, $\mathbf{x}_1$). We exploit these desirable characteristics below.

4. Diversification

We want to investigate whether the multiproduct firm would benefit (or lose) from reorganizing its production/investment activities in a more specialized way. The reorganization involves breaking up the firm into K specialized firms, $2 \leq K < m$. To analyze the economics of diversification, we start with the certainty equivalent of the original firm CE($\mathbf{y}_1$, $\mathbf{x}_1$, $\boldsymbol{\alpha}$) defined in (9), with $\boldsymbol{\alpha} \equiv (w, \mathbf{L}_1, c_1)$. Proposition 1 implies that ($\mathbf{y}_1$, $\mathbf{x}_1$) is chosen as follows:

$$\text{Max}_{\mathbf{y}_1, \mathbf{x}_1} \{CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})\} \equiv \text{Max}_{\mathbf{y}_1} \{\mathbf{p}_{y1}^T \mathbf{y}_1 - DC(\mathbf{y}_1, \boldsymbol{\alpha})\} \tag{11a}$$

where

$$DC(\mathbf{y}_1, \boldsymbol{\alpha}) \equiv -\text{Max}_{\mathbf{x}_1} \{CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})\} + \mathbf{p}_{y1}^T \mathbf{y}_1. \tag{11b}$$

As further discussed in section 6.1 below, the function DC($\mathbf{y}_1$, $\boldsymbol{\alpha}$) can be interpreted as a “discounted cost” function, which is conditional on period-one outputs $\mathbf{y}_1$.

Next, consider the K specialized firms created from the breakup of the original firm. Let the k-th specialized firm produce period-one outputs $\mathbf{y}_1^k = (y_{11}^k, \ldots, y_{m1}^k)$ while facing $\boldsymbol{\alpha}^k \equiv (w^k, \mathbf{L}_1^k, c_1^k)$, k = 1, …, K. To guarantee that each of the K firms exhibit some form of specialization (compared to the original firm), we assume below that $\mathbf{y}_1^k \neq \mathbf{y}_1/K$, k = 1, ..., K.

Definition 1: Economies of diversification (diseconomies of diversification) are said to exist if

$$S \equiv CE^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} CE^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k) > 0 \ (< 0), \qquad (12)$$

where $CE^c(\mathbf{y}_1, \boldsymbol{\alpha}) \equiv \text{Max}_{x_1} \{CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})\}$, and the $\mathbf{y}_1^k$'s satisfy $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$.

The restriction $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$ in (12) guarantees that the evaluation of diversification involves the same aggregate period-one outputs $\mathbf{y}_1$, whether it is produced by the original firm or by the K specialized firms. From equation (12), economies of diversification exist (with $S > 0$) if the certainty equivalent of producing period-one outputs $\mathbf{y}_1$ is higher from an integrated firm as opposed to K specialized firms. This identifies the presence of synergies or positive externalities in the production of outputs. Alternatively, diseconomies of diversification exist (with $S < 0$) if the certainty equivalent of producing $\mathbf{y}_1$ is lower when such outputs are obtained from an integrated firm as opposed to K specialized firms. This reflects the presence of negative externalities in the production process among period-one outputs.

Note that an alternative formulation for S in (12) exists. It is:

$$S \equiv \sum_{k=1}^{K} DC(\mathbf{y}_1^k, \boldsymbol{\alpha}^k) - DC(\mathbf{y}_1, \boldsymbol{\alpha}) > 0 \ (< 0), \qquad (12')$$

Since $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$, it is clear from (11b) that expressions (12) and (12') are equivalent. Then, from equation (12'), economies of diversification exist (with $S > 0$) if and only if the cost of producing period-one outputs $\mathbf{y}_1$ is lower from an integrated firm as opposed to K specialized firms. We will show in section 5 below that S in (12') reduces to the standard measure of diversification in the absence of risk and dynamics (e.g., as discussed by Baumol et al.).

Note that S in equation (12) or (12') is measured in monetary units. Given $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$, relative measures of economies of diversification can be defined as

$$S' \equiv S/CE^c(\mathbf{y}_1, \boldsymbol{\alpha}) = [CE^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} CE^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k)/CE^c(\mathbf{y}_1, \boldsymbol{\alpha}), \qquad (13)$$

assuming that $CE^c(\mathbf{y}_1, \boldsymbol{\alpha}) > 0$, and

$$S'' \equiv S/DC(\mathbf{y}_1, \boldsymbol{\alpha}) = [\sum_{k=1}^{K} DC(\mathbf{y}_1^k, \boldsymbol{\alpha}^k) - DC(\mathbf{y}_1, \boldsymbol{\alpha})]/DC(\mathbf{y}_1, \boldsymbol{\alpha}), \qquad (14)$$

assuming that $DC(\mathbf{y}_1, \alpha) > 0$. Economies (diseconomies) of diversification corresponds to S' > 0 (< 0) in (12'), and S" > 0 (< 0) in (14). S' in (13) and S" in (14) are a unit-free measures. S' reflects the proportional increase in the certainty equivalent obtained by producing outputs $\mathbf{y}_1$ in a single integrated firm (as compared to K specialized firms). Similarly, S" reflects the proportional decrease in cost obtained by producing outputs $\mathbf{y}_1$ in a single integrated firm.

Given $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$ and $\mathbf{y}_1^k \neq \mathbf{y}_1/K$, k = 1, ..., K, equations (12) or (12') allow for various forms of specialization among the K firms. For example, at one extreme, the k-th firm can be completely specialized in the j-th output in period one if $y_{j1}^k = y_{j1}$, and $y_{j1}^{k'} = 0$ for $k' \neq k$. In this case, the k-th firm is the only specialized firm producing the j-th ouput. Alternatively, our definition of economies of diversification in (12) or (12') allows for partial specialization. Assuming that $y_{j1} \neq 0$, j = 1, ..., m, this occurs for the k-th firm when $y_{j1}^k \neq 0$, j = 1, ..., m. Then, while $\mathbf{y}_1^k \neq \mathbf{y}_1/K$ implies some form of specialization for the k-th firm, this firm continues to produce non-zero quantities of all period-one outputs. In general, economies of specialization S in (12) or (12') will depend on the patterns of specialization among the K firms.

5. A decomposition of the certainty equivalent

In this section, we investigate the sources of benefit/cost of diversification. This is done by identifying the components of the certainty equivalent $CE(\mathbf{y}_1, \mathbf{x}_1, \alpha)$ defined in (9).

5.1. The value of information

As discussed above, the information structure $\mathbf{P}^0 \equiv (P^-, \ldots, P^-)$ is associated with no learning, with P^- having only one element. This means that, under the information structure $\mathbf{P}^0$, the period-two choices $\mathbf{x}_2$ and $\mathbf{L}_2$ are made *ex ante*. A monetary evaluation of the change from $\mathbf{P}^* \in \text{argmax}_{\mathbf{P}} \{u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})\}$ to $\mathbf{P}^0$ is given by the conditional selling price of information $V(\mathbf{y}_1, \mathbf{x}_1, \alpha)$, where $\alpha = (w, \mathbf{L}_1, c_1)$ (LaValle, chapter 8). This <u>conditional value of information</u> $V(\mathbf{y}_1, \mathbf{x}_1, \alpha)$ is given by the monetary value V which satisfies

$$u^*(w + V, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P}^0) = \text{Max}_{\mathbf{P}} \{u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})\}. \tag{15}$$

$V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ defined in (15) is the smallest amount of money the manager is willing to receive *ex ante* to give up the information structure $\mathbf{P}^* \in \text{argmax}_{\mathbf{P}} \{u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})\}$ and replace it by $\mathbf{P}^0$. It is a <u>conditional</u> value of information since it depends on the period-one decisions, including $(\mathbf{y}_1, \mathbf{x}_1)$. Under non-satiation in $(c_1, \tilde{c}_2)$, $u^*(w, \cdot)$ is necessarily increasing in w. It follows from (15) that

$$V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) \geq 0, \tag{16}$$

Equation (16) states that the conditional value of information is always non-negative. This result applies for any risk preferences and any reference information structure $\mathbf{P}^0$. Given $\boldsymbol{\alpha} = (w, \mathbf{L}_1, c_1)$, the properties of the conditional value of information $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ provide useful insights on the role of the period-one decisions $\mathbf{z}_1 \equiv (\mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1) \in \Re^{m+n+2}$. Of special interest are the effects z_{i1} (the i-th element of $\mathbf{z}_1$) on $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$. If z_{i1} has a positive effect on $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$, then the i-th netput would increase the value of information. This can happen under two conditions: 1/ under active learning, z_{i1} is part of the firm's information gathering activities; or 2/ the use of z_{i1} increases the options for the firm to adjust its period-two decisions in response to new information. Note that this latter effect can be present with or without active learning. Alternatively, if z_{i1} has a negative effect on $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$, then the i-th netput would decrease the value of information, again under two conditions: 1/ using z_{i1} has adverse effects on the learning process; or 2/ the use of z_{i1} decreases the options for the firm to adjust its period-two decisions in response to new information. This latter effect would arise when z_{i1} is an irreversible decision that cannot be undone either because reversing the decision is not feasible (Henry, and Arrow and Fisher) or because of sunk costs (Pindyck and Dixit). For example, when $(\mathbf{y}_1, \mathbf{x}_1)$ involves choosing between a reversible and an irreversible decision, the associated change in the value of information $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ reduces to Arrow and Fisher's "quasi-option value" under the reversible scenario.

5.2. Risk premium

Using (7) and (15), the first period decisions $(\mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1)$ in the optimization problem (5) or (6) can be written as

$$W(w) = \text{Max}_{y_1, x_1, L_1 \geq 0, c_1} \{u^*(w + V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}), \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P}^0)\}. \quad (17)$$

Note that in equation (17), the manager makes period-two decisions without learning (as reflected by $\mathbf{P}^0$) while being compensated for it (through $V(\cdot)$). However, the manager still faces price and production uncertainty. In general, the manager may want to manage his/her risk exposure using insurance contracts. Here, we focus our attention on the case of profit insurance and an actuarially neutral risk. Thus, we consider a profit insurance contract which replaces the period-two discounted state-contingent profit $[\mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/[1 + \rho(s)]$ by its expected value $M(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2) \equiv \sum_{r=1}^{R} \sum_{s=1}^{S} \Pr(r, s) [\mathbf{p}_{y2}(s)^T \mathbf{y}_2(r, s) - \mathbf{p}_{x2}(s)^T \mathbf{x}_2(r, s)]/[1 + \rho(s)]$. Let $\boldsymbol{\alpha} = (w, \mathbf{L}_1, c_1)$. The <u>risk premium for profit insurance</u> $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ is defined as the sure amount of money Q which satisfies[11]

$$\begin{aligned}
&\text{Max}_{\mathbf{x}_2, \mathbf{L}_2 \geq 0} \{u[c_1, T - L_{a1} - L_{w1}, \ldots, [w + V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) - Q + M(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2) + p_{L1} L_{w1} \\
&\quad + \mathbf{p}_{x1} \cdot \mathbf{x}_1 - p_{c1} c_1][1 + \rho(s)]/p_{c2}(s) + p_{L2}(s) L_{w2}(r, s)/p_{c2}(s), \ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \\
&\quad \ldots]: \text{equation (4) evaluated at } \mathbf{P}^0; (\mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P}^0)\} \\
&= u^*(w + V(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1), \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P}^0). \quad (18)
\end{aligned}$$

The risk premium $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ defined in (18) measures the smallest amount of money the manager is willing to pay *ex ante* to replace period-two profit by its expected value. Note that the risk premium $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ is conditional on the period-one decisions, including $\mathbf{y}_1$ and $\mathbf{x}_1$. Since the risk premium $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ measures the willingness-to-pay to eliminate profit risk, its sign can be used to characterize the nature of the manager's risk preferences: the manager is said to be risk averse, risk neutral, or risk lover with respect to a profit risk when $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) > 0, = 0$, or < 0, respectively. Under risk aversion, the risk premium $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ measures the implicit cost of risk bearing for profit risk.

The properties of the risk premium $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ provide useful insights on the role of the period-one netputs $\mathbf{x}_1$ in risk management. Of special interest are the effects x_{i1} (the i-th intput in $\mathbf{x}_1$) on $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$. If x_{i1} has a positive (negative) effect on $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$, then the i-th input would increase (decrease) the implicit cost of risk bearing. For a risk averse decision maker, if $Q(\cdot) > 0$, the i-th input is risk increasing, and the manager has an incentive to decrease the use of x_{i1}.. Alternatively, for a risk averse decision maker, if $Q(\cdot) < 0$, the i-th input is risk decreasing, and the manager has an incentive to increase the use of x_{i1}. Note that similar interpretations apply to period-one outputs $\mathbf{y}_1$.

5.3. The components of CE under uncertainty and learning

Combining equations (7a)-(7b) and (18) gives

$$\text{Max}_{\mathbf{P}}\ u^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1, \mathbf{P})$$

$$= \text{Max}_{y_2, x_2, L_2 \geq 0}\ \{u[c_1, T - L_{a1} - L_{w1}, \ldots, [w + V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) - Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) + M(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2)$$

$$+ p_{L1} L_{w1} + \mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1 - p_{c1} c_1][1 + \rho(s)]/p_{c2}(s) + p_{L2}(s) L_{w2}(r, s)/p_{c2}(s),$$

$$\ldots, T - L_{a2}(r, s) - L_{w2}(r, s), \ldots]$$

$$: \text{equation (4) evaluated at } \mathbf{P}^0;\ (\mathbf{y}_1, \mathbf{x}_1, L_{a1}, L_{w1}, \tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2, \tilde{L}_{a2}, \tilde{L}_{w2}) \in F(\mathbf{P}^0)\}. \quad (19)$$

Denote by $(\tilde{\mathbf{y}}_2^*, \tilde{\mathbf{x}}_2^*, \tilde{\mathbf{L}}_2^*)$ the solution of the maximization problem on the right-hand side of (19). Then, under non-satiation in c_2, comparing equations (9) and (19), we obtain the following result.

Proposition 2: The certainty equivalent $CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ satisfies

$$CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) = M^*(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) + \mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1 + V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) - Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}), \quad (20)$$

where $M^*(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) \equiv M(\tilde{\mathbf{y}}_2^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1), \tilde{\mathbf{x}}_2^*(w, \mathbf{y}_1, \mathbf{x}_1, \mathbf{L}_1, c_1))$ is the expected period-two discounted profit, and $\boldsymbol{\alpha} = (w, \mathbf{L}_1, c_1)$.

Proposition 2 shows that the certainty equivalent $CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ is the sum of four components. From equation (20), $CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ equals the expected period-two discounted profit $M^*(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$, plus the period-one profit $\mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1$, plus the conditional value of information $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$, minus the risk premium $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$. In addition to expected profit, $M^*(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha}) + \mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1$, this shows that both the value of information $V(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ and the cost of private risk bearing $Q(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ affect the welfare of the firm and its owner-manager. The former has a positive effect, stressing the importance of information processing in managerial decisions. And under risk aversion, the latter has a negative effect: it provides risk-averse managers an incentive to reduce their risk exposure.

6. A decomposition of economies of diversification

Combining equations (12) and (20) gives the following result.

Proposition 3: Economies of diversification (diseconomies of diversification) exist if

$$S \equiv S_\pi + S_Q + S_V > 0\ (< 0), \tag{21}$$

with

$$S_\pi \equiv \pi^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} \pi^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k), \tag{22a}$$

$$S_Q \equiv -Q^c(\mathbf{y}_1, \boldsymbol{\alpha}) + \sum_{k=1}^{K} Q^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k), \tag{22b}$$

$$S_V \equiv V^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} V^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k), \tag{22c}$$

where $\pi^c(\mathbf{y}_1, \boldsymbol{\alpha}) \equiv M^*(\mathbf{y}_1, \mathbf{x}_1^c(\mathbf{y}_1, \boldsymbol{\alpha}), \boldsymbol{\alpha}) - \mathbf{p}_{x1}^T\, \mathbf{x}_1^c(\mathbf{y}_1, \boldsymbol{\alpha})$, $Q^c(\mathbf{y}_1, \boldsymbol{\alpha}) \equiv Q(\mathbf{y}_1, \mathbf{x}_1^c(\mathbf{y}_1, \boldsymbol{\alpha}), \boldsymbol{\alpha})$, $V^c(\mathbf{y}_1, \boldsymbol{\alpha}) \equiv V(\mathbf{y}_1, \mathbf{x}_1^c(\mathbf{y}_1, \boldsymbol{\alpha}), \boldsymbol{\alpha})$, $\mathbf{x}_1^c(\mathbf{y}_1, \boldsymbol{\alpha}) \in \text{argmax}_{x_1} \{CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})\}$, $\boldsymbol{\alpha} = (w, \mathbf{L}_1, c_1)$, and the $\mathbf{y}_1^k$'s satisfy $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$.

Note that $\pi^c(\mathbf{y}_1, \boldsymbol{\alpha})$ in (22a) is a measure of expected profit defined as the discounted period-two expected profit $M^*(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})$ minus the period-one cost $\mathbf{p}_{x1}^T\, \mathbf{x}_1$, both evaluated at $\mathbf{x}_1^c(\mathbf{y}_1, \boldsymbol{\alpha}) \in \text{argmax}_{x_1} \{CE(\mathbf{y}_1, \mathbf{x}_1, \boldsymbol{\alpha})\}$ (as in (11b)).

Proposition 3 shows that S, the economy of diversification measure, has three additive components: S_π in (22a) reflecting the effects on expected profit; S_Q in (22b) reflecting the effects on the cost of risk bearing; and S_V in (22c) reflecting the effects of information and learning. In the absence of uncertainty and risk, $Q^c(\mathbf{y}_1, \boldsymbol{\alpha}) = 0$ and $V^c(\mathbf{y}_1, \boldsymbol{\alpha}) = 0$, implying that $S_Q = 0$ and $S_V = 0$. It follows from (21) that, in the absence of risk, $S \equiv S_\pi$. Thus, without risk, the profit effect S_π in (22a) captures all the economic effects of diversification. Such effects have been analyzed in detail in previous literature (e.g., Baumol et al.). We will show below that, in a riskless situation, our analysis indeed includes as a special case well-known results on the economics of diversification. However, proposition 3 goes beyond previous literature by showing how risk (through the term S_Q in (22b)) and learning (through the term S_V in (22c)) can affect the economies of diversification. Next, we present a further decomposition of the terms in (21)-(22) into scale effects, concavity/convexity effects, and income effects which we use to develop conjectures about the nature of these effects and their implications for diversification strategies.

6.1. Expected profit effects

Proposition 3 identifies the role of diversification on expected profit through the term $S_\pi \equiv \pi^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} \pi^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k)$. It shows that profit effects contribute to economies of diversification if $S_\pi \equiv \pi^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} \pi^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k) > 0$, where $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$. This corresponds to a scenario where expected profit $\pi^c(\mathbf{y}_1, \boldsymbol{\alpha})$ is higher under an integrated firm than under K specialized firms.

The following decomposition of S_π in (22a) will prove useful.

<u>Lemma 1</u>: The profit effect S_π in (22a) can be written as

$$S_\pi \equiv S_{\pi 1} + S_{\pi 2} + S_{\pi 3}, \tag{23}$$

where $$S_{\pi 1} \equiv \pi^c(\mathbf{y}_1, \boldsymbol{\alpha}) - K\, \pi^c(\mathbf{y}_1/K, \boldsymbol{\alpha}), \tag{24a}$$

$$S_{\pi 2} \equiv K\, \pi^c(\mathbf{y}_1/K, \boldsymbol{\alpha}) - \sum_{k=1}^{K} \pi^c(\mathbf{y}_1^k, \boldsymbol{\alpha}), \tag{24b}$$

$$S_{\pi 3} \equiv \sum_{k=1}^{K} \pi^c(\mathbf{y}_1^k, \boldsymbol{\alpha}) - \sum_{k=1}^{K} \pi^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k). \tag{24c}$$

Lemma 1 decomposes the profit effect S_π into three additive components: $S_{\pi 1}$ reflecting scale effects (24a), $S_{\pi 2}$ reflecting trans-ray concavity effects (24b), and $S_{\pi 3}$ capturing income effects (24c). First, consider $S_{\pi 1}$. Note from (24a) that $S_{\pi 1} = 0$ when $\pi^c(\mathbf{y}_1, \boldsymbol{\alpha})$ is linear homogeneous in $\mathbf{y}_1$. Then, $S_{\pi 1} = 0$ corresponds to situations where $[\pi^c(\lambda\, \mathbf{y}_1, \boldsymbol{\alpha})/\lambda]$ is a constant for all $\lambda > 0$. Define the ray-average profit as $RAP(\lambda, \mathbf{y}_1) \equiv \pi^c(\lambda\, \mathbf{y}_1, \boldsymbol{\alpha})/\lambda$, where λ is a positive scalar reflecting the scale of period-one outputs. Define

$\left\{\begin{matrix}\text{increasing returns to scale (IRTS)}\\ \text{constant returns to scale (CRTS)}\\ \text{decreasing returns to scale (DRTS)}\end{matrix}\right\}$ as situations where the ray-average profit $RAP(\lambda, \mathbf{y}_1)$ is

$\left\{\begin{matrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{matrix}\right\}$ in $\lambda > 0$. It follows that $S_{\pi 1} = 0$ under CRTS. Alternatively, $S_{\pi 1}$ in (24a) is non-zero only when there is a departure from CRTS.

We make the following conjecture about $S_{\pi 1}$:

<u>Conjecture $C_{\pi 1}$</u>: For each $\mathbf{y}_1 > 0$, there is a scale $\lambda_0(\mathbf{y}_1) > 0$ such that the scale term is positive, $S_{\pi 1}[\lambda\, \mathbf{y}_1, \cdot] > 0$, for all $\lambda \in (0, \lambda_0(\mathbf{y}_1)]$.

$C_{\pi 1}$ states that the term $S_{\pi 1}$ is positive for small scales of operation. As just discussed, this means that the technology exhibits increasing returns to scale (IRTS) in the region of "small scales." It is well known that a sufficient condition for a small firm to exhibit IRTS is the existence of fixed cost. Our conjecture $C_{\pi 1}$ can be motivated by the prevalence of fixed cost in production/marketing/investment decisions. Since $S_{\pi 1} > 0$ contributes to economies of diversification, it follows from the conjecture $C_{\pi 1}$ that the scale effect $S_{\pi 1}$ provides an incentive for small firms to diversify. It remains possible for "large firms" to produce at a scale where CRTS applies, i.e. where the scale effect vanishes (with $S_{\pi 1} = 0$).

The term $S_{\pi 2}$ in (23) and (24b) reflects a concavity effect. To see that, note that $\pi^c(\sum_{k=1}^{K} \theta_k\, \mathbf{y}_1^{k}, \boldsymbol{\alpha})$ $\left\{\begin{matrix} \geq \\ = \\ \leq \end{matrix}\right\}$ $\sum_{k=1}^{K} \theta_k\, \pi^c(\mathbf{y}_1^{k}, \boldsymbol{\alpha})$ if the function $\pi^c(\mathbf{y}_1, \cdot)$ is $\left\{\begin{matrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{matrix}\right\}$ in $\mathbf{y}_1$, for any $\theta_k \in [0, 1]$ satisfying $\sum_{k=1}^{K} \theta_k = 1$. Choosing $\theta_k = 1/K$ and using $\sum_{k=1}^{K} \mathbf{y}_1^{k} = \mathbf{y}_1$, it follows from (24b) that $S_{\pi 2}$ $\left\{\begin{matrix} \geq \\ = \\ \leq \end{matrix}\right\}$ 0 if the function $\pi^c(\mathbf{y}_1, \cdot)$ is $\left\{\begin{matrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{matrix}\right\}$ in $\mathbf{y}_1$. In other words, from (23), the concavity of $\pi^c(\mathbf{y}_1, \cdot)$ in $\mathbf{y}_1$ contributes to economies of diversification. The concavity of $\pi^c(\mathbf{y}_1, \cdot)$ in $\mathbf{y}_1$ reflects diminishing marginal productivity with respect to period-one outputs. This means that diminishing marginal productivity contributes to economies of diversification. In addition, note that the concavity of $\pi^c(\mathbf{y}_1, \cdot)$ in (24b) is evaluated along a hyperplane (since $\mathbf{y}_1 = \sum_{k=1}^{K} \mathbf{y}_1^k$). Following Baumol et al. (p. 81), a function is said to be trans-ray concave (trans-ray convex) if it is concave (convex) along a hyperplane. Thus, the concavity (convexity) properties just discussed are in fact trans-ray concavity (trans-ray convexity) of the expected profit function $\pi^c(\mathbf{y}_1, \cdot)$ along the hyperplane defined by $\mathbf{y}_1 = \sum_{k=1}^{K} \mathbf{y}_1^k$. It follows from (24b) that trans-ray concavity of $\pi^c(\mathbf{y}_1, \cdot)$ (along the hyperplane satisfying $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$) contributes to economies of diversification.

Finally, consider the term $S_{\pi 3}$ in (23) and (24c). It reflects the effects of heterogeneity in the $\boldsymbol{\alpha}^k$'s in (24c). Indeed, if $\boldsymbol{\alpha}^k = \boldsymbol{\alpha}$, k = 1, …, K, then $S_{\pi 3} = 0$. Thus, it is only when the $\boldsymbol{\alpha}^k$'s differ among specialized firms that the $S_{\pi 3}$ can be non-zero. In addition, note that the effects of $\boldsymbol{\alpha} = (w, \mathbf{L}_1, c_1)$ reflect income effects (e.g., as captured by initial wealth w). This means that the term $S_{\pi 3}$ in (23) and (24c) capture the heterogeneity of income effects in the evaluation of economies of diversification. This term could be of special importance in certain contexts including ones with high poverty incidence and variation in wealth/income.

How does Lemma 1 relate to previous research? To answer that question, consider a situation where dynamics are neglected, i.e. where $M^*(\cdot) = 0$. If, in addition, risk is neglected, then $V(\cdot) = 0$ and $Q(\cdot) = 0$. In this case, it follows from (20) that $CE = \mathbf{p}_{y1}^T \mathbf{y}_1 - \mathbf{p}_{x1}^T \mathbf{x}_1$, and from (11b) that $DC(\mathbf{y}_1, \boldsymbol{\alpha}) = \text{Min}_{\mathbf{x}_1} \{\mathbf{p}_{x1}^T \mathbf{x}_1 : (\mathbf{y}_1, \mathbf{x}_1) \text{ feasible}\} = -\pi^c(\mathbf{y}_1, \boldsymbol{\alpha})$. Then, $DC(\mathbf{y}_1, \boldsymbol{\alpha})$ in (11b) becomes the standard cost function. And S in (12') and S/DC in (14) reduce to the standard cost-based measures of economies of diversification found in the literature (e.g., Evans and Heckman; Baumol; Baumol et al.). In this case, note that $S_{\pi 3} = 0$, implying that $S_\pi \equiv S_{\pi 1} + S_{\pi 2}$ from (23). Then, the roles of scale effect and of trans-ray convexity reduce to the analysis of diversification presented by Baumol and Baumol et al. In particular, Baumol and Baumol et al. showed that complementarity among outputs contributes to the trans-ray convexity of the cost function and the presence of economies of scope. Using (11), this means that output complementarity contributes to the trans-ray concavity of profit $\pi^c(\mathbf{y}_1, \cdot)$, thus providing incentives to diversify. This shows how our approach extends previous literature on the economics of the multiproduct firm.

6.2. Risk effects

Proposition 3 shows that risk effects contribute to economies of diversification if $S_Q = -Q^c(\mathbf{y}_1, \boldsymbol{\alpha}) + \sum_{k=1}^{K} Q^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k) > 0$, where $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$. This corresponds to a scenario where the risk premium Q is lower under an integrated firm than under K specialized firms. In a way similar to Lemma 1, we have the following result.

<u>Lemma 2</u>: The risk effect S_Q in (22b) can be written as

$$S_Q \equiv S_{Q1} + S_{Q2} + S_{Q3}, \tag{25}$$

where $$S_{Q1} \equiv -Q^c(\mathbf{y}_1, \boldsymbol{\alpha}) + K\, Q^c(\mathbf{y}_1/K, \boldsymbol{\alpha}), \tag{26a}$$

$$S_{Q2} \equiv -K\, Q^c(\mathbf{y}_1/K, \boldsymbol{\alpha}) + \sum_{k=1}^{K} Q^c(\mathbf{y}_1^k, \boldsymbol{\alpha}), \tag{26b}$$

$$S_{Q3} \equiv -\sum_{k=1}^{K} Q^c(\mathbf{y}_1^k, \boldsymbol{\alpha}) + \sum_{k=1}^{K} Q^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k). \tag{26c}$$

Lemma 2 decomposes the risk effect S_Q into three additive components: S_{Q1} reflecting scale effects (26a), S_{Q2} reflecting trans-ray convexity effects (26b), and S_{Q3} capturing income effects (26c). First, consider S_{Q1}. Note from (26a) that $S_{Q1} = 0$ when $Q^c(\mathbf{y}_1, \boldsymbol{\alpha})$ is linear homogeneous in $\mathbf{y}_1$. Then, $S_{Q1} = 0$ corresponds to situations where $[Q^c(\lambda\, \mathbf{y}_1, \boldsymbol{\alpha})/\lambda]$ is a constant for all $\lambda > 0$. Define the ray-average risk premium as $RAR(\lambda, \mathbf{y}_1) \equiv Q^c(\lambda\, \mathbf{y}_1, \boldsymbol{\alpha})/\lambda$, where λ is a positive scalar reflecting the scale of period-one outputs. It follows that $S_{Q1} = 0$ when the ray-average risk premium $RAR(\lambda, \cdot)$ is constant. And under a U-shape $RAR(\lambda, \cdot)$, being in the region where the ray-average risk premium is declining (increasing) implies $S_{Q1} > 0$ (< 0). Thus, an increasing ray-average risk premium implies that $S_{Q1} < 0$, i.e. that the scale of operation provides a disincentive for risk diversification. We make the following conjecture:

<u>Conjecture C_{Q1}</u>: $S_{Q1} < 0$.

To motivate C_{Q1}, consider situations where the risk premium takes the form

$$Q^c(\mathbf{y}_1, \cdot) = \beta \begin{bmatrix} y_{11} & y_{12} & \cdots \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{12} & \sigma_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \end{bmatrix}, \tag{27}$$

where $\beta > 0$ reflects risk aversion and $\begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{12} & \sigma_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$ is the variance-covariance matrix of net returns per unit of $\mathbf{y}_1 = (y_{11}, y_{12}, \ldots)$. Under the expected utility model, the specification (27) corresponds to situations of normal distributions and constant absolute risk aversion (see Freund; Pratt). More generally, (27) applies as a "local measure" of the risk premium in the neighborhood of the riskless case (Pratt). Under the specification (27), we have $S_{Q1} = Q^c(\mathbf{y}_1, \cdot)\,(1\text{-}K)/K \leq 0$. Thus, under risk aversion, a local measure of the risk premium implies $S_{Q1} \leq 0$. The conjecture C_{Q1} simply states that this local result may hold in general. It indicates that scale effects have in general a negative effect on diversification incentives.

The term S_{Q2} in (25) and (26b) reflects a trans-ray convexity effect. To see that, note from (26b) that $S_{Q2} \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} 0$ if the function $Q^c(\mathbf{y}_1, \cdot)$ is $\begin{Bmatrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{Bmatrix}$ in $\mathbf{y}_1$.[12] Since the concavity/convexity is evaluated along a hyperplane (where $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$), it follows from (26b) that the trans-ray convexity (trans-ray concavity) of $Q^c(\mathbf{y}_1, \cdot)$ in $\mathbf{y}_1$ implies that $S_{Q2} \geq 0$ (≤ 0). We make the following conjecture:

<u>Conjecture C_{Q2}</u>: $S_{Q2} > 0$.

To motivate C_{Q2}, consider the case where the risk premium takes the form (27). As noted above, equation (27) provides at least a local measure of the risk premium (Pratt). Noting that $Q^c(\mathbf{y}_1, \cdot)$ in (27) is (trans-ray) convex in $\mathbf{y}_1$, it follows that $S_{Q2} \geq 0$, implying that risk exposure provides an incentive to diversify. The conjecture C_{Q2} simply states that this local result may be expected to hold in general. In other words, under C_{Q2}, the risk premium $Q^c(\mathbf{y}_1, \cdot)$ is expected to be trans-ray convex (along the hyperplane satisfying $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$), implying that risk aversion generates incentives for diversification.

Under conjectures C_{Q1} and C_{Q2}, the scale effect S_{Q1} and the trans-ray convexity effect S_{Q2} work against each other: the latter in favor of diversification, the former against it. Which one dominates depends on the nature of risk exposure. To illustrate, consider the specification (27) where the $\mathbf{y}_1^k$'s involve complete specialization (with $\mathbf{y}_1^1 = (y_{11}, 0, 0\ldots)$, $\mathbf{y}_1^2 = (0, y_{12}, 0, \ldots)$, etc.). This gives $S_{Q1} + S_{Q2} = -\beta \sum_j \sum_{j'\neq j} \sigma_{jj'}\, y_{ij}\, y_{1j'}$. Given $\beta > 0$ and $\mathbf{y}_1 > \mathbf{0}$, it follows that $(S_{Q1} + S_{Q2})$ is positive (negative) when all covariances $\sigma_{jj'}$ are negative (positive). This gives the well-known result that, among risk-averse decision makers, negative (positive) covariances tend to stimulate (dampen) the incentive to diversify (e.g., Markowitz; Tobin; Samuelson, 1967). Thus, the net effect of S_{Q1} and S_{Q2} on diversification incentives is largely an empirical matter, but the result also shows that our state contingent approach extends previous analyses of the role of risk aversion in diversification strategies.

Finally, the term S_{Q3} in (25) and (26c) captures how the heterogeneity of income effects contributes to the risk premium and the incentive to diversify.

6.3. Information effects

Proposition 3 shows that information effects contribute to economies of diversification if $S_V = V^c(\mathbf{y}_1, \boldsymbol{\alpha}) - \sum_{k=1}^{K} V^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k) > 0$, where $\sum_{k=1}^{K} \mathbf{y}_1^k = \mathbf{y}_1$. This corresponds to a scenario where the value of information V is higher under an integrated firm than under K specialized firms.

<u>Lemma 3</u>: The information effect S_V in (22c) can be written as

$$S_V \equiv S_{V1} + S_{V2} + S_{V3}, \tag{28}$$

where $S_{V1} \equiv V^c(\mathbf{y}_1, \boldsymbol{\alpha}) - K\, V^c(\mathbf{y}_1/K, \boldsymbol{\alpha})$, (29a)

$$S_{V2} \equiv K\, V^c(\mathbf{y}_1/K, \boldsymbol{\alpha}) - \sum_{k=1}^{K} V^c(\mathbf{y}_1^k, \boldsymbol{\alpha}), \tag{29b}$$

$$S_{V3} \equiv \sum_{k=1}^{K} V^c(\mathbf{y}_1^k, \boldsymbol{\alpha}) - \sum_{k=1}^{K} V^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k). \tag{29c}$$

Lemma 3 decomposes the information effect S_V into three additive components: S_{V1} reflecting scale effects (29a), S_{V2} reflecting trans-ray concavity effects (29b), and S_{V3} capturing income effects.

First, consider S_{V1}. Again, note from (29a) that $S_{V1} = 0$ when $V^c(\mathbf{y}_1, \boldsymbol{\alpha})$ is linear homogeneous in $\mathbf{y}_1$. Then, $S_{V1} = 0$ corresponds to situations where $[V^c(\lambda\, \mathbf{y}_1, \boldsymbol{\alpha})/\lambda]$ is a constant for all $\lambda > 0$. Define the ray-average value of information as $RAV(\lambda, \mathbf{y}_1) \equiv V^c(\lambda\, \mathbf{y}_1, \boldsymbol{\alpha})/\lambda$, where λ is a positive scalar reflecting the scale of period-one outputs. It follows that $S_{V1} = 0$ when the ray-average value of information $RAV(\lambda, \cdot)$ is constant. And under an inverted U-shape $RAV(\lambda, \cdot)$, being in the region where the ray-average value of information is increasing (decreasing) implies that $S_{V1} > 0$ (< 0), meaning that the scale of operation strengthens (weakens) the information incentive for diversification. We make the following conjecture:

<u>Conjecture C_{V1}</u>: For each $\mathbf{y}_1 > 0$, there is a scale $\lambda_0(\mathbf{y}_1) > 0$ such that the scale term is positive, $S_{V1}[\lambda\, \mathbf{y}_1, \cdot] > 0$, for all $\lambda \in (0, \lambda_0(\mathbf{y}_1)]$.

C_{V1} states that the term S_{V1} is positive for small scales of operation, which means that the ray-average value of information $RAV(\lambda, \cdot)$ is increasing in the region of "small scales." Similar to the conjecture $C_{\pi1}$, this conjecture can be motivated by the presence of fixed costs in learning activities. Since $S_{V1} > 0$ contributes to economies of diversification, it follows from the conjecture C_{V1} that scale effects related to learning provide an incentive for small firms to diversify. It remains possible for "large firms" to produce at a scale where $S_{V1} = 0$ (where the scale effect vanishes).

The term S_{V2} in (28) and (29b) reflects a trans-ray concavity effect. Note from (29b) that S_{V2} $\begin{Bmatrix} \geq \\ = \\ \leq \end{Bmatrix}$ 0 if the function $V^c(\mathbf{y}_1, \cdot)$ is $\begin{Bmatrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{Bmatrix}$ in $\mathbf{y}_1$. Since the concavity/convexity is evaluated along a hyperplane (where $\sum_{k=1}^{K} \mathbf{y}_1^{\,k} = \mathbf{y}_1$), it follows from (29b) that the trans-ray concavity (trans-ray convexity) of $V^c(\mathbf{y}_1, \cdot)$ in $\mathbf{y}_1$ implies that $S_{V2} \geq 0$ (≤ 0). Thus, a trans-ray concave (convex) value of information $V^c(\mathbf{y}_1, \cdot)$ would contribute positively (negatively) to economies of diversification. While identifying the exact nature of such effects remains an empirical matter, we make the following conjecture:

<u>Conjecture C_{V2}</u>: $S_{V2} < 0$.

C_{V2} states that S_{V2} is likely to be negative. This is motivated as follows. In complex environments, bounded rationality is expected to limit the ability of decision makers to process information. Greater diversification typically exposes the decision maker to additional sources of risk, implying more complex uncertainty. This suggests that, under bounded rationality, the value of information would tend to be higher under specialized activities and would decline under increased diversification, implying $S_{V2} < 0$. Under such a conjecture, the value of information $V^c(\mathbf{y}_1, \cdot)$ tends to be trans-ray convex in $\mathbf{y}_1$, in which case information provides a disincentive for diversification. This issue is further discussed in section 7.2.

Finally, the term S_{V3} in (28) and (29c) captures how the heterogeneity of income effects shapes the value of information and the incentive to diversify.

6.4. Combined effects

Using Lemma 1, 2 and 3, Propositions 3 generates our main result.

Proposition 4: Economies of diversification (diseconomies of diversification) exist if

$$S \equiv S_1 + S_2 + S_3 > 0 \ (< 0), \tag{30}$$

with
$$S_1 = S_{\pi 1} + S_{Q1} + S_{V1}, \tag{31a}$$

$$S_2 = S_{\pi 2} + S_{Q2} + S_{V2}, \tag{31b}$$

$$S_3 = S_{\pi 3} + S_{Q3} + S_{V3}, \tag{31c}$$

where $S_1 \equiv CE^c(\mathbf{y}_1, \boldsymbol{\alpha}) - K\ CE^c(\mathbf{y}_1/K, \boldsymbol{\alpha})$ represents a scale effect, $S_2 \equiv K\ CE^c(\mathbf{y}_1/K, \boldsymbol{\alpha}) - \sum_{k=1}^{K} CE^c(\mathbf{y}_1^k, \boldsymbol{\alpha})$ represents a trans-ray concavity effect, and $S_3 \equiv \sum_{k=1}^{K} CE^c(\mathbf{y}_1^k, \boldsymbol{\alpha}) - \sum_{k=1}^{K} CE^c(\mathbf{y}_1^k, \boldsymbol{\alpha}^k)$ is an income effect.

Proposition 4 decomposes the economies of diversification S into all its components. Along the lines discussed in Lemma 1-3, equation (30) shows that S can be decomposed into three components: the scale effect S_1, the trans-ray concavity effect S_2, and the income effect S_3. Equation (31a) shows that the scale effect S_1 is the sum of the scale effects associated with expected profit $S_{\pi 1}$, risk S_{Q1}, and information S_{V1}. This indicates that the scale of operation can affect the motivation for diversification in multiple ways: through their effects on expected profit, on the risk premium, as well as the value of information. From conjectures $C_{\pi 1}$, C_{Q1} and C_{V1}, we expect $S_{\pi 1} > 0$ for small firms, $S_{Q1} < 0$, and $S_{V1} > 0$ for small firms, which means that the net effect of scale on diversification incentives is indeterminate. For small firms, if the terms $S_{\pi 1}$ and S_{Q1} are sufficiently large, then the scale of small firms would give them an extra incentive to diversify. Alternatively, for large firms, if the terms $S_{\pi 1}$ and S_{Q1} are close to zero, then $S_{Q1} < 0$ would imply that scale gives large firms an extra incentive to specialize.

Similarly, equation (31b) implies that the trans-ray concavity effect S_2 is the sum of the corresponding effects associated with expected profit $S_{\pi 2}$, risk S_{Q2}, and information S_{V2}. Again, this means that the trans-ray concavity effects matter in multiple ways: through their effects on expected profit, on the risk premium, as well as the value of information. While the role of trans-ray concavity (or rather trans-ray convexity of the cost function) has been identified in the literature on scope economies (e.g., Baumol; Baumol et al.), our analysis shows that such effects are relevant as well in assessing the role of risk and information in diversification strategies. From conjectures C_{Q2} and C_{V2}, we expect $S_{Q2} > 0$, and $S_{V2} < 0$ which means that the net effect of trans-ray convexity on diversification incentives is indeterminate. If the term S_{Q2} is sufficiently large, then the certainty equivalent $CE(\mathbf{y}_1, \cdot)$ may be trans-ray concave, implying a stronger incentive to diversify. This could occur in situations of extreme risk aversion or strong negative covariance among distinct activities. Alternatively, if the term S_{V2} is negative and sufficiently large, then the certainty equivalent $CE(\mathbf{y}_1, \cdot)$ may be trans-ray convex, implying an extra incentive to specialize. Under bounded rationality, this would occur in situations where the decision maker finds it difficult to obtain and/or process information.

Finally, equation (31c) shows that the income effects S_3 is the sum of the income effects associated with expected profit $S_{\pi 3}$, risk S_{Q3}, and information S_{V3}.[13] Overall, Proposition 4 shows that the economies of diversification haves nine components, each reflecting different effects (as discussed above). Interestingly and conveniently, each effect takes a simple additive form in (30) and (31). The broader economic significance of these results for diversification outcomes is discussed next.

7. Economic Implications of an Integrated Approach to Diversification

Our economic model provides a nuanced picture of the potential factors influencing diversification outcomes in an owner-operated firm. Previous analyses have typically focused on one of the three components of scope, risk, and learning, and up to two of the effects. For example, the analysis of economies of scope has focused on the scale effect $S_{\pi 1}$ and trans-ray concavity/convexity effect $S_{\pi 2}$ (see Baumol; Baumol et al.). Yet, we have identified (in Proposition 4) up to nine separate effects that can influence the diversification decision of an owner-operator under uncertainty and learning. Thus, the economics of diversification may often be more complex than previously assumed. This creates a significant challenge to economic analysis. Among the nine factors, which ones are likely to be economically important? The answer will depend both on the refinement of applied theoretical models and corresponding empirical analysis. To the extent that the nature and magnitude of these nine effects may vary depending on the firm, the industry and the economic context, their evaluation is in large part an empirical issue. However, in section 6, we developed specific conjectures about several of these effects. Below, we discuss the implications of these conjectures in the analysis of diversification strategies, with applications to the fields of finance, development economics, agricultural economics, and entrepreneurship. Since our analysis has stressed the presence of three possible rationales (scope, risk and learning), we begin our discussion by considering the increased explanatory power obtained from linking initially linking two of the three rationales at a time.

First, what new insights can be obtained from considering the joint effects of risk and learning on diversification strategies? We find that our integrated approach can help to resolve important debates in the financial economics and risk management literature. Consider again the "Samuelson puzzle" that, under constant-relative risk aversion, age should not affect risk management/diversification decisions. If, however, diversification choices are analyzed from the perspective of both learning and risk rationales, then we obtain a new explanation for why entrepreneurs might find it optimal to undertake specialized investments. It can be deduced from our conjecture C_{V2}: under bounded rationality, limitations in information processing lead to the trans-ray convexity of the value of information $V(\mathbf{y}_1, \cdot)$, which provides extra incentives for specialization or away from diversification. Assume that the value of information is more important for entrepreneurs (compared to other investors) as entrepreneurs have a superior ability to process information (Zeckhauser). This implies that the portfolios of entrepreneurs are expected to be more specialized (compared to other investors) because of a stronger trans-ray convexity effect. Thus, introducing learning in the analysis of diversification provides a solution to the Samuelson puzzle: under bounded rationality, learning may provide a disincentive for diversification and especially so for entrepreneurs.

Learning can also help to explain the prevalence of under-diversification in portfolio selection. Indeed, poorly-diversified portfolios are commonly observed in both developed countries (e.g., Blume and Friend; Calvet et al.; Campbell; Curcuru et al.; Goetzmann and Kumar; Kelly) as well as in developing countries (e.g., (Banerjee and Newman; Barrett et al.; Binswanger; Dercon; Eswaran and Kotwal; Rosenzweig and Binswanger; Udry; Townsend; Zimmerman and Carter). Our analysis shows that, under bounded rationality, learning provides incentives for specialization (from our conjecture C_{V2}). This outcome is consistent with the evidence that links investor cognitive abilities, financial market participation, and under–diversification decisions (e.g., Calvet et al.). Another explanation for under-diversification in the activity portfolio of poor households in developing countries might be related to the scale effect of risk S_{Q1}. As stated in conjecture C_{Q1}, small scales of operation are expected to have adverse effects on the incentive to diversify (with $S_{Q1} < 0$). We note that this effect is likely to become more important under high levels of risk aversion. Many farms in developing countries are small and support poor households who are likely to be highly risk averse. This empirical regularity suggests that the scale effect related to risk, S_{Q1}, could help to explain why small farms in developing countries often exhibit relatively little diversification. Note that this effect of scale and the cost of risk aversion on diversification choices appear to be new, and illustrates the usefulness of our decomposition of economies of diversification.

Consider next the interactions of scope and risk on diversification decisions. Agricultural contexts are fraught with the presence of significant weather and market risks. This risk exposure has long been linked to the fact that farms are typically diversified multi-output enterprises (Schultz). In the absence of complete risk markets, risk-averse farmers have an incentive to diversify to reduce their risk exposure. Introducing farmer's risk management strategies helps to explain observed farm diversification choices (e.g., Lin et al.). Yet, many farm activities also involve considerable potential for scope economies based on ecological, logistical, and management considerations (Chavas; Chavas and Aliber; Fernandez-Cornejo et al.). One example is given by crop rotations and the potential benefit of reducing pest damages (by suppressing pest populations). Another rotation benefit is the productivity-enhancing effect between nitrogen-using and nitrogen-fixing crops (e.g., corn-soybeans, corn-alfalfa). Scope economies in agriculture are also likely on integrated livestock-crop operations. Grain and forage produced on the farm can be fed to livestock that produce meat, milk, off-spring and manure, the last of which can be returned as soil nutrients to improve land productivity for the next round of crops. In addition, management and labor can be spread between crop and livestock activities across days and seasons, which may increase management and labor productivity. Of course, there is also potential for these multiple farm activities to be risk-mitigating. Hence, it is surprising to note that an integrated analysis of risk and scope effects in agriculture has yet to be done. Our analysis suggests that proper attention given to scale and trans-ray concavity/convexity effects, for both risk and scope, are likely to be crucial to understand optimal diversification strategies in agriculture.

Another potentially fruitful link between scope and risk issues could emerge from a reappraisal of the diversification issues surrounding seasonal migration, especially by rural people in developing countries.[14] Seasonal migration could have significant scope effects if the operator can realize better returns on production, marketing, and investment activities by improving market access or the direct sale of its products. Note that such scope effects may vary with the scale of the operation of the owner-operator. Again, as suggested by our analysis, scale effects and trans-ray concavity/convexity effects related to both risk and scope may be important aspects of seasonal migration decisions.

Seasonal migration can also be used to illustrate the role played by our third rationale: learning. Indeed, beyond generating higher expected returns and modifying risk exposure, seasonal migration might also involve learning about technological options and market conditions. To the extent that the value of information plays a role in diversification strategies, this suggests that learning would also affect seasonal migration decisions. More specifically, our analysis indicates that both scale effects and trans-ray convexity effects related to learning may also shape migration choice.

Finally, consider the case of entrepreneurship. It provides a good example where all three of our rationales for diversification (expected profit, risk and learning) appear to be important. As such, our integrated analysis provides new and useful insights into the economics of entrepreneurs. The role of risk in entrepreneurship has been analyzed by Kihlstrom and Laffont. They point out that entrepreneurs would self-select out of the least risk averse individuals in society, since this gives them some advantage in facing entrepreneurial risk (because of a lower risk premium Q). However, to the extent that risk aversion is commonly found among most decision makers, this feature may not explain why many entrepreneurs are found to be overly specialized (e.g., Gentry and Hubbard; Goetzmann and Kumar; Moskowitz and Vissing-Jorgensen).

Our analysis suggests that their over-specialization can be explained through the learning dimension. Note that concern with the role of learning in entrepreneurial activities is not new. It dates back to Schumpeter, but also to Schultz and Kirzner. What is new is that, even if entrepreneurs are seen as having a superior ability to process information, such ability remains constrained by bounded rationality issues. This is at the heart of our conjecture C_{V2} (stating that the value of information $V(\mathbf{y}_1, \cdot)$ is trans-ray convex, implying an incentive to specialize), and is important for three reasons. First, bounded rationality helps to explain why entrepreneurs tend to hold very specialized portfolios. Second, it suggests that information effects may typically work against scope and risk effects in diversification decisions, which indicates the need for an explicit analysis of the role of learning in investment decisions. Third, it puts the role of entrepreneurs under a new light. While managing their risk exposure, entrepreneurs must integrate knowledge (suggesting the importance of scope economies of management a la Lazear's 'jack-of-all-trades' conceptualization) while dealing with their own bounded rationality (which likely pushes them toward some specialization). The key to entrepreneurial success seems to involve finding proper tradeoffs between these conflicting directions. Note that such tradeoffs likely vary across firms and industries (depending on the quality of human capital and the complexity of the underlying technology). Yet, understanding these tradeoffs should help to assess the relative economic efficiency of alternative patterns of diversification or specialization. To the extent that entrepreneurship is one of the engines of technological progress and economic growth, refined analyses of these tradeoffs should help to generate new insights into the process of economic growth. In this context, our analysis of the role of expected profit, risk and learning in diversification strategies (along with their scale and trans-ray convexity components) should prove useful.

8. Concluding remarks

We have developed a model of economic behavior of a firm owner-manager under bounded rationality, with implications for the assessment of diversification strategies. The model relies on a general two-period model of an owner-managed firm facing uncertainty. The analysis is based on a general state-contingent representation of uncertainty and learning. We analyze economies of diversification based on a certainty equivalent, and investigate the role of expected profit, the risk premium (measuring the cost of risk aversion), and the value of information associated with learning. The influence of scale effects, "trans-ray concavity" effects, and income effects on economies of diversity are examined in detail. We argue that, while scope economies and risk aversion can provide general incentives for diversification, learning processes can have opposite effects. By identifying the role of information and integrating these three rationales for diversification, our analysis provides new insights on diversification and specialization choices with applications to a wide range of fields in economic study.

Our analysis of a firm owner-manager could be extended in several directions. First, it would be useful to extend it to more than two periods to deepen the dynamics. Second, introducing agency issues (such as separation of ownership and control or interactions in learning within a team) would be valuable for understanding efficiency and other welfare concerns that are likely to arise with multiple agents under bounded rationality, including for example the question of scope versus specialization in firms undergoing mergers or acquisitions. Finally, further investigation of the implications of scope, risk and information for economic efficiency in a general equilibrium context could enhance our models of endogenous growth by focusing on the value of learning and specialization/diversification independent of increasing returns (Schultz). These appear to be good topics for future research.

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Footnotes

[1] Note that Baumol et al. characterized economies of scope involving complete specialization schemes. Below, we interpret economies of scope in a broader context that also allows for partial specialization (as discussed by Evans and Heckman, Berger et al., Ferrier et al. and others).

[2] This has shed new lights on the existence, nature and role of synergies and complementarities between production processes (e.g., Anbarci et al.; Antoneli; Arora and Gambardella; Baumol et al.; Desruelle et al.; Milgrom and Roberts; Topkis).

[3] While portfolio theory was originally developed in a mean-variance context (Markowitz; Tobin), it has been extended to capture the role of skewness (e.g., Mitton and Vorkink). Also, while risk aversion was first characterized in the context on the expected utility model (e.g., Arrow, 1965; Pratt), extensions to non-expected utility models have been developed (e.g., Chambers and Quiggin; Chew; Kahneman and Tversky; Machina; Quiggin).

[4] Risk management has provided useful insights into financial and investment decisions under uncertainty. For example, the presence of significant weather and price risk in agriculture has helped explain why most farms are multi-output enterprises (e.g., Lin et al.).

[5] As further discussed below, our conjecture that learning has adverse effects on the incentive to diversify is supported by the empirical evidence that some investors under-diversify when they have superior information (e.g., Goetzman and Kumar).

[6] Note that other explanations have been explored in the literature. They include the presence of liquidity constraints and their effects on diversification choices (e.g., Gollier).

[7] Thus, wage income at time t is $p_{Lt}\, L_{wt} = p_{Lt}\, (T - L_{et} - L_{at})$. It follows that when L_{wt} is positive, the wage rate p_{Lt} measures the unit opportunity cost of both L_{et} and L_{at}.

[8] In our notation, vectors are treated as column vectors. The superscript "T" denotes the transpose (e.g., implying that $p_{x1}^T\, x_1 \equiv \sum_{i=1}^{n} p_{xi1}\, x_{i1}$).

[9] How individuals process information is complex. While neuroscience is making significant progress on this issue (e.g., Camerer et al.), developing a scientific understanding of how the brain processes information and makes decisions remains a very challenging task. In this context, our state-contingent approach is interpreted simply as a reduced-form representation of individual learning.

[10] In the case where probability assessment is not possible, note the analysis presented below can still hold after replacing $M(\tilde{\mathbf{y}}_2, \tilde{\mathbf{x}}_2)$ by some measure of central tendency of discounted period-two profit.

[11] This is consistent with the characterization of the risk premium proposed by Arrow (1965) and Pratt under the expected utility model. Equation (18) is a generalization of the Arrow-Pratt measure under a state-contingent approach.

[12] Note that $Q^c(\sum_{k=1}^{K} \theta_k\, y_1^k, \alpha) \begin{Bmatrix} \geq \\ = \\ \leq \end{Bmatrix} \sum_{k=1}^{K} \theta_k\, Q^c(y_1^k, \alpha)$ if the function $Q^c(y_1, \cdot)$ is $\begin{Bmatrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{Bmatrix}$ in y_1, for any $\theta_k \in [0, 1]$ satisfying $\sum_{k=1}^{K} \theta_k = 1$. Choosing $\theta_k = 1/K$ and using $\sum_{k=1}^{K} y_1^k = y_1$ yield the desired result.

[13] Note that we do not have strong a priori information on the income effects $S_{\pi 3}$, S_{Q3} and S_{V3}. The exact nature and magnitude of these effects seem to be largely an empirical matter.

[14] Rural-urban migration by family members could be treated analogously to the seasonal migration example. However, household migration choices involve multiple agents and raise contracting and learning dynamic issues that reach beyond the scope of the current model.