CAEPR

CENTER FOR APPLIED ECONOMICS AND POLICY RESEARCH

INDIANA UNIVERSITY

Endogenous Timing of Actions under Conflict between Two Types of Second Mover Advantage

Young-Ro Yoon
Indiana University Bloomington

July 30, 2007

Endogenous Timing of Actions under Conflict between Two Types of Second Mover Advantage

Young-Ro Yoon[1]

Abstract

In a model, two players, heterogeneous in their information quality, compete with each other with perfect information about the other player's information quality. If they can decide their timings of actions endogenously, the less-informed player has an incentive to delay her action for learning. On the other hand, the more-informed player wants to delay her action to prevent her information from being revealed, not to enable her to learn. The conflict of these two types of second mover advantages yields a war of attrition. Although both players can benefit from acting as the follower, the gain from a delay for learning is greater than that for preventing the other's learning. Therefore, a cost for the delay in action plays an important role in characterizing the equilibrium. In contrast to the literature, in which only informational externalities are considered, this article shows that the introduction of payoff externalities contributes to different procedures and reasoning processes through which the heterogeneous players' timings of actions are decided endogenously.

Keywords and Phrases: Payoff Externalities, Informational Externalities, Endogenous Timing of Actions, Waiting Option

JEL classification Numbers: D81, D82

1   INTRODUCTION

When actions are taken sequentially, the order of the actions is important for the possibility of learning. When someone takes an action, her action reflects the information on which it is based. Others who observe her action can then infer her information, which may in turn affect their own decisions and actions. This phenomenon defined what are called informational externalities, which gives agents an incentive to delay their actions for learning if the waiting option is available. Regarding this, much of the literature has focused on the topic of delay in an endogenous timing model in which informational externalities are present. However, in the real world, there exist many economic settings in which the payoff externalities, along with the informational externalities, play an important role in agent's decision on the timing of the action.

Take for example the R&D race between firms of heterogeneous quality. Suppose that other firms can imitate the leading firm's innovations as a result of the insufficient enforcement of the patent law. Then, the leading firm's payoffs associated with R&D success can be reduced. Alternatively, other firms can have an advantage by acting as the followers and incurring a smaller R&D cost as a result of learning from the leading firm's technology. Moreover, the following firms can make improvements by both enhancing existing features and adding new features as a result of the observing prevailing technology.[2] In both cases, the leading firm's incentive for innovation may decrease due to the possibility of technology spillover, and the development of new technology might be suppressed or delayed.

Another example involves the uncertainty in return rates for asset investment.[3] It is common for investors with insufficient information to mimic the choice of a reputable agent. Thus, when a reputable agent makes an investment in a specific asset, other investors may likely follow that choice. This imitating behavior may result in a decrease in the dividend received by the reputable agent, therefore she might have an incentive to delay her investment, and thereby prevent others' from imitating her behavior.

The above examples involve the learning aspect of the second mover advantage. When there is an uncertainty, the follower can take advantage of the leader's knowledge, experience or action. With regard to this, a growing body of evidence suggests the learning aspect of the second mover advantage. For example, according to Golder and Tellis (1996), the failure rate of pioneers in a new market was very high and the firms that dominated the market were the second entrants into the market. It can be conjectured that this second mover advantage might have its source in learning made possible by observing the leader's experience.[4]

---

[2] The case of Web browsers, Internet Explorer vs. Netscape, mentioned in Zhang and Markman (1998), is a good example of this. Netscape was the pioneer in the Web browser market, while Internet Explorer was a late entrant. When Internet Explorer was introduced, it had features that Netscape did not have, which was one reason for its success. This was referred to by Bill Gates as the strategy of "embrace and extend," i.e., embracing current Internet standards and then extending them.

[3] In this example, we have to assume that the action, which denotes the choice of asset, is irreversible and can only be taken once. If not, the manipulation by the more-informed agent or reputable agent can be available, which is not the main interest of this article.

[4] The Harvard Business School case about Wal-Mart also shows how the imitation of the second mover can be successful. In this case, when Wal-Mart was in second place in the wholesale industry, one of its strategies was precisely to imitate what the firms in first place did. That approach was very successful, and in 1993, Wal-Mart had

The above examples also implicitly imply another aspect of the second mover advantage. In both the R&D race and the asset investment examples, the delays in actions of the leading firm and the reputable agent are not motivated by the advantage of learning. Rather, their delays in action are done in order to prevent the spillover of their superior information or technology, which can then cause a decrease in their payoffs. That is, their second mover advantage comes from preventing others' free-riding behaviors by acting as the followers, not from what they can learn from others. Up to now, this type of second mover advantage has not received much attention. To my knowledge, there is no empirical literature which deals with this topic directly. However, some articles can be mentioned as the ones which show the related results. Moser (2001) and Lerner (2002) find that the patent system stimulates R&D and innovation. Additionally, Lanjouw and Cockburn (2002) and Arora, Ceccagnoli and Cohen (2001) show that the innovations are concentrated in the pharmaceutical and biotechnology industries because the patents in these areas are relatively easy to define and imitation can be easily detected. The results of these studies conversely imply that if achievement as a leader cannot be firmly protected, the incentive for innovation may decrease. Extending this result further, it can be argued that whether a leader's gain can be protected or not plays an important role in an agent's decision on the timing of their actions.

In the real world, there are many cases in which the second movers can easily free-ride on the leader because of the absence of any system that protects the leader's initiative. The cases mentioned above can be the good examples. Also, generally in business, other firms are allowed to follow or imitate a leading firm's successful business strategy. In this way, if the follower can freely turn the leader's experience or inferred information to her own advantage, and if the leader can be damaged by it, then the agent who would otherwise act as a leader may have an incentive to delay an action. Then, this may result in a conflict between two types of second mover advantage, the one from learning and the other from preventing the other's learning. In this kind of situation, if the action timings are decided endogenously, which type of second mover advantage will succeed in being taken up the follower?

The much of current literature that deals with the endogenous timing of actions does not provide an answer to this question because the studies discussed focus only on the learning aspect of the second mover advantage. Consequently, in this article, I introduce a model in which the payoff externalities, along with the informational externalities, are considered together in analyzing the incentives of delay in actions and the endogenous timings of actions of the heterogeneous players.

The model that I consider in this paper is as follows. Two players, A and B, make a forecast (action) about the true state of a forthcoming period when that state is as yet unknown.[5] There are two rounds during which each player can take an irreversible action only once. Each player observes her own signal, which is correlated with the true state and private information. The players are heterogeneous in that their observed signals differ in precision, which is public information. For the competitive environment of the common task, their reputations or monetary wages depend on both players' performances. Hence, with being successful in doing a task, the relative performance

succeeded in becoming the leader in the wholesale industry. There is general consensus that one of the main reasons why Wal-Mart was able to succeed was its strategy of imitation.

[5] Throughout this paper, the act of forecasting is indicated by the phrase "the action."

compared to her rival is also essential. Therefore, the success or failure of each player's performance endogenously determines whether the other player's identical action causes a positive or a negative payoff externality. Also, it is assumed that a follower can observe a leader's action if the actions are taken sequentially. Thus, informational externalities are also embedded in the model. Under this setup, each player should decide her own timing of action and whether to act according to her information.

Throughout this paper, it is assumed that the precision of B's signal is greater than that of A. Thus, in the following discussion, A denotes the less-informed player and B denotes the more-informed player. The main results can be summarized as follows.

Compared to the literature in which only the informational externalities are considered, the learning incentive is not the unique reason for a delay in action. In the case of A who knows that she is the less-informed player, she has an incentive to delay her action in order to infer the more precise information by observing B's action. The more-informed player B also has an incentive to delay her action, but her delay is not for the sake of learning. B knows that when she acts as a leader, her true signal will be inferred perfectly by A, which will induce A's same action always. As B knows that she is the more-informed player, she regards A's identical action as a strategic substitute. Hence, she wants to prevent her information from being revealed to A and this is why she intends to delay her action. In this way, the presence of payoff externalities yield the conflict between two types of second mover advantage, the one from learning and the other from preventing other's learning.

The key result of this article is that, even though each players can earn gain from a delay in action, A's gain from a delay in action is greater than that of B. In other words, the gain from a delay for learning is greater than that for preventing the other's learning. Hence, when cost is imposed for a delay of action, as it increases, the first one who is driven out of the delay race is B. Therefore, if sequential actions are derived in a pure strategy equilibrium, the leader is always the more-informed player. If a delay cost allows both players to have positive net gains from a delay, there exists a mixed strategy equilibrium. Regarding this equilibrium, the comparative statics yield the possibility that the more-informed player B can act voluntarily without using a waiting option, although it is available, in order to minimize a risk of being penalized by inducing the less-informed player's identical action. Regarding the derived equilibrium, the further analysis about the effects of a waiting option on each player's welfare and the effects of a delay cost on the efficient ordering of action are also provided.

In brief, the analysis of this article shows that the introduction of payoff externalities provides a different rationale for the heterogeneous players' endogenous timing of actions than the model in which only the informational externalities are considered. If only the informational externalities are considered, the delay in action is initiated from an incentive of learning and the more-informed player acts as the leader because she has less to gain in learning than the other player. On the other hand, in this model, the consideration of payoff externalities yields the conflict between two types of second mover advantage because the more-informed player intends to prevent the less-informed player's learning. However, the more-informed player acts as the leader because she is less-patient to the payoff discount for a delay in action. Moreover, in the mixed strategy equilibrium, the

more-informed player can act as the leader voluntarily in order to induce the less-informed player's imitation. These kinds of incentives for the more-informed player, one which prevents the other's learning and one which induces the other's learning, are the results which can be derived from considering payoff externalities along with informational externalities. In this way, this study enriches the analysis of the heterogeneous players' endogenous ordering of actions when payoff externalities and informational externalities both matter.

The rest of this paper is organized as follows. In Section 2, I review the related literature. In Section 3, I introduce the model. In Section 4, I derive each player's best response according to the timing of action. In Section 5, as the benchmark case, I characterize the equilibrium of action timing when only the informational externalities matter. In Section 6, I characterize the equilibrium of action timing when payoff externalities matter along with informational externalities. Finally, Section 7 is a concluding remark.

2 RELATED LITERATURE

The following articles can be introduced as the ones which deal with the topic of endogenous timing of actions in various environments. Chamley and Gale (1994) and Zhang (1997) discuss the strategic delay and the endogenous timing of action when only the informational externalities are present. In Chamley and Gale (1994), a player has an incentive to delay her action in order to observe other players' decisions for information updating. Zhang (1997) links this result to the informational cascade topic. He asserts that, if players are heterogeneous, the most-informed player has the least patience in regards to the cost of delay because she has the least to learn than other players. Thus, she acts as the leader and other players mimic her action immediately. In these models, although the action timing is decided endogenously, no payoff externalities are considered. Thus, each player's main concern in deciding the delay in action is whether to infer other players' information for learning.

Damme and Hurkens (1998), (2004) consider a linear quantity setting and price setting duopoly game with differentiated products when the timing of commitment is decided endogenously. They show that in a risk-dominant equilibrium, the high cost firm will choose to wait and the low cost firm will emerge as the endogenous Stackelberg leader and price leader. Although the timing of action is decided endogenously, in both models, the informational externalities do not play a role in characterizing the equilibrium.

The most relevant paper on this topic with this article is Frisell (2003) which deals with a case in which two firms decide a product design and when to enter the market when the true value of product design is not known. The signal and action space are continuous and it assumes imperfect information about the other firm's information quality. By allowing the learning and direct payoff externalities of each firm's decision, it analyzes the effects of both informational and payoff externalities on the endogenous timing of actions as this article does. According to the result, if the payoff externalities are positive or relatively weakly negative, the more-informed firm acts as the leader because she has the less to learn. On the other hand, if the externalities are

strongly negative, the more-informed firm acts as the follower because it has the more to gain from outwaiting the other firm.

This model is distinct from Frisell (2003) in following points. In Frisell (2003), whether both players' similar actions cause the positive or the negative payoff externalities is assumed exogenously and, moreover, it is common to both players.[6] On the other hand, in my model, what is given exogenously is each player's information quality and the competition environment endogenously results in whether the other player's same action cause the positive or the negative payoff externalities. Furthermore two players evaluates the other player's same action differently. Hence, compared to Frisell (2003), this model specifies when the other player's same action is evaluated as the strategic substitute and complement and why it is. Also, the endogenously derived direct conflict of two types of the second mover advantage, which is not captured in Frisell (2003), yields the main result that the gain from delay for learning is greater than that for preventing other's learning. Although this model assumes that each agent's information quality is public information, the sufficient information for this type of conflict is who the more- and the less-informed player is. Thus, the analysis of this model can also be extended into the case in which only the comparative degree of being informed is known, which is more applicable to the real world. By providing the complementary results to Frisell (2003), the model in this study enriches the analysis of the heterogeneous players' endogenous timing game when payoff externalities and informational externalities both matter.

3   MODEL

Suppose there are two players, A and B, $i \in \{A, B\}$ whose job is to provide a forecast about the true state of forthcoming period. The true state is $w \in \{H, L\}$ and those are mutually exclusive. To both players, it is known that the prior probability of each state is $\Pr(w = H) = \Pr(w = L) = \frac{1}{2}$. Before making a forecast, each player observes her own signal $\theta_i \in \{h, l\}$ which is correlated with the true state. The draws of their signals are conditionally independent given the true state. Each player's signal $\theta_i \in \{h, l\}$ is private information, so that each player does not know which signal is observed by the other player. The signal θ_i partially reveals the information about the true state in following way

$$\Pr(\theta_i = h \,|\, w = H) = \Pr(\theta_i = l \,|\, w = L) = p_i$$
$$\Pr(\theta_i = h \,|\, w = L) = \Pr(\theta_i = l \,|\, w = H) = 1 - p_i$$

where $p_i \in \left(\frac{1}{2}, 1\right)$. Here, p_i measures the precision of player i's signal θ_i, so that it can also be interpreted as information quality. As p_i approaches $\frac{1}{2}$, it means her signal becomes less informative. As p_i approaches 1, it means her signal becomes more informative about the true state. I assume

---

[6]In Frisell (2003), each agent's payoff function is given as $\pi_i = -(\theta_i - \rho)^2 - \alpha(\theta_i - \theta_j)^2 - \delta t_i$, $i \neq j$ where ρ is the unknown true state, θ_i and θ_j are each player's selection of action, δ is payoff discount for delay and t_i is i's timing of action. Here, α measures the degree of payoff externalities. If $\alpha > (<)0$, both players' same actions cause the positive (negative) externalities. It is assumed that α is given exogenously and, for both players i and j, α is common.

that the players are heterogeneous in information quality. Without loss of generality, it is assumed that A is the less-informed player and B is the more-informed player.

Assumption 1 $p_A < p_B$

Player i's action set is denoted by $A = \{a_i, t_i\}$. Here, $a_i \in \{h, l\}$ denotes player i's action in regards to forecasting the true state. If $a_i = h$ ($a_i = l$), it denotes that player i's forecast is $w = H$ ($w = L$). Also, $t_i \in \{t_1, t_2\}$ denotes player i's timing of action. Each player has two rounds during which she can take one irreversible action. If player i acts in round 1 (round 2), then it is denoted by $t_i = t_1$ ($t_i = t_2$). If $t_i = t_1$ and $t_{-i} = t_2$, then it means that i is a leader and $-i$ is a follower. If the actions are taken sequentially, the follower can observe the leader's action before taking her own action. However, if $t_i = t_{-i}$, that is if both players act simultaneously, then each player has no chance to observe the other player's action.

Each player's net payoff is defined by

$$\tilde{\pi}_i(a_i, a_{-i}, t_i, t_{-i}) = \pi_i(\cdot) - f(t_i) \quad \text{where} \quad f(t_i) = \begin{cases} 0 & \text{if } t_i = t_1 \\ c & \text{if } t_i = t_2 \end{cases}$$

where $\pi_i(\cdot)$ denotes the gross payoff and $\tilde{\pi}_i(\cdot)$ denotes the net payoff after considering a penalty for a delay in action. If she acts in round 2, she has to pay a cost c for a delay in action. We assume that c is determined exogenously and can be interpreted as the discount in reputation for delaying action. The gross payoff π_i is determined after the realization of the true state w conditional on both players' actions, a_i and a_{-i}, as follows where $\gamma \geq 1$.

w	$a_B = w$	$a_B \neq w$
$a_A = w$	$1, 1$	$\gamma, -\gamma$
$a_A \neq w$	$-\gamma, \gamma$	$-1, -1$

This payoff structure is designed in order to incorporate the competitive environment between two players. Here, γ ($-\gamma$) is the payoff earned when a player is the unique one who took a correct (wrong) action. When both players act identically to reveal the true state correctly, both players get $+1$, and if not, they earn -1. When her action reveals the true state correctly, if the other player acts identically, then a negative payoff externality results because the good reputation should be shared. On the contrary, when her action fails in revealing the true state, if the other player acted identically, then a positive payoff externality results because the blame or the penalty can be shared. In this way, the accuracy of a player's action endogenously determines whether the other player's identical action is a strategic substitute or a strategic complement. However, as the true state w is not revealed until both players act, each player does not know certainly whether the other player's same action is affirmative or not.

In addition, the value of γ, whether it is $\gamma > 1$ or $\gamma = 1$, plays an important role. If $\gamma = 1$, then as each player's gross payoff depends only on the correctness of her own action, it corresponds to the case in which no payoff externalities are present. However, if $\gamma > 1$, then her gross payoff depends on the correctness of both her and the rival's actions. Thus, it corresponds to the case in which the payoff externalities are present.

Under this setup, player i should decide her timing of action t_i and the truthfulness of her action. Especially, the latter decision depends on the timings of actions of both her and the other player. Finally, below are the definitions which will be used throughout this paper.

Definition 1 *Truthful action: If $a_i = \theta_i$, then we say that player $i's$ action is truthful.*

Definition 2 *Herding: When $\theta_i \neq a_{-i}$, if $a_i = a_{-i} \neq \theta_i$, then we say that player i exhibits herding.*

4   DERIVING THE BEST RESPONSE

In this section, we derive each player's best response according to her timing of action. Below, it is shown that the derived results are quite intuitive. The detailed procedure of deriving the best responses is summarized in the Appendix. Instead, we provide a brief sketch of the procedure and focus on the intuition of the derived results.

First, assume that i acts as the follower, i.e., $t_i = t_2$ and $t_{-i} = t_1$ where $i \in \{A, B\}$. As i has her own information θ_i and can observe a_{-i}, her posterior belief is $\Pr(w|\theta_i, \theta_{-i})$. However, i does not know whether -i's action is truthful or not because she has no chance to observe θ_{-i}. Hence, θ_{-i} should be inferred according to her belief about the truthfulness of a_{-i}. Then, her best response is derived from

$$\sum_w \Pr(w|\theta_i, \theta_{-i})\pi\,(\theta_i = a_i, a_{-i}) \gtrless \sum_w \Pr(w|\theta_i, \theta_{-i})\pi\,(\theta_i \neq a_i, a_{-i}) \tag{1}$$

where LHS denotes the expected payoff when i reveals her signal θ_i truthfully and $E\pi_i(a_i \neq \theta_i)$ denotes the one when she deviates from her signal θ_i.

Second, assume that i acts as the leader, i.e., $t_i = t_1$ and $t_{-i} = t_2$ where $i \in \{A, B\}$. As i acts as the leader, she cannot observe -i's action before taking her own action, which means that i has no chance to infer θ_{-i}. Thus, i's posterior beliefs should be about the true state and -i's true signal, i.e., $\Pr(w, \theta_{-i}|\theta_i)$. From the best response of -i derived from (1), i can expect a_{-i} conditional on the conjectured θ_{-i}. Then, her best response is derived from

$$\sum_w \sum_{\theta_{-i}} \Pr(w, \theta_{-i}|\theta_i)\pi\,(\theta_i = a_i, a_{-i}) \gtrless \sum_w \sum_{\theta_{-i}} \Pr(w, \theta_{-i}|\theta_i)\pi\,(\theta_i \neq a_i, a_{-i}) \tag{2}$$

where LHS denotes the expected payoff when player i reveals her signal θ_i truthfully and $E\pi_i(a_i \neq \theta_i)$ denotes the one when she deviates from her signal θ_i.

Finally, assume that both players act simultaneously, i.e., $t_i = t_{-i}$. By assumption, as i has no chance to infer θ_{-i}, i's posterior beliefs should be $\Pr(w, \theta_{-i}|\theta_i)$. However, unlike the case in which she acts as the leader, -i's best response is not known. Therefore, a_{-i} should be expected according to her belief about the truthfulness of a_{-i} conditional on the conjectured θ_{-i}. Then, her best response is derived from (2).

The above analysis yields that each player's best response, according to both players' timings of action, can be summarized as follows.

Proposition 1

1) Player A:

1-1) Suppose that she acts as the leader or both players act simultaneously. Then, she reveals her signal truthfully.

1-2) Suppose that she acts as the follower. If $\theta_A = a_B(= \theta_B)$, she reveals her signal truthfully and if $\theta_A \neq a_B(= \theta_B)$, she exhibits herding.

2) Player B:

She always reveals her signal truthfully.

Proof

In the Appendix.

Suppose that each player acts as the follower. Then, in the case of B, she reports her signal truthfully always. Intuitively, from assumption $p_B > p_A$, B gives more credit to the correctness of her information. Thus, although $\theta_B \neq \theta_A$, she ignores θ_A and sticks to θ_B. In the case of A, she knows that B is the more-informed player. Hence, when $\theta_A \neq \theta_B$, she gives more weight to the possibility that B's signal is correct. Thus, she imitates B's action while ignoring her own signal.[7] If each player acts as the leader or actions are taken simultaneously, it is intuitive that a player reveals her signal truthfully as the only available information is her own signal. As the follower knows that leader's true signal is revealed truthfully, she assigns the zero probability to the possibility that the leader deviates from her signal. Therefore, the action of the leader reveals the leader's true signal and the follower can infer it perfectly through observing the leader's action.

In following, π_i^L denotes player i's expected gross payoff when she acts as the leader, π_i^F denotes the one when she acts as the follower, and π_i^S denotes the one when both players act simultaneously. Note that the timing of action is decided endogenously and each player should decide her timing of action in advance. As the available information to player i is only her own signal θ_i, the posterior belief should be about the true state and the other player's true signal $\Pr(w, \theta_{-i}|\theta_i)$. How a_{-i} will actually be realized depends on $-i$'s best responses which are demonstrated in Proposition 1. Then, each player's expected gross payoffs are calculated as follows,

$$
\begin{aligned}
\pi_A^L &= \pi_A^S = -\left(\gamma p_B - p_B - \gamma p_A - p_A + 1\right), \, \pi_A^F = (2p_B - 1) \\
\pi_B^L &= (2p_B - 1), \, \pi_B^F = \pi_B^S = (p_A + p_B - \gamma p_A + \gamma p_B - 1)
\end{aligned}
\tag{3}
$$

from $\sum_w \sum_{\theta_{-i}} \Pr(w, \theta_{-i}|\theta_i)\pi(a_i, a_{-i})$.

5   WHEN NO PAYOFF EXTERNALITIES ARE PRESENT

As a bench mark case, we assume the case in which no payoff externalities are present. This case corresponds to the one in which $\gamma = 1$ because her gross payoff depends only on the correctness

---

[7]In this case, herding is efficient because B has the more precise information than A does. Also, as B does not ignore her information, in this model, the inefficient herding does not happen.

of her own action. As the first step, suppose $c = 0$.[8] Then, from (3), each player's expected gross payoffs $\gamma = 1$ are

$$
\begin{aligned}
\pi_A^L &= \pi_A^S = (2p_A - 1), \; \pi_A^F = (2p_B - 1) \\
\pi_B^L &= \pi_B^F = \pi_B^S = (2p_B - 1)
\end{aligned}
\tag{4}
$$

The comparison of (4) yields that

$$
\pi_A^L = \pi_A^S < \pi_A^F \text{ and } \pi_B^L = \pi_B^F = \pi_B^S
$$

In the case of A, the best case is when she acts as a follower because A's perfect inference of θ_B is possible. When $\theta_B = a_B \neq \theta_A$, as A ignores her true signal and imitate B's action, she can prevent the case in which $a_B \neq a_A$ by acting as the follower. That is, A wants to delay her action for learning. On the other hand, whether she acts as a leader or both players' actions are taken simultaneously, it is same that A has no chance to infer B's signal before taking her own action. Therefore, $\pi_A^L = \pi_A^S$ is derived. On the other hand, in the case of B, $\pi_B^L = \pi_B^F = \pi_B^S$ is derived because her best response is always to reveal her signal truthfully and A's action does not affect her payoff.

Suppose that $c > 0$. Then, the equilibrium for the timing of action when $\gamma = 1$ can be characterized as follows.

Proposition 2
Suppose that no payoff externalities are present, i.e., $\gamma = 1$.
1) Suppose $c > 2(p_B - p_A)$. Then, $(t_A, t_B) = (t_1, t_1)$.
2) Suppose $c < 2(p_B - p_A)$. Then, $(t_A, t_B) = (t_2, t_1)$.

We denote $V_i^d(\gamma = 1)$ as player i's gain from a delay in action when $\gamma = 1$. Then,

$$
V_A^d(\gamma = 1) = 2(p_B - p_A) \text{ and } V_B^d(\gamma = 1) = 0
$$

As B has no reason to delay her action with paying a cost when she earns no gain from a delay, always $t_B = t_1$, i.e., $V_B^d(\gamma = 1) = 0 < c$. On the contrary, A wants to delay her action for learning and whether she actually can delay her action or not depends on the value of c. If the expected gain from a delay in action is greater than the cost of delay, i.e., $c < 2(p_B - p_A) = V_A^d$, then she delays her action and acts at $t = t_2$. If not, i.e., $c > 2(p_B - p_A) = V_A^d$, then A should give up delaying her action and act at $t = t_1$. Thus, if the sequential actions are derived endogenously, the leader is always the more-informed player. Also, from $\frac{\partial V_A^d(\gamma=1)}{\partial p_B} > 0$ and $\frac{\partial V_A^d(\gamma=1)}{\partial p_A} < 0$, the following result is derived.

---

[8]The analysis under the assumption $c = 0$ clarifies whether the agent's incentive to delay her action is caused by the cost of delay. For example, suppose that $\pi_i^F > \pi_i^L$ when $c = 0$. When $c > 0$, if she acts at round 1, it means that she should act without delay although she wants to use a waiting option. On the other hand, suppose that when $c = 0$, $\pi_i^L > \pi_i^F$. Then, although $c > 0$, she will act in round 1 because she wants to act in round 1 voluntarily. It can be checked that her action as the leader is not caused by the cost of a delay in action.

Corollary 1

Suppose that $\gamma = 1$. Then, as p_B increases or p_A decreases, the probability that the sequential actions are derived endogenously increases.

6 WHEN PAYOFF EXTERNALITIES ARE PRESENT

6.1 EQUILIBRIUM

Now, assume that $\gamma > 1$. Then, as player i's gross payoff depends on the correctness of both players' actions a_i and a_{-i}, the payoff externalities are present. If we assume that $c = 0$, then the comparison of the gross expected payoffs yields

$$\begin{aligned} \pi_A^L &= \pi_A^S < \pi_A^F \\ \pi_B^L &< \pi_B^F = \pi_B^S \end{aligned}$$

In the case of A, although payoff externalities matter, there should be no change in that the best case to her is when she acts as the follower for the possibility of learning. Meanwhile, in the case of B, unlike the case in which no payoff externalities are present ($\pi_B^L = \pi_B^F = \pi_B^S$), the worst case is when she acts as the leader. This result can be explained from the following reasoning. From B's viewpoint, if she acts as the leader, her true signal is revealed to A perfectly and it induce A's identical action. Assume that θ_B reveals the true state correctly. In this case, if B is the unique player who took the correct action, B's payoff is $\pi_B = \gamma$. However, if A imitates her action, B's payoff is $\pi_B = 1 < \gamma$. Of course, if it turns out that $\theta_B \neq w$, A's imitation can work in B's favor because she earns $-1 > -\gamma$. However, from the assumption $p_A < p_B$, B gives more weight to the possibility that her signal is correct. Thus, she gives more weight to the possibility that A's imitation causes the negative payoff externalities. In other words, B regards A's same action as a strategic substitute. Therefore, it can be checked that B intends to delay her action not for the sake of learning, but for the sake of preventing her information from being revealed to A. Whether she acts as a follower or both players act simultaneously, for either case, B's true signal is not revealed to A. Thus, $\pi_B^F = \pi_B^S$ is derived.

Remark 2

Suppose that the payoff externalities are present, i.e., $\gamma > 1$. Then,

1) The less-informed player wants to delay her action for learning.

2) The more-informed player wants to delay her action in order to prevent her information from being revealed. In other words, she wants to delay her action in order to prevent the less-informed player's learning.

As Remark 2 shows, the competitive environment between players results in a conflict between two types of second mover advantages. Hence, if the timing of actions is decided endogenously, then our game turns out to be the war of attrition and the cost for a delay in action plays an important role in characterizing the equilibrium.

Under the payoff discount $c > 0$, the payoff matrix which describes our game is as follows

	$t_B = t_1$	$t_B = t_2$
$t_A = t_1$	$\tilde{\pi}_A^{S1}, \tilde{\pi}_B^{S1}$	$\tilde{\pi}_A^{L}, \tilde{\pi}_B^{F}$
$t_A = t_2$	$\tilde{\pi}_A^{F}, \tilde{\pi}_B^{L}$	$\tilde{\pi}_A^{S2}, \tilde{\pi}_B^{S2}$

where $\tilde{\pi}_i^{S1} = \pi_i^S$, $\tilde{\pi}_i^L = \pi_i^L$, $\tilde{\pi}_i^F = \pi_i^F - c$ and $\tilde{\pi}_i^{S2} = \pi_i^S - c$ for $i \in \{A, B\}$.

If we recall (3), each player's gain from a delay in action is

$$V_A^d (\gamma > 1) = (p_B - p_A)(\gamma + 1) \text{ and } V_B^d (\gamma > 1) = (p_B - p_A)(\gamma - 1) \tag{5}$$

where $V_i^d (\gamma > 1)$ denotes i's value from a delay in action when $\gamma > 1$. Especially, $V_A^d (\gamma > 1)$ is A's expected gain attained from learning through observing B's action. Also, $V_B^d (\gamma > 1)$ is B's expected gain attained through preventing A's learning. Then, from (5), following can be proposed.

Proposition 3

The gain of delay for learning is greater than that for preventing the other's learning, i.e.,
$$V_B^d (\gamma > 1) < V_A^d (\gamma > 1)$$

That is, although both players receive gains from a delay in action, the gain of the less-informed player is greater than that of the more-informed player. Then, for given $c > 0$, the equilibrium of the action timing can be characterized as follows.

Proposition 4

Suppose that the payoff externalities are present, i.e., $\gamma > 1$. Then, the equilibrium of the timing of action can be characterized as follows.

1) Suppose $0 < c < V_B^d (\gamma > 1)$. Then, $(z, q) = \left(\frac{(p_B - p_A)(\gamma - 1) - c}{(\gamma - 1)(p_B - p_A)}, \frac{c}{(\gamma + 1)(p_B - p_A)} \right)$ where $z = \Pr(t_A = t_1)$ and $q = \Pr(t_B = t_1)$.

2) Suppose $V_B^d (\gamma > 1) < c < V_A^d (\gamma > 1)$. Then, $(t_A, t_B) = (t_2, t_1)$.

3) Suppose $c > V_A^d (\gamma > 1)$. Then, $(t_A, t_B) = (t_1, t_1)$.

Proof

The straightforward computation yields the equilibrium for following two cases, $c_2 < c < c_1$ and $c > c_1$. Below, we focus on the mixed strategy equilibrium for $0 < c < c_2$. In the following, we denote $z = \Pr(t_A = t_1)$ and $q = \Pr(t_B = 1)$. Then, for player A, $V_A(t_A = t_1) = q\tilde{\pi}_A^{S1} + (1 - q)\tilde{\pi}_A^L$ and $V_A(t_A = t_2) = q\tilde{\pi}_A^F + (1 - q)\tilde{\pi}_A^{S2}$ where $V_A(t_A = t_n)$ is the value of taking action at t_n, $n = 1, 2$. Then, from $V_A(t_A = t_1) = V_A(t_A = t_2)$,

$$q = \frac{\tilde{\pi}_A^{S2} - \tilde{\pi}_A^L}{\tilde{\pi}_A^{S1} - \tilde{\pi}_A^L - \tilde{\pi}_A^F + \tilde{\pi}_A^{S2}} = \frac{c}{(\gamma + 1)(p_B - p_A)}$$

In case of B, $V_B(t_B = t_1) = z\tilde{\pi}_B^{S1} + (1 - z)\tilde{\pi}_B^L$ and $V_B(t_B = t_2) = z\tilde{\pi}_B^F + (1 - z)\tilde{\pi}_B^{S2}$. Then from

12

$$V_B(t_B = t_1) = V_B(t_B = t_2),$$

$$z = \frac{\tilde{\pi}_B^{S2} - \tilde{\pi}_B^L}{\tilde{\pi}_B^{S1} - \tilde{\pi}_B^L - \tilde{\pi}_B^F + \tilde{\pi}_B^{S2}} = \frac{(p_B - p_A)(\gamma - 1) - c}{(\gamma - 1)(p_B - p_A)}$$

Finally, if $0 < c < c_2$, $(z, q) = \left(\frac{(p_B - p_A)(\gamma - 1) - c}{(\gamma - 1)(p_B - p_A)}, \frac{c}{(\gamma + 1)(p_B - p_A)} \right)$ where $z = \Pr(t_A = t_1)$, $q = \Pr(t_B = 1)$. ∎

First, if $c > V_A^d (\gamma > 1)$, both players act simultaneously at $t = t_1$ because both players' gains from a delay in action are dominated by the sufficiently high cost for a delay in action. Second, if $V_B^d (\gamma > 1) < c < V_A^d (\gamma > 1)$, then A acts as the follower and B acts as the leader. This explains the situation in which, as the cost for a delay in action increases, the first player driven out of the delay race is B because her gain is less than that of A.

Finally, if $0 < c < V_B^d (\gamma > 1)$, both players can delay their actions because a cost of delay in action is even less than the more-informed player's gain of delay. In this case, there exists no pure strategy Nash equilibrium from following reasoning. Suppose that $t_B = t_2$. Then A's best response is $t_A = t_1$ because there is no need for her to delay her action with paying cost when she cannot observe a_B. Then, for $t_A = t_1$, B's best response is $t_B = t_1$ because if $t_A = t_1$, her information is not revealed to A whether $t_B = t_1$ or $t_B = t_2$. Then, for $t_B = t_1$, A's best response is again $t_A = t_2$ because she can observe a_B by delaying her action. Then, again, B's best response is $t_B = t_2$ to prevent her information from being revealed to A. In this way, each player wants to outguess the other player, which yields no pure strategy Nash equilibrium.

Regarding the mixed strategy equilibrium, the comparative statics yield the following result.

Corollary 2

Consider the mixed strategy equilibrium derived when $\gamma > 1$ and $0 < c < (p_B - p_A)(\gamma - 1)$.

1) As γ increases, there is more possibility that $(t_A, t_B) = (t_1, t_2)$.

2) As c Increases, there is more possibility that $(t_A, t_B) = (t_2, t_1)$.

3) As p_A increases, there is more possibility that $(t_A, t_B) = (t_2, t_1)$.

4) As p_B increases, there is more possibility that $(t_A, t_B) = (t_1, t_2)$.

Proof

Note that $(z, q) = \left(\frac{(p_B - p_A)(\gamma - 1) - c}{(\gamma - 1)(p_B - p_A)}, \frac{c}{(\gamma + 1)(p_B - p_A)} \right)$ where $z = \Pr(t_A = t_1)$ and $q = \Pr(t_B = t_1)$. Then, $\frac{\partial q}{\partial \gamma} < 0$, $\frac{\partial z}{\partial c} < 0$, $\frac{\partial q}{\partial c} > 0$, $\frac{\partial q}{\partial p_A} > 0$ and $\frac{\partial q}{\partial p_B} < 0$ are obvious from given z and q. Also, $\frac{\partial z}{\partial \gamma} = \frac{c}{(p_B - p_A)(\gamma - 1)^2} > 0$, $\frac{\partial z}{\partial p_A} = -\frac{c}{(\gamma - 1)(p_B - p_A)^2} < 0$, and $\frac{\partial z}{\partial p_B} = \frac{c}{(\gamma - 1)(p_B - p_A)^2} > 0$. ∎

First, if γ increases, there is more of a possibility that $(t_A, t_B) = (t_1, t_2)$. Here, γ denotes the degree of the payoff externalities. Thus, as γ increases, B's payoff loss caused from A's imitation increases if her signal reveals the true state correctly. Therefore, B's desire to prevent her information from being revealed will increases. Player A will also have a greater incentive to imitate B's action because, as γ increases, the penalty which she earns when her action turns out to be wrong increases. The result expects that, as the degree of the payoff externalities increases, the desire for preventing the other's learning is greater than that of learning.

13

Second, as c increases, there is more possibility that $(t_A, t_B) = (t_2, t_1)$. Compared to A who intends to delay for learning, the opportunity cost of B's delay will be greater because her delay is only for preventing her information from being revealed without any learning. Thus, as c increases, there will be more of a possibility that B becomes less patient to the penalty for a delay in action.

Third, as A's information quality increases, there is more of a possibility that $(t_A, t_B) = (t_2, t_1)$. Here, the high value of p_A implies a greater possibility that $\theta_A = w$. From B's viewpoint, as p_A increases, she should also consider the case in which $\theta_A = w \neq \theta_B$. When it actually is, if A acts as the leader and B acts as the follower, then $\pi_A = \gamma$ and $\pi_B = -\gamma$, which is the worst case to B. However, if B acts as the leader and A does as the follower, always $a_A = a_B$ is derived because of A's imitation. Then, although it turns out that $a_A = a_B \neq w$, B's payoff is $\pi_B = -1 > -\gamma$. In this way, by acting as the leader, B can prevent the lowest payoff by inducing A's imitation. This is why B can act as the leader voluntarily without using a waiting option, although it is available. Of course, the high value of p_A implies the high value of p_B from our assumption $p_A < p_B$. Thus, B may also have the greater incentive to act as the follower not to reveal her information. Regarding these two conflicts, the result predicts that the former dominates the latter. In the case of A, the high value of p_A means a high possibility of $\theta_A = w$. Then, this gives her less incentive to delay her action because her learning incentive decreases. However, the high value of p_A also means a high possibility of $\theta_B = w$ from our assumption $p_A < p_B$. Thus, acting as the follower can still be attractive to her. For these two conflicts, the result predicts that the latter dominate the former, which makes A more likely to delay her action.

Finally, as B's information quality decreases, there is more of a possibility that the equilibrium is $(t_A, t_B) = (t_2, t_1)$. The low p_B implies a greater possibility of $\theta_B \neq w$. Then, from the same reasoning as found in the previous case, it can be better for B to act as the leader to induce A's imitation. From A's viewpoint, the low p_B also implies a low p_A from our assumption $p_A < p_B$. Thus, she can be more likely to delay her action.

Corollary 3

In a mixed strategy equilibrium, where $\gamma > 1$ and $0 < c < (p_B - p_A)(\gamma - 1)$, if the difference between two informed players' information quality is relatively small, B regards A's identical action as a strategic complement. That is, B can act in round 1 voluntarily without using a waiting option, although it is available, in order to minimize a risk of payoff loss.

6.2 EFFICIENCY

For given c, the consideration of the pure strategy equilibrium yields that

$$\sum_{i \in \{A,B\}} \tilde{\pi}_i(t_2, t_2) < \sum_{i \in \{A,B\}} \tilde{\pi}_i(t_1, t_2) = \begin{cases} < \sum_{i \in \{A,B\}} \tilde{\pi}_i(t_1, t_1) \\ < \sum_{i \in \{A,B\}} \tilde{\pi}_i(t_2, t_1) \end{cases}$$

14

where $\tilde{\pi}_i(t_A, t_B)$ denotes the expected payoff of $i \in \{A, B\}$ when $t_A, t_B \in \{t_1, t_2\}$. Also it can be checked that

$$\sum_{i \in \{A,B\}} \tilde{\pi}_i(t_A, t_B) = (2p_A - c + 2p_B - 2) = \begin{cases} < \sum_{i \in \{A,B\}} \tilde{\pi}_i(t_1, t_1) \\ < \sum_{i \in \{A,B\}} \tilde{\pi}_i(t_2, t_1) \end{cases}$$

where LHS is the sum of both players' expected payoffs in a mixed strategy equilibrium. Hence, the efficient ordering of action depends on the comparison of $\sum_{i \in \{A,B\}} \tilde{\pi}_i(t_1, t_1)$ and $\sum_{i \in \{A,B\}} \tilde{\pi}_i(t_2, t_1)$ where $\sum_{i \in \{A,B\}} \tilde{\pi}_i(t_1, t_1) = 2(p_A + p_B - 1)$ and $\sum_{i \in \{A,B\}} \tilde{\pi}_i(t_2, t_1) = 2(2p_B - 1) - c$. Then, it can be shown that if $c > 2(p_B - p_A)$, $(t_A, t_B) = (t_1, t_1)$ is efficient and if $0 < c < 2(p_B - p_A)$, $(t_A, t_B) = (t_2, t_1)$ is efficient.

Alternatively, we can also consider the role of a cost for a delay in action on the efficient ordering of action when it is measured taking into consideration the gross expected payoffs, the one before considering a payoff discount for delay. Then, the efficiency is obtained when A acts as the follower and B acts as the leader, i.e., $(t_A, t_B) = (t_2, t_1)$ because A can imitate B's action which is based on the more precise information. Recall Proposition 3 and interpret c as a control variable. Then, if $0 < c < c_2$, in a mixed strategy equilibrium, $(t_A, t_B) = (t_2, t_1)$ is derived with probability $w(1-q)$. If $c_2 < c < c_1$, the unique pure strategy equilibrium is $(t_A, t_B) = (t_2, t_1)$. Hence, the efficient ordering of action is always derived endogenously. Finally, if $c > c_1$, $(t_A, t_B) = (t_1, t_1)$. Thus, the efficient ordering of action cannot be derived at all.

If we assume the case in which no cost is imposed for a delay of action, i.e., $c = 0$, B certainly delays her action in order to prevent her information from being revealed to A. Then, in this case, there exist the multiple equilibria $(t_A, t_B) = (t_1, t_2)$ and $(t_A, t_B) = (t_2, t_2)$. Although she delays her action, as A has no chance to observe B's true signal, she has the same expected payoff regardless of her timing of action. This shows that, if no cost is imposed for a delay in action, then the efficient ordering of action cannot be derived at all. Therefore, when the timing of action is decided endogenously, a cost for a delay in action should be imposed in order to derive the efficient ordering. However, it should be noted that imposing a cost does not guarantee efficient ordering. Therefore, in order not to yield the suboptimal outcome, the knowledge about both players' information quality is essential.

6.3   EFFECT OF WAITING OPTION

In this section, we address the problem of whether the availability of a waiting option makes a player ex-ante better off or worse off compared to the case in which both players are forced to take actions simultaneously without using a waiting option. According to Proposition 2, if $c > c_1$, then a waiting option is not available to any player because c is greater than both players' gains from having a delay. Thus, we will focus on the following two cases, where $c_2 < c < c_1$ and $0 < c < c_2$ within which a waiting option is available to at least one of both players.

First, if $c_2 < c < c_1$, the equilibrium is $(t_A, t_B) = (t_2, t_1)$ and the waiting option is available only to A. Then, from $\tilde{\pi}_A^F > \tilde{\pi}_A^L = \tilde{\pi}_A^{S1} > \tilde{\pi}_A^{S2}$, it is easy to check that A becomes ex-ante better-off from using a waiting option compared to the simultaneous actions at $t = t_1$. Second, if $0 <$

$c < c_2$, the waiting option is available to both players and the mixed strategy equilibrium $(z, q) = \left(\frac{(p_B - p_A)(\gamma - 1) - c}{(\gamma - 1)(p_B - p_A)}, \frac{c}{(\gamma + 1)(p_B - p_A)} \right)$ exists. In the case of A, the value of taking action at $t = t_1$ without delay is given as $V_A(t_A = t_1, q) = q\tilde{\pi}_A^{S1} + (1 - q)\tilde{\pi}_A^L$ and the value of using waiting option is given as $V_A(t_A = t_2, q) = q\tilde{\pi}_A^F + (1 - q)\tilde{\pi}_A^{S2}$. Then, from the existence of $0 < q^* < 1$ such that $V_A(t_A = t_1; q^*) = V_A(t_A = t_2; q^*)$, if we let

$$V(q^*) = V_A(t_A = t_1; q^*) = V_A(t_A = t_2; q^*)$$

where $q^*(\gamma, c, p_A, p_B) = \frac{c}{(\gamma + 1)(p_B - p_A)}$, then

$$\begin{aligned} \tilde{\pi}_A^{S1} - V(q^*) &= \tilde{\pi}_A^{S1} - \left(q^*\tilde{\pi}_A^{S1} + (1 - q^*)\tilde{\pi}_A^L \right) \\ &= \left(\tilde{\pi}_A^{S1} - \tilde{\pi}_A^L \right)(1 - q) = 0 \end{aligned}$$

which yields $\tilde{\pi}_A^{S1} = V(q^*)$ because $\tilde{\pi}_A^{S1} = \tilde{\pi}_A^L$. Therefore, the availability of a waiting option gives no effect on A's welfare.

In the case of B, the value of taking action at $t = t_1$ without delay is given by $V_B(t_B = t_1, w) = z\tilde{\pi}_B^{S1} + (1 - z)\tilde{\pi}_B^L$ and the value of using waiting option is given as $V_B(t_B = t_2, w) = z\tilde{\pi}_B^F + (1 - z)\tilde{\pi}_B^{S2}$. Then, from the existence of $0 < z^* < 1$ such that $V_B(t_B = t_1, z^*) = V_B(t_B = t_2; z^*)$, if we let

$$V(z^*) = V_B(t_B = t_1, z^*) = V_B(t_B = t_2; z^*)$$

where $z^*(\gamma, c, p_A, p_B) = \frac{(p_B - p_A)(\gamma - 1) - c}{(\gamma - 1)(p_B - p_A)}$, then

$$\begin{aligned} \tilde{\pi}_B^{S1} - V(z^*) &= \tilde{\pi}_B^{S1} - \left(z\tilde{\pi}_B^{S1} + (1 - z)\tilde{\pi}_B^L \right) \\ &= \left(\tilde{\pi}_B^{S1} - \tilde{\pi}_B^L \right)(1 - z) > 0 \end{aligned}$$

which yields $\tilde{\pi}_B^{S1} > V(z^*)$ because $\tilde{\pi}_B^{S1} > \tilde{\pi}_B^L$. Thus, in the case of B, the availability of waiting option makes her worse-off compared to the simultaneous action at $t = t_1$.

Corollary 4

1) The availability of a waiting option makes the less-informed player weakly better-off compared to the case in which the actions are taken simultaneously in round 1.

2) The availability of a waiting option makes the more-informed player worse-off compared to the case in which the actions are taken simultaneously in round 1.

In other words, above results can be interpreted as follows. If a waiting option is used for learning (preventing the other's learning), it makes a player weakly better-off (strictly worse-off) compared to the case in which both players should act simultaneously in round 1.

6.4   WHEN ONLY THE FOLLOWER'S PAYOFF IS DISCOUNTED

At the above sections, it was assumed that a cost for a delay in action is imposed if a player acts in round 2. In this section, we offer a slight change to this assumption. Consider the case in which two

16

agents are competing against each other in the same organization and a duty or a task is given to them by the chief of the organization. Then, the assumption that a discount is imposed for a delay in action is reasonable because the chief may know that the option of action timing is available to both agents and a delay in action can be detected. However, the more common case is the one in which the options for the timing of action are not likely to be known. For example, assume the situation in which two competing firms are planning to launch new products or use a new marking strategy. Then, what the consumers usually observe is who acts as the leader and who acts as the follower and they will give a discount only to the follower. If we apply this consideration to the model, then it corresponds to the case in which no penalty is imposed for either player, even though both players act in round 2 if they act simultaneously.

Then, our game can be described by the following payoff matrix,

	$t_B = t_1$	$t_B = t_2$
$t_A = t_1$	π_A^S, π_B^S	$\pi_A^L, \tilde{\pi}_B^F$
$t_A = t_2$	$\tilde{\pi}_A^F, \pi_B^L$	π_A^S, π_B^S

where $\pi_A^L = \pi_A^S = -(\gamma p_B - p_B - \gamma p_A - p_A + 1)$, $\tilde{\pi}_A^F = (2p_B - 1) - c$, $\pi_B^L = (2p_B - 1)$, $\pi_B^S = (p_A + p_B - \gamma p_A + \gamma p_B - 1)$ and $\tilde{\pi}_B^F = (p_A + p_B - \gamma p_A + \gamma p_B - 1) - c$.

Proposition 5

Suppose that the penalty for a delay in action is imposed only if a player acts as the follower. If $c > (p_B - p_A)(\gamma + 1)$, $(t_A, t_B) = (t_1, t_1)$, if $c = (p_B - p_A)(\gamma + 1)$, $(t_A, t_B) = (t_1, t_1)$ and (t_2, t_2) and if $c < (p_B - p_A)(\gamma + 1)$, $(t_A, t_B) = (t_2, t_2)$.

Proof

Assume that $z = \Pr(t_A = t_1)$ *and* $q = \Pr(t_B = t_1)$. First, for A, $V_A(t_A = t_1) = q\pi_A^S + (1-q)\pi_A^L$ and $V_A(t_A = t_2) = q\tilde{\pi}_A^F + (1-q)\pi_A^S$ where $V_A(t_A = t_n)$ is the value of taking action at t_n, $n = 1, 2$. Then,

$$V_A(t_A = t_1) - V_A(t_A = t_2) = -q(p_B - p_A - c - \gamma p_A + \gamma p_B)$$

So if $c < (p_B - p_A)(\gamma + 1)$, from $V_A(t_A = t_1) < V_A(t_A = t_2)$, the best response is $q = 0$ and if $c > (p_B - p_A)(\gamma + 1)$, from $V_A(t_A = t_1) < V_A(t_A = t_2)$, the best response is $q = 1$. Also, for B, $V_B(t_B = t_1) = z\pi_B^S + (1-z)\pi_B^L$ and $V_B(t_B = t_2) = z\tilde{\pi}_B^F + (1-z)\pi_B^S$. Then,

$$\begin{aligned} V_B(t_A &= t_1) - V_B(t_A = t_2) \\ &= z(c + p_A - p_B - \gamma p_A + \gamma p_B) + p_B - p_A + \gamma p_A - \gamma p_B \end{aligned}$$

where $c + p_A - p_B - \gamma p_A + \gamma p_B = (p_B - p_A)(\gamma - 1) + c > 0$. So, if $z > z^*$, from $V_B(t_A = t_1) > V_B(t_A = t_2)$, $q = 1$, if $z < z^*$, from $V_B(t_A = t_1) < V_B(t_A = t_2)$, $q = 0$ and if $z = z^*$, $V_B(t_A = t_1) = V_B(t_A = t_2)$ where $z = \frac{\tilde{\pi}\pi_B^S - \pi_B^L}{\tilde{\pi}_B^{S1} - \pi_B^L - \tilde{\pi}_B^F + \tilde{\pi}_B^{S2}} = \frac{(\gamma-1)(p_B-p_A)}{c+(p_B-p_A)(\gamma-1)}$. From these, if $c < (p_B - p_A)(\gamma + 1)$, $(q, z) = (0, 0)$, if $c = (p_B - p_A)(\gamma + 1)$, $(q, z) = (0, 0)$ and $(1, 1)$ and if $c > (p_B - p_A)(\gamma + 1)$, $(q, z) = (1, 1)$. ∎

Therefore, although $c > 0$ is imposed, if a payoff is discounted only when a player acts as the follower, always the simultaneous actions are derived endogenously. This also implies that, the efficient ordering of action, under which the less-informed player can imitate the more-informed player's action, cannot be derived endogenously.

7 CONCLUDING REMARK

In this paper, we analyze the effects of both payoff externalities and informational externalities on the endogenous timing of actions. If each player's information quality is public information, so if it is known who the more- and the less-informed player, the less-informed (more-informed) player regards the more-informed (less-informed) player's same action as a strategic complement (substitute). Hence, the less-informed player wants to delay her action in order to learn and the more-informed player delays her action in order to prevent the less-informed player's learning. This conflict between two types of second mover advantage yields a delay race. However, the gain from a delay for learning is greater than that for preventing other's learning. Hence, when a penalty is imposed for a delay in action, as it increases, the first one driven out of the delay race is the more-informed player. Therefore, in pure strategy equilibrium, if the sequential actions are derived, the more-informed player acts as a leader. Moreover, if a cost for delay of action is sufficiently low, there exists a mixed strategy equilibrium in which the more-informed player can act without using a waiting option voluntarily in order to minimize a risk by inducing the less-informed player's same action. In this way, in contrast to the literature in which only informational externalities are considered, this article shows that the introduction of payoff externalities contributes to different procedures and reasoning processes through which the heterogeneous players' timings of actions are decided endogenously.

The available extensions of the current model are as follows. The analysis of above sections was based on the assumption that each player's information quality is public information, which can be a too strong assumption to be applicable to the real world. Instead, the available common case will be the one in which who the more- and the less-informed agent are is known to both players although the exact value of each player's information quality is still private information. For example, when two agents are competing against each others for the common task, if one is the reputable agent and the other one is not, then there can be a consensus in two players' beliefs for who the more- and the less-informed agents are. Regarding this situation, the analysis of this article provides a clue to the analysis about that kind of situation. If we recall the procedure of deriving each player's equilibrium strategy and equilibrium of timing of action, the sufficient information is not the exact values of those, but whose information is more precise, i.e., $p_A < p_B$. Therefore, although the assumption that information quality is bot public information, if it is known who the more- and the less-informed player are, the conflict between two types of second mover advantage will be derived. However, the equilibrium will be characterized differently because the exact value of information quality is missing. As the other available extension, the analysis about the case in which each player's information quality is private information will be worthwhile. In this case,

we can conjecture that each player can use the cut-off strategy which depends on her information quality. If it is sufficiently low, she can have an incentive to delay her action for learning. On the other hand, if it is sufficiently high, she can also have an incentive to delay her action in order to prevent the other's learning. Furthermore, there can also be the possibility that, if her information quality is intermediate, she can act voluntarily without using a waiting option in order to minimize her risk of payoff loss by inducing the other agent's identical action. These types of extensions will enrich the analysis of the endogenous timing game of heterogeneous players and would be good topics for the future research studies.

8 APPENDIX

8.1 PROOF OF PROPOSITION 1

8.1.1 BEST RESPONSE AS THE FOLLOWER

PLAYER A Assume that A acts in round 2 when B already has acted in round 1. As A can observe B's action, she faces one of following two cases: $\theta_A = a_B$ or $\theta_A \neq a_B$. However, in each case, A does not know whether B's action is truthful or not because she has no chance to observe B's true signal. Thus, A's best response as a follower should be derived according to her belief for the truthfulness of B's action. Without a loss of generality, it is assumed that $\theta_A = h$. In following, A follows the decision rule (1).

Case 1) When A believes that B's announcement is truthful.

Suppose that A believes that $a_B = \theta_B$. First, if A's signal is same as B's action, i.e., $\theta_A = a_B = h$, her posterior beliefs are $Pr(w = H \mid h_A, h_B) = \frac{p_A p_B}{p_A p_B + (1-p_A)(1-p_B)}$ and $Pr(w = L \mid h_A, h_B) = \frac{(1-p_A)(1-p_B)}{p_A p_B + (1-p_A)(1-p_B)}$. Then, $E\pi_A(a_A = \theta_A) = \frac{p_A + p_B - 1}{2 p_A p_B - p_B - p_A + 1} > 0$ and $E\pi_A(a_A \neq \theta_A) = -\frac{(p_A + p_B - 1)\gamma}{(2 p_A p_B - p_B - p_A + 1)} < 0$. Thus, if $\theta_A = a_B$, A's best response is to reveal her signal truthfully. Second, suppose that A's signal is not same as B's action, i.e., $\theta_A = h$ and $a_B = l$. Then, A's posterior beliefs are $Pr(w = H \mid h_A, l_B) = \frac{p_A(1-p_B)}{p_A(1-p_B) + (1-p_A)p_B}$ and $Pr(w = L \mid h_A, l_B) = \frac{(1-p_A)p_B}{p_A(1-p_B) + (1-p_A)p_B}$. Then, $E\pi_A(a_A = \theta_A) = \frac{(p_B - p_A)\gamma}{2 p_A p_B - p_B - p_A} < 0$ and $E\pi_A(a_A \neq \theta_A) = -\frac{(p_B - p_A)}{(2 p_A p_B - p_B - p_A)} > 0$. Thus, if $\theta_A \neq a_B$, A's best response is to imitate B's action ignoring her own signal.

Case 2) When A believes that B's action is not truthful

Suppose that A believes that $a_B \neq \theta_B$. First, if A's signal is same as B's action, i.e., $\theta_A = a_B = h$. Then, A believes that B's true signal is $\theta_B = l$. Thus, player A's posterior beliefs are $Pr(w = H \mid h_A, l_B) = \frac{p_A(1-p_B)}{p_A(1-p_B) + (1-p_A)p_B}$ and $Pr(w = L \mid h_A, l_B) = \frac{(1-p_A)p_B}{(1-p_A)p_B + p_A(1-p_B)}$. Then, $E\pi_A(a_A = \theta_A) = \frac{p_B - p_A}{2 p_A p_B - p_B - p_A} < 0$ and $E\pi_A(a_A \neq \theta_A) = -\frac{(p_B - p_A)\gamma}{2 p_A p_B - p_B - p_A} > 0$. Hence, A's best response is to deviate from her true signal. Second, suppose that A's signal is not same as B's action, i.e., $\theta_A = h, a_B = l$. In this case, A believes that B's true signal is $\theta_B = h$. So, A's posterior beliefs are $Pr(w = H \mid h_A, h_B) = \frac{p_A p_B}{p_A p_B + (1-p_A)(1-p_B)}$ and $Pr(w = L \mid h_A, h_B) = \frac{(1-p_A)(1-p_B)}{p_A p_B + (1-p_A)(1-p_B)}$. Then, $E\pi_A(a_A = \theta_A) = \frac{(p_A + p_B - 1)\gamma}{2 p_A p_B - p_B - p_A + 1} > 0$ and $E\pi_A(a_A \neq \theta_A) = -\frac{p_A + p_B - 1}{2 p_A p_B - p_B - p_A + 1} < 0$, which yields that A's best response is to reveal her signal truthfully.

Lemma A.1

Suppose that A acts in round 2 and B has already acted in round 1. Then A's best response can be described as follows.

1) Suppose that A believes that B's action is truthful. Then if $\theta_A = a_B$, she reveals her signal truthfully, but if $\theta_A \neq a_B$, she exhibits herding.

2) Suppose that A believes that B's action is not truthful. Then if $\theta_A = a_B$, she exhibits herding, but if $\theta_A \neq a_B$, she reveals her signal truthfully.

PLAYER B Assume that B acts in round 2 when A already acted in round 1. Again, B is lack of the chance to observe A's true signal. Thus, B's best response should be derived according to

her belief for the truthfulness of A's action. Without a loss of generality, it is assumed that $\theta_B = h$ and she follows the decision rule (1).

Case 1) When B believes that A's truthful action

Suppose that B believes that $a_A = \theta_A$. First, if B's signal is same as A's action, i.e., $a_A = \theta_B = h$, B's posterior beliefs are $Pr(w = H| h_A, h_B) = \frac{p_A p_B}{p_A p_B + (1-p_A)(1-p_B)}$ and $Pr(w = L| h_A, h_B) = \frac{(1-p_A)(1-p_B)}{p_A p_B + (1-p_A)(1-p_B)}$. Then, $E\pi_B(a_B = \theta_B) = \frac{p_A + p_B - 1}{2p_A p_B - p_B - p_A + 1} > 0$ and $E\pi_B(a_B \neq \theta_B) = -\frac{(p_A + p_B - 1)\gamma}{(2p_A p_B - p_B - p_A + 1)} < 0$. Thus, B's best response as the follower is to reveal her signal truthfully. Second, suppose that B's signal is not same with A's action, i.e., $a_A = l$ and $\theta_B = h$. Then, B's posterior beliefs for true state are $Pr(w = H| l_A, h_B) = \frac{(1-p_A)p_B}{p_A(1-p_B) + (1-p_A)p_B}$ and $Pr(w = L| l_A, h_B) = \frac{p_A(1-p_B)}{p_A(1-p_B) + (1-p_A)p_B}$. Then, $E\pi_B(a_B = \theta_B) = -\frac{(p_B - p_A)\gamma}{2p_A p_B - p_B - p_A} > 0$ and $E\pi_B(a_B \neq \theta_B) = \frac{p_B - p_A}{2p_A p_B - p_B - p_A} < 0$, which yields that B's best response as the follower is to reveal her signal truthfully.

Case 2) When B believes that A's action is not truthful.

Suppose that $a_A = \theta_B = h$. In this case, B believes that A's true signal is $\theta_A = l$. Thus, B's posterior beliefs are $Pr(w = H| l_A, h_B) = \frac{(1-p_A)p_B}{(1-p_A)p_B + p_A(1-p_B)}$ and $Pr(w = L| l_A, h_B) = \frac{p_A(1-p_B)}{(1-p_A)p_B + p_A(1-p_B)}$. Then, $E\pi_B(a_B = \theta_B) = -\frac{p_B - p_A}{2p_A p_B - p_B - p_A} > 0$ and $E\pi_B(a_B \neq \theta_B) = \frac{(p_B - p_A)\gamma}{2p_A p_B - p_B - p_A} < 0$. Thus, B's best response is to reveal her signal truthfully. Second, suppose $a_A = l$ and $\theta_B = h$. Then B believes that A's true signal is $\theta_A = h$. Thus, player B's posterior beliefs are $Pr(w = H| h_A, h_B) = \frac{p_A p_B}{p_A p_B + (1-p_A)(1-p_B)}$ and $Pr(w = L| h_A, h_B) = \frac{(1-p_A)(1-p_B)}{p_A p_B + (1-p_A)(1-p_B)}$. Then, $E\pi_B(a_B = \theta_B) = \frac{(p_A + p_B - 1)\gamma}{2p_A p_B - p_B - p_A + 1} > 0$ and $E\pi_B(a_B \neq \theta_B) = -\frac{(p_A + p_B - 1)}{2p_A p_B - p_B - p_A + 1} < 0$. Hence, B's best response is to reveal her signal truthfully.

Lemma A.2

Suppose that B acts in round 2 and A has already acted in round 1. Then B's best response is to reveal her signal truthfully always.

8.1.2 BEST RESPONSE AS THE LEADER

In following, both players follow the decision rule (2).

PLAYER A We derive A's best response as the leader using the backward induction. In following, without a loss of generality, it is assumed that $\theta_A = h$. If A acts as the leader, she cannot observe B's action before taking her own action. Thus, A's posterior beliefs should be $Pr(w, \theta_B | h_A)$. Although A has no chance to observe a_B and infer θ_B, she knows that B's best response is to reveal θ_B truthfully always. Then, from (2), $E\pi_A(a_A = \theta_A) = -(\gamma p_B - p_B - \gamma p_A - p_A + 1)$ and $E\pi_A(a_A \neq \theta_A) = -(p_A - \gamma - p_B + \gamma p_A + \gamma p_B)$, so

$$E\pi_A(a_A = \theta_A) - E\pi_A(a_A \neq \theta_A) = (\gamma + 1)(2p_A - 1) > 0$$

Therefore, A's best response as the leader is to reveal her signal truthfully.

PLAYER B   In following, assume that $\theta_B = h$. As B cannot observe A's true signal, her posterior belief should be $\Pr(w, \theta_A | h_B)$. However, she knows that A's best response as the follower depends on her belief for the truthfulness of B's action. First, suppose B expects that A believes $a_B = \theta_B$. Then, A always takes the same action as B. Then, $E\pi_B(a_B = \theta_B) = (2p_B - 1) > 0$ and $E\pi_B(a_B \neq \theta_B) = -(2p_B - 1) < 0$. Thus, B's best response is to reveal her signal truthfully. Second, suppose B expects that A believes $a_B \neq \theta_B$. Then if $\theta_A = a_B$, A deviates from her signal and takes a different action from her. However, if $\theta_A \neq a_B$, A reveals her signal truthfully. Then, $E\pi_B(a_B = \theta_B) = \gamma(2p_B - 1) > 0$ and $E\pi_B(a_B \neq \theta_B) = -\gamma(2p_B - 1) < 0$. Therefore, B's best response is to reveal her signal truthfully.

Lemma A.3

Each players' best response as the leader is to reveal her signal truthfully.

8.1.3   BEST RESPONSE UNDER SIMULTANEOUS ACTIONS

In following, both players follow the decision rule (2). If actions are taken simultaneously, each player cannot observe the other player's action. Therefore, she has no chance to infer the true signal of the other player. Also, she should consider the truthfulness of the other player's action. In following, assume that $\theta_A = h$. Then, her posterior beliefs should be $\Pr(w, \theta_B \mid h_A)$. First, suppose A believes that B's action is truthful. Then, $E\pi_A(a_A = \theta_A) = -(\gamma p_B - p_B - \gamma p_A - p_A + 1)$ and $E\pi_A(a_A \neq \theta_A) = -(p_A - \gamma - p_B + \gamma p_A + \gamma p_B)$, which yields

$$E\pi_A(a_A = \theta_A) - E\pi_A(a_A \neq \theta_A) = (\gamma + 1)(2p_A - 1) > 0$$

Thus, A's best response is to reveal her signal truthfully. Next, suppose that A believes that B's action is not truthful. Then, $E\pi_A(a_A = \theta_A) = (p_A - \gamma - p_B + \gamma p_A + \gamma p_B)$ and $E\pi_A(a_A \neq \theta_A) = (\gamma p_B - p_B - \gamma p_A - p_A + 1)$, which yields

$$E\pi_A(a_A = \theta_A) - E\pi_A(a_A \neq \theta_A) = (\gamma + 1)(2p_A - 1) > 0$$

Thus, A's best response is to reveal her signal truthfully.

Also from the above, it can be shown that B's best response is also to reveal her signal truthfully because

$$E\pi_B(a_B = \theta_B) - E\pi_B(a_B \neq \theta_B) = (\gamma + 1)(2p_B - 1) > 0$$

regardless of her belief in the truthfulness of A's action.

Lemma A.4

Suppose that both players act simultaneously. Then each player's best response is to reveal her signal truthfully.

∎

REFERENCES

Arora, A., Ceccagnoli, M. and W. Cohen, 2001, "R&D and the Patent Premium," *NBER working paper* No. W9431.

Chamley, C. and D. Gale, 1994, "Information Revelation and Strategic Delay in a Model of Investment," *Econometrica* 62, 1065-1085.

van Damme. E. and S. Hurkens, 1999, "Endogenous Stackelberg Leadership," *Games and Economic Behavior* 28, 105-209.

_____, 2004, "Endogenous Price Leadership," *Games and Economic Behavior* 47, 404-420.

Frisell, L., 2003, "On the interplay of informational spillover and payoff externalities," *RAND Journal of Economics* 34, 582-592.

Golder, P.N. and G.J. Tellis, 1996, "First to Market, First to Fail? Real Causes of Enduring Market Leadership," *Sloan Management Review* 37, 65-75.

Lanjouw, J.O. and I.M. Cockburn, 2000, "Do Patents Matter?: Empirical Evidence after GATT," *NBER Working Paper* No W7495.

Lerner, J., 2002, "150 Years of Patent Protection," *American Economic Review* 92, 221-225.

Moser, P., 2003, "How do Patent Laws Influence Innovation? Evidence from Nineteenth Century World Fairs," *NBER working paper* No W9909.

Rob, R., 1991, "Learning and Capacity Expansion under Demand Uncertainty," *Review of Economic Studies* 58, 655-675.

Shankar, V., Carpenter. G.S. and L. Krishnamurthi, 1998, "Late Mover Advantage: How Innovative Late Entrants Outsell Pioneers," *Journal of Marketing Research* 35, 54-70.

Zhang, J., 1997, "Strategic Delay and the Onset of Investment Cascades," *RAND journal of economics* 28, 188-205.

Zhang, S., and A.B. Markman, 1998, "Overcoming the Early Entrant Advantage: The Role of Alignable and Nonalignable Difference," *Journal of Marketing Research* 35, 413-426.