CAEPR
INDIANA UNIVERSITY
CENTER FOR
APPLIED ECONOMICS
AND POLICY RESEARCH

CAEPR Working Paper
#2007-010

Effects of Asymmetric Payoffs and Information Cost in Sequential Information Revelation Games

Young-Ro Yoon
Indiana University Bloomington

June 19, 2007

The Center for Applied Economics and Policy Research resides in the Department of Economics at Indiana University Bloomington. CAEPR can be found on the Internet at: http://www.indiana.edu/~caepr. CAEPR can be reached via email at caepr@indiana.edu or via phone at 812-855-4050.

Effects of Asymmetric Payoffs and Information Cost in Sequential Information Revelation Games

Young-Ro Yoon[1]

Abstract

This paper explores the effects of costly information and asymmetry in reward and penalty on an agent's strategic behavior in acquiring and revealing information. Whether information is costly to acquire or not, in order to induce truthfulness in an agent's action, the penalty should not be stressed more than the reward to avoid herding or imitation. When the reward is greater than the penalty, if information is not costly, for the relatively low quality of information, the agent exhibits anti-herding. However, an equilibrium – in which she acts truthfully for all parameters of information quality – can be induced by managing the reward and penalty. If information is costly, within certain parameter sets of information quality, the agent exhibits deviation and imitation. Also, for the moderate quality of information, the agent acquires her information although it is costly and reveals it truthfully. The derived results can provide the reasoning behind agents' behavior trends in information revelation according to reputation and the difficulty of a given task.

JEL classification: D81; D82

Keywords: Asymmetry in reward and penalty; Information Cost; Truthfulness in information revelation; Herding; Anti-Herding; Imitation; Deviation

[1]Department of Economics, Indiana University Bloomington, Wylie Hall 211, 100 South Woodlawn, Bloomington, IN 47405, USA, E-mail: yoon6@indiana.edu, Tel: 1-812-345-8247, FAX: 1-812-855-3736

I gratefully acknowledge the kind and beneficial guidance of David Easley, Tapan Mitra, and Michael Waldman. Also, the generous and helpful comments of Fwu-Rang Chang, Syngjoo Choi, Roy Gardner, Jinyong Kim, Eric Rasmusen, and the participants of the 2004 Far Eastern Econometric Society Summer Meeting, 2004 Midwest Economic Theory Conference are much appreciated. All remaining errors are the author's responsibility.

1 Introduction

The agents, who are in competitive environments, are frequently evaluated based on their performance while fulfilling their duties, and this evaluation in turn affects their wages and promotions. The agents' accumulated evaluations then form the basis for their reputation in the market. While the evaluation of the agents' performance is affected by many factors, the main factor is their performance in relation to other competitors in the market. Therefore, if an agent is less successful than her peers, even if she herself is successful, then she will be given a less positive evaluation than her competitors. However, if she alone was successful in relation to the other agents, she will be given a high evaluation. As a result of such an evaluation scheme, when carrying out a given task each agent must consider not only her own performance, but also the performance of other agents in the market. This presence of payoff externality is the result of a competitive environment.

This situation is extremely applicable to agents working in the financial sector, such as fund managers or financial analysts. The most important aspect of such jobs is the management of information to which access is relatively limited. Due to the agents' access to such restricted information, the agents' truthfulness in revealing this information is a key concern in the field. Much of the current literature address the question of agent's truthfulness, and proposes that the agents have an incentive to reveal distorted information. Herding and anti-herding are good examples of the consequences of an agent's dishonesty in revealing information.[2]

Scharfstein and Stein (1990) and Effinger and Polborn (2001), for example, are representative of the literature that deals with the topics of herding and anti-herding. The models used in these studies commonly assume that two types of agents exist, the smart type and the dumb type. Both of these types of agents work toward the same common goal. While involved in the completion of a task, neither the evaluator (the market) nor the agent know which type of agent she is. Therefore, during an evaluation, each agent strives to be evaluated as the smart type. In addition, it is assumed that the smart type of agent observes the correlated signal, while the dumb type of agent observes only the noisy signal. Thus, from the follower's standpoint, taking the same action as the leader attains the greater probability of being evaluated as the smart type. It can also minimize her risk of being penalized by herself even if her actions lead to incorrect choices. Thus, herding is derived (Scharfstein and Stein, 1990). If, under similar assumptions, the concept of competition between the agents is introduced into the model, then each agent's main objective is to be evaluated as the unique smart type, not simply as one of potentially several smart types. Therefore, if the value of being evaluated as the unique smart type is sufficiently larger than being evaluated as the smart type, anti-herding can be derived (Effinger and Polborn, 2001). In both models, the method by which the market updates the agent's type plays an important role in deriving the equilibrium.

Although the models described above effectively describe the production of herding and anti-

[2] For literature which provides a theoretical model, see Avery and Chevalier (1999), Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), Effinger and Polborn (2001), Gale (1996), Levy (2004), Scharfstein and Stein (1990) and Trueman (1994). In empirical studies, Clement and Tse (2005), De Bondt and Forbes (1999), Gallo, Granger and Jeon (2002), Lamont (2002), Hong, Kubik and Solomon (2000), and Welch (2000) find evidence of herding. On the other hand, Bernhardt, Campello and Kutsoati (2006), Chen and Jiang (2006), and Zitzewtiz (2001a) find evidence of anti-herding.

herding, they fall short of describing real-world situations in two important areas. The primary assumption of the above two models is that the agent's main interest is to be evaluated as the (unique) smart type of agent instead of taking the correct action. However, in real world situations, it is also common to find that an agent's reputation is directly related to the outcome of her job. For example, in addition to a base salary, many agents receive a performance-based bonus, which makes up a large portion of the money that an individual earns. This bonus is detailed in her contract and, therefore, the detailed agreement is composed in advance before the contract starts. Such a bonus means that the agent, naturally, should give a great deal of weight to her performance during the duration of the contract. A good performance will be closely related to having a good reputation in the market and a continued good performance will affirm such a reputation. In addition, a contract generally contains clauses for both a reward for a successful result and a penalty for a disappointing result in regard to the given task. Or, although it is not stated explicitly, an agent can interpret the given payoff structure subjectively as one biased toward either reward or penalty. Given these real world conditions, it is important to asses how agents act strategically according to the given or the subjectively interpreted payoff structure when the payoff directly depends on the correctness of the agents' actions.

Also, it is not difficult to find real world examples that demonstrate how costly it is to acquire information. This cost may be especially pertinent for financial analysts and fund managers because the information acquisition process for these jobs is very expensive. If an agent must sacrifice a large amount of money, time or effort in order to acquire information, then it would be rational for her to not pay the cost and, instead, make use of existing information. In this way, the existence of costly information can cause an agent to behave strategically during the process of information acquisition. Thus, it will be essential to consider the process of both information acquisition and revelation if the agent's truthfulness in action is to be studied. However, much of current literature assumes that information is given exogenously to the agent, and therefore it focuses on an agent's strategic behavior in revealing the given information in various environments. As a result, the analysis of the agent's strategic behavior in *acquiring* the information is not dealt with in depth.[3]

This article introduces a model that is distinct from the current literature in several regards. First, we consider the case in which information is costly to acquire, and hence the acquisition of information is endogenous and is subject to the agent's strategic decision. Next, we explicitly incorporate an asymmetry in the reward and penalty into the model. Also, we assume that agents are homogenous in that the precision of their observed signals are same and that this precision is public information. As a result, how the market forms and updates its belief about an agent's

[3]Swank and Visser (2006) deals with the topic of endogenous information acquisition directly by introducing the concept of costly information. As it follows the assumptions found in traditional reputational concern literature, their model's set-up and procedure are quite different than those used in this model. In Swank and Visser (2006), the equilibrium strategy depends on how much an agent cares about her reputation. In this model, on the other hand, the equilibrium strategy depends on the quality of information and the asymmetry in reward and penalty. Also, in Swank and Visser (2006), the critical value of information cost is described as a function of the market's prior belief that an agent is a smart type of agent. In this model, however, it is described as functions of the reward and the penalty. This allows this model to analyze the affects of a change in the payoff on the agent's strategic behavior in acquiring and revealing information. In this way, this model provides a complementary approach to the topic of endogenous information acquisition by providing an analysis of different aspects than those covered by extant literature.

type does not need to be considered. Instead, we can focus on the direct effects of asymmetry in payoffs on agents' strategic behaviors in a situation in which their payoffs depend only upon the correctness of the agents' actions.

The questions which we seek to answer in this article are as follows: 1) When information is given for free, how does asymmetry in regard to the reward and penalty affect an agent's incentive to acquire information and be truthful in revealing the information?, 2) If the information is costly to acquire and an agent can make use of existing information, in what circumstances does she choose to acquire her own costly information?, 3) If she does choose to acquire such information, does she decide to reveal it truthfully? The results of these questions are summarized as follows.

In the case of the leader, she will always reveal her signal truthfully regardless of whether the information was given exogenously to the follower. However, there are several variations in how the follower may act. If the payoff structure is biased toward the reward, the follower has an incentive to act differently than the leader. On the other hand, if the payoff structure is biased toward the penalty, the follower has an incentive to avoid the worst case scenario, in which she is penalized by herself and she therefore acts in the same manner as the leader. Particularly interesting is the fact that the follower's best response shows an asymmetric feature although the given condition of the payoff structure is symmetric, i.e., $\gamma > \phi$ and $\gamma < \phi$ where γ denotes the reward and ϕ denotes the penalty.

Consider the situation in which information is given exogenously even to the follower. If the penalty is greater than the reward, the follower will take the same action as the leader for all parameter sets of information quality. That is, she exhibits herding if there is a conflict between her information and the leader's inferred information. If, however, the reward is greater than the penalty, the follower's best response varies according to the quality of information. Under such a payoff structure, the follower has an incentive to be differentiated from the leader by acting differently. However, since the reward can only be earned if her action is correct, then the quality of information affects her strategic behavior: if the information quality is relatively low, she has little faith in the correctness of the given information, and therefore always takes a different action than the leader; that is, she exhibits anti-herding even though her information is the same as what she infers the leader's information to be. Contrarily, if the information quality is relatively high, she believes that the information is more likely to be correct, and therefore reveals her signal truthfully; that is, although her information is not same as the inferred leader's information, she does not exhibit herding.

Next, suppose that it is costly for the follower to observe her own signal. As the follower can infer the leader's true signal perfectly, her decision becomes whether to acquire the costly information or make use of existing information correlated with the true state. According to the results, there exists a critical value of information cost above which the follower gives up observing her costly signal. Even when the information cost is less than this critical value, her decision to acquire the costly information is still affected by the quality of information. If the quality of information is sufficiently high or low, she makes use of the inferred leader's information without acquiring her own costly information. Extreme information quality, whether high or low, is a strong indicator of the possibility that the leader's information is correct or incorrect. This causes the follower to consider not acquiring her own costly signal. However, if the information quality is neither high nor low, then the follower observes her signal even though it is costly to acquire, because the quality of information does not strongly signal as to whether the leader's information will be correct. Moreover, if an agent does acquire costly information, then the strategic distortions of information, herding or anti-herding, do not occur and instead the agent reveals her information truthfully. This is quite intuitive. As the information is costly to acquire, if she had any incentive to disregard her observed signal, she would have no reason to observe it at the expense of a high cost.

These results provide an interesting argument that low or high quality information can be socially good or bad. When the payoff structure is strongly biased toward the reward, if the quality of information is relatively low, an agent always takes an action different from that of the other agent. Hence, the efficient case, in which one agent is correct and the other agent is wrong, is derived endogenously. Yet, when information is costly to acquire, if the quality of information is sufficiently high then the efficient outcome cannot be derived at all because an agent exhibits herding or imitation. In a similar logic, if the payoff structure is weakly biased toward the reward, rather than strongly biased, a relatively low quality of information is socially bad.

The above results provide the reasoning behind the results of some empirical studies that discuss an agent's strategic behavior with regard to her reputation and the difficulty of the given task. The key intuition is as follows. If an agent has a good reputation or if the given task is evaluated as an easy task, an agent would have a great deal to lose from a wrong action and very little to gain from a correct action. If, on the other hand, the agent does not have a good reputation, or if the given task is evaluated as a hard task, an agent would have much to gain from a correct action and relatively little to lose from a wrong action. Therefore, the first case will correspond to one in which the payoff structure is biased toward the penalty and the second case will correspond to a situation in which the payoff is biased toward the reward. Section 5 details how the results derived in this model can be used to explain the findings of some previous studies.

The remainder of this paper proceeds as follows. Section 2 explains the model used during this study. Section 3 focuses on a case in which the information is not costly to either player, while Section 4 focuses on a case in which the information is costly for the follower to acquire. Section 5 discusses the application of the results. Section 6 contains the concluding remarks.

2 Model

There are two players A and B, $i \in \{A, B\}$, whose jobs are to provide forecasts about the unknown true state $w \in \{H, L\}$ which are mutually exclusive. To both players, the prior probability of each state is $\Pr(w = H) = \Pr(w = L) = \frac{1}{2}$. Before making a forecast, each player has a chance to observe her own signal $\theta_i \in \Theta = \{h, l\}$ which is correlated with the true state. If θ_i is observed, the draws of their signals are conditionally independent given the true state. Also, as θ_i is private information, each player does not know which signal is observed by the other.

The signal θ_i partially reveals information about the true state in the following way:

$$\Pr(\theta_i = w|\, w) = p_i \quad \text{and} \quad \Pr(\theta_i \neq w|\, w) = 1 - p_i$$

where $p_i \in \left(\frac{1}{2}, 1\right)$. Here, p_i measures the precision of player i's signal θ_i, so can be interpreted as i's information quality. As p_i approaches $\frac{1}{2}$, it indicates that θ_i is becoming less informative and as it approaches 1, it means that θ_i is becoming more informative about the true state. Throughout this paper, we assume that both players are homogenous in that $p_A = p_B = p$, which is public information.

Player i's action, which denotes a forecast about the true state, is denoted by $a_i \in \Psi_i = \{h, l\}$ where $a_i = h$ ($a_i = l$) means that i's forecast is $w = H$ ($w = L$). The ordering of both players' actions is decided exogenously. Without loss of generality, we assume that A is the leader and B is the follower. In these sequential actions, B has a chance to observe A's action before taking her own action a_B.

Each player's payoff is defined by:

$$\pi_A\left(a_A, a_B\right) = \dot{\pi}_A\left(a_A, a_B\right) \text{ and } \pi_B\left(a_A, a_B\right) = \dot{\pi}_B\left(a_A, a_B\right) - c$$

where $c \geq 0$. In a model, we assume that θ_A is always given exogenously to A. On the other hand, for B, we consider both cases in which θ_B is given exogenously and the acquisition of θ_B is endogenous. If $c = 0$, it denotes the case in which θ_B is given exogenously to B. If $c > 0$, it denotes the case in which θ_B is costly to acquire and c should be paid in order to observe θ_B. Hence, if B observes θ_B, her net payoff is $\pi_B\left(a_A, a_B\right) = \dot{\pi}_B\left(a_A, a_B\right) - c$ and if she does not, it is $\pi_B\left(a_A, a_B\right) = \dot{\pi}_B\left(a_A, a_B\right)$.

Player i's gross payoff $\dot{\pi}_i$ is defined by the following payoff matrix where $\gamma > 1$, $\phi > 1$, and $\gamma \neq \phi$.

w	$a_B = w$	$a_B \neq w$
$a_A = w$	$1, 1$	$\gamma, -\phi$
$a_A \neq w$	$-\phi, \gamma$	$-1, -1$

(P)

This payoff structure is designed in order to incorporate the competitive environment of two players. Suppose that both players act identically. Then, if their actions reveal the true state correctly, both players earn $+1$ and if not, both earn -1. On the other hand, if both players take different actions, the player who takes the correct action gets $\gamma > 1$ and the other player who takes a wrong action gets $-\phi < -1$. In other words, if an agent's action turns out to be correct (wrong), the other agent's same action causes the negative (positive) externality because $1 < \gamma$ ($-\phi < -1$). However, as whether $a_i = w$ or $a_i \neq w$ cannot be verified in advance, uncertainty is embedded within the system. Also, note that we assume that $\gamma \neq \phi$. Hence, our case is either $\gamma > \phi$ or $\gamma < \phi$ where the first (second) denotes the case in which the payoff structure is biased toward the reward (penalty). Through this approach, we can analyze the effects of asymmetry in reward and penalty on each player's strategic behavior in information acquisition and revelation.

Consider now each player's strategy. In the case of A, θ_A is always given for free. Hence, A's pure strategy is $s_A : \Theta \longrightarrow \Psi_A$ where $\Theta = \{h, l\}$ and $\Psi_A = \{h, l\}$. That is, for given θ_A, A should decide whether to reveal it truthfully or not. For B, if θ_B is given exogenously to her, $s_B : \Theta \times \Psi_A \longrightarrow \Psi_B$ where $\Theta = \{h, l\}$ and $\Psi_i = \{h, l\}$ for $i \in \{A, B\}$ as she has a chance to observe a_A before she takes her own action. On the other hand, if the acquisition of θ_B is endogenous, whether to acquire θ_B while paying a cost or not is also B's decision problem. So, $s_B : \Psi_A \longrightarrow \Xi_B \times \Psi_B$ where $\Psi_i = \{h, l\}$ and $\Xi_B = \{\text{Observe } \theta_B \text{ paying } c, \text{ Don't observe } \theta_B\}$.

The equilibrium concept used is the perfect Bayesian Nash equilibrium. Note that, as θ_A is private information, whether $\theta_A = a_A$ or $\theta_A \neq a_A$ cannot be verified. Let λ be B's belief that A reveals θ_A truthfully. Then, the strategy profile $s = \{s_A, s_B\}$ and λ constitute a Perfect Bayesian Nash equilibrium if each player's expected payoffs are maximized for a given λ, the other firm's strategy, and especially if λ is consistent with s_A in terms of Bayesian updating.

Finally, below are the definitions used throughout this paper.

Definition 1 *Truthful action: When player i observes her signal θ_i, if $a_i = \theta_i$, we say that $i's$ action is truthful.*

Definition 2 *Herding: Suppose player i observes her signal θ_i. When $\theta_i \neq \theta_{-i}$, if $a_i = \theta_{-i} \neq \theta_i$, we say that i exhibits herding.*

Definition 3 *Anti-herding: Suppose player i observes her signal θ_i. When $\theta_i = \theta_{-i}$, if $a_i \neq \theta_i$, we say that i exhibits anti-herding.*

Definition 4 *Imitation & Deviation from the other player's action: When player i does not observe θ_i, if $a_i = a_{-i}$ ($a_i \neq a_{-i}$), we say that i imitates (deviates from) player $-i$'s action.*

3 Bench mark case: when $c = 0$

As a bench mark, we consider the case in which c is given for free to both players, i.e. $c = 0$. In this case, the timing of the game can be represented as follows:

T1) Nature decides the true state w. The payoff structure (P) and p are announced.

T2) A observes θ_A. Then, she takes an action after deciding whether to reveal θ_A truthfully or not.

T3) B observes θ_B and a_A. Then, she takes an action after deciding whether to reveal θ_B truthfully or not.

T4) True state w is revealed and each player earns her payoff.

Now, each player's expected payoff can be represented as:

$$E\pi_A = \sum_{\theta_B \in \{h,l\}} \sum_{w \in \{H,L\}} \Pr(w, \theta_B | \theta_A) \pi_A(a_A, a_B) \tag{1}$$

$$E\pi_B = \sum_{w \in \{H,L\}} \Pr(w | \tilde{\theta}_A, \theta_B) \pi_A(a_A, a_B) \tag{2}$$

Here, it should be noted that A's posterior belief should be about the true state and B's signal, i.e., $\Pr(w, \theta_B | \theta_A)$ because A has no chance to observe a_B and infer θ_B. In the case of B, as θ_B is given exogenously and she has a chance to infer θ_A from observing a_A, the posterior belief should be $\Pr(w | \tilde{\theta}_A, \theta_B)$. Here, $\tilde{\theta}_A$ denotes the inferred A's signal, not the true A's signal θ_A. As θ_A is private information, although B has a chance to observe a_A, the inference of θ_A depends on B's belief on the truthfulness of A's action. Hence, it can be either $\theta_A = \tilde{\theta}_A$ or $\theta_A \neq \tilde{\theta}_A$.[4]

Then, the analysis yields that the perfect Bayesian Nash equilibrium thus can be characterized as follows.

Proposition 1

Consider the case where θ_i is given exogenously to both players, i,e, $c = 0$.

1) Suppose $\gamma > \phi$. Then, A always reveals her signal truthfully. In the case of B, there exists a critical value $p^* \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1}{2}, p^*\right)$, she acts differently from i (that is, she exhibits anti-herding when $\theta_B = a_A$) and if $p \in (p^*, 1)$, she reveals her signal truthfully.

2) Suppose $\gamma < \phi$. Then, A reveals her signal truthfully and B always acts identically to A (that is, she exhibits herding when $\theta_B \neq a_A$).

Here, $p^* = \frac{1}{\gamma - \phi}\left((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1}\right)$

Proof of Proposition 1

In the appendix.

According to Proposition 1, A always reveals her signal θ_A truthfully. Note that B can observe A's action before she takes her own action. Hence, A's truthfulness in action means that B can infer θ_A perfectly from observing a_A. In addition, this means that B assigns zero probability to the possibility that A deviates from her signal. However, B's best response in the subgame depends on the asymmetry in the reward and the penalty and the information quality as follows.

First, suppose that the reward is greater than the penalty, i.e., $\gamma > \phi$. In order to earn γ, B should consider both taking the correct action and being the only one who does so. For example, if $a_B = a_A$, although B's actions turns out to be correct, her payoff is $+1 < \gamma$. Hence, B has an incentive to be differentiated from A by taking a different action. However, always taking a different action from A cannot be optimal. For example, if it turns out that $a_B \neq w$, then B's payoff is $-\phi < -1$, which is the worst case. Hence, B should balance both incentives, the one to be differentiated and the one to take the correct action. The information quality p plays an important role and this is why B uses the cut-off strategy according to her information quality as described below. If the information quality is relatively low, i.e., $p \in \left(\frac{1}{2}, p^*\right)$, B always takes a different action from A. That is, although $\theta_B = \theta_A$, she exhibits anti-herding. Although both players observe the same signal, B has a weak belief in the correctness of the given signal. So, the incentive to be differentiated increases her incentive to take a different action from A, which yields anti-herding. On the other hand, suppose that the information quality is relatively high, i.e.,

[4] However, it is shown that, in equilibrium, A's best response is to reveal her signal truthfully, which should be consistent with B's belief. Hence, in equilibrium, $\theta_A = \tilde{\theta}_A$.

$p \in (p^*, 1)$. Although B still has an incentive to be differentiated, she has a relatively strong belief in the correctness of the given signal, due to the relatively high quality of information. Hence, the incentive to be differentiated is dominated and she reveals her signal truthfully.

Second, suppose that the penalty is greater than the reward, i.e., $\gamma < \phi$. Then, B always takes the same action as A. That is, although $\theta_B \neq \theta_A$, she ignores her signal and exhibits herding. If $\gamma < \phi$, B should be concerned about the possibility that she alone can be penalized. Hence, B always takes the same action as A without regard to the information quality. By acting in such a manner, she can prevent the worst case scenario, in which she gets $-\phi$ although it turns out that $w \neq a_B$.

In the current literature, it is already proposed that, when agents' types are not known, if the reward is stressed, then the agent has an incentive to be differentiated, which yields anti-herding. Also, it is proposed that if the penalty is stressed, the agent has an incentive to blame a sharing, which yields herding. In addition to informing the extant literature, this model provides the unified model through which both herding and anti-herding are derived according to asymmetry in reward and penalty when the agents' types are known. Moreover, interestingly, it shows that an agent who acts as the follower reacts asymmetrically although the given condition of the payoff structure is symmetric, i.e., $\gamma > \phi$ and $\gamma < \phi$. That is, compared to her extreme reaction, which is to exhibit herding for all $p \in \left(\frac{1}{2}, 1\right)$, when $\gamma < \phi$, if the reward is stressed more than the reward, i.e., $\gamma > \phi$, she exhibits anti-herding only if p is relatively low.

4 Extension: when $c > 0$

In this section, we extend the model into the case in which the follower B should pay the information cost in order to observe her own signal. That is, whether to observe θ_B or not is B's decision problem. However, we still assume that A can access to θ_A for free. If B considers observing her costly signal, she must also decide whether to reveal θ_B truthfully or not. If B considers not observing her signal, she should decide whether to imitate or deviate from A's action. Below, it is shown that A's best response is always to reveal θ_A truthfully. Thus, our problem can be identified as one in which B makes a strategic decision about whether to acquire the costly information and be truthful in revealing it when she can make use of existing information correlated with the true state.

The timing of the game of this case can be represented as follows.

T1) Nature decides the true state w. Also, the payoff structure *(P),* the quality of information p and the information cost c are announced.

T2) A observes θ_A. Then, she acts after deciding whether to reveal θ_A truthfully or not.

T3) B observes a_B. Then, she makes a decision whether to observe her costly signal θ_B or not. If she observes θ_B, she also decides whether to reveal it truthfully or not. If she decides not to observe her signal, she decides whether to imitate or deviate from a_A. Then, she acts.

T4) The true state w is revealed and each player earns the payoff.

In the case of A, she has no chance to observe whether B will observe the costly signal θ_B or not. Hence, A's expected payoff should differ according to A's expectation about B's strategic decision on observing the costly signal θ_B. If A expects that B will observe θ_B,

$$E\pi_A = \sum_{\theta_B \in \{h,l\}} \sum_{w \in \{H,L\}} \Pr(w, \theta_B | \, \theta_A) \pi_A (a_A, a_B) \tag{3}$$

On the other hand, if A expects that B will not observe θ_B,

$$E\pi_A = \sum_{w \in \{H,L\}} \Pr(w | \, \theta_A) \pi_A (a_A, a_B) \tag{4}$$

Using (3) and (4), A decides whether to reveal her signal θ_A truthfully or not.

In the case of B, she does not know which signal will be observed although she decides to observe θ_B. Hence, B's expected payoff when she observes θ_B is

$$E\pi_B = \sum_{\theta_B \in \{h,l\}} \sum_{w \in \{H,L\}} \Pr(w, \theta_B | \, \tilde{\theta}_A) \pi_A (a_A, a_B) - c \tag{5}$$

Here, if she considers observing θ_B, it should be decided in advance whether she will reveal θ_B truthfully or not for given $\tilde{\theta}_A$. B's this decision can be anticipated by comparing the following expected payoffs

$$\sum_{w \in \{H,L\}} \Pr(w | \, \tilde{\theta}_A, \theta_B) \pi_A (a_A, a_B = \theta_B) - c \gtreqless \sum_{w \in \{H,L\}} \Pr(w | \, \tilde{\theta}_A, \theta_B) \pi_A (a_A, a_B \neq \theta_B) - c \tag{6}$$

for both cases where $\theta_B = \tilde{\theta}_A$ or $\theta_B \neq \tilde{\theta}_A$. After deriving the best response for each case from (6), then B computes (5). On the other hand, if she decides not to observe θ_B, her expected payoff is

$$E\pi_B = \sum_{w \in \{H,L\}} \Pr(w | \, \tilde{\theta}_A) \pi_A (a_A, a_B) \tag{7}$$

B's decision whether to observe the costly signal θ_B or not is finally derived from the comparison of (5) and (7). As explained before, in (5), (6), and (7), $\tilde{\theta}_A$ denotes the inferred A's signal according to B's belief in the truthfulness of A's action. The detailed procedure is provided in the proof of Proposition 2 and 3.

Then, the perfect Bayesian Nash equilibrium can be characterized as follows.

Proposition 2

Suppose that $\gamma > \phi$ and θ_B *is costly to acquire, i.e.* $c > 0$.

1) A reveals her signal truthfully for all $p \in \left(\frac{1}{2}, 1\right)$.

2) In the case of B,

2-1) Suppose $c > c^*$. *If* $p \in \left(\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, *B deviates from A's action without observing her signal and if* $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, *she imitates A's action without observing her signal.*

2-2) Suppose $0 < c < c^*$. *If* $p \in \left(\frac{1}{2}, \check{p}\right)$, *B deviates from A's action without observing her signal,* *if* $p \in (\check{p}, \hat{p})$ *, she observes her costly signal and reveals it truthfully, and if* $p \in (\hat{p}, 1)$ *, she imitates A's action without observing her signal.*

Here, $c^* = \frac{(\phi+1)(\gamma-\phi)(\gamma+1)}{(\gamma+\phi+2)^2}$, $\check{p} = \check{p}(\gamma, \phi, c)$ *and* $\hat{p} = \hat{p}(\gamma, \phi, c)$.[5]

Proposition 3

Suppose that $\gamma < \phi$ and θ_B *is costly to acquire, i.e.* $c > 0$.

1) A reveals her signal truthfully for all $p \in \left(\frac{1}{2}, 1\right)$.

2) B imitates A's action without observing her signal for all $p \in \left(\frac{1}{2}, 1\right)$.

Proof of Proposition 2 and 3

In the appendix.

First, consider the case where $\gamma > \phi$. For the simple notation, d denotes B's strategy to deviate from A's action without observing θ_B, m denotes B's strategy to imitate A's action without observing θ_B, s denotes B's strategy to observe θ_B and reveal it truthfully and σ_B denotes B's strategy.

If $c > c^*$, $\sigma_B \in \{d, m\}$. This case corresponds to one where the expected gain of observing the costly signal is dominated by the sufficiently high information cost. Hence, B makes use of existing information θ_A. Whether she imitates or deviates from a_A depends on the quality of information. If it is relatively high, i.e., $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, $\sigma_B = m$ and if not, i.e., $p \in \left(\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, $\sigma_B = d$. The relatively high p means that θ_A has a high probability of being correct. Thus, $\sigma_B = m$. On the other hand, a relatively low p implies that θ_A has a high probability of being incorrect. Thus, $\sigma_B = d$.

If $0 < c < c^*$, $\sigma_B \in \{d, s, m\}$. If the information quality is sufficiently high (low), B has a strong belief that $\theta_A = w$ ($\theta_A \neq w$) with a high probability. Thus, although the information cost is not sufficiently high, the expected net gain of observing θ_B is less than the expected payoff of making use of existing information. Hence, $\sigma_B = m$ ($\sigma_B = d$). That is, if the information quality is extreme, it signals strongly whether θ_A can be correct or wrong. Hence, compared to observing the costly signal θ_B, making use of θ_A attains the greater expected payoff. Meanwhile, if the information quality is intermediate, i.e., $p \in (\check{p}, \hat{p})$, it does not signal strongly whether $\theta_A = w$ or $\theta_A \neq w$. Hence, the expected net gain of observing θ_B is greater than the expected payoff of making use of existing information. So, B has an incentive to observe θ_B although it is costly and reveals it truthfully. Therefore, $\sigma_B = s$.

Second, suppose that $\gamma < \phi$. Then, $\sigma_B = d$. Here, it should be noted that, B's strategy, which is to imitate A's action, is a dominant strategy not because information is costly to acquire, but because the payoff structure is biased toward the penalty. Recall that, although $c = 0$, if $\gamma < \phi$, she exhibits herding to avoid being penalized individually. In this case, it is obvious that B has no incentive to acquire the costly information because, although the different information is observed, she will ignore it and exhibit herding.

[5] $\hat{p} = \frac{1}{\gamma-\phi}\left(\frac{1}{2}\gamma - \frac{1}{2}\phi + \frac{1}{2}\sqrt{-4c\gamma + 4c\phi - 2\gamma\phi + \gamma^2 + \phi^2}\right)$ *and* $\check{p} = \frac{1}{\gamma-\phi}\left(\gamma + 1 - \sqrt{\gamma + \phi - c\gamma + c\phi + \gamma\phi + 1}\right)$

In addition, the procedure of deriving Proposition 2 and 3 yields that if B observes costly θ_B, she always reveals it truthfully.

Corollary 1

Suppose that information is costly to acquire, i.e., $c > 0$. Then, the strategic distortion of the acquired information, herding and anti-herding, does not occur.

This is an intuitive result. As information is costly to acquire, before making a decision as to whether to observe θ_B, B compares the expected gain that will result from observing the costly θ_B and the one from making use of θ_A. B observes θ_B only if the former is greater than the latter. Hence, if she observes her costly signal, she has no incentive to neglect it regardless of θ_B. If she had any incentive to neglect costly θ_B, she would have no reason to pay a cost in order to observe it.

5 Discussion

5.1 Ex-post efficiency and information quality

Recall the given payoff structure (P). According to it, the (ex-post) efficient case, in which the sum of both agents' payoffs is maximized, depends on the values of the reward and the penalty. If $\gamma - \phi > 2$, the efficient case is the situation in which one agent takes the correct action and the other takes the wrong action. That is, the different actions between agents is a necessary and sufficient condition for efficiency. On the other hand, if $\gamma - \phi < 2$, the case in which both agents take the correct action is efficient.

Consider the case where $\gamma - \phi > 2$. When a signal is given exogenously to B, i.e., $c = 0$, if $p \in \left(\frac{1}{2}, p^*\right)$, B always takes a different action from A by exhibiting anti-herding and if $p \in (p^*, 1)$, B reveals her signal truthfully. Hence, the efficiency can always be attained if $p \in \left(\frac{1}{2}, p^*\right)$. If $p \in (p^*, 1)$, although there still exists a possibility that the efficient outcome can be derived, it cannot be guaranteed.[6] Also, when $c > 0$, if $p \in \left(\frac{1}{2}, \check{p}\right)$, B always takes an action different from that of A by deviating from A's action without observing θ_B. Therefore, efficiency can always be attained if $p \in \left(\frac{1}{2}, \check{p}\right)$. If $p \in (\check{p}, \hat{p})$, according to θ_B, sometimes the efficient outcome can be attained and sometimes not. However, if $p \in (\hat{p}, 1)$, the efficient outcome cannot be derived at all because B always imitates A's action.

Next, consider the case where $\gamma - \phi < 2$. If $0 < \gamma - \phi < 2$, then it is necessary for both agents' actions to be the same to achieve efficiency because both agents should take the correct action. Hence, what is sure is that if $p \in \left(\frac{1}{2}, p^*\right)$ when $c = 0$ and if $p \in \left(\frac{1}{2}, \check{p}\right)$ when $c > 0$, the efficient outcome cannot be derived at all. On the other hand, if $\gamma - \phi < 0$, B invariably takes the same action as A regardless of c. Hence, although the efficient outcome cannot be guaranteed, the necessary condition is satisfied.

[6] For example, if $\theta_A \neq \theta_B$, the outcome is efficient.

In brief, if the payoff structure is strongly biased toward the reward, i.e., $\gamma-\phi > 2$, the relatively low quality of information is socially good because it makes an agent intend to be differentiated from the other agent, which causes the efficient outcome. Moreover, if information is costly to acquire, a sufficiently high quality of information is socially bad because it prevents the socially efficient case. If, on the other hand, the payoff structure is weakly biased toward the reward, i.e., $0 < \gamma-\phi < 2$, then relatively low quality of information is socially bad. In this case, for the efficient outcome to be derived, it is necessary that the agent's quality of information should be sufficiently high or at least moderate.

Corollary 2

1) If the payoff structure is strongly biased toward the reward, i.e., $\gamma - \phi > 2$, a relatively low quality of information is socially good. Moreover, if $c > 0$, a sufficiently high quality of information is socially bad.

2) If the payoff structure is weakly biased toward the reward, i.e., $0 < \gamma-\phi < 2$, then a relatively low quality of information is socially bad.

5.2 Application of the results

5.2.1 Difficult task and easy task

The results derived in this model provide a reasoning that explains an agent's strategic behavior in regard to the degree of hardness of a given task. Suppose that a given task is evaluated as being relatively easy. Then, although the outcome of the task is successful, only a small reward will be given because the successful outcome is taken for granted. Alternately, if an agent fails a task which is deemed to be relatively easy, then a large penalty will be given. However, if a given task is evaluated as being difficult, then, even though the outcome is unsuccessful, the penalty will not be as large as for the easy task because less is expected of the agent. In addition, an agent successfully completing a difficult task will have a larger reward than for completing an easier task. Therefore, if the task is evaluated as being relatively easy (hard), then the agent can evaluate her situation as one in which the payoff structure is biased toward the penalty (reward). The results of the model predict that there will be a high tendency for the same or similar action among the agents involved in the easy task and a high tendency toward discrepancy in the actions among the agents involved in the hard task.

5.2.2 Reputation

The above reasoning can also be used to describe the relationship between the agent's strategic behavior and her reputation. If an agent's reputation is relatively low, then even if her action turns out to be incorrect, the penalty will be small because not much is expected of her. However, a large reward will be given to her if she is correct. That is, an agent with a low reputation would have little to lose from taking a wrong action and much to gain from taking a correct action. On the other hand, if an agent has a good reputation, then her correct action is more likely to be taken for granted and she will receive a large penalty for taking a wrong action. Therefore, such an agent would have much to lose from a wrong action and very little to gain from a correct action. The first case may correspond to a case in which the reward is greater than the penalty and the second case may correspond to a case in which the penalty is greater than the reward. Then, this model predicts that an agent with a low reputation is more likely to act differently than her peers and an agent with a high reputation is more likely to act in accordance to her peers.

In Graham (1999), he conducts an empirical test for herding among investment newsletters. His results show that a newsletter analyst herds more if her reputation is high. Also, according to Zitzewitz (2001b), the empirical data on equity analyst's earning forecast shows that agents exaggerate more when they are underrated by their clients.[7] These are the findings consistent with the above reasoning. In addition, Chevalier and Ellison (1999) finds that younger managers deviate less from the mean risk levels and sector weightings of funds in their objective class than older managers and therefore have a higher tendency to herd. As younger managers are more likely to have low reputations, this finding seems to be contradictory to what this model predicts. However, Chevalier and Ellison (1999) also finds that the managerial termination is more performance-sensitive for younger managers, which means that, for the same degree of unsatisfactory performance, younger managers are more likely to be fired than older managers. Therefore, younger managers are more likely to interpret their situation as one in which the payoff structure is biased toward the penalty. Then, their finding is consistent with the results of this study, which state that there is a higher tendency of similar actions in such a case.

5.3 Truthfulness in actions

Note that A and B are homogenous in that both players observe the signals with the same precision. Hence, no one's information should have the priority. However, as proposed in current literature and in this model, the sequential ordering of action can suppress the truthfulness of subsequent player. In this sense, how to design the reward and the penalty to produce both players' truthfulness in actions when the timings of actions are sequential can be a meaningful question. Now we define the equilibrium in which both agents' truthfulness in information revelation is guaranteed for all parameter set of information quality.

Definition 5 *Truthful equilibrium*

We say the equilibrium is truthful, if $a_i = \theta_i$ can be "verified" for $\forall p \in \left(\frac{1}{2}, 1\right)$ and $i \in \{A, B\}$ in equilibrium.

Note that both notions, a) $a_i = \theta_i$ can be verified and b) $a_i = \theta_i$ is available, should be differentiated. As θ_i is private information and only a_i can be observed, truthfulness in action cannot be verified. Hence, the truthful equilibrium is defined as the one in which it is guaranteed

[7] In Zitzewitz (2001b), the author explains that it is due to the agent's incentive to increase the weighting of future observations of her ability. However, in this model, we assert that this model's reasoning, which is that the underrated agents has nothing to lose for the wrong action and much to gain for the correct action, can be an alternative explanation to the empirical finding of his article.

that both players have no incentive to deviate from θ_i endogenously. We have already checked that A, who acts as the leader, always reveals her signal truthfully. Hence, whether or not both players' true signals can be revealed without distortion depends on whether B reveals θ_B truthfully or not.

In addition to our model, we can think about the role of the manager who hires both players and assigns the reward and penalty as a contract to both players. Then, γ and ϕ can be interpreted as control variables. In the following, we check whether it is possible to derive the truthful equilibrium by controlling γ and ϕ. We assume that the manager does not know each player's information quality.[8] What he knows are that p is drawn from uniform distribution defined over from $\frac{1}{2}$ to 1 and both players are homogenous. Then, as the manager has no reason to give any priority to anyone's information, the case the most valuable to the manager will be the one where both players' truthfulness in actions can be induced for all $p \in \left(\frac{1}{2}, 1\right)$.

Now consider the case where θ_B is given to B exogenously, i.e., $c = 0$. According to Proposition 1, if $\gamma < \phi$, the truthful equilibrium cannot be attained at all because B exhibits herding whenever $\theta_B \neq a_A$. Therefore, in order to derive the truthful equilibrium, it is essential that the penalty should not be greater than the reward. However, even when the reward is stressed more than the penalty, *B's* truthfulness in action is guaranteed only if $p \in (p^*, 1)$. Here, as $p^* = p^*(\gamma, \phi)$, we can think about the effects of the reward and the penalty on the critical value p^*. The comparative statics yield the following result.

Corollary 3

Suppose that $c = 0$ *and* $\gamma > \phi$. *Then,* $\frac{\partial p^*}{\partial \gamma} > 0$ and $\frac{\partial p^*}{\partial \phi} < 0$

Proof

In the appendix.

From Corollary 2, as γ decreases or ϕ increases, the parameter set of p in which B reveals her signal truthfully increases. In other words, as γ increases or ϕ decreases, the parameter set of p in which B exhibits anti-herding increases. The interesting point is that, while the penalty cannot be greater than the reward in order to induce B's truthfulness in action, if γ is stressed too much, B has a too strong incentive to be differentiated from A, which yields excessive anti-herding. What this proposes is that, in order to maximize the parameter set of p for which B reveals her signal truthfully, the difference between γ and ϕ should be as small as possible. This argument can also be verified by the following. Note that if $p \in (p^*, 1)$, B reveals her signal truthfully where $p^* = \frac{1}{\gamma - \phi}\left((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1}\right)$. Then,

$$\Pr\left(p \in (p^*, 1)\right) = \frac{2\left(\gamma - \phi\right) - 2\left((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1}\right)}{\gamma - \phi} \tag{8}$$

[8] We want especially to consider the case where the ordering of action cannot be controlled by a manager. If a manager can control the ordering of action, in order to derive the truthfulness in actions, it is best to make the agents act simultaneously.

The L'Hospital's theorem yields that,

$$\lim_{\gamma\to\phi^+} \Pr\left(p\in(p^*,1)\right) = \lim_{\phi\to\gamma^-} \Pr\left(p\in(p^*,1)\right) = \frac{\phi+1}{\sqrt{(\phi+1)^2}} = 1 \tag{9}$$

Corollary 4
Suppose that $c=0$. Then, there exist γ and ϕ such that the truthful equilibrium can be induced.

Next, consider the case where θ_B is costly to acquire, i.e., $c>0$. From Proposition 2 and 3, it can be verified that, like the case in which $c=0$, if $\gamma<\phi$, the truthful equilibrium cannot be attained at all. Therefore, in order to derive the truthful equilibrium, it is essential that the penalty should not be greater than the reward. Although $\gamma>\phi$, note that $\sigma_B=s$ is derived only if $p\in(\check{p},\hat{p})$. Then, the problem of finding the optimal payoff structure under which $\sigma_B=s$ can be described as follows:

$$\begin{aligned} &\underset{\gamma,\phi}{Max}\left(\frac{\hat{p}(\gamma,\phi,c)-\check{p}(\gamma,\phi,c)}{\frac{1}{2}}\right) \\ =\ &\frac{2}{\gamma-\phi}\left(\left(\frac{1}{2}\gamma-\frac{1}{2}\phi+\frac{1}{2}\sqrt{-4c\gamma+4c\phi-2\gamma\phi+\gamma^2+\phi^2}\right)-\left(\gamma+1-\sqrt{\gamma+\phi-c\gamma+c\phi+\gamma\phi+1}\right)\right) \\ &s.t.\ \gamma>\phi \text{ and } c<\frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2} \end{aligned} \tag{10}$$

For the highly nonlinear functional forms of the given problem, deriving the optimal payoff structure explicitly is demanding. Hence, in the following, we provide numerical examples. The following two graphs, Figure 1 and Figure 2, show the effects of the changes in γ and ϕ on the parameter set in which $\sigma_B=s$ for given $c=1$ and $c=10$ respectively. The graphs in each figure were derived as follows. In Figure 1, the graph from the bottom to the top respectively corresponds to the case where $\phi=5$, $\phi=50$, $\phi=500$, and $\phi=1000$ when $c=1$. Also, in Figure 2, the graph from the bottom to the top respectively corresponds to the case where $\phi=100$, $\phi=500$, $\phi=1000$, and $\phi=5000$ when $c=10$. Then, for each fixed c and ϕ, the effects of the change in γ, which satisfies both constraints, are checked. The X-axis describes the reward γ and Y-axis describes $\Pr(\sigma_B=s)$.[9]

[9] Note that we assume $p\tilde{\ }U\left(\frac{1}{2},1\right)$.

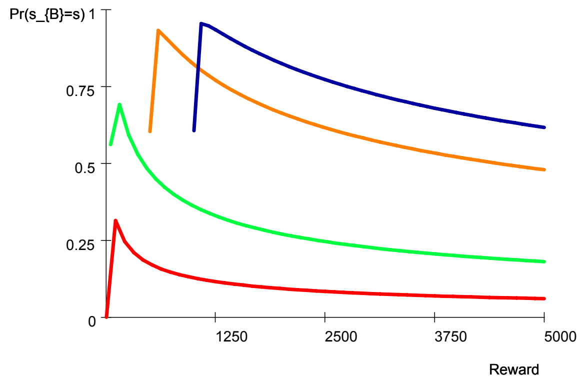

Figure 1: The effects of change in γ and ϕ on $\Pr(\sigma_B = s)$ when $c = 1$

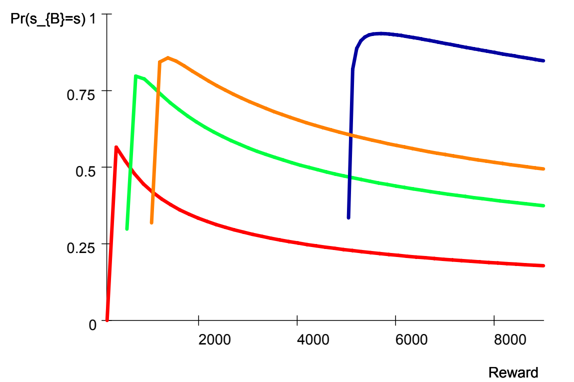

Figure 2: The effects of change in γ and ϕ on $\Pr(\sigma_B = s)$ when $c = 10$

Some common observations from the above two figures can be described as follows.

1) For given c and ϕ, there exists $\bar{\gamma}$ such that if $\phi < \gamma < \bar{\gamma}$, the increase in γ induces an increase in $\Pr(\sigma_B = s)$ but if $\gamma > \bar{\gamma}$, the increase in γ induces a decrease in $\Pr(\sigma_B = s)$ where $\bar{\gamma} > \phi$.

2) For given c, as ϕ increases, $\bar{\gamma}$ increases where $\bar{\gamma} > \phi$.

3) For given c, as $\bar{\gamma}$ and ϕ increase, the maximum value of $\Pr(\sigma_B = s)$ increases.

Compared to the case in which $c = 0$, the main differences are as follows. When $c = 0$, $\Pr(\sigma_B = s)$ increases as $\gamma - \phi$ decreases where $\gamma > \phi$. In other words, what matters is not the absolute values of γ and ϕ, but the relative difference in both values, $\gamma - \phi$. On the other hand, if $c > 0$, the values of γ and ϕ matter. Also, when $c = 0$, as the difference between γ and ϕ decreases and converges to 0, the truthful equilibrium can be attained. However, if $c > 0$, it can be conjectured that $\bar{\gamma} - \bar{\phi} > 0$ where $\bar{\gamma}$ and $\bar{\phi}$ the optimal reward and penalty which attains the maximum of $\Pr(\sigma_B = s)$ for given c. This is intuitive if we note that signal is costly to acquire. To give an agent an incentive to observe a costly signal and distinguish herself, the reward for being the only agent who took the correct action should be relatively greater than that of the case where $c = 0$.

In a related empirical study, Massa and Patgiri (2005) tests the corporate theory of the managerial herding based on the reputational and the career concerns by using data from the mutual fund industry. This data shows that a high incentive contract induces managers to take a strategy that is the most likely to yield an extreme performance realization and therefore take more risks than their peers by adopting trading strategies different than those used by their peers. This finding is consistent with the above discussion. When $c = 0$, the payoff structure excessively biased toward the reward induces the excessive anti-herding. When $c > 0$, for given c and ϕ, the excessively high reward decreases $\Pr(\sigma_B = s)$, which is certainly due to B's deviation from A's action.

6 Concluding Remarks

In this article, we have explored how asymmetry in the reward and penalty as well as costly information affects an agent's strategic behavior in regard to her information acquisition and revelation. If the penalty is greater than the reward, then an agent focuses only on avoiding the penalty. Hence, regardless of whether information is costly to acquire, she will take the same action as the other agent in order to share the penalty, if incurred. On the other hand, if the reward is greater than the penalty, the agent has an incentive to act differently than her peers to be differentiated. If a signal is also given exogenously to the follower, parameter sets of information quality for which she exhibits anti-herding and truthfulness exist. However, the truthful equilibrium, in which agents' truthfulness can be verified endogenously for the whole parameter set of information quality, can be induced by managing the reward and the penalty. If it is costly for the follower to acquire a signal, then parameter sets of information quality for which she exhibits imitation, deviation and truthfulness exist. In particular, if costly information is observed then it is revealed truthfully. Hence, in that situation, providing the payoff structure under which an agent observes her costly signal is important if we want to induce her truthfulness in action.

To focus our attention and deliver the main message in a clear manner, we confine our analysis into a simple environment. Hence there are some possible extensions of the present analysis. For example, we can consider a situation in which players' information qualities are private information and therefore their types matter. Another possibility is to consider the case in which the quality of information depends on the cost paid to acquire. That is, we can assume a case in which more precise information can be attained as she pays more. Finally, when we posit information that is costly to acquire, for the highly nonlinear functional form of critical values of information quality, we did not derive the optimal payoff structure explicitly which maximizes the parameter set of information quality for which the follower's truthfulness in action is guaranteed endogenously. Analyzing this part more in detail, including the case where agents' information qualities are heterogeneous, will be a meaningful extension of the analysis completed here. These awaits the future work.

7 Proof

7.1 Proof of Proposition 1

7.1.1 STEP 1: Deriving B's best response as the follower

Because $c = 0$, B can always observe θ_B. Also, she observes the leader's action a_A before taking her own action. However, due to the lack of the opportunity to observe A's true signal θ_A, she does not know whether a_A is truthful or not. Thus her best response as the follower depends on her belief in the truthfulness of A's action. In following, $E\pi_B(a_B = \theta_B)$ denotes B's expected payoff when she reveals her signal truthfully and $E\pi_B(a_B \neq \theta_B)$ denotes the one when she deviates from her signal.

Case 1) When B believes A's action is truthful

In this case, B's expected payoff is calculated from $E\pi_B(a_B = \theta_B) = \sum_w \Pr(w|\,\tilde{\theta}_A, \theta_B)\pi_B(a_B = \theta_B, a_A = \tilde{\theta}_A)$ and $E\pi_B(a_B \neq \theta_B) = \sum_w \Pr(w|\,\tilde{\theta}_A, \theta_B)\pi_B(a_B \neq \theta_B, a_A = \tilde{\theta}_A)$. In the subgame, B can face either $\theta_B = a_A$ or $\theta_B \neq a_A$. First, without loss of generality, assume that $\theta_B = a_A = h$. As B believes that A's action is truthful, B expects that $\theta_A = h$. Then, from $\Pr(w = H|\,h_A, h_B) = \frac{p^2}{2p^2-2p+1}$ and $Pr(w = L|\,h_i, h_j) = \frac{(1-p)^2}{2p^2-2p+1}$, $E\pi_B(a_B = \theta_B) = \frac{2p-1}{2p^2-2p+1}$ and $E\pi_B(a_B \neq \theta_B) = \frac{(1-p)^2\gamma - p^2\phi}{2p^2-2p+1}$. Then,

$$E\pi_B(a_B = \theta_B) - \ E\pi_B(a_B \neq \theta_B) = \frac{(\phi - \gamma)p^2 + 2(1+\gamma)p - (1+\gamma)}{(2p^2 - 2p + 1)} \tag{A1}$$

Here, the denominator $\left(2p^2 - 2p + 1\right) > 0$ for all $p \in \left(\frac{1}{2}, 1\right)$. Now, if we let $f(p) \equiv (\phi - \gamma)p^2 + 2(1+\gamma)p - (1+\gamma)$, whether $E\pi_B(a_B = \theta_B) \gtreqless E\pi_B(a_B \neq \theta_B)$ depends on $f(p) \gtreqless 0$. First, assume that $\gamma < \phi$. Then, $f(p) = (\phi - \gamma)p^2 + (2p-1)(1+\gamma) > 0$ for $\forall p \in \left(\frac{1}{2}, 1\right)$. So always $E\pi_B(a_B = \theta_B) > \ E\pi_B(a_B \neq \theta_B)$. Second, assume that $\gamma > \phi$. In this case, we can check followings: 1) $f(p)$ is a strictly concave function, 2) It attains the max at $p = \frac{2(1+\gamma)}{2(\gamma-\phi)} > 1$ where $f\left(\frac{2(1+\gamma)}{2(\gamma-\phi)}\right) = \frac{(\phi+1)(\gamma+1)}{(\gamma-\phi)} > 0$, 3) $f\left(\frac{1}{2}\right) = \frac{\phi-\gamma}{4} < 0$, and 4) $f(1) = 1+\phi > 0$. Then, for $p \in \left(\frac{1}{2}, 1\right)$, $f(p)$ is a monotone increasing function, which means that there exists $p^* \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1}{2}, p^*\right)$, $f(p) < 0$ and if $p \in (p^*, 1)$, $f(p) > 0$. That is, if $p \in \left(\frac{1}{2}, p^*\right)$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B \neq \theta_B)$ and if $p \in (p^*, 1)$, $E\pi_B(a_B = \theta_B) > E\pi_B(a_B \neq \theta_B)$ where $p^* = \frac{1}{\gamma-\phi}\left(\gamma + 1 - \sqrt{\gamma + \phi + \gamma\phi + 1}\right)$.

Lemma A.1

Suppose that B believes that A's action is truthful and $\theta_B = a_A$.

1) If $\gamma > \phi$, there exists a critical value $p^* \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1}{2}, p^*\right)$, she exhibits anti-herding and if $p \in (p^*, 1)$, she reveals her signal truthfully.

2) If $\gamma < \phi$, she reveals her signal truthfully for $\forall p \in \left(\frac{1}{2}, 1\right)$.

Here, $p^* = \frac{1}{\gamma-\phi}\left((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1}\right)$

Second, without loss of generality, assume that $\theta_B = l \neq a_A = h$. Then, from $\Pr(w = H|\,h_A, l_B) = \Pr(w = L|\,h_A, l_B) = \frac{1}{2}$, $E\pi_B(a_B = \theta_B) = \frac{\gamma-\phi}{2}$ and $E\pi_B(a_B \neq \theta_B) = 0$. Then, the following result is easily derived.

Lemma A.2

Suppose that B believes that A's action is truthful and $\theta_B \neq a_A$.

1) If $\gamma > \phi$, B reveals her signal truthfully for $\forall p \in \left(\frac{1}{2}, 1\right)$.

2) if $\gamma < \phi$, B exhibits herding for $\forall p \in \left(\frac{1}{2}, 1\right)$.

Case 2) When B believes that A's action is not truthful

In this case, B's expected payoff is calculated from $E\pi_B(a_B = \theta_B) = \sum_w \Pr(w|\,\tilde{\theta}_A, \theta_B)\pi_j(a_B = \theta_B, a_A \neq \tilde{\theta}_A)$ and $E\pi_B(a_B \neq \theta_B) = \sum_w \Pr(w|\,\tilde{\theta}_A, \theta_B)\pi_j(a_B \neq \theta_B, a_A \neq \tilde{\theta}_A)$. First, without loss of generality, assume that $\theta_B = a_A = h$. In this case, B believes that $\theta_A = l$. Then, from $\Pr(w = H|\,l_A, h_B) = \Pr(w = L|\,l_A, h_B) = \frac{1}{2}$, $E\pi_B(a_B = \theta_B) = 0$ and $E\pi_B(a_B \neq \theta_B) = \frac{\gamma-\phi}{2}$. Then, the following result is derived easily.

Lemma A.3

Suppose that B believes that A's action is not truthful and $a_A = \theta_B$.

1) If $\gamma > \phi$, B exhibits anti-herding for $\forall p \in \left(\frac{1}{2}, 1\right)$.

2) If $\gamma < \phi$, B reveals her signal truthfully for $\forall p \in \left(\frac{1}{2}, 1\right)$.

Second, without loss of generality, assume that $\theta_B = h \neq a_A = l$. In this case, B believes that A's true signal is $\theta_A = h$. Then, from $\Pr(w = H|\,h_A, h_B) = \frac{p^2}{2p^2-2p+1}$ and $\Pr(w = L|\,h_A, h_B) = \frac{(1-p)^2}{2p^2-2p+1}$, $E\pi_B(a_B = \theta_B) = \frac{p^2(\gamma-\phi)+2p\phi-\phi}{(2p^2-2p+1)}$ and $E\pi_B(a_B \neq \theta_B) = -\frac{(2p-1)}{(2p^2-2p+1)}$, which yields that

$$E\pi_B(a_B = \theta_B) - E\pi_B(a_B \neq \theta_B) = \frac{p^2\left(\gamma - \phi\right) + p\left(2\phi + 2\right) - \phi - 1}{(2p^2 - 2p + 1)} \tag{A2}$$

We have already checked that $2p^2 - 2p + 1 > 0$ for all $p \in \left(\frac{1}{2}, 1\right)$. Hence, if we denote $k\left(p\right) \equiv p^2\left(\gamma - \phi\right) + p\left(2\phi + 2\right) - \phi - 1$, $E\pi_B(a_B = \theta_B) \gtreqless E\pi_B(a_B \neq \theta_B)$ depends on $k\left(p\right) \gtreqless 0$. First, assume that $\gamma > \phi$. Then we can check the following points: 1) $k\left(p\right)$ is a convex function, 2) $k\left(0\right) < 0$, 3) $k\left(1\right) > 0$, and 4) $k\left(\frac{1}{2}\right) > 0$. So for $p \in \left(\frac{1}{2}, 1\right)$, $f(p)$ is monotone increasing and $k\left(p\right) > 0$ for $\forall p \in \left(\frac{1}{2}, 1\right)$. Hence, $E\pi_B(a_B = \theta_B) > E\pi_B(a_B \neq \theta_B)$ for $\forall p \in \left(\frac{1}{2}, 1\right)$. Second, assume that $\gamma < \phi$. Then we can check the following points: 1) $k\left(p\right)$ is a concave function, 2) $k\left(0\right) < 0$, 3) $k\left(1\right) > 0$, and 4) $k\left(\frac{1}{2}\right) < 0$. So for $p \in \left(\frac{1}{2}, 1\right)$, $f(p)$ is monotone increasing and there exists $\tilde{p} \in \left(\frac{1}{2}.1\right)$ such that if $p \in \left(\frac{1}{2}, \tilde{p}\right)$, $k\left(p\right) < 0$ and if $p \in \left(\tilde{p}, 1\right)$, $k\left(p\right) > 0$. In other words, if $p \in \left(\frac{1}{2}, \tilde{p}\right)$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B \neq \theta_B)$ *and if* $p \in \left(\tilde{p}, 1\right)$, $E\pi_B(a_B = \theta_B) > E\pi_B(a_B \neq \theta_B)$ where $\tilde{p} = \frac{1}{\phi-\gamma}\left(\phi + 1 + \sqrt{\gamma + \phi + \gamma\phi + 1}\right)$.

Lemma A.4

Suppose that B believes that A's action is not truthful and $a_A \neq \theta_B$.

1) Suppose $\gamma > \phi$. Then, B reveals her signal truthfully for $\forall p \in \left(\frac{1}{2}, 1\right)$.

2) Suppose $\gamma < \phi$. Then there exists $\tilde{p}$ such that if $p \in \left(\frac{1}{2}, \tilde{p}\right)$, B exhibits herding and if $p \in \left(\tilde{p}, 1\right)$, she reveals her signal truthfully.

Here, $\tilde{p} = \frac{1}{\phi-\gamma}\left(\phi + 1 + \sqrt{\gamma + \phi + \gamma\phi + 1}\right)$.

7.1.2 STEP 2: Deriving A's best response as the leader

Now, using backward induction, we derive A's best response. Because A acts as the leader, she has no chance to observe θ_B before taking her action. Therefore, A's posterior belief should be about the true state and B's true signal, $\Pr(w, \theta_B | \theta_A)$. Also, she does not know what A's belief will be about the truthfulness of a_A. Thus, A's best response should be derived according to A's expectation for B's belief in the truthfulness of A's action. In following, $E\pi_A(a_A = \theta_A)$ denotes the expected payoff when A reveals her signal truthfully and $E\pi_A(a_A \neq \theta_A)$ denotes the one when she deviates from her signal. In this case, $E\pi_A(a_A = \theta_A) = \sum_w \sum_{\theta_B} \Pr(w, \theta_B | \theta_A)\pi_A(a_A = \theta_A, \cdot)$ and $E\pi_A(a_A \neq \theta_A) = \sum_w \sum_{\theta_B} \Pr(w, \theta_B | \theta_A)\pi_A(a_A \neq \theta_A, \cdot)$.

Case 1) When A expects that B believes that A's action is truthful

Recall Lemma A.1 and Lemma A.2. First, consider the case where $\gamma > \phi$. If $p \in \left(\frac{1}{2}, p^*\right)$, A knows that $a_A \neq a_B$ because B takes the different action from a_A always. Then, $E\pi_A(a_A = \theta_A) = (p\gamma - \phi + p\phi)$ and $E\pi_A(a_A \neq \theta_A) = -(p\gamma - \gamma + p\phi)$. Then,

$$E\pi_A(a_A = \theta_A) - E\pi_A(a_A \neq \theta_A) = (\gamma + \phi)(2p - 1) > 0 \tag{A3}$$

If $p \in (p^*, 1)$, A knows that B reveals her signal truthfully always. Then, $E\pi_A(a_A = \theta_A) = p^2(\phi - \gamma) + p(\gamma - \phi + 2) - 1$ and $E\pi_A(a_A \neq \theta_A) = p^2(\gamma - \phi) - 2p\gamma + \gamma$. Hence,

$$E\pi_A(a_A = \theta_A) - E\pi_A(a_A \neq \theta_A) = (2p - 1)(\gamma - p\gamma + p\phi + 1) > 0 \tag{A4}$$

Second, consider the case where $\gamma < \phi$. Then, from Lemma A.1 and A.2, B's best response as the follower is as follows: *If $\theta_B = a_A$, B reveals her signal truthfully for $\forall p \in \left(\frac{1}{2}, 1\right)$* and if $\theta_B \neq a_A$, *B exhibits herding for $\forall p \in \left(\frac{1}{2}, 1\right)$*. That is, B takes the same action as A always. Then,

$$E\pi_A(a_A = \theta_A) = (2p - 1) > -(2p - 1) = E\pi_A(a_A \neq \theta_A) \tag{A5}$$

Case 2) When A expects that B believes A's action is not truthful

First, consider the case where $\gamma > \phi$. Recall Lemma A.3 and Lemma A.4 which yields that B takes the different action from A always. In this case, it was already checked that A's best response is to reveal her signal truthfully. Second, consider the case where $\gamma < \phi$. Recall Lemma A.3 and Lemma A.4 which states the following: *1) Suppose $\theta_j = a_i$. Then she reveals her signal truthfully for $\forall p \in \left(\frac{1}{2}, 1\right)$. 2) Suppose $\theta_j \neq a_i$. Then there exists $\tilde{p} \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1}{2}, \tilde{p}\right)$, j exhibits herding and if $p \in (\tilde{p}, 1)$, she reveals her signal truthfully.* Now, if $p \in \left(\frac{1}{2}, \tilde{p}\right)$, A knows that B takes the same action as her always. Also, if $p \in (\tilde{p}, 1)$, A knows that B reveals her signal truthfully always. In Case 1), it was already checked that, for both cases, A's best response is to reveal her signal truthfully.

Finally, from Case 1) and Case 2), A's best response is to reveal her signal truthfully always regardless of B's belief.

Lemma A.5

A's best response as the leader is to always reveal her signal truthfully.

Then, from A.5, B assigns a zero probability to the possibility that A deviates from θ_A. Therefore, the perfect Bayesian Nash equilibrium can be characterized as Proposition 1. ■

7.2 Proof of Proposition 2 and 3

In following, $E\pi_i(a_i = \theta_i)$ denotes i's expected payoff when she reveals her signal truthfully and $E\pi_i(a_i \neq \theta_i)$ denotes the one when she deviates from her observed signal. For B, if she does not observe her signal, $E\pi_B(a_B = a_A)$ denotes the expected payoff when she imitates A's action and $E\pi_B(a_B \neq a_A)$ denotes the one when she deviates from A's action.

7.2.1 STEP 1: Deriving B's best response as the follower

CASE 1: When B believes that A's action is truthful

Suppose that B believes A's action is truthful. Then, for given a_A, she must decide whether to observe her costly signal θ_B. If she observes θ_B, the situation can be either $\theta_B = a_A$ or $\theta_B \neq a_A$. If she does not observe θ_B, she should take an action based on θ_A inferred from a_A according to her belief for the truthfulness of a_A.

First, we assume that B observes her costly signal. Then, her best response is same as that of the case where θ_B is given to her exogenously. Thus, her best response of this case is Lemma A.1 and A.2.

Second, suppose that B does not observe her signal. Then, she should make use of θ_A inferred from observing a_A. Assume that $a_A = h$. Then, B should decide whether to imitate or deviate from a_A based on the posterior belief $\Pr(w \mid \theta_A)$ where she believes $a_A = \theta_A$. Then, from $E\pi_B(a_B = a_A) = 2p - 1$ and $E\pi_B(a_B \neq a_A) = -p\phi + (1-p)\gamma$,

$$E\pi_B(a_B = a_A) - E\pi_B(a_B \neq a_A) = p(2 + \gamma + \phi) - (1 + \gamma) \tag{A6}$$

So, if $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, $E\pi_B(a_B = a_A) > E\pi_B(a_B \neq a_A)$ and if $p \in \left(\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, $E\pi_B(a_B = a_A) < E\pi_B(a_B \neq a_A)$. Here, it can be checked that $\frac{1+\gamma}{2+\gamma+\phi} < 1$ for $p \in \left(\frac{1}{2}, 1\right)$. However, if $\gamma > \phi$, $\frac{1+\gamma}{2+\gamma+\phi} > \frac{1}{2}$ and if $\gamma < \phi$, $\frac{1+\gamma}{2+\gamma+\phi} < \frac{1}{2}$. Therefore, when $\gamma > \phi$, if $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, $E\pi_B(a_B = a_A) > E\pi_B(a_B \neq a_A)$ and if $p \in \left(\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, $E\pi_B(a_B = a_A) < E\pi_B(a_B \neq a_A)$. However, if $\gamma < \phi$, $E\pi_B(a_B = a_A) > E\pi_B(a_B \neq a_A)$ for $\forall p \in \left(\frac{1}{2}, 1\right)$.

Lemma A.6

Suppose that B does not observe her signal θ_B when she believes $a_A = \theta_A$.

1) Suppose $\gamma > \phi$. If $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, she imitates A's action and if $p \in \left(\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, she deviates from A's action.

2) Suppose $\gamma < \phi$. Then she imitates A's action always.

Now, using the above results, Lemma A.1, A.2 and A.6, we derive B's strategic decision in regards to observing her costly signal and being truthful in revelation. It should be noted that, when B considers observing her costly signal, she does not know which signal $\theta_B \in \{h, l\}$ will be observed. Thus, the posterior belief should be $\Pr(w, \theta_B | \theta_A)$.

1) When $\gamma > \phi$

If we consider the case where $\gamma > \phi$, B's best response as the follower varies according to whether $p \in \left(\frac{1}{2}, p^*\right), p \in \left(p^*, \frac{1+\gamma}{2+\gamma+\phi}\right)$, and $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$.

Case 1-1) When $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$

If $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, the available strategy to B is to imitate a_A without observing θ_B or reveals to observe θ_B truthfully after observing θ_B. Then

$$E\pi_B(a_B \;=\; \theta_B) = \sum_w \sum_{\theta_B} \Pr(w, \theta_B | \tilde{\theta}_A)\pi_B(\cdot) = p^2(\phi - \gamma) + p(\gamma - \phi + 2) - 1 - c \quad \text{(A7)}$$

$$E\pi_B(a_B \;=\; a_A) = \sum_w \Pr(w | \tilde{\theta}_A)\pi_B(\cdot) = 2p - 1 \quad \text{(A8)}$$

So,

$$E\pi_B(a_B = \theta_B) - E\pi_B(a_B = a_A) = -p^2(\gamma - \phi) + p(\gamma - \phi) - c \quad \text{(A9)}$$

We denote $h(p) \equiv -p^2(\gamma - \phi) + p(\gamma - \phi) - c$. Then, the following points can be checked: 1) $h(p)$ is a strictly concave function and it attains the maximum value at $p = \frac{1}{2}$, 2) $h(0) = -c < 0$, 3) $h(1) = -c < 0$, 4) $h\left(\frac{1}{2}\right) = \frac{1}{4}(\gamma - \phi) - c$, and 5) $h\left(\frac{1+\gamma}{2+\gamma+\phi}\right) = \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2} - c$. From 4), if $c > \frac{\gamma-\phi}{4}$, $h\left(\frac{1}{2}\right) < 0$. Then, for all $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B = a_A)$. On the other hand, if $c < \frac{\gamma-\phi}{4}$, $h\left(\frac{1}{2}\right) > 0$. Then, as $h(p)$ is a monotone decreasing function for $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, $h\left(\frac{1+\gamma}{2+\gamma+\phi}\right) \gtrless 0$ decides $h(p) \gtrless 0$. If $\frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2} < c$, $h(p) < 0$ for all $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$. If $\frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2} > c$, there exists $\hat{p} \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, \hat{p}\right)$, $h(p) > 0$ and if $p \in (\hat{p}, 1)$, $h(p) < 0$. Here, note that $\frac{\gamma-\phi}{4} > \frac{(\phi+1)(\gamma-\phi)(\gamma+1)}{(\gamma+\phi+2)^2}$. Hence, the above can be summarized as follows: If $c > \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B = a_A)$ for $\forall p \in \left(\frac{1}{2}, 1\right)$. If $c < \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$, there exists $\hat{p} \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, \hat{p}\right)$, $E\pi_B(a_B = \theta_B) > E\pi_B(a_B = a_A)$ and if $p \in (\hat{p}, 1)$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B = a_A)$. Here, $\hat{p} = \frac{1}{\gamma-\phi}\left(\frac{1}{2}\gamma - \frac{1}{2}\phi + \frac{1}{2}\sqrt{-4c\gamma + 4c\phi - 2\gamma\phi + \gamma^2 + \phi^2}\right)$.

Lemma A.7

Suppose that $\gamma > \phi$ and $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$.

1) Suppose $c > \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$. Then, B deviates from A's action without observing her signal.

2) Suppose $c < \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$. Then there exists $\hat{p} \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$ such that if $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, \hat{p}\right)$, B observes her signal and reveals it truthfully and if $p \in (\hat{p}, 1)$, B imitates A's action without observing her signal.

Here, $\hat{p} = \frac{1}{\gamma-\phi}\left(\frac{1}{2}\gamma - \frac{1}{2}\phi + \frac{1}{2}\sqrt{-4c\gamma + 4c\phi - 2\gamma\phi + \gamma^2 + \phi^2}\right)$

Case 1-2) When $p \in \left(\frac{1}{2}, p^*\right)$

If $p \in \left(\frac{1}{2}, p^*\right)$, the strategy which B considers is either to observe her signal or to deviate from the leader's action without observing her signal. Her best response in this case can be derived easily from the following reasoning. Suppose that she observes her signal. In this case, if $\theta_B = a_A$, then she exhibits anti-herding and if $\theta_B \neq a_A$, she reveals her signal truthfully. Thus, it is derived that $a_B \neq a_A$. Also, if she does not observe her signal, she deviates from A's action, which yields that $a_B \neq a_A$. That is, whether or not she observes her signal, always it is induced that $a_B \neq a_A$. Then, it is obvious that B has no incentive to observe her costly signal. Thus, in this case, her best response is to deviate from A's action without observing θ_B.

Lemma A.8

Suppose that $\gamma > \phi$ and $p \in \left(\frac{1}{2}, p^*\right)$. Then, B deviates from A's action without observing her signal.

Case 1-3) When $p \in \left(p^*, \frac{1+\gamma}{2+\gamma+\phi}\right)$

In this case, the strategy which B considers is either to observe θ_B and reveals it truthfully or to deviate from A's action without observing θ_B. Then,

$$\begin{aligned} E\pi_B\left[a_B = \theta_B\right] &= \sum_w \sum_{\theta_B} \Pr(w, \theta_B | \, \tilde{\theta}_A)\pi_B(a_B = \theta_B, \cdot) && \text{(A10)} \\ &= p^2\left(\phi - \gamma\right) + p\left(\gamma - \phi + 2\right) - 1 - c && (1) \\ E\pi_B\left[a_B \neq a_A\right] &= \sum_w \Pr(w | \, \tilde{\theta}_A)\pi_B(a_B \neq a_A, \cdot) = \gamma - p\left(\gamma + \phi\right) && \text{(A11)} \end{aligned}$$

Then,

$$E\pi_B(a_B = \theta_B) - E\pi_B(a_B \neq a_A) = p^2\left(\phi - \gamma\right) + p\left(2\gamma + 2\right) - c - \gamma - 1 \quad \text{(A12)}$$

We denote $g\left(p\right) \equiv p^2\left(\phi - \gamma\right) + p\left(2\gamma + 2\right) - c - \gamma - 1$. Then, followings can be checked. 1) $g\left(p\right)$ is a strictly concave function. 2) $g\left(p\right)$ attains the maximized value at $p = \frac{2\gamma+2}{2\gamma-2\phi} > 1$, 3) $g\left(p = 0\right) < 0$, 4) $g\left(p = \frac{1}{2}\right) < 0$, and 5) $g\left(p = p^*\right) < 0$. Now consider the value of $g\left(p = \frac{1+\gamma}{2+\gamma+\phi}\right)$. It can be checked that, if $c > \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$, $g\left(p\right) < 0$ and if $c < \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$, $g\left(p = \frac{1+\gamma}{2+\gamma+\phi}\right) > 0$. Hence, if $c > \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B \neq a_A)$ for $\forall p \in \left(p^*, \frac{1+\gamma}{2+\gamma+\phi}\right)$. Also, if $c < \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$, there exists $\check{p}$ such that if $p \in \left(p^*, \check{p}\right)$, $E\pi_B(a_B = \theta_B) < E\pi_B(a_B \neq a_A)$ and if $p \in \left(\check{p}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, $E\pi_B(a_B = \theta_B) > E\pi_B(a_B \neq a_A)$. Here, $\check{p} = \frac{1}{\gamma-\phi}\left(\gamma + 1 - \sqrt{\gamma + \phi - c\gamma + c\phi + \gamma\phi + 1}\right)$.

Lemma A.9

Suppose that $\gamma > \phi$ and $p \in \left(p^*, \frac{1+\gamma}{2+\gamma+\phi}\right)$.

1) Suppose $c > \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$. Then, B deviates from A's action without observing θ_B.

2) Suppose $c < \frac{(\gamma-\phi)(\phi+1)(\gamma+1)}{(\gamma+\phi+2)^2}$. Then, there exists $\check{p} \in \left(p^*, \frac{1+\gamma}{2+\gamma+\phi}\right)$ such that if $p \in \left(p^*, \check{p}\right)$, B deviates from a_A without observing θ_B and if $p \in \left(\check{p}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, B observes θ_B and *reveals it truthfully.*

Here, $\check{p} = \frac{1}{\gamma-\phi}\left(\gamma + 1 - \sqrt{\gamma + \phi - c\gamma + c\phi + \gamma\phi + 1}\right)$

Then, Lemma A.7, A.8 and A.9 yield the following result.

Lemma A.10

Suppose $\gamma > \phi$ and B believes that $a_A = \theta_A$.

1) Suppose $c > \frac{(\phi+1)(\gamma-\phi)(\gamma+1)}{(\gamma+\phi+2)^2}$. If $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$, B imitates A's action without observing her signal and if $p \in \left(\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}\right)$, she deviates from A's action without observing her signal.

2) Suppose $0 < c < \frac{(\phi+1)(\gamma-\phi)(\gamma+1)}{(\gamma+\phi+2)^2}$. If $p \in \left(\frac{1}{2}, \check{p}\right)$, B deviates from A's action without observing her signal, if $p \in (\check{p}, \hat{p})$, she observes her costly signal and reveals it truthfully, and if $p \in (\hat{p}, 1)$, she imitates A's action without observing her signal.

2) When $\gamma < \phi$

Suppose $\gamma < \phi$. Then, B's best response can be derived easily from following reasoning. When B observes θ_B, if $a_A = \theta_B$, B reveals her signal truthfully and if $a_A \neq \theta_B$, she exhibits herding. Also if B does not observe θ_B, B imitates A's action always. Thus, whether or not she observes her costly signal, it is derived that $a_A = a_B$. Therefore, B has no incentive to observe her costly signal.

Lemma A.11

Suppose that $\gamma < \phi$ and *j believes that $a_i = \theta_i$. Then, j always imitates i's action without observing her signal.*

CASE 2: When B believes that A's action is not truthful

The procedure through which we derive B's best response when she believes that A's action is not truthful is analogous with those used for Lemma A.10 and A.11. Also, below, it is shown that A's best response is always to reveal θ_A truthfully. Hence, the detailed procedure of proof is skipped.

Lemma A.12

Suppose that B believes $a_A \neq \theta_A$.

1) Suppose $\gamma > \phi$. Then, B deviates from A's action without observing her signal for $\forall p \in \left(\frac{1}{2}, 1\right)$.

2) Suppose $\gamma < \phi$. Then, if $p \in \left(\frac{1}{2}, \frac{\phi+1}{\gamma+\phi+2}\right)$, B imitates A's action without θ_B and if $p \in \left(\frac{\phi+1}{\gamma+\phi+2}, 1\right)$, she deviates from A's action without observing θ_B.

7.2.2 STEP 2: Deriving A's best response as the leader

By assumption, θ_A is given exogenously to A. Thus, the strategy which A considers is either to reveal θ_A truthfully or not. Note that, B's belief about the truthfulness in A's action affects B's best response as the follower. However, B's finalized action is to imitate A's action, deviate from A's action without observing θ_B or reveal θ_B truthfully after observing it. In following, $E\pi_A\left(a_A = \theta_A\right)$ denotes A's expected payoff when A reveals her signal truthfully and $E\pi_A\left(a_A \neq \theta_A\right)$ denotes the one when she deviates from her signal. Also, note that, if A expects that B makes use of θ_A without observing θ_B, the posterior belief should be $\Pr(w|\,\theta_A)$. On the other hand, if A expects that B observes θ_B, the posterior belief should be $\Pr(w, \theta_B|\,\theta_A)$.

Consider the case where A expects that B imitates a_A without observing θ_B. Then, for given θ_A, under $\Pr(w|\,\theta_A)$,

$$E\pi_A\left(a_A = \theta_A\right) = 2p - 1 > -\left(2p - 1\right) = E\pi_A\left(a_A \neq \theta_A\right) \tag{A13}$$

Consider the case where A expects that B deviates from a_A without observing θ_B. Then, for given θ_A, under $\Pr(w|\,\theta_A)$, $E\pi_A\left(a_A = \theta_A\right) = \left(p\gamma - \phi + p\phi\right)$ and $E\pi_A\left(a_A \neq \theta_A\right) = -\left(p\gamma - \gamma + p\phi\right)$. Hence,

$$E\pi_A\left(a_A = \theta_A\right) - E\pi_A\left(a_A \neq \theta_A\right) = \left(\gamma + \phi\right)\left(2p - 1\right) > 0 \tag{A14}$$

Consider the case where A expects that B observes θ_B and reveals it truthfully. Then, for given θ_A, under $\Pr(w, \theta_B|\,\theta_A)$, $E\pi_A\left(a_A = \theta_A\right) = \left(2p + p\gamma - p\phi - p^2\gamma + p^2\phi - 1\right)$ and $E\pi_A\left(a_A \neq \theta_A\right) = -\left(2p\gamma - \gamma - p^2\gamma + p^2\phi\right)$. Hence,

$$E\pi_A\left(a_A = \theta_A\right) - E\pi_A\left(a_A \neq \theta_A\right) = \left(2p - 1\right)\left(\gamma - p\gamma + p\phi + 1\right) > 0 \tag{A15}$$

Finally, for all cases, A's best response as the leader is to reveal her signal truthfully. Moreover, this holds for all $p \in \left(\frac{1}{2}, 1\right)$, $\gamma > 1$ and $\phi > 1$. Therefore, regardless of B's belief in the truthfulness of A's action, A's best response as the leader is to reveal θ_A truthfully.

Lemma A.13
A's best response is to reveal θ_A truthfully.

Then, from Lemma 13, in the subgame, B assigns a zero probability to the possibility that A deviates from θ_A. Therefore, the perfect Bayesian Nash equilibrium can be characterized as Proposition 3 and 4. ■

7.3 Proof of Corollary 3

This can be proved if following can be checked: $\frac{\partial p^*}{\partial \gamma} > 0$ and $\frac{\partial p^*}{\partial \phi} < 0$. Note that

$$p^* = \frac{1}{\gamma - \phi}\left(\left(\gamma + 1\right) - \sqrt{\gamma + \phi + \gamma\phi + 1}\right) \tag{A16}$$

Then,

$$\frac{\partial p^*}{\partial \gamma} = \frac{\left(\phi + 1\right)\left(\gamma + \phi + 2 - 2\sqrt{\gamma + \phi + \gamma\phi + 1}\right)}{2\left(\phi - \gamma\right)^2\sqrt{\gamma + \phi + \gamma\phi + 1}} \tag{A17}$$

Here, $\left(\gamma + \phi + 2\right)^2 - \left(2\sqrt{\gamma + \phi + \gamma\phi + 1}\right)^2 = \left(\phi - \gamma\right)^2 > 0$ where $\gamma + \phi + 2 > 0$ and $2\sqrt{\gamma + \phi + \gamma\phi + 1} > 0$. Thus, $\gamma + \phi + 2 > 2\sqrt{\gamma + \phi + \gamma\phi + 1}$, which yields $\frac{\partial p^*}{\partial \gamma} > 0$. Also

$$\frac{\partial p^*}{\partial \phi} = -\frac{\left(\gamma + 1\right)\left(\gamma + \phi + 2 - 2\sqrt{\gamma + \phi + \gamma\phi + 1}\right)}{2\left(\phi - \gamma\right)^2\sqrt{\gamma + \phi + \gamma\phi + 1}} < 0 \tag{A18}$$

Thus, $\frac{\partial p^*}{\partial \phi} < 0$. $\blacksquare$

REFERENCES

Avery, C.N., Chevalier, J.: Herding over the career. Economics Letters 63, 327–333 (1999)

Banerjee, A.V.: A Simple-model of herd behavior. The Quarterly Journal of Economics 107, 797-817 (1992)

Bernhardt, D., Campello, M., Kutsoati, E.: Who herds?. Journal of Financial Economics 80, 657-675 (2006)

Bikhchandani, S., Hirshleifer, D., Welch, I.: A theory of fads, fashion, custom and cultural Change as Informational Cascades. Journal of Political Economy 100, 992-1026 (1992)

Brandenburger, A., Polak, B.: When managers cover their posteriors: making the decisions the market wants to see. *RAND* Journal of Economics 27, 523-541 (1996)

Chen, Q., Jiang, W.: Analysts' weighting of private and public information. Review of Financial Studies 19, 319-355 (2006).

Chevalier, J., Ellison, G.: Career concerns of mutual fund managers. The Quarterly Journal of Economics 114, 389-432 (1999)

Clement, M.B., Tse, S.Y.: Financial analyst characteristics and herding behavior in forecasting. Journal of Finance 60, 307-341 (2005).

De Bondt. W.M., Forbes, W.P.: Herding in analyst earnings forecasts: evidence from the United Kingdom. European Financial Management 5, 143-154 (1999)

Effinger, M.R., Polborn, M.K.: Herding and anti-herding: a model of reputational differentiation. European Economic Review 45, 385-403 (2001)

Gale, D.: What we have learned form social learning?. European Economic Review 40, 617-628 (1996)

Gallo. G., Granger. C.W., Jeon. Y.: Copycats and common swings: the impact of the use of forecasts in information sets. IMF Staff Papers 49 (1), 1-18 (2002)

Graham, J.R.: Herding among investment newsletters: theory and evidence. Journal of Finance 54, 237-268 (1999)

Hong, H., Kubik, J., Solomon A.: Security analysts' career concerns and herding of earnings forecasts. RAND Journal of Economics 31, 121-144 (2000)

Lamont. O.: Macroeconomic forecasts and microeconomic forecasters. Journal of Economics Behavior and Organization 48, 265-280 (2002)

Levy, G.: Anti-herding and strategic consultation. European Economic Review 48, 503-525 (2004)

Massa, M., Patgiri, R.: Compensation and managerial herding: evidence from the mutual fund industry. Available at SSRN: http://ssrn.com/abstract=756885 (2005)

Scharfstein, D.S., Stein, J.C.: Herd behavior and investment. American Economic Review 80, 465-479 (1990)

Swank, O.H., Bauke, V.: The consequences of endogenizing information for the performance of a sequential decision procedure, forthcoming in Journal of Economic Behavior and Organization.

Trueman, B.: On the incentives for security analysts to revise their earnings forecasts. Contemporary Accounting Research 7, 203-222 (1990a)

——————**-:** Analyst forecasting and herding behavior. Review of Financial Studies 7, 97-124 (1994b)

Welch, I.: Herding among security analysts. Journal of Financial Economics 58, 369-396 (2000)

Zitzewitz, E.: Measuring herding and exaggeration by equity analysts and other opinion sellers. Stanford GSB Working Paper (2001a).

——————: Opinion-producing agents: career concerns and exaggeration. Stanford GSB Working Paper (2001b).