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Strategic Information Spillover to be Imitated: Incentive to Make Use of Relative Performance Evaluation

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Strategic information spillover to be imitated: Incentive to make use of relative performance evaluation

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Abstract

In this article, we deal with the topic of intentional information spillover using a model in which both informational- and payoff-externalities are present and the timing of agents' actions is endogenous. In this model, three players, who are heterogeneous in the quality of their information, compete with one another in a common task. According to the results, the weakly-informed players may voluntarily relinquish an option to wait, although no cost is imposed for a delay of action. When acting without a delay, they reveal their information with the hope that others will imitate them. This type of information spillover is due to their incentive, which is to make use of the relative performance evaluation structure under which a bad reputation can be shared if others are also wrong.

JEL classification: D81; D82

Keywords: Blame sharing; Endogenous timing of actions; Herding; Information spillover; Informational externalities; Payoff externalities

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"*Worldly wisdom teaches that it is better for reputation to fail conventionally than to succeed unconventionally.*" - John Maynard Keynes.

1 Introduction

Agents in competitive environments are frequently evaluated based on their performance while fulfilling their duties, and this evaluation in turn affects their wages and promotions. While the evaluation of an agent's performance is affected by many factors, her relative performance in relation to other competitors in the market is often the primary factor.[2] For example, if an agent is less successful than her peers, even though she herself was successful, she will be given a less positive evaluation than her competitors. However, if she alone is successful when compared to her peers, she will be given a high evaluation. This comparative evaluation affects even those situations in which the agent is unsuccessful: if other competitors were also not successful, the responsibility and the blame can be shared, resulting in a less negative evaluation than if she alone had been unsuccessful. As a result of such an evaluation scheme, when carrying out a given task each agent must consider not only her own performance, but also the performance of other agents in the market. That is, a relative performance evaluation results in payoff externalities.

This situation is extremely applicable to agents working in the financial sector, such as analysts and fund managers. The most important aspect of such jobs is the management of information to which access is relatively limited. Due to the agents' access to such restricted information, the agents' truthfulness in revealing this information is a key concern. Much of the current literature addresses the question of agents' truthfulness, and proposes that the agents have an incentive to reveal distorted information. As is well known, herding is a good example of the consequences of an agent's dishonesty in revealing information.[3] It is derived primarily from the fact that the revealed information of early movers prevails over and suppresses the late movers' private information.

In much of the literature which deals with the topic of herding, it is assumed that the timing of the agents' actions is given exogenously. Additionally, this literature primarily addresses the situation in which prevailing information, revealed by the exogenously-given ordering of action, is available when agents make a decision. Hence, herding is a result derived from the strategic behaviors exhibited by the subsequent agents in making use of given available information. However, due to the assumption that the ordering of action is exogenous, an analysis of agents' strategic decisions on the timing of their actions is overlooked, and thus it encounters a limit in fully analyzing the

[2] Kutsoati and Bernhardt (1999) provide a summary of evidences suggesting that analysts' compensation depends on their relative performance. They state that "*For example, the Institutional Investor (II) publishes an annual poll of analysts ranked by their forecasting record, and in many cases an analyst's salary is affected by his rank on the II poll. (see Stickel (1990, 1992)). In the 1970s, Merrill Lynch hired top-ranked analysts using the II poll as a guide*" (p 1). Also, they state that "*Mikhail et al, (1999) finds that analysts who are relativey less accurate than their peers are more likely to lose their jobs, but that absolute forecast accuracy does not affect layoff probabilities*" (p 2).

[3] In empirical studies, Clement and Tse (2005), De Bondt and Forbes (1999), Gallo, Granger and Jeon (2002), Lamont (2002), Hong, Kubik and Solomon (2000), and Welch (2000) find evidence of herding.

situation in which agents compete with one another and act strategically in regard to information. For example, if an agent knows and is displeased by the fact that others can mimic her actions – exhibiting herding behaviors – she may try to hide her information from the others. If, on the other hand, she determines that herding by others is helpful to her for some reasons, she may try to reveal her information to the others. As an action reveals the information on which it is based, the other agents who observe her action can thus infer her information. Given these conditions, controlling the timing of an action is the one way in which an agent can control the flow of her information. Thus, allowing the endogenous timing of action gives an agent the opportunity to make a strategic decision on the flow of her information through a decision on the timing of action.

The aim of this paper is to deepen the understanding of the forces which yield herd behavior by allowing endogenous timing of actions. Of particular interest to this study is the role of timing decisions made by agents who are involved in a competitive environment in which their payoffs depend on a relative performance evaluation. We find that a situation is possible in which the first mover can have an advantage by being imitated by the second mover. To be more specific, this article presents the situation in which an agent wants to act as the leader, voluntarily revealing information for the sake of inducing herding behaviors in the other agents. This interesting result is based on the agents' incentives to make the relative performance evaluation scheme work in their own interest. While extant literature deals with herding, as it is induced by subsequent agents' strategic imitation, we also, by allowing exogenous timing of action in the model, pay attention to the *generation* of the information to which subsequent agents exhibit herding.

The model that we deal with in this article can be described as follows. In this model, three heterogeneous players – the most-informed one (M), the less-informed one (L) and uninformed one (U) – make forecasts (actions) about the unknown true state. Two players are partially informed about the true state, but the third player is uninformed. The signals of the two informed players are private information. Both informed players observe signals which differ in precision, which is public information. There are two rounds, but each player can take an action only once. All actions are irreversible. It is assumed that no cost is imposed for a delay of action. By assuming no penalty for a delay, we can rule out the possibility that an agent, despite her desire to delay, avoids doing so because of negative consequences associated with delaying. Therefore, in this model, an agent's strategic decision to act without a delay can be understood as a voluntary decision unconstrained by considerations other than the usefulness of the delay to her strategy. After all players act, the true state is revealed and each player earn her payoff following the relative performance evaluation system. Hence, in addition to the correctness of her own action, how others perform is also a factor in her payoff. In addition, we assume that, when the timing of the agents' actions is sequential, the follower can observe the leader's action and therefore infer information. Hence, in this model, informational externalities are present along with payoff externalities. Under these conditions, each player should decide strategically both whether to act in round 1 or round 2 and how to act. Throughout this paper, "M" denotes the most-informed player, "L" denotes the less-informed player and "U" denotes the uninformed player.

According to results, players' endogenous decisions on the timings of actions result in multiple equilibria in which the leader is never M. Also, for these multiple equilibria, the considerations of the payoff- and the risk-dominance criteria proposed by Harsanyi & Selten (1988) yield that, in dominant equilibrium, L uses the cut-off property according to the relative quality of her information in deciding the timing of her action. The derived dominant equilibrium can be explained with the following reasoning.

M, who knows that she is the most-informed player, gives a great deal of weight to the possibility that her signal reveals the true state correctly. Hence, she regards other players' identical actions as strategic substitutes and delays her action to prevent her information from being revealed to other less-informed players. On the other hand, whether L delays her action or not depends on the relative quality of her information. That is, unlike M, she considers acting voluntarily without a delay even though a waiting option is available. As no cost is imposed for a delay of action, L expects that M will delay and therefore M's action would not be observable even if she delays. To compensate for this loss of an opportunity to infer M's information, L considers making use of U's incentive to learn, but not always. If L's information quality is relatively high, she has a relatively strong belief in the correctness of her information. Thus, she regards U's identical action as a strategic substitute and delays her action to prevent her information from being revealed to U. On the other hand, if the quality of her information is relatively low, L has a weak belief in the correctness of her information. Then, as L is concerned about being penalized, to minimize a loss in payoff, she regards U's identical action as a strategic complement and wants to induce U's identical action through a spillover of information. This is why L acts without a delay even though a waiting option is available at no cost. Although it is sorted out as the dominated equilibrium, if L's information quality is relatively low, there also exists an equilibrium in which U acts in round 1 and L acts in round 2. Furthermore, interestingly, L ignores her own more precise information and exhibits herding toward U's action although she knows that it delivers no meaningful information. This equilibrium is also derived by the incentives to minimize a risk by sharing an identical action.

In brief, the results show that how agents make strategic decisions on the timing of their actions when considering both informational and payoff externalities. As the timing of action is endogenous, agents can control the flow of information through a decision about the timing of their actions. As one way to pursuit her own interest, information can be revealed intentionally if it is of low quality. The agent who reveals information does so with intent to induce others to imitate her, minimizing the risk by making use of the relative performance evaluation system. Hence, if herding is derived, it is toward the action based on the low quality information. In this way, in addition to the incentives to learn and prevent other's learning which are proposed in the literature, this article provides a new strategic incentive regarding information: to reveal information in order to induce others to take identical actions.

The rest of this chapter is organized as follows: In the next section, we introduce the related literature. In Section 3, we introduces a model. In Section 4, we derive the players' best responses according to the timing of the actions. In Section 5, we characterize the equilibrium of action timing. Section 5 contains the concluding remarks.

2 Related literature

Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), Gale (1996), and Scharfstein and Stein (1990) are representative articles which deal with the topic of herding from a theoretical perspective. The main common assumption of these models is that the timing of agents' actions are given exogenously. Hence, their analyses do not address the impact of agents' strategic decisions regarding the timing of their actions. Also, these models focus primarily on the subsequent agents' strategic decisions in making use of existing prevailing information, and as a result, do not address the leading agent's strategic incentive to reveal information in order to induce subsequent players to exhibit herding. The "sharing the blame effect," which is crucial in our analysis, is explicitly mentioned in Scharfstein and Stein (1990) and is the one primary factor which yields herding in their model.[4] However, in their model, the "sharing the blame effect" is used by the agent who acts as the subsequent player for given information. Thus, the point that information can be revealed in order to make use of the "sharing the blame effect" is not addressed.

Conner and Remelt (1991) and Conner (1995) are also relevant articles in that they explicitly propose the possibility that a strategic advantage can be attained from being imitated, using firms' activities in the market as models. In both articles, allowing others to imitate or copy can be a dominant strategy if the agents are functioning within a positive network externality. However, the network externality defined in their models is positive in the sense that inducing other firms' activities can increase the size of market. Hence, the positive externality defined in both models is quite different from the one presented in this model.

Endogenous timing of actions is addressed, in articles such as Chamley and Gale (1994) and Zhang (1997). Chamley and Gale (1994), using a model in which agents are homogenous, explore the situation in which an agent intends to delay her action using a waiting option for the sake of learning. Zhang (1997) extends the model into the case in which agents are heterogeneous, and shows that the most-informed agent acts as the leader because she has the least to learn compared to the other, less-informed players and is therefore least patient with the cost of delay. Both models consider a situation in which only informational externalities are present. Hence, the above results are based on the assumption that if an agent has any incentive to delay, it is only for the sake of learning.[5]

[4] In detail, in Scharfstein and Stein (1990), the "sharing the blame effect" is crucially based on the assumption that smart type agents observe the correlated signal and dumb type agents observe the noisy signal. Hence, when the subsequent agent exhibits herding, although her actions turn out to be wrong, she can claim that her mistake is due to the systemetically unpredictable shock. In this model, however, the "sharing the blame effect" is caused by the fact that in this payoff structure a bad reputation can be shared as more people make the same mistake.

[5] If we consider the case in which an agent's payoff depends only upon the correctenss of her own action, it corresponds to the case in which only informational externalities are present. Then, as the agent has no reason to consider others' actions, if she has any intent to delay her action, it must be for the sake of learning from observing

To my knowledge, Frisell (2003) is the first article to address the topic of endogenous timing of action in a model incorporating payoff externalities as well. In his model, two heterogeneous firms compete with one another in a common task, and each firm's type is private information. It shows that if the payoff externalities are positive, the more-informed agent acts as the first mover to induce the other agents to take similar actions. As it assumes that each firm's type is unknown, this model provides a deeper insight into the analysis of games in which the timing of action is endogenous. However, in his model, whether the payoff externality is positive or negative is given exogenously, and therefore the model does not address the question of when the externality can be either positive or negative and why. In this paper, however, whether the payoff externality is positive or negative is decided endogenously. Also, this model shows that the equilibrium in which information is deliberately revealed to induce other's learning can be derived even though agents' types are public information. In this way, this article complements the analysis of the case in which the timing of action is endogenous and payoff externalities are present.

This article is an extension of Yoon (2006) in which, in a similar model, only two types of players are considered. According to its results, if two types of players – the more-informed one and the less-informed one – are considered, only two incentives – to learn and to prevent the other's learning – are initiated. Because of the conflict between these incentives, a delay race is induced. Hence, a cost for a delay of action plays an important role in characterizing equilibrium. Although both types of agents gain from a delay, the gain achieved deploying a strategic delay in order to prevent the other from learning is smaller than the gain achieved by learning. Hence, in pure strategy equilibrium, if the sequential timing of action is derived, the leader is the more-informed player. The incentive to induce other's imitation is captured only in a mixed strategy equilibrium when the cost of delay is sufficiently low to allow both agents to have positive net gains from a delay. Compared to Yoon (2006), this extension shows that the introduction of variety in agents' types can yield a pure strategy equilibrium in which information can be revealed intentionally in order to induce imitation on the part of the other players. Also, although it is dominated, the equilibrium in which the least-informed player (uninformed player) can act as the leader and the more-informed player voluntarily herds toward the less-informed player's action is derived.

3 Model

Suppose that there are three players L, M and U, $i \in \{L, M, U\}$, whose jobs are to provide a forecast about the unknown true state. The true state is $w \in \{H, L\}$, which are mutually exclusive. To all players, it is known that the prior probability of each state is $\Pr(w = H) = \Pr(w = L) = \frac{1}{2}$. Before making a forecast, both L and M have opportunities to observe their own signals $\theta_j \in \Theta = \{h, l\}$ which are correlated with the true state respectively where $j \in \{L, M\}$. The draws of their signals

others' actions in order to make a better (more correct) decision.

are conditionally independent given the true state and each player's signal θ_j is private information. The signal θ_j partially reveals information about true state in the following way

$$\begin{aligned}\Pr(\theta_j = h|\, w &= H) = \Pr(\theta_j = l|\, w = L) = p_j \\ \Pr(\theta_j = h|\, w &= L) = \Pr(\theta_j = l|\, w = H) = 1 - p_j\end{aligned} \tag{1}$$

where $p_j \in \left(\frac{1}{2}, 1\right)$. From above, p_j measures the precision of θ_j or the quality of information. As p_j approaches $\frac{1}{2}$, the signal becomes less informative about the true state. As it approaches 1, the signal becomes more informative. Throughout this article, we assume that L and M are heterogeneous in that $p_L < p_M$ where the exact values of p_L and p_M are public information. In the case of U, it is assumed that she has no chance to observe her own signal correlated with the true state. Instead, U knows that other two players L and M are informed players and the exact values of p_L and p_M are known to her. Throughout this paper, L denotes the less-informed player, M denotes the most-informed player and U denotes the uninformed player.

Player i's action set is $A = \{a_i, t_i\}$. First, $a_i \in F = \{h, l\}$ denotes player i's action of forecasting about the true state. Hence, $a_i = h$ ($a_i = l$) denotes the case in which player i forecasts that $w = H$ ($w = L$). Next, each player has two rounds during which she can take an irreversible action only once and $t_i \in T = \{t_1, t_2\}$ denotes player i's timing of action where t_1 denotes round 1 and t_2 denotes round 2. Although i acts in round 2, we assume that no cost is imposed for a delay of action. That is, a waiting option is given to all players. If the actions are taken sequentially, the player who acts in round 2 can observe the action taken in round 1 before taking her own action. However, if the actions are taken simultaneously, each player has no chance to observe other players' actions.

Each player's payoff π_i is determined by the following

$$\pi_i(a_i, a_{-i}, n, w) = \begin{cases} \frac{\gamma}{n} & \text{if } a_i = w \\ \frac{-\gamma}{n} & \text{if } a_i \neq w \end{cases} \tag{2}$$

after two periods are over and the true state is revealed. Here, $\gamma > 0$ and it can be interpreted as the reputation or monetary compensation which depends on the correctness of action. Also, $n \in \{1, 2, 3\}$ denotes the total number(s) of player(s) who took the same action, including herself. The important feature of this payoff structure is that

$$\frac{\triangle \pi_i(n, \cdot)}{\triangle n} = \begin{cases} < 0 & \text{if } a_i = w \\ > 0 & \text{if } a_i \neq w \end{cases} \tag{3}$$

This payoff structure is designed to incorporate the relative performance evaluation induced in the competitive environment in which players are involved. When her action reveals the true state correctly, if other players act identically then negative payoff externalities result because the good reputation the agent earned for taking the correct action must be shared with the others. If, on the other hand, her action turns out to be wrong in revealing the true state, other players' identical actions cause positive payoff externalities because the blame for taking the wrong action will be shared, and therefore the payoff loss caused from failing in revealing the true state will decrease. In this way, the accuracy of player's actions endogenously determines whether other players' identical actions are strategic substitutes or strategic complements. However, as the true state w is not revealed until all players act, each player does not know for certain whether other players' identical actions will be affirmative or not.

As θ_j is private information, even if L and M reveal distorted information that does not align with their observed signals, it cannot be verified. Regarding this, we use the following definitions throughout this article.

Definition 1 *Truthful action: For $j \in \{L, M\}$, if $a_j = \theta_j$, we say that j's action is truthful.*

Definition 2 *Herding: For $j \in \{L, M\}$, 1) when $\theta_j \neq \theta_{-j}$, if $a_j = \theta_{-j} \neq \theta_j$, or 2) when $a_U \neq \theta_j$,* if *$a_j = a_U \neq \theta_j$, we say that j exhibits herding.*

Definition 3 *Imitation & Deviation: For U, when a_j* is observable, *if $a_U = a_j$ ($a_U \neq a_j$), we say that C imitates (deviates from) player j's action.*

In addition, if L and M have no chance to observe a_U and U should act without observing any informed player's action, we assume that both informed players L and M believe that U decides whether $a_U = h$ or $a_U = l$ after flipping a fair coin.[6]

Assumption 1

Suppose U has no chance to observe any informed player's action. In this case, both informed players believe that $\Pr(a_U = h) = \Pr(a_U = l) = \frac{1}{2}$.

Consider each player's pure strategy.[7] All players are informed about p_L and p_M, $s_j : \Theta \longrightarrow T \times F$ and $s_U : T \times F$ where $\Theta = \{h, l\}$, $T = \{t_1, t_2\}$, $F = \{h, l\}$ and $j \in \{L, M\}$. Note again that, as θ_j is private information, it can be interpreted as type. Let λ_{-j} and λ_U, respectively, be -j's and U's posterior beliefs regarding j's truthfulness in revealing θ_j. Then, the strategy profile $S = \{s_L, s_M, s_U\}$ and $\lambda = \{\lambda_j, \lambda_{-j}, \lambda_U\}$ constitute a Perfect Bayesian Nash equilibrium if $E\pi_i(s_L, s_M, s_U)$ where $i \in \{L, M, U\}$ is maximized for given λ, the other player's strategies, and moreover if λ is consistent with S in terms of Bayesian updating.

Finally, the timing of our model is as follows:

T1) Nature decides the true state w. The payoff structure and p_j where $j \in \{L, M\}$ are announced.

T2) $j \in \{L, M\}$ observes her private signal θ_j.

T3) $i \in \{L, M, U\}$ decides both her timing of action and how to act before round 1 starts. Then, she acts according to her decision after round 1 starts.

[6] Although U randomizes her action because she has no chance to observe a_j, if j can observe a_U, j decides her best response for given a_U. Hence, j believes that $\Pr(a_U = h) = \Pr(a_U = l) = \frac{1}{2}$ only if she cannot observe a_U.

[7] In this paper, the mixed strategy equilibrium will not be considered to stick closely to the solution procedure of risk- and payoff-dominance proposed in Harsanyi and Selten (1988).

T4) After two rounds are over, the true state w is revealed. Then, each player earns her payoff according to the correctness of all players' actions.

4 Deriving the best response

In this section, we derive each player's best response according to her timing of action. Below, we provide the brief sketch of procedure and focus on the intuition of the derived result. The detailed procedures are available in the supplementary materials.

4.1 Best response in round 2

4.1.1 Uninformed player

Assume that $t_j = t_1$ and $t_{-j} = t_2$ where $j \in \{L, M\}$. In words, either L or M, but not both, acted in round 1. Then, U can observe a_j and she should decide whether to imitate or deviate from a_j. As she cannot observe a_{-j}, the posterior belief should be $\Pr(w, \theta_{-j}|\tilde{\theta}_j)$ and U's best response is decided by

$$\sum_{w}\sum_{\theta_{-J}} \Pr(w, \theta_{-j}|\tilde{\theta}_j)\pi_U\left(a_U = a_j, \cdot\right) \gtrless \sum_{w}\sum_{\theta_{-j}} \Pr(w, \theta_{-j}|\tilde{\theta}_j)\pi_U\left(a_U \neq a_j, \cdot\right) \tag{4}$$

Here, $\tilde{\theta}_j$ denotes the inference of j's signal according to U's belief in the truthfulness of j's action. As θ_j is private information, whether $a_j = \theta_j$ or not cannot be verified. Hence, U's posterior belief $\Pr(w, \theta_{-j}|\tilde{\theta}_j)$ should be based on the inferred θ_j according to her belief. Next, assume that $t_j = t_{-j} = t_1$. Then, as U has a chance to infer θ_L and θ_M, U's posterior belief should be $\Pr(w|\tilde{\theta}_L, \tilde{\theta}_M)$. Also, she faces one of the following two cases: $a_U = a_M$ and $a_U \neq a_M$. If she observes that $a_U = a_M$, U's best response is decided by

$$\sum_{w}\left[\Pr(w|\tilde{\theta}_L, \tilde{\theta}_M)\pi_U\left(a_U = a_L = a_M, \cdot\right)\right] \gtrless \sum_{w}\left[\Pr(w|\tilde{\theta}_L, \tilde{\theta}_M)\pi_U\left(a_U \neq a_L = a_M, \cdot\right)\right] \tag{5}$$

On the other hand, if she observes that $a_L \neq a_M$, U's best response is decided by

$$\sum_{w}\left[\Pr(w|\tilde{\theta}_L, \tilde{\theta}_M)\pi_U\left(a_U = a_L, \cdot\right)\right] \gtrless \sum_{w}\left[\Pr(w|\tilde{\theta}_L, \tilde{\theta}_M)\pi_U\left(a_U = a_M, \cdot\right)\right] \tag{6}$$

Finally, if no informed player acted in round 1, a_U can be either h or l. Both $a_U = h$ and $a_U = l$ attain the same expected payoff.

4.1.2 Informed players

When $j \in \{L, M\}$ acts in round 2, the following situations are possible: Case 1) $t_{-j} = t_1$, $t_U = t_2$, Case 2) $t_{-j} = t_2$, $t_U = t_1$, Case 3) $t_{-j} = t_U = t_1$, and Case 4) No action was taken in round 1. If j has a chance to observe a_{-j}, her posterior belief should be $\Pr(w|\theta_j, \tilde{\theta}_{-j})$. But if not, it should be

$\Pr(w, \theta_{-j}|\, \theta_j)$. Also, according to whether U acted in round 1 or not and whether U has a chance to observe any informed player's action or not, j's best response is derived from a different decision rule for the following reason. When U did not act in round 1, if U has a chance to observe a_j and therefore infer θ_j, it is obvious that U will imitate or deviate from a_j according to her belief in the truthfulness of a_j, instead of randomizing her action between h and l. Then, j can anticipate U's best response in this case. However, if U did not act in round 1 and U has no chance to infer any informed player's signal, j expects that $\Pr(a_U = h) = \Pr(a_U = l) = \frac{1}{2}$. Finally, although U has no chance to observe a_j, if U acted prior to j and therefore j can observe a_U, j decides her best response for given a_U.

Then, in Case 1) and Case 3), j's best response is decided by

$$\sum_w \left[\Pr(w|\, \theta_j, \tilde{\theta}_{-j})\pi_j\,(a_j = \theta_j, \cdot)\right] \gtreqless \sum_w \left[\Pr(w|\, \theta_j, \tilde{\theta}_{-j})\pi_j\,(a_j \neq \theta_{-j}, \cdot)\right] \tag{7}$$

Next, in Case 2), j's best response is decided by

$$\sum_w \sum_{\theta_{-j}} [\Pr(w, \theta_{-j}|\, \theta_j)\pi_j\,(a_j = \theta_j, \cdot)] \gtreqless \sum_w \sum_{\theta_{-j}} [\Pr(w, \theta_{-j}|\, \theta_j)\pi_j\,(a_j \neq \theta_j, \cdot)] \tag{8}$$

Finally, in Case 4), j's best response is decided by

$$\begin{aligned} & \frac{1}{2}\sum_w \sum_{\theta_{-j}} \Pr(w, \theta_{-j}|\, \theta_j)\,[\pi_j\,(a_j = \theta_j, a_U = h, \cdot) + \pi_j\,(a_j = \theta_j, a_U = l, \cdot)] \\ \gtreqless\ & \frac{1}{2}\sum_w \sum_{\theta_{-j}} \Pr(w, \theta_{-j}|\, \theta_j)\,[\pi_j\,(a_j \neq \theta_j, a_U = h, \cdot) + \pi_j\,(a_j \neq \theta_j, a_U = l, \cdot)] \end{aligned} \tag{9}$$

4.2 Each player's best response in round 1

If U acts in round 1, it is obvious that a_U can be either h or l because both $a_U = h$ and $a_U = l$ attain the same expected payoff. Now consider the best response of the informed player $j \in \{L, M\}$ when she acts in round 1. If $t_j = t_1$, j faces one of the following cases: Case 1) in which $t_{-j} = t_1$ and $t_U = t_1$, Case 2) in which $t_{-j} = t_1$ and $t_U = t_2$, Case 3) in which $t_{-j} = t_2$ and $t_U = t_1$, and Case 4) in which $t_{-j} = t_2$ and $t_U = t_2$. If $t_j = t_1$, she has no chance to observe a_{-j}. Hence, the posterior belief should always be $\Pr(w, \theta_{-j}|\, \theta_j)$. Also note that according to whether U has a chance to observe any informed player's action or not, j's best response is derived from a different decision rule. In Case 2) and Case 4), j's best response is derived from (8). On the other hand, in Case 1) and 3), j's best response is derived from (9).

4.3 Result

From the above procedures, each player's best response according to the players' timing of their actions can be derived as follows. The first key feature of the following results is that if $j \in \{L, M\}$ acts in round 1, she always reveals her signal truthfully. That is, a_j delivers j's true signal θ_j.

Therefore, if $-j$ or U has a chance to observe a_j, θ_j is inferred perfectly. In other words, the player assigns zero probability to the possibility that $a_j \neq \theta_j$.

Lemma 1: *Player L (The less-informed player)*

1) Suppose $t_L = t_1$. Then L always reveals her signal truthfully.

2) Suppose $t_L = t_2$.

2-1) If $t_M = t_1$, regardless of t_U, she always takes the same action as M.

2-2) Suppose $t_M = t_2$ and $t_U = t_1$. If $\theta_L = a_U$, L reveals her signal truthfully. However, when $\theta_L \neq a_U$, if $p_M - 7p_L + 3 > 0$, she exhibits herding and if $p_M - 7p_L + 3 < 0$, she reveals her signal truthfully.

If L acts in round 2 – in other words, if she has a chance to observe M's action – L's best response is always to take the same action as M. As she knows that $p_M > p_L$, if $\theta_L \neq \theta_M$, she gives more weight to the possibility that $w = \theta_M$. Hence, she exhibits herding. Interestingly, when only U acts in round 1, a_U can affect A's best response in round 2 according to the values of p_L and p_M even though L knows that a_U delivers no information about the true state. This is due to the presence of the payoff externality induced by the given payoff structure. Although U's action does not reveal any meaningful information about the true state, sometimes it can be matched with M's action and moreover it can be that $a_L \neq a_M = a_U = w$. If this happens, L is penalized by herself and earns the lowest payoff $-\gamma$. Therefore, L should be concerned about a_U and she becomes more concerned as her belief in the correctness of her information becomes weaker. Hence, when her information quality is relatively low (i.e. $p_M - 7p_L + 3 > 0$, which means that $p_L < \frac{p_M+3}{7}$, if $\theta_L \neq a_U$), L exhibits herding, ignoring her own meaningful information although it is correlated with the true state. However, if she has a relatively strong belief in the correctness of her information (i.e., $p_M - 7p_L + 3 < 0$, which means that $p_L > \frac{p_M+3}{7}$), although $\theta_L \neq a_U$, she does not exhibit herding and instead reveals her signal truthfully.

Lemma 2: *Player M (The most-informed player)*

M reveal her signal truthfully always.

M's best response, which is always to reveal her information truthfully, is intuitive because she knows that she is the most-informed player. It should be particularly noted that, unlike L, when only U acted in round 1, U's action does not affect M's best response. As M knows that she is the most-informed player, although she can be concerned about the case in which $a_M \neq a_L = a_U$, she gives more weight to the possibility that $a_M = w$. Hence, M is less concerned about being penalized by herself. This is why, in contrast to L, she does not exhibit herding.

Lemma 3: *Player U (The uninformed player)*

1) Suppose $t_U = t_1$. Then, U announces either h or l.

2) Suppose $t_U = t_2$ where $j \in \{L, M\}$.

2-1) If $t_j = t_1$ and $t_{-j} = t_2$, she imitates that a_j.

2-2) If $t_j = t_1$ and $t_{-j} = t_1$, she imitates a_M regardless of a_L.

2-3) If $t_j = t_{-j} = t_2$, U announces either h or l.

The result that U imitates the observed action of any informed player is intuitive because, by doing so, she can free-ride on the information correlated with the true state. It is also natural for U to imitate M's action when she can observe both informed players' actions because U knows that M is more informed than L.

4.4 Expected payoffs

Note that, as each player's timing of action is decided endogenously, each player should decide her timing of action before round 1 starts. Hence, in the case of $j \in \{L, M\}$, the posterior belief should be about the true state and the other informed player's true signal, i.e., $\Pr(w, \theta_{-j}|\, \theta_j)$. In the case of U, as she has no chance to observe any informed player's action, her belief should be about the true state and both informed players' true signals, i.e., $\Pr(w, \theta_{-j}, \theta_j)$. The realization of a_j, a_{-j} and a_U in each case depends on each player's best response, proposed in Lemma 1,2 and 3.

Now, each player's expected payoffs according to the players' timings of actions are derived in the following. In the case of $j \in \{L, M\}$, if (t_L, t_M, t_U) does not allow U to observe any informed player's action, then j's expected payoff is derived by

$$E\pi_j(t_L, t_M, t_U) = \frac{1}{2} \sum_{w} \sum_{\theta_{-j}} \Pr(w, \theta_{-j}|\, \theta_j) \left[\pi_j(\cdot, a_U = h) + \pi_j(\cdot, a_U = l)\right] \tag{10}$$

On the other hand, if (t_L, t_M, t_U) allows U to observe either a_j or a_{-j} or both, j's expected payoff is derived by

$$E\pi_j(t_L, t_M, t_U) = \sum_{w} \sum_{\theta_{-j}} \left[\Pr(w, \theta_{-j}|\, \theta_j) \pi_j(a_L, a_M, a_U)\right] \tag{11}$$

In the case of U, her expected payoff is derived by

$$E\pi_U(t_L, t_M, t_U) = \sum_{w} \sum_{\theta_{-j}} \sum_{\theta_j} \Pr(w, \theta_{-j}, \theta_j) \pi_j(a_L, a_M, a_U) \tag{12}$$

Then, each player's expected payoff according to (t_L, t_M, t_U) can be derived as follows.

1) Player L (less-informed player)

$E\pi_L(t_1, t_1, t_1) = E\pi_L(t_1, t_2, t_1) = E\pi_L(t_2, t_2, t_2) = \left(-\frac{1}{12}\right)\gamma\,(4p_M - 14p_L + 5)$

$E\pi_L(t_1, t_1, t_2) = \left(-\frac{1}{3}\right)\gamma\,(2p_M - 4p_L + 1)$

$E\pi_L(t_1, t_2, t_2) = \left(-\frac{1}{6}\right)\gamma\,(p_M - 5p_L + 2)$

$E\pi_L(t_2, t_1, t_1) = \frac{5}{12}\gamma\,(2p_M - 1)$

$E\pi_L(t_2, t_1, t_2) = \frac{1}{3}\gamma\,(2p_M - 1)$

$$E\pi_L(t_2, t_2, t_1) = \begin{cases} \left(-\frac{1}{12}\right)\gamma\,(2p_M - 1) & \text{if } p_M - 7p_L + 3 > 0 \\ \left(-\frac{1}{12}\right)\gamma\,(4p_M - 14p_L + 5) & \text{if } p_M - 7p_L + 3 < 0 \end{cases}$$

2) Player M (Most-informed player)

$E\pi_M(t_1, t_1, t_1) = E\pi_M(t_1, t_2, t_1) = E\pi_M(t_2, t_2, t_2) = \frac{1}{12}\gamma\,(14p_M - 4p_L - 5)$

$E\pi_M(t_1, t_1, t_2) = \frac{1}{6}\gamma\,(5p_M - p_L - 2)$
$E\pi_M(t_1, t_2, t_2) = \frac{1}{3}\gamma\,(4p_M - 2p_L - 1)$
$E\pi_M(t_2, t_1, t_1) = \frac{5}{12}\gamma\,(2p_M - 1)$
$E\pi_M(t_2, t_1, t_2) = \frac{1}{3}\gamma\,(2p_M - 1)$

$$E\pi_M(t_2, t_2, t_1) = \begin{cases} \frac{2}{3}\gamma\,(2p_M - 1) & \text{if } p_M - 7p_L + 3 > 0 \\ \frac{1}{12}\gamma\,(14p_M - 4p_L - 5) & \text{if } p_M - 7p_L + 3 < 0 \end{cases}$$

3) Player U (Uninformed player)

$E\pi_U(t_1, t_1, t_1) = E\pi_U(t_1, t_2, t_1) = E\pi_U(t_2, t_2, t_2) = \left(-\frac{1}{3}\right)\gamma\,(p_L + p_M - 1)$
$E\pi_U(t_1, t_1, t_2) = \frac{1}{6}\gamma\,(5p_M - p_L - 2)$
$E\pi_U(t_1, t_2, t_2) = \left(-\frac{1}{6}\right)\gamma\,(p_M - 5p_L + 2)$
$E\pi_U(t_2, t_1, t_1) = \left(-\frac{1}{3}\right)\gamma\,(2p_M - 1)$
$E\pi_U(t_2, t_1, t_2) = \frac{1}{3}\gamma\,(2p_M - 1)$

$$E\pi_U(t_2, t_2, t_1) = \begin{cases} \left(-\frac{1}{12}\right)\gamma\,(2p_M - 1) & \text{if } p_M - 7p_L + 3 > 0 \\ \left(-\frac{1}{3}\right)\gamma\,(p_L + p_M - 1) & \text{if } p_M - 7p_L + 3 < 0 \end{cases}$$

5 Equilibrium

5.1 Multiple equilibria

In the following, we consider the pure strategy equilibrium. The computations yield the following result.

Proposition 1

The pure strategy equilibrium of the timing of action can be characterized as follows.

1) Suppose $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$. *Then, there exist multiple equilibria* $(t_L, t_M, t_U) = (t_1, t_2, t_2)$ and $(t_L, t_M, t_U) = (t_2, t_2, t_1)$.

2) Suppose $\frac{2p_M+1}{4} < p_L < p_M$. *Then, there exist multiple equilibria* $(t_L, t_M, t_U) = (t_2, t_2, t_2)$ *and* $(t_L, t_M, t_U) = (t_2, t_2, t_1)$.

First, it can be checked that M, using the option to wait, always acts in round 2. Because M knows she is the most-informed player, she gives more weight to the possibility that her action reveals the true state correctly. When she forecasts the true state correctly, she regards the other players' identical actions as ones which cause a negative payoff externality, because the good reputation her correct action earned must be further divided as more players take identical actions. If her information is revealed to any less-informed player, her action will be imitated, which is what she wants to avoid. Therefore, she always uses a waiting option to prevent revealing her information to other less-informed players.

Next, L's decision on the timing of action depends on the values of p_M and p_L as follows. If her information quality is relatively low, i.e. $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$, she acts in either round 1 or round 2. On the other hand, if her information quality is relatively high, i.e., $\frac{2p_M+1}{4} < p_L < p_M$, she always acts in round 2 using a waiting option. If $\frac{2p_M+1}{4} < p_L < p_M$, it corresponds to the case in which p_L is relatively high for given p_M. Hence, in this case, L expects that θ_L has a relatively high probability of revealing the true state correctly. L knows that when she acts in round 1 and U acts in round 2, U imitates her action and this prevents the worst case scenario in which L earns the lowest payoff. In that sense, U's identical action can be affirmative to L. However, when L has a relatively strong confidence in the correctness of her information, she does not intend to induce U's imitation. That is, L gives more weight to the possibility that U's imitation can cause a negative payoff externality. Therefore, L wants to prevent her information from being revealed to U, which is why she delays her action using a waiting option.

On the other hand, if $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$, L evaluates that the quality of her information is relatively low. Hence, she gives more weight to the possibility that θ_L fails to reveal the true state correctly. Then, it is natural that she should be more concerned about the penalty imposed when her action turns out to be wrong. L knows that if she can induce U to take an action identical to her own, although her action turns out to be wrong, she can earn at least $\pi_L = -\frac{\gamma}{2}$ and therefore prevent the worst case in which she earns the lowest payoff $-\gamma$. In this sense, L evaluates U's imitative action as one which will cause a positive payoff externality. So, she acts in round 1 to induce U to take the same action. By acting in round 1, she can reveal her information to U, which induces U's imitation in round 2. This is why L renounces the option to wait even though no cost is imposed for a delay.

Then, what is the reasoning for the equilibrium in which L acts in round 2 when U acts in round 1? This equilibrium is supported by L's best response, which is to take the same action as U, ignoring her information, when her information quality is relatively low and only U acted in round 1. As explained previously, if she can share an identical action with someone, she can prevent earning the lowest payoff. Hence, to compensate for the loss of the opportunity to learn, she takes the same action as U although she knows that a_U delivers no information about the true state.

Finally, consider U's decision on the timing of action. If L's information quality is relatively high, i.e., $\frac{2p_M+1}{4} < p_L < p_M$, both informed players delay their actions. Even when U acts in round 1, for relatively high p_L, L always reveals her signal truthfully. Therefore whether U acts in round 1 or round 2, all players earn the same expected payoffs. If L's information quality is relatively low, i.e., $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$, U can act either in round 1 or round 2. For given $(t_L, t_M) = (t_1, t_2)$, it is obvious that U has no incentive to deviate from $t_U = t_2$ to $t_U = t_1$ because, by acting in round 2, she can imitate a_L which is based on information correlated with the true state. Also, for given $(t_L, t_M) = (t_2, t_2)$, U has no incentive to deviate from $t_U = t_1$ to $t_U = t_2$. If U acts in round 1, she can induce L's identical action, which make her earn at least $\pi_U = -\frac{\gamma}{2}$. As acting in round 2 can yield the possibility that $\pi_U = -\gamma$, U has no incentive to act in round 2.

5.2 Risk dominance and Payoff dominance

Now, for the derived multiple equilibria, we consider the payoff- and the risk-dominance criterion proposed by Harsanyi & Selten (1988). The concept of risk dominance captures the idea that, when the multiple equilibria exist, players will measure the risk involved in playing each of these equilibria and coordinate expectations on the equilibrium which is less risky.

Proposition 2

Suppose $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$*. Then, for given* $t_M = t_2$*,* $(t_L, t_U) = (t_1, t_2)$ *payoff-dominates and risk-dominates* $(t_L, t_U) = (t_2, t_1)$*.*

Proof

We consider the case where $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$. For multiple equilibria, the timing of M's action is fixed as $t_M = t_2$ for both equilibria. Hence, for given $t_M = t_2$, we can focus only on A's and C's timings of actions, $(t_L, t_U) = (t_1, t_2)$ and $(t_L, t_U) = (t_2, t_1)$. In the following, we use the notion that for a two-player game, the risk-dominant equilibrium is the one with the largest Nash product, which means it is the equilibrium for which the product of the losses from deviation is largest. If we recall Lemma 1, L's best response when $t_M = t_2$ *and* $t_U = t_1$ depends on the condition $p_M - 7p_L + 3 \gtreqless 0$. Here, this condition corresponds to $p_L \lesseqgtr \frac{p_M+3}{7}$ where $\frac{p_M+3}{7} < \frac{2p_M+1}{4}$. Therefore, in the following, we must analyze our case $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$ after dividing into $\frac{1}{2} < p_L < \frac{p_M+3}{7}$ and $\frac{p_M+3}{7} < p_L < \frac{2p_M+1}{4}$.

Suppose that $\frac{1}{2} < p_L < \frac{p_M+3}{7}$. Then the corresponding payoff matrix can be represented as follows.

	$t_U = t_1$	$t_U = t_2$
$t_L = t_1$	$-\frac{\gamma(4p_M-14p_L+5)}{12}, -\frac{\gamma(p_L+p_M-1)}{3}$	$-\frac{\gamma(p_M-5p_L+2)}{6}, -\frac{\gamma(p_M-5p_L+2)}{6}$
$t_L = t_2$	$-\frac{\gamma(2p_M-1)}{12}, -\frac{\gamma(2p_M-1)}{12}$	$-\frac{\gamma(4p_M-14p_L+5)}{12}, -\frac{\gamma(p_L+p_M-1)}{3}$

Table 1: Payoff matrix when $\frac{1}{2} < p_L < \frac{p_M+3}{7}$

Consider the payoff dominance. From the given payoff matrix, it is verified that

$$\pi_L(t_2, t_1) - \pi_L(t_1, t_2) = \pi_U(t_2, t_1) - \pi_U(t_1, t_2) = \left(-\frac{5}{12}\right)\gamma(2p_L - 1) < 0 \tag{13}$$

So, $(t_L, t_U) = (t_1, t_2)$ payoff-dominates $(t_L, t_U) = (t_2, t_1)$. Next, for the risk-dominance, the computation yields that

$$\begin{aligned} & [\pi_L(t_2, t_1) - \pi_L(t_1, t_1)][\pi_U(t_2, t_1) - \pi_U(t_2, t_2)] \\ & - [\pi_L(t_1, t_2) - \pi_L(t_2, t_2)][\pi_U(t_1, t_2) - \pi_U(t_1, t_1)] \\ = & \left(-\frac{5}{72}\right)\gamma^2(2p_L - 1)(2p_M - 1) < 0 \end{aligned} \tag{14}$$

So $(t_L, t_U) = (t_1, t_2)$ risk dominates $(t_L, t_U) = (t_2, t_1)$.

Now, suppose that $\frac{p_M+3}{7} < p_L < \frac{2p_M+1}{4}$. The corresponding payoff matrix can be represented as follows.

	$t_U = t_1$	$t_U = t_2$
$t_L = t_1$	$-\frac{\gamma(4p_M-14p_L+5)}{12}, -\frac{\gamma(p_L+p_M-1)}{3}$	$-\frac{\gamma(p_M-5p_L+2)}{6}, -\frac{\gamma(p_M-5p_L+2)}{6}$
$t_L = t_2$	$-\frac{\gamma(4p_M-14p_L+5)}{12}, -\frac{\gamma(p_L+p_M-1)}{3}$	$-\frac{\gamma(4p_M-14p_L+5)}{12}, -\frac{\gamma(p_L+p_M-1)}{3}$

Table 2: Payoff matrix when $\frac{p_M+3}{7} < p_L < \frac{2p_M+1}{4}$

First, if we consider the payoff dominance, it is checked that

$$\pi_L(t_2,t_1) - \pi_L(t_1,t_2) = \left(-\frac{1}{12}\right)\gamma(2p_M - 4p_L + 1) < 0 \tag{15}$$

$$\pi_U(t_2,t_1) - \pi_U(t_1,t_2) = \left(-\frac{1}{6}\right)\gamma(7p_L + p_M - 4) < 0 \tag{16}$$

because $2p_M - 4p_L + 1 > 0$ under the condition that $p_L < \frac{2p_M+1}{4}$. So, $(t_L, t_U) = (t_1, t_2)$ payoff-dominates $(t_L, t_U) = (t_2, t_1)$. Second, for the risk-dominance, the computation yields that

$$\begin{aligned} & [\pi_L(t_2,t_1) - \pi_L(t_1,t_1)][\pi_U(t_2,t_1) - \pi_U(t_2,t_2)] \\ & -[\pi_L(t_1,t_2) - \pi_L(t_2,t_2)][\pi_U(t_1,t_2) - \pi_U(t_1,t_1)] \\ = & \left(-\frac{1}{72}\right)(2p_M - 4p_L + 1)(7p_L + p_M - 4)\gamma^2 < 0 \end{aligned} \tag{17}$$

So $(t_L, t_U) = (t_1, t_2)$ risk dominates $(t_L, t_U) = (t_2, t_1)$. ■

In brief, the consideration of the payoff- and risk-dominance criterion yields that whether L uses a given waiting option or not depends on her relative information quality. In payoff- and risk-dominant equilibrium, if p_L is relatively low, she regards U's imitative action as a strategic complement and wants to induce U to take the same action in order to minimize the risk. Therefore, she acts in round 1 to voluntarily reveal her information although a waiting option is available. On the other hand, if p_L is relatively high, she regards U's same action as a strategic substitute and therefore uses a waiting option to prevent her information from being revealed to U. Especially when p_L is relatively low, i.e., $\frac{1}{2} < p_L < \frac{2p_M+1}{4}$, the result that $(t_L, t_U) = (t_1, t_2)$ is a dominant equilibrium is intuitive. When both U and L intend to minimize a risk by sharing an identical action, the equilibrium in which $(t_L, t_U) = (t_2, t_1)$ is not likely to be derived because sharing the action based on information correlated with the true state will be more attractive than sharing the action which delivers no meaningful information. The considerations of the payoff- and the risk-dominance criterion sort out this equilibrium as the dominated one.

In addition, when $\frac{2p_M+1}{4} < p_L < p_M$, L's and M's timings of actions are fixed by $t_L = t_2$ and $t_M = t_2$ for both equilibria, and what differs is only U's timing of action. However, note that $\frac{2p_M+1}{4} < p_L < p_M$ corresponds to the case in which $p_M - 7p_L + 3 > 0$. Thus, if $t_U = t_1$ and $t_L = t_2$, although $\theta_L \neq a_U$, U reveals her signal truthfully without exhibiting herding. Therefore, whether U's timing of action is either $t_U = t_1$ or $t_U = t_2$, all players' expected payoffs are same. Hence, there should no payoff dominance between two equilibria. Also, it can be checked that the computation yields no risk-dominance between two equilibria. However, in both equilibria, there is no difference in that θ_L and θ_M are revealed truthfully and U randomizes her action.

5.3 Discussion

In a risk-dominant equilibrium, it is easily verified that as M's information quality increases, the probability that L reveals her information to U by acting in round 1 increases because the critical value of p_L, $p_L^* = \frac{2p_M+1}{4}$, increases as p_M increases. As p_M increases, because the cost of losing the opportunity to learn increases, L will have the greater incentive to compensate for it by inducing U to take an identical action. However, although p_M converges to 1, p_L^* converges to $p_L = \frac{3}{4}$. That is, although M's information quality is almost perfectly correlated with the true state, still L can act in round 2 with a positive probability, not to reveal her information to U. On the other hand, if p_M converges to $\frac{1}{2}$, p_L^* converges to $\frac{1}{2}$. That is, as M's signal is less informative about the true state, the probability that L will use a waiting option increases and, in an extreme case, it converges to 1. As the precision of M's signal decreases, the cost of losing the opportunity of learning decreases. Hence, it will be less attractive for L to induce U's identical action because it means that her relative information quality increases.[8]

The key feature of the derived result is that there exists an equilibrium in which the information spillover can be made intentionally for the sake of minimizing a risk, although each player's information quality is public information. When the most-informed player's information is not acquirable, the relatively less-informed player acts strategically to take advantage of the relative performance evaluation. Here, the network externality is induced by the existence of the payoff externality along with the informational externality. When the less-informed player's information quality is relatively low, she wants to induce the least-informed player's identical action intentionally and U also imitates L's action in order to minimize a payoff loss by sharing a blame (payoff externality). The least-informed player's identical action can be induced because action delivers information and therefore learning is available (informational externality).

In brief, if a player has a strong confidence in her information, she wants to hold it and spillover does not occur. This corresponds to the behavior of the less-informed player when the quality of her information is relatively high as well as to the behavior of the most-informed player. On the contrary, however, if a player has a weak confidence in her information, she wants to reveal it intentionally to create a positive network externality. Therefore, in this situation, the revealed information is of low quality and if herding is exhibited, it is toward the action based on information of low quality. That is, the less-informed player's strategic decision on the timing of action is based on the incentive not to pursue meaningful information, but to minimize a risk using the payoff externality.

This model also shows how the number of players' types affect their strategic decisions in revealing information through a decision on the timing of action. This model can be interpreted as an extension of Yoon (2006), in which the endogenous action of only two types players is considered. According to its results, the less-informed player wants to delay her action in order to observe the

[8] By assumption, as p_M decreases, p_L also decreases. Hence, $p_M - p_L$ decreases, which means that the relative quality of M's information increases.

more-informed player's action for the sake of learning. On the other hand, the more-informed player intends to delay her action in order to prevent the less-informed player from inferring her signal. For these two incentives, the conflict of two different types of the second mover advantage is initiated, which yields both players' delay race. If we apply this reasoning to this current model, it can be conjectured that, if M is not considered in our game, L may evaluate U's action only as the one which always causes a negative externality because she is more informed than U. Hence, L will delay her action to prevent her information from being revealed and her incentive to induce U's identical action will not be derived. In this way, although each player's information quality is public information, this model shows that the variety of types of players can yield the possibility that the more-informed agent can intend to reveal information to the less-informed agent intentionally in order to induce herding.[9]

In addition, in this model, it is assumed that the least-informed player is the uninformed one. If, however, instead of this assumption we assume that the least-informed player can observe the least-precise signal correlated with the true state, the main feature of the results will not be changed. In the above procedures, the key factor which yields the equilibrium is the less-informed player's incentive to make use of the learning incentive of the player who is less informed than she is (the least-informed or uninformed player). If the least-informed player can observe a signal correlated with the true state, the less-informed player's incentive to share information through a spillover may increase because the least-informed player's signal can reveal the true state correctly. As the risk of being penalized by herself increases, the critical value of her information quality under which she reveals her information intentionally may increase, which yields the greater possibility of intentional information spillover in the hopes of inducing imitation. From the standpoint of the least-informed player's view, although she can observe her own signal correlated with the true state, she will likely exhibit herding toward the less-informed player's action because she knows that it is based on more precise information.

5.4 Quality of information and ex-ante efficiency

According to the given payoff structure, the first best case, in which all three players' payoffs are maximized, is the one in which all players take correct actions. But, as the correctness of each player's action cannot be checked ex-ante, instead we check the ex-ante efficiency of the derived equilibrium. In the following, $T_1 \succ_E T_2$ means that T_1 is more efficient than T_2 in the sense that the sum of each player's expected payoff is strictly greater in T_1 than in T_2. Also, $T_1 \sim_E T_2$ means that T_1 and T_2 are equivalent in the sense that the sum of each player's expected payoff is equal both in T_1 and in T_2.

[9] In Yoon (2006), in the mixed strategy equilibrium the more-informed player's incentive to act as the leader to induce the less-informed player's identical action is derived. However, as is well known, a mixed strategy equilibrium in a game of complete information can be interpreted as a pure strategy Bayesian Nash equilibrium of the incomplete information case. In this model, however, there exists a pure strategy equilibrium in which information is revealed intentionally by the more-informed player to induce the other's learning.

Corollary 1

1) Suppose that $p_M - 7p_L + 3 > 0$. Then, $(t_2, t_1, t_2) \succ_E (t_1, t_2, t_2) \succ_E (t_2, t_1, t_1) \sim_E (t_2, t_2, t_1) \succ_E (t_1, t_1, t_1) \sim_E (t_1, t_2, t_1) \sim_E (t_2, t_2, t_2) \succ_E (t_1, t_1, t_2)$.

2) Suppose that $p_M - 7p_L + 3 < 0$. Then, $(t_2, t_1, t_2) \succ_E (t_1, t_2, t_2) \succ_E (t_2, t_1, t_1) \succ_E (t_1, t_1, t_1) \sim_E (t_1, t_2, t_1) \sim_E (t_2, t_2, t_2) \sim_E (t_2, t_2, t_1) \succ_E (t_1, t_1, t_2)$.

Here, $(t_n, t_n, t_n) = (t_L, t_M, t_U)$ where $n \in \{1, 2\}$.

As Corollary 1 shows, the first best case is the one in which M acts in round 1 and both L and U act in round 2. This is intuitive if we note that M is the most-informed player. If the most precise information is available and the less-informed players can follow it, this ordering of action will attain the greatest possibility that all players' actions will be correct. However, as we checked, this first best case is not attainable in equilibrium because M always delays her action to prevent her information from being revealed to other, less-informed players. Although the first best case cannot be attained, we can still check the efficiency of the equilibrium and see the relation between the quality of information and efficiency.

Corollary 2

As L's quality of information increases, the equilibrium becomes less socially efficient.

Recall Proposition 1. If $\frac{1}{2} < p_L < \frac{p_M+3}{7}$, the equilibria are (t_1, t_2, t_2) and (t_2, t_2, t_1). (Here, note that $\frac{p_M+3}{7} < \frac{2p_M+1}{4}$.) Then, from 1), it can be verified that the risk-dominant equilibrium is the second best and the risk-dominated equilibrium is the third best. Next, if $\frac{p_M+3}{7} < p_L < \frac{2p_M+1}{4}$, still the equilibria are (t_1, t_2, t_2) and (t_2, t_2, t_1). Then, from 2), the risk-dominant equilibrium is the second best and the risk-dominated equilibrium is the fourth best. Finally, if $\frac{2p_M+1}{4} < p_L < p_M$, the equilibria are (t_2, t_2, t_2) and (t_2, t_2, t_1). Then, from 2), both equilibria are fourth best. Hence, to summarize: as p_L increases, the derived equilibrium becomes less efficient. If we think about the following reasoning, this result is intuitive. If the most-precise information is not available, the next best case will be the one in which the next precise information is available. Hence, L should act in round 1 and U should have a chance to infer θ_L. However, as p_L increases, L wants to hold her information without revealing it. Hence, the socially less-efficient outcome is derived as p_L increases.

6 Concluding remarks

In this article, we explore the incentive of information spillover using a model in which agents compete with each other in a common task and earn a reward or a penalty under a relative performance evaluation system. In this model, the competition of agents is incorporated into the payoff structure, so that payoff externalities are present along with an informational externality. It is assumed that no cost is imposed for a delay of action. Hence, a waiting option is available to all agents and whether each agent will delay her action or not is decided endogenously. According to the derived results, there exist multiple equilibria in which the leader is never the most-informed player. As the most-informed player regards other less-informed players' same actions as strategic substitutes, she wants to prevent her information from being revealed to other less-informed players. This is why she uses a waiting option to delay her action. Whether the sequential actions of the informed players are derived or not depends on the less-informed player's strategic decision on her timing of action. If her information quality is relatively high, she has a strong belief in the correctness of her information. Thus, she regards the uninformed player's imitative action as a strategic substitute. Therefore, to prevent her information from being revealed to the least-informed player, she delays her action. On the other hand, if her information quality is relatively low, she may have a weak belief in the correctness of her information. Hence, she can regard the uninformed player's identical action as a strategic complement and may intend to induce, by revealing her information, the least-informed player to take an identical action. From this reasoning, the intentional information spillover driven by a desire to be imitated can be derived.

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7 Supplementary Material

This part is for the proof of Lemma 1, 2 and 3. In following, first we derive each player's best response in round 2. Then, using the backward induction, we derive her best response in round 1.

7.1 Each player's best response in round 2

7.1.1 Uninformed player U's best response

Lemma A.1

Suppose that U acts in round 2 when only L acted in round 1. Then U's best response can be described as follows.

1) Suppose U believes $\theta_L = a_L$ *and* $\theta_M = a_M$. *Then,* $a_U = a_L$.

2) Suppose U believes $\theta_L = a_L$ *and* $\theta_M \neq a_M$. *Then, if* $p_M - 7p_L + 3 > 0$, $a_U \neq a_L$ *and if* $p_M - 7p_L + 3 < 0$, $a_U = a_L$.

3) Suppose U believes $\theta_L \neq a_L$ *and* $\theta_M = a_M$. *Then, if* $p_M - 7p_L + 3 > 0$, $a_U = a_L$ *and if* $p_M - 7p_L + 3 < 0$, $a_U \neq a_L$.

4) Suppose U believes $\theta_A \neq a_A$ *and* $\theta_B \neq a_B$. *Then,* $a_C \neq a_A$.

Proof of Lemma A.1

Without loss of generality, assume that $a_L = h$. In this case, U's best response is derived from the decision rule (4). a) Assume that U believes that $\theta_L = a_L$ and $\theta_M = a_M$. Then, $E\pi_U(a_U = a_L) = \left(-\frac{1}{6}\right)\gamma\left(p_M - 5p_L + 2\right)$ and $E\pi_U(a_U \neq a_L) = \left(-\frac{1}{2}\right)\gamma\left(3p_L + p_M - 2\right)$. As $E\pi_U\left[a_U = a_L\right] - E\pi_U\left[a_U \neq a_L\right] = \frac{1}{3}\gamma\left(7p_L + p_M - 4\right) > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, her best response is to imitate L's action. b) Assume that U believes that $\theta_L = a_L$ *and* $\theta_M \neq a_M$. Then, $E\pi_U\left[a_U = a_L\right] = \frac{1}{6}\gamma\left(5p_L + p_M - 3\right)$ and $E\pi_U\left[a_U \neq a_L\right] = \frac{1}{2}\gamma\left(p_M - 3p_L + 1\right)$. As $E\pi_U(a_U = a_L) - E\pi_U(a_U \neq a_L) = \left(-\frac{1}{3}\right)\gamma\left(p_M - 7p_L + 3\right)$, if $p_M - 7p_L + 3 > 0$, she deviates from L's action and if $p_M - 7p_L + 3 < 0$, she imitates L's action. c) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M = a_M$. Then, $E\pi_U\left[a_U = a_L\right] = \left(-\frac{1}{6}\right)\gamma\left(5p_L + p_M - 3\right)$ and $E\pi_U\left[a_U \neq a_L\right] = \left(-\frac{1}{2}\right)\gamma\left(p_M - 3p_L + 1\right)$. As $E\pi_U(a_U = a_L) - E\pi_U(a_U \neq a_L) = \frac{1}{3}\gamma\left(p_M - 7p_L + 3\right)$, if $p_M - 7p_L + 3 > 0$, she imitates L's action and if $p_M - 7p_L + 3 < 0$, she deviates from L's action. d) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U\left[a_U = a_L\right] = \frac{1}{6}\gamma\left(p_M - 5p_L + 2\right)$ and $E\pi_U\left[a_U \neq a_L\right] = \frac{1}{2}\gamma\left(3p_L + p_M - 2\right)$. As $E\pi_U(a_U = a_L) - E\pi_U(a_U \neq a_L) = \left(-\frac{1}{3}\right)\gamma\left(7p_L + p_M - 4\right) < 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she deviates from L's action. ■

Lemma A.2

Suppose that U acts in round 2 when only M acted in round 1. Then, if U believes $\theta_M = a_M$, $a_U = a_M$ and *if U believes* $\theta_M \neq a_M$, $a_U \neq a_M$.

Proof of Lemma A.2

Without loss of generality, assume that $a_M = h$. In this case, U's best response is derived from the decision rule (4). a) Assume that U believes that $\theta_L = a_L$ and $\theta_M = a_M$. Then, $E\pi_U\left[a_U = a_M\right] = \frac{1}{6}\gamma\left(5p_M - p_U - 2\right)$ and $E\pi_U\left[a_U \neq a_M\right] = \left(-\frac{1}{2}\right)\gamma\left(p_L + 3p_M - 2\right)$. As

$E\pi_U[a_U = a_M] - E\pi_U[a_U \neq a_M] = \frac{1}{3}\gamma(p_L + 7p_M - 4) > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she imitates M's action. b) Assume that U believes that $\theta_L = a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U[a_U = a_M] = \left(-\frac{1}{6}\right)\gamma(p_L + 5p_M - 3)$ and $E\pi_U[a_U \neq a_M] = \frac{1}{2}\gamma(3p_M - p_L - 1)$. As $E\pi_U[a_U = a_M] - E\pi_U[a_U \neq a_M] = \left(-\frac{1}{3}\right)\gamma(7p_M - p_L - 3) < 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she deviates from M's action. c) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M = a_M$. Then, $E\pi_U[a_U = a_M] = \frac{1}{6}\gamma(p_L + 5p_M - 3)$ and $E\pi_U[a_U \neq a_M] = \left(-\frac{1}{2}\right)\gamma(3p_M - p_L - 1)$. As $E\pi_U[a_U = a_M] - E\pi_U[a_U \neq a_M] = \frac{1}{3}\gamma(7p_M - p_L - 3) > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she imitates M's action. d) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U[a_U = a_M] = \left(-\frac{1}{6}\right)\gamma(5p_M - p_L - 2)$ and $E\pi_U[a_U \neq a_M] = \frac{1}{2}\gamma(p_L + 3p_M - 2)$. As $E\pi_U[a_U = a_M] - E\pi_U[a_U \neq a_M] = \left(-\frac{1}{3}\right)\gamma(p_L + 7p_M - 4) < 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she deviates from M's action. From the above, it is checked that U's belief for the truthfulness of L's action in round 2 does not affect U's best response. Her best response only depends on her belief for the truthfulness of M's action. ■

Lemma A.3

Suppose that U acts in round 2 when both L and M acted in round 1. Then, if U believes $\theta_M = a_M$, $a_U = a_M$ and if she believes $\theta_M \neq a_M$, $a_U \neq a_M$.

Proof of Lemma A.3

First, we consider the case in which $a_L = a_M$. In this case, U's best response is derived from the decision rule (5). Without loss of generality, assume that $a_L = a_M = h$. a) Assume that U believes that $\theta_U = a_U$ and $\theta_M = a_M$. Then, $E\pi_U(a_U = a_M = a_L) = \frac{(p_L+p_M-1)\gamma}{3(2p_Lp_M-p_M-p_L+1)} > 0$ and $E\pi_U(a_U \neq a_M = a_L) = -\frac{(p_L+p_M-1)\gamma}{(2p_Lp_M-p_M-p_L+1)} < 0$ where $2p_Lp_M - p_M - p_L + 1 > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$. So, she imitates both players' actions. b) Assume that U believes that $\theta_L = a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U(a_U = a_M = a_L) = \frac{(p_M-p_L)\gamma}{3(2p_Lp_M-p_M-p_L)} < 0$ and $E\pi_U(a_U \neq a_M = a_L) = -\frac{(p_M-p_L)\gamma}{(2p_Lp_M-p_M-p_L)} > 0$ where $2p_Lp_M - p_M - p_L < 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$. So she deviates from both players' actions. c) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M = a_M$. Then, $E\pi_U(a_U = a_M = a_L) = -\frac{(p_M-p_L)\gamma}{3(2p_Lp_M-p_M-p_L)} > 0$ and $E\pi_U(a_U \neq a_M = a_L) = \frac{(p_M-p_L)\gamma}{(2p_Lp_M-p_M-p_L)} < 0$. So, she imitates both players' actions. d) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U(a_U = a_M = a_L) = -\frac{(p_L+p_M-1)\gamma}{3(2p_Lp_M-p_M-p_L+1)} < 0$ and $E\pi_U(a_U \neq a_M = a_L) = \frac{(p_L+p_M-1)\gamma}{(2p_Lp_M-p_M-p_L+1)} > 0$. So, she deviates from both players' actions.

Second, we consider the case in which $a_L \neq a_M$. In this case, U's best response is derived from the decision rule (6). Without loss of generality, assume that $a_L = h$ and $a_M = l$. a) Assume that U believes that $\theta_L = a_L$ and $\theta_M = a_M$. Then, $E\pi_U(a_U = a_L) = \frac{(p_{M,} - p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} < 0$ and $E\pi_U(a_U = a_M) = -\frac{(p_M-p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} > 0$. So, she imitates M's action. b) Assume that U believes that $\theta_L = a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U(a_U = a_L) = \frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} > 0$ and $E\pi_U(a_U = a_M) = -\frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} < 0$. So, she imitates L's action. c) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M = a_M$. Then, $E\pi_U(a_U = a_L) = -\frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} < 0$ and $E\pi_U(a_U = a_M) = \frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} > 0$. So she imitates M's action. d) Assume that U believes that $\theta_L \neq a_L$ and $\theta_M \neq a_M$. Then, $E\pi_U(a_U = a_L) = -\frac{(p_M-p_L)\gamma}{3(2p_Lp_M-p_M-p_L)} > 0$ and $E\pi_U(a_U = a_M) = \frac{(p_M-p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} < 0$. So she imitates L's action.

From the above, it is checked that U's best response depends only on her belief for the truthfulness of M's action. ■

7.1.2 Informed player L's and M's best response

Player L Lemma A.4

Suppose that L acts in round 2 when only M acted in round 1. Then, L's best response can be described as follows.

1) Suppose that L believes that $\theta_M = a_M$. Then if $\theta_M = a_M$, she reveals her signal truthfully and if $\theta_M \neq a_M$, she exhibits herding.

2) Suppose that L believes that $\theta_M \neq a_M$. Then if $\theta_M = a_M$, she exhibits herding and if $\theta_M \neq a_M$, she reveals her signal truthfully.

Proof Lemma A.4

In following, recall Lemma A.2 which states U's best response in round 2 when only M acted in round 1. In this case, L's best response is derived from the decision rule (7).

Case 1) When L believes that a) M's action is truthful and b) expects that U believes that M's action is truthful: 1) Assume that $\theta_L = a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_L+p_M-1)\gamma}{3(2p_Lp_M-p_M-p_L+1)} > 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_L+p_M-1)\gamma}{(2p_Lp_M-p_M-p_L+1)} < 0$. So, she reveals her signal truthfully. 2) Assume that $\theta_L = l$, $a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_M-p_L)\gamma}{(2p_Lp_M-p_M-p_L)} < 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_M-p_L)\gamma}{3(2p_Lp_M-p_M-p_L)} > 0$. So she exhibits herding.

Case 2) When L believes that a) M's action is truthful and b) expects that U believes that M's action is not truthful: 1) Assume that $\theta_L = a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} > 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} < 0$. So, she reveals her signal truthfully. 2) Assume that $\theta_L = l$, $a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_M-p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} < 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_M-p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} > 0$. So she exhibits herding.

Case 3) When L believes that a) M's action is not truthful and b) expects that U believes that M's action is truthful: 1) Assume that $\theta_L = a_M$, assume $\theta_L = a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_M-p_L)\gamma}{3(2p_Lp_M-p_M-p_L)} < 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_M-p_L)\gamma}{(2p_Lp_M-p_M-p_L)} > 0$. So she exhibits herding. 2) Assume that $\theta_L \neq a_M$, assume that $\theta_L = l$, $a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_L+p_M-1)\gamma}{(2p_Lp_M-p_M-p_L+1)} > 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_L+p_M-1)\gamma}{3(2p_Lp_M-p_M-p_L+1)} < 0$. So, she reveals her signal truthfully.

Case 4) When L believes that a) M's action is not truthful and b) expects that U believes that M's action is not truthful: 1) Assume $\theta_L = a_M = h$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_M-p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} < 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_M-p_L)\gamma}{2(2p_Lp_M-p_M-p_L)} > 0$. So she deviates from her signal. 2) Assume that $\theta_L = h$ and $a_M = l$. Then, $E\pi_L\left[a_L = \theta_L\right] = \frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} > 0$ and $E\pi_L\left[a_L \neq \theta_L\right] = -\frac{(p_L+p_M-1)\gamma}{2(2p_Lp_M-p_M-p_L+1)} < 0$. So she reveals her signal truthfully. ■

Lemma A.5

Suppose that L acts in round 2 and only U acted in round 1. Then L's best response can be described as follows.

1) Suppose that she expects $\theta_M = a_M$ in round 2. *Then if $\theta_L = a_U$, she reports her signal truthfully. However, when $\theta_L \neq a_U$, if $p_M - 7p_L + 3 > 0$, she deviates from her signal and if $p_M - 7p_L + 3 < 0$, she reports her signal truthfully.*

2) Suppose that she expects $\theta_M \neq a_M$ in round 2. Then, when $\theta_L = a_U$, if $p_M - 7p_L + 3 > 0$, she deviates from her signal and if $p_M - 7p_L + 3 < 0$, she reports her signal truthfully. However, if $\theta_L \neq a_U$, she reports her signal truthfully.

Proof of Lemma A.5

In this case, L's best response is derived from the decision rule (8).

Case 1) When L believes that M's action in round 2 is truthful: First, assume that $\theta_L = a_U = h$. Then, $E\pi_L[a_L = \theta_L] = \left(-\frac{1}{6}\right)\gamma(p_M - 5p_L + 2)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{2}\right)\gamma(3p_L + p_M - 2)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{1}{3}\gamma(7p_L + p_M - 4) > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she reveals her signal truthfully. Second, assume that $\theta_L = h$, $a_U = l$. Then, $E\pi_L[a_L = \theta_L] = \left(-\frac{1}{2}\right)\gamma(p_M - 3p_L + 1)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{6}\right)\gamma(5p_L + p_M - 3)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{3}\right)\gamma(p_M - 7p_L + 3)$. So if $p_M - 7p_L + 3 > 0$, she exhibits herding and if $p_M - 7p_L + 3 < 0$, she reveals her signal truthfully.

Case 2) When L believes that M's action is not truthful in round 2: First, assume that $\theta_L = a_U = h$. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{6}\gamma(5p_L + p_M - 3)$ and $E\pi_L[a_L \neq \theta_L] = \frac{1}{2}\gamma(p_M - 3p_L + 1)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{3}\right)\gamma(p_M - 7p_L + 3)$, if $p_M - 7p_L + 3 > 0$, she deviates from her signal and if $p_M - 7p_L + 3 < 0$, she reveals her signal truthfully. Second, assume that $\theta_L = h$, $a_U = l$. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{2}\gamma(3p_L + p_M - 2)$ and $E\pi_L[a_L \neq \theta_L] = \frac{1}{6}\gamma(p_M - 5p_L + 2)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{1}{3}\gamma(7p_L + p_M - 4) > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, she reports her signal truthfully. ■

Lemma A.6

Suppose that L acts in round 2 and both M and U already acted in round 1. Then L's best response is as follows.

1) Suppose that L believes that M's action in round 1 is truthful. Then if $\theta_L = a_M$, she reveals her signal truthfully, but if $\theta_L \neq a_M$, she deviates from signal and exhibits herding.

2) Suppose that L believes that M's action in round 1 is not truthful. Then if $\theta_L = a_M$, she deviates from her signal and exhibits herding, but if $\theta_L \neq a_M$, she reveals her signal truthfully.

Proof of Lemma A.6

In this case, L's best response is derived from the decision rule (7).

Case 1) When L believes that M's action in round 1 is truthful: First, assume that $a_M = a_U = \theta_L = h$. Then, $E\pi_L[a_L = \theta_L] = \frac{(p_L + p_M - 1)\gamma}{3(2p_Lp_M - p_M - p_L + 1)} > 0$ and $E\pi_L[a_L \neq \theta_L] = -\frac{(p_L + p_M - 1)\gamma}{(2p_Lp_M - p_M - p_L + 1)} < 0$. So her best response is to reveal her signal truthfully. Second, assume that $a_M = a_U = h$ and $\theta_L = l$. Then, $E\pi_L[a_L = \theta_L] = \frac{(p_M - p_L)\gamma}{(2p_Lp_M - p_M - p_L)} < 0$ and $E\pi_L[a_L \neq \theta_L] = -\frac{(p_M - p_L)\gamma}{(2p_Lp_M - p_M - p_L)} > 0$. So her best response is to deviate from her signal. Third, assume that $a_M = \theta_L = h$ and $a_U = l$. Then, $E\pi_L[a_L = \theta_L] = \frac{(p_L + p_M - 1)\gamma}{2(2p_Lp_M - p_M - p_L + 1)} > 0$ and $E\pi_L[a_L \neq \theta_L] = -\frac{(p_L + p_M - 1)\gamma}{2(2p_Lp_M - p_M - p_L + 1)} < 0$. So her best response is to report her signal truthfully. Finally, assume that $a_M = h$ and $\theta_L = l = a_U$. Then, $E\pi_L[a_L = \theta_L] = \frac{(p_M - p_L)\gamma}{2(2p_Lp_M - p_M - p_L)} < 0$ and $E\pi_L[a_L \neq \theta_L] = -\frac{(p_M - p_L)\gamma}{2(2p_Lp_M - p_M - p_L)} > 0$. So her best response is to deviate from her signal. Therefore, from the above, if $a_M = \theta_L$, she reveals her signal truthfully and if $a_M \neq \theta_L$, she exhibits herding.

Case 2) When L believes that M's action in round 1 is not truthful: The detailed procedure is analogous. So it is skipped. ■

Lemma A.7

Suppose that L acts in round 2 and no action was taken in round 1. Then L's best response is to reveal her signal truthfully.

Proof of Lemma A.7

In this case, L's best response is derived from the decision rule (9). First, assume that L expects that M's action is truthful in round 2. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{12}\gamma(14p_L + 4p_M - 9)$ and $E\pi_L[a_L \neq \theta_L] = \frac{1}{12}\gamma(4p_M - 14p_L + 5)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{7}{6}\gamma(2p_L - 1) > 0$, L's best response is to reveal her signal truthfully. Although L expects that M's action is not truthful in round 2, the same result is derived. ■

Player M The detailed procedure of deriving M's best response in round 2 is analogous with that used for L's case. So it is skipped.

Lemma A.8

Suppose that M acts in round 2. Then her best response is to reveal her signal truthfully always.

7.2 Each player's best response in round 1

7.2.1 Player U

In the case of U, as she has no chance to observe any informed player's action, her action can be either h or l. Whether $a_U = h$ or $a_U = l$, the same expected payoffs are derived.

7.2.2 Player L

Lemma A.9

Suppose that L acts in round 1. Then L's best response is to reveal her signal truthfully always.

Proof of Lemma A.9

In following, without loss of generality, assume $\theta_L = h$.

Case 1) When U can observe any informed player's action

Case 1-1) When L is the unique leader in round 1

In this case, L's best response is derived from the decision rule (8). From Lemma A.8, M's best response in round 2 is always to reveal her signal truthfully. Then, from Lemma A.1, U's best response in round 2 depends only on U's belief for the truthfulness of L's action.

a) Assume that L expects that U believes that L's action is truthful. Then, L expects that U imitates L's action always. Then, $E\pi_L[a_L = \theta_L] = \left(-\frac{1}{6}\right)\gamma(p_M - 5p_L + 2)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{6}\right)\gamma(5p_L + p_M - 3)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{5}{6}\gamma(2p_L - 1) > 0$, L's best response is to reveal her signal truthfully. b) Assume that L expects that U believes that L's action is not truthful. In this case, if $p_M - 7p_L + 3 > 0$, L expects that U's best response is to imitate L's action. Then, from (a), L's best response is to reveal her signal truthfully. If $p_M - 7p_L + 3 < 0$, L expects that U deviates from L's action. Then, $E\pi_L[a_L = \theta_L] = \left(-\frac{1}{2}\right)\gamma(p_M - 3p_L + 1)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{2}\right)\gamma(3p_L + p_M - 2)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{3}{2}\gamma(2p_L - 1) > 0$, L's best response is to reveal her signal truthfully.

Case 1-2) Both L and M act in round 1

In this case, L's best response is derived from the decision rule (8). From Lemma A.3, U's best response in round 2 depends on her belief for the truthfulness of M's action. 1) Assume that both L believes that M's action is truthful and expects that U believe that M's action is truthful. Then, L expects that U imitates M's action in round 2. Then, $E\pi_L[a_L = \theta_L] = \left(-\frac{1}{3}\right)\gamma(2p_M - 4p_L + 1)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{3}\right)\gamma(4p_L + 2p_M - 3)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{4}{3}\gamma(2p_L - 1) > 0$, L's best response is to reveal her signal truthfully. 2) Assume that L believes that M's action is truthful and expects that U believes that M's action is not truthful. In this case, L expects that U deviates from M's action in round 2. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{2}\gamma(2p_L - 1) > 0$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{2}\right)\gamma(2p_L - 1) < 0$. So L's best response is to reveal her signal truthfully. 3) Assume that L believes that M's action is not truthful and expects that U believes that M's action is truthful. In this case, L expects that U deviates from M's action in round 2. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{3}\gamma(4p_L + 2p_M - 3)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{3}\right)\gamma(4p_L + 2p_M - 3)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{2}{3}(4p_L + 2p_M - 3)\gamma > 0$ for all $p_L, p_M \in \left(\frac{1}{2}, 1\right)$, L's best response is to reveal her signal truthfully. 4) Assume that L believes that M's action is not truthful and expects that U believes that M's action is not truthful. In this case, L expects that U deviates from M's action in round 2. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{2}\gamma(2p_L - 1) > 0$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{2}\right)\gamma(2p_L - 1) < 0$. So L's best response is to reveal her signal truthfully.

Case 2) When U has no chance to observe any informed player's action.

Case 2-1) When both L and U act in round 1

In this case, L's best response is derived from the decision rule (9). From Lemma A.8, both L and U expect that M acts truthfully in round 2. Then, $E\pi_L[a_L = \theta_L] = \left(-\frac{1}{12}\right)\gamma(4p_M - 14p_L + 5)$ and $E\pi_L[a_L \neq \theta_L] = \left(-\frac{1}{12}\right)\gamma(14p_L + 4p_M - 9)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{7}{6}\gamma(2p_L - 1) > 0$, L's best response is to reveal her signal truthfully.

Case 2-2) When all players act in round 1

In this case, L's best response is derived from the decision rule (9). First, assume that L believes that M's action is truthful. Then, this case corresponds to case 2-1). So L's best response is to reveal her signal truthfully. Second, assume that L believes that M's action is not truthful. Then, $E\pi_L[a_L = \theta_L] = \frac{1}{12}\gamma(14p_L + 4p_M - 9)$ and $E\pi_L[a_L \neq \theta_L] = \frac{1}{12}\gamma(4p_M - 14p_L + 5)$. As $E\pi_L[a_L = \theta_L] - E\pi_L[a_L \neq \theta_L] = \frac{7}{6}\gamma(2p_L - 1) > 0$, L's best response is to reveal her signal truthfully.

Finally, L's best response is always to reveal her signal truthfully. ■

7.2.3 Player M

Lemma A.10

Suppose M takes action in round 1. Then M's best response is to reveal her signal truthfully always.

Proof of Lemma A.10

The proof procedure is analogous to that used for the proof of Lemma A.9. So it is skipped.

7.3 Summary of results

From Lemma A.9 and A.10, if $t_j = t_1$, always $a_j = \theta_j$ for $j \in \{L, M\}$. Also, from Lemma A.8, if $t_M = t_2$, always $a_M = \theta_M$. Then, these results simplify our derived each player's best response as follows: 1) In Lemma A.1, if U acts in round 2 when only L acted in round 1, $a_U = a_L$. 2) In Lemma A.2, if U acts in round 2 and only M acted in round 1, $a_U = a_M$. 3) In Lemma A.3, if U acts in round 2 when both L and M acted in round 1, $a_U = a_M$. 4) In Lemma A.4, if L acts in round 2 when only M acted in round 1, if $\theta_L = a_M$, $a_L = \theta_L$ and if $\theta_L \neq a_M$, $a_L = a_M \neq \theta_L$. 5) In Lemma A.5, if L acts in round 2 and only U acted in round 1, if $\theta_L = a_U$, $a_L = \theta_L$. However, when $\theta_L \neq a_U$, if $p_M - 7p_L + 3 > 0$, $a_L = a_U \neq \theta_L$ and if $p_M - 7p_L + 3 < 0$, $a_L = \theta_L$. 6) In Lemma A.6, when L acts in round 2 and both M and U acted in round 1, if $\theta_L = a_M$, $a_L = \theta_L$, but if $\theta_L \neq a_M$, $a_L = a_M \neq \theta_L$. Then, these are the desired results which are Lemma 1, Lemma 2 and Lemma 3.