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On the impact of heterogeneity on indeterminacy.

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Summary. Some recent research indicates that the occurrence of indeterminacy in models with externalities may be overstated because these models ignore agents' heterogeneity. We consider a neoclassical two-sector growth model with technological externalities. Agents are heterogenous in respect to their shares of the initial stock of capital and in labor endowments. We find that the sign of the effect of inequality on indeterminacy is not pinned down by the standard properties of preferences. However, when the inverse of absolute risk aversion is a convex (respectively concave) function homogeneity (heterogeneity) tends to neutralize the external effects and eliminate indeterminacy.

Key words: Endogenous growth, Heterogeneity, Indeterminacy, Inequalities, Income distribution.

JEL-classification numbers: D30, D50, D90, O41.

Running Head: Heterogeneity and indeterminacy

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1 Introduction

Recently, there have been claims that the occurrence of indeterminacy in models with externalities "is overstated in representative agent models, as these ignore the potential stabilizing effect of heterogeneity" (Herrendorf et al. (2000)). Such a general statement is surprising from a general equilibrium viewpoint because heterogeneity provides many additional degrees of freedom to the economy. One may then suspect that the result rests on some specific feature of the model. The aim of the present paper is to analyze the link between heterogeneity and indeterminacy in a standard general equilibrium model and therefore explore the robustness of the results in Herrendorf et al. (2000).

We adopt a dynamic general equilibrium model of the type used by Bewley (1982) but with technological externalities. In order to investigate the dynamic properties of the model, the technology is specified analytically while preferences are kept general. In fact, the model we consider is similar to Example 2.1 in Boldrin and Rustichini (1994). We consider this example because it allows closed solutions. The major innovation is that we abandon the representative agent assumption and admit non-linear utility functions. Agents may be heterogeneous in respect to the share of the initial stock of capital and in labor endowments as well as in preferences. The supply of labor is inelastic. Due to the structure of the model, individual characteristics and heterogeneity do not affect the steady state itself as far as aggregate variables are considered. However, this model is sufficient to analyze the effects of heterogeneity on indeterminacy.

In the model heterogeneity has an effect on indeterminacy. The occurrence of indeterminacy depends on the distribution in labor endowments and in shares of initial capital among the agents as well as on preferences and technology. However, when agents have identical preferences, we find that the sign of the effect of wealth heterogeneity on indeterminacy is not pinned down by the usual axioms imposed on preferences. Indeed, our characterization shows that when the inverse of absolute risk aversion, i.e. risk tolerance, is a convex function homogeneity tends to neutralize the external effects in generating indeterminacy while in the opposite case, homogeneity favors indeterminacy. Note that data and indirect evidence seems to support the convexity of absolute risk aversion but this is not enough to determine the concavity of risk tolerance and therefore the sign of the impact of heterogeneity on indeterminacy.

The introduction of externalities in a model with heterogenous agents presents some well known technical difficulties (see Kehoe, Levine and Romer (1990), Santos (1992) or Section 3 in Ghiglino (2002)). However, the analysis can still be pursued in a way similar to the one which is standard for convex economies. First, the Pseudo-Pareto Optimal (PPO) allocations are obtained as solutions to a central planner's problem in which the objective is a weighted sum of individual utility functions. Second, the dynamic properties of the PPO are analyzed for each given set of welfare weights. Finally, the property of the competitive equilibrium is obtained from the PPO by picking the welfare weights such that all the individual budget constraints are satisfied and binding. Note that the welfare weights are functions of the initial conditions, but because of the externality these functions need not to be continuous. However, we will obtain several results pertaining to the existence of indeterminacy without assuming continuity.

To our knowledge, this paper is the first attempt to analyze the link between indeterminacy and the redistribution of capital shares and labor endowments in a general equilibrium model with external effects. The scope of the present paper is similar to Herrendorf et al. (2000). However, the two frameworks are very different as these authors consider a continuous time overlapping generations model with exogenous prices (similar to Matsuyama (1991)). Ghiglino and Sorger (2002) consider a continuous time, endogenous growth model with externalities and heterogeneous agents. Indeterminacy is shown to occur but their analysis fails to qualify the effects of redistributions on the occurrence of indeterminacy and preferences are bound to be log-linear. Finally, a version without externalities of the present model is also considered in Ghiglino and Olszak-Duquenne (2001) and Ghiglino (2003). In those papers it is shown that with no externalities the distribution of labor endowments and capital shares matters in the stability properties of the steady state.

The paper is organized as follows: In section 2 the model is introduced while the equilibria are defined in Section 3. In section 4 the occurrence of indeterminacy in the model with heterogenous agents is analyzed. Finally, in Section 5 the link between heterogeneity and indeterminacy is obtained.

2 The model

In the present paper we consider a competitive two-sector economy with heterogeneous agents and technology externalities. Since we focus on dynamics, the model need to be kept as tractable as possible. The technology is formalized as in Example 2.1 in Boldrin and Rustichini (1994) but we introduce heterogeneity across agents. The externalities are of the labor-augmenting type as detailed below. There is no joint-production and firms produce according to constant returns production functions so that at the optimum, profits are zero. There are two produced goods, a consumption good and a capital good. The consumption good cannot be used as capital so it is entirely consumed. The capital good cannot be consumed. There are two inputs, capital and labor. We also suppose that there is instantaneous capital depreciation and that labor is inelastically used in production.

There are two firms, one for each sector. The firm in the first sector produces a consumption good with two inputs, capital k^1 and raw labor $\tilde{l}^1$, according to a production function that includes externalities from capital, $\hat{F}^1(k^1, \tilde{l}^1, k)$. The externality is assumed to be a labor-augmenting technological progress, i.e. $\hat{F}^1(k^1, \tilde{l}^1, k) = F^1(k^1, k^\eta \tilde{l}^1)$. Let $l^1 = k^\eta \tilde{l}^1$ be the "effective" labor force and assume that F^1 is a Cobb-Douglas production function. Then $F^1(k^1, l^1) = (l^1)^\alpha (k^1)^{1-\alpha}$ with $\alpha \in (0,1)$ where k^1 is the amount of capital and l^1 the amount of "effective" labor used by the firm of the consumption sector.

In a decentralized economy, the firm maximizes profit

$$Max\ p_t^1\ F^1(k_t^1, k_t^\eta \tilde{l}_t^1) - p_{t-1}^2 k_t^1 - w_t \tilde{l}_t^1$$

where p_t^1 is the present price of the consumption good at period t, p_{t-1}^2 is the present price of the capital good bought at period $t-1$ and w_t the present price of raw labor at period t. The optimal production plan satisfies the first order conditions

$$p_t^1 \frac{\partial F^1}{\partial k} = p_{t-1}^2$$

$$p_t^1 k_t^\eta \frac{\partial F^1}{\partial l} = w_t$$

In the second sector, the externality is also a labor-augmenting technological progress so it can be treated as above. The representative firm produces a capital good according to a Leontief function $F^2(k^2, l^2) = Min\ (l^2, k^2/\gamma)$ with $\gamma \in (0,1)$. The optimal production plan for this firm is then

$$l^2 = k_t^\eta \tilde{l}_t^2 = \frac{k_t^2}{\gamma}$$

There are n agents. In each period consumers provide inelastically a constant amount of labor e_i, $i = 1, ..., n$ with $\sum_{i=1}^n e_i = 1$. A model in which the amount of labor provided is endogenously determined could be analyzed but at a much higher cost. At the beginning of the economy, each agent i is endowed with a fixed share θ_i of the initial stock of capital k_0 with $\sum_{i=1}^{n} \theta_i = 1$. Consumer's preferences are characterized by a discounted utility function of the form

$$U^i(x^i) = \sum_{t=0}^{\infty} \delta^t \quad u_i(x_{it})$$

where x_{it} is the consumption of agent i at time t and x^i is its intertemporal consumption stream. In order to ensure existence of the interior steady state we assume $\delta > \gamma$. The instantaneous utility function fulfills the Inada condition

$$\lim_{x_{it} \to 0} u_i'(x_{it}) = +\infty.$$

In a decentralized economy, an agent i maximizes his utility function subject to a single budget constraint

$$\sum_{t=0}^{\infty} p_t^1 x_{it} = \sum_{t=0}^{\infty} w_t e_i + \theta_i k_0 \quad \text{with } i = 1, ..., n.$$

where the price of k_0 has been normalized to one.

3 Equilibria and steady states

In the present economy the first welfare theorem does not necessary hold. However, as was recognized by Kehoe, Levine and Romer (1992), every competitive equilibrium obtained in a decentralized economy is a Pseudo-Pareto Optimum (PPO) in the sense that is the solution to the maximization of a social welfare function (see Ghiglino (2002) for some applications of this approach). This function could be considered as the objective of a constrained central planner. In the current section we first define competitive equilibria and then characterize the set of Pseudo-Pareto Optima.

3.1 The competitive equilibrium

A competitive equilibrium can be defined as a sequence satisfying the following definition. Note that, due to the form of the externality the total "effective" labor at time t is the product of the work force with k_t^η.

Definition 1 *A competitive equilibrium is a sequence of prices* $(p_t^1, p_t^2, w_t)_{t=0}^\infty$ *such that markets clear for every* $t \geq 0$*:*

- $l_t^1 + l_t^2 = k_t^\eta \sum_{i=1}^n \omega_i = k_t^\eta$
- $k_{t+1}^1 + k_{t+1}^2 = F^2(k_t^2, l_t^2)$
- $\sum_{i=1}^n x_{it} = F^1(k_t^1, l_t^1)$
- $k_0^1 + k_0^2 = k_0$ *with* k_0 *given*

where

- (x_{it}) *is a solution to the individual maximization program of agent* i, $i = 1, ..., n$ *for* $(p_t^1, p_t^2, w_t)_{t=0}^\infty$.
- (k_t^j, l_t^j) *is a solution to profit maximization for firm* j, $j = 1, 2$ *for* $(p_t^1, p_t^2, w_t)_{t=0}^\infty$ *with* $l_t^j = k_t^\eta \tilde{l}_t^j$ *and* k_t^η *given.*

In the present model, competitive equilibria are Pseudo-Pareto Optimal allocations, i.e. solutions to the maximization of a "social" welfare function (see example 2.1, Kehoe, Levine and Romer (1992))

3.2 The planner's optimum

A Pseudo-Pareto Optimal (PPO) allocation is a solution to the planner's problem for a given vector of welfare weights $\mu \in [0,1]^{n-1}$:

$$
\begin{aligned}
Max \quad & \sum_{i=1}^{n-1}(\mu_i \sum_{t=0}^{\infty} \delta^t \; u_i(x_{it})) + (1 - \sum_{i=1}^{n-1} \mu_i) \sum_{t=0}^{\infty} \delta^t \; u_n(x_{nt}) \\
s.t. \quad & \sum_{i=1}^{n} x_{it} = F^1(k_t^1, l_t^1) \;\; \text{for all } t \\
& k_{t+1}^1 + k_{t+1}^2 = F^2(k_t^2, l_t^2) \;\; \text{for all } t \\
& l_t^1 + l_t^2 = z_t^{\eta} \;\; \text{for all } t \\
& k_0 \;\; \text{given}
\end{aligned}
$$

together with the side condition $z_t = k_t$.
The set of PPO is obtained when μ spans $[0,1]^{n-1}$ with $\sum_{i=1}^{n-1} \mu_i \leq 1$. A given competitive equilibrium is obtained for a μ such that the budget constraints of all the consumers bind.

For the case with no externalities, i.e. $\eta = 0$, the solutions to the above program are interior as soon as $e_i \neq 0$ or $\theta_i \neq 0$ for $i = 1, ..., n$. As shown in Ghiglino and Olszak-Duquenne (2001) this is a consequence of the Inada conditions on preferences and technology.

Let u_μ be a social utility function defined by

$$
\begin{aligned}
u_\mu(x) \;\; = \;\; & Max \; \sum_{i=1}^{n-1} \mu_i u_i(x_{it}) + (1 - \sum_{i=1}^{n-1} \mu_i) \; u_n(x_{nt}) \\
& s.t \;\; \sum_{i=1}^{n} x_{it} = x
\end{aligned}
$$

Let $T(k,y,z)$ be the usual transformation function giving the maximal output in the capital good compatible with total capital input k and to consumption output at least equal to y. It is defined as the solution to

$$
\begin{aligned}
Max \qquad & F^1(k^1,l^1) \\
s.t. \qquad & F^2(k^2,l^2) \geq y \\
& k^1+k^2=k \\
& l^1+l^2=z^\eta
\end{aligned}
$$

Then the planner's problem is seen to be equivalent to

$$
\begin{aligned}
Max \qquad & \sum_{t=0}^{\infty} \delta^t u_\mu(T(k_t,k_{t+1},z_t)) \\
s.t. \qquad & F^2(k_t,z_t^\eta) \geq k_{t+1} \\
& k_0 \ \text{ given}
\end{aligned}
$$

The solution depends on z_t and k_0. However, there is still to take into account the side condition $z_t = k_t$. With the specification of production adopted through the paper the transformation function can be written as

$$T(k,y,z) = (z^\eta - y)^\alpha (k - \gamma\ y)^{1-\alpha}$$

In the sequel we use the return function $V : R_+ \times R_+ \times R_+ \to R$ defined by

$$V_\mu(k,y,z) = u_\mu(T(k,y,z))$$

The function V_μ is concave in (k,y), because u_μ and T are concave.

3.3 The Euler conditions

Using the return function $V_\mu(k, y, z)$ the maximization program can be written as

$$\begin{array}{rll} Max & & \sum_{t=0}^{\infty} \delta^t \; V_\mu(k_t, k_{t+1}, z_t) \\ s.t. & (k_t, k_{t+1}) \in D_t, & k_0 \text{ given} \\ \text{side condition:} & & z_t = k_t. \end{array}$$

where D_t is the set $\{(k_t, k_{t+1}) | F^2(k_t, z_t^\eta) \geq k_{t+1}\}$. Let $V_{\mu 1}(k, y, z) = \partial V_\mu(k, y, z)/\partial k$ and $V_{\mu 2}(k, y, z) = \partial V_\mu(k, y, z)/\partial y$.

In the present framework it is a standard result that the set of interior Pseudo-Pareto Optima is the set of $\{k_t\}_t$ that satisfies the transversality condition $\lim_{t \longrightarrow \infty} \delta^t V_{\mu 1}(k_t, k_{t+1}, k_t) k_t = 0$ and are solutions to the system

$$V_{\mu 2}(k_t, k_{t+1}, k_t) + \delta V_{\mu 1}(k_{t+1}, k_{t+2}, k_{t+1}) = 0, \forall t \geq 0$$

3.4 The steady state in the capital good

At a steady state, $k_t = k^*$ for every $t \geq 0$. The capital k^* is implicitly defined by the equation

$$V_{\mu 2}(k^*, k^*, k^*) + \delta V_{\mu 1}(k^*, k^*, k^*) = 0$$

In models with a unique consumption sector, aggregate steady state variables depend only on the technology. Indeed, using the definition of the return function, the Euler condition can be written as

$$T_2(k^*, k^*, k^*) + \delta T_1(k^*, k^*, k^*) = 0$$

where $T_1(x,y,z) = \partial T(x,y,z)/\partial x$ and $T_2(x,y,z) = \partial T(x,y,z)/\partial y$. Some easy calculations gives the steady state capital as a function of the discount factor and the technology parameters only

$$k^* = \left[\frac{(1-\alpha)(\gamma-\delta)}{\gamma-\alpha-\delta(1-\alpha)}\right]^{\frac{1}{1-\eta}}$$

In the present paper, total labor supply is normalized to one. In more general models k^* would represent capital at the stead state normalized by total labor supply. Finally, aggregate consumption x^* is given by

$$x^* = T(k^*, k^*, k^*) = k^*(k^{*\eta-1} - 1)^{\alpha}(1-\gamma)^{1-\alpha}$$

3.5 The steady state in individual consumptions

At the steady state, aggregate capital does not depend on the return function. Consequently, the welfare weights are irrelevant and both the individual preferences and the way endowments are distributed among individuals do not matter. In more general models this is not true. In particular, when there are two consumption goods the steady state values of aggregate consumption depend on the individual welfare weights and therefore on the heterogeneity in preferences and endowments.

As opposed to aggregate variables, the steady state values of individual consumption do depend on individual characteristics through the welfare weights. The exact relationship is provided by the following Lemma.

Lemma 1 *At a steady state k^* the individual demands are*

$$\left\{ \; {x_i}^* = \frac{x^*}{1-\gamma} \, [\; (\delta(1-\alpha)+\alpha-\gamma)e_i + (1-\delta)(1-\alpha)\theta_i \;] \right.$$

where $x^* = k^*({k^*}^{\eta-1}-1)^\alpha(1-\gamma)^{1-\alpha}$.

Proof: See the Appendix. Q.E.D.

Lemma 1 implies that for given technology parameters and discount factor, steady state values of individual consumption depend linearly on initial holdings in capital and labor endowments. This also means that there is a linear manifold of $(\theta_i, e_i)_{i=1,\ldots,n}$ associated to each equilibrium allocation.

In general equilibrium convex models the limit point depends on the distribution of initial capital and labor, even when the turnpike property holds. However, Yano (1984, 1991, 1998) shows that the sensitivity to the shares of initial capital tends to disappear as the time discount factor δ approaches one. A similar result concerning the steady states holds in the present model. Note that individual consumptions have limit points that depend on the individual endowments in labor.

Lemma 2 *When δ is sufficiently close to one the steady states associated to different distributions of individual holdings of initial capital lie in a neighborhood of $\overline{k^*} = (1-\alpha)^{1/1-\eta}$ and this neighborhood shrinks as $\delta \to 1$. Similarly, the individual steady state consumptions lie close to*

$$\overline{x_i^*} = \alpha^\alpha[(1-\alpha)(1-\gamma)]^{1-\alpha}e_i$$

Proof: From Lemma 1 with straightforward calculations. Q.E.D.

4 Indeterminacy with heterogenous agents

The dynamics of the heterogeneous agents model can in principle be deduced from the dynamics of the Pseudo-Pareto Optima (PPO). Indeed, once the dynamics of these is known the only thing that remains to be done is to pick the PPO that corresponds to the given distribution of endowments. However, this construction doesn't imply that the local stability and determinacy properties of the steady state can be deduced from the properties of the PPO allocations with the welfare weights fixed at the steady state values (see Ghiglino (2002)). When the welfare weights are continuous functions of the initial conditions, the dynamic and determinacy properties of the general equilibrium model with heterogeneous agents and those of the model with fixed weights are identical. Ghiglino and Olszak-Duquenne (2001) have shown that when there are no externalities, continuity holds. Because of the externality the continuity property cannot be assumed here. However, the analysis of indeterminacy can be pursued to a large extent without this strong property. The fundamental property is that indeterminacy of the solutions to the planner's problem implies the existence of disaggregate economies with indeterminate competitive equilibria. This result is contained in the following Lemma.

Lemma 3 *A sufficient condition for the existence of local indeterminacy in the general equilibrium economy with heterogeneous agents is indeterminacy in the model with the welfare weights fixed at their steady state values.*

Proof: Assume that indeterminacy occurs in the aggregated model with fixed welfare weights. Then, for a given aggregate initial stock k_0 and welfare weights $(\mu_i)_{i=1}^n$ there is a continuum of paths $(k_t)_{t=1}^\infty$ converging to the steady state. For each of these paths, the first order condition associated to

$$Max \sum_{i=1}^{n-1} \mu_i u_i(x_{it}) + (1 - \sum_{i=1}^{n-1} \mu_i)\ u_n(x_{nt})$$

$$s.t \sum_{i=1}^{n} x_{it} = T(k_t, k_{t+1,} k_t)$$

gives the individual consumption allocations $((x_{it})_{t=0}^{\infty})_{i=1}^{n}$. Finally, $(\theta_i)_{i=1}^{n}$ is obtained from the individual budget constraints and the values of the prices and the wages. Due to the indeterminacy there is an open interval of feasible $(\theta_i)_{i=1}^{n}$ with the same k_0 and $(\mu_i)_{i=1}^{n-1}$. On the other hand, in a neighborhood of the steady state, a small perturbation of the welfare weights will only slightly affect the path $(k_t)_{t=1}^{\infty}$. Due to the continuity of the functions involved and the fact that prices converge to the steady state price, the interval of feasible $(\theta_i)_{i=1}^{n}$ is also only slightly affected. Therefore, there is an open set of $(\theta_i)_{i=1}^{n}$ such that for each element in this set there is a continuum of equilibrium paths. Therefore the steady state is indeterminate in the individual variables. Q.E.D.

Lemma 3 allows to focus on the properties of the PPO in order to prove indeterminacy in the model with heterogeneous agents. For fixed μ, the behavior of k_t near the steady state k^* is obtained from the linearization of the Euler equation near (k^*, k^*, k^*) . The stability and the local determinacy properties of the steady state depend on how the modulus of the eigenvalues λ_1 and λ_2, obtained as solutions to the characteristic equation, compares to one. For a given discount factor and technology parameters, the two eigenvalues are shown to depend on $\rho_\mu(x^*) = -u'_\mu(x)(u''_\mu(x))^{-1}$, i.e. the absolute risk tolerance associated to the social utility function at the steady state. Finally, the effect of heterogeneity on dynamics can be analyzed because the welfare weights μ depend on the distribution (θ, e). The formal result follows.

Proposition 1 *There exist open set of economies, defined by utility functions and parameters (α, γ, η, δ), such that the occurrence of local indeterminacy at the competitive steady state depends on the shape of the distribution (θ, e).*

Proof: See Appendix. Q.E.D.

Remarks: Proposition 1 also holds when the agents have identical preferences and identical labor endowments or capital shares. On the other hand, for a sufficiently weak externality the usual turnpike property applies to this economy, i.e. the steady state becomes determinate and stable as $\delta \to 1$. Finally, there exists $\overline{\eta}$ such that for $\eta > \overline{\eta}$ the stability is not implied by a high discount factor δ. (see the Result 3 in the Appendix for a proof).

5 The impact of heterogeneity on indeterminacy

Heterogeneity is one of the main macroeconomic indicators of the microeconomic structure of the economy. When agents have identical preferences, heterogeneity can be characterized by the spread of shares of capital and/or labor endowments. If we furthermore assume that only shares of initial capital (or only labor endowments) differ, the agents can be distributed on the real line. In a homogeneous economy the distribution of shares is picked around some intermediate value while in a heterogenous economy the shares are widely spread and so are the equilibrium individual allocations. Several criteria can be used to rank distributions. The formal definition we use is given below.

Definition 2 *Assume there are N types of consumers ordered according to their steady state allocation, i.e. $x_i \leq x_j$ for $i < j$. Let $n_i(J)$ be the number of consumers of type i in economy J and let $n(J)$ be the corresponding distribution. Furthermore, assume that* *the mean of the distribution* $\sum_{i=1}^{N} n_i(J)x_i$ is *independent of J. Then Economy A is said to be no more unequal than Economy B iff* $\sum_{i=1}^{N} n_i(A)f(x_i) \geq \sum_{i=1}^{N} n_i(B)f(x_i)$ *for all continuous and concave functions* f, *noted* $n(A) \preceq_I n(B)$.

In equilibrium, $\sum_{i=1}^{N} n_i(J)x_i$ is equal to x^* regardless of the distribution because of market clearing. Therefore, assuming that $\sum_{i=1}^{N} n_i(J)x_i$ is independent of J is not restrictive. Furthermore, when considering the effect of a redistribution at most $N = 2n$ types of consumers need to be considered as there are at most n types in the initial configuration and at most n types in the final configuration. Distributions with the same mean can be ranked using second-order stochastic dominance. Definition 2 follows from Rothschild and Stiglitz (1970) and defines a ranking based on continuous instead of increasing functions. The ranking $\preceq_I$is shown to be equivalent to other intuitive notions of spread. In particular, they show that the property $n(A) \preceq_I n(B)$ is equivalent to the fact that *economy B has not less weight in the tails than economy A*.

From Proposition 1 follows that heterogeneity may have an effect on indeterminacy. In fact, the occurrence of indeterminacy depends on the value of $\rho_\mu(x^*)$. Furthermore, social risk tolerance is shown to be the weighted sum of individual risk tolerance, $-u'(x_i)/u''(x_i)$. Then, the effect of heterogeneity clearly depends on the concavity properties of individual risk tolerance.

A difficulty with Proposition 1 is that it does not concern the effects of *all redistributions* of initial endowments on dynamics and determinacy but only of some well chosen (θ, e). This is a consequence of the lack of continuity of the welfare weights as functions of the initial conditions. Indeed, even if the steady state is determinate for fixed welfare weights, without continuity there can be paths originating from the same initial capital but with completely different welfare weights that eventually converge to the steady state. We then need to define a weaker definition of indeterminacy.

Definition 3 *The steady state is said to be* **locally determinate** *when there is a unique path converging to it for any initial capital taken in a neighborhood of its steady state value. The steady state is said to be* **weakly locally determinate** *when there is a unique path converging to the steady state for any initial n+1-tuple of capital and individual wealth taken in a neighborhood of the steady state values.*

In other words, if k^* is the steady state value of capital and $(w_i^*)_{i=1}^n$ are the associated individual incomes, the steady state is weakly determinate if for any k_0 close to k^* there doesn't exist another path with $(w_i)_{i=1}^n$ close to $(w_i^*)_{i=1}^n$. We can now state our main result.

Proposition 2(i) When homogeneity is good for determinacy *Assume that individual risk tolerance is a strictly convex function. Assume that the parameters are chosen in the open non-empty sets defined in Proposition 1. Then there exists a distribution* n_0 *such that steady state is weakly locally determinate for any economy* J *with* $n(J) \preceq_I n_0$*. On the other hand, there exists a distribution* $n_1 \succeq_I n_0$ *such that the steady state is indeterminate for any economy* H *with* $n(H) \succeq_I n_1$*.*

Proposition 2(ii) When heterogeneity is good for weak determinacy *Assume that individual risk tolerance is a strictly concave function. Assume that the parameters are chosen in the open non-empty sets defined in Proposition 1. Then there exists a distribution* n_0 *such that the steady state is locally weakly determinate for any economy* J *with* $n(J) \succeq_I n_0$*. On the other hand, there exist a distribution* n_1*,* $n_1 \preceq_I n_0$*, such that the steady state is indeterminate for any economy* H *with* $n(H) \preceq_I n_1$*.*

Proof: See Appendix.

Proposition 2(ii) states that whenever the wealth distribution affects indeterminacy and individual absolute risk aversion $R(x) = -u''(x)/u'(x)$ is a strictly concave function, homogeneity produces indeterminacy. The same result holds if absolute risk aversion is a strictly convex function, provided the second derivative of absolute risk aversion is sufficiently small, $2R'^2(x)/R(x) > R''(x), \forall x > 0$. On the other hand, Proposition 2(i) states that when absolute risk aversion is a strongly convex function the opposite result holds. Indeed, since $(-u'(x)/u''(x))'' = ((R^2(x))^{-1}R'(x))' = -2(R^3(x))^{-1}(R'(x))^2 + ((R^2(x))^{-1}R''(x))$ if $(R(x))^{-1}(R'(x))^2 < R''(x)$ then $(-u'(x)/u''(x))'' > 0$, i.e. a strictly convex function. On the other hand, if $2(R(x))^{-1}(R'(x))^2 > |R''(x)|$ then $-u'(x)/u''(x)$ is a convex function.

An implication of Proposition 2 is that whenever heterogeneity affects indeterminacy the usual axioms on preferences don't limit the sign of this effect. The reason is that the characterization involves third and other high order derivatives of the utility functions. Standard assumptions on preferences do not put any limitation on these and empirical data is also lacking. Models of precautionary saving usually require the third derivative to be positive. Recent research suggests that a positive third order derivative is not sufficient for the expected wealth accumulation to be increasing with the earning risks while a sufficient condition is that $u'(x)u'''(x)(u''(x))^{-2}$ is a constant k with $k > 0$ (see Huggett and Vidon (2002)). Other indirect evidence seems to suggest that absolute risk aversion is convex but whether or not it is sufficiently convex to produce a strictly concave inverse is an open question (see Gollier (2001)).

The following result concern preferences with the HARA property. Note that this class include most of the commonly used specifications, as the CARA and CRRA.

Corollary 1 HARA preferences *Assume that preferences can be represented by a utility function of the HARA class, i.e.* $u(x) = (1-\gamma)\gamma(ax(1-\gamma)^{-1} + b)^{\gamma}$ *with* a, b *and* γ *as* *parameters. Then the degree of heterogeneity plays no role in the stability and determinacy of the steady state.*

Indeed, HARA utility functions are characterized by $u'(x)u'''(x)(u''(x))^{-2} = k$ with $k > 0$, so that $(R^{-1}(x))''=(u'(x)u''(x)^{-1})''=(((u''(x))^2 - (u'(x)u'''(x))(u''(x))^{-2})' =$
$= (-u'(x)u'''(x)(u''(x))^2)' = 0$ (see Caroll and Kimball (1996)).

6 Conclusion

The present paper identifies conditions on consumer's heterogeneity sufficient to generates indeterminacy. It also gives conditions ensuring determinacy. The circumstances for which heterogeneity eliminates indeterminacy are plausible. However, there is also a large set of economies such that heterogeneity is neutral or even favors indeterminacy. As the crucial variable is the concavity of the inverse of risk aversion, it is not clear whether the importance of indeterminacy in models with externalities is overstated due to the representative consumer assumption.

The results have a wide range of validity within the considered technology. It is an open question whether the results can be extended to a more general specification of technology and to more sectors. It should be pointed out that in our model heterogeneity in individual productivity is taken into account only through heterogeneity in labor endowments. A more satisfactory formulation would endogeneise labor supply.

7 References

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8 Appendix

8.1 Proof of Lemma 1

At a steady state $k_t = k^*, x_{it} = x_i^*$ and $p_t^1 = \delta^t p_0^1$ where p_0^1 is the price of the consumption good in period 0. Let $\widetilde{l}_j^* = k^{*\eta} l_j^*$ be the effective labor employed in firm j. The budget constraint, together with the conditions on profit maximization, imply

$$x_i^* = \frac{1-\delta}{p_0^1}\left(\theta_i k^* + \sum_{t=0}^{\infty} w_t e_i\right) = e_i k^{*\eta} \frac{\partial F^1}{\partial \widetilde{l}}(k_1^*, \widetilde{l}_1^*) + (1-\delta)\theta_i k^* \frac{\partial F^1}{\partial k}(k_1^*, \widetilde{l}_1^*)$$

The values of k_1^* and $\widetilde{l}_1^*$ are obtained by solving the system

$$\begin{aligned} \widetilde{l}_1^* + \widetilde{l}_2^* &= k^{*\eta} \\ k_1^* + k_2^* &= k^* = F^2(k_2^*, \widetilde{l}_2^*) \end{aligned}$$

and using the fact that at the optimum $\widetilde{l}_2^* = k_2^*/\gamma$. Q.E.D.

8.2 Proof of Proposition 1

The proof involves some preliminary results. In the first step (Result 1 and 2) we analyze the dynamic stability of the P.O. allocations as a function of $\rho_\mu(x^*)$ for given μ. For notational convenience we drop the subscript μ and the argument x^* whenever this is possible. The details of the calculations can be found in Ghiglino and Olszak-Duquenne (2002).

8.2.1 Result 1: Eigenvalues

The stability and the local determinacy properties of the steady state depend on the modulus of the eigenvalues λ_1 and λ_2. For given discount factor and technology parameters, the two eigenvalues depend on $\rho = -u'(x^*)(u''(x^*))^{-1}$.

Result 1 *The eigenvalues associated to the dynamic system expressed in terms of the inverse of social risk aversion* ρ *are*

$$\lambda_{1,2} = \frac{1}{2}\Big[-B \pm \sqrt{B^2 - 4C} \Big]$$

with

$$B = \frac{\rho(T_{22} + \delta(T_{11} + T_{13})) - \delta T_1((1+\delta)T_1 + T_3)}{\delta(\delta T_1^2 + \rho T_{12})}$$

$$C = \frac{1}{\delta}\left[1 + \frac{\delta T_1 T_3 + \rho T_{23}}{\delta T_1^2 + \rho T_{12}} \right]$$

where $T_{ij} = T_{ij}(k^*, k^*, k^*),\ i, j = 1, 2, 3$ *are the second order derivatives of* T.

Proof of Result 1 The definition of V implies that for $i, j = 1, 2, 3$,

$$V_{ij}(k^*, k^*, k^*) = u''(T(k^*, k^*, k^*))T_iT_j + u'(T(k^*, k^*, k^*))T_{ij}$$

so that

$$\frac{V_{ij}(k^*, k^*, k^*))}{u''(T(k^*, k^*, k^*))} = T_iT_j + \frac{u'(T(k^*, k^*, k^*))}{u''(T(k^*, k^*, k^*))}T_{ij} = T_iT_j + \rho T_{ij}.$$

The result is obtained from the linearized version of the Euler equation and the fact that $T_2(k^*, k^*, k^*) = -\delta T_1(k^*, k^*, k^*)$. Q.E.D.

8.2.2 Result 2: Parameter's values allowing for indeterminacy

Define the eigenvalues so that $\mid \lambda_1 \mid < \mid \lambda_2 \mid$. From Result 1 follows that at most one of the two graphs $\lambda_1(\rho)$ and $\lambda_2(\rho)$ intersect the horizontal line drawn at -1, because the branches of $\lambda_i(\rho)$ are monotonous function of ρ. In order to prove indeterminacy two cases have to be considered: 1) The eigenvalue λ_1 intersects the line $\lambda = -1$ while $\lambda_2 < -1$ in which case stability and instability are possible but indeterminacy is ruled out; 2) The eigenvalue λ_2 intersects the line $\lambda = -1$ while $1 > \lambda_1 > -1$ in which case both (determinate) stability and indeterminacy are possible. The following result gives open sets of parameters $(\alpha, \gamma, \eta, \delta)$ for which there exists a (positive) solution ρ_c to the equation $\lambda_i = -1$ for $i = 1$ or $i = 2$, i.e. changes in the curvature ρ may bring a change in the dynamic behavior of the economy or/and in determinacy.

Result 2 (i) *If*

$$\frac{1}{3} < \alpha < \frac{1}{2} \quad ; \quad \gamma > 1 - 2\alpha \quad ; \quad \eta > \overline{\eta} = \frac{2\alpha + \gamma - 1}{1 - \gamma}$$

or

$$\alpha > \frac{1}{2} \quad ; \quad \gamma < 1 - \alpha \quad ; \quad \eta > \overline{\eta}$$

then there exists $\overline{\delta}$ such that for all δ in $I_\delta =]\,\overline{\delta}, 1\,[$, there exists $\rho_c > 0$ such that $\lambda_2(\rho_c) = -1$. There is indeterminacy for $\rho > \rho_c$ and determinate stability otherwise.

(ii) If

$$\alpha < \frac{1}{3} \quad \text{and} \quad \gamma \in \left] \frac{\alpha(1-2\alpha)}{\sqrt{(1-\alpha)^2 + (1-2\alpha)^2}}, \alpha \right[$$

or

$$\frac{1}{3} < \alpha < \frac{1}{2} \quad \text{and} \quad \gamma \in \left] \frac{\alpha(1-2\alpha)}{\sqrt{(1-\alpha)^2 + (1-2\alpha)^2}}, 1 - 2\alpha \right[$$

then there exist δ_c and δ_{cc} in $]\,0, 1\,[$ such that for all δ in $I_{\delta c} =]\,\delta_c, \delta_{cc}\,[$, there exists $\rho_c > 0$ such that $\lambda_1(\rho_c) = -1$ or $\lambda_2(\rho_c) = -1$.

(iii) For any given set of admissible parameters $(\alpha, \gamma, \eta, \delta)$, there exists a value of ρ_0 such that for all economies with a higher curvature, $\rho < \rho_0$, the steady state of the reduced model is stable and determinate.

Proof of Result 2: Existence of the roots First, remark that $\lambda_1 = -1$ and $\lambda_2 = -1$ are satisfied simultaneously only if $\lambda_1 = (-B + \sqrt{B^2 - 4C})/2 = \lambda_2 = (-B - \sqrt{B^2 - 4C})/2 = -1$. This implies $\sqrt{B^2 - 4C} = B - 2 = 0$ and $C = 1$, a non generic situation. Then, $\lambda_{1,2} = -1$ implies $B - C = 1$. Direct computation shows that

$$B - C = 1 \Leftrightarrow \frac{-(T_{22} - T_{12} - T_{23} + \delta(T_{11} + T_{13}))\rho + \delta T_1(\delta T_1 + 2T_1 + 2T_3)}{\delta(-\rho T_{12} + T_1 T_2)} = 1$$

Solving for ρ gives the solution

$$\rho_c = \frac{2\delta T_1(T_1(1+\delta) + T_3)}{T_{22} - T_{12} - T_{23} + \delta(T_{11} + T_{13} - T_{12})} = -\alpha\delta x^* \frac{Q(\delta)}{f(\delta)}$$

with

$$Q(\delta) = \delta(1-\alpha)(1+\eta) + (1-\alpha) + \eta(\alpha - \gamma)$$

$$f(\delta) = a\delta^2 + b\delta + c$$

and

$$\begin{aligned}
a &= (1-\alpha)(1-2\alpha)(1+\eta) \\
b &= -[\alpha(1-\eta)(1-2\alpha) + \gamma(1+\eta)(2-3\alpha)] \\
c &= \gamma[\gamma(1+\eta) + \alpha(1-\eta)]
\end{aligned}$$

The value ρ_c is an acceptable solution for the equation $\lambda_i = -1$ provided it is strictly positive. As the numerator is always strictly negative, $f(\delta)$ need to be negative. When $\alpha < 1/2$, f is convex with $a > 0$, $b < 0$ and $c > 0$ implying that $f(0) > 0$.

- **Proof of Result 2(ii)**: The sufficient condition for the existence of an interval included in $]0,1[$ on which $f < 0$ is a positive discriminant for f with roots δ_c and δ_{cc} between 0 and 1. When $\gamma < 1-2\alpha$ (here $0 < \alpha < 1/2$) we have that $f(1) > 0$ and that $\delta_c\delta_{cc} < 1$. Then there exist δ_c and δ_{cc} are in $]\ 0,1\ [$. The first set of conditions in Lemma 2 (ii) is due to the fact that the assumption $\gamma < \alpha$ is binding for $\alpha < 1/3$.

- **Proof of Result 2(i), first set of conditions)**: The existence of a value $\overline{\delta}$ in $]0,1[$ such that $f(\delta) < 0$ on $]\overline{\delta},1[$ requires a positive discriminant and $f(1) < 0$. When $\gamma > 1-2\alpha$ the externality must be chosen larger than $\overline{\eta}$ in order to have a positive solution ρ_c in which case it exists $\overline{\delta}$ in $]0,1[$ such that $f(\delta) > 0$ on $]\overline{\delta},1[$. Otherwise, ρ_c is strictly negative. Note that $\overline{\eta}$ is smaller than 1 under the previous assumptions.

- **Proof of Result 2(i), second set of conditions)** In this case, f is a concave function with $f(0) > 0$. This is sufficient to prove the existence of the roots δ_c and δ_{cc}. Let $\overline{\delta}$ be the unique positive root. If $f(1)$ is negative then $\overline{\delta}$ is in $]0,1[$. First, note that α larger than $1/2$ implies $2\alpha+\gamma-1 > 0$. Then, from the expression for $f(1)$ given above, $f(1) < 0$ for $\eta > (2\alpha+\gamma-1)/(1-\gamma) = \overline{\eta}$. Finally, $\overline{\eta}$ is smaller than 1 only if $\gamma < 1-\alpha$.

- **Proof of Result 2(iii).** Follows from Result 1. Q.E.D.

Proof of Result 2: Identification of the roots Since for $\rho \to \infty$ both eigenvalues are negative, B is clearly positive. Then, the convention $|\lambda_1| < |\lambda_2|$ implies $\lambda_1 = (-B+\sqrt{B^2-4C})/2$ and $\lambda_2 = (-B-\sqrt{B^2-4C})/2$. From the fact that the graph of λ_1 is upward sloping as $\rho \to \infty$ (and inversely for λ_2) it follows that $\lim_{\rho\to\infty} \lambda_1 < -1 \Longleftrightarrow$

$\lim_{\rho\to\infty}\sqrt{B^2-4C} < \lim_{\rho\to\infty} B-2$ and $\lim_{\rho\to\infty}\lambda_2 > -1 \iff \lim_{\rho\to\infty} -\sqrt{B^2-4C} > \lim_{\rho\to\infty} B-2$. If $\lim_{\rho\to\infty} B-2$ is negative then $\lim_{\rho\to\infty}\lambda_1 > -1$ and any solution ρ_c is a solution to $\lambda_2 = -1$. Similarly, if $\lim\limits_{\rho\to\infty} B-2 > 0$ then $\lim\limits_{\rho\to\infty}\lambda_2 < -1$, and ρ_c is a solution to $\lambda_1 = -1$. The limit $\lim_{\rho\to\infty} B-2$ can be considered as a function of δ, all others parameters (α,γ,η) being fixed (indeed, the eigenvalues p_i are functions of ρ members of a family generated by the parameter δ). Using the definition we obtain,

$$\lim_{\rho\to\infty} B-2 = \frac{T_{11}}{\alpha^2\delta T_{12}}g(\delta)$$

with $g(\delta) = (1-\alpha)(1-3\alpha-\alpha\eta)\delta^2+(\alpha^2-\gamma(1-2\alpha)-\eta\alpha(\alpha-\gamma))\delta+\gamma^2$. As T_{11} and T_{12} are both negative, we focus the analysis on the sign of $g(\delta)$ on $]0,1[$. Some straightforward but tedious calculations lead to the result. Q.E.D.

8.2.3 Proof of Proposition 1

$i)$ In Ghiglino (2003) it is shown that the value of the curvature at the steady state is given by

$$\rho_{(\theta,\omega)}(x^*) = -\sum_{i=1}^{n} n_i \frac{u_i'}{u_i''}(x_i^*(\theta_i,\omega_i))$$

Without loss of generality, we may assume that preferences are identical so that the subscripts can be dropped.

$ii)$ Assume that the coefficients α, γ and η satisfy the assumptions in case (i) of Result 2, and let δ be in I_δ. According to Result 2, these assumptions imply that $\rho_c > 0$ and that indeterminacy for the reduced model occurs for any $\rho_I < \rho_c$. Since there are no structural constraints on the first and second derivatives of the individual utility function, except the usual sign conditions, the previous expression implies that ρ_I can be obtained with a suitable choice of preferences. Furthermore, since $\rho_{(\theta,\omega)}(x^*)$ depends on the derivatives of the individual utility functions evaluated at different points, the result still holds when the preferences for all agents are identical.

iii) As in *ii*) assume that the parameters fulfill the conditions stated in *i*) of Result 2. Consider now a redistribution of initial capital shares such that all agents becomes identical. The economy trivially admits a representative agent. In this situation, for any $\rho_s < \rho_c$ the steady state is stable (and determinate). Since, there are no structural constraints on the first and second order derivatives of the individual utility function, except the usual sign conditions, the preferences chosen in *i*) can be perturbed as to satisfy also $\rho = \rho_s$. Q.E.D.

8.2.4 Turnpike properties

Result 2 concerns the dynamic properties of the steady state when the technology, the welfare weights (or the individual preferences and endowments) and the time discount factor are fixed parameters. A classical result concerns the asymptotic stability properties when the time discount tends to zero, i.e. $\delta \longrightarrow 1$.

Result 3

i) *If $\alpha < 1/2$ and $\gamma < 1 - 2\alpha$ then for given technology, preferences and initial endowments there exists $\delta_s < 1$ such that the steady state is stable for $\delta_s < \delta < 1$.*

ii) *If α and γ satisfy the hypothesis (i) of Result 2, then there exists a value $\eta = \overline{\eta}$ such that:*

- *if $\eta < \overline{\eta} \Longrightarrow$ there exists δ_s such that the steady state is stable for all $\delta > \delta_s$.*
- *if $\eta > \overline{\eta} \Longrightarrow$ the stability and determinacy can be influenced by the curvature of the social utility function for all $\delta > \overline{\delta}$, with $\overline{\delta}$ defined in Result 2.*

iii) *If* $\alpha > 1/2$ *and* $\gamma > 1 - \alpha$*, there exists* δ_s *such that the steady state is stable for all* $\delta > \delta_s$*.*

Proof: A straightforward application of Result 2. Q.E.D.

8.3 Proof of Proposition 2

The distinction between n_0 and n_1 needs to be explained first. In Proposition 2(i), $n(H) \succ_I n_1$ ensures that the aggregate economy exhibits indeterminacy. Due to Lemma 3 this imply indeterminacy in the disaggregated economy. For $n \preceq_I n_1$ the aggregated model is determinate. However, this doesn't exclude that the disaggregated model may be indeterminate. The existence of n_0 such that for $n \preceq_I n_0$ the disaggregate economy is determinate is ensured by the fact that at least when all consumers are identical the steady state is determinate. A similar distinction holds for Proposition 2(ii), i.e. determinacy holds for $n \succeq_I n_0$ in the aggregate model and indeterminacy holds for $n \prec_I n_1$ in the disaggregate model.

The rest of the proof is similar to Ghiglino (2003). Provided $T(x) = -u'(x)/u''(x)$ is a concave function, Definition 2 and the discussion thereafter implies that B is more heterogeneous than A iff $\sum_{i=1}^{N} n_i(A)T(x_i) \geq \sum_{i=1}^{n} n_i(B)T(x_i)$. If we define $\rho(J)$ as the value of $\rho_{(\theta,\omega)}(x^*)$ associated to the distribution $n_i(J)$, the previous condition becomes iff $\rho(A) \geq \rho(B)$. Indeed, $\rho_{(\theta,\omega)}(x^*) = -\sum_{i=1}^{n} n_i T(x_i^*(\theta_i, \omega_i))$. On the other hand, according to Result 2 in the proof of Proposition 1 an increase in ρ favors indeterminacy. Therefore, when individual absolute risk tolerance $T(x)$ is a concave function homogeneity favors indeterminacy (Proposition 2(ii)). The result holding for $T(x)$ convex is obtained similarly.

Q.E.D.