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Abstract

Procedures based on the Generalized Method of Moments (GMM) (Hansen, 1982) are basic tools in modern econometrics. In most cases, the theory available for making inference with these procedures is based on first order asymptotic theory. It is well-known that the (first order) asymptotic distribution does not provide accurate p-values and confidence intervals in moderate to small samples. Moreover, in the presence of small deviations from the assumed model, p-values and confidence intervals based on classical GMM procedures can be drastically affected (nonrobustness). Several alternative techniques have been proposed in the literature to improve the accuracy of GMM procedures. These alternatives address either the first order accuracy of the approximations (information and entropy econometrics (IEE)) or the nonrobustness (Robust GMM estimators and tests). In this paper, we propose a new alternative procedure which combines robustness properties and accuracy in small samples. Specifically, we combine IEE techniques as developed in Imbens, Spady, Johnson (1998) to obtain finite sample accuracy with robust methods obtained by bounding the original orthogonality function as proposed in Ronchetti and Trojani (2001). This leads to new robust estimators and tests in moment condition models with excellent finite sample accuracy. Finally, we illustrate the accuracy of the new statistic by means of some simulations for three models on overidentifying moment conditions.

Keywords: Exponential tilting, Generalized method of moments, Information and entropy econometrics, Monte Carlo, Robust tests, Saddlepoint techniques.

JEL Classification : C12, C13, C14

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1 Introduction

Procedures based on the Generalized Method of Moments (GMM) (Hansen, 1982) are important tools in econometrics to estimate the parameters and make inference in moment condition models. In general, the inferential tools (p-values and confidence intervals) are based on first order asymptotic theory. More specifically, under appropriate regularity conditions, GMM estimators are asymptotically normal and the standard classical statistics for hypothesis testing are asymptotically χ^2- distributed. These results provide the tools used routinely in econometric analysis. However, there is evidence in the econometric literature that these asymptotic distributions do not provide accurate approximations to p-values and confidence intervals when the sample size is moderate to small; see for instance Altonji and Segal (1996), Burnside and Eichenbaum (1996), Hansen, Heaton, and Yaron (1996) among others in the July 1996's special issue of the *Journal of Business and Economic Statistics*.

To alleviate this problem, several proposals have been put forward in the literature. An overview is presented in the July 1996's special issue of *Journal of Business and Economic Statistics*. For instance, Hansen, Heaton, and Yaron (1996), opted for continuous updating estimators. Other authors such as Christiano and Haan (1996) found that imposing certain restrictions leads to substantial improvements in the small-sample properties of the statistical tests. Andersen and Sørenson (1996) stressed that it is generally not optimal to include many moments in the estimation procedure if the sample size is moderate to small. Bootstrap techniques have also been suggested to improve the approximation of the finite sample distribution of GMM statistics. Hall and Horowitz (1996) gave conditions under which the bootstrap provides asymptotic refinements to the critical values of t-tests and to the tests for overidentifying moment restrictions.

More recently, so-called information and entropy econometric (IEE) techniques have been used to improve the finite sample accuracy of GMM estimators and tests; see Imbens, Spady, and Johnson (1998) (ISJ thereafter) and for an overview, the March 2002 special issue of the *Journal of Econometrics*. The basic idea is to "tilt" the empirical distribution to the nearest distribution satisfying the moment conditions, where the distance is measured by a power divergence statistic (Cressie and Read, 1984) such as the Kullback-Leibler distance. These techniques are related to saddlepoint methods developed in the statistical literature for the fully identified case (M-estimators); see for instance Field and Ronchetti (1990), Spady (1991), Robinson, Ronchetti, and Young (2003).

In spite of their good finite sample accuracy when the model and the moment conditions are exactly satisfied, p-values and confidence intervals based on IEE techniques can be drastically affected as the original GMM procedures by small deviations from the underlying distribution of the model and from the corresponding moment conditions. Ronchetti and Trojani (2001) investigated this problem for the classical GMM procedures and derived robust alternatives to GMM estimators and tests. The goal of this paper is to extend these results to IEE techniques in order to obtain new estimators and tests which combine both robustness properties and good accuracy in moderate to small samples.

The paper is organized as follows. In section 2, we review IEE techniques by focusing in particular on exponential tilting (ET) techniques and provide a link with saddlepoint methods. Section 3 is devoted to the definition and the construction of a robust version of the exponential tilting estimator and corresponding test. In particular, we show that a necessary condition for the robustness of the ET estimator and test is the boundedness of the orthogonality function and its derivative with respect to the parameter. This implies a bounded influence function for the estimator and for the level of the corresponding test. When this condition is not satisfied by the original orthogonality function, we apply the technique developed in Ronchetti and Trojani (2001) to truncate the original orthogonality function and we use this modified orthogonality function in the ISJ procedure. This leads to new robust ET estimators and tests which are discussed in subsection 3.2. Section 4 presents a Monte Carlo study for three benchmark models which shows the excellent finite sample behavior of the new techniques both at the model and in the presence of small deviations from the model. Finally, section 5 provides some concluding remarks and suggestions for further research. The algorithm and the computational aspects are discussed in the appendix.

2 Exponential tilting

Let $(Z_n)_{n\in\mathbb{N}}$ be a stationary ergodic sequence defined on an underlying probability space and taking values in $\mathbb{R}^N$ and let $\mathcal{P}$=$\{P_\theta,\ \theta\in\Theta\subset\mathbb{R}^k\}$ be a family of distributions in $\mathbb{R}^N$ corresponding to the model distribution (or reference model). Further, let us define a function $h:\mathbb{R}^N \text{x}\, \Theta\rightarrow\mathbb{R}^H$ that enforces a set of orthogonality conditions

$$E[h(Z;\theta_0)]=0 \tag{1}$$

on the structure of the underlying model. We assume that θ_0 is the unique solution of (1) and we consider the case where the number of conditions H is larger than the number of parameters k.

The GMM estimator $\hat{\theta}^{gmm}$ of θ_0 (Hansen, 1982) is defined by

$$\hat{\theta}^{gmm}=\arg\min_\theta Q_W(\theta) \tag{2}$$

where $Q_W(\theta)=\left(\frac{1}{N}\sum_{i=1}^N h(Z_i;\theta)\right)' W^{-1}\left(\frac{1}{N}\sum_{i=1}^N h(Z_i;\theta)\right)$ for some positive semi-definite matrix W. Moreover, under (1) $N\cdot Q_W(\hat{\theta}^{gmm})$ is asymptotically χ^2_{H-k}

distributed and can be used to test overidentifying conditions (Hansen's test).

To improve the finite sample properties of the GMM estimator and Hansen's test, ISJ proposed a class of alternative estimators based on the following idea. Given two discrete distributions $\tilde{\pi}$ and π with common support and for a fixed scalar parameter λ, define the power-divergence statistic by (Cressie and Read, 1984)

$$I_\lambda(\tilde{\pi}; \pi) = \frac{1}{\lambda \cdot (1+\lambda)} \sum_{i=1}^{N} \tilde{\pi}_i \Big[\Big(\frac{\tilde{\pi}_i}{\pi_i}\Big)^\lambda - 1\Big]. \tag{3}$$

The estimator $\hat{\theta}$ of θ, for a given λ, is then defined by the closest distribution to the empirical distribution, as measured by the Cressie-Read statistic, within the set of distributions admitting a solution to the moment equations, i.e. $\hat{\theta}$ is the solution of the problem

$$\min_{\pi,\ \theta} I_\lambda(\tilde{\pi}; \pi),\ \text{subject to} \sum_{i=1}^{N} h(Z_i; \theta) \cdot \pi_i = 0 \text{ and } \sum_{i=1}^{N} \pi_i = 1, \tag{4}$$

where $\tilde{\pi}$ is the vector of empirical frequencies $\tilde{\pi}_i = \frac{1}{N}$ for $i = 1, ..., N$.

Different values of λ lead to different estimators as discussed in ISJ. We focus on an important special case of this family of estimators, namely when $\lambda \longrightarrow -1$. In this case, the optimization in (4) leads to the exponential tilted (ET) estimator $\hat{\theta}^{et}$ which is defined as the minimizer of the Kullback-Leibler information criterion:

$$\min_{\pi,\ \theta} \sum_{i=1}^{N} \pi_i \cdot \log(\pi_i) \quad \text{subject to} \quad \sum_{i=1}^{N} h(Z_i; \theta) \cdot \pi_i = 0 \,\text{and} \sum_{i=1}^{N} \pi_i = 1. \tag{5}$$

It turns out that π_i is given by

$$\pi_i = \frac{e^{t' h(Z_i; \theta)}}{\sum_{j=1}^{N} e^{t' h(Z_j; \theta)}}, \tag{6}$$

and by defining the empirical cumulant generating function of $h(Z_i; \theta)$,

$$K(t; \theta) = \log\Big(\frac{1}{N} \sum_{i=1}^{N} e^{t' h(Z_i; \theta)}\Big), \tag{7}$$

we obtain

$$-K(t,\theta) = \sum_{i=1}^{N} \pi_i log(\pi_i) + log(N). \tag{8}$$

Therefore (5) can be rewritten more compactly as

$$\max_{t,\ \theta} K(t;\theta) \qquad \text{subject to} \qquad \frac{\partial}{\partial t} K(t;\theta) = 0\,, \tag{9}$$

where π_i is defined by (6).

Under regularity conditions, the tilted estimator $\hat{\theta}^{et}$ is asymptotically (first order) equivalent to the GMM estimator, i.e. $\sqrt{N}(\hat{\theta}^{et} - \theta_0)$ has the same asymptotic normal distribution as $\sqrt{N}(\hat{\theta}^{gmm} - \theta_0)$.

The corresponding test for overidentifying moment restrictions is based on the test statistic $-2 \cdot N \cdot K(t;\hat{\theta}^{et})$ $(= 2 \cdot N \cdot KLIC(\hat{\pi}^{et};\tilde{\pi})$ in ISJ, p. 342). Under the null hypothesis, this test statistic has the same asymptotic distribution as the classical Hansen test statistic, i.e. χ^2_d, where $d = H - k$.

ISJ provide convincing evidence that $\hat{\theta}^{et}$ and the corresponding test have better finite sample properties than $\hat{\theta}^{gmm}$ and Hansen's test. Furthermore, by (6), (7) and (9),

$$\begin{aligned} \frac{\partial}{\partial t} K(t;\theta) &= e^{-K(t;\theta)} \cdot \frac{1}{N} \sum_{i=1}^{N} h(Z_i;\theta) e^{t' h(Z_i;\theta)} \\ &= \sum_{i=1}^{N} h(Z_i;\theta)\pi_i(\theta) = E_\pi\big[h(Z;\theta)\big] = 0\,, \end{aligned}$$

i.e. the empirical distribution $(\frac{1}{N},...,\frac{1}{N})$ is tilted to $(\pi_1,...,\pi_N)$ in order to satisfy the orthogonality conditions under $(\pi_1,...,\pi_N)$. This is the key procedure to obtain saddlepoint approximations of the distribution of estimators and test statistics which are well known to be highly accurate; cf. for instance Daniels (1954), Field and Ronchetti (1990), and Spady (1991) for the fully identified case

(M-estimtors). Indeed the empirical version used here corresponds to the so-called empirical saddlepoint approximation; see Ronchetti and Welsh (1994) and for a connection with empirical likelihood, Monti and Ronchetti (1993).

3 Robust Exponential Tilting

The tilted estimator $\hat{\theta}^{et}$ is an attractive alternative to the GMM estimator $\hat{\theta}^{gmm}$ when the moment conditions (1) are exactly specified. In this section, we want to investigate the behavior of the tilted estimator and the corresponding tests in the presence of slight misspecifications of the moment conditions.
Let us first review these aspects for $\hat{\theta}^{gmm}$.

3.1 Robust alternatives to the GMM

The lack of robustness of the GMM estimator and tests in the presence of small deviations from the underlying distribution has already been studied extensively; see Ronchetti and Trojani (2001) and references therein. In particular, in that paper, it is shown that the influence function of the GMM estimator is proportional to the orthogonality function h. When $h(z;\theta)$ is unbounded in z, this leads to non robust estimators. An alternative robust version was proposed as follows.

Consider the Huber function

$$\mathcal{H}_c : \mathbb{R}^H \to \mathbb{R}^H, \qquad y \mapsto y \cdot w_c(y) = \begin{cases} y & \text{if } \| y \| \leq c \\ \\ c \cdot \frac{y}{\|y\|} & \text{if } \| y \| > c \,, \end{cases} \tag{10}$$

where $w_c(y) = \min(1, \frac{c}{||y||})$ for $y \neq 0$ and $w_c(0) = 1$, and a new mapping $h_c^{A,\tau} : \mathbb{R}^N \times \Theta \to \mathbb{R}^H$ defined by

$$h_c^{A,\tau}(z\,;\theta) = \mathcal{H}_c\big(A(\theta)[h(z\,;\theta) - \tau(\theta)]\big)\,, \tag{11}$$

where the nonsingular matrix $A \in \mathbb{R}^{H \times H}$ and the vector $\tau \in \mathbb{R}^H$ are determined through the implicit equations :

$$\begin{cases} E_\theta\big[h_c^{A,\tau}(Z\,;\theta)\big] = 0 \\ \\ \frac{1}{N}\sum_{i=1}^N \big[h_c^{A,\tau}(Z_i\,;\theta)\big]\cdot\big[h_c^{A,\tau}(Z_i\,;\theta)\big]' = I\,. \end{cases} \tag{12}$$

Then, the GMM estimator $\hat{\theta}_c^{gmm}$ and the corresponding tests defined by the modified bounded orthogonality conditions $h_c^{A,\tau}$ have an influence function bounded by c ($\geq \sqrt{H}$) and are robust in the sense of Hampel, Ronchetti, Rousseeuw, and Stahel (1986). An iterative algorithm for the computation of $\hat{\theta}_c^{gmm}$ is provided by Ronchetti and Trojani (2001, $p.\,47$); see also Appendix A.

3.2 Robust exponential tilting estimator and test

In view of section 2 and subsection 3.1, it seems natural at this point to try and derive an estimator (and the corresponding tests) with the good finite sample properties of $\hat{\theta}^{et}$ and the robustness properties of $\hat{\theta}_c^{gmm}$. This can be achieved by solving (9), with $h(z\,;\theta) = h_c^{A,\tau}(z\,;\theta)$.
More specifically, by writing $K_c(t\,;\theta) = log[\frac{1}{N}\sum_{i=1}^N e^{t'h_c(Z_i\,;\theta)}]$ and $h_c(.\,;.)$ instead of $h_c^{A,\tau}(.\,;.)$ for simplicity, the new robust tilting estimator $\hat{\theta}_c^{et}$ is defined by the optimization problem :

$$\max_{t,\,\theta}\; K_c(t\,;\theta), \tag{13}$$

subject to

$$\begin{cases} \sum_{i=1}^N h_c(Z_i\,;\theta)e^{t'h_c(Z_i\,;\theta)} = 0 & (13.a) \\ \\ E_\theta\big[h_c(Z\,;\theta)\big] = 0 & (13.b) \\ \\ \frac{1}{N}\sum_{i=1}^N h_c(Z_i\,;\theta)h_c'(Z_i\,;\theta) = I\,. & (13.c) \end{cases}$$

$\hat{\theta}_c^{et}$ is asymptotically equivalent to $\hat{\theta}_c^{gmm}$, the robust GMM estimator defined by $h_c(.\,;.)$. Moreover, when $c \longrightarrow \infty$, we recover the classical estimator $\hat{\theta}^{et}$. Notice that even in the case where the cumulant generating function of $h(Z\,;\theta)$ does not exist and $\hat{\theta}^{et}$ is not defined, $\hat{\theta}_c^{et}$ with a finite c exists and is an alternative to the classical estimator; see subsections 4.2 and 4.3. Finally, the corresponding robust test for overidentifying restrictions is defined by the test statistic $-2 \cdot N \cdot K_c(t_c\,; \hat{\theta}_c^{et})$ which is asymptotically χ^2_d under the null hypothesis, where $d = H - k$.

Let us now investigate in more details the robustness properties of $\hat{\theta}_c^{et}$. The tilting estimator can be viewed as an M-estimator (ISJ, p. 337) with estimating equations $\sum_{i=1}^N \rho^{et}(Z_i\,; \hat{\theta}^{et}, \hat{t}^{et}) = 0$, where

$$\rho^{et}(z\,;\theta, t) = \begin{pmatrix} t' \frac{\partial h}{\partial \theta'}(z\,;\theta) \cdot \exp(t'h(z\,;\theta)) \\ h(z\,;\theta) \cdot \exp(t'h(z\,;\theta)) \end{pmatrix}. \tag{14}$$

The influence function of estimators defined by estimating equations (M-estimators) is proportional to the estimating function (Huber, 1981), i.e.

$$IF(z\,; \hat{\theta}^{et}, P_\theta) = E\Big[- \frac{\partial \rho^{et}}{\partial(\theta', t')}(z\,; \hat{\theta}^{et}, t)\Big]^{-1} \rho^{et}(z\,; \hat{\theta}^{et}, t). \tag{15}$$

The boundedness of the influence function implies a bounded bias of the estimator and of the level of the corresponding test when the underlying distribution lies in a neighborhood of the model (see Heritier and Ronchetti, 1994 and Ronchetti and Trojani, 2001). Here, the IF for ET estimators is bounded if and only if ρ^{et} is bounded with respect to z. Therefore, we can focus our analysis on the function ρ^{et} to determine the robustness properties of the corresponding estimators and tests.

Generally, in the classical version, ρ^{et} is not bounded. In fact, both $h(.\,;.)$ and $\frac{\partial h}{\partial \theta}$ are not necessarily bounded. So the resulting estimators are not guaranteed to be robust. For $\hat{\theta}_c^{et}$, $h_c(.\,;.)$ is bounded by construction, and therefore this estimator is robust if $\frac{\partial h_c}{\partial \theta}$ is bounded.

Consider the robust ET estimator defined by the orthogonality function (11). It follows, with $y = A(\theta)[h(z\,;\theta) - \tau(\theta)]$,

$$\frac{\partial}{\partial \theta} h_c(z\,;\theta) \;=\; \frac{\partial}{\partial \theta}\mathcal{H}_c(y) =$$

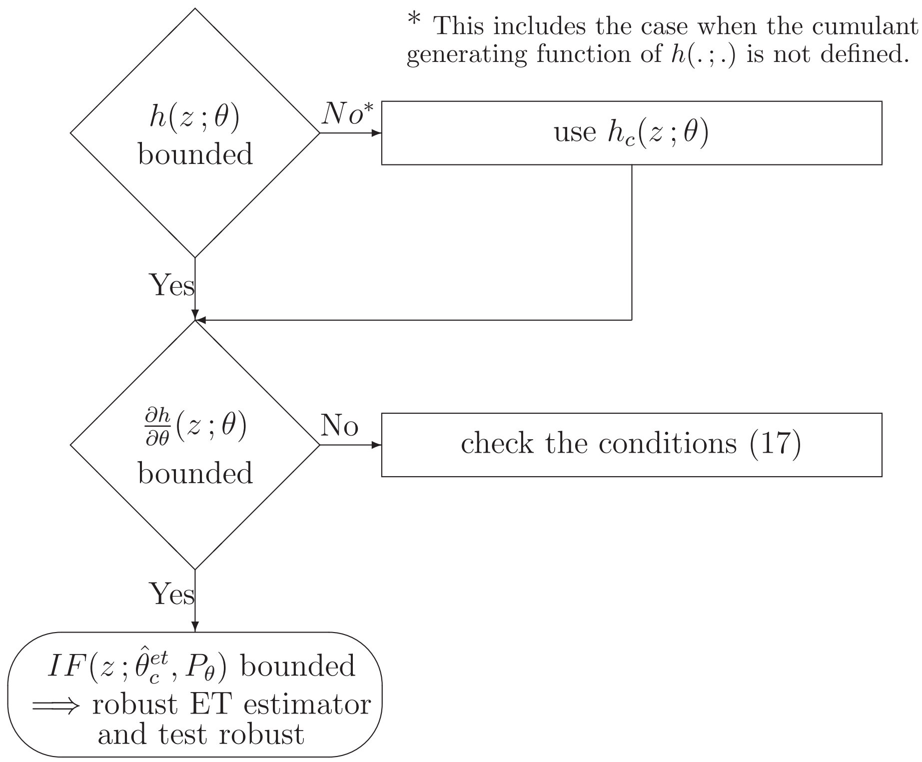
$$\begin{cases} A'A^{-1}y + A[\frac{\partial}{\partial \theta}h(z\,;\theta) - \tau'] & \text{if } \| \, y \, \| \leq c \\ \\ c\{I - \frac{y}{\|y\|} \cdot \frac{y^T}{\|y\|}\} \cdot \{A'A^{-1}\frac{y}{\|y\|} + A\frac{1}{\|y\|}[\frac{\partial}{\partial \theta}h(z\,;\theta) - \tau']\} & \text{if } \| \, y \, \| > c\,. \end{cases} \tag{16}$$

Thus $\frac{\partial}{\partial \theta}h_c(z\,;\theta)$ is bounded with respect to z if and only if

$$\begin{cases} \frac{\partial h}{\partial \theta} & \text{is bounded when } \| \, y \, \| \leq c \\ \\ \frac{1}{\|y\|}\frac{\partial h}{\partial \theta} & \text{is bounded when } \| \, y \, \| > c. \end{cases} \tag{17}$$

The last two conditions are satisfied when $\frac{\partial h}{\partial \theta}$ is bounded everywhere. Then, for a given model, when the derivative of the moment vector with respect to the parameters is bounded, the robustness properties of the estimator $\hat{\theta}_c^{et}$ follow. If this is not the case, we have to check the boundedness of the conditions defined by (17) to determine the robustness properties of the estimator and of the test for the specific model.

Figure 1: *Schematic illustration of the robustness properties of the exponential tilting estimator and test.*



4 Monte Carlo Investigation

To illustrate and compare the behavior of classical and robust ET estimators and tests, we perform a Monte Carlo experiement for three benchmark models (Chi-squared moments, Hall-Horowitz, stochastic lognormal volatility model). In each case we work with data generated from the model and from various slight perturbations of the model. We compute $\hat{\theta}^{et}$ and $\hat{\theta}_c^{et}$ and their corresponding tests for overidentifying moment restrictions based on the test statistics $-2 \cdot N \cdot K(t\,;\hat{\theta}^{et})$ and $-2 \cdot N \cdot K_c(t_c\,;\hat{\theta}_c^{et})$ respectively. Under the null hypothesis , these tests statistics are asymptotically distributed as χ_d^2, where $d = H - k$. We also report, where they are available, the best results obtained by ISJ by means of other tilted test statistics.

In each experiment, we simulate 5000 samples and we report the actual sizes $P[T > v_\alpha]$ for each test based on a test statistic T corresponding to the nominal sizes $\alpha = 0.001, 0.005, 0.01, 0.025, 0.05, 0.1, 0.2$, where v_α is the critical value of the test, i.e. $P[\chi^2_d > v_\alpha] = \alpha$. QQ-plots with respect to χ^2_d quantiles and relative errors $(P[T > v_\alpha] - \alpha)/\alpha$ for each tests are also reported.

4.1 Model 1: Chi-squared Moments

The first Monte Carlo experiment focuses on a two moments, one parameter problem defined by the moment vector:

$$h(Z\,;\theta) = \begin{pmatrix} Z - \theta \\ \\ Z^2 - \theta^2 - 2\theta \end{pmatrix}.$$

The distribution of Z is χ^2_1, $\theta_0 = 1$, and the data are generated from this model. Here $h(z\,;\theta)$ is unbounded in z and $\frac{\partial h}{\partial \theta}(z\,;\theta) = -\begin{pmatrix} 1 \\ 2(\theta+1) \end{pmatrix}$ is constant with respect to z. Therefore, we can use $h_c(z\,;\theta)$ and we can expect good robustness and finite sample accuracy from $\hat{\theta}^{et}_c$ and its corresponding tests.

The results of our simulations, for two sample sizes $N = 500, 250$, are presented in Table 1. In the first case ($N = 500$), we can compare the results of our new robust test to those obtained by the test statistics in ISJ. The first column shows the actual size of the test based on the tilted estimator of the Lagrange multipliers ($ISJ.500$), cf. ISJ p.343. The following two columns give the results for the classical ET ($classET.500$) and for the robust ET ($robET.500$) respectively. We notice that the nominal sizes of $robET.500$ are the closest to the actual size. Notice that $classET.500$ test is very similar to the classical GMM specification test and shows a very liberal behavior in terms of size. The $ISJ.500$ is between the two ET statistics in terms of accuracy.

For a better evaluation of the small sample properties of the robust ET statistics, we also tested a reduced sample size of 250. The results of the classical and robust ET, $classET.250$ and $robET.250$, are reported in the last two columns of Table 1. Even with such a small sample size, the robust ET outperforms the classical test, and the corresponding nominal sizes are very close to the actual sizes. These conclusions are confirmed by the graphical analysis in Figure 2.

Finally, we plot the relative errors, a more stringent measure than absolute errors, for $robET.500$ in Figure 3 (a) and $robET.250$ in Figure 3 (b). Again, these plots demonstrate the high accuracy of the robust ET test. In fact, the relative error in the tail for the robust ET test for $N = 250$ is smaller than 4% down to $\alpha = 0.02$ and still reasonable for smaller sizes. The relative errors of the classical statistics are not reported because they exceed 100% already for $\alpha = 0.05$. These results show that even in the case of no contamination, the robust ET test has a very high finite sample accuracy and is an interesting alternative to classical GMM and ET tests.

Table 1: Comparison of actual and nominal size of the tests applied to *Chi-Squared Moments* (Model 1) without contamination i.e. $Z \sim \chi^2_1$, $H = 2$, $k = 1$ and 5000 replications. $ISJ.N=$ best test statistics from ISJ; $classET.N=$ classical ET test; $robET.N=$ robust test. $.N$ indicates the sample size. The tuning constant for the robust test was set to $c = 2$.

$nom.size$	$ISJ.500$	$classET.500$	$robET.500$	$classET.250$	$robET.250$
0.200	0.237	0.2552	0.2108	0.2772	0.1996
0.100	0.125	0.1554	0.1048	0.1820	0.0986
0.050	0.068	0.1044	0.0488	0.1266	0.0486
0.025	0.038	0.0712	0.0246	0.0930	0.0260
0.010	0.019	0.0474	0.0100	0.0642	0.0078
0.005	0.010	0.0354	0.0048	0.0524	0.0040
0.001	0.003	0.0180	0.0004	0.0293	0.0006

Figure 2: *QQ-plots of overidentifying ET statistics versus* χ^2_1.

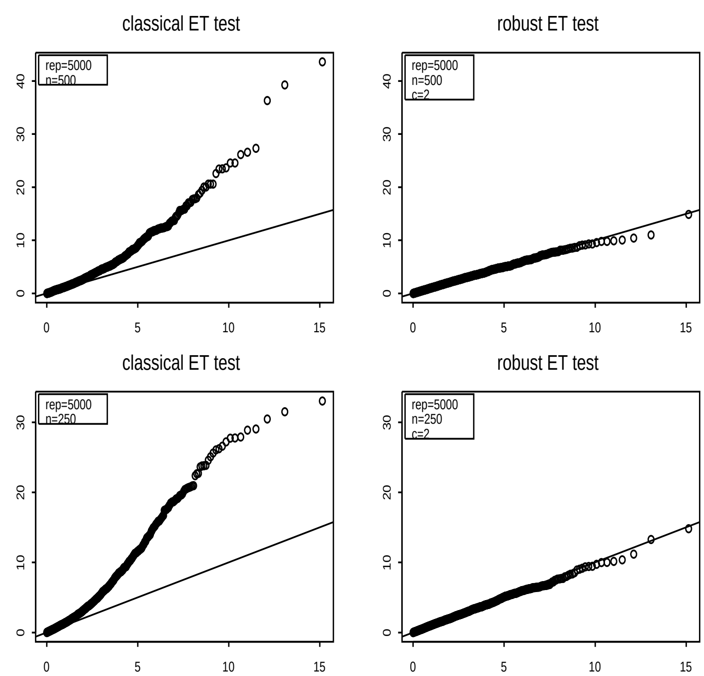


Figure 3: *Relative errors for robET.500 (a) and robET.250 (b)*

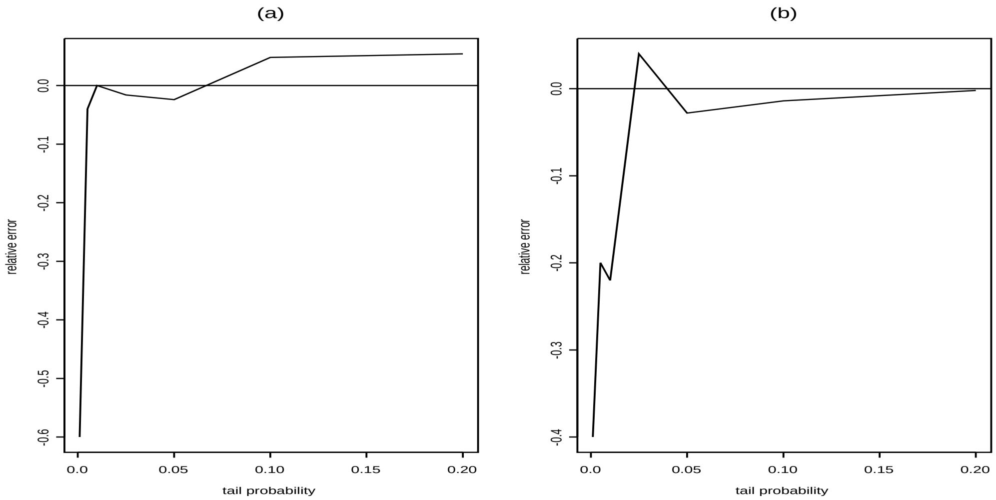
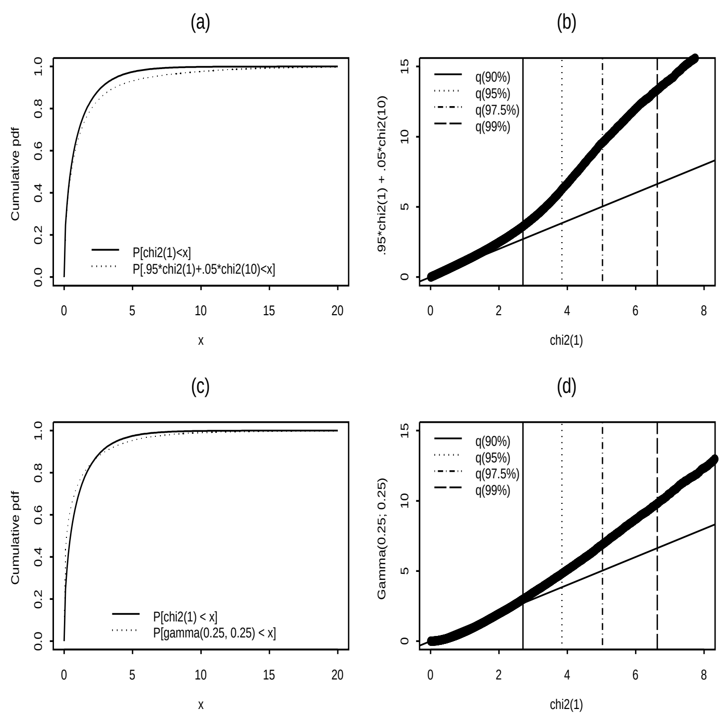


Figure 4: *Probability distribution functions (pdf) and QQ-plots versus* χ^2_1 *for* $100'000$ *simulated observations of two distributions (a) & (b)* $.95\chi^2_1 + 0.05\chi^2_{10}$ *and (c) & (d)* $\Gamma(\frac{1}{4}; \frac{1}{4})$



Let us now investigate the behavior of the different procedures when the data follow a slightly perturbed model distribution. To illustrate the effects, we assume that Z does not follow the model distribution χ^2_1 but two contaminated distributions

$$Z \sim 0.95 \cdot \chi^2_1 + 0.05 \cdot \chi^2_{10} \tag{18}$$

and

$$Z \sim \Gamma(\frac{1}{4}; \frac{1}{4}). \tag{19}$$

In the first case, the Kolmogorv distance between the model and the contaminated distribution (i.e. the maximum difference between the distribution functions of the two distributions) is less than 0.05. In the second case, the Kolmogorov distance is 0.19. This means that (18) can be viewed as a small perturbation of the model distribution and (19) a slightly larger perturbation. Figure 4 shows the probability distribution functions (pdf) and the QQ-plots of the distributions with respect to the model (χ^2_1). We do not argue that (18) or (19) should replace the original model χ^2_1. These are just illustrations of potential small deviations from the model. We still assume the original model with its moment conditions but we take into account the fact that in reality, the data might come from a slightly different unknown distribution with slightly different moment conditions. Thus, our goal is to have procedures based on the original model and moment conditions which still behave reasonably well in the presence of unknown small deviations.

The results of Tables 2, 3 and Figures 5, 6 show that the classical ET test is very inaccurate whereas the robust ET test is stable and very accurate even in the presence of small deviations from the underlying model.

Table 2: Comparison of actual and nominal size of the tests applied to *Chi-Squared Moments* (Model 1) with contaminated data (18), $H = 2$, $k = 1$ and 5000 replications. $classET.N=$ classical ET test; $robET.N=$ robust ET test. $.N$ indicates the sample size. The tuning constant for the robust test was set to $c = 2$.

$nom.size$	$classET.500$	$robET.500$	$classET.250$	$robET.250$
0.200	0.2640	0.2028	0.2742	0.2074
0.100	0.1624	0.0952	0.1688	0.1118
0.050	0.1006	0.0466	0.1108	0.0530
0.025	0.0666	0.0240	0.0750	0.0270
0.010	0.0378	0.0084	0.0504	0.0110
0.005	0.0280	0.0042	0.0352	0.0060
0.001	0.0154	0.0006	0.0178	0.0006

Table 3: Comparison of actual and nominal size of the tests applied to *Chi-Squared Moments* (Model 1) with contaminated data (19), $H = 2$, $k = 1$ and 5000 replications. $classET.N=$ classical ET test; $robET.N=$ robust test. $.N$ indicates the sample size. The tuning constant for the robust test was set to $c = 2$.

$nom.size$	$classET.500$	$robET.500$	$classET.250$	$robET.250$
0.200	0.2870	0.1940	0.2830	0.2178
0.100	0.1748	0.0960	0.1764	0.1168
0.050	0.1054	0.0462	0.1138	0.0576
0.025	0.0728	0.0240	0.0812	0.0310
0.010	0.0436	0.0078	0.0568	0.0132
0.005	0.0312	0.0044	0.0440	0.0076
0.001	0.0148	0.0008	0.0244	0.0022

Figure 5: *QQ-plots of overidentifying ET statistics versus* χ^2_1 *with* $Z \sim 0.95\chi^2_1 + 0.05\chi^2_{10}$.

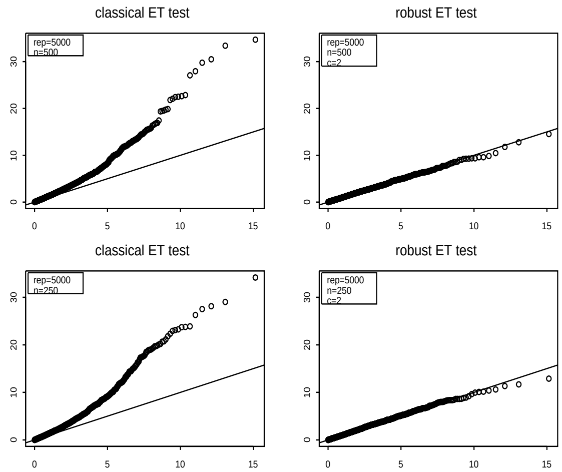
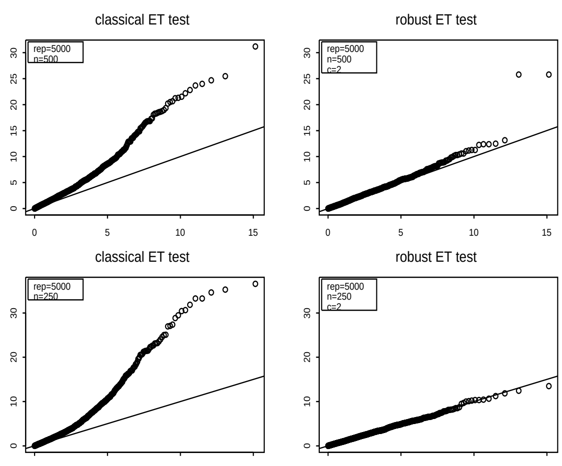


Figure 6: *QQ-plots of overidentifying ET statistics versus* χ^2_1 *with* $Z \sim \Gamma(\frac{1}{4}\,; \frac{1}{4})$.



4.2 Model 2: Hall-Horowitz (1996)

In this experiment, we consider a design investigated by Hall and Horowitz (1996), where the moment vector has the form:

$$h(Z\,;\theta) = \begin{pmatrix} e^{-0.72-\theta\cdot(Z^{(1)}+Z^{(2)})+3\cdot Z^{(2)}} - 1 \\ \\ Z^{(2)} \cdot [e^{-0.72-\theta\cdot(Z^{(1)}+Z^{(2)})+3\cdot Z^{(2)}} - 1] \end{pmatrix}.$$

The vector $(Z^{(1)}, Z^{(2)})'$ follows a bivariate normal distribution with means zero $(0,0)'$, variances 0.16 and correlation coefficient zero. The true value of θ is $\theta_0 = 3$.

We follow the same approach as in subsection 4.1. The simulation results are reported in Table 4. In order to compare our analysis to the ISJ results, we simulate data with two different sample sizes, 200 and 100. The columns $ISJ.N$ represent the closest nominal size from the ISJ investigation with sample sizes of 200 and 100 respectively (cf. ISJ p.345). The columns $robET.N$ report the simulations result of the robust ET, where the constant c is fixed to 2.

Since, for this model, the cumulant generating function of the score vector does not exist, the classical ET test cannot be defined. However, we can "simulate" this case by means of our robust ET test with a large tuning constant c (for example $c = 80$). We call this test a "classical" ET test ("classET"). Notice however, that we do not recommend using this test, the accuracy of the robust ET test with $c = 2$ being so much better.

Inspection of Table 4 reveals the high accuracy of the robust ET test and its better performance compared to the best statistics from ISJ for the two sample sizes. This results are confirmed by the QQ-plots in Figure 7 and the analysis of the relative error in Figure 8 (a) and (b).

Table 4: Comparison of actual and nominal size of the tests applied to *Hall-Horowitz's design* (Model 2) without contamination i.e.

$$\begin{pmatrix} Z^{(1)} \\ Z^{(2)} \end{pmatrix} \sim \mathcal{N}\Big(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} .16 & 0 \\ 0 & .16 \end{bmatrix} \Big),$$

$H = 2$, $k = 1$ and 5000 replications. $ISJ.N=$ best test statistics for ISJ; "$classET$".$N=$ "classical" ET test; $robET.N=$ robust ET test. $.N$ indicates the sample size. The tuning constant for the robust test was set to $c = 2$.

$nom.size$	ISJ.200	"$classET$".200	$robET$.200	ISJ.100	"$classET$".100	$robET$.100
0.200	0.228	0.2486	0.2020	0.250	0.2807	0.2092
0.100	0.125	0.1459	0.0972	0.128	0.1776	0.1022
0.050	0.065	0.0923	0.0468	0.070	0.1175	0.0524
0.025	0.035	0.0582	0.0270	0.043	0.0800	0.0286
0.010	0.016	0.0338	0.0110	0.022	0.0509	0.0134
0.005	0.008	0.0231	0.0042	0.013	0.0376	0.0070
0.001	0.002	0.0012	0.0008	0.004	0.0194	0.0010

Figure 7: *QQ-plots of overidentifying ET statistics versus* χ^2_1.

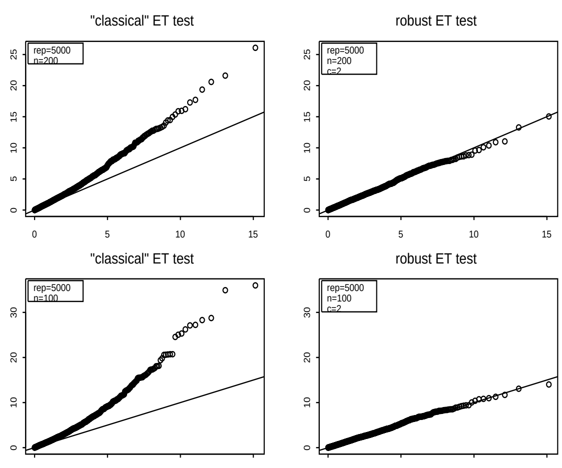
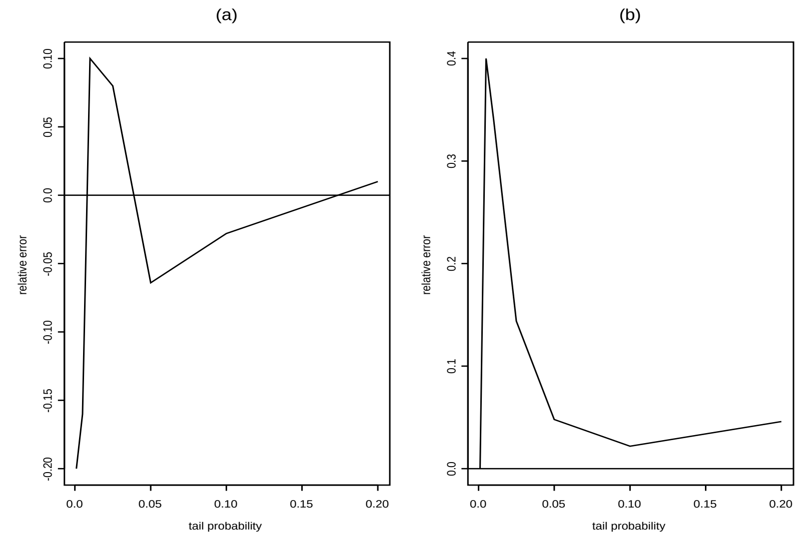


Figure 8: *Relative errors for* $robET.200$ *(a) and* $robET.100$ *(b)*



Similarly to Model 1, we studied the robustness of our new statistics for Hall-Horowitz's model when the data are contaminated according to the following distribution

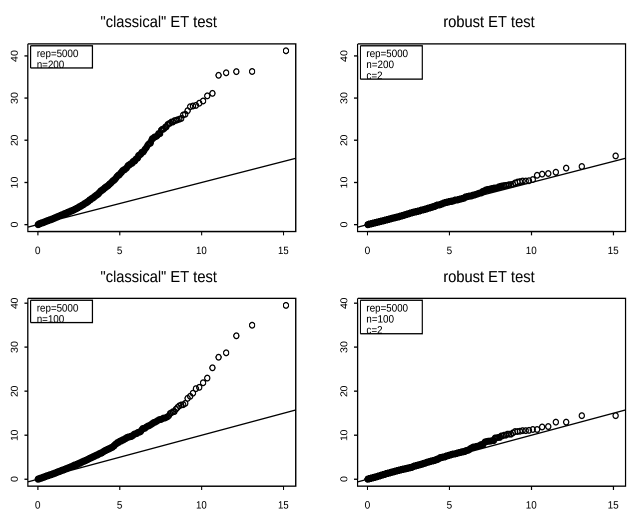
$$\begin{pmatrix} Z^{(1)} \\ Z^{(2)} \end{pmatrix} \sim 0.95 \cdot \mathcal{N}\Big(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} .16 & 0 \\ 0 & .16 \end{bmatrix} \Big) + 0.05 \cdot \mathcal{N}\Big(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Big)$$

In spite of this perturbation, the results for the robust ET statistics are only slightly modified compared to results from non-contaminated data. In contrast, the results for the "classical" ET tests are markedly worse cf. Table 5 and Figure 9.

Table 5: Comparison of actual and nominal size of the tests applied to *Hall-Horowitz's design* (Model 2) with contaminated data. $H = 2$, $k = 1$ and 5000 replications. "*classET*".N= "classical" ET test; *robET*.N= robust ET test. .N indicates the sample size. The tuning constant for the robust test was set to $c = 2$.

nom.size	"*classET*".200	*robET*.200	"*classET*".100	*robET*.100
0.200	0.2692	0.2160	0.3010	0.1988
0.100	0.1620	0.1010	0.1908	0.1072
0.050	0.1022	0.0564	0.1236	0.0554
0.025	0.0678	0.0314	0.0902	0.0320
0.010	0.0412	0.0130	0.0630	0.0136
0.005	0.0300	0.0078	0.0506	0.0080
0.001	0.0122	0.0024	0.0298	0.0014

Figure 9: *QQ-plots of overidentifying ET statistics versus* χ^2_1 *with perturbed data.*



4.3 Model 3: Stochastic Lognormal Volatility Model

The stochastic lognormal volatility (SLV) model offers a powerful alternative to GARCH-type models to explain the well-documented time varying volatility. Moreover, the SLV model provides a reasonable first approximation to model the properties of most financial return series.

During the last ten years, a number of Monte Carlo studies have explored the small sample properties of these estimators. Since, the maximum likelihood approach is difficult to implement, this has left the field open to competition among alternative procedures such as GMM (Melino and Turnbull, 1990), maximum likelihood Monte Carlo (Sandmann and Koopman, 1996), quasi-maximum likelihood, Bayesian Markov Chain Monte Carlo (Jacquier, Polson, and Rossi, 1994), maximum likelihood through numerical integration (Fridman and Harris, 1998) and efficient method of moments (EMM) (Gallant and Tauchen, 1996). Andersen, Chung, and Sørenson (1999) have investigated the finite sample comparison of various methods for estimating SLV. Out of the six alternative methods mentioned above, they found that EMM completely overshadows the others with its flexibility and efficiency.

Consider the simple version of SLV model defined by:

$$\begin{cases} y_t = \sigma_t Z_t \\ \\ \ln \sigma_t^2 = w + \beta \ln \sigma_{t-1}^2 + \sigma_u u_t \end{cases}$$

where $t = 1, ..., N$, $\theta = (w, \beta, \sigma_u)$ is the parameter vector, and (Z_t, u_t) are iid $N(0, I_2)$, that is, the error terms are mutually independent and distributed according to a standard normal distribution. In the model, returns display zero serial correlation but the dependence in the higher-order moments is induced through the stochastic volatility term, σ_t, the logarithm of which follows a first order autoregressive $[AR(1)]$ model. The volatility persistence parameter, β, is estimated to be less than unity, but quite close to it in most empirical studies. Finally, the assumption of lognormality of the volatility process is a convenient parameterization that allows for closed-form solutions of the moments and is consistent with the evidence of excess kurtosis or "fat tails" in the unconditional return distribution.

When we impose the inequality constraints $0 < \beta < 1$ and $\sigma_u \geq 0$ to the model, the return innovation series y_t becomes strictly stationary and ergodic, and unconditional moments of any order exist. Throughout, we work with parameter values that satisfy these additional inequalities. To implement the robust ET procedure, we use 5 orthogonality conditions used by Andersen and Sørenson (1996). The moment vector is defined by

$$h(y, \theta) = \begin{pmatrix} |y_t| - \sqrt{\frac{2}{\pi}} \exp\left(\frac{\mu}{2} + \frac{\sigma^2}{8}\right) \\ y_t^2 - \exp\left(\mu + \frac{\sigma^2}{2}\right) \\ |y_t y_{t-1}| - \frac{2}{\pi} \exp\left(\mu + \frac{\sigma^2}{4}\right) \exp\left(\beta \frac{\sigma^2}{4}\right) \\ |y_t y_{t-3}| - \frac{2}{\pi} \exp\left(\mu + \frac{\sigma^2}{4}\right) \exp\left(\beta^3 \frac{\sigma^2}{4}\right) \\ |y_t y_{t-5}| - \frac{2}{\pi} \exp\left(\mu + \frac{\sigma^2}{4}\right) \exp\left(\beta^5 \frac{\sigma^2}{4}\right)] \end{pmatrix},$$

where $\mu = \frac{w}{1-\beta} \quad \sigma^2 = \frac{\sigma_u^2}{1-\beta^2}$ and $\theta = (w, \beta, \sigma_u)$.

We simulate 5000 samples of 500 observations from the SLV model. The vector of parameters fixed for the simulation of the data vector Z is $\theta_0 = (-.368, .95, .260)$. These values correspond to those used in the empirical study of the SLV by Jacquier et al. (1994) and Andersen et al. (1999), among others.

The samples of size 500 are small by the standards of high-frequency financial time series analysis so the results presented here show the small sample properties of the ET method. Table 6 show in this case the good accuracy of the robust ET method for the overidentifying moments test. The nominal size of the robust ET method is close to the actual size even in the extreme tail. The QQ-plot in Figure 10 (a) confirms these results. Even when the data is contaminated (according to the configuration given in Table 7), the accuracy of the robust ET test is good. Figure 10 (b) confirms these results.

Finally, we estimate the SLV model (without contamination) by EMM and compare the results obtained with our new robust ET estimator. We choose EMM because of its flexibility and efficiency. The EMM computations are based on the procedure outlined in Gallant and Tauchen (2001), implemented in Finmetrics, with the optimal auxiliary model chosen automatically. Table 8 shows the bias, the variance and the associated root mean squared errors (RMSE) for each parameter and for both EMM and robust ET method. With respect to bias, variance, and RMSE, the robust ET method dominates EMM for all parameters except for σ_u, where the bias of EMM is smaller than that of the robust ET method. In particular, the reduction in RMSE is substantial.

Table 6: Comparison of actual and nominal size of the robust ET statistic applied to *SLV's design* (Model 3) without contamination i.e.

$$\begin{pmatrix} Z_t \\ u_t \end{pmatrix} \sim \mathcal{N}\Big(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Big)$$

$H = 5$, $k = 3$ and 5000 replications. $robET.N$= robust ET test. $.N$ indicates the sample size. The tuning constant was set to $c = 2.5$

$nom.size$	$robET.500$
0.200	0.2276
0.100	0.1186
0.050	0.0634
0.025	0.0318
0.010	0.0100
0.005	0.0050
0.001	0.0012

Figure 10: *QQ-plots of overidentifying ET statistics versus* χ^2_2 *with normal (a) and contaminated data (b).*

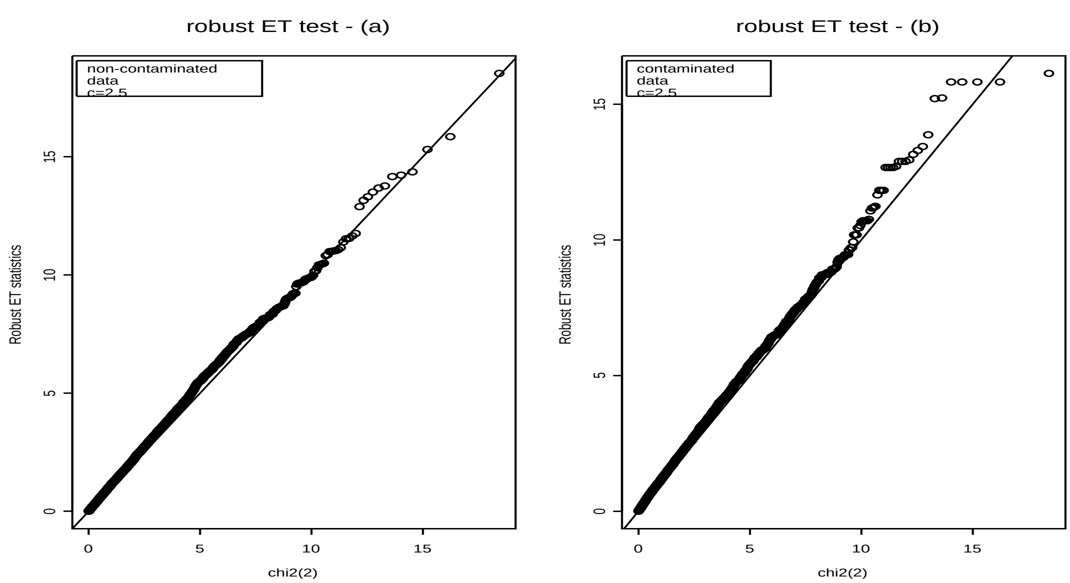


Table 7: Comparison of actual and nominal size of the robust ET statistic applied to *SLV's design* (Model 3) with contaminated data i.e.

$$\begin{pmatrix} Z_t \\ u_t \end{pmatrix} \sim 0.95 \cdot \mathcal{N}\Big(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Big) + 0.05 \cdot \mathcal{N}\Big(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \Big)$$

$H = 5$, $k = 3$ and 5000 replications. $robET.N$= robust ET test. $.N$ indicates the sample size. The tuning constant was set to $c = 2.5$

$nom.size$	$robET.500$
0.200	0.2330
0.100	0.1186
0.050	0.0604
0.025	0.0300
0.010	0.0114
0.005	0.0068
0.001	0.0016

Table 8: Comparison of RMSEs of EMM and ET method with 5000 replications; true parameters $(w, \beta, \sigma_u) = (-.368, .95, .260)$.

Method	w			β			σ_u		
	bias	var	RMSE	bias	var	RMSE	bias	var	RMSE
EMM	-.1640	.3367	.6030	-.0150	.0063	.0810	.0150	.0398	.2002
robust ET	.1020	.0380	.2220	.0144	.0006	.0299	-.0346	.0118	.1142

5 Conclusion

The Robust ET method is a useful procedure which provides attractive alternative estimators and tests to standard GMM methods. Our analysis shows that the new test statistic for overidentifying restrictions has excellent small sample properties for inference. Moreover, by its robustness, the procedure provides reliable estimators and tests even when the model does not hold exactly. Furthermore, the robust ET method is as flexible as GMM because it requires only a modified moments vector. Future research directions include the application of this method to other more complex models and the development of more efficient computational procedures.

A APPENDIX

Here we provide the algorithm and the computational aspects for solving (13) under the constraints (13.a), (13.b) and (13.c). For the particular models studied in this paper, Matlab's code is available from the authors upon request.

A.1 The Algorithm

To develop the algorithm for a general robust ET, we extend the procedure presented in Ronchetti and Trojani (2001, p. 47).

Specifically, for a given bound $c > \sqrt{H}$, the computation of the robust ET estimator can be performed by the following four steps:

i. Fix a starting value θ_0 for θ and initial values $\tau_0 = 0$ and A_0 such that

$$A_0' A_0 = \left[\frac{1}{N} \sum_{i=1}^{N} h(Z_i\,; \theta_0) h(Z_i\,; \theta_0)' \right]^{-1}$$

ii. Compute new values τ_1 and A_1 for τ and A defined by

$$\tau_1 = \frac{E_{\theta_0}[h(Z\,; \theta_0) w_c(A_0(h(Z\,; \theta_0) - \tau_0))]}{E_{\theta_0} w_c(A_0(h(Z\,; \theta_0) - \tau_0))} \tag{20}$$

and

$$(A_1' A_1)^{-1} = \frac{1}{N} \sum_{i=1}^{N} [(h(Z_i\,; \theta_0) - \tau_0))(h(Z_i\,; \theta_0) - \tau_0)' \times w_c^2(A_0(h(Z_i; \theta_0) - \tau_0))]. \tag{21}$$

iii. Compute the optimal ET estimator θ_1 associated to the orthogonality function $h_c^{A_1,\tau_1}$ by solving (13) subject to (13.a).

iv. Replace τ_0 and A_0 by τ_1 and A_1, respectively, and iterate the second and the third step described above until convergence.

A.2 Computational aspects

We used the ***fmincon()*** procedure for optimization in MATLAB 6.5. This algorithm is based on a Sequential Quadratic Programming (SQP) method, in which a Quadratic Programming (QP) subproblem is solved at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula.

A particular point in this algorithm is the calculation of the vector τ defined by (20). The expectation in (20) is easily computed by simulating a sample of size 75000.

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