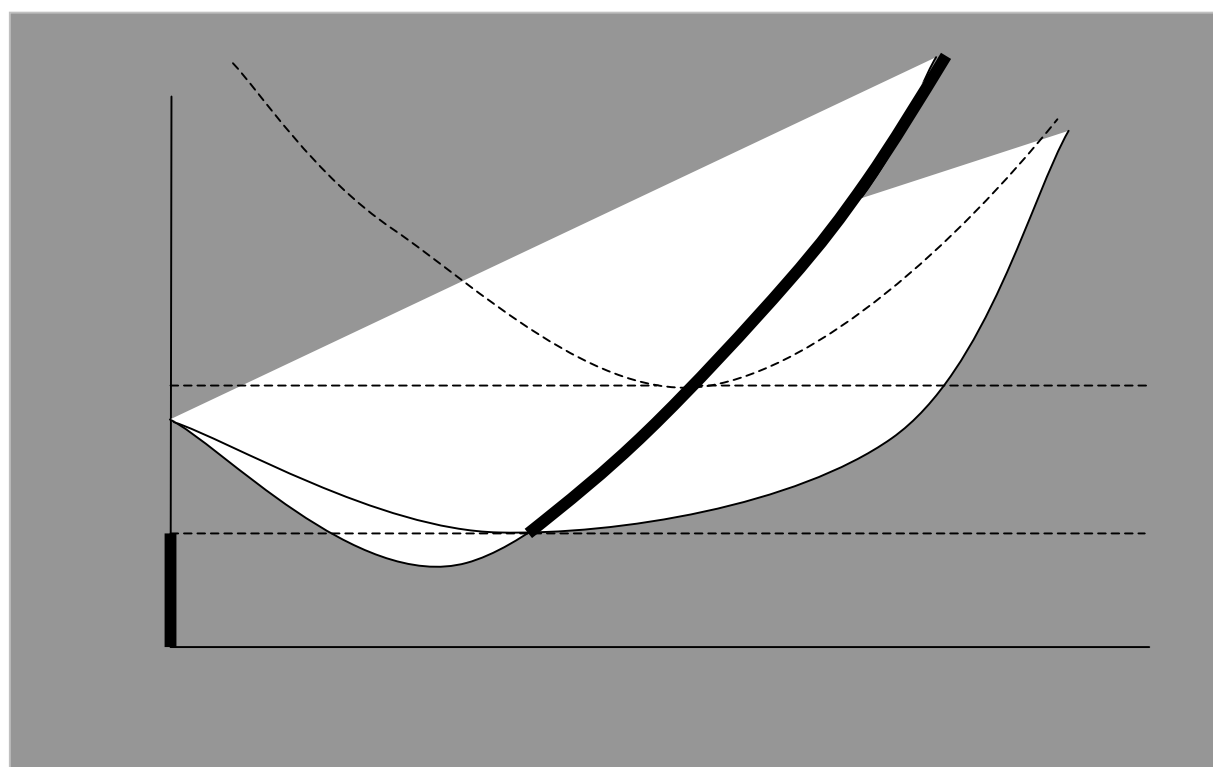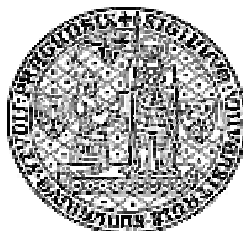# Working paper UK FSV - IES

## *No. 45*

## Hlaváček, Michal; Hlaváček, Jiří: Models of Economically Rational Donators



**2004**

# *Content*

## 1. Altruism and belonging to the community

Standard microeconomics deals with the egoistic subject *homo oeconomicus*, who achieves his own welfare. In the economic behaviour of some subjects there is also a concern in the welfare of others.

Many people are - to some extent - ready to give up their own benefit (for example financial) in favour of their fellows. We refer here to the belonging to a community, when the individual is (or feels to be) a part of this community, for example belonging to family, firm, country or just the community of the fair people. Slightly more general concept is the concept of altruism as an unselfish foregoing of personal profit in favour of the other person (or persons), for example the financial donation.

Altruism and belonging to the community is the natural part of human ethics. Adam Smith, a classicist of economics, is astonished about the human emphasise on the ethical part of human behaviour even in the economic sense which is not only determined by personal benefit[1]. Important issues (even though not dominant in last time) are demonstrations, principles and incentives of human mutuality[2].

Altruism motivated by expected compensation is being denoted as "soft". If the donator does not expect any compensation, we speak about the "hard" altruism[3]. Hard altruist is abstractedly of the norms of individual social groups actuated to do "good" and he/she feels happy if he can offer the donation. Characteristic of hard altruist is the non-calculating feeling of satisfaction from welfare of the other persons. The hard altruist donates without expectation of reciprocity.

Altruistic behaviour can be also viewed as a part of a viable community, higher level of the feel of belonging is usually revealed by a higher donation.[4] This increases ceteris paribus chance for survival of this community in competition with the other communities.

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[1] His Theory of Moral Sentiments was published in 1759.

[2] The other important and now already classical reflection to the incentives of human to offer the donation is Mauses essay about donation (Mauss, M. [1966])

[3] The distinction between "soft" and "hard" altruism was done by the founder of sociobiology E.O. Wilson (1978).

[4] Also not necessarily. Altruist could by non rational self-sacrifice in some case threaten the whole community. For example extreme self-sacrifice of one member of family could threaten his survival and destroy the family by this. For more details see Hlaváček J. et all [1999], paragraph 10.4

Altruism and even the "hard" altruism could be perceived within economic paradigm of homo oeconomicus if we perceive it as a special case of belonging to the community[5]. If the altruist feels mutuality with the other subjects and if he perceives them as part of himself ( "Me"), the donation of financial support could be explained by (basically neo-classical) utility maximisation. If the objective of the subject is minimisation of the threat of downfall than this subject who is a symphatisant or a member of community embodies aversion not only to situations with high level of economic threat to his person, but also to situations with high economic threat to other members of the community.

Let's suppose, that the decision criterion of the donator is his subjective probability of his (economic) survival including survival of "his" community. He is averse to situations threatening survival of members of that community or survival of the community as a whole. He compares that threats with the threat of his own survival.

In the text ahead we will give several examples of the models, in which the donator offers a financial subsidy in order to increase probability of recipient's survival. He takes into account the danger of his own self destruction as a result of too high financial expenses for subsidies.

## 2. Probability of (economical) survival proportional to the relative reserve: Pareto probability distribution

Let us suppose that the survival of two subjects, the donator and the recipient of donation, depends only on their income. In addition let us suppose that the probability of survival (felt by the donator) of each subject is proportional to the ratio of his reserve (measured according to the zone of unavoidable downfall) to his income.

Then the distribution function is given by:

$F(x)=(x-b)/x$      for     $x \geq b$

$F(x)= 0$      for     $x < b$

and the probability density by:

$f(x)=b/x^2$      for     $x \geq b$

$f(x)= 0$      for     $x < b$

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[5] See Etzioni, A. [1999]. "Me" in his approach contains "We", which is part of every individual. Social and ethical dimension of human preferences according to Etzioni necessarily strengthens stability and usually also quality of economic decision making .
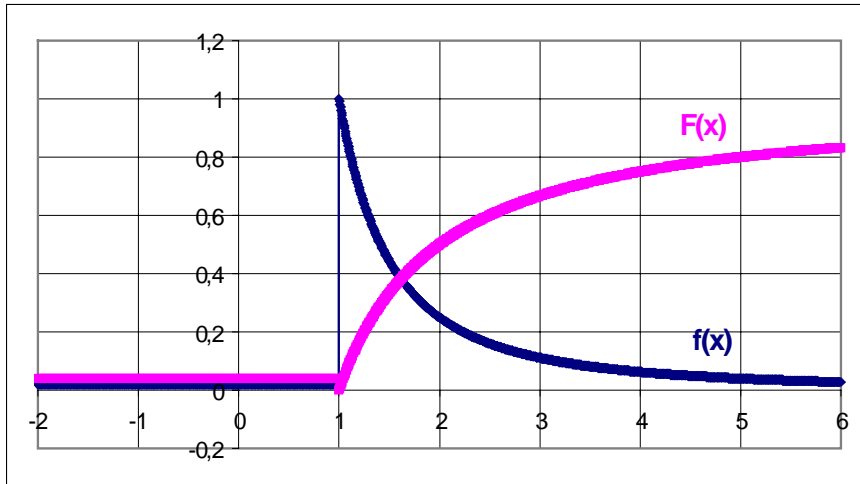
*Fig. 1 : Distribution of the probability according to the relative reserve: Pareto distribution of first degree with the boundary of zone of unavoidable downfall for b = 1: probability density f(x), distribution function F(x).*

This distribution function refers to the Pareto distribution of the first degree[6]. The Pareto distribution embodies zero probability for income on the boundary of unavoidable downfall and probability converging to one with increasing the income to infinity. This is true for both subjects.

Let us suppose for simplicity only two subjects in the first two models: recipient of donation (we will denote his income by d) and the donator (with starting income $A_0$, we will denote his income after donation by a). If we suppose, that the donation is the only income of recipient, his income d is also the level of offered donation.

We interpret the probability of survival as an subjective felt idea about ability to survive. We will suppose that the starting income of the donator $A_0$ enables survival of both subjects with non-zero probability (subjectively felt by the donator), so that it will hold:

$A_0 > D + A$

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[6] Pareto distribution is used for example to analyse incomes of consumers. Pareto distribution of the second degree was used in Hlaváček J., Hlaváček M. (2004). General Pareto distribution of degree a and with boundary b has an distribution function

$F(x) = 1 - (b/x)^a$       for       $x \geq b$,

$F(x) = 0$       for       $x < b$,

and function of probability density

$f(x) = (a/b).(b/x)^{a+1}$       for       $x \geq b$

$f(x) = 0$       for       $x < b$ .

D is a boundary of zone of unavoidable downfall of recipient of donation, A is the boundary of unavoidable downfall of the donator (under these levels subjects lose the ability to survive in economic sense).

By application of Pareto distribution of the first degree for both considered probabilities we will get the following:

$$p_1(d) = 0 \qquad \text{for} \quad d < D$$
$$= (d - D) / d \qquad \text{for} \quad d \geq D$$
$$p_2(a) = 0 \qquad \text{for} \quad a < A$$
$$= (a - A) / a \qquad \text{for} \quad a \geq A$$

where $a$, $A$ is income and boundary of zone of unavoidable downfall of the donator and $d$, $D$ is income and boundary of zone of unavoidable downfall of recipient of the donation.

The appropriate non-symmetric distribution functions are:

$$\eta(d) = 0 \qquad \text{for} \quad d < D$$
$$= D/d^2 \qquad \text{for} \quad d \geq D$$
$$\eta(a) = 0 \qquad \text{for} \quad a < A$$
$$= A/a^2 \qquad \text{for} \quad a \geq A.$$

Let us also suppose that the donator is convinced that both subjects have non-zero probability of survival, so that $d > D \wedge a > A$.

For probability of survival of both subjects we suppose that it is zero for income on boundary of unavoidable downfall and that it converges to one with increasing the income to infinity. This is again true for both subjects.

### 3. Model of maximisation of probability of simultaneous survival of both subjects (the donator and the donation recipient)

The donator here altruistically evaluates his own threat of downfall equally as the threat of downfall of donation recipient. He also supposes independence of both individual threats. Let us suppose that the only threat to both subjects is the threat by low level of financial assets.

The criterion of the donator is in this case the function:

$$p(d,a) = p_1(d) \cdot p_2(a) = (D.A - d.A - a.D + d.a) / (d.a)$$

The donator thus solves optimisation problem:

$$\max_{d+a\leq A_0} p(d,a)$$

The solution of this optimisation problem could be modified with help of the Lagrange function to this problem:

$$\max L(d,a,\lambda) = \max [p(d,a) + \lambda(A_0-d-a)]$$

By differentiation of function $L$ with respect to $d,a,\lambda$ we get fist order conditions for optimum:

$$d.A.(d-D)= a.D(a-A)$$
$$d + a= A_0$$

If the threat of downfall is the same for both subjects ($D=A$), the solution is $d = a = A_0/2$ (uniform division of income)[7].

If it holds that $D > A$, the solution is in favour of more threatened recipient of donation ($d>a$), so it leads to the "hard" altruistic favouring the recipient of donation[8]. At the same time the rate of favouring of more threatened donated subject falls with growing $A_0$ (for income of the donator equal to $A_0=D+A$ it is $d/a= D/A$ for income $A_0$ converging to infinity it goes to $d/a=\sqrt{D/A}$ , so the higher starting income of the donator leads to more uniform division ). However, if $A_0<A+D$ , when the starting income of the donator is so low, that it excludes the survival of both subjects at the same time, then the problem does not have solution and the decision has to be made according to some different criterion. Similar outcomes hold in case when the more threatened subject is the donator. Dependence of level of donation on starting level of income of the donator is shown for both cases in figures 2 and 3:

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[7] For $D=A$ and $d+a=A_0$ we could rewrite first optimality condition as $(d-a).(A_0-A)=0$. From $A_0>D+A$ it is obvious that $A_0>A$ so $A_0-A>0$. The only solution is thus $d=a=A_0/2$.

[8] Optimum must hold that marginal transfer of resources from first subject to the second increases probability of its downfall in the same rate, as it decreases probability of downfall of the second subject. In other words, partial derivations of probabilities of downfall of both subject with respect to income must be the same. From here it is possible to derive the optimal rate in which the donator splits up his income $M_0$. For example for Pareto distribution of second degree (a=2,b=1) the optimal rate is following

$$\frac{d_2}{d_1}=\sqrt[3]{\left(\frac{D_2}{D_1}\right)^2} \; ,$$

thus for $D_1 \neq D_2$, the solution is to favour the more threatened subject.
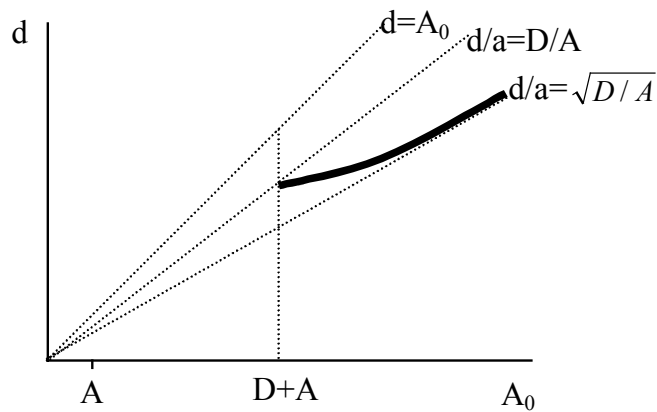
*Figure. 2: Level of donation in dependence on starting level of income of the donator in model of maximisation of probability of simultaneous survival of both subjects in case of $D>A$*
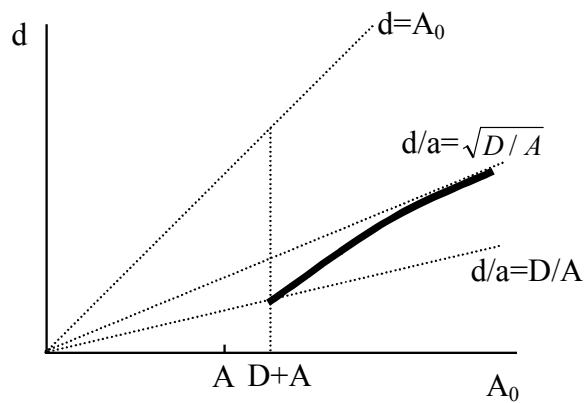


*Figure 3: Level of donation in dependence on starting level of income of the donator in model of maximisation of probability of simultaneous survival of both subjects in case of $D<A$*

The behaviour of the donator is thus given not only by his altruistic criterion, but also on the situation of individual subjects (in our case in relation of their resistance or threat of downfall). Hard economic behaviour could be result of decision making of the "hard" altruistic donator.

## 4. Model of minimisation of probability of simultaneous downfall of both subjects

The donator is again the "hard" altruist in this model, also he has a different decision criterion, than the hard altruist in the preceding model. In this model the donator maximises probability of survival of at least one member of the community. This alternative of "hard"

altruist is concerned only about survival of one of two subjects, but he does not care whether the surviving subject would be him or his subsidy recipient.

Probability of survival of at least one subject could be expressed as:

$$p = p(d, a) = p_1 + p_2 - p_1.p_2 = (d-D)/d + (a-A)/a - (d-D)/d \cdot (a-A)/a$$

The first summand reflects probability of survival of first subject and downfall of the second, second summand reflects probability of survival of the second subject and downfall of the first and the third reflects probability of survival of both at one time.

We again suppose that initial income of the donator does not rule out simultaneous survival of both subjects (the donator and the recipient of donation), thus:

$$A_0 > D + A$$

Again we could model the behaviour of the donator by:

$$\max_{d+a \leq A_0} p(d,a)$$

which could be again modified:

$$\max_{d+a \leq A_0} L(d,a,\lambda) = \max_{d+a \leq A_0} [(p(d,a) + \lambda.(A_0 - d - a)]$$

which gives first order conditions:

$$d = a \wedge d + a = A_0$$

The optimal distribution of income is:

$$d = a = A_0/2.$$

If the starting income $A_0$ of the donator disables simultaneous survival of both subjects, the optimal strategy for the donator changes significantly: unlike the identical support of both he moves all the resources to a more resistant member of the community. If the donator is the more resistant, i.e. if it holds that $D>A$, the donation falls to zero and the recipient of donation is in this case (similarly as in first model) sacrificed even by the altruist of the "hard" type. If the recipient is more resistant subject ($D<A$), the donator chooses self destruction, which enables survival of the subsidy recipient.

Dependence of the optimal level of donation on the starting income of the donator $A_0$ for both mentioned cases is graphed in following figures 4 and 5:
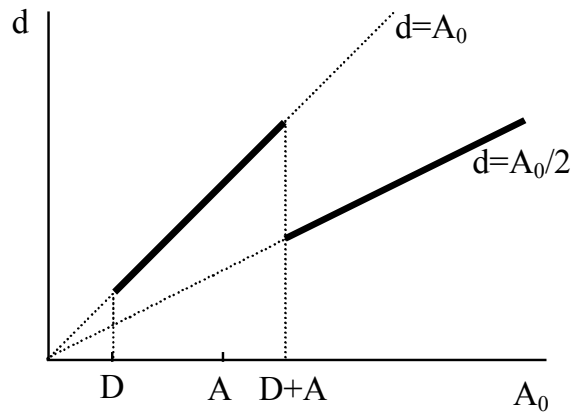
*Figure 4: Level of donation depending on the level of starting income of the donator $A_0$ in the model of minimising the threat of simultaneous downfall of both subjects, in the case where the more threatened is the recipient of donation.*
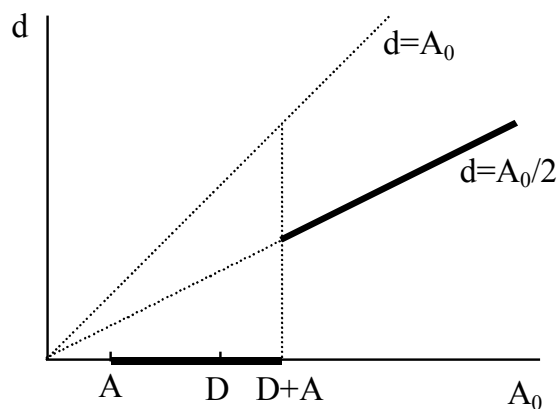


*Figure 5: Level of donation depending on the level of starting income of the donator $A_0$ in the model of minimising the threat of simultaneous downfall of both subjects, in the case where the more threatened is the donator.*

In this model the rational altruist could be forced to behave either as a "homo oeconomicus" or as an altruist of the hard type. It depends on parameters of the model.

## 5. Model of minimisation of threat of downfall of bigger part of the community

In this model of altruistic behaviour of the donator we will suppose his criterion as maximisation of probability of survival of the bigger part of the community. We will suppose that the community has three members (one donator and two recipients of donation), which

would dissolve in case of downfall of two members of this community. This model will enable us to describe the type of the strategy, which is called "gambit" in chess: one of members of the community could be sacrificed in favour of survival of the other two members.

Let us suppose different level of individual threat of downfall of three subjects, while the most resistant is the donator (level of threat $A$), recipient $i$ downfalls if he has income equal or lower than $D_i$. We will number the recipients of donation so that

$A < D_1 < D_2,$

thus the second recipient of donation is most threatened by downfall from all three subjects.

We will again suppose that the initial income enables survival of at least two most resistant members of the community with non-zero probability:

$A_0 > A + D_1$

Probability of survival of the community (according to assumptions, the probability of simultaneous survival of at least two members of the community) could be expressed by following way[9]:

$$p = p(d_1, d_2, d_3) = p_1 . p_2 (1-p_3) + p_1 . p_3 (1-p_2) + p_2 . p_3 (1-p_1) +$$
$$+ p_1 . p_2 . p_3 = p_1 . p_2 (1-p_3) + p_1 . p_3 (1-p_2) + p_2 . p_3$$

Problem of the donator could be again modelled by:

$$\max_{a+d_1+d_2 \leq A_0} p(a, d_1, d_2)$$

We could take into account two possible strategies: either try to equalise individual probabilities of survival of all three members of the community (Strategy A), or sacrifice the weakest member of the community and equalise the probability of survival of two stronger members (Strategy B). Any different strategy would make possible to transfer resources from a less threatened to a more threatened member of the community and thus increasing the maximised criterion of the donator.

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[9] We again suppose, that the downfall of each member does not depend directly on downfall of the other. The dependence is given only by survival of the whole community. For example, this model would not work in case of family with three members (father, mother, child) when the mother dies of sorrow in case of death of the child.

**Strategy A** (effort for survival of all three members)

To make the survival of all three members of the community at one time possible it must hold that the initial income of the donator has to be higher than the sum of boundaries of the zone of the unavoidable downfall of all three subjects:

$A_0 > A + D_1 + D_2$

The conditions for optimal choice of the donator ensures that the individual probabilities of the survival would be equal for all three subjects (otherwise it would be possible to increase the donator's objective function by transfer of resources). Equality of probabilities of survival for all three members of the community (with use of whole income $A_0$, which is the condition fulfilled in optimum automatically for this criterion) will hold for following division of total income $A_0$:

$a = A_0 . A/(A+D_1+D_2)$

$d_i = A_0 . D_i/(A+D_1+D_2)$   for   $i = 1,2$

Probability of survival of at least two members of the community could thus expressed as:

$p_A = (a - A).(d_1 - D_1) / (a.d_1) + (a - A).(d_2 - D_2) / (a.d_2) + (d_1 - D_1) .(d_2 - D_2) / (d_2.d_1)$

Because in this case it holds that $p_1 = p_2 = p_3 = p$ we could calculate probability of survival of the community under strategy A as:

$p_A = 3p_1^2(1 - p_1) + p_1^3 = 3p_1^2 - 2p_1^3$, where $p_1 = (a-A)/a = 1 - (A+D_1+D_2)/A_0$

**Strategy B** (effort for survival of two strongest members of the community)

Equalisation of probabilities of survival of two strongest members of the community (again with use of whole income $A_0$, which condition is fulfilled automatically in optimum) leads to following division of income:

$a = A_0 . A/(A+D_1)$

$d_1 = A_0 . D_1/(A+D_1)$

$d_2 = 0$

Because in this case it holds that $p_1 = p_2$ and $p_3 = 0$, we could calculate the probability of survival of the community as:

$p_B = (a - A)(d_1 - D_1)/(a.d_1) = [1 - (A+D_1)/A_0]^2$

where we use expressions for  $a$ and $d_1$ in the second step.

Let us compare both strategies. Strategy A increases number of favourable cases of survival, however at the cost of lower allowance to every subject and thus at the cost of the higher risk of individual downfall. On the contrary, the strategy B limits the favourable cases only to one (survival of the two strongest members), but their individual probabilities of the survival are now higher (they have higher income thanks to the sacrifice of the weakest member).

Comparison of the probabilities of survival of at least two members of the community under both strategies is shown in the Figure 6 :
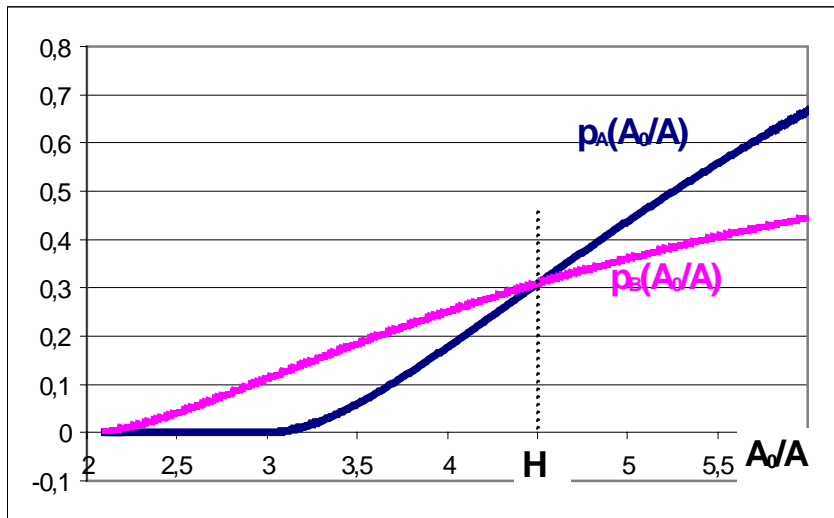


*Figure 6:* *Comparison of the strategies A,B - the case of the same threat of all subjects $A = D_1 = D_2$. The donator switches his strategy when $A_0 = H$.*

On the other hand, if $A_0 > H$, the optimal strategy is to sustain all three members. If one of recipients has lower resistance then the other agents, $H$ is higher[10].

The allocation of the resources ensures that the probability of survival is same for all three members of the community so the optimal allocation is:

$a = A.A_0/(A+D_1+D_2)$

$d_1 = D_1.A_0/(A+D_1+D_2)$

$d_2 = D_2.A_0/(A+D_1+D_2)$

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[10] H could be calculated algebraically if we let $p_A = p_B$ , which gives H as an expression dependent on $A_0$, $A$, $D_1$ and $D_2$. However the calculation is quite complicated (cubic equation) and even the result is not particularly insightful.

This means, that in a case when the optimal strategy A is chosen, the most threatened member of the community receives the greatest part from total income of the community, while the smallest part of income is obtained by the strongest (safest) member.

Solution of this optimisation problem thus depends on the level of disposal income of the whole community $A_0$. For high $A_0/A>H$ it is optimal to favour the third (most threatened) member of the community. For low $A_0/A<H$ it is optimal to sacrifice the initially favoured member and divide the whole income between the first two members, favouring the second member over the first one, to equalise their individual probabilities for survival. This is illustrated in figure 6:



*income of subject*

| | donator |
| --- | More resistant recipient of donation |
| ...... | Less resistant recipient of donation |

$D_2$    $A+D_1$    $A/H$    $A_0$
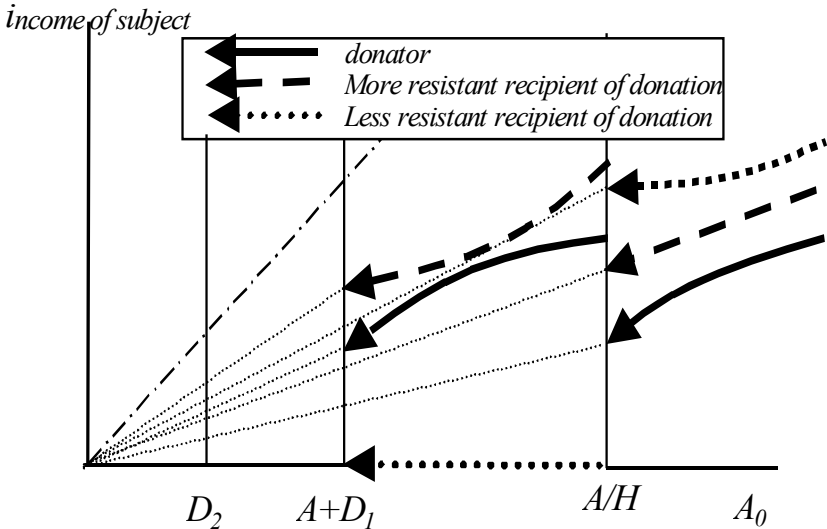
*Figure 6: Distribution of incomes if individual members*

If the both recipients have the same resistance (but less resistant than the donator), the donator is indifferent whom of them sacrifice in case of fall of income $A_0$ under H. The donator would accept draw between both recipient on the donation. If the donator is the least resistant member of the community (i.e. if $A<min (D_1,D_2)$), the optimal solution (in case when $A_0<A$) for him is self destruction.

Our "hard" altruistic donator is totally identified with the community as a hole. He either loses impulse of self-preservation and be ready to self-destruct or decides which member of the community will be sacrificed. The altruist can be cruel.

## 6. The model of maximal extent of survival

In this model, the criterion of the altruistic donator is the maximisation of the expected value of the number of community members who will survive.

Again, let us suppose the community consisting of three members with their boundaries of unavoidable individual downfall $A$, $D_1$, $D_2$, while the community is threatened by the low number of surviving members. The altruistic donator is fully identified with the goal of the community. The sum of the initial incomes of all members of the community again does not rule out the survival of all three members of the community at one time:

$$A_0 > A + D_1 + D_2$$

We will suppose the greater resistance of the donator and the same resistance of both recipients of the donation:

$$A < D_1 = D_2$$

We will again suppose Pareto distribution of the first degree, when the probability of survival of the individual converges to one if its income growth to infinity and equals zero if income is below the boundary of the zone of the unavoidable downfall. Probabilities of survival of the members of the community thus equal:

$$p_1(a) = (a-A)/a$$
$$p_2(d_1) = (d_1-D_1)/d_1$$
$$p_3(d_2) = (d_2-D_2)/d_2 ,$$

The criteria (maximised) function is in case of maximisation of the expected value of the number of the members of the community:

$$p(a, d_1, d_2) = p_1(a) + p_2(d_1) + p_3(d_2)$$

Decision making of the donator could be thus described by:

$$\max_{a+d_1+d_2 \leq A_0} p(a, d_1, d_2)$$

This problem could be modified with help of Lagrange function:

$$\max L(a, d_1, d_2, \lambda) = \max [p(a, d_1, d_2) + \lambda(A_0 - a - d_1 - d_2)]$$

In optimum, the partial derivatives of the Lagrange function are equal to zero, first order conditions are thus:

$$A/a^2 = D_1/d_1^2 = D_2/d_2^2$$
$$a + d_1 + d_2 = A_0$$

From this we could figure out the optimal division of the resources, or by other words the optimal strategy of the donator:

$$a = \frac{A_0 \cdot \sqrt{A}}{\sqrt{A} + \sqrt{D_1} + \sqrt{D_2}}$$

$$d_1 = \frac{A_0 \cdot \sqrt{D_1}}{\sqrt{A} + \sqrt{D_1} + \sqrt{D_2}}$$

$$d_2 = \frac{A_0 \cdot \sqrt{D_2}}{\sqrt{A} + \sqrt{D_1} + \sqrt{D_2}}$$

If the criterion of the donator is the maximisation of the expected value of the number of surviving members, the donator allocates the resources in favour of the more threatened members of the community, however the solution leads to the lower level of differentiation than in preceding models:

$$a : d_1 : d_2 = \sqrt{A} : \sqrt{D_1} : \sqrt{D_2} .$$

Similarly to the preceding models the fall of the income leads to discontinuous change of an allocation, while in this point of discontinuity the donator sacrifices the least resistant of the community and divides the income between him and more resistant recipient of the donation.

\* \* \*

We have shown that the altruism of donators could have many forms. The particular form of his criterion influences significantly his decision about allocation of his resources between recipients of the donation. Moreover, even under one criterion we could mention sudden change in strategy of the rational donator: This donator, even he is a "hard" altruist and favours the interests of the community over his own individual interests, could be forced by situation to "gambit", so to the sacrifice of one subject in favour of the whole community.

Preference of the interests of the community over interests of the individuals (including interests of the donator by himself) could lead the donator to the liquidation of the (in other situation preferred) least resistant members of the community. Interesting is also the discontinuity in the behaviour of the donator: even small change of disposal income leads to strong change of the allocation, including the liquidation of the preferred subject.

From the point of view of this interpretation we could generalise the results of our analysis to the behaviour of the donator- state. This donator decides about existence of the particular projects and the comprehension of the patterns in its donation policy fall in rank of the theoretical economics.

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