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Karel Janda: Bankruptcy Procedures with Ex Post Moral Hazard



The optimal design of credit contracts and bankruptcy procedures is an important policy question both in developed market economies and in countries with emerging markets. In this paper I deal with several theoretical considerations related to these important policy problems. My main concern is with the impact of relaxation of bankruptcy procedures providing for a possibility of a renegotiation of the debt instead of strictly imposing bankruptcy whenever the debtor falls into a default on his debt. I deal with this problem in a context of collateralized debt contracts in the conditions of imperfect information about the prospects of the entrepreneur and about the results of his project.

## Keywords

Bankruptcy, Moral Hazard, Adverse Selection, Soft Budget Constraint

## 1. Introduction

This paper has two goals. One is to analyze the commitment problem in collateralized debt contracts under asymmetric information. The other is to contribute to the discussion of the soft budget constraint (SBC) problem in transition economies. These goals are interrelated since commitment to imposing liquidation on non-performing firms is a key aspect of hardening the budget constraint. This paper shows that the view from the SBC literature that hardness is „good" and softness „bad" is questionable. Renegotiation and debt forgiveness in some cases improve welfare relative to liquidation of the defaulting firm. This is true despite the cheating and lower financial discipline created by the soft approach of creditors to enforcing credit contracts.

This paper presents a simple incomplete contract model of a credit market with ex ante asymmetric information and ex post costly state verification. I consider an entrepreneur who has private knowledge about his probability of success and who borrows from a lender in order to undertake a risky project. The borrower is able to pledge collateral to secure his loan. After the realization of the outcome of the investment project there is ex post asymmetric information between the borrower and the lender, who is not able to observe the project's outcome. The lender is able to observe it only if he imposes costly bankruptcy on the borrower. I compare the hard budget constraint situation when the lender is committed to imposing bankruptcy upon the borrower's announcement of default with the SBC situation where the lender may renegotiate the debt.

The fact that the outcome of the investment project is costlessly observable only by the entrepreneur leads to a possible ex post moral hazard and enforcement problem. The successful borrower may either repay his debt or falsely report failure of the investment project. One possible way to solve this problem is to collateralize the project fully with outside collateral. In the case where the borrower does not have sufficient wealth to pledge as outside collateral, the payment incentives have to be provided by a different arrangement. A widely used enforcement mechanism is to allow the lender to impose bankruptcy on the defaulting entrepreneur and to take over the project.

The resolution of the situation following a default by a borrower is a subject of many debtor-credit laws. The most well known is the US Bankruptcy Code which covers two basic types of bankruptcy proceedings. Firms that enter liquidation under Chapter 7 of the US Bankruptcy Code are typically shut down and their assets are dispersed under the control of the bankruptcy trustee (the liquidator) who represents the creditors. Under Chapter 11, firms reorganize as going concerns under the control of their management, protected from their creditors and able to operate. The US Bankruptcy Code provides creditors with basic protection against borrowers who would hide the project's revenue and refuse to pay the lender. At the
same time it is very much concerned with protecting a bankrupt firm's going-concern value while it is in bankruptcy and with minimizing inefficiencies caused by liquidation of the defaulted firm.

The fact that many countries are currently modifying their bankruptcy laws towards the US Bankruptcy Code could be taken as an indirect indication of the revealed success of the US debtor-oriented-approach. Nevertheless the legal norms used in a developed market economy with a strong legal culture may not be the best to use in a developing or transition country. Maskin and Xu (2001) argue that Chapter 11 has given rise to soft budget constraint (SBC) problems. They also review some proposals for reforming Chapter 11 bankruptcy procedures and suggest that more theoretical analysis should be done in this area.

The literature on transition economies has frequently stressed the hardening of the firm's budget constraint as a necessary condition for a successful transition to a market economy. The implementation of strict bankruptcy procedures was viewed as an essential element of this hardening of the budget constraint. A prime example of a tough approach to bankruptcy is the Hungarian automatic bankruptcy trigger experiment.

In September 1991 the Hungarian Parliament passed a major change of the Bankruptcy Act which introduced an automatic trigger on bankruptcies. This provision required the managers of firms which held overdue debts of any size to any creditor to initiate bankruptcy proceedings to avoid prosecution under the civil code. The introduction of the compulsory bankruptcy proceedings led to a sharp increase in the number of bankruptcies in Hungary. Kornai (2002) reports that the number of bankruptcy filings per firm reached 24 percent in Hungary in 1992. This is a quite high rate since Claessens and Klapper (2002) show that for their world-wide sample of 37 countries the corresponding average rate was between 0.02 percent (Spain) and 8 percent (Sweden) with the majority of countries having this rate in the 1 to 4 percent range. The Hungarian automatic bankruptcy trigger improved the state of payment discipline, overcame creditor passivity, and hardened the budget constraint of Hungarian firms. As argued by Bonin and Schaffer (2002) the high efficiency costs of forced bankruptcies led to the abolishment of the automatic trigger in September 1993.

Based on the Hungarian experience, June 1996’s amendment to the Czech Bankruptcy Act added a requirement that the debtor has to file for bankruptcy if he is overdebted. As opposed to Hungarian case there was not specified any criminal liability for failure to comply with this requirement in the Czech law. This was changed in 2000 when the criminal penalty of up to three years in prison was instituted for a manager of a defaulted enterprise who does not file for bankruptcy. The announcement of this civil law penalty led to a lot of discussion in the business community in the Czech Republic. In May 2000 The Prague Post newspaper wrote „'There is nothing like Chapter 11 [of the U.S. Bankruptcy Acts] in the Czech Economy, ' said Weston Stacey, executive director of the American Chamber of Commerce. 'Now bankruptcy means liquidation and going out of business.' " Nevertheless in July 2001 Janosik and Lizal concluded that „Criminal and financial sanctions (liability) for negligence in respect of the breach of the obligation (managerial duties of statutory bodies) to file for the bankruptcy, when required to do so by the Bankruptcy Act, are not yet fully known and obeyed. ... The solution is to really start to prosecute the legally responsible management for criminal behavior." Lizal (2002) reports that as of January 2002 no one has been held responsible for not filing.

These stylized facts about the use of the automatic bankruptcy trigger versus allowing for debt renegotiation and forgiveness serve as a motivation for my model in which I compare credit contracts with soft (lender is not committed to initiate bankruptcy upon default) and hard (bankruptcy automatically follows the default) bankruptcy procedures. My model contri-
butes to the literature on the soft budget constraint (SBC), which was started by Kornai (1979, 1980) and subsequently developed by Schaffer (1989) and Dewatripont and Maskin (1995). While Kornai, Maskin, and Roland (2003) discuss different approaches and interpretations of SBC, I will follow Maskin and Xu (2001) in treating SBC as a financial commitment problem of not imposing bankruptcy on the defaulted entrepreneur.

Dewatripont and Maskin (1995) consider a case in which only an entrepreneur knows the quality of his project and the lender does not have any instruments through which he would be able to screen the projects. In my analysis I allow for the inclusion of outside collateral as a screening instrument. I extend the SBC model of Dewatripont and Maskin (1995) by connecting the standard adverse selection model of collateralized debt as described by Schmidt-Mohr (1997) with the debt renegotiation models of Bester (1994), Scheepens (1995), Choe (1998), Khalil and Parigi (1998), Boyer (2001), and Menichini and Simmons (2003). I show how the introduction of ex ante asymmetric information influences results obtained under an assumption of ex ante symmetric information and how ex post costly state verification interacts with the sorting role of collateral in the incentive compatibility problem of adverse selection.

My model is close to the simplest version of the Dewatripont and Maskin (1995) SBC model, as presented by Kornai, Maskin, and Roland (2003), because I do not allow for any exertion of effort by lender or borrower which would influence the outcome of the project. Some of my results could also be compared with the results obtained by Bester (1994) who has only one type of borrower in his model and who does not include in his model the adverse selection effect of the ex ante asymmetric information. He repeatedly emphasizes that the results of his model are conditional on ex ante symmetric information and he conjectures that when the entrepreneur knows more about the project's ex ante profitability than the creditor, the results of the model might change significantly and the renegotiation may in fact be harmful. I will show, that in my setup the renegotiation under adverse selection increases welfare. My model also provides a qualification for Bester's (1994) conjecture that renegotiation may seriously undermine the role of collateral as a screening device.

My model allows me to consider collateral both as a signaling instrument and as a factor determining the likelihood of bankruptcy. I show that the screening effect of collateral prevails but that the major qualitative result of a welfare-enhancing role of renegotiation survives the introduction of the ex ante asymmetric information into the model. My explicit analysis of the interaction between credit renegotiation and the screening role of the collateral brings in this way a new contribution to an extensive literature dealing with adverse selection models of the credit market in general as summarized by Schmidt-Mohr (1997) and with the use of collateral in particular as documented in a survey by Coco (2000). To my knowledge, my model is the first attempt to model together adverse selection with costly state verification and renegotiation in the presence of collateralized debt contracts.

The rest of this paper is organized in the following way. Section 2 introduces my model. The situation under the commitment to the original contract is analyzed in the section 3 . The commitment assumption is relaxed in the section 4 . Section 5 concludes the paper.

## 2. The Model

I consider a risk neutral entrepreneur who wants to undertake a project. The project is either a failure, with return $\tilde{X}$ normalized to $\tilde{X}=1$, or a success with the return $\tilde{X}=X$. The return $\tilde{X}$ includes the market value of the productive assets used in the project evaluated at the time of the conclusion of the project. This means in particular that the failure return can be interpreted as a scrap value of the assets used in the process which are legally attachable. The
project requires an investment $I \in(1, X)$. The entrepreneur can be either of type $L$ or type $H$. The probability of a success depends on the type of entrepreneur. It is $0<p_{L}<p_{H}<1$ for a „low" and a „high" type respectively. This is the only difference between these two types.

The entrepreneur has a collateralizable wealth $W$ consisting of the assets which would normally not be legally attachable and he borrows the investment finance $I$ from a risk neutral lender. The lender does not know the type of the borrower and the lender does not observe the return realization of the project. He learns the return realization only if he imposes bankruptcy upon a borrower and takes over the project. When the lender takes over the project or the outside collateral $C \leq W$, his valuation of these is $\alpha \tilde{X}$ and $\alpha C$, respectively, where $0<\alpha<1$. I assume that the expected foreclosure value of the project exceeds the investment cost for both types of borrower:

$$
\begin{equation*}
\alpha\left(p_{i} X+1-p_{i}\right)>I, i \in\{L, H\} . \tag{1}
\end{equation*}
$$

I also assume that in the case of a project failure the lender cannot recover the loan $I$ even when he takes over both project and collateral and the cost of taking over are negligible, that is $\alpha \rightarrow 1$ :

$$
\begin{equation*}
I>W+1 . \tag{2}
\end{equation*}
$$

As a lower bound on the available collateralizable outside wealth $W$ I assume that it is higher than the value of the failed project when taken over by the lender. That is, I assume

$$
\begin{equation*}
W>\alpha \tag{3}
\end{equation*}
$$

The debt contract ( $R, C$ ) requires the borrower to pay the amount of $R$ upon a completion of the project. The assumption (2) implies that $R>1$. I also assume that $R \leq X$ since the borrower would never be able to pay more than the successful return of the project. If the borrower does not pay $R$ the lender has a right to force the borrower into a bankruptcy. Bankruptcy means that the lender takes over the project and the collateral $C$. Instead of forcing the borrower into bankruptcy the lender can renegotiate the contract after the borrower announces the failure of the project. If the project really failed the lender would maximize his payoff by making a renegotiated offer of $(1, C)$.

I model this situation through the following game. There are many lenders and one borrower. A borrower knows his type ( $L$ or $H$ ), but a lender only knows that the probability of type $L$ is $\theta$. Lenders compete by offering contracts $(R, C)$. Each lender also has the option of not making an offer, which guarantees him a zero profit. If the borrower does not accept any contract, the game ends and everybody gets a zero payoff. If the borrower accepts one contract, the borrower and his lender play the following subgame.

In the first stage of this subgame the project is realized either as a success or as a failure. This realization is observed by the borrower but remains unknown to the lender.

In the second stage only the successful entrepreneurs can pay $R$ as $1<R \leq X$. Thus after observing failure outcome of the project, the borrower has to default. In the case of success the borrower has two choices. Either pay $R$ or to claim that the project failed and default. The borrower can choose the mixed strategy according to which he defaults with probability $0 \leq d \leq 1$ and pays $R$ with probability $1-d$. In the case of repayment the game ends with payoffs $X-R$ for the borrower and $R-I$ for the lender. In the case of default the subgame continues to the third stage.

In the third stage after observing default the lender either imposes bankruptcy or offers a renegotiated contract ( $1, C$ ). The lender can randomize by imposing bankruptcy with the probability $0 \leq b \leq 1$. When bankruptcy happens, the lender takes over the project with the payoff
being $\alpha(X+C)-I$ or $\alpha(1+C)-I$ according to the realization of the project. The borrower's payoff is $-C$. By renegotiating the contract the lender gets payoff $1+\alpha C-I$ and the borrower gets $X-1-C$ if the project was a success or $-C$ if the project was a failure.

I assume that the lender and borrower do not have verifiable random device available. This means that it is not possible for them to write a contract which would specify a (nontrivial) probability with which a lender would impose bankruptcy on defaulting borrower. As a result of this assumption the initial contract is incomplete.

## 3. Credit Contract without Renegotiation

When the lender is committed not to renegotiate and automatically impose bankruptcy on the defaulting borrower, I have a standard model of screening in a credit market.

The expected utility of a borrower of a type $i$ is

$$
\begin{equation*}
U_{i}=p_{i}\left(X-R_{i}\right)-\left(1-p_{i}\right) C_{i} . \tag{4}
\end{equation*}
$$

The expected profit of a lender from a separating contract with a borrower of a type $i$ is

$$
\begin{equation*}
\rho_{i, N R}=p_{i} R_{i}+\left(1-p_{i}\right) \alpha\left(1+C_{i}\right)-I . \tag{5}
\end{equation*}
$$

I assume that the collateralizable wealth is high enough

$$
\begin{equation*}
C_{H, N R}^{*}<W, \tag{6}
\end{equation*}
$$

and the proportion $\theta$ of low type borrowers is high enough so that the indifference curve of the high type borrower through his equilibrium contract does not intersect the pooling zero profit line.

Under these assumptions the separating contracts form the unique separating equilibrium of the Rothschild and Stiglitz (1976) type.

The equation (5) shows that the slope of a zero profit line of a borrower of a type $i$ is $\frac{-\alpha\left(1-p_{i}\right)}{p_{i}}$ and the intercept is $\frac{I-\alpha\left(1-p_{i}\right)}{p_{i}}$. This implies that the zero profit line of a low type borrower is steeper than the zero profit line of a high type borrower and that it has a higher intercept. The slope of an indifference line of a borrower of a type $i$ is

$$
\frac{d R_{i}}{d C_{i}}=-\frac{1-p_{i}}{p_{i}}
$$

This means that the indifference lines of a low type borrower are steeper than those of a high type borrower and that the indifference lines are steeper than the zero profit line for each type of borrower.

Under these conditions the low type obtains in the equilibrium his complete information contract. The equilibrium contract of the high type is given by a binding incentive compatibility constraint of a low type.

The restriction on the relative shares of both types of borrowers is satisfied as long as

$$
\begin{equation*}
\theta \geq \frac{(1-\alpha)\left(1-p_{H}\right) p_{H}}{p_{H}-p_{L}+(1-\alpha)\left(1-p_{H}\right) p_{H}} \tag{7}
\end{equation*}
$$

The values of repayment and collateral at the equilibrium separating contracts are provided in the following theorem.

Proposition 1. The equilibrium solution is given by the following separating contracts:

$$
\begin{gather*}
C_{L, N R}^{*}=0,  \tag{8}\\
R_{L, N R}^{*}=\frac{I-\left(1-p_{L}\right) \alpha}{p_{L}}, \tag{9}
\end{gather*}
$$

for a low type borrower and

$$
\begin{align*}
C_{H, N R}^{*} & =\frac{\left(p_{H}-p_{L}\right)(I-\alpha)}{p_{H}\left(1-p_{L}\right)-\alpha p_{L}\left(1-p_{H}\right)},  \tag{10}\\
R_{H, N R}^{*} & =\frac{I-\left(1-p_{H}\right) \alpha\left(1+C_{H, N R}^{*}\right)}{p_{H}}, \tag{11}
\end{align*}
$$

for a high type borrower.
Proof. The lender's maximization problem is

$$
\begin{align*}
& \max _{\left(R_{L}, C_{L}, R_{H}, C_{H}\right)} M=\theta U_{L}+(1-\theta) U_{H} \\
& \quad=\theta\left[p_{L}\left(X-R_{L}\right)-\left(1-p_{L}\right) C_{L}\right] \\
& +(1-\theta)\left[p_{H}\left(X-R_{H}\right)-\left(1-p_{H}\right) C_{H}\right] \tag{12}
\end{align*}
$$

subject to

$$
\begin{gather*}
p_{i}\left(X-R_{i}\right)-\left(1-p_{i}\right) C_{i} \geq p_{i}\left(X-R_{j}\right)-\left(1-p_{i}\right) C_{j}  \tag{13}\\
U_{i} \geq 0  \tag{14}\\
p_{i} R_{i}+\left(1-p_{i}\right) \alpha\left(1+C_{i}\right)=I  \tag{15}\\
0 \leq C_{i} \leq W \tag{16}
\end{gather*}
$$

where $i, j \in\{L, H\}$.
I consider a case in which collateralizable wealth $W$ is sufficiently high to cover required collateral. That is I assume that there always exists feasible value of $W$ which is higher than the equilibrium collateral $C_{i, N R}^{*}$. I also assume that incentive compatibility condition (13) for high type and individual rationality conditions (14) for both types are satisfied in the equilibrium. I check these assumptions after I obtain the solution of the less restricted optimization problem.

Given these assumptions I substitute for

$$
R_{i}=\frac{I-\left(1-p_{i}\right) \alpha\left(1+C_{i}\right)}{p_{i}}
$$

from lender's zero profit condition (15), do some algebraic simplification and form the following Lagrangian

$$
\begin{aligned}
\max _{\left(C_{L}, C_{H}\right)} L & =\theta\left[p_{L} X-I+\alpha\left(1-p_{L}\right)-\left(1-p_{L}\right)(1-\alpha) C_{L}\right] \\
& +(1-\theta)\left[p_{H} X-I+\alpha\left(1-p_{H}\right)-\left(1-p_{H}\right)(1-\alpha) C_{H}\right] \\
& +\mu\left\{C_{H}\left[\left(1-p_{L}\right) p_{H}-\alpha p_{L}\left(1-p_{H}\right)\right]\right. \\
& \left.-p_{H}\left(1-p_{L}\right)(1-\alpha) C_{L}-\left(p_{H}-p_{L}\right)(I-\alpha)\right\} \\
& +\tau_{L} C_{L}+\tau_{H} C_{H} .
\end{aligned}
$$

Kuhn-Tucker conditions are FOC

$$
\begin{gather*}
\frac{\partial L}{\partial C_{L}}=-\theta\left(1-p_{L}\right)(1-\alpha)-\mu p_{H}\left(1-p_{L}\right)(1-\alpha)+\tau_{L}=0  \tag{17}\\
\frac{\partial L}{\partial C_{H}}=-(1-\theta)\left(1-p_{H}\right)(1-\alpha)-\mu\left[p_{H}\left(1-p_{L}\right)-\alpha p_{L}\left(1-p_{H}\right)\right]
\end{gather*}
$$

$$
\begin{equation*}
+\tau_{H}=0 \tag{18}
\end{equation*}
$$

and incentive compatibility condition for low type of borrower ( $I C_{L}$ ), $C_{i} \geq 0$, complementary slackness conditions and nonnegativity of multipliers.

First I show that $C_{L}=0$. Suppose by contradiction that $C_{L}>0$. Complementary slackness then implies $\tau_{L}=0$. FOC (17) then implies $\left(1-p_{L}\right)(1-\alpha)\left(\theta+\mu p_{H}\right)=0$, which is a contradiction. Therefore $C_{L}=0, \tau_{L}>0$.

Next I show that $C_{H}>0$. Suppose by contradiction that $C_{H}=0$. Then $\left(I C_{L}\right)$ implies $-\left(p_{H}-p_{L}\right)(I-\alpha) \geq 0$, which is a contradiction. Therefore $C_{H}>0, \tau_{H}=0$.

Finally I show that $\left(I C_{L}\right)$ is binding. Suppose by contradiction that $\left(I C_{L}\right)$ is not binding. Then by complementary slackness $\mu=0$. FOC (18) then implies $(1-\theta)\left(1-p_{H}\right)(1-\alpha)=0$, which is a contradiction. Therefore $\left(I C_{L}\right)$ is binding. The solution of the less constrained problem is then given by $C_{L}=0, C_{H}$ obtained from binding $\left(I C_{L}\right)$ and $R_{i}$ obtained from lenders' zero profit conditions.

As a last step I check that the solution to the less constrained problem in fact satisfies the additional conditions I assumed to hold when I formed the Lagrangian. The expected utility of a low type borrower is

$$
U_{L}=p_{L} X+\left(1-p_{L}\right) \alpha-I .
$$

The assumption (1) guarantees that $U_{L}>0$. The expected utility of a high type borrower is

$$
\begin{aligned}
U_{H} & =p_{H} X-p_{H} R_{H}-\left(1-p_{H}\right) C_{H} \\
& =p_{H} X-I+\alpha\left(1-p_{H}\right)-\left(1-p_{H}\right)(1-\alpha) C_{H} .
\end{aligned}
$$

Rewrite $p_{H} X=\alpha p_{H} X+(1-\alpha) p_{H} X$ and obtain

$$
U_{H}=\alpha\left[p_{H} X+1-p_{H}\right]-I+(1-\alpha)\left[p_{H} X-\left(1-p_{H}\right) C_{H}\right] .
$$

Since by assumption (2) $C_{H}<W<I-1$,

$$
\begin{align*}
& U_{H}>\alpha\left[p_{H} X+1-p_{H}\right]-I+(1-\alpha)\left[p_{H} X-\left(1-p_{H}\right)(I-1)\right] \\
& =\alpha\left[p_{H} X+1-p_{H}\right]-I+(1-\alpha)\left[p_{H} X+1-p_{H}-\left(1-p_{H}\right) I\right] . \tag{19}
\end{align*}
$$

Since the right hand side of (19) is positive by assumption (1), the individual rationality condition of high type borrower is satisfied.

The incentive compatibility condition of high type borrower is

$$
\begin{equation*}
p_{H} X-p_{H} R_{H}-\left(1-p_{H}\right) C_{H}-p_{H} X+p_{H} R_{L}+\left(1-p_{H}\right) C_{L} \geq 0 . \tag{20}
\end{equation*}
$$

The left hand side of (20) is equal to

$$
\begin{aligned}
& =p_{H} R_{L}-p_{H} R_{H}-\left(1-p_{H}\right) C_{H} \\
& =p_{H} \frac{I-\left(1-p_{L}\right) \alpha}{p_{L}}-\left[I-\alpha\left(1-p_{H}\right)\left(1+C_{H}\right)\right]-\left(1-p_{H}\right) C_{H} \\
& =\frac{p_{H}-p_{L}(I-\alpha)+\left(1-p_{H}\right)(\alpha-1) C_{H}}{p_{L}} \\
& =\frac{\left(p_{H}-p_{L}\right)^{2}(I-\alpha)}{p_{L}\left[p_{H}\left(1-p_{L}\right)-\alpha p_{L}\left(1-p_{H}\right)\right]} \\
& >0 .
\end{aligned}
$$

This proves that the incentive compatibility condition of high type borrower is satisfied. Q.E.D.

The equation (10) immediately implies that the required collateral increases with the size of the project $I$. The increase in the efficiency $\alpha$ leads nonambiguously to lower repayment $R_{L, N R}^{*}$ of the low type borrower and it also has direct negative effect on the high type borrower's repayment $R_{H, N R}^{*}$. There is also indirect effect through collateral requirement $C_{H, N R}^{*}$. Differentiating $C_{H, N R}^{*}$ with respect to $\alpha$ shows that increase in efficiency $\alpha$ leads to higher collateral for all projects of a sufficient size, such that

$$
\begin{equation*}
I>\frac{p_{H}\left(1-p_{L}\right)}{p_{L}\left(1-p_{H}\right)} . \tag{21}
\end{equation*}
$$

As long as intuitively plausible relation of higher use of collateral with the decrease of the deadweight loss of collateral transfer $1-\alpha$ holds true, the restrictions on the size of collateralizable wealth $W$ (2) and (6) are mutually consistent for any $\alpha$.

In the equilibrium only high type of a borrower posts collateral. As common in this type of credit market models, which do not allow for renegotiation, the collateral serves only as a screening instrument for separating the safer borrower from the more risky borrower. Since the probability of success of the high type borrower is lower than one, there will be unsuccessful collateralized projects in the equilibrium despite the fact that the low type borrower does not pledge any collateral. The share of collateralized failed projects in all failed projects is

$$
\begin{equation*}
\frac{\left(1-p_{H}\right)(1-\theta)}{\left(1-p_{L}\right) \theta+\left(1-p_{H}\right)(1-\theta)} . \tag{22}
\end{equation*}
$$

For some values of the parameters it could even happen that the majority of failed projects would be collateralized. This would happen if the proportion of low type borrowers were sufficiently low, such that $\theta<\frac{1-p_{H}}{1-p_{H}+1-p_{L}}$. This restriction is consistent with the lower bound on $\theta$ given by (7) as long as the deadweight cost of bankruptcy and ownership transfer are low enough such that $\alpha>\frac{p_{L}\left(1-p_{H}\right)}{p_{H}\left(1-p_{L}\right)}$.

While the borrower with a low probability of success obtains the same outcome as under complete information, the good borrower with the high probability of success is harmed by a presence of a bad borrower. In the next section I investigate whether relaxing the full commitment assumption can improve the situation of the good borrower. I also answer the question of the possibility of a Pareto improvement through renegotiation.

## 4. Credit Contract with Renegotiation

I will assume that the credit contract at the first part of the game achieved the separation of the low and high type of the borrower. Then I analyze the subgame following the signing of the contract using the fact that the type of borrower was already revealed by his acceptance of the separating contract. Then I go back and I show that the optimal contract at the first part of the game was indeed the separating one. Because of assumption (2), I consider only repayments $R_{i}$ satisfying $1+W<R_{i}<X$.

The subgame following the signing of contract can be solved using a perfect Bayesian equilibrium. The Bayesian updating requires that conditional on default the lender believes that the probability of a successful realization of the project is

$$
\begin{equation*}
\pi_{i}=\frac{p_{i} d_{i}}{p_{i} d_{i}+1-p_{i}} \tag{23}
\end{equation*}
$$

If the lender never imposes bankruptcy after the entrepreneur's default, then the borrower always declares default. On the other hand, if the lender always imposes bankruptcy on defaulted entrepreneur, then the successful entrepreneur never defaults. This leaves a possibility of an equilibrium where both players use mixing strategy. However, if the probability of a successful outcome is relatively low or the costs of bankruptcy are relatively high, then the lender might impose bankruptcy only with small probability or not even bother to initiate bankruptcy proceedings because the expected gains from detecting false default may not compensate the costs of bankruptcy. In that case the successful entrepreneur in equilibrium would always default. However the proof of the following lemma shows that such equilibria would not satisfy the assumption (1). This means that the unique equilibrium will indeed be in mixed strategies.

Lemma 1. Let $i \in\{L, H\}$. A successful entrepreneur of type $i$ defaults with a probability

$$
\begin{equation*}
d_{i}=\frac{\left(1-p_{i}\right)(1-\alpha)}{p_{i}(\alpha X-1)} \tag{24}
\end{equation*}
$$

Following a default bankruptcy is imposed with a probability

$$
\begin{equation*}
b_{i}=\frac{R_{i}-1-C_{i}}{X-1} . \tag{25}
\end{equation*}
$$

This equilibrium of the signaling subgame is unique.
Proof.
I consider first the borrower's problem. When the project is successful, the borrower maximizes his expected utility

$$
\max _{d_{i}} U_{B}=\left(1-d_{i}\right)\left(X-R_{i}\right)+d_{i}\left[-b_{i} C_{i}+\left(1-b_{i}\right)\left(X-1-C_{i}\right)\right] .
$$

After some simplifications this leads to

$$
U_{B}^{\prime}=R_{i}-1-C_{i}-b_{i}(X-1),
$$

where $U_{B}$ ' is a derivative of $U_{B}$ with respect to $d_{i}$. Therefore

$$
d_{i}\left\{\begin{array}{cl}
=0 & \text { if } R_{i}<1+C_{i}+b_{i}(X-1) \\
\in[0,1] & \text { if } R_{i}=1+C_{i}+b_{i}(X-1) \\
=1 & \text { if } R_{i}>1+C_{i}+b_{i}(X-1)
\end{array}\right.
$$

Next I consider the lender's problem. When he observes the default, the sequential rationality requires him to maximize his expected utility

$$
\begin{aligned}
\max _{b_{i}} U_{L} & =\pi_{i}\left\{b_{i}\left[\alpha\left(x+C_{i}\right)-I\right]+\left(1-b_{i}\right)\left(1+\alpha C_{i}-I\right)\right\} \\
& +\left(1-\pi_{i}\right)\left\{b_{i}\left[\alpha\left(1+C_{i}\right)-I\right]+\left(1-b_{i}\right)\left(1+\alpha C_{i}-I\right)\right\} .
\end{aligned}
$$

After some simplifications this leads to

$$
U_{L}{ }^{\prime}=\alpha \pi_{i}(X-1)-(1-\alpha),
$$

where $U_{L}$ ' is a derivative of $U_{L}$ with respect to $b_{i}$. This implies

$$
b_{i} \begin{cases}=0 & \text { if } \pi_{i}<\frac{1-\alpha}{\alpha(X-1)} \\ \in[0,1] & \text { if } \pi_{i}=\frac{1-\alpha}{\alpha(X-1)} \\ =1 & \text { if } \pi_{i}>\frac{1-\alpha}{\alpha(X-1)} .\end{cases}
$$

I will consider separately all three cases for possible values of $\pi_{i}$ and $b_{i}$.
Case $1 \quad \pi_{i}<\frac{1-\alpha}{\alpha(X-1)}$ : Then $b_{i}=0$ implies that $\frac{\partial U_{\mathrm{B}}}{\partial d_{i}}=R_{i}-\left(1+C_{i}\right)$. Because $R>1+W$, the sequential rationality of the borrower requires $d_{i}=1$. Then according to (23) $\pi_{i}=p_{i}$. This means that

$$
p_{i}<\frac{1-\alpha}{\alpha(X-1)} .
$$

This is equivalent to

$$
\alpha\left(p_{i} X+1-p_{i}\right)<1,
$$

which is a contradiction.
Case $2 \pi_{i}>\frac{1-\alpha}{\alpha(X-1)}$ : Then $b_{i}=1$ implies that $\frac{\partial U_{B}}{\partial d_{i}}=R_{i}-\left(X+C_{i}\right)$. Because $R<X$, the sequential rationality of the borrower requires $d_{i}=0$. Then according to (23) $\pi_{i}=0$, which is a contradiction.

Case $3 \pi_{i}=\frac{1-\alpha}{\alpha(X-1)}$ :
Case 3.1 $d_{i}=0$ : Then by (23) $\pi_{i}=0$, which is a contradiction.
Case $3.2 \quad d_{i}=1$ : Then by (23) $\pi_{i}=p_{i}$ and consequently $p_{i}=\frac{1-\alpha}{\alpha(X-1)}$. This is equivalent to $\alpha\left(p_{i} X+1-p_{i}\right)=1$, which is a contradiction.

Case 3.3 $d_{i} \in(0,1)$ : Then $R_{i}=1+C_{i}+b_{i}(X-1)$. This leads to $b_{i}=\frac{R_{i}-1-C_{i}}{X-1}$. Since $1+W<R_{i}<X$ the lender's strategy $b_{i}$ satisfies $b_{i} \in(0,1)$. Finally (23) and $\pi_{i}=\frac{1-\alpha}{\alpha(X-1)}$ imply that $d_{i}=\frac{\left(1-p_{i}\right)(1-\alpha)}{p_{i}(\alpha X-1)}$.

## Q.E.D.

The equilibrium values of mixing probabilities $d_{i}$ and $b_{i}$ imply the following results, which I summarize in Proposition 2.

## Proposition 2.

1.An increase in the collateral requirement and a decrease in the contractual repayment lead to a lower probability of bankruptcy.
2.The higher the deadweight cost of a transfer of collateral or ownership of the project, the higher a probability that a successful entrepreneur will default.
3.Higher value of a successful outcome decreases the probability that the successful entrepreneur will default.
4.The share of defaulting successful entrepreneurs is higher for borrowers with ex ante lower probability of success.
Proof. It follows from Lemma 1.
Using the solution of the renegotiation subgame obtained in Lemma 1, I solve for the equilibrium contracts in the screening game.

Given the equilibrium probability of default $d_{i}$, the lender's profit from the separating contract to a type $i$ borrower is

$$
\begin{equation*}
\rho_{i, R}=p_{i}\left(1-d_{i}\right) R_{i}+\left(p_{i} d_{i}+1-p_{i}\right)\left(1+\alpha C_{i}\right)-I \tag{26}
\end{equation*}
$$

Similarly as in the case without a renegotiation, the zero profit line of a low type is steeper and has a higher intercept than the zero profit line of a high type.

The comparison of the slopes of indifference lines and a zero profit line for a borrower of a type $i$ shows that the indifference lines are steeper if the probability of success is sufficiently high such that

$$
\begin{equation*}
p_{i}>\frac{1}{\alpha(X-1)} \tag{27}
\end{equation*}
$$

By assumption (1) this condition is automatically satisfied as long as the investment cost is sufficiently higher than the investment return in the failure state, that is if $I>1+\alpha$. Given the restriction (2) on the minimal size of the investment cost, it follows that the condition (27) is automatically satisfied when $W>\alpha$. This is satisfied by assumption (3).

Given the condition (27) which states that the probabilities of success will be sufficiently high for both types, the relative positions of indifference lines and zero profit lines determine a separating equilibrium. In this equilibrium the low type gets his most preferred contract on the low type zero profit line and the high type's contract is determined by a binding incentive constraint for a low type. The values of equilibrium repayment and collateral are provided in the following Proposition.

## Proposition 3.

The collateral and repayment requirements for the equilibrium credit contract without commitment are:

$$
\begin{gather*}
C_{L, R}^{*}=0  \tag{28}\\
R_{L, R}^{*}=\frac{\alpha\left[X I-(X-1)\left(1-p_{L}\right)\right]-I}{\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-1} \tag{29}
\end{gather*}
$$

for a low type borrower and

$$
\begin{gather*}
C_{H, R}^{*}=\frac{p_{L} \alpha(X-1)\left(p_{H}-p_{L}\right)(\alpha X-1)(I-1)}{\left(1-p_{L}\right)\left[\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1\right]-p_{L} \alpha^{2}\left(1-p_{H}\right)(X-1)} \\
\frac{1}{\left[\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-1\right]}  \tag{30}\\
R_{H, R}^{*}=\frac{\alpha\left[X I-(X-1)\left(1-p_{H}\right)\right]-I}{\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1}-
\end{gather*}
$$

$$
\begin{equation*}
\frac{\alpha^{2}\left(1-p_{H}\right)(X-1)}{\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1} C_{H, R}^{*}, \tag{31}
\end{equation*}
$$

for a high type borrower.
Proof.
The lender's maximization problem is

$$
\begin{align*}
& \max _{\left(R_{L}, C_{L}, R_{H}, C_{H}\right)} M=\theta U_{L}+(1-\theta) U_{H} \\
& \left.=\theta \theta p_{L}\left(X-R_{L}\right)-\left(1-p_{L}\right) C_{L}\right] \\
& +(1-\theta)\left[p_{H}\left(X-R_{H}\right)-\left(1-p_{H}\right) C_{H}\right] \tag{32}
\end{align*}
$$

subject to

$$
\begin{gather*}
p_{i}\left(X-R_{i}\right)-\left(1-p_{i}\right) C_{i} \geq p_{i}\left(X-R_{j}\right)-\left(1-p_{i}\right) C_{j}  \tag{33}\\
U_{i} \geq 0  \tag{34}\\
p_{i}\left(1-d_{i}\right) R_{i}+\left(p_{i} d_{i}+1-p_{i}\right)\left(1+\alpha C_{i}\right)=I  \tag{35}\\
0 \leq C_{i} \leq W, \tag{36}
\end{gather*}
$$

where $i, j \in\{L, H\}$.
I consider a case in which collateralizable wealth $W$ is sufficiently high to cover required collateral. That is I assume that there always exists feasible value of $W$ which is higher than the equilibrium collateral $C_{i, R}^{*}$. I also assume that incentive compatibility condition (33) for high type and individual rationality conditions (34) for both types are satisfied in the equilibrium. I check these assumptions after I obtain the solution of the less restricted optimization problem.

I substitute for $d_{i}$, express the repayment $R_{i}$ from lender's zero profit condition

$$
R_{i}=\frac{(\alpha X-1) I-\alpha\left(1-p_{i}\right)(X-1)}{\alpha-1+\alpha p_{i}(X-1)}-\frac{\alpha^{2}\left(1-p_{i}\right)(X-1)}{\alpha-1+\alpha p_{i}(X-1)} C_{i}
$$

and substitute it in $U_{i}$

$$
U_{i}=p_{i} X-p_{i} \frac{(\alpha X-1) I-\alpha\left(1-p_{i}\right)(X-1)}{\alpha-1+\alpha p_{i}(X-1)}-\frac{\left(1-p_{i}\right)(1-\alpha)\left[\alpha p_{i}(X-1)-1\right]}{\alpha-1+\alpha p_{i}(X-1)} C_{i} .
$$

Since the assumptions (1)-(3) imply that $\alpha p_{i}(X-1)-1>0$, I see that increase in collateral $C_{i}$ indeed decreases utility of borrower as I would intuitively expect.

I also substitute for $R_{i}$ in the incentive compatibility condition of low type borrower $\left(I C_{L}\right)$ and I express ( $I C_{L}$ ) as

$$
\begin{aligned}
& p_{L}\left[\frac{(\alpha X-1) I-\alpha\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)}-\frac{(\alpha X-1) I-\alpha\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)}\right. \\
& \left.-\frac{\alpha^{2}\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)} C_{H}+\frac{\alpha^{2}\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)} C_{L}\right] \\
& +\left(1-p_{L}\right)\left(C_{H}-C_{L}\right) \geq 0 .
\end{aligned}
$$

Now I form the Lagrangian

$$
\begin{aligned}
& \max _{\left(C_{L}, C_{H}\right)} L=\theta\left\{p_{L} X-p_{L} \frac{(\alpha X-1) I-\alpha\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)}\right. \\
& \left.-\frac{\left(1-p_{L}\right)(1-\alpha)\left[\alpha p_{L}(X-1)-1\right]}{\alpha-1+\alpha p_{L}(X-1)} C_{L}\right\} \\
& +(1-\theta)\left\{p_{H} X-p_{H} \frac{(\alpha X-1) I-\alpha\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)}\right. \\
& \left.-\frac{\left(1-p_{H}\right)(1-\alpha)\left[\alpha p_{H}(X-1)-1\right]}{\alpha-1+\alpha p_{H}(X-1)} C_{H}\right\} \\
& +\mu\left\{p _ { L } \left[\frac{(\alpha X-1) I-\alpha\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)}-\frac{(\alpha X-1) I-\alpha\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)}\right.\right. \\
& \left.-\frac{\alpha^{2}\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)} C_{H}+\frac{\alpha^{2}\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)} C_{L}\right] \\
& \left.+\left(1-p_{L}\right)\left(C_{H}-C_{L}\right)\right\}+\tau_{L} C_{L}+\tau_{H} C_{H} .
\end{aligned}
$$

Kuhn-Tucker conditions are FOC

$$
\begin{gather*}
\frac{\partial L}{\partial C_{L}}=-\theta \frac{\left(1-p_{L}\right)(1-\alpha)\left[\alpha p_{L}(X-1)-1\right]}{\alpha-1+\alpha p_{L}(X-1)} \\
+\mu\left[p_{L} \frac{\alpha^{2}\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)}-\left(1-p_{L}\right)\right]+\tau_{L}=0  \tag{37}\\
\frac{\partial L}{\partial C_{H}}=-(1-\theta) \frac{\left(1-p_{H}\right)(1-\alpha)\left[\alpha p_{H}(X-1)-1\right]}{\alpha-1+\alpha p_{H}(X-1)} \\
-\mu\left[p_{L} \frac{\alpha^{2}\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)}-\left(1-p_{L}\right)\right]+\tau_{H}=0 \tag{38}
\end{gather*}
$$

and incentive compatibility condition for low type of borrower ( $I C_{L}$ ), $C_{i} \geq 0$, complementary slackness conditions and nonnegativity of multipliers.

First I show that $C_{L}=0$. Suppose by contradiction that $C_{L}>0$. Complementary slackness then implies $\tau_{L}=0$. FOC (37) then implies

$$
-\theta \frac{\left(1-p_{L}\right)(1-\alpha)\left[\alpha p_{L}(X-1)-1\right]}{\alpha-1+\alpha p_{L}(X-1)}+\mu\left[p_{L} \frac{\alpha^{2}\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)}-\left(1-p_{L}\right)\right]=0 .
$$

This simplifies as

$$
-\frac{\left(1-p_{L}\right)(1-\alpha)\left[\alpha p_{L}(X-1)-1\right]}{\alpha-1+\alpha p_{L}(X-1)}(\theta+\mu)=0,
$$

which is a contradiction.
Next I show that $C_{H}>0$. Suppose by contradiction that $C_{H}=0$. Then ( $I C_{L}$ ) implies

$$
p_{L}\left[\frac{(\alpha X-1) I-\alpha\left(1-p_{H}\right)(X-1)}{\alpha-1+\alpha p_{H}(X-1)}-\frac{(\alpha X-1) I-\alpha\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)}\right] \geq 0 .
$$

After a tedious algebra this can by simplified as

$$
\frac{\alpha p_{L}\left(p_{H}-p_{L}\right)(\alpha X-1)(X-1)(1-I)}{\left[\alpha-1+\alpha p_{H}(X-1)\right]\left[\alpha-1+\alpha p_{L}(X-1)\right]} \geq 0,
$$

which is a contradiction. Therefore $C_{H}>0, \tau_{H}=0$.
Finally I show that $\left(I C_{L}\right)$ is binding. Suppose by contradiction that $\left(I C_{L}\right)$ is not binding. Then by complementary slackness $\mu=0$. FOC (38) then implies

$$
-(1-\theta) \frac{\left(1-p_{H}\right)(1-\alpha)\left[\alpha p_{H}(X-1)-1\right]}{\alpha-1+\alpha p_{H}(X-1)}=0,
$$

which is a contradiction. Therefore $\left(I C_{L}\right)$ is binding. The solution of the less constrained problem is then given by $C_{L}=0, C_{H}$ obtained from binding ( $I C_{L}$ ) and $R_{i}$ obtained from lenders' zero profit conditions.

As a next step I check that the solution to the less constrained problem in fact satisfies the additional conditions I assumed to hold when I formed the Lagrangian. The expected utility of a low type borrower is

$$
\begin{aligned}
U_{L} & =p_{L} X-p_{L} \frac{(\alpha X-1) I-\alpha\left(1-p_{L}\right)(X-1)}{\alpha-1+\alpha p_{L}(X-1)} \\
& =p_{L} \frac{-(1-\alpha) X+\alpha X p_{L}(X-1)+\alpha\left(1-p_{L}\right)(X-1)-(\alpha X-1) I}{\alpha-1+\alpha p_{L}(X-1)}
\end{aligned}
$$

I rewrite $(\alpha X-1) I$ as $(\alpha X-X+X-1) I$ and I obtain

$$
\begin{aligned}
U_{L} & =p_{L} \frac{-(1-\alpha) X+(X-1) \alpha\left(p_{L} X+1-p_{L}\right)+X(1-\alpha) I-(X-1) I}{\alpha-1+\alpha p_{L}(X-1)} \\
& =p_{L} \frac{(1-\alpha) X(I-1)+(X-1)\left[\alpha\left(p_{L} X+1-p_{L}\right)-I\right]}{\alpha-1+\alpha p_{L}(X-1)}>0 .
\end{aligned}
$$

The incentive compatibility constraint of high type borrower ( $I C_{H}$ ) is

$$
\begin{equation*}
p_{H}\left(R_{L}-R_{H}\right)-\left(1-p_{H}\right) C_{H} \geq 0 . \tag{39}
\end{equation*}
$$

From binding ( $I C_{L}$ ) I substitute $C_{H}=\frac{p_{L}\left(R_{L}-R_{H}\right)}{1-p_{L}}$ to (39) and after some simplifications I obtain

$$
\left(R_{L}-R_{H}\right) \frac{p_{H}-p_{L}}{1-p_{L}} \geq 0
$$

This means that ( $I C_{H}$ ) is satisfied if and only if $R_{L}-R_{H} \geq 0$. After substitution for $R_{L}$ and $R_{H}$ I get

$$
\begin{gather*}
R_{L}-R_{H}=\frac{\alpha\left[X I-(X-1)\left(1-p_{L}\right)\right]-I}{\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-1}-\frac{\alpha\left[X I-(X-1)\left(1-p_{H}\right)\right]-I}{\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1} \\
+\frac{\alpha^{2}\left(1-p_{H}\right)(X-1)}{\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1} C_{H, R}^{*} . \tag{40}
\end{gather*}
$$

It is enough to show that the difference of the first two right-hand-side terms in equation (13) is positive. After some simplifications I obtain

$$
\begin{aligned}
& \frac{\alpha\left[X I-(X-1)\left(1-p_{L}\right)\right]-I}{\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-1}-\frac{\alpha\left[X I-(X-1)\left(1-p_{H}\right)\right]-I}{\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1}= \\
& \frac{\alpha\left(p_{H}-p_{L}\right)(X-1)(I-1)(\alpha X-1)}{\left\{\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-1\right\}\left\{\alpha\left[p_{H} X+\left(1-p_{H}\right)\right]-1\right\}}>0 .
\end{aligned}
$$

Therefore $\left(I C_{H}\right)$ is satisfied.
As a last step I show that the expected utility of a high type borrower

$$
U_{H}=p_{H}\left(X-R_{H}\right)-\left(1-p_{H}\right) C_{H}
$$

is positive. The positive expected utility of a low type borrower implies that $X>R_{L}$. Therefore it is enough to prove that

$$
\begin{equation*}
p_{H}\left(R_{L}-R_{H}\right)-\left(1-p_{H}\right) C_{H} \geq 0 . \tag{41}
\end{equation*}
$$

Since the (41) is the $\left(I C_{H}\right)$ as expressed in (39), I proved that $U_{H}>0$. Q.E.D.
In the case when the success probability for a low type or for both types would be lower than the critical value given by a condition (27), which is implied by assumptions (1) - (3), the separating contracts given by equations (28)-(31) would no longer be optimal. The equilibrium contract would be in this case a pooling contract in which both types of borrower have to provide the maximum available amount of collateral $C=W$. The interest rate is then determined by a lender's pooling zero profit line:

$$
R_{L H, R}^{*}=\frac{\alpha\left[X I-(X-1)\left(1-p_{L H}\right)\right]-I}{\alpha\left[p_{L H} X+\left(1-p_{L H}\right)\right]-1}-\frac{\alpha^{2}\left(1-p_{L H}\right)(X-1)}{\alpha\left[p_{L H} X+\left(1-p_{L H}\right)\right]-1} W,
$$

where $p_{L H}=\theta p_{L}+(1-\theta) p_{H}$.
In the case of a success probability for both types being lower than the critical value given by a condition (27), this pooling equilibrium always exists. This is guaranteed by the following relations between indifference and zero profit lines. A high type indifference line is flatter than the pooling zero profit line, which precludes a profitable pooling deviation. A low type indifference line is flatter than the slope of a low type zero profit line, which precludes a profitable separating deviation for a low type.

When only a low type has a lower-than-critical probability of success, the pooling equilibrium exists as long as the probability $\theta$ of the low type is sufficiently high such that the high type indifference line is flatter than the pooling zero profit line. Due to the fact that the binding zero profit lines are in this case determined by pooled probabilities of success, these lines move closer to the pooling zero profit line than under the separating equilibrium. For a pathological case of $W$ close to zero and $p_{L}$ only slightly lower than the critical value (27), the nonexistence of an equilibrium caused by a separating deviation of a low type could arise .

Following this clarification of what could happen if the assumptions ( $1-3$ ) implying a condition (27) were not satisfied, I will restrict my further attention to the cases satisfying these assumptions.

The proportion of collateralized defaulted projects on the total number of defaulted projects will be the same as in the case with a commitment. This is because the probability of announcing a default for a borrower of a type $i$ is $d_{i} p_{i}+1-p_{i}=\frac{\alpha(X-1)}{\alpha X-1}\left(1-p_{i}\right)$. The constant factor $\frac{\alpha(X-1)}{\alpha X-1}>1$ measures the increase in the probability of default due to a strategic default. The ratio of collateralized defaults on the total number of defaults is then equal to this ratio in the commitment case, which is given by an expression (22).

Instead of focusing on defaults, I may consider actually imposed bankruptcies instead. In this case the probability of imposing a bankruptcy on a borrower of a type $i$ is
$b_{i}\left(d_{i} p_{i}+1-p_{i}\right)$. This means that the ratio of collateralized bankruptcies on the total number of bankruptcies is

$$
\begin{equation*}
\frac{\left(1-p_{H}\right)(1-\theta)}{\frac{b_{L}}{b_{H}}\left(1-p_{L}\right) \theta+\left(1-p_{H}\right)(1-\theta)} . \tag{42}
\end{equation*}
$$

Since in equilibrium $b_{L}>b_{H}$, the comparison of expressions (22) and (42) shows that the share of collateralized bankruptcies on all bankruptcies is lower than the share of collateralized defaults on all defaults.

Since the probability of default is a decreasing function of the successful project outcome $X$, the equilibrium contract depends on the value of the successful outcome. In particular, it can be shown that without commitment the equilibrium repayment $R_{L, R}^{*}$ of a low type borrower is a decreasing function of the value of successful outcome $X$. This is different from the situation without a renegotiation when the value of successful outcome does not influence the equilibrium size of a collateral and repayment.

The main result on the welfare consequences of relaxing the commitment assumption in my model follows now.

Proposition 4. The possibility of renegotiation increases welfare.
Proof. The expected utility of the lender is both with and without commitment determined by a binding zero profit condition. Therefore the welfare comparison depends on the expected utilities of both types of borrower. Suppose that renegotiation does not increase welfare. Therefore at least one type of borrower obtains in the commitment case greater utility than in the renegotiation case or both types of borrower obtain in the commitment case greater or equal utility than in the renegotiation case. This implies that in the renegotiation case there exists an equilibrium of the Bayesian subgame following the signing of contract in which the lender plays a pure strategy of imposing bankruptcy whenever the borrower announces default. This contradicts the uniqueness of non-degenerate mixing strategy established in Lemma 1. Q.E.D.

My conclusion that renegotiation increases welfare is consistent with the results of Bester's (1994) costly state verification model. On the other hand Scheepens (1995) provides an extension of Bester's model in which he shows that the renegotiation may decrease the entrepreneur's welfare. Scheepens assumes in his model that the bankruptcy court performs an imperfect stochastic test of whether the borrower is able to fulfill his debt payment. In the case the borrower is found guilty of cheating he is punished by a nonpecuniary penalty. If this penalty is not sufficiently high then the possibility of renegotiation decreases the entrepreneur's payoff as compared to the commitment case.

The influence of renegotiation on the size of collateral required of the high type borrower is in general ambiguous in my model. This happens because for the very low success probabilities the collateral required with the renegotiation is higher than without renegotiation while for other parameter values it is lower.

In particular,

$$
\lim _{p_{H} \rightarrow 1}\left(C_{H, N R}^{*}-C_{H, R}^{*}\right)=\frac{(1-\alpha)\left\{\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-I\right\}}{\alpha\left[p_{L} X+\left(1-p_{L}\right)\right]-1}>0 .
$$

My results could be compared to the results of Bester (1994), who analyzes the renegotiation with ex ante symmetric information. Under the restriction corresponding to my assumption (27) of sufficiently high probabilities of success he concludes that neither of the borrowers provides collateral and the repayment is given by a lender's zero profit condition.

## 5. Conclusions

In this paper I have analyzed an adverse selection model in a credit market with the possibility of renegotiation. I have provided a possible explanation for widespread softness of the budget constraint in the case of defaulting entrepreneurs. I have shown that the inclusion of the possibility of renegotiation increases welfare since the utility of the lender remains the same and the utilities of both types of borrowers increase. This means that when the lender and borrower are not able to write and enforce complete contracts allowing for randomization, the soft budget constraint is welfare enhancing. I have also shown that renegotiation does not preclude the use of collateral as a screening device when the lender and the borrower have ex ante asymmetric information about the chances of success of the intended investment project.

My model of SBC features a positive probability of willful default by a successful entrepreneur in equilibrium. This possibility is not considered by Kornai, Maskin and Roland (2003), but it is consistent with empirical studies of banking in transition and developing countries by Dietz, McNaughton and Carlson (1992) and Brixi et al. (2001).

Since the probability of default in the model with renegotiation is a decreasing function of the successful project outcome $X$, I obtain the intuitively plausible result that the equilibrium contract depends on the value of the successful outcome. This is different from the situation of pure adverse selection without renegotiation when the value of the successful outcome does not influence the equilibrium collateral and repayment requirements.

My result of a welfare increase by relaxing the commitment of the lender to impose bankruptcy on the defaulting entrepreneur supports the theoretical argument provided by Manove, Padilla and Pagano (2001) in favor of more lenient bankruptcy procedures. They show in their model of collateral and project screening that the strict enforcement of bankruptcy decreases the efficiency of the credit market. This is consistent with my welfare results. My results also provide additional theoretical support to the empirical studies by Schaffer (1998) and Bonin and Schaffer (2002). They argue that the strict imposition of bankruptcies on defaulting entrepreneurs is not a good way to improve the efficiency of credit markets in transition economies. While my central motivating example was a SBC problem in the transition economies, I have to note that the problem of SBC and commitment in credit contracts in general is relevant to emerging markets, developing economies and developed markets economies too.

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