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Option Pricing: The empirical tests of the Black-Scholes pricing formula and the feed-forward networks

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IES Working Paper:16/2009

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Bibliographic information:
Vlasáková Baruníková, M. (2009). " Option Pricing: The empirical tests of the Black-Scholes pricing formula and the feed-forward networks " IES Working Paper 16/2009. IES FSV. Charles University.

Option Pricing: The empirical tests of the Black-Scholes pricing formula and the feed-forward networks

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April 2009

Abstract:
In this article we evaluate the pricing performance of the rather simple but revolutionary Black-Scholes model and one of the more complex techniques (neural networks) on the European-style S&P Index call and put options over the period of 1.6.2006 till 8.6.2007. Our results on call options show that generally Black-Scholes model performs better than simple generalized feed-forward networks. On the other hand neural networks performance is improving as the option goes deep in the money and as days to expiration increase, compared to the worsening performance of the BS models. Neural networks seem to correct for the well-known Black-Scholes model moneyness and maturity biases.

Keywords: option pricing, neural networks

JEL: C45, G13

Since the famous Black, Scholes, Merton formula substantial progress has been made in the option pricing theory. The aim of this paper is to evaluate the difference between the rather simple but revolutionary Black-Scholes model and one of the more complex techniques (neural networks) on the European-style S&P Index call and put options over the period of 1.6.2004 till 8.6.2007. Our results on call options show that generally Black-Scholes model with historical volatility performs better than simple generalized feed-forward networks. On the other hand neural networks performance is improving as the option goes deep in the money and as days to expiration increase, compared to the worsening performance of the BS model. Neural networks seem to correct for the well-known Black-Scholes model moneyness and maturity biases. Both models have much lower explanatory power for put options compared to calls. Since options are real indicators of the market movements we assign this fact to the expectations of the market participants about the market growth during the evaluated period.

Introduction

Options have been traded for centuries. The first option contract that resembles today option contracts dates back to the seventeenth century (see e.g. *Gibson (1991)*). However the option markets were not regulated and as such were often manipulated untill the estabilishment of the first listed option exchange (Chcago Board Options Exchange; CBOE) in 1973. Since then the option trading recorded unusual expansion in trading volume; variety of option contracts; and geographical coverage. The development of option markets in 90´s is considered to be a *"most striking financial innovation"*[1].

[1] Gibson,R. (1991): *Option valuation. Analyzing and Pricing Standardized Option Contract,* Georg Editeur, Geneva, Switzerland, preface.

The increase in popularity of option trading gave rise to the immense volume of literature on the option pricing theory – see *Bates (2003)* amongst others for comprehensive review and discussion on option pricing techniques and their empirical testing. For great evidence on the more recent contributions see e.g. *Garcia, Ghysels and Renault (2004)*. The option pricing theory defines the relations between the factors that influence the option price and the option price itself in order to formalize the option pricing formulae or mechanism. The theory dates back to the very beginning of the 20-th century, when the French mathematician *Louis Bachelier (1990)* deduced an option pricing formula. It was based on the assumption that the stock price follows a Brownian motion with zero drift. However, the greatest improvement was triggered by the work of *Robert C. Merton (1973) and F.Black and M.Scholes (1973)* in 1970´s. The authors presented the first complete equilibrium option pricing model under the assumption of risk-neutrality. Although their formula violates the reality in number of ways (see the discussion later in the paper), it is still the most well-known option pricing model. It is widely used in practice and constitutes the fundamentals for many subsequent academic researches. Since Black, Scholes and Merton´s work many extensions of the model and vast number of other pricing and hedging techniques have been developed (again, see *Bates (1995)* or *Garcia, Ghysels and Renault (2004)* amongst others). The theory of option pricing is useful not only in the risk management but in the theorethical understanding of the financial markets.

The aim of this work is to test empirically the revolutionary Black-Scholes model and the modern method of generalized feed-forward networks on the S&P 500 Index options during the period of 1.6.2004 to 8.6.2007. The use of the neural networks in finance modeling is growing in the last decades (looking at the Czech literature, see e.g. *Barunik (2008)*). We aim to compare the pricing performance of these methods with emphasis on the **moneyness and days to expiration structure of the options**. We use the S&P 500 Index options covering the period from 1.6.2006 to 8.6.2007 for the out-of-sample performance. In the **Black Scholes** model we relax assumptions of constant volatility and interest rate. We use the 3-month interest rate and the historical volatility as the inputs for the model. Both inputs are used on purpose as they are likely to be available in the daily trading situation. **We compare the BS model to the simple generalized feed-forward networks with one hidden layer and tanh transformation function**. The spot price to strike price ration and time to maturity enter the networks as inputs. We do not use volatility input on purpose as we suppose the neural networks will be able to approximate arbitraly well on their own. Our first hypothesis is that the **BS model will in general outperform the neural networks** since the S&P option index market is said to be the home-ground of the Black-Scholes model (see e.g. *Corrado, Su (1997)*); and the BS model uses the inputs (historical volatility above all) that reflect the real situation in the markets pretty well. **On the other hand we believe that neural networks will be able to correct for some well-known maturity and moneyness biases of the BS model**. We suppose that the BS model will have problem to price correctly the deep ITM or OTM options with increasing days to expiration. We are further interested in the relative performance of the models on the calls and puts.

1.1. Black, Scholes and Merton model

BS model is often set as a benchmark model for the empirical comparison in the option pricing literature (see e.g. *Dumas, Flemming, Whaley (1997), Bakshi, Cao, Chen (1997) or Amilon (2003)* amongst many others). **We set the BS formula as the reference model as well. However we relax the basic assumptions that are known to violate the real conditions in the financial markets – the constant volatility and constant risk-free interest rate**.

Robert Merton, Fischer Black ad Myron Scholes[2] derived the first simple closed-form solution for pricing of the **European-style** call options on non-dividend paying stock. Their formula obtains only five variables (the spot price of the underlying, the exercise price of the option, the risk-free interest rate, the volatility and the time to maturity). The model belongs to the family of the **parametric continuous-time models with closed form solution**. The stock price is assumed to follow the geometric Brownian motion and it is based on the following assumptions: the stock price follows Wiener process such that: $dS = mSdt + s\,Sdz$, where μ and σ are constant (the change in lnS is normally distributed with the mean $\left(\mu - \frac{\sigma^2}{2}\right)T$ and the variance $\sigma^2 T$, e.i. the returns are assumed to be log-normally distributed.); the short selling of securities with full use of proceeds is permitted; there are no transaction costs or taxes; all securities are perfectly divisible; there are no dividends during the life of the derivative and there are no riskless arbitrage opportunities; security trading is continuous; the risk-free rate of interest (r) is constant and the same for all maturities.

[2] See *Merton (1973) and Black and Scholes (1973).*

Using Ito´s lemma; the no-arbitrage condition and the replicating portfolio authors derived following partial differential equation:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}s^{2}S^{2}\frac{\partial^{2} f}{\partial S^{2}} = rf \qquad (1.1.1)$$

The BS pricing formula is the solution of the BS PDE under the boundary conditions $f = max(0, S_t - X)$ for call and $f = max(X - S_t, 0)$ for put when $t = T$ and it is then defined as follows:

$$c = S_0N(d_1) - Xe^{-rT}N(d_2) \qquad (1.1.2)$$

$$p = Xe^{-rT}N(-d_2) - S_0N(-d_1) \qquad (1.1.3)$$

where $d_1 = \frac{\ln\left(\frac{S_0}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, $d_2 = \frac{\ln\left(\frac{S_0}{X}\right)-\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$. $N(x)$ is the cumulative probability distribution function for a standardized normal distribution, c and p are the European-style call and put prices and other variables are familiar.

In our work we use the **historical volatility as an input into the formulae** (see e.g. *Amilon (2003)*). We calculate the volatility from historical S&P Index returns returns such that:

$$\hat{\sigma} = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i-\bar{r})^2}}{\sqrt{\tau}} \qquad (1.1.4)$$

where $n + 1$ is number of observations, r is the natural logarithm of the stock return, τ is the length of interval in years. To ease the computational burden we set n to 21.It would be more accurate to set n equal to the days to maturity of each option. For **3 months risk free interest rate** proxy we use the average of the annualized 7 to 500 days continuously-compounded interest rates for the options with the maturity less than 500; and 500 to 1000 days continuously-compounded interest rates for the options with more than 500 days to expiration. **We have to point out that the BS model is based on the data are that likely to be available in a trading situation and as such is believed to predict the option prices with sufficient accuracy.**

Even though the BS model and its various extensions are widely used amongst practitioners, it is generally assumed that the BS model lies on several highly questionable asumptions. **Assets**

returns are assumed to be log-normally distributed and the stock price follows continuous path through time. In practice the evidence (see e.g. *Bates (1998)*) shows that returns follow leptokurtic distribution. Based on the empirical tests (see *Macbeth and Merville (1979)* or *Dumas, Flemming, Whaley (1996)* amongst others) the cross-sectional properties of option prices indicate another weakness of the model – **the instantaneous volatility is not identical across the strike prices (moneyness) and the maturities**. There is abundant empirical evidence that the BS model exhibits strong pricing biases across both moneyness and maturity – **volatility smiles, skews or smirks**. Volatility smile is anomalous pattern that can be derived by calculating implied volatility of the option across a range of strike prices. The smile is related to the degree to which the option is ITM or OTM. Typically the steepness of the skew decreases with the increasing days to expiration. It is generally agreed on that the volatility smile is the consequence of empirical violations of the BS model assumptions of constant volatility and normality of log-prices. Such evidence is clearly indicative of implicit stock return distributions that are negatively skewed with higher kurtosis. The shape of the smile differs according to the underlying asset. For the discussion on volatility smiles see *Macbeth and Merville (1979), Rubinstein (1985), Dummas, Flemming and Whaley (1996) or Corrado and Su (1997)* amongst others.

Many generalizations and extensions of the BS pricing model have emerged since its publication; attempting to correct for the imperfections. These are based on relaxing of the BS model´s most stringent assumptions as described above. The group of the models includes the constant elasticity of variance models, stochastic volatility models, GARCH models and jump diffusion models amongst others. See e.g. *Hull(2006), Merton(1975), Hull and White (1987), Heston (1993), Heston and Nandi (2000) or Bakshi, Cao, Chen (1997)* for further evidence.

1.2. Generalized feed-forward networks

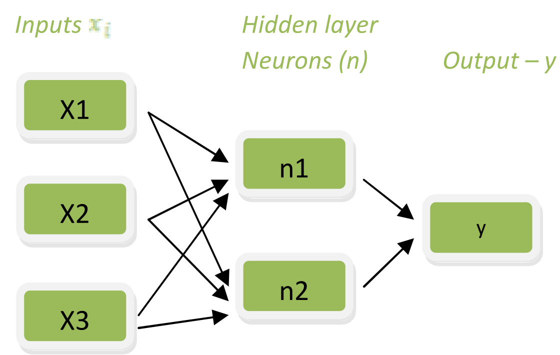
Since most financial theoretical models remain in spite of their complexity misspecified, **semi-parametric** methods seem to be promising tool for pricing and hedging of derivatives. **Neural network (NN)** is data-driven, semi-parametric pricing method in which **data is allowed to determine both the dynamics of the price of the underlying asset and its relation to the price of**

the derivatives. This method imposes minimal assumptions on the price dynamics of the underlying asset as compared to the benchmark BS model. The option pricing formula is believed to be embedded in the noisy market prices. NN is statistic model based on the data processing units. Through processing information in currently available data NN make generalizations for the future events. NN are recently becoming more and more popular with practitioners in financial markets.

We will resort ourselves to the description and use of the **multilayer perceptrons** (MLP or feed-forward) neural networks, which is most widely popular in finance (see e.g. *Anders et al. (1998)*). **Radial basis functions, projection pursuit regression, probabilistic and generalized regression** and others (see e.g. *Hutchinson, Lo and Poggio (1994)*) are the other common methods used for derivatives pricing amongst others.

The **neural networks structure** is described as follows. The **activation functions** (neurons or the transformation functions) are organized in layers. **Input layer** contains the inputs. **Output layer** contain outputs. There can be a number of **hidden layers** between the input and the output layer. Based on the number of hidden layers the network is **single layer or multilayer**. In the hidden layers the input variables are transformed by a special activation function. However, finding the number of hidden layers is more art of experiment than a science. One has to find the correct number of hidden layers so that the function is well approximated and one refrains over-fitting. The activation functions process inputs by forming linear combinations of the neurons and then transform them through logsigmoid; tanh hyperbolic tangent; Gaussian or other transformation functions. We use hyperbolic tangent activation function in this paper.

Figure 1 –Feed-forward neural network with one hidden layer



The neural network is than described by the following equations:

$$y_{k,t} = \gamma_0 + \sum_{k=1}^{k^*} \gamma_k N_{k,t} \tag{1.2.1}$$

$$N_{k,t} = T(n_{k,t}) = \frac{e^{n_{k,t}} - e^{-n_{k,t}}}{e^{n_{k,t}} + e^{-n_{k,t}}} \tag{1.2.2}$$

$$n_{k,t} = \omega_{k,0} + \sum_{i=1}^{i^*} \omega_{k,i} x_{i,t} \tag{1.2.3}$$

There are i^* input variables and k^* neurons; $\omega_{k,i}$ is the weight (coefficient) vector. In the neural network parlance, the variable $n_{k,i}$ is transformed by the transformation function and becomes a neuron $N_{k,i}$ at time t. The set of neurons are then combined linearly with the coefficient vector $y_{k,t}$ and form the final forecast of the output.

Given the network structure and the functional forms for the activation functions the unknown parameters ω are estimated through various techniques, called "**learning algorithms**". The weights (or coefficient) are adjusted during the process of the learning. There exist for example **stochastic gradient descent backpropagation, conjugate gradient or Levenberg-Marquardt learning** algorithms[3]. We use conjugate gradient learning algorithm in this paper (for more details see e.g. *P.D.McNelis (1995))*.

According to *Hutchinson,Lo and Poggio (1994*) amongst others the main advantages of **NN approach over the traditional parametric models are as follows**. NN models **do not rely on restrictive parametric assumptions** like the lognormality of the underlying returns, the sample-path continuity or the constant volatility. They are **robust to the specification errors** that are

[3] See for example *P.D.McNelis (1995).*

common for the parametric models. They are flexible to **encompass a wide range of derivative securities.** They have good out-of-sampling and delta-hedging performance. On the other hand the nonparametric pricing methods are **highly (historical) data-intensive**. These models are thus not appropriate for newly-created derivatives (in case they have no similar counterparts among existing securities or cannot be replicated by a combination of existing derivatives) or thinly-traded derivatives. The "**black-box" criticism** of the neural network approach is often based on the fact that using this approach researches let the data determine the relationships instead of specifying how the inputs affect the prices and supporting the results with the theorethical fundamental. Another problem is that since errors in fitting the option prices are likely to be correlated across options and over time the **statistical tests are difficult to formulate** (see e.g. *Amilon (2003) or Hutchinson et al. (1994)*). *Gradojevic et al. (2007)* further points out that NN may feature the **"recency effect"** where parameters are unduly adopted in favor of the most recent trading data. The crucial question with the NN technique is to **specify the network correctly**. Some authors follow rather heuristic approach to build the architecture, however, *Anders and Korn (1999)* amongst others provide the reader with the simple guide how to specify the network architecture. Authors propose selection strategies that combine a top-down (irrelevant input connections are removed) and bottom-up (the number of hidden units is determined) approach.

In order to identify the type of neural networks we should apply on our data we first **performed preliminary analysis**. We further report only on the best performing neural network. The preliminary analysis showed that **Generalized feed-forward networks[4] (GFN hereafter) with 1 layer, tanh hyperbolic tangent transformation function and the conjugate gradient learning algorithm fits the data best**. *Hutchinson, Lo and Poggio (1994), Amilon (2003), Anders, Korn, Schmitt (1998)* amongst others tested the same type of networks on the option data. It has been showed (see e.g. *Anders, Korn, Schmitt (1998)*) that simple networks can approximate the unknown functions arbitrary well.

Our option pricing formula is then defined (similarly as e.g. *Gradojevic, Gencay, Kukolj (2007)*) as:

$$C_t = \emptyset(S_t, X, \tau) \tag{1.2.4}$$

[4] MLP is a special case of GFN.

where C_t is the price of the option, other inputs are familiar. τ is the days to expiration. Assuming the homogeneity of degree one of the option pricing formula we further sample the data in order to capture the large differences in the values of the inputs and decrease the number of inputs[5] and the complexity of the network, the function is than as follows:

$$\frac{C_t}{X} = \emptyset\left(\frac{S_t}{X}, 1, \frac{\tau}{365}\right) \qquad (1.2.5)$$

For the empirical comparison we **need to split the data in order to train the neural networks**.

1.3. Data description

In this paper we test the performance of the Black-Scholes and Merton option pricing model and the generalized feed-forward networks with 1 layer and tanh transformation function. We test the European-style S&P Index call and put options from the period from 1.6.2004 till 8.6.2007 with emphasis on the moneyness and days to expiration categories.

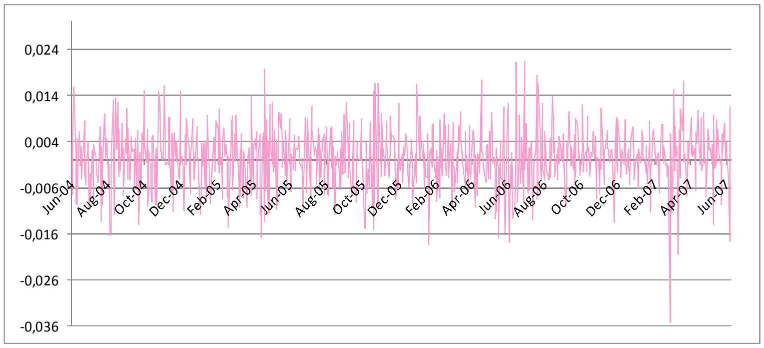
The S&P 500 Index is a capitalization-weighted index of 500 stocks from a broad range of industries. The component stocks are weighted according to the total market value of their outstanding shares. The S&P Index option[6] market is one of the most liquid and active option markets in the United States. Many financial economists have therefore used options on the S&P 500 Index for their empirical analysis (let us mention *Bakshi, Cao, Chen (1997); Heston and Nandi (2000), Dummas, Flemming and Whaley (1996) or Garcia and Gencay (1998)* amongst many others).

The data are obtained from the IvyDB Optionmetrics database. It consists of the daily close S&P 500 Index price adjusted for dividends, the date, the total contract volume and the call/put flag, the option ID, the option expiration date, the daily best bid and best offer. The sample contains 491 819 unique option prices and 761 unique index prices in the period from 1.6.2004 till 8.6.2007. Following the empirical practice we use the midpoint of the bid-offer as the option price. We further use the continuously-compounded interest rate from the IvyDB that is

[5] We as well did not include the interest rate in the network as it worsened its performance due to its complexity
[6] For more information about the S&P 500 Index option visit the CBOE webpage (www.cboe.com).

calculated from the continuously-compounded zero-coupon interest rates at various maturities. The zero-coupon curve is derived from BBA LIBOR rates and settlement prices of CME Eurodollar futures.

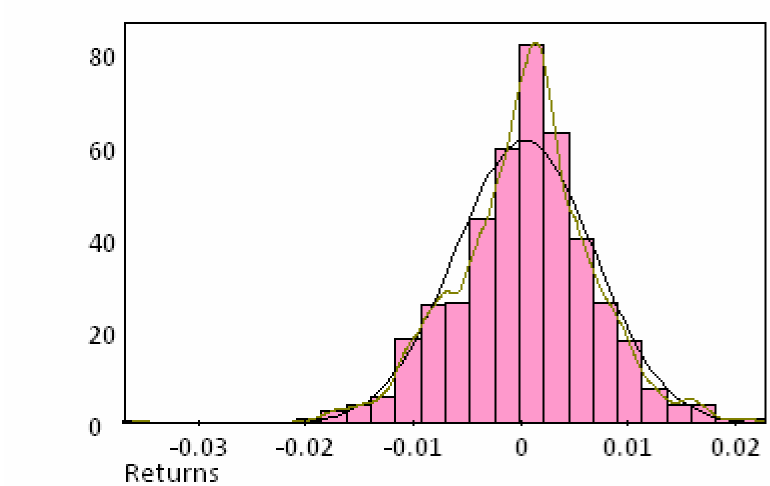
Figure 2 – The Index S&P spot returns



The *figure 1* shows the **S&P Index returns** (calculated as the difference of the natural logarithm of the spot price between the days t and $t-1$). The *figure 2* shows the histogram of the returns covering the whole period. Compared to the normal distribution the returns are leptokurtics with negative skewness; have higher peak and are skewed to the right – the sign of the frequent occurrence of larger and more positive price movements. The yearly subsamples showed similar shape.

Figure 3 – The distribution of S&P 500 returns

The index returns are calculated as follows: $ln(S_t) - ln(S_{t-1})$*. The black line indicates the normal distribution. Actual distribution has higher peak; slightly heavier tail and is a bit skewed to the right. The whole period is covered.*



We follow the work of *Bakshi, Cao, Chen (1997)*[7] to apply the exclusion filters on the dataset. **First,** options with less **than six days to expiration** are excluded in order to prevent the liquidity-related biases. Second, price **quotes lower than 0,375 $** are excluded in order to mitigate the impact of price discreteness on option valuation. Third, the quotes that do not satisfy the no-arbitrage condition: $\mathbf{C \geq \max(0, S_t - X_t)}$ **for calls and** $\mathbf{P \geq \max(0, X_t - S_t)}$ **for puts** are taken out of the sample**.** As we have the index close price adjusted for dividends, we do not need to adjust the S&P 500 spot price series for dividends. The next table shows the **number and type of excluded quotations**; almost 30 % of the data were excluded. The biggest impact has had the quotes-related filter (11.84%). The final dataset consist of 57.56% calls and 44.44% puts. In relative terms puts do not meet the no-arbitrage condition and quote filter more often than calls. Generally options that do not satisfy the no-arbitrage condition are mostly ITM.

[7] Their work is followed by many subsequent researches and their criterions are reasonable.

Table 1 – The dataset of the S&P Index options

The reported values are respectively the number of the calls and the puts in the original datased, excluded due to the exclusion filters and the after-filter dataset. The first column is measured in the units of quotations. The second states the share of the calls and the puts within each category. The third and fourth columns always compare the excluded quote to the original dataset with regard to the type of the option (calls or put options). The sample period extends from 1.6.2004 till 8.6.2007.

		In units	*% within each category*	*% of excluded dataset*	*% of the original dataset*
Original dataset					
	Calls	245783	49.97%	-	-
	Puts	246036	50.03%	-	-
	Total	*491819*	*100.00%*	-	-
Exclusion filters					
TM<6					
	Calls	11261	50.12%	8.16%	4.58%
	Put	11206	49.88%	8.12%	4.55%
	subtotal	*22467*	*100.00%*	*16.29%*	*4.57%*
quote <0.375					
	Calls	18402	31.61%	13.34%	7.49%
	Puts	39814	68.39%	28.86%	16.18%
	subtotal	*58216*	*100.00%*	*42.20%*	*11.84%*
no arbitrage					
	Calls	12446	21.73%	9.02%	5.06%
	deep ITM calls	12186	21.28%	8.83%	4.96%
	Puts	44829	78.27%	32.49%	18.22%
	deep ITM puts	34861	60.87%	25.27%	14.17%
	subtotal	*57275*	*100.00%*	*41.52%*	*11.65%*
	Total	*137958*	-	*100.00%*	*28.05%*
Modified dataset					
	Calls	203674	57.56%	-	82.87%
	Puts	150187	44.44%	-	61.04%
	Total	*353861*	*100.00%*	-	*71.95%*

We further divide the dataset into the categories according to the **moneyness and the time to maturity.** We again follow the work of *Bakshi, Cao, Chen (1997)* and define the moneyness and time to maturity as follows; **the call (put) is** said to be **ITM (OTM) if the spot price of the underlying to strike price ratio** $\frac{S}{X} \geq 1.03$**; ATM if the ratio** $\in (0.97, 1.03)$ and **OTM (ITM) if** $\frac{S}{X} < 0.97$**.** The **short – term maturity option** expires in less than 60 days; **long-term** in more than

180 days and **mid-term** has more than or equal to 60 and less than 180 days to expiration. The following table shows the sample properties of the options.

Table 2 – The sample properties of the S&P Index options.

The reported values are respectively the dollar value of the average bid-offer mid–point price and the number of observations within each category defined according to the moneyness and the days to expiration.

		Calls					Puts			
Moneyness, S/X		Days to expiration					Days to expiration			
		< 60	$[60,180)$	≥ 180	*Subtotal*		< 60	$[60,180)$	≥ 180	*Subtotal*
< 0.94	deep OTM	$0.96	$3.69	$23.57		deep ITM	$140.01	$120.54	$123.52	
		753	4937	21467	*27157*		1497	1462	3887	*6846*
$[0.94,0.97)$	OTM	$2.52	$11.30	$61.21		ITM	$53.72	$59.97	$83.35	
		5750	4615	4936	*15301*		2639	2714	4751	*10104*
$[0.97,1.00)$	ATM	$9.51	$26.05	$86.70		ATM	$25.48	$36.89	$67.05	
		9767	5608	5504	*20879*		8875	5583	5500	*19958*
$[1.00,1.03)$	ATM	$30.70	$48.93	$110.56		ATM	$9.78	$22.49	$53.92	
		9330	4963	5543	*19836*		9334	4963	5536	*19833*
$[1.03,1.06)$	ITM	$61.71	$78.33	$133.20		OTM	$4.35	$14.91	$43.13	
		7942	3980	5105	*17027*		7711	3980	5105	*16796*
≥ 1.06	deep ITM	$221.67	$272.46	$333.07		deep OTM	$1.86	$4.82	$14.99	
		31386	25272	46816	*103474*		13560	17929	45161	*76650*
	Subtotal	*64928*	*49375*	*89371*	*203674*	*Subtotal*	*43616*	*36631*	*69940*	*150187*

The summary statistics is obtained for the **daily average bid-ask mid-point option price**. Note that **the price of the option is increasing with the deepness of the option being in the money** (as there is higher chance for the spot to move in desirable direction) **and increasing days to the expiration** (as its time value increases). The price of call goes from 0.96 $ for short-term deep OTM call to 333.07$ for long-term deep ITM call. Put has narrower boundaries; it goes from 1.86$ to 140$.

To get a sense of the frequently discussed **moneyness and time to maturity biases** the following table shows the BS implied volatility within each category of the S&P Index options sample. We use the implied volatility obtained from the Ivy DB as it uses the standard procedure for calculating the implied volatility; the theoretical BS price is set equal to the averaged best bid-offer mid-point option price and the formula is inverted using the numerical search technique.

Table 3 – The BS implied volatility

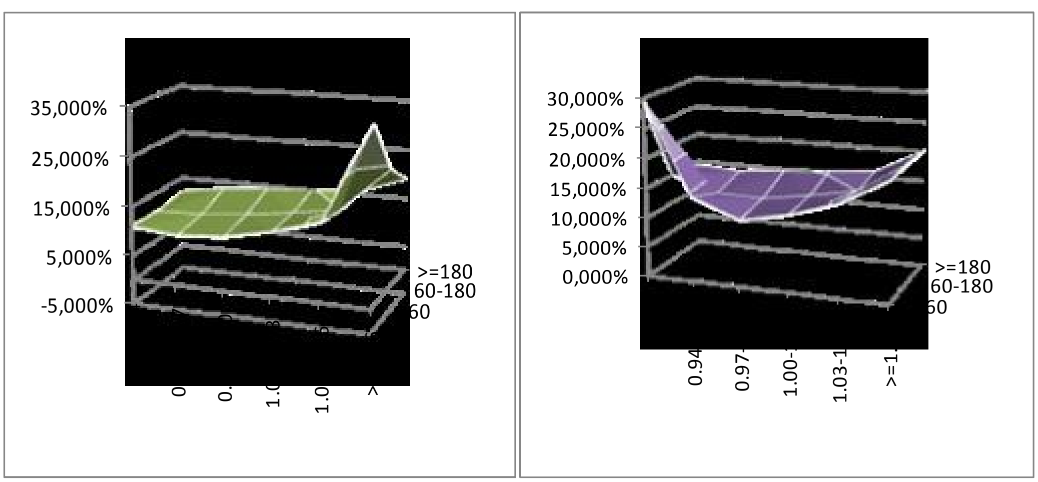
The reported values are the averaged implied volatilities for each of the moneyness and tie to maturity categories for both calls and puts.

		Calls				Puts		
Moneyness, S/X		Days to expiration				Days to expiration		
		< 60	(60,180)	≥ 180		< 60	(60,180)	≥ 180
< 0.94	deepOTM	11,041%	10,905%	12,035%	deepITM	29,437%	14,980%	14,340%
(0.94,0.97)	OTM	10,274%	11,138%	13,516%	ITM	14,242%	12,351%	13,891%
(0.97,1.00)	ATM	10,974%	12,100%	14,343%	ATM	11,298%	12,169%	14,622%
(1.00,1.03)	ATM	13,002%	13,396%	14,931%	ATM	12,942%	13,446%	15,217%
(1.03,1.06)	ITM	15,956%	14,976%	15,633%	OTM	15,527%	15,038%	15,924%
≥ 1.06	deepITM	37,606%	23,894%	19,129%	deepOTM	20,181%	20,304%	20,076%

The equally weighted averaged implied volatility is calculated for each subsample with regard to moneyness and days to expiration. The results confirms the well-known BS bias – **regardless the term to expiration the implied volatility exhibits U-shaped pattern across the moneyness as the option goes from deep OTM to deep ITM**. Calls exhibits rather "sneer-like" pattern, while puts follow the traditional smile. These findings indicate the most severe BS misspricing for the deep ITM option. However the maturity-related bias is not so clear, probably because the long-term option category contains rather nonhomogenous option sample from 180 to more than 500 days to expiration. The following simple graphs display the patterns. The subsample yearly periods showed similar patterns.

Figure 4 – The volatility smirk and smile for call and put options

The green figure shows the volatility smirk for the call option and the purple one the volatility smile for the put options. Moneyness is on the horizontal axis, the volatility on the vertical axis and the days to maturity on the deepness axis.



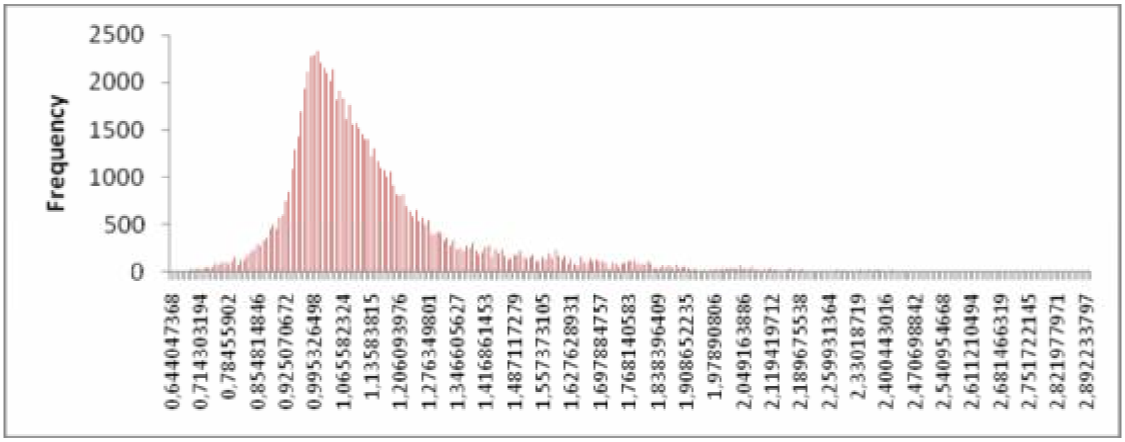
The period from the 1.6.2004 to 1.6.2006 is used for training of the neural networks. The period from 1.6.2006 to 8.6.2007 is used for the model comparison. We have 57 702 unique put prices and 83 716 call prices with 6 to 1094 days to expiration and 141 strike prices ranging from 500 to 2000 in 257 trading days. When we look at the days to expiration, the most frequently occurred are **short-term options** with mode equal to 30. When we look at the moneyness, the trading activity occurs most for **around ATM** options and the distribution is leptokurtic with positive skewness (long right tail). The graph covers puts as calls have very similar characteristics.

Table 4 – Moneyness sample statistics for calls and puts

The moneyness sample statistics for the testing period from 1.6.2006 till 8.6.2007.

	Calls	Puts
Mean	1,143834	1,127632
Standard Error	0,000874	0,000784
Median	1,073549	1,069571
Mode	0,990836	0,990836
Standard Deviation	0,252947	0,188291
Sample Variance	0,063982	0,035453
Kurtosis	5,818722	8,597069
Skewness	2,088777	2,533901
Minimum	0,644047	0,782688
Maximum	2,90004	2,5653

Figure 5 – Moneyness histogram for puts



1.4. Performance evaluation

For the interpretation of the individual model we use the commonly used statistical criteria as described in the table. They are based on the comparison of the model errors - the difference between the actual values of the predicted variable and their estimates.

Table 5 – Criteria for the models comparison

N is the number of observation, y_t is the estimated variable, $\hat{y}_t$ is the estimate of the variable, w is the number of model parameters and $\bar{y}$ is the average of the estimated variable..

Mean squared error (MSE)	$\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2$
R – squared	$1 - \frac{\sum_{t=1}^{N}(y_t - \hat{y}_t)^2}{\sum_{t=1}^{N}(y_t - \bar{y})^2}$
Root Mean Square Error (RMSE)	$\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2}$
Schwartz Information Criterion (SIC)	$\log\left[\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2\right] + \frac{w}{N}\log(N),$
Mean Absolute Percentage Error (MAPE)	$\frac{\sum_{t=1}^{N}\left(\frac{y_t - \bar{y}}{y_t}\right)}{N} * 100$

To assess the out-of-sample performance of the two non-nested models we use the **Diebold-Mariano (DM) test[8].** It tells us how we should determine that the out-of-sample fit of one model is significantly better than the other. **The statistics tests for the null hypothesis of equal predictive ability against the alternative of the non equal predictive ability for the two nonnested models**.

The statistics is as follows:

$$DM_t = \frac{\frac{1}{T}\sum_{t=1}^{T} d_t}{\sqrt{\frac{1}{T}\sum_{\tau=-(T-1)}^{T-1} 1\left\{\frac{\tau}{S(T)}\right\}\hat{\gamma}(\tau)}} \overset{a}{\sim} N(0,1) \qquad (1.3.1)$$

where, $\left\{\varepsilon_{t+h|t}^{i}\right\}_{t=1}^{T}$ and $\left\{\varepsilon_{t+h|t}^{j}\right\}_{t=1}^{T}$ are h-step ahead prediction errors; $d_t = L\left(\varepsilon_{t+h|t}^{i}\right) - L\left(\varepsilon_{t+h|t}^{j}\right)$, $1\left\{\frac{\tau}{S(T)}\right\}$ is the lag window, $S(T)$ is the truncation lag, $\hat{\gamma}(\tau) = \frac{1}{T}\sum_{t=|\tau|+1}^{T}\left(d_t - \bar{d}\right)\left(d_{t-|\tau|} - \bar{d}\right)$ and $L\left(\varepsilon_{t+h|t}^{j}\right)$ is the positive loss function.

1.5. The empirical results

Let us denote the Generalized feed-forward neural networks as NN(1); and the Black Scholes model BS(HIS).

Let as first look at the **Diebold-Mariano test**. Since the *p-values* are very small, **we reject the null hypothesis of equal predictive accuracy of the neural networks and BS** models. As the statistics for **calls** are positive for all quadrants we can say **that the overall predictive ability of BS model is better than for the neural network model**. This confirms our first hypothesis stated in the introduction that BS will outperform the neural networks since the S&P Index option market is based on the BS valuation. However as the test statistics are below the critical value of -1.69 at the 5 % critical level for the BS(HIS) model for puts. **For the puts NN(1) produces predictive errors that are significantly lower than those of BS(HIS)** model.

[8] For more details see for example McNelis (2005), pg.97.

Table 6 The Diebold-Mariano Test

The Diebold-Mariano statistics for calls and puts between the neural networks and Black Scholes, p-value is in parenthesis

NN(1)		Calls	Puts
		BS(HIS)	BS(HIS)
	DM(0)	331.80 (0.000)	-7.12 (0.000)
	DM(1)	321.30 (0.000)	-6.78 (0.000)
	DM(2)	313.30 (0.000)	-6.47 (0.000)
	DM(3)	304.21 (0.000)	-6.23 (0.000)
	DM(4)	298.33 (0.000)	-6.00 (0.000)

The table 7 shows the overall **out-of-sample performance of our models for the whole testing dataset using the common statistical measures**. For **calls** the models fit the data very well according to R-squared statistics (as R^2 approaches one). However, **BS model has better results in almost all categories**, having lower Schwarz Information Criterion and root-mean square error. With mean absolute percentage error neural networks outperform BS model.

Table 7 – The overall out-of-sample performance of the examined model on the option prices.

Average stands for the average of the estimated output, other sample statistics are defined as above.

	Call options					Put options				
	Average	MAPE	RMSE	SIC	R^2	Average	MAPE	RMSE	SIC	R^2
NN(1)	$192.39	191.39	$40.10	7.38	96.79%	$26.20	232.68	$14.45	5.34	65.43%
BS(HIS)	$190.52	223.51	$14.60	5.36	99.35%	$10.13	73.02	$18.78	5.87	41.58%
Sample	$185.22					$21.64				

More interesting is the **put** options performance. This time **BS(HIS)** fails to fit the data at all (R^2 = 41.58%) and neural networks outperform the BS(HIS). Compared to call prices, the **put options have much worse results. We assign this feature to the fact, that during the testing period market participants have been very positive about the growth of the financial markets.** We again must to point out, that very good performance of the BS model is due to the used inputs of volatility and interest rate.

When we have a closer look at the results within the categories we get more detailed picture about the performance of the models. The table 8 shows the statistics for the **call options**. The overall performance of both models with regard to the R^2 has quite high explanatory power with all values being higher than 95 % . **The performance of neural network model is improving as the days to the expiration increase and as option goes ITM; compared to the worsening performance of the BS model. Short-term and mid-term options are priced best with BS model**

in all moneyness categories with regard to all criteria**.** The result is quite natural as the historical volatility is monthly volatility and matches the short term options quite well. For **mid-term option** the BS model still outperforms the neural network model even though BS model performance is worsening (see Figure 6). **For long-term options evidence is rather mixed, depending on the criterion.** Neural networks have better SIC and R^2 criteria as option goes from ATM to deep ITM compared to BS model. When we look in more details on ME, **NN(1) fits the data best** compared BS model in **the ATM and deep ITM category respectively. We have to point out again that the relatively good performance of the BS models is caused by the accuracy of volatility and interest rate inputs that reflect the real condition in the markets and by the fact that S&P options market is said to be the home-ground of BS.**

BS(HIS) over prices the option when it is deep OTM, OTM and deep ITM, other options are underpriced. On the other hand, NN overprice all options except for deep ITM.

Within the put **options we can see different patterns** (see the table 9). **The best performing model based on the goodness of fit statistics, ME, RMSE and SIC criterions is always NN(1)**. **BS(HIS)** model seems to fail to predict the option prices correctly. **Regardless the days to expiration BS model under-price the options while NN over prices the options** (see Figure 8). Both models show pretty small goodness of fit statistics (lower than 75 %). As stated earlier, the worse put results compared to call outcomes may be the result of the expectations of the market participants about the growth of the financial markets.

Table 8 – The neural network and BS model performance according to the moneyness and maturity on the S&P Index Call options.

Moneyness and days to maturity categories are defined as above, 1 stands for short-term, 2 for mid-term and 3 for longer option, av is the averae price and ME stands for mean error. Other statistics are defined above.

		Deep OTM			OTM			ATM			ATM			ITM			Deep ITM		
		1	2	**3**	**1**	2	**3**	**1**	2	**3**	**1**	2	**3**	**1**	2	**3**	**1**	2	**3**
	av	$0.96	$3.68	$29.17	$2.63	$11.98	$70.15	$10.54	$29.43	$98.98	$34.26	$54.61	$126.02	$67.89	$85.52	$151.47	$226.99	$291.18	$376.61
NN(1)	av	$45.22	$43.51	$62.75	$58.37	$62.26	$99.50	$72.32	$77.41	$119.52	$87.83	$93.06	$138.08	$104.30	$110.34	$153.50	$209.06	$269.04	$357.53
BS(HIS)	av	$2.62	$6.48	$31.25	$4.13	$12.87	$69.20	$10.46	$27.71	$97.80	$32.67	$51.25	$125.29	$67.07	$82.92	$153.03	$228.41	$295.72	$397.48
NN(1)	ME	-$44.26	-$39.82	-$33.58	-$55.74	-$50.28	-$29.35	-$61.78	-$47.99	-$20.54	-$53.57	-$38.46	-$12.07	-$36.40	-$24.82	-$2.03	$17.93	$22.14	$19.09
BS(HIS)	ME	-$1.65	-$2.80	-$2.08	-$1.49	-$0.89	$0.95	$0.08	$1.72	$1.18	$1.60	$3.35	$0.73	$0.82	$2.60	-$1.56	-$1.42	-$4.54	-$20.87
NN(1)	RMSE	$44.66	$40.69	$35.38	$56.00	$50.71	$31.04	$62.13	$48.75	$23.11	$54.19	$39.35	$15.16	$37.09	$25.99	$8.02	$35.04	$40.03	$40.65
BS(HIS)	RMSE	$2.96	$6.06	$16.73	$3.85	$6.86	$18.06	$2.96	$7.73	$17.05	$3.90	$7.20	$14.91	$2.39	$4.94	$11.61	$2.24	$5.93	$26.52
NN(1)	SIC	7.64	7.42	7.13	8.06	7.86	6.88	8.26	7.78	6.29	7.99	7.35	5.44	7.23	6.52	4.17	7.11	7.38	7.41
BS(HIS)	SIC	2.28	3.63	5.64	2.71	3.87	5.81	2.86	4.11	5.69	2.73	3.97	5.64	1.76	3.21	4.92	1.62	3.56	6.56
NN(1)	Rsqr	94.13%	94.98%	95.02%	94.13%	91.45%	93.40%	87.37%	90.26%	94.06%	87.19%	91.01%	95.38%	90.11%	93.33%	97.27%	93.88%	95.79%	97.54%
BS(HIS)	Rsqr	99.97%	99.89%	98.89%	99.96%	99.84%	97.77%	99.94%	99.75%	96.77%	99.93%	99.70%	95.53%	99.96%	99.76%	94.28%	99.97%	99.91%	98.95%

Table 9 - The neural network and BS model performance according to the moneyness and maturity on the S&P Index Put options.

Moneyness and days to maturity categories are defined as above, 1 stands for short-term, 2 for mid-term and 3 for longer option, av is the average price and ME is the mean error, other measures are explained earlier in the chapter.

		Deep ITM			ITM			ATM			ATM			OTM			Deep OTM		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
	av	$23.17	$22.63	$21.49	$23.01	$22.01	$21.64	$22.49	$21.16	$22.28	$21.46	$21.10	$22.29	$21.40	$21.78	$20.69	$21.40	$21.48	$21.69
NN(1)	av	$24.85	$26.97	$25.51	$27.70	$26.14	$25.29	$27.36	$26.13	$26.57	$25.42	$24.81	$26.89	$25.97	$26.88	$25.04	$26.18	$26.41	$26.32
BS(HIS)	av	$13.71	$11.20	$10.34	$11.21	$11.61	$10.42	$10.58	$10.35	$10.63	$9.95	$10.25	$10.35	$9.99	$10.78	$10.08	$9.42	$10.26	$9.93
NN(1)	ME	-$1.67	-$4.34	-$4.03	-$4.69	-$4.12	-$3.66	-$4.87	-$4.97	-$4.29	-$3.96	-$3.71	-$4.60	-$4.57	-$5.09	-$4.36	-$4.78	-$4.94	-$4.63
BS(HIS)	ME	$9.47	$11.43	$11.15	$11.80	$10.40	$11.22	$11.91	$10.81	$11.65	$11.51	$10.85	$11.94	$11.42	$11.00	$10.61	$11.98	$11.22	$11.76
NN(1)	RMSE	$11.91	$11.99	$11.91	$11.55	$11.84	$11.39	$12.41	$11.81	$11.43	$12.21	$12.21	$11.72	$11.86	$12.13	$11.39	$12.20	$12.44	$11.75
BS(HIS)	RMSE	$12.44	$15.47	$15.46	$15.57	$14.28	$13.98	$12.44	$15.08	$14.71	$15.84	$15.76	$15.71	$15.64	$15.25	$13.78	$16.02	$15.90	$15.65
NN(1)	SIC	5.00	4.98	4.96	4.90	4.95	4.87	5.04	4.94	4.88	5.01	5.01	4.93	4.95	5.00	4.87	5.00	5.04	4.93
BS(HIS)	SIC	5.15	5.50	5.48	5.51	5.34	5.29	5.55	5.44	5.39	5.54	5.53	5.48	5.51	5.47	5.26	5.55	5.54	5.50
NN(1)	Rsqr	73.43%	66.66%	68.45%	73.43%	67.84%	66.99%	65.72%	61.96%	68.22%	67.91%	69.33%	66.97%	63.89%	65.80%	63.36%	63.41%	63.65%	65.16%
BS(HIS)	Rsqr	71.04%	44.48%	46.79%	41.43%	53.23%	50.31%	43.22%	38.00%	47.38%	46.07%	48.92%	40.70%	37.24%	45.95%	46.42%	36.95%	40.59%	38.20%

Figure 6 – The actual and models predicted prices for the deep in-the-money call option

The graph shows the actual and models predicted prices for the deep ITM (with moneyness around 1,6) call option. As the days go from 60 (mid-term option) to 193 (long term option) the predictive capability of the neural networks is improving while the BS model performance is worsening.

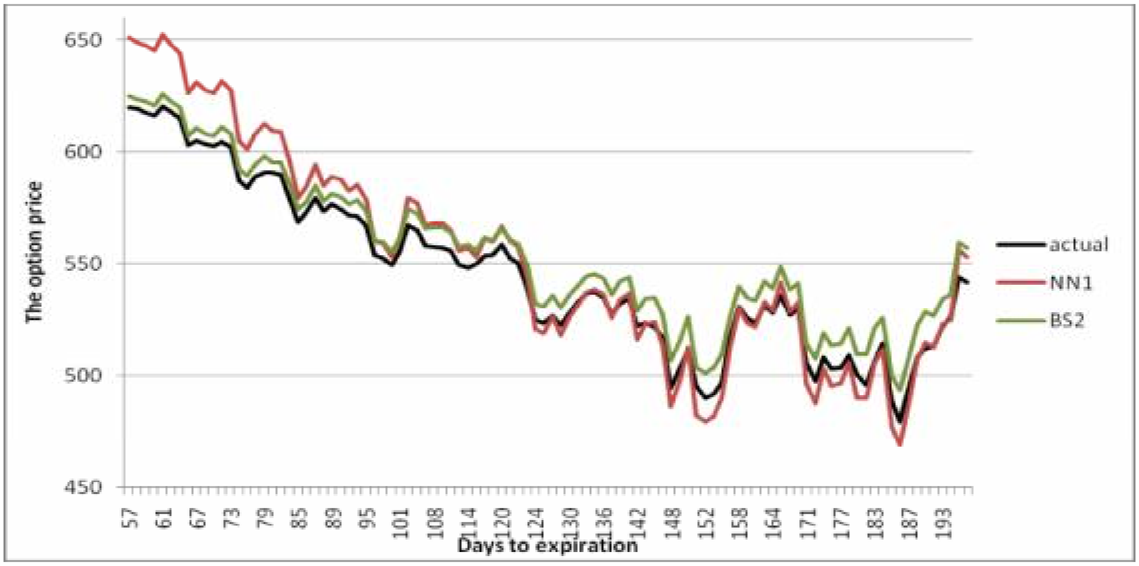


Figure 7 - The actual and models predicted prices for the deep ITM short-term call option

The graph shows the actual and models predicted prices for the deep ITM (with moneyness around 1,74) and short-term (9 to 40 days to expiration) call option. BS model predicts the actual prices very well, while neural networks over-price the option.

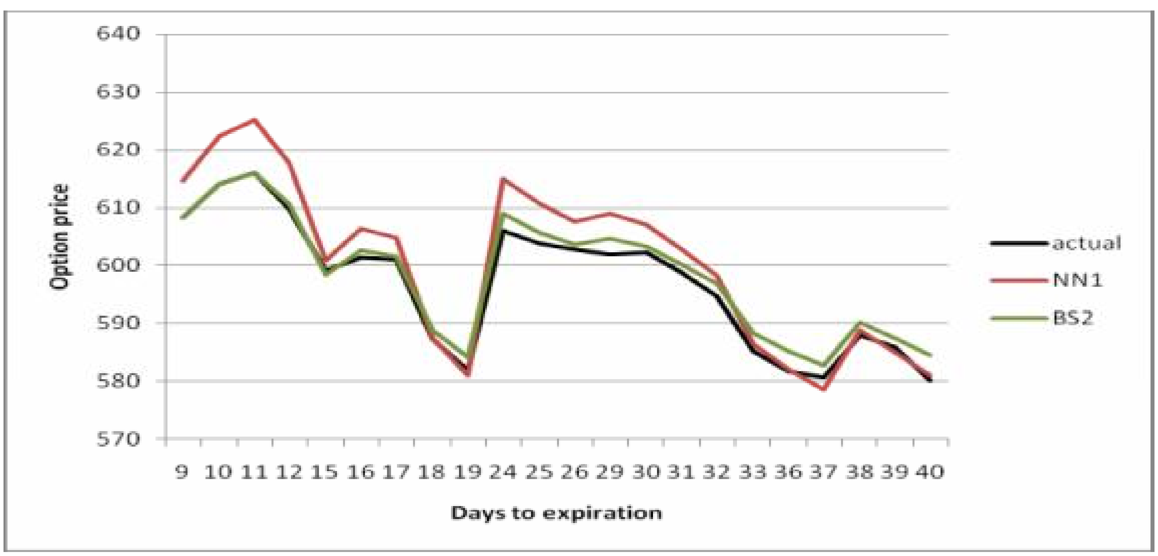
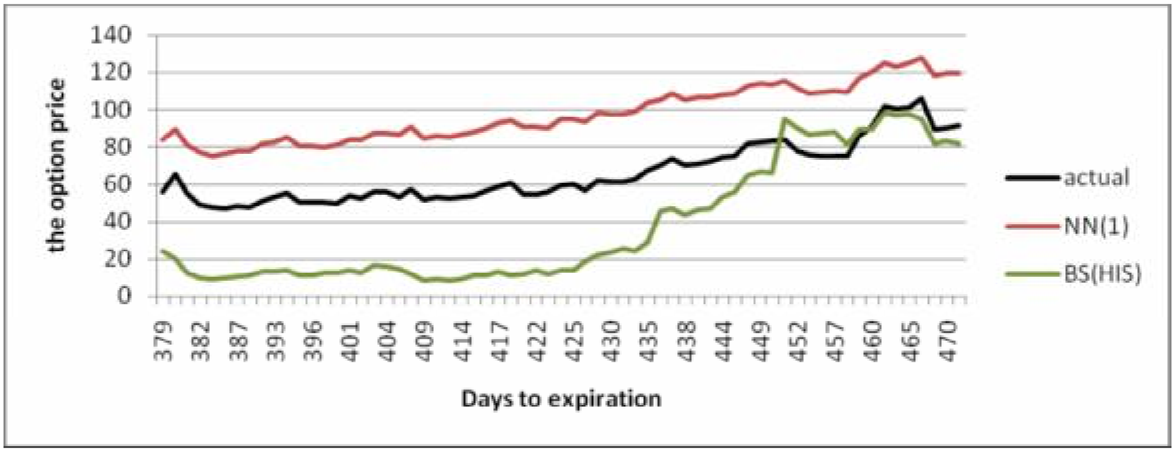


Figure 8 – The Actual and model predicted prices for the ATM long-term put option

The graph shows the actual and models predicted prices for the ATM (with moneyness around 1,02) and long-term (379-470) put option. BS model under-prices the option and neural network overprices it.



The above mentioned results confirm the conclusions derived by *Corrado and Su (1997)* amongst others about the **existence of moneyness and maturity biases of the BS model. BS models performance is worsening as time to maturity increases and the moneyness deepens compared to the improving NN performance on call options**. To correct for the weaknesses of the networks and to improve their overall performance we should use better volatility estimate as input; for example GARCH model as proposed amongst other by *Ritchken and Trevor (1999)* or to simple use the BS implied volatility per each day and moneyness caregory from the previous day. Moreover we can use more complex Levenberg-Marquart learning algorithm or train networks separately in each category.

There are many other challenges for the future work. We can compare the performance of the more challenging **GARCH or stochastic volatility models or work on the better neural networks identification**. We have omitted the **hedging analysis** in this work as hedging and pricing performance may differ substantially in the model comparison. Since the **American-style** derivatives are more frequently traded than European-style derivatives one should compare the performance of the neural networks and techniques for pricing the American-style options.

1.6. Conclusion

Since famous Black-Scholes option pricing formula immense volume of literature on the option pricing was issued. Soon after the model was proposed it was realized that it lies on the highly unrealistic assumptions like the lognormality of the asset returns and constant volatility or interest rate. It therefore exhibits strong pricing biases. It moreover cannot price more complex contingent claims (e.g. American-style derivatives).

The paper is devoted to the **empirical comparison** of the complex neural networks and simple Black-Scholes model. **We evaluate the performance of the generalized feed-forward neural networks and Black-Scholes model on the European-style S&P Index call and put options covering the out-of-sample period 1.6.2006 till 8.6.2007**. We use the historical volatility as an input to the BS model. In order to make the BS model more competitive we use the data that are likely to be available in common trading situation instead of the constant volatility and interest rate since these assumptions are known to violate the real situation in the market. We compare them **to the Generalized feed-forward networks with 1 layer, tanh hyperbolic tangent transformation function and the conjugate gradient learning algorithm model.**

The explanatory power of both models is sufficiently high for the **call options**. **The overall performance of the Black-Scholes model with historical volatility dominates the neural networks. However as the option goes long-term and deep in-the-money the neural networks improve their performance**. These results acknowledge the well-known BS maturity and moneyness biases (known as volatility smiles).

For **put options the explanatory power of the option pricing models is rather low, however the neural networks always perform better than BS model.** Regardless the days to expiration BS model under-prices the options while neural networks over-price the options. We assign the worse results of models performance for puts (compared to calls) to the expectations of the market participants about the future growth of the markets during the evaluated period.

For the future work the empirical issues addressed in this paper can also be reexamined using data from American-style options, individual stock options or other more complex derivatives. Moreover, hedging performance may be evaluated with respect to each model.

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