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Fairness and Squareness: Fair Decision Making Rules in the EU Council?

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Fairness and Squareness: Fair Decision Making Rules in the EU Council?

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Abstract:
The concept of fair representation of voters in a committee representing different voters' groups, such as national representations in union of states, is discussed. This concept, introduced into discussion about voting rights in the Council of European Union in 2004, was narrowed to proposal of distribution of voting weights among the member states proportionally to square roots of population. Such a distribution should guarantee the same indirect voting power to each EU citizen, measured by Penrose-Banzhaf index of voting power. In this paper we attempt to clarify this concept.

Keywords: Council of Ministers, indirect voting power, Penrose-Banzhaf power index, Shapley-Shubik power index, square root rule, simple voting game

JEL: C71, D72, H77

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1. Introduction

In the late spring of 2004 the following draft of open letter of European scientists to the governments of the EU member states was distributed in European academic community:

> *In the last few years there has been an intense discussion on the voting procedure in the Council of Ministers of the European Union. With 25 member states (and two more in the near future) it is not a simple task to make reliable judgements on the implications of the various voting systems that have been suggested.*
>
> *We the undersigned wish to draw the attention of EU Governments to the fact that scientific methods can be used and need to be used to analyse, understand and design complex voting systems. In particular:*
>
> *1) From a scientific point of view there are obvious drawbacks to the systems of voting in the European Council discussed so far. The experts on voting theory agree that the Treaty of Nice gives too much power to a number of countries while others obtain less power than appropriate. On the other hand, the draft European Constitution assigns too much power to the biggest and the smallest states in a systematic way, while the middle size countries do not get their due share of influence (see the tables attached). Moreover, the Nice system will be extremely ineffective due to its high quotas.*
> *2) The 'compromises' proposed recently to change the quota in the draft Constitution either to 65% of the population and 55% of the states or to 55% of the population and 55% of the states make the situation for several countries even worse than in the draft Constitution. As can be shown by mathematical analysis, it is not the quotas that are mainly at fault, but rather the system of proposed weights.*
> *3) The basic democratic principle that the vote of any citizen of a Member State ought to be worth as much as for any other Member State is strongly violated both in the voting system of the Treaty of Nice and in the rules given in the draft Constitution. It can be proved rigorously that this principle is fulfilled if the influence of each country in the Council is proportional to the square root of its population. This is known as 'Penrose's Square Root Law'. Such a system may be complemented by a simple majority of states.*
> *4) A voting system that obeys the Square Root Law, i.e., which gives equal power to all citizens, is easily implemented. It is representative, objective, transparent, and effective. Such a system was proposed by Swedish diplomats already in 2000, and recently endorsed in a number of scientific articles.*
>
> *We urge our politicians to take into consideration the contribution of the scientific community to this issue. We are highly concerned that any system implemented without due regard to the scientific analysis of voting power may become a major drawback to a democratic development in the European Union.*

Open letter was originally signed by the group of nine distinguished scientists from six EU countries, calling themselves "Scientists for a democratic Europe", later cosigned by 38 other colleagues, and submitted to the governments of member states and to Commission[1]. In this paper we want to explore the statements.

The basic idea of the proposal supported by the open letter is the following concept of "fairness": *If the European Union is a union of citizens, then it is fair when each citizen (independently on her national affiliation) exercises the same influence over the union issues. It is achieved when voting weight of each national representation in Council of Ministers is proportional to the square root of population.*

So called square root rule is attributed to British statistician Lionel Penrose (1946) and is closely related to indirect voting power measured by Penrose-Banzhaf power index. Different aspects of square root rule are analysed in Felsenthal and Machover (1998, 2007), Laruelle and Widgrén (1998), Baldwin and Widgrén (2004), Plechanovová (2004), Słomczyński and Życzkowski (2006, 2007), Hosli (2008) and Leech and Aziz (2008).

Concept of indirect voting power is based on the following rather artificial construction: Assume n districts (e.g. regions) with different size of population (voters), represented in a super-regional committee that decides different agendas relevant for the whole entity. Each district representation in the committee has some voting weight (number of votes). Decision making process is performed by series of referenda in each district and districts' representations in the committee are voting according results of referenda. In each district an individual citizen has the same voting weight (one vote) that provides him with a voting power (each citizen from one district has the same voting power). Also each district representation has some voting power in the committee that follows from its voting weight. Then indirect voting power of a citizen from particular district is given by product of her voting power in local referenda and voting power of her representation in the committee. The representation of districts in the committee is considered fair, if each citizen has the same indirect voting power independently of the district he belongs to.

2. Model

Let N be a set of members of a committee and $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ be a nonnegative vector of weights (e.g. votes or shares) of committee members. A subset $S \in N$ of committee members voting uniformly (YES or NO) is called a voting configuration. Let us denote by $w(S) = \sum_{i \in S} w_i$ .Voting rule is defined by quota q, satisfying

$$\frac{1}{2}\sum_{i\in N} w_i < q \le \sum_{i\in N} w_i$$

(quota q represents minimal total weight necessary to approve the proposal).The triple [N, q, **w**] is called a simple weighted committee. Voting configuration S is called a winning one if $w(S) \ge q$ and a losing one in opposite case.

[1] The letter (including added tables) and list of its signatories see e.g. at the following web address: http://www.esi2.us.es/~mbilbao/pdffiles/letter.pdf

Voting power analysis seeks an answer to the following question: Given a simple weighted committee, what is an influence of its members over the outcome of voting? Voting power of a member i is a probability that i will be decisive in the sense that such situation appears in which she would be able to reverse the outcome of voting by reversing her vote. To define a particular power measure means to identify some qualitative property (decisiveness) whose presence or absence in voting process can be established and quantified (e.g. Nurmi (1997)). Generally there are two such properties related to committee members' positions in voting, that are being used as a starting point for quantification of voting power: swing position and pivotal position of committee members.

Let S be a winning configuration in a simple weighted committee [N, q, **w**]. A member $i \in S$ has a swing in configuration S if w(S) ≥ q and w(S\{i}) < q. Assuming all configurations equally likely, it makes sense to evaluate a priori voting power of each member of the committee by probability to have a swing. This probability is measured by absolute Penrose-Banzhaf power index

$$\Phi_i^{PB}(N,q,\mathbf{w}) = \frac{s_i}{2^{n-1}}$$

(where s_i is the number of swings of the member i and 2^{n-1} is the number of coalitions with i as a member). To compare relative power of different members of the committee, the relative form of Penrose-Banzhaf power index is used:

$$f_i^{PB}(N,q,\mathbf{w}) = \frac{s_i}{\sum_{k\in N} s_k}$$

Let the numbers $1,2,...,n$ be the fixed names of committee members and $(i_1, i_2,..., i_n)$ be a permutation of the members of the committee, and let us assume that member k is in a position r in this permutation, i.e. $k = i_r$. A member k of the committee is in a pivotal situation (has a pivot) with respect to a permutation $(i_1, i_2,..., i_n)$, if $w(i_1,i_2,\ldots,i_{r-1}) < q$ and $w(i_1, \ldots,i_{r-1},i_r) \geq q$. what implies $w(i_{r+1},i_{r+2},\ldots,i_n) < 0$ and $w(i_r,i_{r+1},\ldots,i_n) \geq 1$. Hence, outcome of voting will be in this case identical with the vote of member $k=i_r$, "yes" if she votes "yes" and "no" if she votes "no". Assume that a strict ordering of members in a given permutation expresses an intensity of their support (preferences) for a particular issue in the sense that, if a member i_s precedes in this permutation a member i_t, then support by i_s for the particular proposal to be decided is stronger than support by i_t. One can expect that the group supporting the proposal will be formed in the order of positions of members in the given permutation. If it is so, then the member k will be in situation when the group composed from preceding members in the given permutation still does not have enough of votes to pass the proposal, and a group of members placed behind her in the permutation has not enough of votes to block the proposal. The group that will manage his support will win. Member in a pivotal situation has a decisive influence on the final outcome. Assuming many voting acts and all possible preference orderings equally likely, under the full veil of ignorance about other aspects of individual members' preferences, it makes sense to evaluate an a priori voting power of each committee member as a probability of being in pivotal situation. This probability is measured by the SS-power index:

$$\Phi_i^{SS}(N,q,\mathbf{w}) = \frac{p_i}{n!}$$

(p_i is the number of pivotal positions of the committee member i, and $n!$ is the number of permutations of the committee members, i.e., number of different orderings of n elements). From

$$\sum_{i=1}^{n} p_i = n!$$

it follows, that

$$f_i^{SS}(N,q,\mathbf{w}) = \frac{p_i}{\sum_{k\in N} p_k} = \Phi_i^{SS}(N,q,\mathbf{w})$$

(i.e. relative SS-power index is equal to absolute one).

3. Penrose-Banzhaf indirect power

Let N be the set of districts (regions) and p_i population of district i, [N, q, **w**] be a committee of districts and w_i be a weight of the district i in the committee. Consider a randomly selected "yes-no" issue and suppose that people in each district decide their approval or disapproval by referendum (each citizen has one vote). For simplicity assume the number of voters participating in referendum in district i is equal to the number of district's population, and the quota (number of votes required to approve proposal) is equal $\frac{1}{2} < m_i < p_i$.

Number of all winning configurations in district i:

$$\binom{p_i}{m_i} = \frac{p_i!}{(p_i - m_i)!\,m_i!}$$

Number of winning configurations without participation of a particular citizen:

$$\binom{p_i - 1}{m_i} = \frac{(p_i - 1)!}{(p_i - 1 - m_i)!\,m_i!} = \frac{p_i!(p_i - m_i)}{(p_i - m_i)!\,m_i!\,p_i}$$

Number of winning configurations a particular citizen is a member of:

$$\binom{p_i}{m_i} - \binom{p_i - 1}{m_i} = \binom{p_i}{m_i}\left[1 - \frac{(p_i - m_i)}{p_i}\right] = \binom{p_i}{m_i}\frac{m_i}{p_i}$$

This gives number of swings of a single citizen in district referendum (changing her vote she will change the outcome). Using a probability of swing as a measure of voting power (assuming that all voting configurations are equally probable), we obtain Penrose-Banzhaf absolute power index of a single citizen in district i as

$$\Phi_{i(citizen)}^{PB} = \frac{1}{2^{p_i - 1}}\binom{p_i}{m_i}\frac{m_i}{p_i}$$

The voting power of each citizen is decreasing function of the size of population and not increasing function of the quota.

If we assume simple majority quota (proposal supported by half of voters plus one) equal (for large p_i)

$$m_i = \text{int}\left(\frac{1}{2}p_i + 1\right) \approx \frac{1}{2}p_i$$

(least integer greater than pi/2), then the number of cases in which the average citizen of district i has a swing (the outcome of district referendum will be identical with her vote) is equal to

$$\frac{1}{2}\left(\begin{array}{c} p_i \\ \left(\frac{p_i}{2}\right) \end{array}\right) = \frac{1}{2}\frac{p_i\,!}{\left(\frac{p_i}{2}\right)!\left(\frac{p_i}{2}\right)!} = \frac{1}{2}\frac{p_i\,!}{\left(\left(\frac{p_i}{2}\right)!\right)^2}$$

and probability to have a swing is

$$\Phi^{PB}_{i\,(citizen\ in\ district\ referendum)} \approx \frac{1}{2^{p_i}}\frac{p_i\,!}{\left(\left(\frac{p_i}{2}\right)!\right)^2}$$

(power of a citizen of i, absolute Penrose-Banzhaf index). Using Stirling’s formula

$$n! \approx \frac{n^n}{e^n}\sqrt{2pn}$$

(Felsenthal and Machover (1998)), for sufficiently large p_i we obtain approximation

$$\Phi^{PB}_{i\,(citizen\ in\ unit\ referendum)} \approx \sqrt{\frac{2}{p\ p_i}}$$

(proof see Laruelle and Widgren (1998)). The larger size of population in district i, the smaller is individual citizen Penrose-Banzhaf power in referendum-type district voting.

Following the same reasoning we can estimate the voting power of a citizen in a global (all districts) referendum as

$$\Phi^{PB}_{(citizen\ in\ all\ districts\ referendum)} \approx \sqrt{\frac{2}{p\sum_{i\in N} p_i}}$$

If the districts’ representations in the committee of districts are voting in each issue according to results of districts’ referenda and $\Phi_{i\ (district\ representation)}$ is the Penrose-Banzhaf power of the district i representation in the districts’ committee, then

$$\Phi^{PB}_{i\,(citizen\ in\ district\ committee)} = \Phi^{PB}_{i\,(district\ representation)}\Phi^{PB}_{i\,(citizen\ in\ district\ referendum)} = \Phi^{PB}_{i\,(district\ representation)}\sqrt{\frac{2}{p\ p_i}}$$

is the i-th district average citizen (indirect) power in the districts’ committee decision making.

To guarantee equal indirect power of citizens from different districts in the districts’ committee, it must hold

$$\Phi^{PB}_{i}\sqrt{\frac{2}{p\ p_i}} = a$$

for all i, where α is a positive constant. It holds if

$$\Phi^{PB}_{i} = a\sqrt{\frac{p\ p_i}{2}}$$

i.e. if voting power of member states is proportional to the square roots of population. It happens when relative voting power of districts is equal to

$$f^{PB}_{i} = \frac{\sqrt{p_i}}{\sum_{i\in N}\sqrt{p_i}}$$

Finally, to fix indirect voting power of an average citizen we require that it is equal to her voting power in a global referendum of citizens of all districts, what means that voting power of representation of district i in the committee should be equal

$$\Phi_i^{PB} = \sqrt{\frac{2}{p\sum_{i\in N} p_i}}\sqrt{\frac{p\,p_i}{2}} = \sqrt{\frac{p_i}{\sum_{i\in N} p_i}}$$

(probability that representation of district i will have a swing in the districts' committee voting).

Substituting European Union member states for districts and Council of Ministers for districts' committee, we can apply the concept of "fair representation" on distribution of member states' voting weights in the Council.

To illustrate concepts discussed above let us consider a simple example of a hypothetical union of four member states A, B, C, and D (see Table 1). Data provided in Table 1 are not based on calculation of member states' PB-power indices with some particular voting quota in hypothetical union Council, they indicate, having the square root estimations of citizens' power in member states, what absolute national voting power in the Council guarantees equals indirect power of citizens of different member states, measured by union citizen power in global referendum.

Table 1
Fair distribution of power in hypothetical union

State	Population p_i	Square root of population $\sqrt{p_i}$	Share of population $\frac{p_i}{\sum_{i\in N} p_i}100$	Share of square root population $\frac{\sqrt{p_i}}{\sum_{i\in N}\sqrt{p_i}}100$	Citizen national absolute PB power	Fair absolute PB power In the Council	Fair relative PB power in the Council	Fair citizen indirect power
A	100000000	10000	56,18	41,67	0,000079788	0,749531689	0,4167	0,000059804
B	49000000	7000	27,53	29,17	0,000113984	0,524672182	0,2917	0,000059804
C	25000000	5000	14,04	20,83	0,000159577	0,374765844	0.2083	0,000059804
D	4000000	2000	2,25	8,33	0,000398942	0,149906338	0.0833	0,000059804
Σ	178000000	24000	100	100	0,000059804	1,798876054	1,0000	

Source: own calculations
p_i denotes population of member state i

4. Voting weights, quota and voting power

It was rigorously proved how fair distribution of voting power in the Council should look like to guarantee equal indirect voting power of all European citizens (providing system of referenda is considered a mechanism of decision making). But the open letter says something more: *It can be proved rigorously that this principle is fulfilled if the influence of each country in the Council is proportional to the square root of its population. This is known as 'Penrose's Square Root Law'.*

There is still one problem to be solved: what allocation of voting weights among member states leads to proportionality of Penrose-Banzhaf power to the square roots of population? Supporters of square root rule are proposing to allocate the weights in the Council proportionally to the square root of population, assuming that in committees with large number of members the distribution of weights is a good proxy of voting power. But a priori voting power seldom reflects exact distribution of voting weights. If [N, q, **w**] is a simple weighted committee and Φ[N, q, **w**] is a vector of power indices of its members, then usually Φ[N, q, **w**] ≠ α**w.**

Originally it was assumed that voting weights proportional to square roots of population together with simple majority quota provide solution of the problem. But it appeared that it is generally not the case. Being aware of this problem, Słomczyński and Życzkowski (2006) formulated the following minimization problem:

Minimize sum of square residuals between the normalized Penrose-Banzhaf power indices and voting weights defined as proportional to the square roots of population according to the quota q

$$s^2(q) = \sum_{i \in N} \left(f_i(N, q, \sqrt{\mathbf{p}}) - \frac{\sqrt{p_i}}{\sum_{k \in N} \sqrt{p_k}} \right)^2$$

for q ∈ (50, 100] in percentage of total weight. They used heuristic and found approximation of optimal quota q ≈ 61.4% for the EU of 27. So, the final proposal, known as "Jagiellonian Compromise", reads as follows: "*The voting weight of each member state is allocated proportionally to the square root of its population, the decision of the Council being taken if the sum of weights exceeds a (certain) quota*" (Słomczyński and Życzkowski (2006)), setting the quota equal to 61,4% of the sum of square roots of population in the member states of the EU. Let us call it SZ distance from equal indirect power.

Later they improved their quota estimation and provided general quota approximation formula

$$q \approx \frac{1}{2}\left(\sqrt{\sum_{i \in N} p_i} + \sum_{i \in N} \sqrt{p_i} \right)$$

minimizing distance between vector of weights and vector of power indices (see Słomczyński and Życzkowski (2007), Leech and Aziz (2008)). Let us call this approximation SZ optimal quota.

Berg and Holler (1986) provide the following property of simple weighted committees: Let [N, Q, **w**] be a family of committees with the same weights **w** and set of different quotas Q = {q_1, q_2, …, q_m} such that 0,5 < q_k ≤ 1, φ is a probability distribution over Q where φ_k is a probability with which a random mechanism selects the quota q_k and Φ_{ik}(N, q_k, **w**) be a power index in the committee [N, q_k, **w**] with a quota q_k ∈ Q, then

$$\overline{\Phi}_i(N, Q, \mathbf{w}) = \sum_{k: q_k \in Q} \Phi_{ik}(N, q_k, \mathbf{w}) j_k$$

is an expected power of the member i in the randomized committee [N, Q(φ), **w**]. For any vector of weights there exist a finite set Q and a probability distribution φ such that

$$\overline{\Phi}_i(N, Q(j), \mathbf{w}) = \sum_{q_k \in Q} \Phi_{ik}(N, q_k, \mathbf{w}) j_k = a\, w_i$$

Randomized voting rule Q(φ) leads to strictly proportional power.

For any simple weighted committee there exists a finite number of different intervals $(\gamma_0, \gamma_1], \ldots, (\gamma_{m-1}, \gamma_m]$ such, that for any quota from a particular interval $(\gamma_{k-1}, \gamma_k]$ the sets of winning and losing voting configurations are the same and for quotas from different intervals the sets of winning and losing configurations are different. These intervals are called quota intervals of stable power.

From final number of quota intervals of stable power it follows that there exists exact solution to Słomczyński and Życzkowski minimization problem (SZ optimal quota)

$$q^* = \arg\min_j \sum_{i\in N}\left(f_i(N, q_j, \sqrt{\mathbf{p}}) - \frac{\sqrt{p_i}}{\sum_{k\in N}\sqrt{p_k}} \right)^2$$

where j = 1, 2, …, m, m is the number of intervals of stable power and $q_j \in (\gamma_{j-1}, \gamma_j]$. Quota q* provides only good approximation, it does not guarantee the exact proportionality of power indices and weights. Moreover, this property is not related specifically to square root rule and holds for any reasonable power index (e.g. Shapley-Shubik) as well. If $(\gamma_{t-1}, \gamma_t]$ is the quota interval of stable power minimizing distance between vector of power indices and vector of weights and approximation

$$q \approx \frac{1}{2}\left(\sqrt{\sum_{i\in N} p_i} + \sum_{i\in N}\sqrt{p_i} \right)$$

is correct, then it must hold

$$g_{t-1} < \frac{1}{2}\left(\sqrt{\sum_{i\in N} p_i} + \sum_{i\in N}\sqrt{p_i} \right) \le g_t$$

To illustrate concepts discussed above let us consider a simple example of a union of four member states A, B, C, and D (basic data see in Table 1).

Let us check relation between weights and quota for our hypothetical union from Table 1. Assume that weights in the Council are equal to square roots of population $\sqrt{p_i}$ and quota is fixed on the level

$$q \approx \frac{1}{2}\left(\sqrt{\sum_{i\in N} p_i} + \sum_{i\in N}\sqrt{p_i} \right) = 18671$$

In Table 2 we provide absolute and relative Penrose-Banzhaf power indices of national representations using this quota, and resulting indirect voting power of citizens of each member state and union citizen. Euclidean distance is used to compare calculated indirect power with theoretically equal one. The “rigorously proved” square root rule remains to be problematic even if we accept “national referenda” mechanism.

Table 2
Calculated voting power by square root rule

	p_i	$\sqrt{p_i}$	Swings	Absolute PB	Relative PB	Citizen national power	Citizen union indirect power	Distance from equal power
A	100000000	10000	3,00	0,3750	0,3750	0,000079788	0,000029921	0,001736111
B	49000000	7000	3,00	0,3750	0,3750	0,000113984	0,000042744	0,006944444
C	25000000	5000	1,00	0,1250	0,1250	0,000159577	0,000019947	0,006944444
D	4000000	2000	1,00	0,1250	0,1250	0,000398942	0,000049868	0,001736111
	178000000	24000	8	1	1	0,000059804		1,73611111E-02

Source: own calculations

In our case square root rule does not lead to equal indirect power: either quota formula is not correct, or square root rule is not as good approximation as it is declared to be (or both).

In Table 3 we provide full list of quota intervals of stable power and apply our exact algorithm for optimal quota. We can see that while SZ optimal quota is equal to 18671 (in weights) or 77,79%, exact optimal quota is from interval (12000, 14000], or any quota between 50,01% and 59,33%. There is no quota granting equal indirect power of citizens, the best SZ distance provided by exact optimal quota is 0,00347222, while SZ distance based on SZ optimal quota approximation is 0,01736111.

Table 3
All possible PB power indices, quota intervals of stable power

State	p_i	$\sqrt{p_i}$	Swings	Absolute PB	Relative PB	Citizen national power	citizen union indirect power	Distance from equal power
quota	(12000, 14000]							
A	100000000	10000	5,00	0,6250	0,4167	0,000079788	0,000049868	0
B	49000000	7000	3,00	0,3750	0,2500	0,000113984	0,000042744	0,001736111
C	25000000	5000	3,00	0,3750	0,2500	0,000159577	0,000059841	0,001736111
D	4000000	2000	1,00	0,1250	0,0833	0,000398942	0,000049868	0
	178000000	24000	12	1,5000	1	0,000059804		3,47222222E-03
quota	(14000, 15000]							
A	100000000	10000	6,00	0,7500	0,6000	0,000079788	0,000059841	0,033611111
B	49000000	7000	2,00	0,2500	0,2000	0,000113984	0,000028496	0,008402778
C	25000000	5000	2,00	0,2500	0,2000	0,000159577	0,000039894	6,94444E-05
D	4000000	2000	0,00	0,0000	0,0000	0,000398942	0,000000000	0,006944444
	178000000	24000	10	1,2500	1	0,000059804		4,90277778E-02
quota	(1500, 17000]							
A	100000000	10000	5,00	0,6250	0,5000	0,000079788	0,000049868	0,006944444
B	49000000	7000	3,00	0,3750	0,3000	0,000113984	0,000042744	6,94444E-05
C	25000000	5000	1,00	0,1250	0,1000	0,000159577	0,000019947	0,011736111
D	4000000	2000	1,00	0,1250	0,1000	0,000398942	0,000049868	0,000277778
	178000000	24000	10	1,2500	1	0,000059804		1,90277778E-02
quota	(17000, 19000]							
A	100000000	10000	3,00	0,3750	0,3750	0,000079788	0,000029921	0,001736111
B	49000000	7000	3,00	0,3750	0,3750	0,000113984	0,000042744	0,006944444
C	25000000	5000	1,00	0,1250	0,1250	0,000159577	0,000019947	0,006944444
D	4000000	2000	1,00	0,1250	0,1250	0,000398942	0,000049868	0,001736111
	178000000	24000	8	1,0000	1	0,000059804		1,73611111E-02
quota	(19000, 22000]							
A	100000000	10000	2,00	0,2500	0,3333	0,000079788	0,000019947	0,006944444
B	49000000	7000	2,00	0,2500	0,3333	0,000113984	0,000028496	0,001736111
C	25000000	5000	2,00	0,2500	0,3333	0,000159577	0,000039894	0,015625
D	4000000	2000	0,00	0,0000	0,0000	0,000398942	0,000000000	0,006944444
	178000000	24000	6	0,7500	1	0,000059804		3,12500000E-02
quota	(22000, 24000]							
A	100000000	10000	1,00	0,1250	0,2500	0,000079788	0,000009974	0,027777778
B	49000000	7000	1,00	0,1250	0,1667	0,000113984	0,000014248	0,015625
C	25000000	5000	1,00	0,1250	0,1667	0,000159577	0,000019947	0,001736111
D	4000000	2000	1,00	0,1250	0,1667	0,000398942	0,000049868	0,006944444
	178000000	24000	4	0,5000	0,75	0,000059804		5,20833333E-02

Source: own calculations

5. Several remarks to square root rule

Model of equalization of indirect Penrose-Banzhaf power applied to distribution of voting weights in the EU is legitimate and scientifically justified. But it is not the unique way how to implement fairness principle and statements from open letter of European scientists are rather exaggerated. They are based on two different premises: a) on the concept of referenda type mechanism of decision making, which is crucial for the used principle of fairness itself, b) on the implicit assumption that Penrose-Banzhaf model is the only way how to quantify voting power, which is crucial for implementation. But exactly the same can be done with Shapley-Shubik model of voting power.

Let us comment first the used model of referenda based decision making mechanism. It contradicts the intuition of representative democracy and introduces as a reality a process of direct democracy. National representation in the Council of EU means government representation. With small exceptions governments do not feel political responsibility to the citizens, even in internal affairs. Citizens in multi-party systems do not elect the government, but they decide about composition of the Parliament. Government formation is based on trade-offs of political parties and individual members of the parliament.

Assume that square root fairness principle is implemented on national level. It means that every voter should have the same indirect voting power independently on which party he voted for (by allocation of seats in the Parliament proportionally to square roots of votes the party obtained in election). Such a proposal would be immediately disqualified as a symptom of mental illness. Just to illustrate the consequences, hypothetical composition of 2006 Lower House of the Czech Parliament under the fairness square root rule is provided in Table 4.

Table 4
Hypothetical square root "fair" composition of the Lower House of Czech Parliament after 2006 election

Party	Number of votes	Square root	% of votes	% SR weights	% of seats	Adjusted seats
Strana zdravého rozumu	24828	157,569	0,474033	2,444744	4,889488	5
České hnutí za národní jednotu	216	14,69694	0,004124	0,228029	0,456057	0
Balbínova poetická strana	6897	83,04818	0,131682	1,288524	2,577049	3
Liberální reformní strana	253	15,90597	0,00483	0,246787	0,493575	0
Právo a spravedlnost	12756	112,9425	0,243546	1,752346	3,504691	5
Nezávislí	33030	181,7416	0,630631	2,81979	5,639581	5
Česká pravice	395	19,87461	0,007542	0,308362	0,616724	1
Koruna česká	7293	85,39906	0,139243	1,324999	2,649998	3
ODS	1892475	1375,673	36,1324	21,34409	42,68818	43
ČSSD	1728827	1314,849	33,00792	20,40038	40,80076	41
Unie svobody	16457	128,2848	0,314208	1,990389	3,980777	4
Helax - Ostrava se baví	1735	41,65333	0,033126	0,646267	1,292535	1
Pravý blok	20382	142,7655	0,389147	2,215062	4,430124	4
VIZE-www.4vize.cz	3109	55,75841	0,059359	0,865113	1,730226	2
Česká strana nár. soc.	1387	37,24245	0,026482	0,577831	1,155662	1
Moravané	12552	112,0357	0,239651	1,738277	3,476554	3
Strana zelených	336487	580,075	6,424435	9,000086	18,00017	18
Humanistická strana	857	29,27456	0,016362	0,454206	0,908412	1
KSČM	685328	827,8454	13,08474	12,84434	25,68868	26
Koalice pro Českou republiku	8140	90,22195	0,155414	1,399828	2,799656	3
Národní strana	9341	96,64885	0,178345	1,499544	2,999088	3
Folklor i společnost	574	23,9583	0,010959	0,371722	0,743444	0
KDU ČSL	386706	621,8569	7,38325	9,648348	19,2967	19
Nezávislí demokraté Železný	36708	191,5933	0,700854	2,972644	5,945288	6
Strana Rovnost šancí	10879	104,3024	0,207709	1,618292	3,236585	3
total	5237612	6445,216	100	100	200	200

Source: www.volby.cz, own calculations

Even if we accept the square root principle of fairness, the implementation has week formal points (optimal quota approximation).

In the case of Shapley-Shubik power fair distribution of power among member states in the Council voting should be equal to the share of population.[2] If p_i is size of population in member state i, than $1/p_i$ is Shapley-Shubik voting power of a single citizen of country i in internal country referendum. Then, if the voting power of country i in the Council is

$$f_i^{SS} = \frac{p_i}{\sum_{k=1}^{n} p_k}$$

the indirect SS voting power of the single citizen of country i is

$$\frac{1}{p_i}\frac{p_i}{\sum_{k=1}^{n} p_k} = \frac{1}{\sum_{k=1}^{n} p_k}$$

(the same for every citizen of EU). For voting weights proportional to share of population we can find a quota minimizing distance between "fair" SS-power distribution and the power distribution generated by population weights as

$$q^* = \arg\min_j \sum_{i\in N}\left(f_i^{SS}(N, q_j, \mathbf{p}) - \frac{p_i}{\sum_{k\in N} p_k}\right)^2$$

The choice of "fairness principle" is a problem of political consensus of member states and cannot be resolved by "scientific community" and by mathematical models, but clarification, clear formulation and representation of the problem can be of help in political decisions. From that point of view square root rule discussion is useful and legitimate. What is wrong is that it is presented as the only correct way how to deal with the problem and creates illusion that "fairness" issue has been solved

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[2] Supporters of square root rule associates its justification exclusively with Penrose-Banzhaf power concept. Their objections to Shapley-Shubik power concept are based on classification of power measures on so called I-power (voter's potential influence over the outcome of voting) and P power (expected relative share in a fixed prize available to the winning group of committee members, based on cooperative game theory) introduced by Felsenthal, Machover and Zwicker (1998). Shapley-Shubik power index was declared to represent P-power and as such unusable for measuring influence in voting. We tried to show (Turnovec (2007)) that objections against Shapley-Shubik power index, based on its interpretation as a P-power concept, are not sufficiently justified. Both Shapley-Shubik and Penrose-Banzhaf measure could be successfully derived as cooperative game values, and at the same time both of them can be interpreted as probabilities of being in some decisive position (pivot,swing) without using cooperative game theory at all.

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