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Fish Wars: Cooperative and Non-Cooperative Approaches

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Abstract Mirman (1979) and Levhari and Mirman (1980) suggested a simple two person multistage game-theoretical model which sheds some light on the economic implications inherent in the fishing conflicts where the decisions of the competitors have an effect on the evolution of the fish population and so, on the future expected profit of the competitors. In this paper we consider a generalization of the Levhari and Mirman Fish War Game for the case of n participants of the conflict for different scenarios of hierarchical and coalition structures of countries. We derive the equilibrium and its steady-state behavior for all these scenarios and analyze the impact which the hierarchical and coalition structures can have on fishery and ecology.

Keywords Nash equilibrium, multistage game, fish war game, cooperative behavior
JEL classification C71, C72, C73, Q22

1. Introduction

In recent years a lot of international conflicts about fishing rights in various seas and water zones have taken place. Mirman (1979) and Levhari and Mirman (1980) suggested a simple two person multistage game-theoretical model which sheds some light on the economic implications inherent in the fishing conflicts where the decisions of the competitors have an effect on the evolution of the fish population and so, on the future expected profit of the competitors. Using logarithmic utility and exponential growth functions, they showed that the noncooperative equilibrium yields a smaller steady-state fish stock than the cooperative solution. Their model has been extended by numerous authors. For example, Benhabib and Radner (1992) incorporated trigger strategies into the resource extraction model. Fischer and Mirman (1992, 1996) allowed for the interaction between two different species of fish. Datta and Mirman (1999) added one more source of interdependence, each country's affection on the market price of fish, and characterized strategic manipulation of the market price as well as the common property resource. One of the standard results that these studies have shown is that the no-coordination equilibrium is Pareto-dominated by the full coordina-

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tion solution. The full coordination equilibrium is compared with the no-coordination equilibrium. Under full coordination, all countries participate in coordination, and there is a central authority that controls each country's volume of catch. Under the no-coordination equilibrium, each country only takes into account its own intertemporal welfare and it does not care about other countries' welfare.

All the above models except that of Datta and Mirman (1999), are based on two-country settings. Nowak (2006) investigated a generalization of the game for n players where the countries have the same facilities, Okoguchi (1991) considered the n countries model for the case of the countries selfish behaviour and Kwon (2006) investigated partial coordination schemes for the game. It is interesting to note that the Levhari and Mirman model of consuming of the exhausted resources was applied by Mohapatra and Venkatasubramania (2004) to develop a dynamic game theoretic approach for choosing power optimization strategies for various components (e.g. cpu, network interface etc.) of a low-power device operating in a distributed environment.

In this paper we consider a generalization of the Levhari and Mirman Fish War Game for the case of n participants of the conflict for different scenarios of hierarchical and coalition structures of countries. We derive the equilibrium and its steady-state behavior for all these scenarios and analyze the impact which the hierarchical and coalition structures can have on fishery.

2. Setup of the game

Suppose that there are n countries (the owners, countries) each of whom can extract a renewable resource, e.g., fish. Following Levhari and Mirman, assume that fish population, if uninterrupted by fishing, changes according to the following biological growth rule $x_{t+1} = x_t^{\alpha}$ where $\alpha \in (0,1)$, $t = 0,1,\ldots$, and x_0 is the initial level of fish at time $t = 0$. The boundary point $x_0 = 1$ is the stable steady state of the resource population after a normalization when there is no extraction.

Agent i ($i = 1,\ldots,n$) has a utility function u_i to estimate the profit for present consumption of the fish in each period. We shall consider the symmetric case where the instantaneous utility function of each country i is logarithmic, i.e. $u_i(\cdot) = \log(\cdot)$. Let c_i^t be the present consumption of country i in period t. So, $\sum_{i=1}^{n} c_i^t \leq x_t$. Let $\beta_i \in (0,1)$ be the discount factor for country i. It is assumed that each country maximizes the total discounted utility over finite or infinite horizon if fishery is managed by individual countries. Assume further that each country get the same amount of fish in the last phase of fishing. So, the payoff to country i in the finite horizon T is given as follows:

$$v_i^T = \sum_{t=0}^{T} \beta_i^t \log(c_i^t)$$

The utility function has an interesting feature. If country i consumes nothing in some period his utility is $-\infty$. Therefore, the players cannot extract everything during the play if the game has to be continued. Whenever they do that everybodys utility is $-\infty$.

We will investigate this model in different scenarios of hierarchical and coalition structures. Our goal is to get the optimal strategies in closed form. The result obtained in the paper essentially depends on the logarithm form and the exponential growth of the utility function. The derived optimal strategies for different scenarios of hierarchical structures and cooperation will allow us to find the impact they produce on ecological situation. We will show the advantage of the cooperation and demonstrate that its impact depends on the number of countries.

3. The main results

In this Section we give a list of theorems which supply the optimal strategies of the considered game for a few following spots of the fish war problem. First we consider the situation of the strong competitive environment where each country tries to maximize own profit and we will find Nash equilibrium for this plot. Then we consider the plot where all the countries make up one coalition and they jointly maximize the sum of each country's payoff. After that,we consider the plot where there is a strong hierarchical structure between countries, namely, all n of them are arranged one by one in a linear hierarchical structure consisting of n levels. The fourth plot deals with the situation where among all the countries there is only one leader and the rest ones compete with each other and they are the followers for the leader, so the hierarchical structure consists of two levels.

The next theorem was proved in Okuguchi (1981) and it supplies the optimal strategy of the countries for the case of their selfish behaviour.

Theorem 1. *In finite horizon game the optimal strategy c_i^1 of country i, $i = 1,\dots,n$, corresponding to Nash equilibrium, on the first stage is given as follows:*

$$c_i^1 = \frac{1/\gamma_i^T}{1+\sum_{j=1}^n (1/\gamma_j^T)} x_0, \text{ where } \gamma_i^T = \sum_{k=1}^{T} (\alpha\beta_i)^k.$$

If T tends to infinite then c_i^1 tends to c_i and the steady-state level of fish is $\bar{x}_{NE}$, where

$$c_i = \frac{1/\gamma_i}{1+\sum_{j=1}^n (1/\gamma_j)} x_0,$$

$$\bar{x}_{NE} = \frac{1}{\left(1+\sum_{j=1}^n (1/\gamma_j)\right)^{\alpha/(1-\alpha)}},$$

and

$$\gamma_i = \sum_{k=1}^{\infty} (\alpha\beta_i)^k = \alpha\beta_i/(1-\alpha\beta_i).$$

Now we consider the plot where all the countries form a coalition and they jointly maximize the sum of their payoffs. The case where all the countries have the same discount factor was investigated in Okuguchi (1981).

Theorem 2. *In finite horizon game for the cooperative plot (i.e all the countries maximize their joint payoff) the optimal strategy of country i on the first stage is given as follows:*

$$c_i^1 = \frac{x_0}{n + \sum_{j=1}^{n} \gamma_j^T} \tag{1}$$

If T tends to infinite then c_i^1 tends to c_i and the steady-state level of fish is $\bar{x_c}$, where

$$c_i = \frac{x_0}{n + \sum_{j=1}^{n} \gamma_j}, \tag{2}$$

$$\bar{x_c} = \left[\sum_{j=1}^{n} \gamma_j \Big/ \Big(n + \sum_{j=1}^{n} \gamma_j\Big) \right]^{\alpha/(1-\alpha)}. \tag{3}$$

Proof. For the one-period horizon ($T = 1$) on the first stage (since the second one is the last one and so, on the second stage the countries just share the fish) the objective is to maximize by $c_1^1, \ldots, c_n^1$:

$$\sum_{i=1}^{n} \left[\log(c_i^1) + \alpha\beta_i \log\left(\frac{1}{n} \Big(x_0 - \sum_{j=1}^{n} c_j^1\Big) \right) \right] \tag{4}$$

The maximum condition is

$$c_i^1 = \frac{1}{1 + \sum_{j=1}^{n} \gamma_j^1} \left[x_0 - \sum_{j=1, j\neq i}^{n} c_j^1 \right], \quad i = 1, \ldots, n.$$

Thus,

$$c_i^1 = \frac{x_0}{n + \sum_{j=1}^{n} \gamma_j^1}. \tag{5}$$

It is interesting that all the countries have the same strategy. The remaining fish population is given as follows:

$$x_0 - \sum_{i=1}^{n} c_i^1 = \left[1 - \frac{n}{n + \sum_{j=1}^{n} \gamma_j^1} \right] x_0 \tag{6}$$

Below we give a general remak which we will employ in the proof of this and next theorems.

Remark 1. *If for one-period horizon the optimal strategies on the first stage are of the form $c_i^1 = C^i x_0$ where C^i $(i = 1, \ldots, n)$ are positive and $\sum_{i=1}^{n} C^i < 1$ then the total* *discounted utility of country i for one-period horizon as a function of* x_0 *is given as follows:*

$$v_i(x_0) = \log(C^i x_0) + \alpha\beta_i \log\left(\left(x_0 - \sum_{i=1}^{n} C^i x_0\right)/n\right) = (1+\alpha\beta_i)\log(x_0) + A_i,$$

where $A_i = \log(C^i) + \alpha\beta_i \log((1-\sum_{i=1}^n C^i)/n)$ *is independent of* x_0.

In the case of two-period horizon by (5), (6) and Remark 1, the objective is to maximize by $c_1^1, \ldots, c_n^1$:

$$\begin{aligned}
&\sum_{i=1}^{n}\left[\log(c_i^1) + \beta_i v_i\left(\frac{1}{n}\left(x_0 - \sum_{j=1}^{n} c_j^1\right)\right)^{\alpha}\right] \\
&= \sum_{i=1}^{n}\left[\log(c_i^1) + \alpha\beta_i(1+\alpha\beta_i)\log\left(x_0 - \sum_{j=1}^{n} c_j^1\right) + B_i\right],
\end{aligned}$$

where $B_i = \beta_i(A_i - \alpha\log(n))$ which can be investigated similarly to the one-period horizon case. Namely, the maximum condition is

$$c_i^1 = \frac{1}{1+\sum\limits_{j=1}^{n}\gamma_j^2}\left[x_0 - \sum_{j=1, j\neq i}^{n} c_j^1\right], \quad i = 1, \ldots, n.$$

Thus,

$$c_i^1 = \frac{x_0}{n + \sum\limits_{j=1}^{n}\gamma_j^2}.$$

By backward induction step by step for the T-period horizon we obtain that the optimal strategy is given by (1). The steady-state catch (2) follows from (1) while T tends to infinity. For the steady-state level of fish we have that $\overline{x_c} = (\overline{x_c} - \sum_{i=1}^n c_i)^{\alpha}$ and the result (3) follows from (2). This completes the proof of Theorem 2. □

In the next theorem we consider the case where there is a strong hierarchical structure between countries (so, there is no direct competition between them), namely, all n of them are arranged one by one in a linear order and they make decision about fishing sequentially.

Theorem 3. *For the strong linear hierarchical structure model Leader-Follower (where, say, the first level leader is country 1, the second level leader is country 2 and so on) in finite horizon game the optimal strategy of country i on the first stage is given as follows:*

$$c_i^1 = \frac{1}{\gamma_i^T}\prod_{j=1}^{i}\frac{\gamma_j^T}{1+\gamma_j^T}x_0 \tag{7}$$

If T *tends to infinite then* c_i^1 *tends to* c_i *and the steady-state level of fish is* $\overline{x}_{LF}$, *where*

$$c_i = \frac{1}{\gamma_i}\prod_{j=1}^{i}\frac{\gamma_j}{1+\gamma_j}x_0, \tag{8}$$

$$\bar{x}_{LF} = \left[1 - \sum_{i=1}^{n} \frac{1}{\gamma_i} \prod_{j=1}^{i} \frac{\gamma_j}{1+\gamma_j}\right]^{\alpha/(1-\alpha)}. \tag{9}$$

Proof. In the case of one-period horizon ($T = 1$) on the first stage of the game the country n tries to maximize by c_n^1:

$$\log(c_n^1) + \alpha\beta_n \log\left(\frac{1}{n}\left(x_0 - \sum_{i=1}^{n} c_i^1\right)\right) \tag{10}$$

Thus, since $\gamma_n^1 = \alpha\beta_n$, the maximum condition is

$$c_n^1 = \frac{1}{1+\gamma_n^1}\left[x_0 - \sum_{i=1}^{n-1} c_i^1\right]. \tag{11}$$

Then, since c_n^1 is already known and given by (11), the country $n-1$ tries to maximize by c_{n-1}^1:

$$\begin{aligned} &\log(c_{n-1}^1) + \alpha\beta_{n-1} \log\left(\frac{1}{n}\left(x_0 - \sum_{i=1}^{n} c_i^1\right)\right) = \\ &= \log(c_{n-1}^1) + \alpha\beta_{n-1} \log\left(\frac{1}{n}\frac{\gamma_n^1}{1+\gamma_n^1}\left(x_0 - \sum_{i=1}^{n-1} c_i^1\right)\right) \end{aligned} \tag{12}$$

So, since $\gamma_{n-1}^1 = \alpha\beta_{n-1}$, the maximum condition is

$$c_{n-1}^1 = \frac{1}{1+\gamma_{n-1}^1}\left[x_0 - \sum_{i=1}^{n-2} c_i^1\right].$$

In similar way for $i = 2, \ldots, n$ we have

$$c_i^1 = \frac{1}{1+\gamma_i^1}\left[x_0 - \sum_{j=1}^{i-1} c_j^1\right] \text{ and } c_1^1 = \frac{x_0}{1+\gamma_1^1}.$$

Then, consequentially substituting c_1^1 into c_2^1, c_1^1 and c_2^1 into c_3^1 and so on, we obtain that the optimal strategy of the country i ($i = 1, \ldots, n$) on the first stage of the game is given as follows:

$$c_i^1 = \frac{1}{\gamma_i^1} \prod_{j=1}^{i} \frac{\gamma_j^1}{1+\gamma_j^1} x_0 \tag{13}$$

Since

$$\sum_{i=a}^{n} \frac{1}{\gamma_i^1} \prod_{j=a}^{i} \frac{\gamma_j^1}{1+\gamma_j^1} = \frac{1}{1+\gamma_a^1} + \frac{\gamma_a^1}{1+\gamma_a^1} \sum_{i=a+1}^{n} \frac{1}{\gamma_i^1} \prod_{j=a+1}^{i} \frac{\gamma_j^1}{1+\gamma_j^1}$$

with $a = 1, \ldots, n-1$ and

$$\frac{1}{\gamma_n^1} \prod_{j=n}^{n} \frac{\gamma_j^1}{1+\gamma_j^1} = \frac{1}{1+\gamma_n^1} < 1,$$

then by induction we have that

$$\sum_{i=1}^{n} \frac{1}{\gamma_i^1} \prod_{j=1}^{i} \frac{\gamma_j^1}{1+\gamma_j^1} < 1.$$

So,

$$\sum_{i=1}^{n} c_i^1 < x_0. \tag{14}$$

In the case of two-period horizon, (15) and (14) and Remark 1 yields that the country n on the first stage tries to maximize by c_n^1:

$$\log(c_n^1) + \beta_n v_n \left[\frac{1}{n}\left(x_0 - \sum_{i=1}^{n} c_i^1\right)^{\alpha}\right],$$

which can be investigated similarly to the one-period horizon case. Namely, the maximum condition is

$$c_n^1 = \frac{1}{1+\gamma_n^2}\left[x_0 - \sum_{i=1}^{n-1} c_i^1\right].$$

Then, since c_n^1 is already known, the country $n-1$ tries to maximize by c_{n-1}^1:

$$\log(c_{n-1}^1) + \beta_{n-1} v_{n-1} \left[\frac{1}{n}\left(x_0 - \sum_{i=1}^{n-1} c_i^1 - \frac{1}{1+\gamma_n^2}\left(x_0 - \sum_{i=1}^{n-1} c_i^1\right)\right)^{\alpha}\right]$$

So, the maximum condition is

$$c_{n-1}^1 = \frac{1}{1+\gamma_{n-1}^2}\left[x_0 - \sum_{i=1}^{n-2} c_i^1\right].$$

In similar way for $i = 1, \dots, n$ we have

$$c_i^1 = \frac{1}{1+\gamma_i^2}\left[x_0 - \sum_{j=1}^{i-1} c_j^1\right]$$

and

$$c_i^1 = \frac{1}{\gamma_i^2} \prod_{j=1}^{i} \frac{\gamma_j^2}{1+\gamma_j^2} x_0. \tag{15}$$

Then, consequentially substituting c_1^1 into c_2^1, c_1^1 and c_2^1 into c_3^1 and so on, we obtain that the optimal strategy of the country i ($i = 1, \dots, n$) on the first stage of the game is given as by (7) for $T = 2$.

Analogously by backward induction step by step we obtain that the optimal strategy of country i on the first stage of the game is given by (7) for any T. (8) and (9) straightforward follow from (7) when T tends to infinity. This completes the proof of Theorem 3. □

The fourth plot deals with the situation Leader-Followers where among all the countries there is only one leader. The rest ones compete with each other and they all are the followers for the leader, so this hierarchical structure consists just of two levels. Of course, here we assume that there are at least three countries since otherwise the plot will coincide with the plot studied in the previous theorem.

Theorem 4. *In finite horizon game with one Leader (say, Leader is country 1, the others are followers) the optimal strategy of country i on the first stage is given as follows:*

$$c_1^1 = \frac{x_0}{1+\gamma_1^T}, \tag{16}$$

$$c_i^1 = \frac{1}{1+1/\gamma_1^T} \frac{1/\gamma_i^T}{1+\sum_{j=2}^{n}(1/\gamma_j^T)} x_0, \quad i=2,\dots,n \tag{17}$$

If T tends to infinite then c_i^1 tends to c_i and the steady-state level of fish is $\bar{x}_{LFs}$, where

$$c_1 = x_0/(1+\gamma_1), \tag{18}$$

$$c_i = \frac{1}{1+1/\gamma_1} \frac{1/\gamma_i}{1+\sum_{j=2}^{n}(1/\gamma_j)} x_0, \quad i=2,\dots,n, \tag{19}$$

$$\bar{x}_{LFs} = \left[1 - \frac{1}{1+\gamma_1} - \frac{\gamma_1}{1+\gamma_1} \frac{\sum_{j=2}^{n}(1/\gamma_j)}{1+\sum_{j=2}^{n}(1/\gamma_j)}\right]^{\alpha/(1-\alpha)}. \tag{20}$$

Proof. In the case of one-period horizon ($T=1$) the country $i, i=2,\dots,n$ (each of them is the follower to the first country who is the leader and so each of them considers the strategy of country 1 as a given one) tries to maximize

$$\log(c_i^1) + \alpha\beta_i \log\left(\frac{1}{n}\left(x_0 - \sum_{j=1}^{n} c_j^1\right)\right).$$

The maximum condition is

$$c_i^1 = \frac{1}{1+\gamma_i^1}\left[x_0 - \sum_{j=1, j\neq i}^{n} c_j^1\right], \quad i=2,\dots,n.$$

Solving this system corresponding to c_i we obtain

$$c_i^1 = \frac{1/\gamma_i^1}{1+\sum_{j=2}^{n}(1/\gamma_j^1)}(x_0 - c_1^1), \quad i=2,\dots,n. \tag{21}$$

Then, Leader (country 1) tries to maximize the following payoff by c_1^1 where c_i^1 for $i = 2, \dots, n$ are given by (21):

$$\log c_1^1 + \alpha\beta_1 \log\left(\frac{1}{n}\left[x_0 - c_1^1 - \frac{\sum_{i=2}^{n}(1/\gamma_i^1)}{1 + \sum_{j=2}^{n}(1/\gamma_j^1)}(x_0 - c_1^1)\right]\right)$$

So,

$$c_1^1 = \frac{x_0}{1 + \gamma_1^1} \tag{22}$$

and by (21) and (22) we have that

$$c_i^1 = \frac{1}{1 + 1/\gamma_1^1} \frac{(1/\gamma_i^1)}{1 + \sum_{j=2}^{n}(1/\gamma_j^1)} x_0, \quad i = 2, \dots, n. \tag{23}$$

It is clear that

$$\sum_{i=1}^{n} c_i^1 = \left[\frac{1}{1 + \gamma_1^1} + \frac{\gamma_1^1}{1 + \gamma_1^1} \frac{\sum_{j=2}^{n}(1/\gamma_j^1)}{1 + \sum_{j=2}^{n}(1/\gamma_j^1)}\right] x_0 < x_0. \tag{24}$$

In the case of two-period horizon, by (22)-(24) and Remark 1, the country $i = 2, \dots, n$ tries to maximize by c_i^1:

$$\log(c_i^1) + \beta_i v_i \left[\frac{1}{n}\left(x_0 - \sum_{i=1}^{n} c_i^1\right)^{\alpha}\right],$$

which can be investigated similarly to the one-period horizon case. Namely, the maximum condition is

$$c_i^1 = \frac{(x_0 - c_1^1)/\gamma_i^2}{1 + \sum_{j=2}^{n}(1/\gamma_j^2)}, \quad i = 2, \dots, n. \tag{25}$$

Then, since c_i^1 for $i = 2, \dots, n$ are already known and given by (25), the country 1 tries to maximize by c_1^1:

$$\log(c_1^1) + \beta_1 v_1 \left[\frac{x_0 - c_1^1}{n\left(1 + \sum_{j=2}^{n}(1/\gamma_j^2)\right)}\right]^{\alpha}$$

So, the maximum condition is

$$c_1^1 = \frac{x_0}{1 + \gamma_1^2}.$$

Thus, (21) implies

$$c_i^1 = \frac{1}{1+1/\gamma_1^2} \frac{1/\gamma_i^2}{1+\sum\limits_{j=2}^{n}(1/\gamma_j^2)} x_0, \quad i = 2,\dots,n.$$

Analogously by backward induction step by step we obtain that the optimal strategy of country i on the first stage of the game is given by (16) and (17) for any T. (18), (19) and (20) straightforward follow from (16) and (17) when T tends to infinity. This completes the proof of Theorem 4. □

4. Conclusions

The results obtained in Theorems 1– 4 allow to compare the behavior of the players under different patterns of hierarchical structures and cooperation. Also, by means the closed form of the steady-state level of fish population we can estimate influence of fishery on ecological situation assuming that the bigger level of fish population means better ecological situation.

It turns out that the steady-state level of fish in the Leader-Follower plot is smaller than the corresponding value for the selfish plot and the last one in turn is smaller than the corresponding value in the cooperative scenario, namely,

$$\bar{x}_{LF} < \bar{x}_{LFs} < \bar{x}_{NE} < \bar{x}_c.$$

The inequality $\bar{x}_{NE} < \bar{x}_c$ by Theorems 1 and 2 is equivalent to the following obvious inequality:

$$\frac{1}{1+\sum\limits_{j=1}^{n}(1/\gamma_j)} < \frac{\sum_{j=1}^{n}\gamma_j}{n+\sum_{j=1}^{n}\gamma_j}$$

The inequality $\bar{x}_{LFs} < \bar{x}_{NE}$ by Theorems 1 and 4 is equivalent to

$$1 - \frac{1}{1+\gamma_1} - \frac{\gamma_1}{1+\gamma_1} \frac{\sum\limits_{j=2}^{n}(1/\gamma_j)}{1+\sum\limits_{j=2}^{n}(1/\gamma_j)} < \frac{1}{1+\sum\limits_{j=1}^{n}(1/\gamma_j)}$$

which follows from the following equivalent inequality:

$$\frac{a+b}{1+a+b} < \frac{a}{1+a} + \frac{1}{1+a}\frac{b}{1+b},$$

where $a = 1/\gamma_1$, $b = \sum_{i=2}^{n}(1/\gamma_i)$.

The inequality $\bar{x}_{LF} < \bar{x}_{LFs}$ by Theorems 3 and 4 is equivalent to

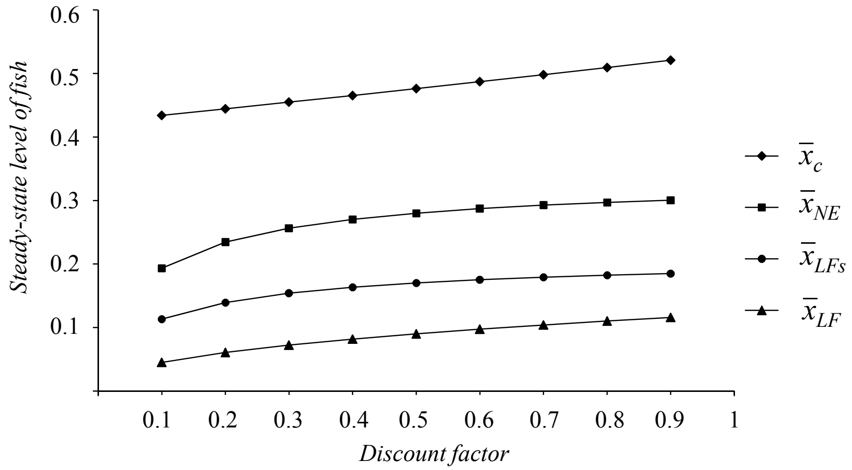
$$1 - \sum_{i=1}^{n} \frac{1}{\gamma_i} \prod_{j=1}^{i} \frac{\gamma_j}{1+\gamma_j} < 1 - \frac{1}{1+\gamma_1} - \frac{\gamma_1}{1+\gamma_1} \frac{\sum_{j=2}^{n}(1/\gamma_j)}{1+\sum_{j=2}^{n}(1/\gamma_j)}$$

which is equivalent to

$$\sum_{i=2}^{n} \frac{1/\gamma_i}{\prod_{j=2}^{i}(1+1/\gamma_j)} > \frac{\sum_{j=2}^{n}(1/\gamma_j)}{1+\sum_{j=2}^{n}(1/\gamma_j)}$$

and the last one can be easily proved by induction.

Figure 1. The steady-state level of fish



Now on a numerical example for three countries ($n = 3$) where $\alpha = 0.3$, $\beta_1 = 0.9$, $\beta_3 = 0.3$ and $\beta_2 = 0.1, 0.2, \ldots, 0.9$ (see Figure 1) we will demonstrate how possible patterns of countries behaviour and also their individual facilities which can be measured in this model by means of discount factors impact on fish population. It is interesting that if the discount factor β_2 is close to zero then cooperative behaviour essential better for ecology than if the discount factor is close to 1. It can be explained by the fact that if the discount factor is small the country takes into account only a few beginning intervals of the game since the profit for the next ones is very small and this short-sighted politics brings a great damage to ecology. Meanwhile if the discount factor is close to 1 then it makes the country to plan its activity for longer periods hoping also on a big profit in the future and so, if the country has a confidence in future it makes it to take care about ecology since without ecology there is no future also.

Now consider situation where there are n countries and they have the same facilities that can be identified by the same discount factor. So, let $\beta_i = \beta$, $i = 1, \dots, n$. Then, the the steady-state levels of fish population are given as follows:

$$\bar{x}_{LF} = \left(\frac{\gamma}{\gamma+1}\right)^{n\alpha/(1-\alpha)}, \quad \bar{x}_{LFs} = \left(\frac{\gamma}{\gamma+1}\frac{\gamma}{\gamma+n-1}\right)^{\alpha/(1-\alpha)},$$
$$\bar{x}_{NE} = \left(\frac{\gamma}{\gamma+n}\right)^{\alpha/(1-\alpha)}, \quad \bar{x}_{c} = \left(\frac{\gamma}{\gamma+1}\right)^{\alpha/(1-\alpha)}$$

It is very interesting that the cooperative plot allows to support the same fish population independents on the number of countries, meanwhile strong hierarchial structure like leader-follower one leads to exponential degradation of the population. If competition between countries or at least a part of them takes place then although some reduction of the the population happens but it takes place not so fast as for the strong hierarchial structure.

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