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Survival, Uncertainty, and Equilibrium Theory: An Exposition

by

Mukul Majumdar
and
Nigar Hashimzade

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Mukul Majumdar and Nigar Hashimzade

Department of Economics, Cornell University, Ithaca, New York, USA
School of Business and Economics, University of Exeter, Exeter, UK

Summary.

Keywords:

1 Introduction: A Historical Note

In his assessment of the important developments of general equilibrium theory in early fifties, T.C. Koopmans ([20], p.59) observed that "most authors have ignored the analytical difficulty of formulating a model that ensures the possibility of survival, blithely admitting any nonnegative rates of consumption as sustainable. Arrow and Debreu face this issue and find it to be a complicated one..." Koopmans gave a verbal description/interpretation of the Arrow-Debreu assumptions (see Section 4 of [2]) that guaranteed the existence of equilibrium, noting that "they assume that each consumer can, if necessary, survive on the basis of the resources he holds and the direct use of his own labor, without engaging in exchange, and still have something to spare of some type of labor which is sure to meet with a positive price in any equilibrium. If, contrary to the authors' indications, their model were given a stationary state interpretation, it would be found best suited for describing a society of self-sufficient farmers who do a little trading on the side. In modern society few of us can indeed survive without engaging in exchange..." Koopmans felt that "there is considerable challenge to further research on the survival problem" and did touch upon some directions and interpretations somewhat informally, and did also recognize the "inadequacies of any model unable to recognize the element of uncertainty in individual survival."

We note that in his celebrated article Nikaido ([26], p.136) assumed that in his exchange economy the initial endowment vector of each agent was strictly positive. However, in his subsequent note [27], he recognized that this was a "rather strict assumption" in his method of proof (relying upon what came to be known as the Debreu–Gale–Nikaido lemma and generated a substantial literature) of the existence of Walrasian equilibrium. Nikaido proceeded to present a crisp and beautiful extension of the existence theorem to the case where the initial endowment vector was positive ("non-zero, non-negative bundle") and the sum of the initial endowment vectors (over all agents) was strictly positive. Concerned with the application of the model to international trade theory , he felt (quite justifiably) that this weaker assumption was "reasonable enough."

For brevity, we touch upon just one more landmark in equilibrium theory. In Debreu's *Theory of Value* [9], the word "survival" did not figure at all, but some remarks relevant for the interpretations and applications of the Walrasian model are worth recalling. First, he provided a treatment of "uncertainty" in which "the contract for the transfer of a commodity ... specifies,in addition to its physical properties, its location and its date, an event on the occurrence of which the transfer is conditional. This .. definition of a commodity allows one to obtain a theory of uncertainty free from any probability concept and formally identical with the theory of certainty...". This treatment of "uncertainty" postulating the existence of a complete set of markets for all "commodities" originated in Arrow's remarkable paper [1]. With such a definition of "commodities" it is clear that the technology or production set Y_j of producer j and the consumption set X_i of consumer i, are "in general, contained in a coordinate subspace of R^l with a relatively small number of dimensions" ([9] p.38 and p.51). Secondly, Debreu (p.50) asserted that the consumption set X_i of all the possible consumption plans reflected "a priori constraints (for example, of a physiological nature)" (a "concrete" example appears on p.51 with diagrams illustrating consumption sets):

> "... the decision for an individual to have during the next year as sole input one pound of rice and as output one thousand hours of some type of labor could not be carried out".

Another example (p.52) concerns consumption at two dates, with a minimal level of consumption at date 1 which allows the consumer to survive until date 2. In this example, by choosing x_i the consumer "implicitly" chooses "his *life span*". Finally, in his existence theorem on pp.83–84 Debreu assumed (assumption (c) on p.84) that "there is some $x_i^0 \in X_i$ such that $x_i^0 \ll \omega_i$". In his Notes following the text of Chapter 5 (on pp.88–89), he indicated (referring to the contributions of Mckenzie and David Gale) how the assumption (c) could be replaced with other weaker(but, in our view, much less transparent) assumptions involving the interior of the asymptotic cone of the aggregate production possibility set. These assumptions play an essential role in establishing the continuity of the budget set correspondence which is a step towards getting the upper semicontinuity of the excess demand correspondence to which the Debreu–Gale–Nikaido lemma is used, and also in ensuring that the wealth of each agent is positive in equilibrium. Newman [25] has commented on the proper interpretation of the consumption possibility set and the difficulties in handling the survival problem.

It is fair to say that survival was not a serious theme in the subsequent progress of Walrasian equilibrium analysis. However, the importance of this problem was stressed in several prominent publications by Amartya Sen, primarily in the context of his study of famines. Sen [32] quoted Koopmans' remark on the Arrow–Debreu model (that we noted above) and felt that "the problem that is thus eliminated by assumption in these general equilibrium models is precisely the one central to a theory of survival and famines." He commented further that " 'the survival problem' for general equilibrium models calls for a solution not in terms of a clever assumption that eliminates it irrespective of realism, but for a reflection of the real guarantees that actually prevent starvation deaths in advanced capitalist economies." The entitlement approach developed by Sen provided the foundation for modern theoretical analysis of famines. However, it is difficult to capture the sweep of this approach by using formal models, particularly when uncertainty has to be explicitly modeled (see [8] for a deterministic analysis). Sen emphasized that for a better understanding of the problems of survival, one must recognize: (i) for a consumer to survive, his wealth at the equilibrium price system must allow him to obtain the basic necessities, and (ii) "starvation can develop for a group of people as its endowment vector collapses, and there are indeed many accounts of such endowment declines on the part of sections of poor rural population in developing countries... but starvation can also develop with *unchanged* asset ownership through movement of exchange entitlement mapping" ([33], pp.47–48).

Among the themes in the subsequent literature on famines, we note the following:

(1) famines often occur without substantial decline in aggregate food availability:

> "⋯ starvation is a matter of some people not *having* enough food to eat, and not a matter of there *being* not enough food to eat. While the latter can be a cause of the former, it is clearly one of many possible influences." [32]

An example is the Bangladesh famine of 1974, where the availability of food per head, including food production and net imports, in 1974 was higher than in any other year during 1971–76 (see [12]). Thus, a partial equilibrium analysis focusing on the food market may be unable to capture the complexity of events leading to an entitlement failure, and may render misleading policy prescriptions.

(2) the impact of famines may differ for distinct groups of population, in particular, different occupational groups. For example, during the Ethiopian famine of 1972–74 agricultural community suffered the most, and within this group nomadic herdsmen were hit the hardest. The famine itself was initiated by droughts, which resulted in reduced food supply. However,the

herdsmen "were affected not merely by the drought but also by the growth of commercial agriculture, displacing some of these communities from their traditional dry-weather grazing land, thereby vastly heightening the impact of the drought"[33]. Other occupational categories that were in the destitution groups included farm servants, rural laborers, craftsmen, women in service occupations. One can see that the most suffering groups were the ones *who did not have command over food production, and whose own production or labor, being less essential for survival, was no longer in demand when food supply dropped*.

> "The characteristics of exchange relations between the pastoral and agricultural economies contributed to the starvation of the herdsmen by making price movements reinforce – rather than counteract – the decline in the livestock quantities. The pastoralist, hit by the drought, was decimated by the market mechanism." ([33], p.112)

Another example is the Bangladesh famine of 1974, during which the families of rural laborers suffered the most, even without decline in the aggregate food availability. The reason in this case was the loss of employment as a result of floods: floods prevented planting of the rice, which "would reduce the food output later, but its impact on employment was immediate and vicious" [12]. To understand this aspect of famines one has to either study the economy at a *disaggregated* level with specialized occupations/endowments or allow for some commodities to play essential role in survival;

(3) expectations about future food prices can play a significant role in market behavior and result in food deprivation of certain groups of population. According to the study by Ravallion [28], a sudden increase in rice prices in Bangladesh during the 1974 famine could be explained by "excessive hoarding" by stockholders, who overestimated the damage to the future crop from floods, and, subsequently, have over-optimistic price expectations. To study the role of expectations one needs to turn to dynamic general equilibrium framework.

A formal approach to the analysis of these themes seems to call for general equilibrium models with specific structures (or, in Lindbeck's words, "concrete" Walrasian systems) and with an explicit recognition of uncertainty. In this paper we review some attempts in this direction.

First, using the earlier works of Hildenbrand ([18], [19]) and Bhattacharya and Majumdar [3] an attempt was made in [4] to define the probability of survival in the presence of *intrinsic uncertainty* affecting the endowments. It is reviewed in Section 2 with an improvement of one of the principal results on the asymptotic behavior of the equilibrium price as the size of the economy increases. It is shown (see (16) that "ruin" may occur either as a result of a collapse of endowments or as a result of an unfavorable movement in the price system. For the case of a weak correlation in endowments and the case of dependence in the form of dependency neighborhoods, the asymptotic results are similar for the ones for the case of independent agents. The issue of modeling a group of agents exposed to a common shock leads to the study of exchangeable random variables, and the problem of characterizing the probability of ruin in a large economy becomes more subtle.

Nest, we turn to the possible role of *extrinsic uncertainty* (a topic to which David Cass has made notable contributions) and give an example of ruin in a sunspot equilibrium. Using a model with overlapping generations and constant endowments we show that there can exist multiple equilibria with inter-generational trade, in some of which all agents survive, and in others old agents are ruined, with all fundamentals being the same. These multiple equilibria are supported by self-fulfilling beliefs, and the agents co-ordinate their beliefs using "sunspots" as a co-ordinating device. Next we turn to a model which links the survival problem to specialization. The main result, in line with [10], is that in an economy with specialization in production a group of agents, whose produce is less essential for survival, is more vulnerable to starvation.

2 The Probability of Survival: Intrinsic Uncertainty in the Cobb–Douglas–Sen Economy

In this and the next section we introduce a survival problem in the Walrasian framework. For simplicity of exposition, we assume that the preferences of the agents can be represented by a Cobb–Douglas utility function. This assumption enables us to compute the Walrasian equilibrium explicitly (see (17): such a computation can be extended to more than two goods). A more general treatment, using Hildenbrand's path-breaking work, of *some* of the issues can be found in [4], although the "language" becomes unavoidably technical. Our exposition focuses on the central economic issues with a rather minimal set of techniques from probability theory, and, hopefully, has pedagogical value.

In what follows, R_{++} is the set of positive real numbers, $x = (x_k) \in R^l$ is non-negative (written $x \geq 0$) if $x_k \geq 0$ for all k, and x is strictly positive (written $x \gg 0$) if $x \in R{+}{+}$. Define $x > 0$ as positive (non-negative and non-zero).

Consider, first, a deterministic Walrasian exchange economy with two goods. Assume that an agent i has an initial endowment $e_i = (e_{i1}, e_{i2}) \gg 0$, and a Cobb–Douglas utility function

$$u(x_{i1}, x_{i2}) = x_{i1}^{\gamma} x_{i2}^{1-\gamma} \tag{1}$$

where $0 < \gamma < 1$ and the pair (x_{i1}, x_{i2}) denotes the quantities of goods 1 and 2 consumed by agent i. Thus an agent i is described by a pair $\alpha_i = (\gamma, e_i)$.

Let $p > 0$ be the price of the first good. In a Walrasian model with two goods, we can normalize prices so that $(p, 1-p)$ is the vector of prices *accepted by all the agents.* The typical agent solves the following maximization problem (P):

$$\text{maximize } u(x_{i1}, x_{i2}) \tag{2}$$

subject to the "budget constraint" defined as

$$px_{i1} + (1-p)x_{i2} = w_i(p)$$

where the income or wealth w_i of the i-th agent is defined as the value of its endowment computed at $(p, 1-p)$:

$$w_i(p) = pe_{i1} + (1-p)e_{i2}. \tag{3}$$

Solving the problem (P) one obtains the *excess* demand for the first good as:

$$\zeta_{i1}(p, 1-p) = [(1-p)/p]\gamma e_{i2} - (1-\gamma)e_{i1} \tag{4}$$

One can verify that

$$p\zeta_{i1}(p, 1-p) + (1-p)\zeta_{i2}(p, 1-p) = 0 \tag{5}$$

The *total excess demand* for the first good at the prices $(p, 1-p)$ in a Walrasian exchange economy with n agents is given by:

$$\zeta_1(p, 1-p) = \sum_{i=1}^{n} \zeta_{i1}(p, 1-p) \tag{6}$$

In view of (5) it also follows that

$$p\zeta_1(p, 1-p) + (1-p)\zeta_2(p, 1-p) = 0 \tag{7}$$

The "market clearing" Walrasian equilibrium price system is defined by

$$\zeta_1(p^*, 1-p^*) = \zeta_2(p^*, 1-p^*) = 0 \tag{8}$$

and direct computation gives us the equilibrium price p_n^* (we emphasize the dependence of equilibrium price on the number of agents by writing p_n^*) as:

$$p_n^* = \left[\sum_{i=1}^{n} X_i\right] \Big/ \left[\sum_{i=1}^{n} X_i + \sum_{i=1}^{n} Y_i\right] \tag{9}$$

where

$$X_i = \gamma e_{i2},\ Y_i = (1-\gamma)e_{i1}. \tag{10}$$

To be sure, one can verify directly that demand equals supply in the market for the second good when the excess demand for the first good is zero.

Finally, let us stress that a Walrasian economy is "informationally decentralized" in the sense that agent i has no information about (e_j) for $i \neq j$. Thus it is *not* possible for agent i to compute the equilibrium price p_n^*.

One of the suggestions made by Sen ([33], Appendix A) to deal with survival explicitly is now recalled using our notation. Let F_i be a (nonempty) closed subset of R^2_{++}. We interpret F_i as the set of all combinations of the two goods that enable the i-th agent to survive. Now, given a price system $(p, 1-p)$, one can define a function $m_i(p)$ as

$$m_i(p) = \min_{(x_{i1}, x_{i2}) \in F_i} \{px_{i1} + (1-p)x_{i2}\} \tag{11}$$

Thus, $m_i(p)$ is readily interpreted as the minimum expenditure needed for survival at prices $(p, 1-p)$.

Example: Let $(a_{i1}, a_{i2}) \gg 0$ be a fixed element of R^2_{++}. Let

$$F_i = \{(x_{i1}, x_{i2}) \in R^2_{++} : x_{i1} \geq a_{i1}, x_{i2} \geq a_{i2}\} \tag{12}$$

Here $m_i(p) = pa_{i1} + (1-p)a_{i2}$.

In our approach we do not deal with the set F_i explicitly. Instead, let us suppose that, in addition to its utility function and endowment vector, each agent i is characterized by a continuous function $m_i(p) : [0,1] \to R_{++}$, and say that for an agent to *survive* at prices $(p, 1-p)$, its wealth $w_i(p)$ (see (3) must exceed $m_i(p)$. Hence, the i-th agent *fails to survive* (or, *is ruined*) at the Walrasian equilibrium $(p_n^*, 1-p_n^*)$ if

$$[p_n^* e_{i1} + (1-p_n^*)e_{i2}] \leq m_i(p_n^*) \tag{13}$$

or, using the definition (3)

$$w_i(p_n^*) \leq m_i(p_n^*) \tag{14}$$

From (13) and (14) one can see that an agent may face ruin due to (a) a possible endowment failure or (b) the equilibrium price system adversely affecting its wealth relative to the minimum expenditure. This issue is linked to the literature on the "price" and "welfare" effects of a change in the endowment on a deterministic Walrasian equilibrium (see the review of the transfer problem by Majumdar and Mitra [22]).

Consider now a case of *intrinsic uncertainty*: suppose that the endowments $\mathbf{e}_i$ of the agents $(i = 1, 2, \cdots n)$ are random variables. In other words, each $\mathbf{e}_i$ is a (measurable) mapping from a probability space$(\Omega, \mathcal{F}, P)$ into the non-negative orthant of R^2. One interprets Ω as the set of all possible states of environment, and $\mathbf{e}_i(\omega)$ is the endowment of agent i in the particular state ω. The distribution of $\mathbf{e}_i(\cdot)$ is denoted by μ_i [formally each μ_i is a probability measure on the Borel σ field of R^2, its support being a nonempty subset of the strictly positive orthant of R^2]. From the expression (9), the "market clearing" equilibrium price $p_n^*(\omega)$ is random, i.e., depends on ω:

$$p_n^*(\omega) = \left[\sum_{i=1}^{n} \gamma e_{i2}(\omega)\right] / \left[\sum_{i=1}^{n} \gamma e_{i2}(\omega) + \sum_{i=1}^{n} (1-\gamma)\, e_{i1}(\omega)\right] \tag{15}$$

The wealth $w_i(p_n^*(\omega))$ of agent i at $p_n^*(\omega)$ is simply $p_n^*(\omega)e_{i1}(\omega) + [1 - p_n^*(\omega)]e_{i2}(\omega)$. The event

$$\mathcal{R}_n^i = \{\omega \in \Omega : w_i(p_n^*(\omega)) \leq m_i(p_n^*(\omega))\} \tag{16}$$

is the set of all states of the environment in which agent i does not survive. Again, from the definition of the event $\mathcal{R}_n^i$ it is clear that an agent may be ruined due to a meager endowment vector in a particular state of environment. In what follows, we shall refer to this situation as a "direct" effect of endowment uncertainty or as an "individual" risk of ruin. But it is also possible for ruin to occur through an unfavorable movement of the equilibrium prices (terms of trade) even when there is no change (or perhaps an increase!) in the endowment vector. A Walrasian equilibrium price system reflects the entire pattern of endowments that emerges in a particular state of the environment. Given the role of the price system in determining the wealth of an agent and the minimum expenditure needed for survival, this possibility of ruin through adverse terms of trade can be viewed as an "indirect" ("terms of trade") effect of endowment uncertainty.

Our first task is to characterize $P(\mathcal{R}_n^i)$ when n is large (so that the assumption that an individual agent accepts market prices as given is realistic). While this task is certainly made easier by the structure of the model that allows us to compute $p_n^*(\omega)$ explicitly (15), the convergence arguments are still somewhat technical, in particular when we attempt to dispense with the assumption of stochastic independence.

To begin with let us make the following assumptions:

A1. $\{X_i\}$, $\{Y_i\}$ are uniformly bounded[1]: there exists $M > 0$ such that $0 \leq X_i < M$, $0 \leq Y_i < M$ for $i = 1, 2, \cdots$.
A2. $\{X_i\}$ are uncorrelated, $\{Y_i\}$ are uncorrelated.
A3. $\left[(1/n)\sum_i EX_i\right]$ converges to some $\pi_1 > 0$, $\left[(1/n)\sum_i EY_i\right]$ converges to some $\pi_2 > 0$ as n tends to infinity.
In the special case when the distributions of $\mathbf{e}_i$ are the same for all i (so that $1/n \sum_i EX_i = \pi_1$, where π_1 is the common expectation of all X_i; similarly for π_2), A3 is satisfied.

Under A1–A3, if the number n of agents increases to infinity, as a consequence of the strong law of large numbers we have the following limiting property of equilibrium prices p_n^*:
Proposition 1. *Under A1–A3, as n tends to infinity, $p_n^*(\omega)$ converges with probability 1 (almost surely) to the constant*

$$p_0 = \pi_1 / [\pi_1 + \pi_2] \tag{17}$$

For the proof, we apply Corollary 6.2 in Bhattacharya and Waymire [5] (p.649) to the sequences $\{\frac{1}{n}\sum_{i=1}^{n}(X_i - E(X_i))\}$ and $\{\frac{1}{n}\sum_{i=1}^{n}(Y_i - E(Y_i))\}$ and

[1] Recall that X_i and Y_i are non-negative by non-negativity assumption on endowments.

conclude that each of these sequences converges to zero with probability 1. Therefore, $\{\frac{1}{n}\sum_{i=1}^{n} X_i\}$ converges with probability 1 to π_1, and $\{\frac{1}{n}\sum_{i=1}^{n} Y_i\}$ converges with probability 1 to π_2 (see, for example, Rohatgi [31], p. 252, Theorem 13). Next, we consider $g(x,y) \equiv x/(x+y)$, a continuous function of a vector (x,y) on R_{++}. By definition of almost sure convergence, $(X_n, Y_n) \xrightarrow{a.s.} (X,Y)$ implies $g(X_n, Y_n) \xrightarrow{a.s.} g(X,Y)$ (see, for example, Ferguson [14], p.9). Therefore, $p_n^* \xrightarrow{a.s.} p_0$. Q.E.D.

Roughly, one interprets (17) as follows: for large values of n, the equilibrium price will not vary much from one state of the environment to another, and will be insensitive to the exact value of n, the number of agents.

In [17] $p_n(\omega)$ was shown to converge *only in probability* to p_0 under weaker assumptions. In our context, the boundedness assumption A1 seems quite innocuous. Since we do not assume stochastic independence, the proof relies on a relatively recent version of the Strong Law of Large Numbers due to Etemadi [13] (see [5] for further discussion).

For the constant p_0 defined by (17), we have the following characterization of the probability of ruin in a large Walrasian economy:

Proposition 2. *If* $p_0 e_{i1}(\omega) + (1-p_0)e_{i2}(\omega)$ *has a continuous distribution function,*

$$\lim_{n\to\infty} \left[P(\mathcal{R}_n^i)\right] = P\{\omega : p_0 e_{i1}(\omega) + (1-p_0)e_{i2}(\omega) \le m_i(p_0)\} \tag{18}$$

Remark: The probability on the right side of (18) does not depend on n, and is determined by μ_i, a characteristic of agent i, and p_0.

2.1 Dependence

Case studies of famines often indicate that a famine is typically confined to a particular geographic region or affects people belonging to the same occupation group (see, for example, [11]). To account for this property one needs to introduce stochastic dependence among the agents in the model. Of course, the most difficult question is how to model the dependence among agents. Also, dependence among random variables complicates asymptotic theory, and, to obtain analytic results, one has to assume a particular structure of the form of dependence. In this section we consider few examples of stochastic dependence among agents in which the limiting results can be derived explicitly. Results parallel to Proposition 1 were obtained in [4] and [16]. Stated informally, three interesting models were tractable: a model involving "weak" correlation among agents ([4], Proposition 1.3); a model with appropriate restrictions on the size of the dependency neighborhood (a concept introduced by Stein [35] and studied by Hashimzade [16] in the present context), and a model with exchangeable (conditionally independent) agents. The last model is particularly important in recognizing that the terms of trade effect may remain significant even in a large economy. We stress the importance of this point with a precise statement of the basic result proved in [4].

2.2 Exchangeability (exposure to a common shock)

To capture the probability of exposure to a common shock to endowments in a simple manner, let us say that μ and ν are two possible probability laws of $\{\mathbf{e}_i(\cdot)\}_{i\geq 1}$. Think of Nature conducting an experiment with two outcomes "H" and "T" with probabilities $(\theta, 1-\theta), 0<\theta<1$. Conditionally, given that "H" shows up, the sequence $\{\mathbf{e}_i(\cdot)\}_{i\geq 1}$ is independent and identically distributed with common distribution μ. On the other hand, conditionally given that "T" shows up, the sequence $\{\mathbf{e}_i(\cdot)\}_{i\geq 1}$ is independent and identically distributed with common distribution ν. Let $\pi_{1\mu}$ and $\pi_{1\nu}$ be the expected values of X_1 under μ and ν respectively. Similarly, let $\pi_{2\mu}$ and $\pi_{2\nu}$ be the expected values of Y_1 under μ and ν. It follows that $p_n(\cdot)$ converges to $p_0(\cdot)$ almost surely, where $p_0(\cdot) = \pi_{1\mu}/[\pi_{1\mu}+\pi_{2\mu}] = p_{0\mu}$ with probability θ and $p_0(\cdot) = \pi_{1\nu}/[\pi_{1\nu}+\pi_{2\nu}] = p_{0\nu}$ with probability $1-\theta$. We now have a precise characterization of the probabilities of ruin as n tends to infinity. To state it, write

$$J = \{(u_1,u_2)\in R^2_+ : p_{0\mu}u_1 + (1-p_{0\mu})u_2 \leq m_i(p_{0\mu})\};$$
$$r_i(\mu) = \int_J \mu(du_1, du_2). \tag{19}$$

Similarly, define $r_i(\nu)$ obtained on replacing μ by ν in (19).

Proposition 4. *Assume that $p_0e_{i1}(\omega) + (1-p_0)e_{i2}(\omega)$ had a continuous distribution function under each distribution μ and ν of $\mathbf{e}_i = (e_{i1}, e_{i2})$.*

(a) *Then, as the number of agents n goes to infinity, the probability of ruin of the i-th agent converges to $r_i(\mu)$, with probability θ, when "H" occurs and to $r_i(\nu)$, with probability $1-\theta$ when "T" occurs.*
(b) *The overall, or unconditional, probability of ruin converges to*

$$\theta r_i(\mu) + (1-\theta) r_i(\nu).$$

Here, the precise limit distribution is slightly more complicated, but the important distinction from the case of independence (or, "near independence") is that the limit depends not just on the individual uncertainties captured by the distributions μ and ν of an agent's endowments, but also on θ that retains an influence on the distribution of prices even with large n.

3 Survival and Extrinsic Uncertainty: An Example with Overlapping Generations.

We now turn to *extrinsic* uncertainty: when the uncertainty affects the *beliefs* of the agents (for example, the agents believe that market prices depend on some "sunspots") but the fundamentals are the same in all states. Clearly, with respect to the probability of survival, the extrinsic uncertainty has no direct effect, because it does not affect the endowments. However, it may have an indirect effect: self-fulfilling beliefs of the agents regarding market prices affect their wealth, and some agents may be ruined in one state of environment and survive in some other state, even though the fundamentals of the economy are the same in all states. To study the indirect, or the adverse term-of-trade effect of extrinsic uncertainty on survival we need a dynamic economy.

Consider a discrete time, infinite horizon OLG economy with constant population. We use Gale's terminology [15] wherever appropriate. For expository simplicity, and without loss of generality we assume that at the beginning of every time period $t = 1, 2, \cdots$ there are two agents: one "young" born in t, and one "old" born in $t-1$. In period $t = 1$ there is one old agent of generation 0. There is one (perishable) consumption good in every period. The agent born in t (generation t) receives an endowment vector $e_t = (e_t^y, e_t^o)$ and consumes a vector $c_t = (c_t^y, c_t^o)$. We consider the Samuelson case [2] and assume, without loss of generality, $e_t = (1, 0)$. We assume that the preferences of the agent of generation t can be represented by expected utility function $\mathcal{U}_t(\cdot) = E\left[U^t(c_t)\right]$ with Bernoulli utility $U^t(c_t)$, continuously differentiable and almost everywhere twice continuously differentiable, strictly concave and strictly monotone in $\mathcal{D}$, compact, convex subset of R^2_{++}. The old agent of generation 0 is endowed with one unit of fiat money, the only nominal asset in the economy. In every period the market for the perishable consumption good is open and accessible to all agents. Denote the nominal price of the consumption good at time t by p_t. Define a *price system* to be a sequence of positive numbers, $\mathbf{p} = \{p_t\}_{t=0}^{\infty}$, a *consumption program* to be a sequence of pairs of positive numbers $\mathbf{c} = \{c_t\}_{t=0}^{\infty}$, a *feasible program* to be a consumption program that satisfies $c_t^y + c_{t-1}^o \leq e_t^y + e_{t-1}^o = 1$. The agent of generation t maximizes his lifetime expected utility in the beginning of period t. In period 1, the young agent gives its saving (s_1^y) of the consumption good, to the old agent in exchange for one unit of money (the exchange rate is determined by p_1). Thus, $p_1 s_1 = 1$. This unit of money is carried into period 2 (the old age of agent born in period 1) and is exchanged (at the rate determined by p_2) for the consumption food saved by the young agent born in period 2 (s_2^y). The process is repeated.

3.1 Perfect Foresight Equilibrium

If there is no uncertainty, with perfect foresight the price-taking young agent's optimization problem is the following:

[2] If a population grows geometrically at the rate γ, so that γ^t agents is born in period t, and there is only one good in each period, the Samuelson case corresponds to marginal rate of intertemporal substitution of consumption under autarky, $U_1(e^y, e^o)/U_2(e^y, e^o)$, being less than γ. In our case $\gamma = 1$.

$$\max U(c_t^y, c_t^o)$$

subject to

$$c_t^y = 1 - s_t^y$$
$$c_t^o = p_t s_t^y / p_{t+1}$$

$(0 \leq s_t^y \leq 1,\ t = 1, 2, \cdots)$.

Here, $s_t^y \equiv e_t^y - c_t^y$ is savings of the young agent (this is the Samuelson case, in Gale's definitions [15]). A *perfect foresight competitive equilibrium* is defined as a feasible program and a price system such that

(i) the consumption program $\bar{\mathbf{c}} = \{\bar{\mathbf{c}}_\mathbf{t}\}$ solves optimization problem of each agent given $\bar{\mathbf{p}} = \{\bar{p}_t\} : (\bar{c}_t^y, \bar{c}_t^o) \in \mathcal{D}$, $\bar{c}_t^y = 1 - s_t$ and $\bar{c}_t^o = \bar{p}_t s_t / \bar{p}_{t+1}$ with

$$s_t = arg \max_{0 \leq s_t^y \leq 1} U\left((1 - s_t^y), s_t^y \frac{\bar{p}_t}{\bar{p}_{t+1}}\right)$$

and

(ii) the market for consumption good clears in every period:

$$\bar{c}_t^y + \bar{c}_{t-1}^o = 1 \text{ (demand = supply for the consumption good)}$$
$$\bar{p}_t s_t = 1 \text{ (demand = supply for money)}$$

for $t = 1, 2, \cdots$.

By strict concavity of the utility function $U(c_t^y, c_t^o)$, the young agent's optimization problem has a unique solution. Hence, we can express s_t as a single-valued function of p_t/p_{t+1}, i.e. we write $s_t = s_t(p_t/p_{t+1})$. This function (called savings function) generates an offer curve in the space of net trades, as price ratios vary. In the perfect foresight equilibrium

$$s_t(p_t/p_{t+1}) = 1/p_t. \tag{20}$$

The stationary perfect foresight monetary equilibrium is a sequence of constant prices p and constant consumption programs $(1 - \bar{s}, \bar{s})$, where $\bar{s} = s(1)$.[3]

3.2 Sunspot equilibrium

Now consider an extrinsic uncertainty in this economy. There is no uncertainty in fundamentals, such as endowments and preferences, but the agents

[3] Given our assumptions on preferences and endowments, the stationary perfect foresight monetary equilibrium exists and is optimal (see, for example, [21], Chap. 8).

believe that market prices depend on realization of an extrinsic random variable (sunspot). We assume that there is one-to-one mapping from the sunspot variable to price of the consumption good. Because the agents cannot observe future sunspots, they maximize expected utility over all possible future realization of the states of nature. We examine the situation with two states of nature, $\sigma \in \{\alpha, \beta\}$, that follow a first-order Markov process with stationary transition probabilities,

$$\Pi = \begin{bmatrix} \pi^{\alpha\alpha} & \pi^{\alpha\beta} \\ \pi^{\beta\alpha} & \pi^{\beta\beta} \end{bmatrix} \tag{21}$$

where $\pi^{\sigma\sigma'} > 0$ is the probability of being in state σ' in the next period given that current state is σ, and $\pi^{\sigma\alpha} + \pi^{\sigma\beta} = 1$. A young agent born in t observes price p_t^σ and solves the following optimization problem:

$$\max \left[\pi^{\sigma\alpha} U(c_t^{y,\sigma}, c_t^{o,\alpha}) + \pi^{\sigma\beta} U(c_t^{y,\sigma}, c_t^{o,\beta})\right]$$

subject to

$$\begin{aligned} c_t^{y,\sigma} &= 1 - s_t^\sigma \\ c_t^{o,\sigma'} &= p_t^\sigma s_t^\sigma / p_{t+1}^{\sigma'} \end{aligned}$$

$(0 \leq s_t^\sigma \leq 1,\ s_t^{\sigma'} \geq 0,\ \sigma, \sigma' \in \{\alpha, \beta\})$.

We restrict our attention to stationary equilibria, in which prices depend on the current realization of the state of nature σ, and do not depend on the calendar time nor the history of σ. A *stationary sunspot equilibrium*, SSE, is a pair of feasible programs and nominal prices, such that for every $\sigma \in \{\alpha, \beta\}$

(i) the consumption programs solve the agents' optimization problem:

$$\begin{aligned} &s^\sigma(p^\sigma / p^{\sigma'}) = \\ arg \max_{0 \leq s^\sigma \leq 1} &\left[\pi^{\sigma\alpha} U\left((1 - s^\sigma), s^\sigma p^\sigma / p^\alpha\right) + \right. \\ &\left. + \pi^{\sigma\beta} U\left((1 - s^\sigma), s^\sigma p^\sigma / p^\beta\right)\right] \end{aligned} \tag{22}$$

and

(ii) markets clear in every period, in every state.

$$\begin{aligned} c^{y,\sigma} + c^{o,\sigma} &= 1 \\ p^\sigma s^\sigma &= 1 \end{aligned}$$

It is easy to see that a stationary sunspot equilibrium exists when the equation

$$\frac{p^\alpha}{p^\beta} s^\alpha \left(\frac{p^\alpha}{p^\beta}\right) - s^\beta \left(\frac{p^\beta}{p^\alpha}\right) = 0 \tag{23}$$

has positive solutions for p^α / p^β other than 1 . Solution $p^\alpha / p^\beta = 1$ corresponds to the equilibrium in which uncertainty does not matter. It can be shown that, if sunspot equilibria exist in this economy, there are at least two of them, with $p^\alpha/p^\beta > 1$ and $p^\alpha/p^\beta < 1$ (see, for example, [7], [34]). This means that in the sunspot equilibrium consumption of old agents is above the certainty equilibrium consumption of olds in one state of nature and below in the other. Suppose, we introduce an exogenous minimal subsistence level of consumption (independent of $\sigma \in \{\alpha, \beta\}$). It may be the case that in one of the states of nature consumption of old agents falls short of minimal subsistence level: old agents are ruined. Note that the endowments are not affected by the uncertainty, and, therefore, there is no *direct* effect of uncertainty on ruin. The event of ruin is caused purely by an *indirect*, or term-of-trade effect: the equilibrium price system is such that the wealth of old agents does not allow them to survive. The following numerical example illustrates this possibility for the case of quadratic utility.

3.3 Ruin in equilibrium.

Let the preferences of the agents be represented by expected utility function with

$$
\begin{aligned}
U(\mathbf{c}) &= u(c^y, c^o) - v(c^o) \\
u(c^y, c^o) &= 2a\sqrt{c^y c^o} + q\, c^y + r\, c^o - \frac{1}{2}b(c^y)^2 - \frac{1}{2}d(c^o)^2 \\
v(c^o) &= \begin{cases} \frac{\theta}{2}\left(A - c^o\right)^2, & 0 < c^o \leq A \\ 0, & c^o > A \end{cases}
\end{aligned}
$$

where a, b, c, q, r, θ, A are positive constants such that the utility function is increasing and jointly concave in its arguments in $\mathcal{D}$. $v(\cdot)$ is the disutility of consuming less than A, the minimal subsistence level.[4] As above, agents in each generation receive identical positive endowments $e = 1$ of consumption good when young and zero endowments when old; the initial olds are endowed with one unit of money.

[4] It may seem odd that the disutility from starvation is finite, but this can be justified by the willingness of the agents to take a risk. Consider the following. In the continuous time, if the consumption of an old agent is above A, he lives to the end of the second period. If his consumption is below A, perhaps, he does not die immediately. Albeit low, the amount consumed allows him to live some time in the second period, and his lifespan in the second period is the longer, the closer is his consumption to A. In the discrete time this translates into probability of survival in the second period as a function of consumption. Thus, the old agent survives with probability 1 if $c^o \geq A$ and with probability less than 1 if $c^o < A$. Suppose, the objective of the agent is to maximize the probability of survival (or maximize his expected lifespan). Then it can be presented equivalently as the objective to minimize the disutility from consumption at the level below A. Clearly, this disutility can be finite, at least in the vicinity of A, if the agent is willing to take a risk. The authors are indebted to David Easley for this argument.

Benchmark case: perfect foresight

For the above preferences, savings function $s_t(p_t/p_{t+1})$ is implicitly defined by

$$\rho_t = \frac{a\sqrt{\rho_t s_t/(1-s_t))} + q - b\,(1-s_t)}{a\sqrt{(1-s_t)/(\rho_t s_t)} + r - d\,\rho_t s_t - v'(\rho_t s_t)}, \tag{24}$$

where $\rho_t \equiv p_t/p_{t+1}$. The offer curve is described by

$$(1-x)\left(a\sqrt{\frac{y}{x}} + q - b\,x\right) - y\left(a\sqrt{\frac{x}{y}} + r - d\,y - v'(y)\right) = 0 \tag{25}$$

In the stationary (deterministic) perfect foresight monetary equilibrium consumption plan of an agent is $(x, 1-x)$, where x solves

$$a\left(\sqrt{\frac{x}{1-x}} - \sqrt{\frac{1-x}{x}}\right) + x\,(b+d) + v'(1-x) + q - r - b = 0 \tag{26}$$

Stationary sunspot equilibria

Two states of nature, α and β evolve according to a stationary first-order Markov process. The states of nature do not affect the endowments. Agents can trade their real and nominal assets. In a stationary sunspot equilibrium with trade s^α, s^β solve the following system of equations:

$$\pi^{\alpha\alpha} a\sqrt{\tfrac{s^\alpha}{1-s^\alpha}} + (1-\pi^{\alpha\alpha})\,a\sqrt{\tfrac{s^\beta}{1-s^\alpha}} + q - b\,(1-s^\alpha) = \tag{27}$$
$$= \pi^{\alpha\alpha} a\left(\sqrt{\tfrac{1-s^\alpha}{s^\alpha}} + r - d\,s^\alpha - v'(s^\alpha)\right) + (1-\pi^{\alpha\alpha})\left(a\sqrt{\tfrac{1-s^\alpha}{s^\beta}} + r - d\,s^\beta - v'(s^\beta)\right)\tfrac{s^\beta}{s^\alpha}$$

and

$$\pi^{\beta\beta} a\sqrt{\tfrac{s^\beta}{1-s^\beta}} + \left(1-\pi^{\beta\beta}\right) a\sqrt{\tfrac{s^\alpha}{1-s^\beta}} + q - b\left(1-s^\beta\right) = \tag{28}$$
$$= \pi^{\beta\beta} a\left(\sqrt{\tfrac{1-s^\beta}{s^\beta}} + r - d\,s^\beta - v'(s^\beta)\right) + \left(1-\pi^{\beta\beta}\right)\left(a\sqrt{\tfrac{1-s^\beta}{s^\alpha}} + r - d\,s^\alpha - v'(s^\alpha)\right)\tfrac{s^\alpha}{s^\beta}$$

It is easy to see that one solution is $s^\alpha = s^\beta = 1-x$, where x solves the equation for the perfect foresight above. This solution does not depend on the transition probabilities, prices and consumption are not affected by the uncertainty: sunspots *do not matter* in this equilibrium. However, there may be more solutions. For example, for $a = 2$, $b = 0.5$, $d = 7$, $q = 0.02$, $r = 0.6\,\theta = 0.05\,A = 0.3$ and $\pi^{\alpha\alpha} = \pi^{\beta\beta} = 0.15$ there are three stationary monetary equilibria in the economy: one coinciding with the perfect foresight equilibrium and two sunspot equilibria. Prices and consumption programs for these equilibria are given in the following table.

State	PFE	1st SSE	2nd SSE
α	(0.6670; 0.3330; 3.00)	(0.5973; 0.4027; 2.48)	(0.7518; 0.2482; 4.03)
β	(0.6670; 0.3330; 3.00)	(0.7518; 0.2482; 4.03)	(0.5973; 0.4027; 2.48)

(In every entry, the first number is consumption of young, the second is consumption of old, and the third is nominal price of consumption good.)

The consumption programs in sunspot equilibria are Pareto inferior to the program in the perfect foresight equilibrium. Furthermore, in two sunspot equilibria old agents survive in one state of nature and fail to survive in another *with the same amount of resources*, because equilibrium price is too high. (We intentionally considered the case where agents survive in the certainty equilibrium to demonstrate that survival is always feasible. Also, in this model young agents always survive, – otherwise, the overlapping generations structure collapses.)

4 Survival and Specialization

The entitlement approach in the study of famines suggests that one has to explicitly take into account: (1) some goods or services produced in the economy are less essential for survival than others, and (2) some groups of agents are involved in production of these "less essential" goods and services and supply them to the market in exchange for "more essential" ones, and, therefore, are more vulnerable to starvation. The first observation can be formalized as the existence of asymmetry in preferences, and the second one – as the existence of specialization in output (or endowments) among agents.

A model of a static economy with asymmetric preferences and complete specialization was introduced by Desai in [10]. The idea is the following. In order to survive an agent needs to consume an essential good (or goods) at or above some minimum level. Only after attaining the minimum level of consumption of the essential good the agent can derive utility from consumption of other, non-essential goods. If some agents in the economy are not initially endowed with the essential good, they have to purchase the essential good in the market. We modify Desai's model to incorporate the probability of survival when the consumption of the essential good falls below the minimum subsistence level[5]. There are two goods, essential ("food", labeled x) and non-essential ("non-food", labeled y) in consumption, and two agents, a food producer (agent 1) and a non-food producer (agent 2). The food producer is endowed with $f > 0$ units of good x, and the non-food producer with

[5] This formulation eliminates the "degeneracy" of the equilibrium with starvation, in which an agent enjoys "minus infinity" utility, and, furthermore, prefers one "minus infinity" level of utility to another "minus infinity", see [10], p.434.

$e > 0$ units of good y. The agents have identical preferences, and each needs a minimum quantity of food, f_i^*, to survive. The probability $\mathcal{P}$ of survival of agent i is the function of i's consumption of food:

$$\mathcal{P}\left[i \text{ survives}\right] = \begin{cases} 0, & x_i \leq \bar{x}_i \\ v(x_i), & \bar{x} < x_i \leq f_i^* \\ 1, & x_i > f_i^* \end{cases}$$

where $v(x)$ is continuous and strictly increasing [6]. Without loss of generality we further assume $\bar{x}_i = 0$ and $f_i^* = f^*$ for $i = 1, 2$. Hence, if $x_i \in (0, f^*)$, i starves, and is ruined with probability $1 - v(x_i)$ because of starvation. Given his budget constraint, an agent i, first, maximizes his probability of survival (or minimizes probability of ruin), and, second, if he survives with probability one, maximizes his utility of consumption *above* the survival level:

$$u_i = u_i\left(\left(x_1 - f_i^*\right), x_2\right)$$

(the survival level of the non-essential good y is zero), where $u_i(\cdot)$ is strictly concave, strictly increasing and twice continuously differentiable. We can formally combine these two consecutive objectives of an agent into one objective function:

$$U_i\left(x_i, y_i\right) = \begin{cases} v\left(x_i\right), & x_i \leq f^* \\ v\left(f^*\right) + u\left(\left(x_i - f^*\right), y_i\right), & x_i > f^* \end{cases}$$

We assume

$$u(\cdot) = \left(x_i - f^*\right)^{\alpha_i} y_i^{\beta_i}, \tag{29}$$

$x_i > f^*$, $y_i > 0$, $\alpha_i, \beta_i \in (0, 1)$, $\alpha_i + \beta_i \leq 1$. The functional form of $v(x_i)$ in a static model is irrelevant, as long as it is strictly increasing in x_i. Let p be the price of food in terms of non-food consumption good. The *market equilibrium* in this economy is the set of consumption vectors $\{(x_1, y_1), (x_2, y_2)\}$ and price p such that

(i) given p, agent i maximizes his objective function U_i subject to his budget constraint:

$$p\, x_i + y_i \leq p\, f_i + e_i$$

$i = 1, 2$, $(f_1, e_1) = (f, 0)$, and $(f_2, e_2) = (0, e)$.

(ii) markets clear:

$$x_1 + x_2 = f,$$
$$y_1 + y_2 = e.$$

[6] More generally, probability function is non-decreasing and right-continuous. We use stronger assumptions to ensure uniqueness.

Because of the asymmetric preferences and complete specialization the non-food producer is more vulnerable to starvation [7]. Consider different cases.

Case 1. Absolute scarcity.
Suppose, the harvest is so low that it cannot feed the food producer himself: $f \leq f^*$. The food producer consumes all his endowment and survives with positive probability. There is no trade, and the non-food producer is ruined with probability 1.

Case 2. Aggregate scarcity.
If the endowment of the food producer exceeds his minimum subsistence level, he sells some of good x to the non-food producer in exchange for good y. However, if the total amount of food is not enough to feed both agents, $f^* < f < 2f^*$, the non-food producer starves and is ruined with positive probability.

Case 3. Aggregate availability.
Suppose now, that the total amount of food is enough to feed both agents, $f > 2f^*$. However, as the analysis below shows, this condition is necessary, but not sufficient to allow the non-food producer to survive with probability one in the equilibrium. Below we derive the necessary and sufficient condition of survival of both agents with probability one. We show that this condition does not involve endowment and preferences of the non-food producer.

Now we proceed to the formal analysis. Case 1 is irrelevant for our purpose, because there is no trade in that case. In two other cases consumption of agent 1 (food producer) is

$$(x_1, y_1) = \left(f^* + \frac{\alpha_1}{\alpha_1 + \beta_1} (f - f^*), \frac{\beta_1}{\alpha_1 + \beta_1} p (f - f^*) \right).$$

At price p the wealth, in terms of food, of agent 2 (non-food producer) is e/p. If this wealth is below f^*, he sells all his endowment in good y for good x, and his consumption is then

$$(x_2, y_2) = \left(\frac{e}{p}, 0 \right)$$

From the market clearance condition the equilibrium price is

$$p^* = \frac{\alpha_1 + \beta_1}{\beta_1} \frac{e}{f - f^*}. \tag{30}$$

Hence, agent 2 starves and is ruined with probability $1 - v\left(\frac{\beta_1}{\alpha_1+\beta_1} (f - f^*) \right)$ (notice, that this probability depends only on the endowment and preferences of agent 1). This happens when $e/p^* < f^* < f$, or, using (30),

$$f^* < f < f^* \left(2 + \frac{\alpha_1}{\beta_1} \right). \tag{31}$$

[7] We assume free disposal for good y.

Clearly, it can happen that the non-food producer starves even when the amount of food is more than enough to feed both agents, i.e.,

$$2f^* < f < f^* \left(2 + \frac{\alpha_1}{\beta_1}\right).$$

This is an example of *exchange entitlement failure.*

If the wealth of agent 2 is above f^*, he survives with probability one, and his consumption is

$$(x_2, y_2) = \left(f^* + \frac{\alpha_2}{\alpha_2 + \beta_2}\left(\frac{e}{p} - f^*\right), \frac{\beta_2}{\alpha_2 + \beta_2}(e - pf^*)\right).$$

Equilibrium price in this case is

$$p^* = \frac{\alpha_2/\left(\alpha_2 + \beta_2\right)}{\beta_1/\left(\alpha_1 + \beta_1\right)} \frac{e}{f - f^*\left(1 + \frac{\beta_2(\alpha_1+\beta_1)}{\beta_1(\alpha_2+\beta_2)}\right)}. \tag{32}$$

Hence, the non-food producer survives with probability 1 when $e \geq p^* f^*$, or, using (32),

$$f \geq f^* \left(2 + \frac{\alpha_1}{\beta_1}\right). \tag{33}$$

This is the necessary and sufficient condition for survival of both types. One can see that, holding the aggregate amount of food fixed, agent 2 is more likely to starve, the more agent 1 prefers his own good over the other good. Neither non-food producer's tastes nor his endowment affect his probability of survival.

An important policy implication is that under condition (31) increase in the endowment of the non-food producer will not improve his food purchasing power. Hence, the famine remedy in this case can be (i) redistribution or (ii) direct food support. An interesting question in this regard is whether universal or targeted food support is more efficient. Drèze and Sen provide an extensive discussion of this issue in [12], Chapter 7. In the context of this model, the universal support means giving equal amount of food to both agents, and targeted support means giving the total amount of food aid to the most vulnerable agent, i.e. to the non-food producer. If the objective of the relief agency is to maximize the probability of survival of all agents, then in situation (31), because type 1 survives with probability 1 given his endowment, the relief agency can do either of the following:

(a) Give food to type 2 only. Type 2 will survive with probability 1 if the amount of food aid is, at least, $f_a \equiv f^* - \dfrac{\beta_1}{\alpha_1 + \beta_1}(f - f^*)$;

(b) Give equal amounts of food to both types. If each type receives $1/2 f_b$, such that $1/2 f_b < f^*$, then type 2 will survive with probability 1 if the value (in terms of food) of his endowment is at least $f^* - 1/2 f_b$. Simple calculations render $f_b = 2\left(f^* - \dfrac{f}{2 + \alpha_1/\beta_1}\right)$.

It is straightforward to show that $f_a < f_b$, i.e. targeted support is more efficient than universal support (requires less resources, holding cost of distribution equal), if and only if $f^* < f < f^*\,(1 + \alpha_1/2\beta_1)$. In other words, the model suggests that at relatively high aggregate amount of food in the economy the food aid from outside should be distributed equally among food- and non-food producers. At relatively low aggregate amount of food in the economy the food aid from outside should be directed to the non-food producers.

5 Concluding remarks

Joan Robinson ([30], p.189) wrote that "the hidden hand will always do its work, but it may work by strangulation." It has of course been an achievement of high order to spell out the conditions under which the price system can play an effective role in coordinating decentralized decisions in order to generate a Pareto efficient allocation of resources. From all indications it appears, sadly enough, that the strangulation by the invisible hand will haunt millions, especially in the century of globalization. General equilibrium analysis is very much relevant in understanding the full implications of economic policy to improve the probability of survival. But help will not come from models in which uncertainty has no essential role to play or in which the consumers have the luxury of "choosing their life spans" explicitly or implicitly. Much remains to be done in developing models that can throw light on the survival issues.

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