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Foreign Trade and Equilibrium Indeterminacy

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Abstract

We show that dependence of production on foreign inputs (or non-producible natural resources) can significantly increase the likelihood of indeterminacy. Payment of imported foreign factors of production may act as a semi-fixed cost, amplifying production externalities and returns to scale, making self-fulfilling expectations driven busyness cycles easier to arise. This is demonstrated using a standard neoclassical growth model. Calibration exercise shows that the required increasing returns to scale can be reduced by as much as 64% based on estimated share of foreign inputs in production for OECD countries.

Keywords: Indeterminacy, Factor Imports, Natural Resources, Capacity Utilization, Externality, Returns to Scale, Open Economy, Sunspots, Self-Fulfilling Expectations.

JEL Classification: E13, E20, E30.

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Based on input-output tables of OECD countries, imports of intermediate goods and raw materials account for a significant fraction of total inputs in domestic production. Table 1 reports the average share of the value of imports in domestic production across 35 production sectors for each of the ten OECD countries considered. These cost shares of foreign inputs range from 5.4% (Japan) to 21.1% (Netherlands). The average cost share of foreign inputs among the ten countries is about 13%. Hence, imported production factors play a non-trivial role in an economy's production process.[1]

Table 1. Cost Share of Foreign Inputs in Domestic Production

Australia	Canada	Denmark	France	Germany	Italy	Japan	Netherlands	U.K.	U.S.
10%	16%	20%	13%	14%	12%	5%	21%	16%	6%

What does the dependence of production on foreign inputs imply for economic fluctuations? Payment to foreign inputs acts as a tax for production, hence it has not only a wealth effect but also a substitution effect. Under the wealth effect, a higher factor price for imported materials reduces domestic income, leading to lower consumption and higher labor supply. Under the substitution effect, the higher factor price leads to lower demand for foreign materials, depressing productivity of capital and labor, resulting in lower employment. The substitution effect can be dramatically amplified if there exist production externalities in an economy.

This paper shows that dependence of production on foreign inputs is an important channel leading to indeterminacy in a standard neoclassical growth model with imperfect competition or production externalities. In particular, we prove that indeterminacy is easier to obtain the larger the share of imported factors is in domestic production. The key is that payments for foreign imports can act as a semi-fixed cost that amplifies returns to scale, rendering the balanced growth path of a neoclassical economy more likely to be indeterminate. In such a model, a fear or speculation of an increase in the imported factor price, say due to political instability in the foreign country, can trigger "pessimisms", making economic recessions self-fulfilling.

The imported foreign factor of production in our model can also be interpreted as non-producible natural resources extracted domestically. Hence the implication of our model is not limited to open economies with trade. We call the non-producible production factor foreign input in this paper because we have better data to calibrate its share in GDP.[2]

Equilibrium indeterminacy in a standard neoclassical growth model with externalities or increasing returns to scale has received significant amount of attention in the recent business cycle literature due to the pioneering work of Benhabib

[1]Data are based on input-output tables from OECD (1995) Reports. Each input-output table has 35 sectors and contains value of imported inputs used by each sector. The figures shown in the text were calculated by dividing the total value of imported inputs by the total value of production of all sectors.

[2]We thank Karl Shell for suggesting this interpretation to us.

and Farmer (1994). It is now widely viewed as a promising vehicle for studying endogenous business cycles and sunspots driven fluctuations.[3] Although this first-generation indeterminate RBC model requires implausibly large degrees of externalities to generate indeterminacy (thereby casting doubt on their empirical relevance, see e.g., Schmitt-Grohe 1997), subsequent work by Benhabib and Farmer (1996), Benhabib and Nishimura (1997), Benhabib, Nishimura and Meng (2000), Bennett and Farmer (2000), Perli (1998), Weder (1998 and 2001) and Wen (1998), among many others, show that adding other standard features of real economies into the model of Benhabib and Farmer (1994) can reduce the degree of externalities required for inducing local indeterminacy.[4] This line of research discovers that features such as additional sectors of production, durable consumption goods, non-separable utility functions, small open economy, or variable capacity utilization can reduce the required externalities for local indeterminacy to a degree that is within empirically admissible range. In this paper we add to this fast growing literature another mechanism for indeterminacy: the dependence of production on foreign inputs. Based our calibrations, we show that when the share of foreign inputs in domestic production increases from zero percent to five percent, the required degree of increasing returns to scale for indeterminacy is reduced by as much as 17%; and if the share increases to 20%, then the corresponding reduction can be as large as 64%.[5]

The rest of the paper is organized as follows. To illustrate the basic mechanism of indeterminacy due to dependence on foreign imports as production factors, we first investigate a benchmark model (Benhabib and Farmer, 1994) by assuming that production of intermediate goods requires not only capital and labor, but also a third factor, say "oil", imported from outside the economy. For simplicity, we assume that this third factor is perfectly elastically supplied. Later on, we will introduce a foreign monopoly power that supplies the third factor with arbitrary elasticity of supply, so as to study the robustness of our results. Finally, we will calibrate a more realistic model with variable capacity utilization (Wen, 1998) and show that dependence of foreign imported factors of production can significantly decrease the degree of externality required for indeterminacy so that the social returns to scale are essentially constant for multiple equilibria to emerge.

[3]For the broader literature on sunspots, please see Shell (1977, 1987), Cass and Shell (1983), Shell and Smith (1992), Azariadis (1981), Azariadis and Guesnerie (1986), and Woodford (1986a, 1986b, 1991).

[4]See Wen (2001) for a recent analysis of this class of models regarding mechanisms giving rise to local indeterminacy from the viewpoint of the permanent-income hypothesis. For open economy models with indeterminacy, see Weder (2001), Meng (2003), and Meng and Velasco (2003).

[5]Our model differs from that of Weder (2001). In Weder (2001), indeterminacy is easier to obtain for a small open economy due to perfect or nearly perfect world capital markets that keep interest rate more or less constant. In his model, the ability to use international credit markets to disconnect savings and investment is an important mechanism for indeterminacy. Weder's model also requires some form of negative externalities to ensure stability of the steady state. In our model, the economy does not need to be small and the foreign factor markets do not need to be perfectly competitive. The mechanism for indeterminacy in our model is through production costs due to payments for foreign factors, such as oil, which are not producible at home.

1. The Benchmark Model

This is a slightly modified version of the Benhabib-Farmer (1994) model. There are two production sectors in the economy, the final goods sector and the intermediate goods sector. The final goods sector is competitive and it uses a continuum of intermediate goods to produce final output according to the production technology,

$$Y = \left(\int_{i=0}^{1} y_i^{\lambda} di \right)^{\frac{1}{\lambda}}$$

where $\lambda \in (0, 1)$ measures the degree of factor substitution among intermediate goods. Let p_i be the relative price of the ith intermediate goods in terms of the final good, the profits of final good producer are given by

$$\Pi = Y - \int_{i=0}^{1} p_i y_i di.$$

First order conditions for profit maximization lead to the following inverse demand functions for intermediate goods:

$$p_i = Y^{1-\lambda} y_i^{\lambda-1}.$$

The technology for producing intermediate goods is given by

$$y_i = k_i^{a_k} n_i^{a_n} o_i^{a_o},$$

where the third factor in production, o, is imported, and $(a_k + a_n + a_o) \geq 1$ measures returns to scale at the firm level. Assuming that firms are price takers in the factor markets, the profits of the ith intermediate good producer are given by

$$\pi_i = p_i y_i - (r + \delta) k_i - w n_i - p^o o_i,$$

where $(r + \delta)$ denotes the user cost of renting capital, w denotes real wage, and p^o denotes the real price of oil (the imported good). The intermediate goods producers are monopolists facing downward sloping demand curves for intermediate goods, hence the profit functions can be rewritten as

$$\pi_i = Y^{1-\lambda} y_i^{\lambda} - (r + \delta) k_i - w n_i - p^o o_i,$$

which is concave as long as $\lambda(a_k + a_n + a_o) \leq 1$. Profit maximization by each intermediate goods producing firm leads to the following first order conditions:

$$r + \delta = \lambda a_k \frac{p_i y_i}{k_i}$$

$$w = \lambda a_n \frac{p_i y_i}{n_i}$$

$$p^o = \lambda a_o \frac{p_i y_i}{o_i}.$$

In a symmetric equilibrium, we have $n_i = n, k_i = k, o_i = o, y_i = y = Y, \pi_i = \pi, p_i = 1$, and

$$\Pi = Y - \left(\int_{i=0}^{1} y_i^{\lambda} di \right)^{\frac{1}{\lambda}} = 0$$

$$\pi = \left(1 - \lambda(a_k + a_n + a_o)\right) Y.$$

In words, perfect competition in the final goods sector leads to zero profit and imperfect competition in the intermediate goods sector leads to positive profit if $\lambda(a_k + a_n + a_o) < 1$.

A representative consumer in the economy maximizes utility,

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t - b \frac{n_t^{1+\gamma}}{1+\gamma} \right)$$

subject to

$$c_t + s_{t+1} = (1 + r_t) s_t + w_t n_t + \pi_t,$$

where s is aggregate saving. Since the aggregate factor payment, $p^o o$, goes to the foreigners, it is not included in the consumer's income. The first order conditions for utility maximization with respect to labor supply and savings are given respectively by

$$b n_t^{\gamma} = \frac{1}{c_t} w_t,$$

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \left(1 + r_{t+1}\right).$$

In equilibrium, $s_t = k_t$, and factor prices equal marginal products, the first order conditions and the budget constraint then become

$$b n_t^{1+\gamma} = \frac{1}{c_t} \lambda a_n y_t \tag{1}$$

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \left(1 - \delta + \lambda a_k \frac{y_{t+1}}{k_{t+1}} \right) \tag{2}$$

$$c_t + k_{t+1} = (1 - \delta) k_t + (1 - \lambda a_o) y_t \tag{3}$$

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_o}. \tag{4}$$

2. Conditions for Indeterminacy

Assuming that the foreign input is perfectly elastically supplied, then the factor price, p^o, is independent of the factor demand for o.[6] Hence we can substitute out o_t in the production function using

$$o_t = \lambda a_o \frac{y_t}{p^o},$$

[6]This assumption will be relaxed in section 3.

to obtain the following reduced-form production function:

$$y_t = A k_t^{\frac{a_k}{1-a_o}} n_t^{\frac{a_n}{1-a_o}}, \tag{4'}$$

where $A = \left(\frac{\lambda a_o}{p^o}\right)^{\frac{a_o}{1-a_o}}$ acts as the technology coefficient in a neoclassical growth model, which is inversely related to the foreign factor price. In this reduced-form production function, the effective returns to scale is measured by

$$\frac{a_k + a_n}{1 - a_o},$$

which exceeds the true returns to scale, $(a_k + a_n + a_o)$, provided that $(a_k + a_n + a_o) > 1$. Hence, the reliance on foreign factors amplifies the true returns to scale.

It can be easily shown that a unique steady state exists in this economy. To study indeterminacy, we substitute y by utilizing equation $(4')$ and log linearize equations (1)-(3) around the steady state. This gives

$$\left(1 + \gamma - \frac{a_n}{1 - a_o}\right) \hat{n}_t = \frac{a_k}{1 - a_o} \hat{k}_t - \hat{c}_t$$

$$-\hat{c}_t = -\hat{c}_{t+1} + (1 - \beta(1 - \delta)) \left(\left(\frac{a_k}{1 - a_o} - 1 \right) \hat{k}_{t+1} + \frac{a_n}{1 - a_o} \hat{n}_{t+1} \right)$$

$$(1 - s)\hat{c}_t + \frac{s}{\delta} \hat{k}_{t+1} = \left(\frac{a_k}{1 - a_o} + s \frac{1 - \delta}{\delta} \right) \hat{k}_t + \frac{a_n}{1 - a_o} \hat{n}_t$$

where s is the adjusted steady-state saving rate (investment-to-national income ratio) given by

$$s = \frac{\delta k}{(1 - \lambda a_o) y} = \frac{\delta \beta \lambda a_k}{(1 - \lambda a_o)(1 - \beta(1 - \delta)}.$$

The above system of linear equations can be reduced to

$$M_1 \begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = M_2 \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}$$

where

$$M_1 = \begin{bmatrix} \frac{1-\beta(1-\delta)}{1-a_o} \left(a_k + a_o - 1 + \frac{a_n a_k}{(1+\gamma)(1-a_o) - a_n} \right) & 1 + \frac{(1-\beta(1-\delta))a_n}{(1+\gamma)(1-a_o) - a_n} \\ s\frac{1}{\delta} & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 0 & 1 \\ \frac{a_k}{1-a_o} \left(1 + \frac{a_n}{(1+\gamma)(1-a_o) - a_n} \right) + s\frac{1-\delta}{\delta} & \frac{(1+\gamma)(1-a_o)}{(1+\gamma)(1-a_o) - a_n} - s \end{bmatrix}$$

Denote $B = M_1^{-1} M_2$, a necessary and sufficient condition for indeterminacy is that both eigenvalues of B are less than one in modulus. This is true if and only if the determinate and the trace of B satisfy

$$\begin{aligned} -1 &< \det(B) < 1 \\ -(1 + \det(B)) &< \mathrm{tr}(B) < 1 + \det(B) \end{aligned}$$

The determinate and the trace of B are given by (see Appendix 1):

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta(1-\delta))\frac{(1-\lambda)}{\lambda(1-a_o)}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}}\right] \tag{5}$$

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta(1-\delta))(1+\gamma)\left(\frac{1-a_o-a_k}{1-a_o}\right)\delta\frac{1-s}{s}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}} \tag{6}$$

Notice that when $\lambda = 1$, then $\det(B) = 1/\beta > 1$, indicating saddle-path-stability as in a standard RBC model. Hence, what is crucial for indeterminacy is not increasing returns to scale per se, but also the degree of market power or imperfect competition.

The common denominator in the second term in expression (5) and the third term in (6) suggests that when the labor's elasticity of output in the reduced-form production function, $\frac{a_n}{1-a_o}$, increases, the model may go through a point of discontinuity at which $1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o} = 0$ and $\det(B)$ and $\mathrm{tr}(B)$ both change sign from $+\infty$ to $-\infty$, if the condition $1-a_o-a_k > 0$ still holds. Clearly, when these terms are negative infinity, the conditions for $\det(B) < 1$ and $\mathrm{tr}(B) < 1 + \det(B)$ are trivially satisfied. But to reach the discontinuity point such that the second term in (5) and the third term in (6) are negative, we need

$$\beta(1-\delta)\frac{a_n}{1-a_o} > 1+\gamma. \tag{7}$$

(7) is an important necessary condition for indeterminacy. Clearly, the larger is a_o, the easier this condition can be satisfied. To facilitate interpreting this condition, we map the monopolistic competition model into a one-sector competitive model with production externalities (see Benhabib and Farmer, 1994), in which the aggregate production function is replaced by

$$y_t = k_t^{\alpha_k(1+\eta)} n_t^{\alpha_n(1+\eta)} o_t^{\alpha_o(1+\eta)},$$

and the reduced-form production function is replaced by

$$y_t = A k_t^{\frac{\alpha_k(1+\eta)}{1-\alpha_o(1+\eta)}} n_t^{\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)}}$$

where $(\alpha_k + \alpha_n + \alpha_o) = 1$ and the parameter η measure the degree of production externalities. This model is identical to the monopolistic competition model if $\lambda a_k = \alpha_k$, $\lambda a_n = \alpha_n$, $\lambda a_o = \alpha_o$, and $(a_k + a_n + a_o) = 1+\eta$. This gives $\lambda(a_k + a_n + a_o) = \lambda(1+\eta) = 1$, implying that in the corresponding monopolistic competition model the intermediate goods producing firms earn zero profits. In the externality version of the model, aggregate returns to scale are measured by $1+\eta$. With this change in framework, equations (5) and (6) become

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta(1-\delta))\frac{\eta}{1-\alpha_o(1+\eta)}}{1+\gamma-\beta(1-\delta)\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)}}\right] \tag{5'}$$

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta(1-\delta))(1+\gamma)\left(\frac{1-(\alpha_o+\alpha_k)(1+\eta)}{1-\alpha_o(1+\eta)}\right)\delta\frac{1-s}{s}}{1+\gamma-\beta(1-\delta)\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)}} \qquad (6')$$

Clearly, indeterminacy is not possible if $\eta = 0$, which implies $\det(B) = 1/\beta > 1$. This shows that monopoly power in the previous version of the model pertains to externality in the current version of the model. Condition (7) thus becomes

$$\beta(1-\delta)\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)} - 1 > \gamma,$$

which can also be expressed as

$$\eta > \frac{(1+\gamma)(1-\alpha_o) - \beta(1-\delta)\alpha_n}{\beta(1-\delta)\alpha_n + (1+\gamma)\alpha_o}. \qquad (7')$$

Condition (7′) is analogous to that derived by Benhabib and Farmer (1994) in a continuous time model when $\delta \to 0$ and $\beta \to 1$. In a continuous time version of the model, this condition simplifies to

$$\eta > \frac{1-(\alpha_o+\alpha_n)+\gamma(1-\alpha_o)}{(\alpha_o+\alpha_n)+\gamma\alpha_o}.$$

If $\alpha_o = 0$ (i.e., production does not require the imported factor), then this condition for indeterminacy is identical to that in Benhabib and Farmer (1994). Since the right hand side is a decreasing function of α_o, this necessary condition for indeterminacy is easier to satisfy than that in the Benhabib-Farmer model.

To further pin down the full set of conditions for indeterminacy, note that as long as $(\alpha_o + \alpha_k)(1+\eta) < 1$, the second term in the determinate of B and the third term in the trace of B must pass through $-\infty$ for large enough η and moves to a finite negative number as η keeps increasing. Since we are interested only in the smallest value of η that gives rise to indeterminacy, we can therefore limit our attention to the following simpler one-sided conditions as necessary and sufficient conditions for indeterminacy:

$$\det(B) > -1 \text{ and } \mathrm{tr}(B) > -(1+\det(B)),$$

assuming the necessary condition (7′) is satisfied.

The condition $\det(B) > -1$ is equivalent to

$$\eta > \frac{(1+\gamma)(1-\alpha_o) - \beta(1-\delta)\alpha_n}{\beta(1-\delta)\alpha_n + (1+\gamma)\alpha_o - \frac{1+\gamma}{1+\beta}(1-\beta(1-\delta))}.$$

Note that if this condition is satisfied, then condition (7′) is also satisfied since they differ only by a positive term, $\frac{1+\gamma}{1+\beta}(1-\beta(1-\delta))$. In a continuous time version of this model ($\delta \to 0, \beta \to 1$), this condition simplifies to

$$\eta > \frac{1-(\alpha_o+\alpha_n)+\gamma(1-\alpha_o)}{(\alpha_o+\alpha_n)+\gamma\alpha_o},$$

which is identical to (7′). Hence, an increase in α_o, either holding α_n constant or holding $(\alpha_o + \alpha_n)$ constant, will decrease the right hand side, making indeterminacy easier to arise.

The condition $\mathrm{tr}(B) > -(1 + \det(B))$ leads to

$$\frac{1+\eta}{1-\alpha_o(1+\eta)} > \frac{2(1+\gamma)(2-\delta) + (1+\gamma)\delta\frac{1-s}{s}(1-\beta(1-\delta))}{2(1+\beta)(1-\delta)\alpha_n - (1+\gamma)\frac{\delta\alpha_k}{s}\left(2-(1-s)(1-\beta(1-\delta))\right)}.$$

Clearly, the presence of α_o on the left-hand side makes the inequality easier to satisfy the larger the value of α_o is. Alternatively, we can consider a continuous time version of the model ($\delta \to 0, \beta \to 1$), then the above condition simplifies to

$$\frac{1+\eta}{1-\alpha_o(1+\eta)} > \frac{1+\gamma}{\alpha_n},$$

which implies

$$\eta > \frac{1-(\alpha_o+\alpha_n)+\gamma(1-\alpha_o)}{(\alpha_o+\alpha_n)+\gamma\alpha_o}.$$

This is identical to the condition implied by $\det(B) > -1$. Hence, the necessary and sufficient conditions for indeterminacy are all easier to be satisfied if $\alpha_o > 0$.

3. Robustness

The necessary and sufficient conditions show that indeterminacy is easier to occur the larger the share of the imported production factor in aggregate output is. However, the above results are obtained under the assumption that the supply of foreign factor is perfectly elastic. This section examines the robustness of our result when imperfectly elastic supply of the imported factor o_t is allowed.

To incorporate a less elastic supply of foreign factors, we assume that oil is supplied by a monopolist foreign country whose objective function is to maximize profit:

$$\Pi^f = p_t^o o_t - \frac{b}{1+\zeta} o_t^{1+\zeta},$$

where the cost function of oil production is concave ($\zeta \geq 0$). Given the inverse aggregate demand function of oil from home country, $p_t^o = \frac{\alpha_o y_t}{o_t}$, profit maximization of the foreign country leads to the following first order condition:

$$\alpha_o \frac{\partial y}{\partial o} = b o_t^{\zeta}.$$

This implies that the supply curve for oil is given by

$$p_t^o = \frac{b}{\alpha_o} o_t^{\zeta},$$

where $1/\zeta$ measures the elasticity of supply.

It is easy to show that when supply meets demand, the home country's reduced form production function becomes:

$$y_t = A k_t^{\frac{\alpha_k(1+\eta)}{1-\frac{\alpha_o(1+\eta)}{1+\zeta}}} n_t^{\frac{\alpha_n(1+\eta)}{1-\frac{\alpha_o(1+\eta)}{1+\zeta}}}, \tag{8}$$

where A is a productivity parameter that depends negatively on the cost parameter b. Note that if $\zeta = 0$, then the model is reduced to the previous one with perfectly elastic supply of the foreign factor. Clearly, as long as the supply elasticity of the foreign factor, $\frac{1}{\zeta}$, is not too small (or ζ not too large), the implication for indeterminacy is the same: namely, the dependence of production factors on foreign imports ($\alpha_o > 0$) makes indeterminacy easier to occur since the share parameter of foreign factor in production continues to magnify the aggregate returns to scale of the home country if $\zeta < \eta$:

$$\frac{\alpha_k(1+\eta)}{1-\frac{\alpha_o(1+\eta)}{1+\zeta}} + \frac{\alpha_n(1+\eta)}{1-\frac{\alpha_o(1+\eta)}{1+\zeta}} > (1+\eta).$$

Hence, as long as $\zeta < \eta$, indeterminacy is easier the larger α_o is. Note that except for the production function, none of the first order conditions of the previous model (equations 1-3) is affected by the fact $\zeta > 0$. For example, the resource constraint of the home country remains the same as before:

$$c_t + k_{t+1} - (1-\delta)k_t = y_t - p_t^o o_t = (1-\alpha_o)y_t,$$

in spite of a less elastic supply curve of o_t. In other words, the reduced form production function (8) is a sufficient indicator for the effect of factor supply elasticity on indeterminacy.

4. Calibration with Capacity Utilization

The above analysis based on a simple benchmark model provides the essential understanding on the mechanism as to how the dependence of production on foreign imported factors can increase the likelihood of indeterminacy under externalities or imperfect competition. Now we calibrate a more realistic neoclassical growth model with variable capital utilization, so as to show that indeterminacy can easily occur under essentially constant aggregate returns to scale.

This is the representative-agent version of the model of Wen (1998)[7] in which a representative agent chooses sequences of consumption (c), hours (n), capacity utilization (e), and capital accumulation (k) to solve

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - b \frac{n_t^{1+\gamma}}{1+\gamma} \right)$$

[7]For a simple proof for the equivalence between a representative-agent version with externality and a monopolistic-competition version of the model with increasing returns to scale, see Benhabib and Wen (2004). This equivalence continues to hold when certain factors of production are imported from foreign countries, as proved in the previous sections.

subject to

$$c_t + [k_{t+1} - (1-\delta_t)k_t] + p_t o_t = y(e_t k_t, n_t, o_t),$$

where the home country pays the amount $p_t^o o_t$ in terms of output to foreigners to receive the amount o_t as factor inputs,[8] and where the production technology is given by

$$y(e_t k_t, n_t, o_t) = \Phi_t \left(e_t k_t\right)^{\alpha_k} n_t^{\alpha_n} o_t^{\alpha_o}, \qquad \alpha_k + \alpha_n + \alpha_o = 1;$$

in which $e_t \in [0,1]$ denotes capital utilization rate, and Φ_t is a measure of production externalities and is defined as a function of average aggregate output which individual firms take as parametric:

$$\Phi_t = \left[(e_t k_t)^{\alpha_k} n_t^{\alpha_n} o_t^{\alpha_o}\right]^{\eta}, \quad \eta \geq 0.$$

The rate of capital depreciation, δ_t, is time variable and is endogenously determined in the model. In particular, it is assumed that capital depreciates faster if it is used more intensively:

$$\delta_t = \frac{1}{\theta} e_t^{\theta}, \quad \theta > 1;$$

which imposes a convex cost structure on capital utilization.[9]

Proposition 4.1. *The necessary and sufficient conditions for indeterminacy under variable capacity utilization are given by*

$$\eta > \frac{\theta\left[(1+\gamma)(1-\alpha_o) - \beta\alpha_n\right] - (1+\gamma)\alpha_k}{\theta\beta\alpha_n + (1+\gamma)(\alpha_k + \alpha_o\theta) - \theta(1+\gamma)\frac{1-\beta}{1+\beta}}, \tag{9}$$

$$1 + \eta > \frac{\left(2(1+\gamma) + (1-\beta)\phi\right)\theta}{\alpha_n(1+\beta)\theta - \alpha_k(1+\beta)\phi(\theta-1) + \left(2(1+\gamma) + (1-\beta)\phi\right)(\alpha_o\theta + \alpha_k)}; \tag{10}$$

where $\phi \equiv \frac{\delta}{2}(1+\gamma)\left((1-\alpha_o)\frac{\theta}{\alpha_k} - 1\right)$.

Proof. See Appendix 2.[10]■

We calibrate the model's structural parameters following Benhabib and Wen (2004) and Wen (1998). Namely, we set the time period in the model to a quarter, the time discounting factor $\beta = 0.99$, the steady-state rate of capital depreciation $\delta^* = 0.025$ (which implies $\theta = 1.404$), the inverse labor supply elasticity $\gamma = 0$, and the labor elasticity of output $\alpha_n = 0.7$. We also assume that the supply of the foreign factor of production is perfectly elastic (i.e., $\zeta = 0$). Given these parameter values, the following table shows that as the share of foreign factor in

[8] Note that trade is balanced in every period since the cost of intermediate goods – energy imports – are paid for with exports of output. Hence national income is given by $y - po$, which equals domestic consumption and capital investment.

[9] See Benhabib and Wen (2004) and Wen (1998).

[10] Note that conditions (9) and (10) are identical in the limit as $\beta \to 1$ (which implies $\theta \to 1$ also).

domestic production increases, the threshold value of the production externality for inducing indeterminacy (η^*) decreases dramatically. For example, when we increase the share parameter of foreign input α_o from zero percent to 10 percent, the reduction in the externality is 33%. And if we increase the share parameter to 20 percent, then the reduction in the externality is 64%.

Table 2. Effect of α_o on Indeterminacy

Factor Share (α_o)	Externality (η^*)	% Reduction of η
0.00	0.1037	0
0.05	0.0864	-0.17
0.10	0.0696	-0.33
0.15	0.0534	-0.49
0.20	0.0378	-0.64

The above table is based on the assumption that the foreign imported factor is mainly a substitute for capital, hence when α_o increases, α_n remains constant but α_k decreases such that $\alpha_k + \alpha_o$ remains constant (assuming constant returns to scale at the firm level). If we assume that the imported foreign factor is mainly a substitute for labor instead (i.e., $\alpha_n + \alpha_o$ is fixed), then a larger α_o also implies a smaller η^* although the reduction of externality is less dramatic as α_o increases. If we fix the ratio of α_k/α_n while increase α_o, then the reduction in η is between the first case and the second case. In the real world, however, a higher value of α_o presumably affects α_k more than it affects α_n, since imported production factors or materials are often treated by the literature as better substitutes for capital goods than for labor. The model, however, can also be applied to the case of foreign labor.

5. Conclusion

In this paper we showed that dependence of domestic production on foreign factors can significantly reduce the required degree of returns to scale for indeterminacy when the supply of foreign factors is sufficiently elastic. As a result, multiple equilibria and self-fulfilling expectations driven fluctuations can arise much more easily under very mild externalities or with essentially constant returns to scale.

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Appendix 1.

Given $B = M1^{-1}M2$, we have $det(B) = \frac{det(M2)}{det(M1)}$. Straightforward re-arrangement shows that

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta(1-\delta))\frac{(1-\lambda)}{\lambda(1-a_o)}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}}\right].$$

Also, given

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

we have $\mathrm{tr}(B) = b_{11} + b_{22}$, where $b_{11} = \frac{\delta}{s}\frac{(1+\gamma)a_k}{(1+\gamma)(1-a_o)-a_n} + 1 - \delta$, and $b_{22} = \frac{(1+\gamma)(1-a_o)-a_n}{(1+\gamma)(1-a_o)-\beta(1-\delta)a_n} - \left[\frac{\delta}{s}\frac{(1+\gamma)a_k}{(1+\gamma)(1-a_o)-a_n} - \delta\right]\left[\frac{(1-\beta(1-\delta))(a_n+(1+\gamma)(a_k+a_o-1))}{(1+\gamma)(1-a_o)-\beta(1-\delta)a_n}\right]$. Re-arrangement gives

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta(1-\delta))(1+\gamma)\left(\frac{1-a_o-a_k}{1-a_o}\right)\delta\frac{1-s}{s}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}}.$$

■

Appendix 2.

Denote λ_t as the Lagrangian multiplier for the budget constraint, the first order conditions with respect to $\{c, n, e, o, k\}$ and the budget constraint are given respectively by

$$\frac{1}{c_t} = \lambda_t \tag{A}$$

$$an_t^{\gamma} = \lambda_t \alpha_n \left(e_t k_t\right)^{\alpha_k(1+\eta)} n_t^{\alpha_n(1+\eta)} o_t^{\alpha_o(1+\eta)} \tag{B}$$

$$\alpha_k \frac{y_t}{k_t} = e_t^{\theta} \tag{C}$$

$$\alpha_o y_t = p_t o_t \tag{D}$$

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha_k\frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta}e_{t+1}^{\theta}\right] \tag{E}$$

$$c_t + k_{t+1} - (1 - \frac{1}{\theta}e_t^{\theta})k_t = (1-\alpha_o)y_t. \tag{F}$$

To simplify the analysis, we use equation (C) to substitute out e in the production function to get

$$y_t = Ak_t^{\alpha_k(1+\eta)\tau_k} n_t^{\alpha_n(1+\eta)\tau_n} o_t^{\alpha_o(1+\eta)\tau_n} \tag{G}$$

where $\tau_k \equiv \frac{\theta-1}{\theta-\alpha_k(1+\eta)}, \tau_n \equiv \frac{\theta}{\theta-\alpha_k(1+\eta)}$. Next, we use equation (D) to substitute out o in the production function (G) to get

$$y_t = \tilde{A}k_t^{\frac{\alpha_k(1+\eta)\tau_k}{1-\alpha_o(1+\eta)\tau_n}} n_t^{\frac{\alpha_n(1+\eta)\tau_n}{1-\alpha_o(1+\eta)\tau_n}}. \tag{H}$$

After similar substitutions in all equations, the above equation system is reduced to

$$c_t = \frac{\alpha_n}{a}\frac{y_t}{n_t^{1+\gamma}} \tag{A'}$$

$$c_{t+1} = \beta c_t\left[(1-\frac{1}{\theta})\alpha_k\frac{y_{t+1}}{k_{t+1}}+1\right] \tag{B'}$$

$$c_t + k_{t+1} - k_t = (1-\alpha_o-\frac{\alpha_k}{\theta})y_t \tag{C'}$$

where the production function is given by (H). Denote $a^* \equiv \frac{\alpha_k(1+\eta)\tau_k}{1-\alpha_o(1+\eta)\tau_n}, b^* = \frac{\alpha_n(1+\eta)\tau_n}{1-\alpha_o(1+\eta)\tau_n}$, log-linearize the above equations (A'-C') around the steady state and substitute out c_t using (A'), we have the following simplified 2-variable system:

$$(1+\beta(a^*-1))\hat{k}_{t+1} + (\beta b^* - (1+\gamma))\hat{n}_{t+1} = a^*\hat{k}_t + (b^*-(1+\gamma))\hat{n}_t$$

$$\hat{k}_{t+1} = \hat{k}_t + \left[(1-\alpha_o)\frac{\theta}{\alpha_k}-1\right]\delta(1+\gamma)\hat{n}_t$$

or

$$M_1\begin{bmatrix}\hat{k}_{t+1}\\ \hat{n}_{t+1}\end{bmatrix} = M_2\begin{bmatrix}\hat{k}_t\\ \hat{n}_t\end{bmatrix}$$

where

$$M_1 = \begin{bmatrix}1+\beta(a^*-1) & \beta b^*-(1+\gamma)\\ 1 & 0\end{bmatrix}$$

$$M_2 = \begin{bmatrix}a^* & b^*-(1+\gamma)\\ 1 & \left[(1-\alpha_o)\frac{\theta}{\alpha_k}-1\right]\delta(1+\gamma)\end{bmatrix}.$$

Hence, the Jacobian is given by

$$B = M_1^{-1}M_2 = \begin{bmatrix}1 & \left[(1-\alpha_o)\frac{\theta}{\alpha_k}-1\right]\delta(1+\gamma)\\ \frac{(1-\beta)(1-a^*)}{1+\gamma-\beta b^*} & \frac{1+\gamma-b^*+(1+\beta(a^*-1))\left[(1-\alpha_o)\frac{\theta}{\alpha_k}-1\right]\delta(1+\gamma)}{1+\gamma-\beta b^*}\end{bmatrix},$$

which implies that the determinate and the trace of B are given by (after simplification and re-arrangement):

$$\det(B) = \frac{1}{\beta}\left[1+\frac{\eta(1+\gamma)(1-\beta)\frac{\tau_n}{1-\alpha_o(1+\eta)\tau_n}}{1+\gamma-\beta b^*}\right]$$

$$\mathrm{tr}(B) = 1+\det(B)+\frac{(1-\beta)(1-a^*)\left[(1-\alpha_o)\frac{\theta}{\alpha_k}-1\right]\delta(1+\gamma)}{1+\gamma-\beta b^*}.$$

Following the same discussions in section 3, it can be shown that the value of η that satisfies the condition, $\det(B) > -1$, also satisfies the condition, $\beta b^* > 1+\gamma$, hence the necessary and sufficient conditions for indeterminacy can be limited to the value of η that satisfy:

$$\det(B) > -1 \text{ and } \mathrm{tr}(B) > -(1+\det(B)).$$

These two conditions imply the conditions in proposition 4.1■