# Communication and Bargaining in the Spatial Model

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August 31, 2005

This paper studies collective choice by participants possessing private information about the consequences of policy decisions in policymaking institutions that involve cheap-talk communication and bargaining. The main result establishes a connection between the extent to which problems of this type posses fully-revealing equilibria that select policies in the full information majority rule core (when it is well-defined) and the extent to which a fictitious sender-receiver game possesses a fully revealing equilibria. This result allows us to extend Banks and Duggan's (2000) core equivalence results to the case of noisy policymaking environments with private information when some combination of nonexclusivity and preference alignment conditions are satisfied.

## 1 Introduction

In many collective choice settings participants face uncertainty about the relationship between the policy levers that they can control and the eventual outcomes that they care about. In the presence of this uncertainty participants may collect information or be chosen on the basis of expertise. Thus, it is likely that collective choice problems also involve private information about the uncertain relationship between policies and outcomes. In addition to asymmetric information the participants may differ in their preferences or ideologies; they may disagree about which information contingent rule for selecting policy is optimal. The presence of divergent preferences opens up the possibility that agents may not be willing to reveal their information.

[1] I appreciate conversations with Kris Ramsay.

Despite the canonical nature of this description, basic questions about the possibility of efficiently aggregating information and preferences remain open. Much is known about strategic behavior in policymaking institutions without uncertainty (e.g., Baron and Ferejohn, 1989; Banks and Duggan; 2000) and questions of information transmission between agents and a principal (Crawford and Sobel, 1982; Gilligan and Krehbiel, 1989; Baron, 2000; Battaglini, 2002). However, little is known about strategic behavior amongst a collective of policymakers in the presence of asymmetric information. One might expect these considerations to surface in the study of deliberative democracy, an area that political scientists have become increasingly preoccupied with (e.g., Guttman and Thompson 1996). Unfortunately, scholars of deliberation tend to ignore the incentives for information transmission in deliberative settings. The few game theoretic works that focus on incentives (Austen-Smith and Feddersen, 2003a,b; Coughlan; 2000, Gerardi and Yariv 2003; Meirowitz 2003,2005) consider problems in which the set of alternatives is binary and exogenously given. This paper moves beyond existing theoretic work on deliberation, specifically, and collective choice, generally. It considers endogenous agendas, voting, and communication in the presence of informational asymmetries and preference divergence. More precisely, this paper investigates the extent to which institutions that allow for cheap talk communication and bargaining over policy can effectively aggregate preferences and information.

The spatial model has become a centerpiece of the literatures on legislative politics, agenda theory, and social choice theory. Because of this, we focus on the problem of simultaneously aggregating information and preferences when agents have spatial preferences over outcomes which admit a non-empty majority rule core and agents possess private information about a policy shock. The first main result connects two distinct literatures – bargaining and signalling. We establish an equivalence between the problem of finding equilibria that reach the full information majority rule core in communication and bargaining games and the problem of finding truthful equilibria in particular cheap-talk signalling games. Motivated by this equivalence, the two results offer a characterization of the preference profiles and informational environments in which cheap-talk signalling games posses truthful equilibria. In the case of private signals that are neither conditionally independent nor identically distributed these results may be of particular interest.

It is amusing to note that this work brings the literature on cheap talk communication full circle. Crawford and Sobel (1982) motivate their path-breaking investigation of cheap-talk signalling with a discussion of bargaining problems.

> Bargainers typically have different information about preferences and even what is feasible. Sharing information makes available better potential agreements but it also has strategic effects that make one suspect that revealing all to an opponent is not usually the most advantageous policy....While our primary motivations stem from the theory of bargaining, we have found it useful to approach these questions in a more abstract setting, which allows us to identify the essential prerequisites for the solution we propose. (p. 1431).

While the literature on cheap talk signalling games is now quite extensive, connections between this literature and the problem of communication and bargaining have not yet been explored. The equivalence result presented here, moves in this direction by showing that in the context of well behaved spatial policymaking models, the solution to a bargaining problem with communication is "the same as" the solution to a cheap-talk signaling game with multiple senders.

Many of the logical steps needed for the development are present in the extant literature. An extension of Banks and Duggan's (2000) core characterization result to the case of common knowledge of a shock to policy leads to the conclusion that when preferences over outcomes satisfy the Plott (1967) conditions no delay stationary Bayesian equilibria reach the expected ideal policy of the participant with the core ideal point. Following the approach of Myerson's (1982) revelation principle for direct coordination mechanisms we augment the endogenous agenda game with a round of communication, and show that the incentive compatibility conditions for information revelation correspond to those in a simple cheap-talk game in which the receiver has the preferences of the core voter.[2] Finally, a result in Baron and Meirowitz (2004), and a generalization of the conditions satisfied in Battaglini (2002) allow us to present necessary and sufficient conditions on the informational environment and preferences for satisfaction of these incentive compatibility conditions.

[2]Baliga and Morris (2002) and Kim (2005) consider the value of pre-play communication in two player-games.

## 2 The Model

We consider the following collective choice problem. A policy $p \in \mathbb{R}^d$ must be chosen. A set of $n$ (odd) participants, $N$, have preferences that depend on the policy, $p$, and a random shock $\varepsilon \in \mathbb{R}^d$. Participants do not observe the shock, but each $i \in N$ observes a private signal $s_i \in \mathbb{R}^d$ that is correlated with $\varepsilon$. In this setting an informational environment is a joint distribution on the random variables $(\varepsilon, \mathbf{s}) := (\varepsilon, s_1, s_2, ..., s_n)$ , Let $F(\varepsilon, \mathbf{s})$ denote such a joint distribution and assume that the informational environment is sufficiently well-behaved that for any sub vectors $a$ and $b$ of the random variables the conditional distribution $F(a \mid b)$ exists. Participant $i$ has an ideal point $y_i \in \mathbb{R}^d$ and preferences representable by the Bernoulli utility function

$$u_i(p, \varepsilon) = -\|p + \varepsilon - y_i\|^2 . \tag{1}$$

The quadratic loss function and additive shock is commonly used in the literature.[3] For our purposes a particularly important property of this representation is the fact that mean-variance analysis is appropriate. Specifically, if $F(\cdot)$ is a distribution function then the extension of preferences to lotteries is representable by the Von Neumann-Morgenstern utility function

$$\int u_i(p, \varepsilon) dF(\varepsilon) = -\|p + \overline{\varepsilon} - y_i\|^2 - v \tag{2}$$

where $\overline{\varepsilon} = \int \varepsilon dF(\varepsilon)$, and $v = \int (\varepsilon - \overline{\varepsilon})'(\varepsilon - \overline{\varepsilon}) dF(\varepsilon)$ are the expectation and variance of $\varepsilon$ under the distribution $F(\cdot)$.

We draw on the extensive literature following Baron and Ferejohn (1989), and assume that policymaking occurs in a sequential process. Specifically, we consider the simple majority rule bargaining game of Banks and Duggan (2000). In period $t = 1, 2.....$ participant $i \in N$ is chosen with probability $\rho_i \in (0, 1)$ to make a proposal $p_i^t \in \mathbb{R}^d$. Following a proposal, the participants simultaneously cast ballots to accept or reject the proposal. If at least $\frac{n+1}{2}$ participants vote to accept then the game ends and the policy $p_i^t$ is enacted. If at least

[3] Bendor and Meirowitz (2004) note that reliance on these assumptions has hurt the delegation literature and demonstrate that the preference assumption is far less relevant than the assumption that randomness is of this form.

$\frac{n+1}{2}$ participants vote to reject then the game moves on to period $t+1$ and the process repeats. A termination history is then characterized by a policy $p_i^t$ and a time $t$, at which the policy is enacted. The results in Banks and Duggan (2000) are most descriptive under the assumption that agents do not discount, so we maintain this assumption. The expected payoff to participant $j$ from such a termination history is

$$\int u_j(p_i^t, \varepsilon, )dF(\varepsilon) = -\left\|p_i^t + \bar{\varepsilon} - y_j\right\|^2 - v. \tag{3}$$

Most scholars choose to focus on stationary equilibria to bargaining games of this form. See Baron and Kalai (1993) for a justification of this selection. A stationary strategy to the bargaining game is a proposal $p_i$ that $i$ will make at any history in which she is recognized to propose and a measurable mapping $v_i : \mathbb{R}^d \to \{accept, reject\}$. Thus, $v_i(p)$ specifies how $i$ will vote if $p$ is proposed. An equilibrium in the Banks and Duggan game involves sequentially rational proposal strategies and voting strategies that satisfy weak dominance. In the presence of the random shock, weak dominance requires that voter $j$ support proposal $p_i^t$ if $\int u_j(p_i^t, \varepsilon, )dF(\varepsilon)$ is higher than the continuation value obtained from the defeat of proposal $p_i^t$ and equilibrium play in subsequent periods. In Banks and Duggan (2000) there is no policy uncertainty, so that $\varepsilon$ is commonly known to be the 0 vector. Banks and Duggan focus on a subset of the stationary equilibria (termed no delay equilibria) that involve agreement on a policy in the first period. Given the simplification afforded by (3) for a fixed distribution $F(\varepsilon)$ we can characterize the no delay equilibria in a bargaining game with common uncertainty about the shock. A few definitions are needed first.

Given a profile of ideal points, $\mathbf{y} = (y_1, y_2, ..., y_n) \in \mathbb{R}^{nd}$ the majority rule core is the set of policies that are unbeatable by pair-wise comparisons and defined as

$$C_m(\mathbf{y}) = \left\{x \in \mathbb{R}^d : \forall z \in X,\ \#\left\{i : \|y_i - x\| \leq \|y_i - z\|\right\} \geq \frac{n+1}{2}\right\} \tag{4}$$

where $\#A$ is the cardinality of the set $A$. In the case of no uncertainty, Plott's (1967) result offers a characterization of the profiles for which the core is non-empty. With $n$ odd, we can restate the characterization in a convenient manner using the notion of a half space. For two policies, $x, t \in \mathbb{R}^d$ the open half space at $x$ including $t$ is

$$H_t^+(x) = \left\{ z \in \mathbb{R}^d : z't > x't \right\}. \tag{5}$$

The number of participants with ideal points in $H_t^+(x)$ is denoted $\#H_t^+(x) = \#\{i \in N : y_i \in H_t^+(x)\}$. We say that $x$ is **a median in all directions** if for every $t$, $\#H_t^+(x) < \frac{n+1}{2}$. With $n$ odd, Plott's result can then be stated in the following simple manner: *the majority rule core coincides with the set of medians in all directions.*

# 3 Intermediate results

With the quadratic loss function and an odd number of voters, Banks and Duggan (2004) use the representation in (3) to show that if $x$ is a median in all directions then the majority preference relation over lotteries on $\mathbb{R}^d$ is the same as the preference relation of the participant with ideal point $x$. It takes only a slight modification of one of the proofs in Banks and Duggan (2000) to extend the core equivalence result to bargaining games in which no agent has private information but there is public uncertainty about the consequences of policy in the form of a distribution $F(\varepsilon)$.

**Lemma 1** *Assume that ideal points* $\mathbf{y}$ *are such that* $x \in \mathbb{R}^d$ *is a median in all directions. Consider a majority rule bargaining game in which, (1) each agent is recognized with positive probability; (2) agents do not discount; (3) there is uncertainty about the shock* $\varepsilon$ *characterized by a commonly known distribution* $F(\varepsilon)$ *–so no agent possesses private information. In this game there exists a no delay stationary equilibrium and all no delay stationary equilibria select the policy* $x - \overline{\varepsilon}$ *with probability 1.*

*Proof:* Existence follows from lemma 1 of Banks and Duggan (2000): To see this, it is sufficient to note that the payoffs are strictly quasi-concave in $p$, majority rule is proper and under the assumption that $x$ is a median in all directions, $\{x\} = C_m(\mathbf{y})$. The fact that all no delay stationary equilibria yield $x - \overline{\varepsilon}$ with probability 1 requires appeal to theorem 6 of Banks and Duggan (2000). With fixed $F(\varepsilon)$, $\int u_i(p, \varepsilon, )dF(\varepsilon) = -\|p + \overline{\varepsilon} - y_i\|^2 - v$ and thus the payoffs over lotteries are quadratic with ideal points $y_i - \overline{\varepsilon}$ plus a constant scaler $-v$. Given this and the fact that majority rule is strong with $n$ odd, theorem 6 applies.■

In the remainder of the paper we assume that a median in all directions exists. Now consider an augmented version of an endogenous agenda bargaining game with uncertainty in which (1) agents have private information in the form of the private signals $s_i$ and (2) prior to bargaining there is a round of simultaneous communication. For a fixed information environment characterized by the joint distribution $F(\varepsilon, \mathbf{s})$ and a profile of preferences characterized by $\mathbf{y}$ we define the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y} \rangle$ **−deliberation game** as follows. In the first period participants make simultaneous public announcements, $m_i \in \mathbb{R}^d$, about $s_i$ and then the bargaining game is played. Analysis of this game requires that we consider an equilibrium concept which involves sequential rationality and consistent belief formation. Accordingly, we are interested in perfect Bayesian Nash equilibria in which conditional on beliefs about $(\varepsilon, \mathbf{s})$ voting and proposing strategies are stationary and voting strategies satisfy weak dominance. A strategy for player $i$ is then a measurable message mapping, $m_i : \mathbb{R}^d \rightarrow \mathbb{R}^d$, a measurable proposal mapping, $p_i : \mathbb{R}^{d(n+1)} \rightarrow \mathbb{R}^d$, and a measurable voting mapping, $v_i : \mathbb{R}^{d(n+2)} \rightarrow \{accept, reject\}$. An equivalent way to conceptualize voting strategies is to think about message contingent acceptance sets. In addition to strategies an equilibrium requires that players have beliefs about $(\varepsilon, \mathbf{s})$ conditional on the observed history. In fact for any period $t$ there must be two types of beliefs, those for proposers that condition on the messages $\mathbf{m}$, as well as play in previous periods and those for voters that condition on $\mathbf{m}$ and the proposal $p_i^t$ and the play in previous periods. For a fixed history of play up to period $t-1$, $h^{t-1}$, and message profile $\mathbf{m}$ a belief for player $i$ is then a joint distribution function on $\mathbb{R}^{d(n+1)}$ and we use the notation $\mu_i(\cdot \mid \mathbf{m}, h^{t-1}), \mu_i(\cdot \mid p_j^t, \mathbf{m}, h^{t-1})$ to denote such conditional beliefs. We sometimes call an equilibrium of this form a **deliberative equilibrium.** If such an equilibrium involves passage of the first period proposal with probability 1 we call it a **no delay deliberative equilibrium.**

In the remainder of the paper we focus on whether or not there exists a no delay deliberative equilibrium in which participants are truthful in the communication stage. We say a **deliberative equilibrium is truthful** if for all $i \in N$, $m_i(s_i) = s_i$. Since our focus is on no delay equilibria in which the messages are truthful very few types of histories need to be studied. We will, however, need to address beliefs following a proposal $p_i^t$ that is not consistent with equilibrium behavior by $i$ given the observed profile of messages, because this type of history occurs if a single agent deviates from a no delay stationary strategy profile.

First, we characterize the relationships between $\mathbf{s}$ and policy that are supportable in truthful no delay deliberative equilibria. The next lemma shows that such an equilibrium policy selection must coincide with the mapping

$$p^*(\mathbf{s}) = x - \int \varepsilon dF(\varepsilon \mid \mathbf{s}). \tag{6}$$

**Lemma 2** *Consider an informational environment characterized by the joint distribution $F(\varepsilon, \mathbf{s})$ and assume that ideal points $\mathbf{y}$ are such that $x \in \mathbb{R}^d$ is a median in all directions. Any no delay truthful deliberative equilibrium to the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}\rangle$ −deliberation game results in policy selection according to the rule $p^*(\mathbf{s}) = x - \int \varepsilon dF(\varepsilon \mid \mathbf{s})$.*

> *Proof:* In any truthful equilibrium consistency of beliefs requires that all agents have the same message conditional posteriors of the form $F(\varepsilon \mid \mathbf{s})$. This and lemma 1 yield the result.■

Our goal is to show that in order to understand whether an informational environment and preference profile admit truthful deliberative equilibria it is sufficient to study a simpler cheap talk signalling game in which the agents in $N$ simultaneously submit messages to a receiver that selects policy. For a fixed information environment $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}\rangle$ and point $x \in \mathbb{R}^d$ we define the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x\rangle$ −**signalling game** as follows. In period 1 each $i \in N$ simultaneously submits a message $m_i \in \mathbb{R}^d$ to a receiver, $r$, with ideal point $x$. In period 2 the receiver selects a policy $p \in \mathbb{R}^d$. So a pure strategy for sender $i$ is a measurable mapping $m_i : \mathbb{R}^d \to \mathbb{R}^d$, a pure strategy for the receiver is a measurable mapping $p : \mathbb{R}^{dn} \to \mathbb{R}^d$ and for each message profile, a belief for the receiver is a joint distribution on $\mathbb{R}^d$ which we denote by $\eta(\cdot \mid \mathbf{m})$. We focus on perfect Bayesian equilibria, requiring that message functions are simultaneous best responses given the policy function, that the policy function is optimal for $r$ given the beliefs $\eta(\cdot \mid \mathbf{m})$ and that given the message functions the beliefs are consistent with Bayes' rule when it applies. We call such an equilibrium a **signalling equilibrium** and say a **signaling equilibrium is truthful** if $m_i(s_i) = s_i$ for all $i \in N$. The analogue to lemma 2 is.

**Lemma 3** *Consider an informational environment characterized by the joint distribution $F(\varepsilon, \mathbf{s})$. Any truthful signalling equilibrium to the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x\rangle$ −signaling game results in* *policy selection according to the rule* $p^*(\mathbf{s}) = x - \int \varepsilon dF(\varepsilon \mid \mathbf{s})$.

*Proof:* The requirement that any equilibrium to the signaling game involves consistent beliefs and sequentially rational policy selection implies that $p^*(\mathbf{m}) = x - \int \varepsilon dF(\varepsilon \mid \mathbf{m})$. In a truthful equilibrium $m_i = s_i$ for each $i \in N$ and thus, the result attains.$\blacksquare$

# 4 The equivalence result

In this section we show that if given the ideal points $\mathbf{y}$ the point $x$ is a median in all directions then the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y} \rangle$ −deliberation game possesses a truthful deliberative equilibrium if and only if there is a truthful signaling equilibria in the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x \rangle$ −signalling game. The subsequent section focuses on signaling games and isolates conditions on preferences and informational environments that are necessary and sufficient for the existence of truthful equilibria in either type of game.

**Proposition 1** *Consider an informational environment characterized by the joint distribution* $F(\varepsilon, \mathbf{s})$ *and assume that ideal points* $\mathbf{y}$ *are such that* $x \in \mathbb{R}^d$ *is a median in all directions. In the* $\langle F(\varepsilon, \mathbf{s}), \mathbf{y} \rangle$ *−deliberation game there exists a truthful no delay deliberative equilibrium if and only if the* $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x \rangle$ *−signalling game possesses a truthful signalling equilibrium.*

*Proof:* Assume that $x$ is a median in all directions.

($\Longrightarrow$) Assume that in the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y} \rangle$ −deliberation game there exists a truthful no delay perfect Bayesian Nash equilibrium. Let $p^e(\mathbf{m}){:}\mathbb{R}^{nd} \rightarrow \mathbb{R}^d$ denote the mapping from message profiles into policies that results from the bargaining strategies in this equilibrium, and let $\mu_i^e(\cdot \mid \mathbf{m})$ denote the message conditional belief of player $i$ in the equilibrium to the deliberation game. Now suppose that in the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x \rangle$ −signalling game the receiver uses the strategy $p^e(\mathbf{m})$. Since $m_i(s_i) = s_i$ is an equilibrium strategy in the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y} \rangle$ −deliberation game, it must be a best response in the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x \rangle$ −signalling game if all participants $j \in N \backslash i$ are truthful and $r$ uses $p^e(m)$. It remains only to verify that there are consistent beliefs for the receiver which make this strategy sequentially rational.

Since the equilibrium to the deliberation game is truthful all participants must form the same beliefs (i.e., $\mu_i^e(\cdot \mid \mathbf{m})$ is almost surely equal to $\mu^e(\cdot \mid \mathbf{m}), \forall i \in N$) following any $\mathbf{m}$ that is feasible (i.e., $\mathbf{m}$ is in the support of $F(\mathbf{s})$.). Let $\eta(\cdot \mid \mathbf{m})$ denote the marginal of $\mu^e(\cdot \mid \mathbf{m})$ with respect to $\varepsilon$. Since beliefs are consistent in the equilibrium to the deliberation game this belief is consistent in the signaling game. By lemma 2 we have $p^e(\mathbf{m}) = p^*(\mathbf{m}) = x - \int \varepsilon dF(\varepsilon \mid \mathbf{s}) = x - \int \varepsilon d\mu^e(\cdot \mid \mathbf{m})$ and thus the receiver's strategy is sequentially rational given the belief.

($\Longleftarrow$) Assume that the$\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x\rangle$ $-$signalling game possesses a truthful perfect Bayesian Nash equilibrium. Let $p^E(\mathbf{m})$:$\mathbb{R}^{nd} \to \mathbb{R}^d$ denote the receiver's strategy and let $\mu^E(\cdot \mid \mathbf{m})$ denote the receiver's posterior in this truthful equilibrium. Consider the strategy profile to the deliberation game in which $m_i(s_i) = s_i$, each participant's proposal strategy is $p^E(\mathbf{m})$ and each participant's message contingent belief is given by $\mu^E(\cdot \mid \mathbf{m})$. Moreover, let the message and proposal contingent beliefs correspond to $\mu^E(\cdot \mid \mathbf{m})$. Finally, let the voting strategies of each $i$ satisfy weak dominance given the utility function $\int u_i(p, \varepsilon, )d\mu^E(\varepsilon \mid \mathbf{m})$. By construction the beliefs are consistent and the voting strategies satisfy sequential rationality. It remains to check that no agent has an incentive to unilaterally deviate in the message and or proposing strategies. Since the proposal strategy selects the unique policy that is in the majority rule core given $\mathbf{m}$, any proposal other than $p^E(\mathbf{m})$ will not be accepted by a majority and, thus, such a proposal will fail. This means that if player $i$ deviates only in her proposing strategy the deviation will not pass and the next proposer (other than $i$) following the equilibrium strategies will propose $p^E(\mathbf{m})$ and it will pass. Since each agents recognition probability is less than one, when agent $i$ is recognized and makes a rejected proposal a different proposer will eventually be recognized and the proposal $p^E(\mathbf{m})$ will eventually be proposed and pass. This implies that the only deviation that agent $i$ can unilaterally make which will affect her payoffs is to deviate in her message, sending $m_i' \neq m_i$. Given that everyone else plays the conjectured strategy profile, following such a deviation the resulting outcome will be $p^E(\mathbf{m}_{-i}, m_i')$ regardless of how $i$ proposes if she is recognized. Specifically, if

$i$ deviates at the message level and is not recognized to propose, the equilibrium strategies will result in the policy $p^E(\mathbf{m}_{-i}, m_i')$. If $i$ is recognized to propose and proposes $p^E(\mathbf{m}_{-i}, m_i')$ this policy will pass. The remaining possibility is that $i$ is recognized and proposes a policy $p' \neq p^E(\mathbf{m}_{-i}, m_i')$.But unless $y_i = x$ this policy will not be supported by a majority of voters and thus the policy $p^E(\mathbf{m}_{-i}, m_i')$ will eventually be passed. If $y_i = x$ then the equilibrium profile result in $i$'s optimum and the deviation cannot be desirable. Thus, the only deviation that $i$ can make which affects her payoff is to cause the policy $p^E(\mathbf{m}_{-i}, m_i')$ to pass instead of the policy $p^E(\mathbf{m})$. But, since the$\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x\rangle$ −signalling game possesses a truthful perfect Bayesian Nash equilibrium, agent $i$ weakly prefers $p^E(\mathbf{m})$ to $p^E(\mathbf{m}_{-i}, m_i')$ as this is the necessary incentive compatibility condition in the signaling game.■

It should be noted that the equivalence does not hold for general policymaking games. Specifically, in representing a game with communication as a mechanism it is generally necessary to include the additional requirement that players are willing to play the game in a prescribed manner. In Myerson (1982) there are two types of incentive compatibility conditions: truthful and obedient. In the current problem, however, it turns out that all unilateral deviations reach payoffs that are achievable through just a deviation at the message stage and thus only the truthful conditions bind.

# 5 Necessary and Sufficient conditions for truthful equilibria

The remainder of the paper highlights conditions under which truthful equilibria (of either type) exist. The exposition focuses on signalling games. In order to impose slightly more structure we now assume that $F(\varepsilon)$ is absolutely continuous with respect to Lebesgue measure on a convex subset of $\mathbb{R}^d$. In addition, we assume that each agent's private signal is given by $s_i = \varepsilon + \delta_i$ where each dimension of $\delta_i$ is itself drawn from a distribution that is absolutely continuous with respect to Lebesgue measure on a convex set of $\mathbb{R}^1$ or concentrated at 0. In otherwords for each dimension, private signals are either perfectly informative or has a nice density. These conditions are, for example, more general than those in Battaglini (2002, 2004), Baron (2000), Gilligan and Krehbiel (1987, 1989) and Krishna and Morgan (2001) as both perfectly and imperfectly informed agents are allowed. We do not assume that the individual disturbances $\delta_i$ are independent or identically distributed. So, some agents may observe the same signals on some dimensions. These conditions allow us to use the spatial structure and investigate the local incentives for agents to move policy.

A well known condition that appears in the mechanism design literature is non exclusivity (Postlewaite and Schmeidler,1986; Palfrey and Srivastava, 1987). Let $\mathbf{s}_{-i}$ denote the profile of private signals for $N\backslash\{i\}$ and let $\mathbf{s}_{-ij}$ denote the profile of private signals for $N\backslash\{i,j\}$. The informational environment $F(\varepsilon,\mathbf{s})$ satisfies **nonexclusivity** if for any $i \in N$, $F(\varepsilon \mid \mathbf{s}_{-i}) = F(\varepsilon \mid \mathbf{s})$ for almost every $\mathbf{s}$.A related but stronger condition, strong nonexclusivity is considered by Baron and Meirowitz (2004). The informational environment $F(\varepsilon,\mathbf{s})$ satisfies **strong nonexclusivity** if for any $i,j \in N$, $F(\varepsilon \mid \mathbf{s}_{-ij}) = F(\varepsilon \mid \mathbf{s})$ for almost every $\mathbf{s}$. Thus, in a nonexclusive environment any coalition of $n-1$ participants have collectively observed all of the available information and in a strongly nonexclusive environment any coalition of $n-2$ participants have collectively observed all of the available information.

In our setting with quadratic loss functions, the receiver only cares about learning the distribution of the mean given $\mathbf{s}$. So for our purposes, we can focus on slightly weaker conditions about the conditional distributions of the expectation of $\varepsilon$. For any subset $A \subset N$, let $\mathbf{s}_A^{'}$ denote the profile of private signals for the participants in $A$.The informational environment $F(\varepsilon,\mathbf{s})$ satisfies **mean nonexclusivity on** $A$ if for any $i \in A$, $\int \varepsilon dF(\varepsilon \mid \mathbf{s}_{A\backslash\{i\}}) = \int \varepsilon dF(\varepsilon \mid \mathbf{s}_A)$ for almost every $\mathbf{s}$. Similarly for any $A \subset N$, **mean strong nonexclusivity on A** is satisfied if for any $i,j \in A$, $\int \varepsilon dF(\varepsilon \mid s_{A\backslash\{i,j\}}) = \int \varepsilon dF(\varepsilon \mid s_A)$ for almost every $\mathbf{s}$. In most published work on the spatial model (Battaglini, 2004 is the exception) it is assumed that informed agents observe perfect signals, and in this case the term mean in the above conditions is extraneous.

We cannot focus just on the informational environment. In addition we need to consider joint conditions on the information environment and the preference profile, $\mathbf{y}$. For any subspace $X$ of $\mathbb{R}^d$ let $proj_X(s_i)$ denote the projection of $s_i$ on $X$. For any $A \subset N$, we say that $\langle F(\varepsilon,\mathbf{s}),\mathbf{y},x\rangle$ satisfy **minimal alignment on** $A$ and $x$ if there exists a set of subspaces $\{X_i\}_{i\in A}$ s.t. (1)

$$\int \varepsilon dF(\varepsilon \mid \{proj_{X_i}(s_i)\}_{i\in A}) = \int \varepsilon dF(\varepsilon \mid \mathbf{s}_A) \tag{7}$$

for each $\mathbf{s}$ and (ii) for each $i \in A$, $proj_{X_i}(y_i) = proj_{X_i}(x)$. In otherwords, minimal alignment on $A$ and $x$ is satisfied if all of the information available to the participants in $A$ about the mean of $\varepsilon$ can be learned by observing for each $i \in A$ only the projection of $s_i$ on the subspace $X_i$, and the receiver and $i$ have aligned preferences on $X_i$. Battaglini (2002) considers the case where all senders observe the same private information (so nonexclusivity is satisfied on $N$) and shows that with two senders possessing ideal points that are not colinear with the receivers', fully revealing equilibria exist. The construction hinges on the fact that each sender has aligned preferences with the receiver over a subspace of the outcome space. In Battaglini's model the requirement that ideal points are not colinear is a special case of minimal alignment. In general, however, it is possible to construct fully-revealing equilibria in which the receiver bases a portion of the policy decision on the information possessed by a sender even when information violates non exclusivity. In fact the non exclusivity is not critical to Battaglini's result. Allowing each sender to have an informational monopoly on the dimension of the shock on which her preferences are aligned with the receiver's preferences does not affect the result.

We begin with a sufficiency result.

**Proposition 2** *Consider an informational environment characterized by the joint distribution $F(\varepsilon, \mathbf{s})$ and assume that the senders have ideal points $\mathbf{y}$ and the receiver has ideal point $x$. Let $A_x$ denote the set of individuals in $N$ with ideal points not equal to $x$. (1) If mean strong non exclusivity on $A_x$ is satisfied then the signalling game possesses a truthful perfect Bayesian Nash equilibrium, and if in addition the ideal points $\mathbf{y}$ are such that $x \in \mathbb{R}^d$ is a median in all directions then the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}\rangle$ −deliberation game possesses a truthful deliberative equilibrium. (2) If minimal alignment on $A$ and $x$ is satisfied then the signalling game possesses a truthful perfect Bayesian Nash equilibrium, and if in addition the ideal points $\mathbf{y}$ are such that $x \in \mathbb{R}^d$ is a median in all directions then the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}\rangle$ −deliberation game possesses a truthful deliberative equilibrium.*

*Proof:*

(1) For signalling games the result is an immediate consequence of proposition 3 in Baron and Meirowitz (2004). To see this consider a game between the senders $A_x$ and a receiver that knows $\mathbf{s}_{N\backslash Ax}$ and assume that the random shock is just the expectation of $\varepsilon$. Mean strong non exclusivity in the signalling game corresponds to strong non exclusivity in this new signaling game, and proposition 3 shows that this game has a truthful perfect Bayesian equilibrium. The construction relies on beliefs which render any unilateral deviation inconsequential. Given this the result for deliberation games follows from proposition 1 above.

(2) The proof is by construction. Assume that minimal alignment on $A$ and $x$ is satisfied. By this condition, there exists a list of subspaces$\{X_i\}_{i\in A_x}$ s.t.

$$\int \varepsilon dF(\varepsilon \mid \{proj_{X_i}(s_i)\}_{i\in A_x}) = \int \varepsilon dF(\varepsilon \mid \mathbf{s}_{A_x}) \tag{8}$$

Let $\eta^E(\varepsilon \mid \mathbf{m}) = F(\varepsilon \mid \{proj_{X_i}(m_i)\}_{i\in A}, \mathbf{m}_{N\backslash A_x})$. By minimal alignment, given truthful strategies this defines a consistent belief for every possible $\mathbf{m}$. By lemma 3, the policy function is $x - \int \varepsilon d\eta^E(\varepsilon \mid \mathbf{m})$, so no participant in $N\backslash A_x$ has an incentive to deviate from a truthful message as the policy is optimal given the group information. To show that no unilateral deviation is desirable assume that for $i \in A_x$ it is the case that $N\backslash\{i\}$ are truthful. Also fix $s_i$ and consider a deviation $m_i' \neq s_i$. By condition 2 of minimal alignment, if

$$x - \int \varepsilon d\eta^E(\varepsilon \mid s_i, \mathbf{s}_{-i}) \neq x - \int \varepsilon d\eta^E(\varepsilon \mid m_i', \mathbf{s}_{-i}) \tag{9}$$

(i.e., the deviation affects the policy outcome) then on the subspace that the deviation affects policy, agent $i$ has preferences that are aligned with $x$,

$$proj_{X_i}(x - \int \varepsilon d\eta^E(\varepsilon \mid s_i, \mathbf{s}_{-i})) \neq proj_{X_i}(x - \int \varepsilon d\eta^E(\varepsilon \mid m_i', \mathbf{s}_{-i})) \tag{10}$$

and on the remaining subspaces on which $i$'s preferences are not aligned there is no policy change,

$$proj_{\mathbb{R}^d \backslash X_i}(x - \int \varepsilon d\eta^E(\varepsilon \mid s_i, \mathbf{s}_{-i})) = proj_{\mathbb{R}^d \backslash X_i}(x - \int \varepsilon d\eta^E(\varepsilon \mid s_i, \mathbf{s}_{-i})). \quad (11)$$

Using mean variance, we can write $i$'s expected payoff from her message as

$$-\left\| y_i - x + \int \int \varepsilon d\eta^E(\varepsilon \mid \mathbf{s}_{-i}, m_i') dF(\mathbf{s}_{-i} \mid s_i) \right\| - v$$

for some scalar, $v$. The above argument shows that if the deviation to $m_i'$ is payoff consequential it increases the distance from policy to $y_i$ for all realizations of $\mathbf{s}_{-i}$ and thus lowers $i$'s expected payoff. So truthful messages form a best response. Given this, the deliberation game result follows from proposition 1.■

The converse of this proposition is not true. Krishna and Morgan (2001) represents a counterexample to the converse in which neither strong non exclusivity nor minimal alignment are satisfied. This example does, however, satisfy non exclusivity and this is critical to the equilibrium construction. The receiver, can detect when at least one sender is lying and chooses policy to punish this behavior. We now focus on a necessity result. This requires combining nonexclusivity and minimal alignment. Again letting $A_x$ denote the agents with ideal points other than $x$, we say $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x \rangle$ satisfy **condition** $\alpha$ if there exists two (not necessarily disjoint) subsets of $A_x$, denoted $B$ and $C$ such that (1) minimal alignment is satisfied on $B$ and $x$ and (2) mean nonexclusivity is satisfied on $C$ and (3) the vectors of private signal profiles from these groups $(\mathbf{s}_B, \mathbf{s}_C)$ and $N \backslash A_x$ are sufficient to predict the expectation of $\varepsilon$, specifically,

$$\int \varepsilon dF(\varepsilon \mid \mathbf{s}_{N \backslash A_x}, \mathbf{s}_B, \mathbf{s}_C) = \int \varepsilon dF(\varepsilon \mid \mathbf{s}). \quad (12)$$

for a.e. $\mathbf{s}$. Note that condition $\alpha$ uses mean nonexclusivity not mean strong non exclusivity.

**Proposition 3** *If the* $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}, x \rangle$ *−signalling game possesses a truthful perfect Bayesian Nash equilibrium then condition* $\alpha$ *is satisfied.*

*Proof:* Suppose that there is a truthful equilibrium in the signalling game and condition $\alpha$ is not satisfied. Since condition $\alpha$ fails, for any $B \subset A_x$ on which minimal alignment is satisfied and $C \subset A_x$ on which mean non-exclusivity is satisfied it is the case that

$$\int \varepsilon dF(\varepsilon \mid \mathbf{s}_{N \backslash A_x}, \mathbf{s}_B, \mathbf{s}_C) \neq \int \varepsilon dF(\varepsilon \mid \mathbf{s}) \tag{13}$$

for a set of profiles $\mathbf{s}$ with positive measure. However, since a truthful equilibrium exists in the $\langle F(\varepsilon, \mathbf{s}), \mathbf{y}\rangle$ −signalling game it is the case that for at least one such selection of $B$ and $C$ it is the case that a non-empty set of agents, $R = N \backslash \{N \backslash A_x \cup B \cup C\}$, are willing to reveal their private information and this information influences the final policy. Since condition $\alpha$ is not satisfied, if we take $B, C, R$ and modify them such that $R'$ is the smallest (by set inclusion) set containing agents that cannot be moved to $B'$ or $C'$ while maintaining the assumption that minimal alignment is satisfied on $B'$ and mean non-exclusivity is satisfied on $C'$ the resulting $R'$ is non empty. Since we have assumed a truthful equilibrium exists it must be the case that in a signaling game between the agents $R'$ and a receiver with ideal point $x$ observing $(\mathbf{s}_{N \backslash A_x}, \mathbf{s}_{B'}, \mathbf{s}_{C'})$ a truthful equilibrium exists. To derive a contradiction from this conclusion we consider such a signalling game and the incentives of the senders in $R'$. Consider an agent $i \in R'$ who's information is not redundant, meaning

$$\int \varepsilon dF(\varepsilon \mid \mathbf{s}_{N \backslash A_x}, \mathbf{s}_{B'}, \mathbf{s}_{C'}, \mathbf{s}_{R' \backslash \{i\}}) \neq \int \varepsilon dF(\varepsilon \mid \mathbf{s}). \tag{14}$$

Such an $i$ must exist or else we will have shown that condition $\alpha$ is satisfied. Assume that all senders $j \in R' \backslash \{i\}$ are truthful. Since $i \in R'$ it is the case that (1) $i \in A_x$ and thus $y_i \neq x$ and (2) it is not possible to find a set of agents that observe the information contained in $s_i$ and on which mean nonexclusivity is satisfied. Given that $s_i = \varepsilon + \delta_i$ these 2 conclusions and lemma 3 imply that for some subspace $X$ of $\mathbb{R}^d$, in the truthful equilibrium

$$p^*(\mathbf{m}_{-i}, m_i) = x - \int \varepsilon dF(\varepsilon \mid \mathbf{s}_{-i}, proj_X(m_i)) \tag{15}$$

Since $i$ has no incentive to be dishonest it must be the case that

$$\left\| y_i - x - \int\int \varepsilon dF(\varepsilon \mid \mathbf{s}_{-i}, proj_X(m_i))dF(\mathbf{s}_{-i} \mid s_i) \right\| \tag{16}$$

is optimized by $m_i = s_i$. Given the assumptions on the distributions, $\int\int \varepsilon dF(\varepsilon \mid \mathbf{s}_{-i}, proj_X(m_i))dF(\mathbf{s}_{-i} \mid s_i)$ is continuous in $m_i$. This and the fact that $m_i = s_i$ is an optimizer means that $proj_X(x) = proj_X(y_i)$. But this implies that $i$ can be added to the set $B$, contradicting the conclusion that $i \in R'$. Thus, condition $\alpha$ is satisfied or the truthful equilibrium does not exist.■

One point worth noting is that proposition 2 does not require $s_i = \varepsilon + \delta_i$ or the absolute continuity assumptions. Proposition 3 does, however, rely on these assumptions. Without them it is possible to come up with settings that posses truthful equilibria but violate condition $\alpha$. One unidimensional example involves one sender with ideal point $y_1 = 1$, a receiver with ideal point $x = 0$, a shock taking possible values $\varepsilon \in \{-3, 3\}$, and a perfectly informative signal $s_1 = \varepsilon$.If the sender is truthful, and the receiver selects a sequentially rational policy given the message the sender's utility is 1 in either state. However, a deviation by the sender results in a sender utility of either $-49$ or $-25$ depending on the realization of $\varepsilon$. Thus, a truthful equilibrium exists but condition $\alpha$ is not satisfied. It should also be noted that proposition 2, sufficiency, can be extended to a modified version of condition $\alpha$ – with the second condition using mean strong nonexclusivity. This result follows from proposition 2, if we consider 2 separate signalling games, one between the receiver and the agents on which mean strong nonexclusivity is satisfied, and the other between the receiver and the agents on which minimal alignment is satisfied.

# 6 Conclusion

Bargaining problems in which agents posses private information and are allowed to communicate may posses large equilibrium sets. Moreover, in these problems it is difficult to asses the extent to which information sharing is possible. However, when agents are patient and preferences satisfy the Plott conditions, we can answer questions about the possibility of full information revelation by analyzing sender-receiver models. It is easy to demonstrate this point in the case of a unidimensional policy space and smooth shocks. Cheap talk communication and bargaining can result in policy selection that is optimal for the median committee member only if all pieces of information are observed by at least two committee members. Equivalently in sender-receiver models like Krishna and Morgan (2001) in which the players have distinct ideal points on the line, fully-revealing equilibria only exist if any information that is observed by one sender and not the receiver is also observed by another sender. A second informative example involves a two dimensional policy space with 5 committee members possessing the ideal points $y_0 = (0, 0)$, $y_1 = (2, 0)$, $y_2 = (1, 2)$, $y_3 = (-1, 0)$, $y_4 = (-2, -4)$. Is there an equilibrium to the communication and bargaining game which aggregates all of the available private information to select agent 0's favorite policy, $-\int \varepsilon dF(\varepsilon \mid \mathbf{s})$? We find that such an equilibrium exists in the bargaining and communication game if and only if there is a truthful equilibrium in a signalling game in which agents 1,2,3,4, are senders and agent 0 is the receiver. This is the case if, for example, each dimension of the shock is observed by three agents or one of the odd indexed agents observes the first coordinate and one of the even indexed agents observes the second coordinate. In general, combinations of non exclusivity and preference alignment conditions are necessary for truthful equilibria.

While progress has been made in understanding when truthful equilibria exist for "nice preference profiles", this paper addresses only a small set of the questions pertaining to policymaking in the spatial model. Since the presence of a non-empty majority rule core is non-generic, questions about the possibility of aggregation when the Plott conditions are not satisfied need to be answered. While Banks and Duggan (2000) present results about the upper hemicontinuity of the equilibrium correspondence, the approach taken here does not seem particularly applicable to the study of communication and bargaining in settings that are "close" to ones in which the Plott conditions are satisfied. Results about profiles that do not satisfy the Plott conditions are likely to hinge on analysis of partially-revealing equilibria, a direction that is left for future work.

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