Growth Outside the Stable Path: 
Lessons from the European Reconstruction

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Abstract

This paper exploits a natural experiment, the large destruction of capital in continental Europe during World War II, to characterize the transitional dynamics of an economy that begins with a capital stock below its steady state level. We use these regularities as a benchmark to discriminate among competing growth specifications. A model that combines non-separabilities in preferences with a technology that restricts the degree of substitutability between inputs outperforms the widely used AK and Cobb-Douglas specifications with time-separable preferences. Our results suggest that policy evaluations based in growth models that overlook non-separabilities in preferences or impose strong restrictions on the technological structure might be grossly misleading.

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1. Introduction.

Exogenous shocks provide an interesting environment for the evaluation of competing economic hypotheses. Wars and natural disasters, with their associated destructions of capital, imply an unambiguous deviation from the equilibrium growth path. This paper exploits the adjustment of Western European economies after World War II as a tool to discriminate among alternative specifications of preferences and technology within the growth context.

Endogeneity, in non-experimental sciences, remains a crucial limitation of empirical attempts to discriminate among alternative theoretical hypotheses. In economics, long lasting controversies on the real effects of monetary policy or the short-run role of aggregate demand in the determination of income are classic examples of this problem. In the growth context, exogenous technological change drives capital accumulation and growth in neoclassical models, while the endogenous growth literature points to alternative forms of capital accumulation as the determinants of technological change and growth\(^1\). The difficulties in identifying exogenous shocks have shifted our attention to special historical events. Economists can learn a lot from the behavior of the economy after a war or natural disaster, since they are the closest things to controlled experiments.

Most macroeconomics textbooks illustrate the transitional dynamics of the neoclassical model relying in the post-World War II growth record of Western Europe or Japan. Nevertheless, we still miss a more comprehensive analysis of the reconstruction process, not only in terms of its outstanding growth record, but also devoting attention to the behavior of other economic indicators. The performance of these variables might provide valuable information to contrast competing growth theories. We attempt to fill this gap by comparing the adjustment process after the war with the transitional dynamics generated by a variety of growth specifications.

\(^1\) Physical capital as in Rebelo (1991), human capital as in Lucas (1988), public capital as in Barro (1990) or technological capital in the form of a increased variety or quality of intermediate inputs as in Romer (1989) and Grossman and Helpman (1991).
In the spirit of Kaldor (1961), our preliminary goal is to identify some empirical regularities that characterize the transition of an economy after a large destruction of capital. We find that the European post-war experience was characterized by high growth rates, non-monotonic adjustment of the saving rate, and a smoothly increasing capital-output ratio and wage share. Then, we develop a flexible model that is parameterized to reproduce some of the most popular specifications found in the growth literature. We use the stylized facts of the reconstruction process as a benchmark to compare the performance of our alternative parameterizations.

This analysis of the transitional dynamics proves to be a promising avenue for the evaluation of competing growth specifications. This issue has become one of increasing relevance given the widespread use of growth models as the standard tool for the analysis of a large variety of problems involving intertemporal tradeoffs\(^2\) and the sensitivity of the results of these studies to the underlying growth structure.

Our main findings suggest that non-separabilities in preferences and deviations from the restrictive Cobb-Douglas technology seem to be essential to reconcile the model predictions with the main patterns in the data. And therefore policy evaluations derived under growth specifications that overlook these issues should be regarded with caution. For instance, Alvarez-Cuadrado et al. (2004) report welfare gains from a permanent increase in the rate of technological change under time non-separable preferences being 40% larger than the conventional estimates derived with time-separable specifications.

From the outset it is worth clarifying that our goal is not to present a model that provides a full account of the events that took place in post-war Europe, but rather to take advantage of this historical event to refine our understanding of some desirable features of preferences and technology.

\(^2\) Over the last decades growth models have become a standard tool for the analysis, not only of long-run issues for which they were originally designed, but also to address a wide variety of problems. For instance, Gourinchas and Jeanne (2003) use a calibrated neoclassical model to measure the welfare gains from financial integration, Turnovsky and Chatterjee (2002) calibrate a non-scale growth model to conduct a numerical analysis of the alternative uses for the U.S. budget surplus projected for the end of Clinton’s administration, and Ireland (1994), using an “AK” growth model, explores whether a permanent tax cut can improve the long run government budget balance.
within a growing economy. Other authors have focused on the consequences of the World War II. Christiano (1989) conducts an analysis similar to ours in an attempt to understand the nature of the increase in the Japanese saving rate that took place during the seventies. Gilchrist and Williams (2001) consider an interpretation of Japan and Germany’s post-war growth that relies in technological change embodied in new vintages of capital.

The paper is organized as follows. Section 2 documents the post-war gap and characterizes some key features of the adjustment process. In sections 3 and 4 we set up a flexible model that could be parameterized to reproduce some of the most popular growth specifications and we characterize its macroeconomic behavior. Section 5 conducts a numerical analysis contrasting the performance of several model specifications with the empirical regularities of the reconstruction process and the conclusions are summarized in Section 6.

2. Europe after the war: Some stylized facts of the reconstruction process.

World War II left a track of devastation in continental Europe. During the war years, the productive efforts of more than an entire generation were lost with per capita income returning to the levels of the turn of the century. As a result, the post-war European situation provides a rare natural experiment for an economy that starts out with a capital stock well below its equilibrium level.

Three causes contributed to the low level of physical capital in post-war Europe. First, as Crafts and Toniolo (1996) point out, Europe as a consequence of the Great Depression experienced a slowdown in the process of accumulation of physical capital not being matched by an equal deceleration of human capital formation. Second, during the war the depletion rate of the two kinds of capital was strongly biased against physical assets; Tables 1 and 2 summarize evidence on the devastating effects of the war. In our sample of economies most directly affected by the conflict (France, Germany, Italy, Austria and the Netherlands) the war destroyed between one third and one

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3 Along these lines, Milward (1984) claims that Europe was reconstructed, not only from the destructive consequences of the war, but also from the economic collapse of 1929-32. “Thus the increase in inputs of education and technology into most Western European economies did not diminish over that time so that if the barriers to grow were lifted a surge of growth would be automatically produced” (pg. 463)
quarter of the pre-war capacity, while the cost of the war in human lives is substantially lower. And third, as we will later argue, technological change didn’t stop during the conflict, and therefore conventional quantifications of the post-war gap tend to understate it\textsuperscript{4}.

Before we proceed with the evaluation of the post-war data we need to assess the duration of the reconstruction period. At first glance it seems sensible to assume that this period ends when output recovers the pre-war level. Nevertheless, Janossy (1971) argues that the reconstruction period ends at the time when the actual level of production equals the level that would have been reached at that point if the war never occurred\textsuperscript{5}.

Our first approach to determine the length of the reconstruction period is based on some recent results of the extensive research program on the time series properties of output initiated by Nelson and Plosser (1982). Ben-David et al. (2003) evaluate the time series properties of 120 years of OECD output data. Their preferred specification that allows for two structural breaks rejects the unit root null hypothesis in favor of the trend stationary alternative for most of our war economies. Specifically their results suggest that the economies affected by a large destruction of capital had a first break during the war years and a second one in the late 60’s. Since pre-war levels were recovered early in the 50’s, this second break can be interpreted as the end of the reconstruction period, in line with Janossy’s argument\textsuperscript{6}. A second approach to assess the length of reconstruction period draws on the work of Islam (1995) and Caselli et al. (1997). Their estimates suggest that economies converge to their balanced growth path at a rate close to 9%. At this speed of

\textsuperscript{4} Recent models that link technological change to the production of ideas are consistent with this hypothesis. Endogenous models that rely in learning-by-doing mechanisms or capital accumulation are more difficult to reconcile with this argument. Saint-Paul (1993) claims “entrepreneurial behavior, the drive for innovation, and all the institutional infrastructure that make a market economy work where here (in France) after the war” (pg. 110-111). Boltho (1982) argues that “turning to technical progress, it is possible that this may also have accelerated after 1945 in view of the investment backlog accumulated during the recession and the war... This abundant supply of technology (developed in the US) increased the steady state gap” (pg. 12).

\textsuperscript{5} In a sense this proposition assumes that the rate of technological change relevant for these European economies kept increasing during the conflict.

\textsuperscript{6} Furthermore, the economies not affected by the war have both structural breaks before the Great Depression, and therefore we find this evidence compelling of our claim that the control group (U.S., Australia and Canada) grew along the stable path during the two decades that followed the war.
convergence it would have taken around 16 years to close 75% of the gap caused by the war. Since both approaches suggest that the recovery from the war was characterized by long lasting dynamics, we extend the scope of our analysis until 1965.

We use data on PPP adjusted real per-capita GDP from Maddison (2001), saving rates are from Maddison (1992), the capital-output ratio data is from King and Levine (1994) and we calculate the labor income share as the share of employee compensation in National Income using OECD statistics (1963, 1969 and 1972). This share is adjusted for self-employment following Gollin (2002). Figure 1 illustrates the evolution of these variables for our sample of economies. In order to identify the stylized facts of the reconstruction process, we reproduce the evolution of those same variables for other OECD economies that were not directly affected by the conflict that we use as a control group in our natural experiment. In an attempt to isolate the response to the large destruction of capital from other shocks taking place during the post-war period figure 2 reproduces the averages for each variable for both samples of economies. Combining both figures, we identify the following stylized facts characterizing the transitional dynamics of an economy after a large destruction of physical capital:

- **Stylized fact 1.** The adjustment process is characterized by high and very slowly decreasing growth rates. If the control group is properly capturing some of the post-war shocks, figure 2 suggests that the growth rate peaks several periods after the end of the conflict. Papageorgiou and Perez-Sebastian (2005) report a similar non-monotonic adjustment for the growth rates of South Korea and Japan.

- **Stylized fact 2.** After 1955 the capital-output ratio smoothly increases.

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7 Given the dispersion in size of the economies, averages constructed using GDP as a weight exhibit a pattern strongly dominated by the large countries. Nonetheless the analysis that follows is robust to this alternative weight. See Alvarez-Cuadrado (2005) on the robustness of the control group to the inclusion of other OECD countries.

8 As we argue in Appendix I, we believe that King and Levine (1994) estimates overstate the true capital-output ratio in the beginning of the sample. Maddison (1994) provides evidence consistent with this claim, and therefore we choose to ignore the early observations of this ratio. Hayashi (1989) finds a similar pattern in the Japanese post-war wealth-income ratio and he argues that measurement errors in the data in the early fifties might be behind this behavior.
- **Stylized fact 3.** The saving rate exhibits a characteristic inverted u-shape. During the first years it monotonically increases reaching its maximum after more than a decade, and thereafter slowly decreases. This pattern is only reinforced by direct comparison with the control group. Christiano (1989) and Hayashi (1989) report similar evidence for the Japanese post-war period. Chari et al. (1996) reports similar saving patterns for South Korea.

- **Stylized fact 4.** The wage share, adjusted for self-employment, exhibits an upward trend, increasing on average above 10% in the period considered. The average unadjusted wage share increased almost 17%.

At first glance, stylized facts 1 and 2 provide interesting information about the underlying production structure. The mild decrease (increase) in the growth rate (capital-output ratio) seems to point to some degree of diminishing returns at work. Stylized fact 4 might contribute to further refinements on the production structure; variations in the inputs shares suggest deviations from the conventional Cobb-Douglas specification and its unitary elasticity of substitution between inputs. Finally, stylized fact 4 points to preferences specifications characterized by some degree of temporal dependence in consumption. As Alvarez-Cuadrado et al. (2004) highlight, differences in the adjustment paths between time separable and time non-separable economies depend crucially in the nature of the shock analyzed. Their results suggest that shocks that lead to sudden decreases in consumption, such as a large destruction of capital, are an ideal environment to bring to light these differences.

### 3. The model

Consider a closed economy\(^9\) populated by \( N \) identical and infinitely lived households that grows at the exogenous rate, \( \dot{N}/N = n \). The individual household’s objective is to maximize:

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\(^9\) Under the strict capital controls of the early Bretton Woods agreement the closed economy assumption seems a sensible one. The restoration of current account convertibility did not occur until 1959 in Europe, and the agreement allowed
\[ \Omega \equiv \int_0^\infty \frac{1}{1-\sigma} \left[ \frac{C_i}{H_i^\gamma} \right]^{1-\sigma} e^{-\beta t} dt = \int_0^\infty \frac{1}{1-\sigma} \left[ C_i^{(1-\gamma)} \left( \frac{C_i}{H_i} \right)^\gamma \right]^{1-\sigma} e^{-\beta t} dt \] (1)

Where \( \beta > 0 \) is the individual discount rate. Instantaneous utility is defined over absolute consumption, \( C_i \), and relative consumption, \( C_i/H_i \), where agents compare their own consumption with a measure of the past economy-wide average level of consumption, \( H_i \), in the tradition initiated by Duesemberry (1949) and later formalized by Pollak (1976) among others. We impose non-satiation in utility, restricting \( \gamma \) to lie in the range \( 0 \leq \gamma < 1 \). If \( \gamma = 0 \), (1) reduces to the conventional specification in which preferences are time independent and therefore only the absolute level of consumption matters. As \( \gamma \to 1 \) only relative consumption matters and the absolute level of consumption becomes irrelevant. In general, \( \sigma \) and \( \gamma \) interact to determine the intertemporal elasticity of substitution.

We model \( H_i \) as a declining weighted average of past consumption of a reference group. Since agents are atomistic, when deciding over current consumption, they ignore the effects of this decision on the future evolution of the reference stock, taking it as exogenous. Therefore under this “catching up with the Joneses” specification the reference stock is determined by,

\[ H_i(t) = \rho \int_{-\infty}^t e^{\rho(t-\tau)} \bar{C}(\tau) d\tau \quad \rho > 0 \] (2)

Where \( \bar{C} = \sum_{i=1}^N C_i/N \) denotes the average consumption of the economy.

Differentiating with respect to time implies the following rate of adjustment of the reference stock,

controls on capital account transactions for an indefinite period. In this line, Saint-Paul (1993) characterizes the French economy as “exchange controls prevailed, imports were severely restricted, the market for foreign exchange was severely regulated and segmented with a complicated system of multiple exchange rates.” (pg. 15) Therefore a close economy that equates domestic saving to investment does not seem as an unrealistic assumption.
With the speed of adjustment, $\rho$, measuring the relative importance of recent consumption in the determination of the reference stock.

An alternative approach commonly referred as “habit formation” assumes that the reference stock is an average of the agent’s own past consumption. Under this assumption, a forward-looking agent fully internalizes the effects of her current consumption decision in the future evolution of the reference stock. This approach requires the introduction of a second co-state variable leading to a higher order dynamical system despite the fact that under exogenously supplied labor the behavior of both specifications is very similar. For these reasons, we restrict our analysis to the external evolution of the reference stock, although our results are easily extended to a habit-forming environment.

Finally, our preferences differ from those in Christiano (1989) which introduces an additive exogenously determined reference level of consumption. His specification has several theoretical problems, apart from the arbitrary determination of the reference level. The most important one is that for plausible parameterizations of that reference level, the low post-war consumption might lead to a negative argument in the utility function.

Individual $i$’s output is produced combining her private capital stock, $K_i$, and the level of inelastically supplied labor, $N_i$. Assuming that the exogenous rate of technological change, $\dot{A}/A = \alpha$, is Harrod neutral, a natural benchmark since this assumption is necessary for the existence of a balanced growth path, individual output is determined according to the following CES production function,

$$ Y_i = \alpha \left[ \mu K_i^{-b} + (1 - \mu) (AN_i)^{-b} \right]^{\frac{1}{b}} \quad b > -1 $$

(4)
Where $\alpha$, might reflect any institutional factors that affect the level of output, $\mu$ determines the functional distribution of income, and $1/(1+b)$ is the elasticity of substitution between capital and augmented labor.

Final output can be either consumed, yielding immediate satisfaction, or saved. Assuming that the existing capital stock depreciates at a rate, $\delta$, household $i$’s capital stock evolves according to the following law of motion,

$$\dot{K}_i = Y_i - C_i - (n + \delta)K_i \quad (5)$$

The combination of a flexible preference specification, (1), and a general production structure, (2), allows us to parameterize a wide variety of models extensively used in the growth literature.

4. Macroeconomic equilibrium.

The representative agent makes her consumption-saving decision to maximize (1) subject to (5) with the evolution of the reference stock taken as given. With $\lambda_i$ being the co-state variable associated with the capital stock, the first order conditions for an optimum are

$$U_{c_i} \equiv \frac{C_i^{-\sigma}}{H_i^{\gamma(1-\sigma)}} = \lambda_i \quad (6a)$$

$$\frac{\mu}{\alpha^b} \left( \frac{Y_i}{K_i} \right)^{1+b} - \delta n = \beta \frac{\lambda_i}{\lambda_i} \quad (6b)$$

together with the transversality conditions

$$\lim_{t \to \infty} \lambda_i K_i e^{-r_k} = 0 \quad (6c)$$
The interpretations of (6a) and (6b) are standard; (10a) equates the utility of an additional unit of consumption, which depends on the reference stock, to the shadow value of capital and (6b) is an intertemporal allocation condition equating the marginal product of capital to the rate of return on consumption.

Taking the time derivative of (6a), combining the result with (6b) and (3) and imposing the equilibrium condition \( C_i = \bar{C} = C/N \), we can obtain the equilibrium path for individual consumption. Summing across households,

\[
\hat{C} \equiv \frac{\dot{C}}{C} = \frac{1}{\sigma} \left[ \frac{\mu}{\alpha^b} \left( \frac{Y}{K} \right)^{1+b} - \beta - \delta - (1-\sigma)n - \rho \gamma (1-\sigma) \left( \frac{C}{H} - 1 \right) \right]
\]

which describes the evolution for the growth rate of aggregate consumption as a function of the capital and reference stocks.

We define a balanced growth path as being one along which all variables grow at a constant rate. With capital being accumulated from final output, the only balanced solution is one in which the capital-output ratio remains constant. Following this definition it is convenient to define variables expressed in units of effective labor, \( k \equiv K/AN \), \( c \equiv C/AN \) and \( h \equiv H/AN \), which will remain constant in steady state. This enables us to rewrite expression (3), (4), (5) and (7) in terms \( k, c \) and \( h \),

\[
\dot{k} = y - c - (\delta + x + n)k \quad (8a)
\]

\[
\dot{c} = \frac{c}{\sigma} \left[ \frac{\mu}{\alpha^b} \left( \frac{y}{k} \right)^{1+b} - \beta - \delta - n - \sigma x - \rho \gamma (1-\sigma) \left( \frac{c}{h} - 1 \right) \right] \quad (8b)
\]

\[
\dot{h} = \rho (c - h) - xh \quad (8c)
\]
where $y$, output per unit of effective labor, is a function of $k$ and other technological parameters of the model. Note that if either $\gamma = 0$, so that the reference stock is irrelevant to utility, or $\rho = 0$, so that the reference stock is fixed, (8a) and (8b) collapse to the system of equations that describes the dynamics under the familiar time-separable utility specification.

Imposing the steady state condition, $\dot{c} = \dot{k} = \dot{h} = 0$, we can solve for the steady state values of capital, consumption, and habit as follows,

$$k^* = \left[ \frac{\gamma (1-\sigma) x + \beta + \delta + n + \sigma x}{\mu \alpha} \right]^{\frac{1}{\mu}} - \mu$$ \hspace{1cm} (9a)

$$c^* = y^* - (\delta + n + x) k^*$$ \hspace{1cm} (9b)

$$h^* = \frac{c^*}{(x/\rho + 1)}$$ \hspace{1cm} (9c)

In the absence of technological change, $x = 0$, (9c) implies $c^* = h^*$, so that the stationary consumption level coincides with the level of the reference stock. Furthermore the equilibrium capital stock of the time non-separable economy will be independent of $\gamma$ and equal to the equilibrium capital stock of an economy populated by agents with time-separable preferences. When $x > 0$, the introduction of the reference stock increases the equilibrium capital stock per unit of effective labor if and only if the short-run intertemporal elasticity of substitution is less than unity, as empirical evidence suggests.
5. Numerical analysis

We parameterize our model to reproduce some of the specifications encountered in the growth literature, specifically\(^\text{10}\); a standard version of the Ramsey (1928) model where preferences are time separable with Cobb-Douglas technology (SCD), Carroll et al. (1997) where preferences are time non-separable and technology exhibits constant returns to capital (NSAK), Alvarez-Cuadrado et al. (2004) where preferences are time non-separable and technology is Cobb-Douglas (NSCD), and finally a specification that combines time non-separable preferences with a CES technology (NSCES).

5.1 Equilibrium values under alternative parameterizations.

The resulting specifications are calibrated to reproduce some of the key features of actual economies. Table 3 summarizes the parameters upon which our simulations are based. The rate of time preference \(\beta = 0.04\) and the rate of depreciation \(\delta = 0.05\) are standard and require no further explanation.

In the economies with neoclassical technology (SCD, NSCD and NSCES) \(\alpha\) is normalized to 1, the distribution parameter, \(\mu\), is set to match an equilibrium capital share of 0.4, the parameter that governs the intertemporal substitutability of consumption, \(\sigma\), is chosen to match an equilibrium saving rate equal to 0.26, the rate of population growth is set to 0.015 and the rate of exogenous technological change, \(x = 0.02\), is consistent with the average growth rate of our sample economies over the last century\(^\text{11}\). In the NSAK economy, we use \(\alpha\) to reproduce a 2% equilibrium growth rate, while \(\sigma\) is set equal to 2.5, consistent with the estimates reported by Ogaki and Reinhart (1998). In the time non-separable specifications (NSCD, NSAK and NSCES) the parameter that controls the importance of the reference stock, \(\gamma\), is set to 0.9 within the range of estimates provided by Fuhrer

\(^{10}\) Even though we can parameterize our model to reproduce Rebelo (1991) where preferences are time separable and technology exhibits constant returns to capital, we abstract from this specification on the basis of its well-known absence of transitional dynamics.

\(^{11}\) Maddison (2001) reports the following average growth rates of per capita real output for the twentieth century; Austria, 1.96\%, France, 2.02\%, Germany, 1.87\%, Italy 2.32\% and Netherlands, 1.84\%. 

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(2000) and Ravina (2005). There is little empirical evidence on the magnitude of the speed of adjustment of the reference stock, $\rho$. Carroll (2000) calibrates a similar model to reproduce the increase in the Japanese saving rate during the seventies, for this purpose he chooses a very slow speed of adjustment. Along these lines, we set $\rho$ equal to 0.35 that implies a half-life of the adjustment of the reference stock to a permanent change in consumption close to 2 years $^{12}$.

The final parametric choice is for the elasticity parameter in the CES technology, $b$. We set the elasticity of substitution between inputs to 0.7, the mid-point of the range of estimates reported by Antras (2004) after allowing for biased technological change $^{13,14}$.

Table 4 presents the equilibrium for the four economies. As a result of our parameterization all models display an equilibrium growth rate of 2%. The models with neoclassical technology yield an output-capital ratio of 0.32, a savings rate of 0.26, a rate of return of capital close to 8% and a steady state capital share of 0.4, all of them in line with the empirical evidence on OCED economies. As expected, the savings rate (output-capital ratio) is substantially higher (lower) in the model that exhibits constant returns to capital, in empirical grounds we can rationalize these values attending to a broad definition of capital. Finally, the speeds of convergence implied by all of our specifications are consistent with the estimates reported by Islam (1995) and Caselli et al. (1997).

$^{12}$ This value is consistent with the estimates reported by Ravn, Schmitt-Grohe and Uribe (2006).

$^{13}$ The initial attempts to estimate the elasticity of substitution provided estimates well below unity in time series studies (Arrow et al. (1961), David and Van de Klundert (1965)) while cross sectional estimates where close to 1 (Arrow et al (1961)). In a seminal contribution Berndt (1976), assuming Hicks-neutral technological change, reconciles both approaches reaching estimates that are not significantly different from one. Antras (2004) argues convincingly that restricting the analysis to Hicks-neutral technological change biases the estimate of the elasticity towards one. When he modifies his specification to allow for biased technical change, he consistently obtains estimates of the elasticity of substitution well below unity.

$^{14}$ We have conducted extensive sensitivity analysis within the range of relevant parameter values. Changes in parameters that are common across specifications lead to parallel changes in the adjustment of the different models, leaving the conclusions of the exercise that follows basically unchanged. As one can expect our results are sensitive to changes in the parameters that determine the importance of the reference stock in preferences and the ability to substitute capital for labor in production, which are used to pin down our different specifications. Intuitively decreases in the importance of the reference stock, measured by lower values of $\gamma$, lead to larger responses of consumption after the destruction of capital; the limiting case, where the reference stock is irrelevant, is captured by our SCD specification. In the production side, as long as the elasticity of substitution between inputs is below unity, increases in this elasticity, captured by lower values of $b$, lead to smaller changes in relative prices and factor shares; the limiting case, where this elasticity is one, is captured by our SCD and NSCD specifications.
5.2 Computational Strategy.

In general, the adjustment path of this class of models is evaluated relying on linear approximations about the steady state. The performance of these approximations depends crucially on the degree of non-linearity of the original problem and on the steady state gap considered. Given the substantial steady state gap caused by the conflict, the use of linear techniques is not recommendable and we resort to polynomial approximations that have been reported to provide very reliable results in similar contexts. The solution to (8) with the initial conditions, $k(0) = k_0$ and $h(0) = h_0$, and the boundary condition (6c) amounts to the solution of a boundary value problem, i.e. a system of nonlinear ordinary differential equations with conditions specified at two different points in time, $t = 0$ and $t \to \infty$. Following Judd’s (1992) collocation method, we use a twelfth degree Chebyshev polynomial basis to approximate this solution. Denoting the approximate solutions for consumption, capital and the reference stock as $\hat{c}$, $\hat{k}$ and $\hat{h}$, respectively, we define our residual functions as the difference between the left hand side and the right hand side of (8), where the true solutions are replaced by our approximation. Thirty three residual conditions are derived from 11 Chebyshev-nodes and three additional conditions are determined by the two initial conditions and the terminal condition. As a result we end up with a system of 36 linear equations on the 36 unknown coefficients of the approximate time paths. We solve this system using Broyden’s method. Santos (2000) demonstrates that the approximation error is of the same order of magnitude as the size of the Euler equation residuals. This residual, that under the true solution is zero, can be interpreted as the one-period optimization error. We evaluate our solution over a set of 10,000 points in an interval of 100 years; the maximum consumption residual is 0.001 which in relative terms implies an error of 5.7 cents out of every $100 worth of consumption. In contrast an agent following the consumption rule derived from a linear approximation deviates by as much as $3 out of $100 from the optimal consumption path.
5.3 Initial conditions for the state variables.

The last factor we need to consider before we proceed with our calibration exercise is the initial conditions for the two state variables, capital and reference stock. The available evidence on capital stocks suggests that by the end of the conflict our war economies had lost more than 30% of their pre-war capital stock. On the other hand, output figures suggest a loss close to 90% of the pre-war stock of capital. We choose as the initial value of the capital stock 50% of its steady state value. Since our objective is to discriminate among the four models rather than to mimic the exact patterns of the data, small deviations from this initial condition do not affect the qualitative behavior of any of our specifications nor the results of our discrimination exercise.

In the models with non-separable preferences, the reference stock would have adjusted during the conflict as consumption decreased. Since there are no available figures for consumption during those years, we rely on indirect estimates derived from the output figures assuming a constant consumption share. Combining these results with the speed of adjustment, \( \rho \), we reach an estimate of the initial reference stock close to 90% of its steady state value. Finally, under time non-separable preferences the gap between the initial capital and reference stocks is an important factor in the first periods of the transition. Along these lines, imbalances between capital and labor inherited from the pre-war period should affect both state variables, and their steady state gaps, in a similar way leaving the results of the analysis that follows basically unchanged.

5.4 Dynamic response after a large destruction of capital.

It is worth noticing that the adjustment of this class of models is driven by the interaction of two forces; the “rate of return effect” and the “status effect”. When capital is below its equilibrium level, its rate of return is high, and therefore the relative price of current relative to future consumption is high and the present value of human wealth is low. These factors contribute to the

\[ \text{It is worth noticing that the largest decreases in output took place during the last years of the conflict. In France output fell by almost 45% between 1940 and 1943, in Germany and Austria output fell by 50% between 1944 and 1946, in Italy output fell around 40% between 1943 and 1945, and finally the Netherlands experienced a 33% decrease in output in 1944.}\]
“rate of return effect” that reduces consumption increasing savings. On the other hand, the presence of a reference stock is behind the “status effect” that constraints the deviations of individual consumption from the predetermined level of consumption of the reference group.

A 50% destruction of capital leads to an immediate decrease in output under any specification. In the SCD economy output decreases by 24%, and with only the “rate of return effect” at work, consumption absorbs the impact of the shock, decreasing by 35%. This large decrease in consumption leads to a 39% increase in the saving rate. This high level of investment combined with the high average productivity of capital leads to a monotonic adjustment path along which capital, output and consumption grow together, gradually restoring their pre-shock levels. The growth and saving rates reach their maxima immediately after the shock converging monotonically towards their unchanged intertemporal equilibrium levels.

In the NSAK economy with capital being the only input of production, its initial destruction is translated into a proportional decrease in output. Since the aggregate technology exhibits constant returns to capital, its marginal product is constant, and therefore the saving-consumption decision is fully driven by the agents’ unwillingness to reduce consumption below her standard of living, determined by their pre-shock reference stock. Under these circumstances the shock is mainly absorbed by a 24% reduction in savings. As the reference stock adjusts, the saving rate increases converging to its intertemporal equilibrium value from below. The unsustainably high level of consumption leads to a transition characterized by below-equilibrium levels of growth and capital accumulation.

In the economies that combine time non-separable preferences with diminishing returns (NSCD and NSCES) the initial destruction of capital leads to a 24%-26% decrease in output, depending on the degree of substitutability implied by the production technology. The presence of the reference stock inhibits the initial decline in consumption, which only falls by 22%-26%, leading to an immediate decrease in the saving rate. Nonetheless the high marginal product of capital
associated with the lower capital stock leads to a transition characterized by substantial growth. With output growing and consumption tied to the past by the “status effect”, savings increase faster than does output so that the saving rate begins to rise, reaching its peak after a decade. Thereafter, as capital accumulates, its marginal product decreases, and consequently the saving rate declines, doing so monotonically until equilibrium is restored.

The most relevant difference between the two time non-separable economies arises from the adjustment of the factor shares. Under Cobb-Douglas technology, a 50% destruction of capital leads to an exactly offsetting increase in its rate of return, and consequently constant factor shares characterize the adjustment of the NSCD economy. On the other hand the initial factor imbalance generates a more than proportional increase in the return to capital under the restrictions imposed by our CES parameterization leading to more than a 10% decrease in the wage share. Along the transition as capital accumulates, its rate of return decreases and the wage share recovers asymptotically its equilibrium value.

Figure 3 reproduces the time series generated by our alternative specifications in order to compare the adjustment of each model with the patterns of the post-war data summarized by our stylized facts. The predictions of the NSAK model are largely at conflict with post-war evidence. The assumption on the returns to capital leads to a transition characterized by low growth, low savings and a constant capital-output ratio, therefore we should focus on the specifications that introduce diminishing returns to capital16.

Under neoclassical technology, the behavior of the capital-output ratio is qualitatively consistent with the patterns in the data. As in post-WWII Europe, the destruction of capital leads to a transition characterized by an increasing capital-output ratio. The hump-shaped behavior of the saving rate (stylized fact 3) and the non-monotonic convergence of the growth (stylized fact 1) rate

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16 For expositional purposes we choose not to plot the transitional dynamics of the saving rate and the capital-output ratio under the NSAK specification.
are easily reproduced by models that introduce non-separabilities in preferences. On the other hand under the NSCD specification both rates reach their maxima right after the shock and thereafter they decrease monotonically.

To further discriminate between the time non-separable specifications we need to turn to the evolution of the wage share. The post-war evolution of the factor shares is easily reproduced by our NSCES specification. If the elasticity of substitution between inputs is below unity, the initial factor imbalance leads to a more than proportional increase in the price of the scarce resource, capital. As a result, an increasing labor share characterizes the adjustment of the economy with CES technology. Table 6 compares the time derivatives of the series generated under each specification with those of the post-war data, the results of this comparison are consistent with our previous discussion.

Finally, King and Rebelo (1993) highlights the counterfactual implications for the real interest rate of technologies that exhibit strong diminishing returns to capital. Assuming that the post-war U.S. and Japanese economy shared a common steady state level of capital, their calculations show that the Japanese real interest rate implied by a neoclassical technology would have been over 500 percent in 1950. In this context, it seems relevant to evaluate the implications for

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17 The non-monotonic adjustment of the saving rate results from the interaction between the initial values of the two state variables. With an initial capital gap of 50%, the hump shape in saving only disappears if the initial reference stock is below 65% of its steady state value.

18 Barro and Sala-i-Martin (1995) prove that under Cobb-Douglas the saving rate converges monotonically to its steady state. Smetters (2003) working with non-separable preferences and CES technology finds overshooting in the convergence of the saving rate of an economy with an initially low capital stock. Nonetheless in quantitative terms this overshooting is one order of magnitude smaller than the observed in the data.

19 We restrict our analysis to competitive markets but, as a referee pointed out, in the presence of monopolistic power the qualitative behavior of the labor share is consistent with the increases in product market competition that surely accompanied the process of European integration. There are three factors that cast some doubts over the quantitative importance of this hypothesis. First, the process of tariff reductions was initiated in 1959 with the most important increases in trade, and therefore competitive pressures, taking place in the late sixties right after the period we have considered. Second, at the same time that most of our war economies signed the Treaty of Rome, Britain, Norway, Sweden, Denmark, Portugal and Switzerland formed the European Free Trade Association (EFTA) with similar calendar and tariff reduction goals. These economies experienced an increase in their average labor share slightly above 3% compared to the 10.5% average increase experienced by our war economies. And third, the decrease in labor shares that began in the eighties, documented by Blanchard et al. (1997), took place despite the fact that the process of product market integration was at its height at that time. These factors suggest that increases in competition, even though are qualitatively consistent with the evolution of the labor share, are not enough to provide a satisfactory account for the substantial redistribution of income that began in the early fifties.
the marginal product of capital of our preferred specification. After the shock the rate of return to capital peaks around 22.5% then it monotonically decreases reaching 16% after a decade. Saint-Paul (1993) estimates the gross return to capital for France. He reports an estimate of 25% for 1949 and 15% for 1959, in line with the predictions of any of our neoclassical specifications.

6. Conclusions

This paper compares the adjustment of European economies after World War II with the time series generated by several growth models after a shock that halves the stock of physical capital. This analysis of the transitional dynamics is a powerful tool to discriminate among competing theoretical specifications that share similar steady state predictions.

Our results suggest that the introduction of non-separabilities in preferences and technologies that depart from the unitary elasticity of substitution implied by the Cobb-Douglas production function improve substantially the ability of the model to reproduce the adjustment observed after a large destruction of capital.

If the economy is better described by our preferred specification, as evidence from post-war Europe suggests, macroeconomic policies developed under other assumptions are likely to be grossly misleading. For instance in the supply side, our results cast some doubts on the conclusions of the literature that explores whether a reduction in marginal tax rates can improve the long-run government budget that heavily relies on a technological structure that exhibits constant returns to capital. In the demand side, policy choices based on welfare analysis should be taken with caution. Along these lines Alvarez-Cuadrado et al. (2004) document substantial differences in the response of welfare between economies with time separable and time non-separable preferences. In their benchmark calibration an increase in productivity leads to an increase in welfare 40% larger in an economy characterized by time non-separable preferences. In empirical grounds, Hsieh (2002)

20 Alvarez-Cuadrado (2005) explores several explanation of the European Golden Age that discount the importance of the conflict and the destruction of capital. His results reinforce our confidence on the relevance of the steady state gap as a key determinant of the post-war growth record.
points out that conventional growth accounting exercises overstate the true contribution of factor accumulation to growth, when technological change is not neutral and the elasticity of substitution between capital and labor is below one.

Furthermore, our results provide some insight into the substantial reduction in the growth rate of output per worker that began in the late 1960s or early 1970s. In the Western European context, conventional quantifications of the productivity slowdown that fail to account for the lasting effects of the war might overstate its true dimension.

Finally, the performance of our preferred model has been only contrasted in the context of the post-war European economies and, therefore, a broader analysis of this specification remains relevant. Along these lines, empirical evidence on the effects of financial liberalizations or important institutional changes can be compared with the predictions of our model.
Tables and Figures

<table>
<thead>
<tr>
<th>Country</th>
<th>Population</th>
<th>Armed Forces</th>
<th>Military</th>
<th>Civilian</th>
<th>%Milit.</th>
<th>%Civil</th>
<th>Tot.%</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>42,000</td>
<td>4,600</td>
<td>122</td>
<td>470</td>
<td>2.65%</td>
<td>1.26%</td>
<td>1.41%</td>
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<tr>
<td>Italy</td>
<td>43,800</td>
<td>227</td>
<td>60</td>
<td>150</td>
<td>3.43%</td>
<td>1.81%</td>
<td>1.88%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>8,700</td>
<td>400</td>
<td>14</td>
<td>150</td>
<td>3.43%</td>
<td>1.81%</td>
<td>1.88%</td>
</tr>
<tr>
<td>Germany/Austria</td>
<td>78,000</td>
<td>17,900</td>
<td>3,250</td>
<td>2,350</td>
<td>18.16%</td>
<td>3.91%</td>
<td>7.18%</td>
</tr>
<tr>
<td>U.S.</td>
<td>129,200</td>
<td>16,354</td>
<td>405</td>
<td>2,350</td>
<td>18.16%</td>
<td>3.91%</td>
<td>7.18%</td>
</tr>
<tr>
<td>Canada</td>
<td>11,100</td>
<td>1,100</td>
<td>39</td>
<td>150</td>
<td>3.43%</td>
<td>1.81%</td>
<td>1.88%</td>
</tr>
<tr>
<td>Australia</td>
<td>6,900</td>
<td>1,340</td>
<td>29</td>
<td>150</td>
<td>3.43%</td>
<td>1.81%</td>
<td>1.88%</td>
</tr>
</tbody>
</table>

Table 2. The effects of the war in European population. Figures in thousands, population measured in 1940.

<table>
<thead>
<tr>
<th>Preference Parameters</th>
<th>SCD</th>
<th>NSAK</th>
<th>NSCD</th>
<th>NSCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.2</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 3. Alternative model parameterization.

21 “Pre-war level” is the year at which output per capita recovered the 1939 level. “Back to trend” is the year at which actual output would have equaled the level of output if the war never occurred. We estimate this level of output as the fitted value from a regression of the log of real per capital GDP on a constant and a time trend.
<table>
<thead>
<tr>
<th>Variable</th>
<th>SCD</th>
<th>NSAK</th>
<th>NSCD</th>
<th>NSCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rate</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Saving rate</td>
<td>0.26</td>
<td>0.62</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Output-Capital ratio</td>
<td>0.32</td>
<td>0.11</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Labor income share</td>
<td>0.60</td>
<td>-</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Net return on capital</td>
<td>7.8%</td>
<td>7.3%</td>
<td>7.8%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Asym. convergence speed</td>
<td>0.08</td>
<td>0.17</td>
<td>0.11</td>
<td>0.11</td>
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</table>

Table 4. Equilibrium values of the most relevant variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>SCD</th>
<th>NSAK</th>
<th>NSCD</th>
<th>NSCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>-50 %</td>
<td>-50 %</td>
<td>-50 %</td>
<td>-50 %</td>
</tr>
<tr>
<td>Output</td>
<td>-24 %</td>
<td>-50 %</td>
<td>-24 %</td>
<td>-26 %</td>
</tr>
<tr>
<td>Consumption</td>
<td>-35 %</td>
<td>-30 %</td>
<td>-22 %</td>
<td>-26 %</td>
</tr>
<tr>
<td>Saving rate</td>
<td>+39 %</td>
<td>-24 %</td>
<td>-10 %</td>
<td>-1 %</td>
</tr>
<tr>
<td>Labor income share</td>
<td>0 %</td>
<td>0 %</td>
<td>0 %</td>
<td>-12 %</td>
</tr>
<tr>
<td>Net return on capital</td>
<td>+85 %</td>
<td>0 %</td>
<td>+85 %</td>
<td>+122 %</td>
</tr>
</tbody>
</table>

Table 5. Initial responses after the shock as a percentage of the equilibrium values.

<table>
<thead>
<tr>
<th>SF1: Growth rate</th>
<th>Data</th>
<th>SCD</th>
<th>NSAK</th>
<th>NSCD</th>
<th>NSCES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \dot{J} &gt; 0, \ddot{J} &lt; 0 )</td>
<td>( \dot{J} &gt; 0 )</td>
<td>( \dot{J} &gt; 0 )</td>
<td>( \dot{J} &gt; 0, \ddot{J} &lt; 0 )</td>
<td>( \dot{J} &gt; 0, \ddot{J} &lt; 0 )</td>
</tr>
<tr>
<td>SF2: Capital-Output</td>
<td>( \dot{J} &gt; 0 )</td>
<td>( \dot{J} &lt; 0 )</td>
<td>( \dot{J} = 0 )</td>
<td>( \dot{J} &gt; 0 )</td>
<td>( \dot{J} &gt; 0 )</td>
</tr>
<tr>
<td>SF3: Saving rate</td>
<td>( \dot{J} &gt; 0, \ddot{J} &lt; 0 )</td>
<td>( \dot{J} &lt; 0 )</td>
<td>( \dot{J} &gt; 0 )</td>
<td>( \dot{J} &gt; 0, \ddot{J} &lt; 0 )</td>
<td>( \dot{J} &gt; 0, \ddot{J} &lt; 0 )</td>
</tr>
<tr>
<td>SF3: Labor income share</td>
<td>( \dot{J} &gt; 0 )</td>
<td>( \dot{J} = 0 )</td>
<td>( \dot{J} = 0 )</td>
<td>( \dot{J} = 0 )</td>
<td>( \dot{J} &gt; 0 )</td>
</tr>
</tbody>
</table>

Table 6. Time derivatives of deviations from steady state; data vs. alternative model specifications.
Figure 1. Evolution of key economic variables after World War II.
Figure 2. Evolution of key economic variables after World War II. Averages.
Figure 3. Comparison between the time series generated by the models and the actual post-war data.
Appendix I. Data Sources

Section 2

a. Data on the effects of the war.

The decrease in income is calculated using data from Maddison (2001). The decreases in the capital stock are calculated from several sources. For Germany we combine the estimates of Maddison (1994) with evidence on the effects of the allied bombings in public infrastructure and residential capital, and anecdotal evidence on the dismantling of the German industry. Maddison (1994) reports in million 1990DM: Total gross non-residential capital stock in 1939, 453,482, and in 1947, 376,310. Some early readers of this manuscript suggested that Germany might have been building up its stock of capital since the raise of the Nazi party, and therefore the pre-war stock might overstate its steady state value. The rate of growth of the capital stock between 1935 and 1939 averaged 1.92% (that same period population growth averaged .88%), a figure that does not signal such a substantial build up. For France we use the direct estimates provided by Saint-Paul (1993). For Netherlands we rely on Griffiths and Van Zanden (1989) estimates. For Italy we base our estimates in data provided by DeCecco and Giavazzi (1993) and Zamagni (1998). The former report the shares of industrial capacity damaged by the war were 11.8%, 88% and 50% in the northern, central and southern regions respectively. The latter reports post-war estimates of 20% of industrial capacity destroyed by the conflict. Finally the evidence for Austria is sparser, Bischof and Pelinka (1995) is the main source for our estimates.

All the figures about the victims of the war are the direct estimates provided by Ellis (1993).

b. Data on the adjustment process

For expository purposes, we choose to report five-year moving averages for each country and variable. Aggregate data is calculated using real GDP Geary-Khamis PPP adjusted US$ 1990 as the aggregation weight.

Data on per capita real GDP is from Maddison (2001). Geary-Khamis PPP adjusted US$ 1990. When we replace GDP per capita with GDP per worker, we find a similar pattern where the average growth rate for the war economies is 5.21% in the fifties and 4.43% in the sixties. In the control groups those figures are 1.91% and 2.29% respectively.

Data on saving rate is from Maddison (1992), except Italy and Austria that are from Heston et al. (2000), where we use investment share of real GDP at constant prices ($k_i / Investment share of real GDP at constant prices$). The measure of investment constructed by Maddison (1992) includes Gross Non-Residential and Residential Fixed Investment, Investment in Inventories and Investment Abroad. Stylized fact 3 is robust to alternative definitions of the saving rate, specifically to exclusions of the last component.

Data on the capital-output ratios is from King and Levine (1994). They compute the gross capital stock based on the Perpetual Inventory Method. Their method requires an initial guess for the capital stock that they base in an estimate of the steady state capital-output ratio. Then the capital stock for 1950 is computed as the product between this estimate of the capital-output ratio and the widely available measure of income per capita for that year. Subsequent values of the capital stock are computed adding investment and allowing for depreciation. As a consequence of the conflict, it is very possible that the steady state ratio that they use as initial condition substantially overstates the capital stock in 1950. As the weight of the initial stock in their calculation for the current capital stock declines, the estimate becomes more reliable. As a result, we believe that the initial dip in the capital-output ratio comes from the unreliable estimation of its initial value since this problem
disappears as the initial stock depreciates we are confident that the data after the mid-fifties becomes a good approximation to the true capital-output ratio. Confirming our argument, Maddison (1994) provides estimates of the capital-output ratio for Germany and France in 1950. His estimate for France is 15% below the one provided by King and Levine (1994), for Germany is 40% below.

References


Heston, A. R. Summers and B. Aten (2002). Penn World Table Version 6.1, *Center for International Comparisons at the University of Pennsylvania (CICUP).*


