## CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS WORKING PAPER SERIES



CDMA07/01

# Is There More than One Way to be E-Stable?* 

Joseph Pearlman ${ }^{\dagger}$<br>London Metropolitan University

JANUARY 2007


#### Abstract

We initially examine two different methods for learning about parameters in a Rational Expectations setting, and show that there are conflicting E-stability results. We show that this conflict also extends to Minimum State Variable (MSV) representations. One of these methods of learning lends itself to the examination of E-stability for the generic forward-looking rational expectations model. This leads to a completely general relationship between saddlepath stability and E-stability, and a generalization of MSV results.


JEL Classification: C6, E00.
Keywords: E-stability, Minimum state variable.

[^0]
## 1 Introduction

Despite the vast quantity of research on E-stability, notably by Evans and Honkapohja (2000), there is little that touches on the general canonical form for rational expectations (RE) models. The latter is described by Blanchard and Kahn (1980), and it differs from most of the work on E-stability because it allows for the possibility of expectations effectively predicated on a variety of differently dated information sets.

Once this information set heterogeneity has been included, it becomes apparent that there is more than one way in which to engage in least-squares learning about the parameter values. The immediate effect of this is that each method of learning results in potentially differing conditions for E-stability. Thus the constraints on parameter values are potentially more demanding than the generic cases usually considered in the E-stability literature.

However a more general rational expectations model, which may involve numerous variables can only be tested for E-stability using one of the methods described above. It turns out that this method allows for a very simple relationship between saddlepath stability and E-stability in the general case.

Section 2 reminds readers of the general form of RE models, and compares these to the models usually addressed by the E-stability literature. We then proceed to examine a model with a particularly simple $\mathrm{AR}(1)$ reduced form, and examine two different proposed ways of least-squares learning. We derive the conditions in each case for E-stability and examine whether they are equivalent. We also briefly address the impact on the minimum state variable solution. Section 3 shows how these two methods are directly linked to making different learning assumptions in a semi-reduced form of an RE model. Section 4 poses the question of whether the two methods of learning should be jointly addressed. Section 5 extends one of the methods to the general case, and derives the main results of the paper. Section 6 concludes.

## 2 The Generic Rational Expectations Model

Following Blanchard and Kahn (1980) we can write the generic model with forward-looking rational expectations (henceforth RE) in the following form ${ }^{1}$ :

$$
\left[\begin{array}{c}
z_{t}  \tag{1}\\
E_{t} x_{t+1}
\end{array}\right]=\left[\begin{array}{ll}
\alpha & \beta \\
\gamma & \delta
\end{array}\right]\left[\begin{array}{c}
z_{t-1} \\
x_{t}
\end{array}\right]+\left[\begin{array}{c}
\varepsilon_{t} \\
0
\end{array}\right]
$$

[^1]where $E_{t}$ denotes expectations formed using information available at time $t, z_{t}$ is a vector of predetermined variables, and $x_{t}$ is a vector of non-predetermined variables, and $\varepsilon_{t}$ is a vector of random noise terms. We omit any noise terms in the second set of equations for ease of exposition only. $\alpha, \beta, \gamma, \delta$ are appropriately dimensioned matrices, conforming to the dimensions of $z_{t}, x_{t}$.

One of the implications of writing RE systems in this way is that it encompasses expectations made at differing time periods. For a particularly compelling example in macroeconomics, see Svensson (2000).

In order to address the issues in as simple a manner as possible, from now on in this section we treat all the above vectors and matrices as scalars. In particular this means that we can write (1) in terms of $x_{t}$ only, in the following form:

$$
\begin{equation*}
E_{t} x_{t+1}-\alpha E_{t-1} x_{t}=\gamma\left(\beta x_{t-1}+\varepsilon_{t-1}\right)+\delta\left(x_{t}-\alpha x_{t-1}\right) \tag{2}
\end{equation*}
$$

This contrasts with the most commonly used models of Evans and Honkapohja (2000), which are in one of the following forms:

$$
\begin{align*}
x_{t} & =a E_{t} x_{t+1}+c x_{t-1}+\varepsilon_{t}  \tag{3}\\
x_{t} & =a E_{t-1} x_{t+1}+c x_{t-1}+d E_{t-1} x_{t}+\varepsilon_{t} \tag{4}
\end{align*}
$$

Conditions for determinacy and E-stability for learning have been studied for the latter in great detail, and are not repeated here. Instead we focus on two alternatives for learning about the reduced form parameters of (1). Before embarking on this, we recall that the condition for determinacy of the system (1) is that there is one stable root, $\lambda_{S}$, and one unstable root, $\lambda_{U}$, of the quadratic:

$$
\begin{equation*}
\lambda^{2}-(\alpha+\delta) \lambda+\alpha \delta-\beta \gamma=0 \tag{5}
\end{equation*}
$$

### 2.1 Learning about the relationship between $x_{t}$ and $z_{t}$

We make the assumption that there is a perceived relationship between $x_{t}$ and $z_{t}$ of the form

$$
\begin{equation*}
x_{t}+n_{k} z_{t-1}=0 \tag{6}
\end{equation*}
$$

Advancing this by one period and taking expectations of the first equation of (1) yields the relationship

$$
\begin{equation*}
\left(\gamma+n_{k} \alpha\right) z_{t-1}+\left(\delta+n_{k} \beta\right) x_{t}=0 \tag{7}
\end{equation*}
$$

From now on, we avoid all the mathematical rigour of linear least-squares learning and stochastic approximation, and merely provide an intuitive explanation of the updating
relationships. However there is no essential departure from the methodology of Evans and Honkapohja (2000). From (7) it follows that a natural way of updating $n_{k}$ is

$$
\begin{equation*}
n_{k+1}=(1-\eta) n_{k}+\eta \frac{\gamma+n_{k} \alpha}{\delta+n_{k} \beta} \tag{8}
\end{equation*}
$$

Evans and Honkapohja (2000) show that this can be naturally expressed in continuous form as

$$
\begin{equation*}
\frac{d n}{d k}=\frac{\gamma+n \alpha}{\delta+n \beta}-n \tag{9}
\end{equation*}
$$

where the value of $\eta$ is immaterial, as the E-stability of this system is independent of it. Estability holds if (9) converges to the equilibrium from a neighbourhood of the equilibrium. To check this, all we need to do is to ascertain that the derivative of the right-hand-side (RHS) of (9) is negative at the equilibrium. Using the Blanchard and Kahn (1980) result that the relationship between $n$ and $\lambda$ is given by

$$
\left[\begin{array}{ll}
n & 1
\end{array}\right]\left[\begin{array}{ll}
\alpha & \beta  \tag{10}\\
\gamma & \delta
\end{array}\right]=\lambda_{U}\left[\begin{array}{ll}
n & 1
\end{array}\right]
$$

we have in particular $\lambda_{U}=\delta+n \beta$. Using this and (5) it follows that the derivative of the RHS of (9) is equal to

$$
\begin{equation*}
\frac{\alpha \delta-\beta \gamma}{(\delta+n \beta)^{2}}-1=\frac{(\alpha+\delta) \lambda_{U}-\lambda_{U}^{2}}{\lambda_{U}^{2}}-1=\frac{\alpha+\delta-\lambda_{U}}{\lambda_{U}}-1 \tag{11}
\end{equation*}
$$

It is clear from (5) that $\lambda_{U}>(\alpha+\delta) / 2$, so it follows that (11) is negative. Hence we have

Result 1: The updating equation for $n$ is E-stable.
Remark: Note that the updating equation for $n$ associated with the stable root $\lambda_{S}$ would not be E-stable, because the latter satisfies $\lambda_{S}<(\alpha+\delta) / 2$. This has a useful implication when both values of $\lambda<1$, because it means that the minimum state variable (MSV) solution is E-stable, and that indeterminacy is not an issue. To rephrase this, although in principle there may appear to be two candidate dynamic solutions to the system, only one of them is learnable.

### 2.2 Learning about the Perceived Law of Motion

Suppose we now assume that we can write $x_{t}$ as an $\operatorname{AR}(1)$ process:

$$
\begin{equation*}
x_{t}=\lambda x_{t-1}+\phi \varepsilon_{t} \tag{12}
\end{equation*}
$$

Advancing this by one period and taking expectations yields $\lambda x_{t}=\gamma z_{t-1}+\delta x_{t}$. Substituting for $z_{t-1}$ in terms of $x_{t}$ and writing the first equation of (1) in terms of $x_{t}$ yields

$$
\begin{equation*}
x_{t+1}=\left(\alpha+\frac{\beta \gamma}{\lambda-\delta}\right) x_{t}+\frac{\gamma}{\lambda-\delta} \varepsilon_{t+1} \tag{13}
\end{equation*}
$$

It follows that the continuous updating equation for $\lambda$ takes the form

$$
\begin{equation*}
\frac{d \lambda}{d k}=\alpha+\frac{\beta \gamma}{\lambda-\delta}-\lambda \tag{14}
\end{equation*}
$$

The E-stability requirement is once again for the derivative of the RHS of this expression to be negative at the equilibrium. The latter is given by $-\frac{\beta \gamma}{(\lambda-\delta)^{2}}-1$, so that

Result 2: If $\beta \gamma>0$, then the updating equation for $\lambda$ is E -stable.
Remark: If both roots $\lambda<1$, the implication is that there is no unique MSV solution. This is because learning about each root is an E-stable process.

Consider now the case when $\beta \gamma<0$; it is easy to see that the two values of $\lambda$ must lie between $\alpha$ and $\delta$. From our earlier discussion we know that $2 \lambda_{S}<\alpha+\delta<2 \lambda_{U}$, from which it follows that $\lambda_{S}-\delta<\alpha-\lambda_{S}$ and $\lambda_{U}-\delta>\alpha-\lambda_{U}$. Noting that

$$
\begin{equation*}
-\frac{\beta \gamma}{(\lambda-\delta)^{2}}-1=\frac{\alpha-\lambda}{\lambda-\delta}-1 \tag{15}
\end{equation*}
$$

we can deduce the following
Result 3: If $\beta \gamma<0$ and $\alpha>\delta$ then the learning process for $\lambda_{S}$ is not E-stable.
Remark: On the other hand, it is easy to see that if $\lambda_{U}$ is also a stable root, then its learning process is E-stable.

Result 4: If $\beta \gamma<0$ and $\alpha<\delta$ then the learning process for $\lambda_{S}$ is E-stable.
Remark: If $\lambda_{U}$ is also a stable root, then its learning process is not E-stable.

## 3 Comparability with the Standard Setup

Suppose that we focus on the semi-reduced form (2) of the model, and on learning about the law of motion expressed by (12). There are then two potential methods of using this for learning. The first is to use the substitutions $E_{t} x_{t+1}=\lambda x_{t}, E_{t-1} x_{t}=x_{t}-\phi \varepsilon_{t}$, while the second is $E_{t} x_{t+1}=\lambda x_{t}, E_{t-1} x_{t}=\lambda x_{t-1}$.

Method 1: $E_{t} x_{t+1}=\lambda x_{t}, E_{t-1} x_{t}=x_{t}-\phi \varepsilon_{t}$. This yields the relationship

$$
\begin{equation*}
\lambda x_{t}=\alpha\left(x_{t}-\phi \varepsilon_{t}\right)+\delta x_{t}+(\beta \gamma-\alpha \delta) x_{t-1}+\gamma \varepsilon_{t} \tag{16}
\end{equation*}
$$

so that the continuous form of the updating equation is given by

$$
\begin{equation*}
\frac{d \lambda}{d k}=\frac{\beta \gamma-\alpha \delta}{\lambda-\alpha-\delta}-\lambda \tag{17}
\end{equation*}
$$

Note that in equilibrium, the law of motion for $z_{t}$ in (1) is given by $z_{t+1}=(\alpha-\beta n) z_{t}+\varepsilon_{t+1}$, so that $\lambda=\alpha-\beta n$. Substituting this into (17) yields a continuous updating equation for $n$ which is exactly (9).

Result 5: Method 1 is equivalent to learning about $n$.

Method 2: $E_{t} x_{t+1}=\lambda x_{t}, E_{t-1} x_{t}=\lambda x_{t-1}$. This yields the relationship

$$
\begin{equation*}
\lambda x_{t}=\alpha \lambda x_{t-1}+\delta x_{t}+(\beta \gamma-\alpha \delta) x_{t-1}+\gamma \varepsilon_{t} \tag{18}
\end{equation*}
$$

which leads to a continuous updating equation which is the same as (14).
Result 6: Method 2 is equivalent to the earlier method of learning about $\lambda$.

## 4 Simultaneous Learning about $\lambda$ and $n$

Does the conflict between the two forms of learning arise because they are not two alternatives, but because they should be considered as part of a joint learning process? This possibility can be very easily addressed.

Suppose we substitute $x_{t}=-n z_{t-1}$ in the $z_{t}$ equation, which yields $\lambda_{k+1}=\alpha-\beta n_{k}$, and $x_{t+1}=\lambda x_{t}+\phi \varepsilon_{t}$ in the $E_{t} x_{t+1}$ equation, which yields $n_{k+1}=\gamma /\left(\delta-\lambda_{k}\right)$. Hence the continuous updating equations are given by

$$
\begin{equation*}
\frac{d \lambda}{d k}=\alpha-\beta n-\lambda \quad \frac{d n}{d k}=\frac{\gamma}{\delta-\lambda}-n \tag{19}
\end{equation*}
$$

Stability is then analysed by assessing whether perturbations $(\Delta \lambda, \Delta n)$ from the steady state are stable. Thus we need the eigenvalues of the matrix below to be stable.

$$
\frac{d}{d k}\left[\begin{array}{c}
\Delta \lambda  \tag{20}\\
\Delta n
\end{array}\right]=\left[\begin{array}{cc}
-1 & -\beta \\
\frac{\gamma}{(\lambda-\delta)^{2}} & -1
\end{array}\right]\left[\begin{array}{c}
\Delta \lambda \\
\Delta n
\end{array}\right]
$$

It is easy to see that the eigenvalues $\mu$ of this are given by $(\mu+1)^{2}+\beta \gamma /(\lambda-\delta)^{2}$. Thus if $\beta \gamma>0$, then the real parts of these eigenvalues are negative, so the system is stable. But if both values of $\lambda$ lie between -1 and 1 , then there is no MSV solution.

On the other hand, suppose that $\beta \gamma<0$, then (20) is unstable if one of the roots $\mu=-1 \pm \sqrt{ }(-\beta \gamma) /(\lambda-\delta)$ is greater than 0 . This is the case when $-\frac{\beta \gamma}{(\lambda-\delta)^{2}}-1>0$. Both of these possibilities replicate what we have seen before:

Result 7: The conditions for E-stability and for an MSV solution to exist are exactly the same as for the case of learning about the perceived law of motion only.

It seems then that criticism about learning about the relationship between $x_{t}$ and $z_{t}$ may possibly be justified. If we wish to have consistency in our approach to learning, then learning about the relationship embodied by $n$ may be questionable. However in order fully to assess this issue, it is useful to address the general RE case.

## 5 A More General Result

So far, so puzzling. We have seen that that the E-stability and MSV results have various exceptions when one is learning about the law of motion, although there was no such
problem when it came to learning about the linear relationship between forward and backward-looking variables. Here we extend this result to the more general case, and now use capital letters to denote matrices. Thus we rewrite (1) and (6) as

$$
\left[\begin{array}{c}
z_{t}  \tag{21}\\
E_{t} x_{t+1}
\end{array}\right]=\left[\begin{array}{cc}
A & B \\
C & D
\end{array}\right]\left[\begin{array}{c}
z_{t-1} \\
x_{t}
\end{array}\right]+\left[\begin{array}{c}
\varepsilon_{t} \\
0
\end{array}\right] \quad x_{t}+N_{k} z_{t-1}=0
$$

The reason for focusing learning on $N$ in (21) is very simple. Firstly, it means that we can directly impose a relationship describing future expectations of the forward-looking variable: $E_{t} x_{t+1}=-N_{k} z_{t}$. Secondly, suppose as an alternative we wish to learn about the perceived law of motion only; the only way to implement this feasibly is when the $z_{t}$ and $x_{t}$ are of the same dimension. For only in this case can we translate the approach of Section 2 to the higher dimensional case. However the method that learns about $N$ is not so restrictive, and indeed translates itself very simply into learning about the perceived law of motion, as we shall see at the end of this section .

Using the standard approach of Section 2, the equation satisfied by the matrix $N$ as learning progresses, and its relationship to the unstable eigenvalues of the system matrix, are given analogously to (9) and (10) by

$$
\frac{d N}{d k}=(D+N B)^{-1}(C+N A)-N \quad\left[\begin{array}{ll}
N & I
\end{array}\right]\left[\begin{array}{cc}
A & B  \tag{22}\\
C & D
\end{array}\right]=\Lambda_{U}\left[\begin{array}{ll}
N & I
\end{array}\right]
$$

where typically the matrix $\Lambda_{U}$ will have only unstable eigenvalues, and the matrix ( $A-$ $B N)$, which represents the reduced-form dynamics, will have only stable eigenvalues.

Thus the approach we use is to learn about the relationship between $x_{t}$ and $z_{t-1}$ as in (21). E-stability is then ascertained by examining deviations $\Delta v e c(N)$ about the steady state of N :

$$
\begin{equation*}
\frac{d}{d k} \Delta \operatorname{vec}(N)=\left((D+N B)^{-1} \otimes A^{T}-(D+N B)^{-1} \otimes(C+N A)^{T}(D+N B)^{-T} B^{T}-I \otimes I\right) \Delta v e c(N) \tag{23}
\end{equation*}
$$

But from (22) we deduce that $(D+N B)^{-1}(C+N A)=N,(D+N B)^{-1}=\Lambda_{U}^{-1}$, so that we can rewrite (23) as

$$
\begin{equation*}
\frac{d}{d k} \Delta v e c(N)=\left(\Lambda_{U}^{-1} \otimes(A-B N)^{T}-I \otimes I\right) \Delta v e c(N) \tag{24}
\end{equation*}
$$

Proposition 1 Suppose that the number of stable eigenvalues of the matrix in (21) is equal to the dimension of $z_{t}$. Then the learning process for $N$ is E-stable.

Proof: The eigenvalues of $\Lambda_{U}^{-1} \otimes(A-B N)^{T}-I \otimes I$ are of the form $\lambda_{S i} / \lambda_{U j}-1$ where $\lambda_{S i}$ represents a (stable) eigenvalue of $A-B N$ and $\lambda_{U j}$ represents an (unstable) eigenvalue of of $\Lambda_{U}$. But $\operatorname{Re}\left(\lambda_{S i} / \lambda_{U j}\right) \leq\left|\lambda_{S i} / \lambda_{U j}\right|<1$, so that the process for $N$ has eigenvalues with negative real part.

Corollary If all eigenvalues of $\Lambda_{U}$ are unstable and real, and all eigenvalues of $A-B N$ are stable and real, then there is no other N that is E-stable.

Proof: Any other $N$ would be associated with a switch of stable and unstable eigenvalues between $\Lambda_{U}$ and $A-B N$, in which case some values of $\lambda_{S i} / \lambda_{U j}$ would be greater than 1.

Note that this argument does not necessarily hold for the real part of $\lambda_{S i} / \lambda_{U j}$ if eigenvalues are complex.

Proposition 2 If the number of stable eigenvalues of the matrix in (21) is greater than the the dimension of $z_{t}$ (i.e. there is a potential for indeterminacy), and all its eigenvalues are real, then there exists an E-stable MSV representation of the system.

The proof is similar to that of the corollary.
Both the corollary and Proposition 2 are potentially even more powerful than they appear. The issue of real eigenvalues arose because of the switch from discrete to continuous time. Suppose we allow ourselves to blur the distinction between least squares learning and the updated path for $N$, and regard the update as occurring in (discrete) real time, with $\eta$ in (8) tending to 1 . Then we can rewrite the path for $N_{t}$ and its deviation $\Delta v e c\left(N_{t}\right)$ as

$$
\begin{equation*}
N_{t+1}=\left(D+N_{t} B\right)^{-1}\left(C+N_{t} A\right) \quad \Delta v e c\left(N_{t+1}\right)=\left(\Lambda_{U}^{-1} \otimes(A-B N)^{T}\right) \Delta v e c\left(N_{t}\right) \tag{25}
\end{equation*}
$$

In this case it is easy to see that the results of the Corollary and Proposition 2 hold for complex eigenvalues as well.

Finally, suppose that we wish to learn about the perceived law of motion, which we write in the form $z_{t}=F z_{t-1}+G \varepsilon_{t}$. It is easy to see by substituting $x_{t}=-N_{k} z_{t-1}$ into the dynamic equation for $z_{t}$ that we end up with a learning relationship for F given by

$$
\begin{equation*}
\frac{d F}{d k}=A-B N-F \tag{26}
\end{equation*}
$$

It follows that for each element $f_{i j}$ of $F$, its learning equation is given by $\frac{d f_{i j}}{d k}=(A-$ $B N)_{i j}-f_{i j}$. It is clear from this expression that if $N$ tends to a limit, then $F$ must tend to the limit $A-B N$.

## 6 Discussion and Conclusion

The most remarkable result that emerges from this analysis is the amount of conflict that there is between different ways of learning about the parameters. For even the simplest generic form of RE models there is no consistency on E-stability for the two methods of learning discussed over the whole range of parameters. When there is no potential indeterminacy in the system there is a wide range of parameters for which there is consistency, although for $\beta \gamma<0, \alpha>\delta$ this is not the case.

Perhaps even more intriguing is the issue of the MSV solution. McCallum (2003), along with various of his papers e.g. McCallum (2006), makes a strong case for the existence of MSV solutions where the root that is picked out is the minimum one. He does this in the context of models of the form (3) and (4), with an emphasis on well-formulated models. In this paper we have shown that there are cases of potential indeterminacy when only the smaller root is E-stable under learning, which conforms to McCallum's results. However we have found cases where both roots can be E-stable under learning, and also where only the larger root is E-stable.

The most important results of all emerge from the general RE model. In this case there is only one obvious way in which to engage in E-learning, and we have shown that saddlepath stability automatically guarantees E-stability. In addition we have fully justified the MSV approach of McCallum.

The next stage in the approach of this paper will address the case of partial information, and in particular will focus on one of the results of Evans and Honkapohja (2006) which shows that there circumstances in which optimal policy is not E-learnable.

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[^0]:    *Financial support for this research is due to the ESRC, project no. RES-000-23-1126. Thanks to Parantap Basu and Martin Ellison for careful reading and corrections.
    $\dagger$ Department of Economics, Finance \& International Business. Email: j.pearlman@londonmet.ac.uk.

[^1]:    ${ }^{1}$ Blanchard and Kahn (1980) actually use $z_{t}$ on the left, and $z_{t-1}$ on the right-hand side of this equation, but this is merely a labelling issue, and is only of significance in the case of partial information.

