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Exchange rate dynamics, asset market structure and the role of the trade elasticity*

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ABSTRACT

This paper shows that a canonical flexible price international real business cycle model with incomplete financial markets can address the exchange rate volatility puzzle, the exchange rate persistence puzzle, the consumption real exchange rate anomaly, as well as the quantity anomaly. Crucial for the success of the model is the choice of the elasticity of substitution between home and foreign produced goods. The paper shows that the range of this parameter which allows the model to address these international macroeconomics anomalies is very narrow. Furthermore, the paper highlights an anomalous relationship between real exchange rate persistence and the elasticity of substitution between home and foreign produced goods.

JEL Classification: F31, F41.

Keywords: real exchange rate dynamics, incomplete financial markets, Backus-Smith puzzle, exchange rate persistence, trade elasticity.

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1 Introduction

How well does the canonical flexible price international real business cycle (IRBC) model fit the data? Early evidence by Backus, Kehoe and Kydland (1994, 1995) suggests that the model departs from the data in number of important dimensions. Compared to the data, the basic flexible price IRBC model generates international relative prices that are neither volatile nor persistent enough. Even with incomplete financial markets, the model generates unrealistically high levels of international risk sharing, as indicated by a near unitary cross-correlation between the real exchange rate and relative consumption. High degrees of international risk sharing and low exchange rate volatility also imply that home and foreign consumption be highly correlated, more so than home and foreign output. In the data, the ordering of these cross-country correlations is reversed.

Successful attempts have been made to address individual shortcomings or puzzles thrown up by the model. Stockman and Tesar (1995) introduce non-traded goods into an otherwise canonical IRBC model and show that such a modification can go some way towards addressing the quantity anomaly, the relative ordering of the cross-correlation between home and foreign consumption and GDP. Benigno and Thoenissen (2008) add incomplete financial markets to the model of Stockman and Tesar (1995) and show that this simple modification can help address the Backus-Smith puzzle, breaking the strong link between the real exchange rate and relative consumption. However, even with a non-traded goods sector, and incomplete financial markets, the model still does not generate enough volatility of the real exchange rate or the terms of trade. Heathcote and Perri (2002) succeed in addressing the issue of relative price volatility and the ordering of cross-country correlations by eliminating trade in financial assets. They show that for low values of the trade elasticity, i.e. the elasticity of substitution between home and foreign-produced goods, their financial autarky model generates realistic levels of relative price volatility while lowering the counterfactually high cross-country correlation of consumption evident in versions of their model with trade in financial assets.

Recent work by Corsetti, Dedola and Leduc (2006a, 2008) takes Heathcote and Perri's work a step further by introducing, amongst other features, consumption home-bias. Their work suggests that the value of the trade elasticity lies at the heart of not just the volatility of relative prices and the ordering of international co-movements, but can be used to explain most of the irregularities thrown up by the canonical IRBC model, without having to assume financial autarky. They show that models with substantial complementarity between imported and exported goods can yield volatile and persistent real exchange rates, address the Backus-Smith puzzle and reduce the correlation between home and foreign consumption below that of home and foreign GDP. Interestingly, because of the assumption of consumption home-bias most of these anomalies can be addressed with two values of the substitution elasticity. The higher of the two elasticities corresponds to the case where the terms of trade depreciate following a positive shock to home total factor productivity. For the lower of the two elasticities, the terms of trade appreciate following a positive home supply shock. In this case, the terms of trade amplify instead of dampen the effects of productivity shocks on home consumption relative to foreign consumption. The implication is a radically different international transmission mechanism for asymmetric supply shocks. Corsetti *et al* (2006b), Kollmann (2006) and Enders and Müller (2008) show that this alternative view of the transmission mechanism, what they call *negative transmission*, is not entirely without empirical support, for the US economy at least.

The purpose of this paper is to analyse if these encouraging results also hold in a canonical flexible price IRBC model and if so, how robust these findings really are. What is interesting is that, depending on the calibration, the canonical flexible price IRBC model performs surprisingly well, and it does so without modelling features designed to address key open economy macroeconomics facts. The surprising performance of the baseline model has to be set against the robustness of the model to changes in key deep parameters. I measure robustness by the size of the parameter space that allows the model to perform in a data congruent fashion. One is more likely to find empirical support for the simple IRBC model if the permissible parameter range is large. The smaller the range of values of elasticity of substitution between home and foreign-produced goods, θ, that support the model, the less likely one is to find empirical support for it. The results of the paper suggest that, for my parsimonious flexible price IRBC model with incomplete financial markets, the range of θ that supports the model is quite narrow indeed. For values outwith this range, for either larger or smaller values of θ, the model displays all the usual exchange rate puzzles. Not just that, but the permissible range of θ is itself a function of, among other factors, the degree of home-bias in consumption and investment expenditure as well as the structure of the financial asset market. For example, if the degree of home-bias in investment is less than that in consumption, or if home and foreign-produced investment goods are better substitutes for one another than are home and foreign-produced consumption goods, then my model performs quite poorly, regardless of the level of θ. Likewise, the ability of the model to generate a negative transmission mechanism of supply shocks depends not just on the nature of investment demand, but also on how one models the asset market. There can be no negative transmission under complete financial markets, or if one rules out unit roots in bond holding via a bond holding cost. Whereas the baseline model under the assumption of financial autarky generates negative transmission for all values of θ below a given threshold, the same is not the case for an incomplete financial markets specification 'closed' by an endogenous discount factor. Here, negative transmission occurs only in the neighbourhood of the threshold level of θ, reverting to the traditional transmission mechanism for smaller values of θ, confining *negative transmission* to a very narrow range of the parameter space. The paper also shows that when the simple model generates realistic levels of exchange rate volatility, it also generates realistic levels of exchange rate persistence. This persistence result is somewhat puzzling, especially since it occurs even when the model is driven only by non-persistent white noise shocks.

The remainder of the paper is structured as follows: Section 2 sets out the baseline model. Section 3 discusses the calibration of the structural parameters as well as the shock processes. In section 4, I first present a selection of second moments generated by the under the baseline calibration put forward in Section 3. I then proceed to choose values of the elasticity of substitution between home and foreign-produced goods, θ, that allow the model to address various discrepancies between the model and the data present under the baseline calibration. Section 5 carries out a number robustness checks and finds that the choice of θ and the model's ability to address the key international macro puzzles is extremely sensitive to the degree of home bias, the composition of investment goods and the structure of asset markets. Finally, Section 6 concludes.

2 The model

I propose, what is essentially an international real business cycle model with flexible prices and incomplete financial markets. For ease of exposition, I choose a decentralised market structure. The representative household in each country consumes a final consumption good, provides labour services and smooths consumption over time by investing in a non state contingent bond paying out in home-produced intermediate goods. The representative household receives a wage and a share of the income generated by the intermediate goods producing sector. The intermediate goods producing sector combines the household's labour with accumulated capital stock to produce intermediate goods that can be used to produce home and foreign consumption as well as investment goods. Final goods producers produce consumption and investment goods using home and foreign-produced intermediate goods. The share of home-produced intermediate goods differs across countries and final consumption versus investment goods. I assume that agents have a relative preference for home produced intermediate goods in their final consumption basket, i.e. have consumption home bias. The consumption-based real exchange rate, which is a key variable in this model deviates from purchasing power parity because of consumption home-bias. This assumption makes the real exchange rate simply a function of the terms of trade.

2.1 Consumer behavior

The world economy is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the country H (Home), while the segment $[n, 1]$ belongs to F (Foreign). The home country consumer obtains utility from consumption, C, and receives dis-utility from supplying labour, h. Following, Mendoza (1991), Schmitt-Grohé and Uribe (2003) as well as Corsetti, *et al* (2008), I specify that preferences for the representative home consumer are described by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \xi_t \left[C_t, (1 - h_t) \right], \tag{1}$$

$$\xi_0 = 1, \tag{2}$$

$$\xi_{t+1} = \beta(\tilde{C}_t, \tilde{h}_t)\xi_t \quad t \geqslant 0, \tag{3}$$

where the discount factor is endogenous and depends on the sequence of consumption and labour effort. Specifically, the agent takes the average per capita levels of consumption and labour effort, $\tilde{C}_t$ and $\tilde{h}_t$ as given so that the representative agent does not internalise the effect of consumption and labour choice on the discount factor. By assuming that $\beta_{\tilde{C}} < 0$ and $\beta_{\tilde{h}} > 0$, this preference specification allows the model to be linearised around a non-stochastic steady state that is independent of initial conditions such as the initial level of financial wealth, capital stock or total factor productivity. These properties are important given the choice of asset market structure.

In my model, I assume that international asset markets are incomplete. The asset market structure in the model is relatively standard in the literature. Home and foreign agents can trade in one non-contingent bond, B_t that pays out one unit of home-produced intermediate goods in period $t+1$. I denote by B_t the quantity and by R_t the price of the bond purchased by home agents at the end of period t.The representative consumer faces the following budget constraint in each period t:

$$P_t C_t + P_{H,t} R_t B_t = P_{H,t} B_{t-1} + P_t w_t h_t + \Pi_t \tag{4}$$

where P_t is the price index of the consumption bundle, defined below, $P_{H,t}$ is the price of home produced intermediate goods and w_t is the real wage. In addition to the wage, the representative household receives dividends, Π_t, from holding a share in the equity of domestic firms. All domestic firms are wholly owned by domestic agents and equity holding within these firms is evenly divided between domestic households. When optimising, the representative household takes the flow of dividends as given.

The maximization problem of the Home representative agent consists of maximizing (1) subject to (4), along with the usual transversality condition:

$$\lim_{T\to\infty} E_t \prod_{s=1}^{T} R_s B_{t+T} = 0 \tag{5}$$

in determining the optimal profile of consumption and bond holding and the labour supply schedule. The corresponding Lagrange multiplier is:

$$\max_{C_t,h_t,B_t,\mu_t} L = E_t \sum_{s=t}^{\infty} \xi_s \left\{ U(C_s,(1-h_s)) + \mu_s \left[\frac{P_{H,s}}{P_s} B_{s-1} + w_s h_s + \frac{\Pi_s}{P_s} - C_s - \frac{P_{H,s}}{P_s} R_s B_s \right] \right\} \tag{6}$$

Using first order conditions for optimal consumption, labour effort and bond holdings one can derive the static efficiency condition for the consumption-labour choice as well as the consumption Euler equation:

$$\frac{U_h(C_t,(1-h_t))}{U_C(C_t,(1-h_t))} = w_t \tag{7}$$

$$U_C(C_t,(1-h_t))\frac{P_{H,t}}{P_t} = \beta(\tilde{C}_t,\tilde{h}_t)\frac{1}{R_t}\mathrm{E}_t \left[U_C(C_{t+1},(1-h_{t+1}))\frac{P_{H,t+1}}{P_{t+1}} \right] \tag{8}$$

In equilibrium, the household and average per capita levels of consumption and effort are the same, such that

$$C_t = \tilde{C}_t \tag{9}$$

$$h_t = \tilde{h}_t \tag{10}$$

2.2 Final consumption goods sector

Home final consumption goods (C) are produced with the aid of home and foreign-produced intermediate goods (c_H and c_F) in the following manner:

$$C_t = \left[v^{\frac{1}{\theta}} c_{H,t}^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} c_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{11}$$

where θ is the elasticity of intratemporal substitution between home and foreign-produced intermediate goods. Final goods in the home and the foreign country differ in terms of their composition of home and foreign-produced intermediate goods ($v > v^*$). Final goods producers maximize (12) subject to (11).

$$\max_{c_H, c_F} P_t C_t - P_{H,t} c_{H,t} - P_{F,t} c_{F,t} \tag{12}$$

This maximization yields the following input demand functions for the home economy (similar conditions hold for Foreign producers)

$$c_{H,t} = v \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t \ , \quad c_{t,F} = (1-v) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t \tag{13}$$

The price index that corresponds to the previous demand function is defined as:

$$P_t^{1-\theta} = [v P_{H,t}^{1-\theta} + (1-v) P_{F,t}^{1-\theta}] \tag{14}$$

An analogous production structure exists for the production of foreign final consumption goods.

2.3 Intermediate goods sectors

Firms in the intermediate goods sector produce output, y_t, that is used in the production of the final consumption and investment goods at home and abroad using capital and labour services employing the following constant returns to scale production function:

$$y_t = A_t f(k_{t-1}, h_t) \tag{15}$$

where A_t is total factor productivity. The cash flow of this typical firm in the intermediate goods producing sector is:

$$\Pi_t = P_{H_t} A_t f(k_{t-1}, h_t) - P_t w_t h_t - P_{x,t} x_t \tag{16}$$

where w_t is the real wage, P_{H_t} is the price of home-produced intermediate goods and P_t and $P_{x,t}$ are the consumption and investment goods deflators, respectively. In this baseline specification, I assume that home firms turn home-produced intermediate goods into capital stock, and the foreign firm uses only foreign-produced intermediate goods for investment. Thus $P_{x,t} = P_{H,t}$ and $P^*_{x,t} = P^*_{F,t}$. The firm faces the following capital accumulation constraint:

$$k_t = (1-\delta)k_{t-1} + (1 - s(\frac{x_t}{x_{t-1}}))x_t \tag{17}$$

where the initial capital stock, k_{-1}, is given, δ is the rate of depreciation of the capital stock and $(1 - s(\frac{x_t}{x_{t-1}}))x_t$ captures investment adjustment costs as proposed by Christiano *et al* (2005), i.e. it summarizes the technology which transforms current and past investment into installed capital for use in the following period. Following Christiano *et al*, I assume that the function $s(\frac{x_t}{x_{t-1}})$ has the following steady-state properties: $s(1) = s'(1) = 0$ and $s''(1) > 0$. Schmitt-Grohé and Uribe (2004) suggest the following functional form: $s(\frac{x_t}{x_{t-1}}) = \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2$. For the purposes of this paper, all that is needed is a value for $s''(1)$, which according to the functional form suggested by Schmitt-Grohé and Uribe is a constant, κ.[1]

The firm maximizes shareholder's value using the household's intertemporal marginal rate of substitution as the stochastic discount factor. The Lagrangian corresponding to the

[1] It is easy to show that whereas the function $(1 - s(x_t/x_{t-1}))x_t$ is not concave for all values of x, it is so in the vicinity of the steady state, thus the problem is standard in the sense that the conditions (19) - (21) plus the constraint and the relevant terminal conditions are necessary as well as sufficient.

maximization problem of the representative domestic intermediate goods firm is thus:

$$\max_{h_t,x_t,k_t} J = E_t \sum_{s=t}^{\infty} \xi_s \mu_s \left\{ \frac{\Pi_s}{P_s} \right\} + E_t \sum_{s=t}^{\infty} \xi_s \lambda_s \left[(1-\delta)k_{s-1} + (1 - s(\frac{x_s}{x_{s-1}}))x_s - k_s \right] \tag{18}$$

The first-order conditions for the choice of labour input, investment and capital stock in period t are:

$$\frac{P_{H,t}}{P_t} A_t F_h(k_{t-1}, h_t) = w_t, \tag{19}$$

$$q_t(1 - s(\frac{x_t}{x_{t-1}})) = q_t s'(\frac{x_t}{x_{t-1}})\frac{x_t}{x_{t-1}} - \beta(\tilde{C}_t, \tilde{h}_t)\mathrm{E}_t q_{t+1}\frac{\mu_{t+1}}{\mu_t} s'(\frac{x_{t+1}}{x_t})\frac{x_{t+1}}{x_t}\frac{x_{t+1}}{x_t} + \frac{P_{x,t}}{P_t}, \tag{20}$$

$$\beta(\tilde{C}_t, \tilde{h}_t)\mathrm{E}_t \frac{\mu_{t+1}}{\mu_t}\left(\frac{P_{H,t+1}}{P_{t+1}} A_t F_{k_t}(k_t, h_{t+1}) + q_{t+1}(1-\delta) \right) = q_t, \tag{21}$$

where I define Tobin's q as: $q_t \equiv \frac{\lambda_t}{\mu_t}$.

2.4 International relative prices

There are two key relative prices in this model. The first, the terms of trade is defined as the ratio of import to export prices expressed in a common currency: $T = \frac{P_F}{SP_H^*}$. Since I assume that the law of one price holds for individual goods, the expression for the terms of trade can be re-written as $T = \frac{P_F}{P_H}$. A depreciation of the terms of trade is a rise in T, whereas an appreciation is defined as a fall in T. The second important relative price is the real exchange rate. Since I have assumed that the law of one price holds for all goods and I have not allowed for a non-traded goods sector not subject to international goods market arbitrage, the only channel through which the consumer price based real exchange rate can deviate from purchasing power parity is via cross-country differences in consumption shares of the two goods. It is assumed that v, the share of home-produced goods in domestic final consumption exceeds v^*, the share of home-produced goods in foreign final consumption. The difference $v - v^*$ captures the degree of consumption home-bias.

Taking a log-linear approximation, to the definition of the real exchange rate: $RS = \frac{SP^*}{P} = \frac{P_H}{P}\frac{P^*}{P_H^*}$ yields a linear relationship between the real exchange rate and the terms of trade:

$$\widehat{RS}_t = (v - v^*)\hat{T}_t, \tag{22}$$

where for any variable z_t, whose steady state value is $\bar{z}$, I define $\hat{z}_t = \frac{z_t - \bar{z}}{\bar{z}}$, thus a "^"

signifies a log-deviation from steady state. The implication of this is that the real exchange rate is perfectly correlated with and less volatile than the terms of trade. Both of these characteristics are at odds with the data.

2.5 Market Equilibrium

The solution to our model satisfies the following market equilibrium conditions must hold for the home and foreign country:

1. Home-produced intermediate goods market clears:

$$y_t = c_{H_t} + c^*_{H_t} + x_{H_t} + x^*_{H_t} \tag{23}$$

2. Foreign-produced intermediate goods market clears:

$$y^*_t = c_{F_t} + c^*_{F_t} + x_{F_t} + x^*_{F_t} \tag{24}$$

3. Bond Market clears:

$$B_t + B^*_t = 0 \tag{25}$$

2.6 Solution technique

Before solving, I log-linearize the model around the nonstochastic steady state. In a neighborhood of the nonstochastic steady state one can analyze the linearization of the model, provided that the random shocks are sufficiently small. This procedure is standard in stochastic rational expectations macroeconomic models and is valid (i.e. yields a close approximation) provided the stochastic disturbances have a sufficiently small support. For a justification see Appendix A.3 of Woodford (2003). The linearization thus yields a set of equations describing the equilibrium fluctuations of the model. The log-linearization yields a system of linear difference equations which can be expressed as a singular dynamic system of the following form:

$$\mathbf{A}\mathrm{E}_t\mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{y}(t)$ is ordered so that the non-predetermined variables appear first and the predetermined variables appear last, and $\mathbf{x}(t)$ is a martingale difference sequence. There are four shocks in **C**: shocks to the home intermediate goods sectors' productivity, shocks to the foreign intermediate goods sectors' productivity, and shocks to home and foreign investment frictions. The variance-covariance as well as the autocorrelation matrices associated with these shocks are described in table 1. Given the parameters of the model, which I describe in the next section, I solve this system using the King and Watson (1998) solution algorithm.

3 Calibration

In this Section, I outline the baseline calibration. The calibration assumes that countries Home and Foreign are of the same size, and that both countries are symmetric in terms of their deep structural parameters. For the calibration, I specify the following functional form for the utility function:

$$U = E_0 \sum_{t=0}^{\infty} \xi_t \left[\frac{C_t^{1-\rho}}{1-\rho} + \chi \frac{(1-h_t)^{1-\rho}}{1-\rho} \right], \quad (26)$$

$$\xi_0 = 1, \quad (27)$$

$$\xi_{t+1} = \left(1 + \vartheta[C_t + \chi(1-h_t)]\right)^{-1} \xi_t, \quad (28)$$

where ρ (risk aversion) is the same for consumption and leisure. To avoid biasing my results through the functional form assumption of the utility function, I'm assuming the simplest functional form, log-utility, for the baseline calibration.[2] For the baseline calibration, I assume moderate amounts of consumption home-bias, $v = (1-v^*) = 0.88$, which corresponds to the share of home-produced traded goods in the US consumption basket, and complete specialization in the production of the final investment good, $\varphi = (1-\varphi^*) = 1$. The latter assumption is unrealistic, but commonly used in the literature (see Corsetti, *et al* 2008) and in the sensitivity analysis below, I allow φ to differ from unity. Following Benigno and Thoenissen (2008), the intratemporal elasticity of substitution between home and foreign-produced intermediate goods in consumption, θ, is set to 2. This is well within the range of estimates provided in the literature which span from about 0.8 (Heathcote and Perri 2002) to 6 in the finance literature. τ, the intertemporal elasticity of substitution between home

[2]The Backus-Smith correlation, defined as the correlation between relative consumption and the real exchange rate, for instance is more easily addressed by models of this type if consumption and leisure are non-separable.

and foreign intermediate goods in investment goods is set to 1. As there is no clear empirical evidence on this parameter, I have experimented with several different values. Our results are however robust to changing τ. As is common in the real business cycle literature, such as Hansen (1985), I set the share of labour in production to 0.64 and assume a 2.5% depreciation rate of capital per quarter. There is considerable uncertainty regarding the curvature of the investment adjustment cost function $s''(.)$. Christiano, *et al* (2005), who first proposed this specification interpret $1/s''(.)$ as the elasticity of investment with respect to a 1 percent temporary increase in the current price of installed capital. Their empirical evidence suggests a value of $s''(.) = 2.5$. Smets and Wouters (2004), estimate this parameter using Bayesian techniques in the context of a model of the US economy. Their median estimate is around 6. Enders and Müller (2008) estimate $s''(.)$ in an international real business cycle model, driven only by productivity shocks. Their estimates are between zero and 0.4. Groth and Kahn (2007) look at disaggregated data for the UK and the US and for the US find a value of κ of 0.17, much lower than Christiano's estimate based on aggregate data. Given this uncertainty in the literature, I have chosen to set κ to 0.1. This is deliberately small, thus ensuring that the results of the model are not unduly influenced by a parameter for which the literature does not have a consistent value. This value of $s''(.)$ allows the calibrated model to come close to matching the relative volatility of investment to GDP. I perform sensitivity analysis below to ascertain whether the results of this paper are robust to my choice of $s''(.)$.

Table 1: Baseline calibration

Preferences	$\beta = 1/1.01, \rho = 1, \bar{h} = 1/3,$
Final goods tech	$\upsilon = (1 - \upsilon^*) = 0.88,\ \theta = 2,\ \tau = 1,\ \varphi = (1 - \varphi^*) = 1$
Intermediate goods	$\alpha = 0.64,\ \delta = 0.025,\ s''= 0.1$
Shocks	$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$
	$V[\mu] = 10^{-4} \begin{bmatrix} 0.726 & 0.187 \\ 0.187 & 0.726 \end{bmatrix}$

The stochastic process for TFP is taken from the seminal work of Backus *et al* (1995) on international real business cycles. The home country in this calibration is assumed to be the United States. Matrix $V[\mu]$ in Table 1 above shows the variance-covariance matrix of our shock processes, and matrix Ω their first-order autocorrelation coefficients.

4 Four puzzles - one answer?

In Table 2, I show Hodrick-Prescott filtered quarterly data for the United States economy and for the model economy under various different calibrations. My baseline international real business cycle model, where I have set the intratemporal elasticity of substitution, θ, to 2, departs from the data in a number of ways. First, note that under this calibration, the model fails to generate sufficient volatility in relative prices. In the data, the consumer price index based real exchange rate is 3.04 and the terms of trade are 1.71 times as volatile as GDP. My baseline model generates series for the real exchange rate and the terms of trade that are 0.21 and 0.28 time as volatile as GDP, respectively. In the literature this discrepancy between model and data is called the volatility puzzle. The second dimension along which this model departs from the data is the cross-correlation between the real exchange rate and relative consumption at home and abroad. In the data, this cross-correlation is small and often negative, -0.45, for this data sample, indicating a low level of international risk sharing. In the baseline model, this correlation is close to unity, suggesting near complete risk sharing. This difference between model and data is sometimes called the Backus-Smith puzzle after Backus and Smith (1993), or the consumption real exchange rate anomaly, following Chari *et al* (2002). The third dimension along which the model departs from the data is the ranking of the international cross-correlations of GDP and consumption. In the data, the correlation between home and foreign GDP is higher than that of home and foreign consumption, for my data sample, the difference is 0.23. In my baseline model, consumption is more highly correlated with its foreign counterpart than is GDP, the difference amounting to -0.72. Following Backus, Kehoe and Kydland (1995), this is often called the quantity anomaly. Finally, I note that the persistence of the real exchange rate, measured by its first order autocorrelation coefficient, is less than half that of the data and that net trade is pro-cyclical, as opposed to counter-cyclical in the data.

Backus *et al* (1995) point out that the relative volatility of the terms of trade, and by construction that of the real exchange rate in this model, rises as the intratemporal elasticity of substitution between home and foreign-produced goods declines. Therefore, a natural way to improve the fit of our model is to calibrate θ to match the relative volatility of the terms of trade. Corsetti *et al* (2008) show that there will be two values of θ that

Table 2: Second Moments

	Data	Baseline	VP^+	VP^-	$B\text{-}SP^+$	$B\text{-}SP^-$	QA^+	QA^-	Min^+	Min^-
σ_y	1.57	1.42	1.40	1.38	1.40	1.39	1.42	1.38	1.40	1.38
σ_c/σ_y	0.78	0.48	0.47	0.63	0.47	0.52	0.68	0.68	0.47	0.63
σ_x/σ_y	3.18	2.63	2.71	2.65	2.72	2.68	2.79	2.64	2.71	2.65
σ_{rs}/σ_y	3.04	*0.21*	1.30	1.30	*1.39*	*0.18*	**4.09**	1.71	1.38	1.32
σ_t/σ_y	1.71	*0.28*	**1.71**	**1.71**	**1.83**	*0.24*	*5.39*	**2.25**	**1.82**	**1.74**
$\rho(y, y^*)$	0.53	*0.11*	*0.13*	**0.16**	*0.13*	*0.15*	**0.10**	**0.17**	*0.13*	**0.17**
$\rho(c, c^*)$	0.30	*0.83*	*0.95*	**0.10**	*0.94*	*0.60*	**-0.11**	**-0.052**	*0.94*	**0.09**
$\rho(rs, c\text{-}c^*)$	-0.45	*0.99*	**-0.27**	**-0.97**	**-0.45**	**-0.45**	**-0.99**	**-0.98**	**-0.44**	**-0.97**
$\rho(nx, y)$	-0.51	*0.53*	**-0.53**	**-0.56**	**-0.52**	**-0.55**	**-0.43**	**-0.56**	**-0.53**	**-0.56**
ρ_{rs}	0.81	*0.30*	**0.65**	**0.78**	**0.66**	**0.73**	**0.70**	**0.76**	**0.66**	**0.77**
ρ_A	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69

Notes: VP=volatility puzzle, VP(+) $\theta = 0.4702$, VP(-) $\theta = 0.4109$; B-SP = Backus-Smith puzzle, B-SP(+) $\theta = 0.4676$, B-SP(-) - $\theta = 0.3242$; QA= quantity anomaly, QA(+) $\theta = 0.4455$, QA(-) $\theta = 0.4163$; Min = minimise the weighted sum on the three puzzles, Min(+) $\theta = 0.4678$ and Min(-) $\theta = 0.4113$. Baseline $\theta = 2$. A '+' ('-') indicates an equilibrium with positive (negative) international transmission of productivity

will allow us to match this second moment. The first value of θ, is found by reducing the parameter from its baseline value of 2. The column labelled VP^+ in table 2 reports on the second moments generated by this calibration. Numbers appearing in bold typeface indicate a statistic that has significantly improved vis-à-vis the baseline calibration. Here, a low elasticity of substitution between home and foreign-produced goods in final consumption results in large changes in the relative price for a given productivity shock. The model generates a large depreciation of the terms of trade and thus the real exchange rate following a rise in the home country's TFP. Calibrating the model in this way also turns out to resolve the Backus-Smith puzzle, as the real exchange rate and relative consumption are now negatively correlated. A trade elasticity of somewhat below unity also implies a counter-cyclical trade balance. For this calibration, the terms of trade/ real exchange rate also displays realistic levels of persistence. Where the model continues to depart from the data in a serious way is in the ranking of cross-country correlations. Since with this calibration, a home productivity increase is associated with a large real depreciation that shifts purchasing power from home to foreign consumers, consumption across countries will be highly correlated. The second value of θ that allows the model to match the relative volatility of the terms of trade is found by increasing the elasticity from the neighbourhood of zero. The column in table 2 labelled VP^{-} reports the selected second moments for this calibration. Here, an increase in home TFP leads to a large appreciation (fall) in the terms of trade that shifts purchasing power from foreign to home agents. Corsetti *et al* (2008) who first pointed out this behaviour of the terms of trade refer to this as 'negative transmission'. Table 2 suggests that for this calibration the model addresses all of the baseline models major short coming. In addition to matching the relative volatility of the terms of trade, the model also appears to solve the Backus-Smith puzzle (although the correlation is now arguable too negative), the quantity anomaly, the persistence puzzle and generates a counter cyclical trade balance.

The columns labelled B-SP and QA, report calibrations of θ that aim to match the cross-correlation between the real exchange rate and relative consumption and the difference between the cross-country correlations between home and foreign GDP and consumption, respectively. Each time I report two values of θ, one for the positive and one for the negative transmission case. In each case, resolving one anomaly also addresses at least one if not two other anomalies.

In the last two columns of table 2, I report results from a calibration strategy that aims to minimise the loss arising from a equal weighting of the model discrepancy from (i) the relative volatility, (ii) the correlation between the real exchange rate and relative consumption and (iii) the difference between the cross-country correlations between GDP and consumption. Again, I report two calibrations, one for positive transmission and one for negative transmission. The results, particularly for the negative transmission case are encouraging for the model. The terms of trade is as volatile as in the data, relative consumption and the real exchange rate are negatively correlated, but the correlation is too high in absolute value, GDP is more highly correlated across countries than is consumption, net trade is counter-cyclical and the real exchange rate is almost as persistent as in the data.

4.1 Some intuition

Having reported on the results of the above calibration exercise, I now outline some of the intuition underlying the findings, starting with the simulations that imply a positive international transmission of productivity shocks. Lowering θ can address the volatility puzzle, because as home and foreign goods become compliments in consumption and thus less substitutable for one another, a larger depreciation of the terms of trade (fall in the relative price of home-produced goods) is required to clear the market following an asymmetric supply shock. Thus as θ declines, the relative volatility of the terms of trade and thus the real exchange rate increases.

When relative terms of trade volatility is high, Table 2 suggests that the cross-correlation between the real exchange rate and relative consumption is low. As is familiar from Cole and Obstfeld (1991), the terms of trade can act to share idiosyncratic risk across countries. In the special case where $\theta = 1$, the model replicates the complete financial markets allocation where risk is perfectly shared between the home and foreign economy. The smaller (larger) is the value of θ, the more (less) the terms of trade respond to an asymmetric supply shock. For the calibration used in column B-SP$^+$ the response of the terms of trade is to depreciate (rise) by so much that relative consumption actually falls following a home TFP shock. Thus relative consumption and the real exchange rate are negatively correlated.

In the column headed QA$^+$ the quantity anomaly is addressed by choosing a particular value of θ that generates a terms of trade depreciation large enough to cause home consumption to actually fall while foreign consumption rises following a home productivity increase (home and foreign consumption are negatively correlated). Since the cross-correlation between home and foreign GDP is determined mostly by the cross-correlation of the TFP process which is positive, I find that home and foreign consumption are not as highly correlated as home and foreign GDP.

Having outlined some of the intuition for the positive transmission case, I now turn the case where the terms of trade appreciate (fall) following a rise in domestic TFP. An appreciation of the terms of trade shifts purchasing power away from foreign to home consumers. Instead of helping to share risk, the terms of trade actually reenforce the effects of an asymmetric shock on relative welfare. Corsetti, *et al* (2008) provide an elegant intuition for this phenomenon, which I now attempt abridge. One can easily decompose the response of domestic and foreign demand for home produced goods to a change in the terms trade into a substitution and an income effect. In the home economy, where the supply shock occurs, the substitution effect and the income effect have opposite signs. For a depreciation of the terms of trade, the substitution effect is positive, since home goods become relatively cheaper if the terms of trade depreciate. The income effect, on the other hand, is negative, since a depreciation reduces the value of the home-produced output. Abroad, both the substitution and the income effects are positive - home produced goods are relatively cheaper and the value of foreign output rises. Negative international transmission of supply shocks can occur when the negative home income effect on demand outweighs both the positive home substitution effect and the positive foreign substitution and income effects. In this case, following an increase in productivity that raises supply of home-produced goods, world demand for home-produced goods actually falls if the terms of trade depreciates. Thus, to clear the market, the terms of trade have to appreciate, so that the dominant home income effect becomes positive. Negative transmission becomes more likely if the home country is the main source of demand for home produced goods, i.e. with strong home-bias and, as I show below with high relative price volatility, to increase the size of the income effect.

As long as we have negative transmission whereby the terms of trade and by implication the real exchange rate appreciate while relative consumption rises, the correlation between the real exchange rate and relative consumption is negative. Thus negative transmission also addresses the Backus-Smith puzzle. The correlation between home and foreign consumption is reduced, as the negative terms of trade, or wealth effect on foreign consumers tends to drive home and foreign consumption in opposite directions. The correlation reported in column QA^{-} of table 2 is only mildly negative due to the positive cross country spill overs of the TFP process.

Whereas the baseline calibration yields a pro-cyclical trade balance, my attempts to resolve various international macroeconomics puzzles also result in a data congruent counter-cyclical trade balance. Sensitivity analysis on this cross-correlation suggests that, for the current specification of preferences, net trade becomes counter-cyclical for values of θ less than one.[3] In the baseline model, net trade is driven by movements in the terms of trade, home consumption of foreign-produced goods and foreign consumption of home produced goods. If imports and exports are highly substitutable, i.e. high θ, then a home supply shock raises home output and depreciates the terms of trade (worsens net trade) raises foreign consumption of home-produced goods (improves net trade) and lowers home consumption of foreign-produced goods (improves net trade). On balance, particularly if the depreciation is not too large which is the case for high values of θ, net trade improves along with home output. If home and foreign-produced goods are compliments, i.e. low θ, then the terms of

[3]The model can also generate counter-cyclical trade balances for large values of θ if preferences are assumed to be of the GHH kind, which eliminates the wealth effect of consumption in labour supply.

trade depreciation will be larger, foreign consumption of home goods will still increase, but so will home consumption of foreign goods, worsening net trade. Overall, net trade worsens as home output increase.

The fact that values of θ that address the volatility and Backus-Smith puzzles also raise the persistence of the real exchange rate has been noted before, see for example Corsetti *et al* (2006a) and Thoenissen (2006) but is not usually rationalised. Indeed it is not straight forward to come up with a convincing economic argument why the persistence of the terms of trade (and by construction the real exchange rate) should rise so dramatically for certain low values of θ. Below, I analyse the robustness of the persistence of the terms of trade further, by among others stripping out any persistence from the shock process, and changing the structure of the asset market.

5 How robust are these results?

The notes to Table 2 suggest that the parameter space of θ that helps the model to address the main international macro puzzles is quite narrow, from 0.3242 to 0.4702. Figure 1 plots the relative volatility of the terms of trade (denoted by σT) the Backus-Smith correlation (denoted by corr(RS - c-c*)), the difference between corr(y,y*) and corr(c,c*) as well as the persistence of the terms of trade (dented by ρT) for values of θ from close to zero to 2. It becomes apparent that for our model and calibration, for most values of θ, including very small ones, the model fails to address any of the major international macroeconomics puzzles. Only in a narrow range centered around $\theta = 0.45$ does the model perform well. Outwith this region, the terms of trade are not volatile or persistent enough, consumption is more highly correlated with its foreign counter part than is GDP and the correlation between the real exchange rate and relative consumption is positive and close to unity. Figure 2 shows a more compact version of Figure 1 where θ goes only up to unity. Note that all major exchange rate puzzles are present for θ less than 0.25 and greater than 0.5.

The implication of this finding is that the success of the model is limited to a very specific region of the parameter space. In the following sub-sections, I illustrate that the choice of θ for which the model performs well is sensitive to, among other parameters, the degree of consumption home-bias and the composition of investment goods.

5.1 The role of consumption home-bias

Heathcote and Perri (2002), although using a similar model to the current one, find only one value of θ that allows the model to match a given volatility of the terms of trade. A key feature of Heathcote and Perri's model is a lack of consumption home-bias. Figure 3 plots the relative volatility of the terms of trade for values of θ from 0.05 to 1.00 for various degrees of consumption home-bias. As is well known from Corsetti *et al* (2008) the value of θ that correspond to the volatility spike is an increasing function of the degree of home bias. The greater the degree of consumption home-bias, the larger the values of θ that corresponds to data congruent levels of relative price volatility. The line in Figure 3 plotting the relative volatility of the terms of trade for $v = 0.51$ shows that for very low (or no) home-bias, there is only one positive level of θ, corresponding to the traditional transmission mechanism, as in Heathcote and Perri. Overall, Figure 3 suggests that the volatility of the terms of trade are quite sensitive to the degree of consumption home-bias. The implication is that empirically observing a sufficiently low level of θ does not on its own sufficient for the model to generate high levels of relative price volatility. It is important to observe the correct level of consumption home-bias, as well as the right level of θ.

5.2 Composition of investment goods

In this section, I ask: does the composition of investment goods matter? The new open economy macroeconomics literature, with its emphasis on short term nominal rigidities, largely ignores capital accumulation, see for example Benigno and Thoenissen (2003). The rest of the literature is broadly arbitrary in its treatment of capital goods. In the baseline specification, I have assumed that all investment is under taken using home-produced goods, an assumption also made in Benigno and Thoenissen (2008). This is arguably the most extreme form of investment home-bias. Assume instead, that similar to final consumption goods, investment goods (x) are produced with the aid of home and foreign-produced intermediate goods (x_H and x_F) in the following manner:

$$x = \left[\varphi^{\frac{1}{\tau}} x_H^{\frac{\tau-1}{\tau}} + (1-\varphi)^{\frac{1}{\tau}} x_F^{\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}} \tag{29}$$

Investment goods producers maximize (30) subject to (29).

$$\max_{x_H, x_F} P_x x - P_H x_H - P_F x_F \tag{30}$$

The investment goods producer's maximization yields the following investment demand functions and price index:

$$x_H = \varphi \left(\frac{P_H}{P_x} \right)^{-\tau} x \ , \quad x_F = (1 - \varphi) \left(\frac{P_F}{P_x} \right)^{-\tau} x \tag{31}$$

$$P_{x,t}^{1-\tau} = \left[\varphi P_{H,t}^{1-\tau} + (1 - \varphi) P_{F,t}^{1-\tau} \right] \tag{32}$$

The investment goods price index is a function of the price of home and foreign-produced intermediate goods prices. It differs from the consumption goods price index due to different substitution elasticities and different degrees of consumption and investment home biases. Specifically, φ, the share of home-produced intermediate goods in the home final investment good can differ from v, the share of home-produced intermediate goods in the final consumption good. The price of investment goods, relative to the price of consumption goods, $\frac{P_{x,t}}{P_t}$, is a function of the terms of trade. One can illustrate this by taking a log-linear approximation of the investment price index

$$\frac{P_{x,t}}{P_t} = \frac{P_{x,t}}{P_{H,t}} \frac{P_{H,t}}{P_t} \tag{33}$$

around its steady-state value making use of the investment and consumption goods price indices.[4]

$$\begin{aligned} \widehat{\frac{P_{x,t}}{P_t}} &= \widehat{\frac{P_{x,t}}{P_{H,t}}} + \widehat{\frac{P_{H,t}}{P_t}} \\ &= (1 - \varphi) \widehat{\frac{P_{F,t}}{P_{H,t}}} + (v - 1) \widehat{\frac{P_{F,t}}{P_{H,t}}} \\ &= (v - \varphi) \hat{T}_t \end{aligned} \tag{34}$$

[4]We make use of the consumption and investment goods price indices and normalise the price of home-produced traded goods such that in the steady state $P_H = P_F$. Because the law of one price holds, we can define the terms of trade as $T = P_F / P_H$

This shows that the log-deviation of the price of investment goods from its steady state value is a linear function of the log-deviation of the terms of trade from its steady state value. If home-bias for investment goods is stronger (weaker) than for consumption goods $v < \varphi$ ($v > \varphi$) then the price of investment goods is negatively (positively) related to the terms of trade.

In Figure 4 and 5, I make the assumption that the degree of home-bias in investment is either somewhat lower than in consumption, $\varphi = 0.75$, or absent all together, $\varphi = 0.5$. This small change in the structure of the model turns out to be of some importance. In Figures 4 and 5, the terms of trade is somewhat less volatile than the data throughout the range of θ, but the correlation between the real exchange rate and relative consumption is positive and close to unity throughout. The volatility of the terms of trade is also not significantly affected by the choice of θ. Interestingly, GDP across borders is more highly correlated than consumption in the no home-bias case (Figure 5), thus addressing the quantity anomaly, whereas in the low home-bias case (Figure 4) the ordering of the correlations is the other way around. Up to now, I assumed that $\tau = 1$. In Figure 9c, I show the selection of second moments when $\varphi = 0.5$ but when τ is allowed to vary with θ, i.e. $\tau = \theta$. Figure 9c shows that the pattern of second moments does not significantly depend on the value of τ.

5.3 Negative international transmission of productivity shocks

One of the most interesting features of the model is that for a range of sufficiently small values of θ, the international transmission mechanism of productivity shocks is reversed. Negative transmission implies that a rise in home productivity is associated with a fall (appreciation) in the terms of trade. This form of the international transmission mechanism is very much at odds with standard theory. Instead of helping to share country-specific risk arising from productivity shocks (see Cole and Obstfeld 1991), terms of trade movements actually hinder risk sharing, by amplifying the effects of a productivity shock. Recent work by Corsetti, Dedola and Leduc (2006a, 2008) and Enders and Müller (2008) has highlighted the phenomenon of negative transmission and its ability to help explain some key puzzles of international macroeconomics. Limited empirical support for the negative transmission mechanism of supply shocks is put forward by Corsetti, Dedola and Leduc (2006b), Enders and Müller (2008) and to a lesser extent by Kollmann (2006). Having shown that the simple IRBC model's ability to address some of the key international macroeconomics puzzles is quite sensitive to the precise choice of θ, I now proceed to analyse how robust the phenomenon of negative transmission is to changes in θ and to changes in the structure of the model.

For illustrative purposes, I initially abstract from the supply side of the model as well as from trade in bonds, assuming instead an endowment economy under financial autarky. Combining the log-linearized home and foreign intermediate goods sector's market clearing conditions with the log-linearized home country's budget constraint under autarky, one can derive the following relationship between relative endowments of output and the terms of trade:

$$\hat{T}_t = \frac{\hat{y}_t - \hat{y}_t^*}{(1 - 2v(1-\theta))} \tag{35}$$

It follows that the correlation between relative output and the terms of trade will be negative, so that relative supply shocks result in terms of trade appreciations, for all values of $\theta < \frac{2v-1}{2v}$. Thus in an endowment economy under autarky, negative transmission occurs for all values of θ less than this threshold.[5] Corsetti *et al* (2008) have shown that, in a two sector model with distribution costs, this range can be quite large, indeed.

Next, I will keep the financial autarky assumption, but re-introduce the supply side of the model:

$$(1-v)\left[\hat{y}_t - \hat{y}_t^*\right] - \frac{x}{y}\left(\varphi - v\right)\left[\hat{x}_t - \hat{x}_t^*\right] = (1-v)(1-\frac{x}{y})(1-2v(1-\theta))\hat{T}_t + \frac{x}{y}\left(1-\varphi\right)\left(1 - 2(v - \varphi\tau)\right)\hat{T}_t \tag{36}$$

The resulting expression now suggests that the relationship between relative output and the terms of trade depends also on the elasticity of substitution between home and foreign produced investment goods, τ, on the share of home-produced investment goods in total investment, φ, and on the dynamics of relative investment. This more complex relationship no longer guarantees negative transmission for sufficiently small values of θ.

An interesting special case arises if one sets the share of home-produced intermediate goods in investment to that of in consumption, i.e. $v = \varphi$. This case illustrates the role of the elasticity of substitution between home and foreign intermediate inputs in investment,

[5] This expression shows why Heathcote and Perri (2002) did not detect a negative transmission mechanism of supply shocks when analysing the effects of varying θ in their financial autarky model. Without consumption home-bias, $v = 1/2$ and thus the transmission mechanism is positive for all positive values of θ.

τ:

$$[\hat{y}_t - \hat{y}_t^*] = \left[(1 - \frac{x}{y})(1 - 2v(1-\theta)) + \frac{x}{y}\,(1 - 2v(1-\tau))\right]\hat{T}_t \tag{37}$$

The range of θ that allows the model to generate negative transmission becomes larger if $\theta > \tau$, and smaller if $\theta < \tau$. For sufficiently large values of τ, there is no positive value of θ that allows the model generate a negative transmission mechanism.

Figure 6 plots a selection of second moments generated by the model for the baseline calibration but under the assumption of financial autarky for values of θ from 0.05 to 1.00. The link between relative output and the terms of trade suggested by this calibration under autarky is:

$$[\hat{y}_t - \hat{y}_t^*] = (1 - \frac{x}{y})(1 - 2v(1-\theta))\hat{T}_t + \frac{x}{y}\,[\hat{x}_t - \hat{x}_t^*] \tag{38}$$

The line labelled corr(T,A) shows the cross-correlation between the terms of trade and domestic TFP, which I use as a proxy for international transmission. A positive correlation implies that the terms of trade depreciate (rise) following an increase in home TFP, i.e. the conventional international transmission mechanism. A negative correlation implies that the terms of trade appreciate (fall) following an increase in home TFP, i.e. negative transmission. Figure 6 suggests, that for our model and calibration under the assumption of financial autarky, the international transmission of productivity shocks is negative for all positive values of θ below the cut off point.

It can easily be shown that negative transmission is not possible under a complete financial market structure. The risk sharing condition arising under complete financial markets rules out wealth effects and puts a restriction on the relative movements of the terms of trade and relative consumption in our model. In log-linearized form, the risk sharing condition implies the following link between the terms of trade and relative consumption:

$$\widehat{RS}_t = (v - v^*)\hat{T}_t = \rho(\hat{C}_t - \hat{C}_t^*) \tag{39}$$

Under complete financial markets, a terms of trade appreciation (fall) can only occur if relative consumption falls, but an appreciation of the terms of trade is associated with a rise, not a fall in relative consumption. Proceeding as in the autarky case (see the appendix for a detailed derivation), one can derive the following expression for relative output (productivity) and the terms of trade:

$$\hat{T}_t = \frac{\hat{y}_t - \hat{y}_t^*}{4\theta v(1-v) + \frac{(2v-1)^2}{\rho}} \tag{40}$$

where the denominator is positive for all positive values of θ.[6]

Somewhere in between the assumption of financial autarky and that of the presence of a complete set of state-contingent claims lies the incomplete financial markets assumption implicit in the baseline model of this paper. With only one tradable bond, the expression corresponding to (35) and (40) becomes:

$$\hat{y}_t - \hat{y}_t^* = \left[4\theta v(1-v) + \frac{(2v-1)^2}{\rho}\right]\hat{T}_t - \frac{(2v-1)^2}{\rho}E_t\hat{T}_{t+1} + (2v-1)E_t\left(\hat{C}_{t+1} - \hat{C}_{t+1}^*\right) \tag{41}$$

where the link between relative supply (productivity) and the terms of trade is rather more complex than in either the autarky or the complete markets case. Initially, I proceed to analyse the link between negative transmission and θ using simulations. Figure 7, reproduces the analysis of Figure 6 for our baseline incomplete markets model. Interestingly, in this case, negative transmission is confined to a narrow range of the parameter space between $\theta = 0.45$ and $\theta = 0.30$. For values of θ below 0.30, the short-run transmission mechanism is once again positive. Implicitly, the home-country income effect is not strong enough to require the terms of trade to appreciate following a home-country supply shock. Instead, a short term depreciation is sufficient to clear the market for home produced goods. In this model, negative transmission does not occur for all values of θ below a defined cut-off as in the autarky model, but is limited to a narrow rage of θ.

It is also noteworthy that if instead of assuming an endogenous discount factor to rule out a unit root in bond holdings, I solve the mode under the assumption that domestic holdings of foreign-currency denominated bonds are subject to a bond holding cost, as in Benigno (2001), the model will not solve for calibrations under which the endogenous discount factor model displays negative transmission. The mechanism, designed to return domestic holdings of foreign-currency denominated bonds to equilibrium now actually destabilises the model. See also Bodenstein (2006) and Thoenissen (2006) on this point.

A tentative conclusion one can draw from this analysis is that in this simple IRBC

[6] One can easily check that when $\theta = \rho = 1$ the autarky model responds the same as the complete markets model.

model the phenomenon of negative international transmission of supply shocks is much less general than would be the case in an endowment economy under financial autarky. Negative transmission is also limited to one of the ways of 'closing' open economy models outlined by Schimdt-Grohe and Uribe (2003).

5.4 A real exchange rate persistence puzzle

In the baseline calibration ($\theta = 2$) of my model, the persistence of the real exchange rate is less than half of what what it is in the data. For values of θ that address the volatility puzzle, the Backus-Smith puzzle or the quantity anomaly, however, the model generates quite realistic levels of relative price persistence. In the light of recent evidence on the role of relative price stickiness in accounting for real exchange rate persistence put forward by Kehoe and Midrigan (2007), the ability of a simple flexible price business cycle model to generate realistic levels of relative price persistence is encouraging.

Figure 2 plots the persistence of the real exchange rate (terms of trade) for different values of θ. The graph suggests that persistence is high, and thus data congruent only in the region of the parameter space where the model also generates negative transmission (to the left of the volatility spike). In contrast, Figures 4 and 5 suggest that in the incomplete markets model for calibrations where the model does not generate negative transmission, the persistence of the terms of trade is largely invariant to the choice of θ. Figure 6 also suggests that in the absence of an international bond market, the persistence of the terms of trade is also low and invariant to the choice of θ.

Given that the persistence of the real exchange rate in Table 2 is quite close to first-order autocorrelation coefficient of the productivity process, it is worth checking if the persistence of the real exchange rate is driven by the persistence of the driving process. Figure 8, repeats the analysis from Figure 2, assuming zero persistence and no cross-country spill overs in the driving process, i.e. $\Omega = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $V[\mu] = 10^{-4} \begin{bmatrix} 0.726 & 0 \\ 0 & 0.726 \end{bmatrix}$. With white noise TFP shocks, the persistence of the Hodrick-Prescott filtered time series for the terms of trade, and thus the real exchange rate is negative for most values of θ. Surprisingly, for values of θ that generate high relative price volatility the real exchange rate also displays realistic levels of persistence. This result is robust to 'turning off' sources of internal persistence such as capital accumulation. It is also worth mentioning that the transmission mechanism of supply shocks is negative when persistence is high, and positive elsewhere. Given that all that is driving the model are non-persistent supply shocks, it is not entirely clear what the economic rationale is for the high persistence of the terms of trade.

Figures 9a and 9b, repeat the analysis of Figure 8 for greater investment adjustment costs ($s''(1) = 2.5$ as suggested by Christiano *et al*) and for the case of capital as opposed to investment adjustment costs ($k_t = (1-\delta)k_{t-1} + \phi(\frac{x_t}{k_{t-1}})k_{t-1}$ where $\phi(x/k) = \delta$, $\phi'(x/k) = 1$ and -$\phi''(\delta) = 2$). This analysis suggest that the puzzling persistence is not a feature of the adjustment cost parameter or how one modells adjustment costs.

When shocks are small and temporary, one would expect the baseline incomplete markets model presented in this paper to closely resemble the complete markets model. Being able to borrow and lend is sufficient to share the risk arising from small and temporary country-specific supply shocks. This can be shown using the link between the terms of trade and relative supply in the incomplete markets model derived above:

$$\hat{y}_t - \hat{y}_t^* = \left[4\theta v(1-v) + \frac{(2v-1)^2}{\rho}\right]\hat{T}_t - \frac{(2v-1)^2}{\rho}E_t\hat{T}_{t+1} + (2v-1)E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right) \quad (42)$$

Assuming that shocks and the effects of shocks are temporary such that $E_t\hat{T}_{t+1} = E_t\hat{C}_{t+1} = E_t\hat{C}^*_{t+1} = 0$, then the above expression reduces to the relationship arising under complete markets:

$$\hat{T}_t = \frac{\hat{y}_t - \hat{y}_t^*}{4\theta v(1-v) + \frac{(2v-1)^2}{\rho}} \quad (43)$$

where the transmission mechanism is positive for all positive values of θ.

It is well known from Heathcote and Perri (2002) that the one-sector incomplete markets model departs form the complete markets allocation when shocks or the effect of shocks are very persistent or permanent. In this case international borrowing and lending is not enough to ensure against such country-specific shocks. The analysis of Figure 8 suggests that even for non-persistent shocks, there exists a very small range of values of θ that cause the model to generate high terms of trade persistence and negative transmission and thus depart significantly from the complete markets allocation. Outwith this range, for either higher or lower values of θ, the model generates very little persistence and shock transmission is positive, suggesting that the persistence of the response of the terms of trade is closely linked to model's ability to generate a negative transmission mechanism. One can show that if the model displays high persistence, then the incomplete markets model can generate negative transmission. Starting with equation (41) and assuming that the effects of temporary shocks are permanent, i.e. $E_t\hat{T}_{t+1} = \hat{T}_t$ and $E_t\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right) = E_t\left(\hat{C}_t - \hat{C}^*_t\right)$ yields:

$$\hat{y}_t - \hat{y}^*_t = 4\theta v(1-v)\hat{T}_t + (2v-1)\left(\hat{C}_t - \hat{C}^*_t\right) \tag{44}$$

Next, I make use of the bond market clearing condition, equation (A8) in Appendix A to obtain:

$$\hat{y}_t - \hat{y}^*_t = 4\theta v(1-v)\hat{T}_t + (2v-1)\left((2v\theta-1)\hat{T}_t - \frac{(\beta b_t - b_{t-1})}{1-v}\right) \tag{45}$$

making the further assumption that the final term is zero or very small, implying a scenario where bond holdings are stationary, we get the same link between relative output and the terms of trade as in the autarky case:

$$\hat{T}_t = \frac{\hat{y}_t - \hat{y}^*_t}{(1-2v(1-\theta))} \tag{46}$$

What this does not, however, show is why the model generates the high persistence in the first place and only in the neighbourhood of θ that also generates negative persistence.

In summary, the baseline flexible price model can generate high levels of real exchange/terms of trade persistence. However, sensitivity analysis suggests that high persistence is a feature of the incomplete markets model, not present in either complete markets models or under financial autarky, and occurs only for certain values of θ regardless of the persistence of the shock process. As there are no obvious strong internal propagation mechanisms, the persistence of relative prices generated by the model represents somewhat of a puzzle.

6 Conclusion

This paper starts with the premise that given a careful choice of parameters, a simple flexible price international real business cycle model is able to address a number of hitherto puzzling discrepancies between data and models. Specifically, I show that a simple one-sector, two-country incomplete financial markets IRBC model can generate volatile and persistent time series for the terms of trade and the real exchange rate, can generate a negative cross-correlation between the real exchange rate and relative consumption, thus addressing the

Backus-Smith puzzle and can address the quantity anomaly whereby the consumption is has a higher cross country correlation than GDP. The key parameter that enables the model to address all these puzzles is the elasticity of substitution between home and foreign-produced intermediate goods, θ.

The main contribution of this paper is not, however, to highlight the success of the model, but to point out how sensitive the model's results are to the choice of parameters and model structure. The model performs in a data congruent fashion only in a narrow range of θ. Finding empirical support for one's choice of θ in this range is, however, not sufficient to justify the model. I show that the range of θ for which the model 'behaves well' is itself a function of the degree of consumption home-bias and the composition of investment demand, as well as the asset market structure.

For certain values of θ the model is able to generate, what Corsetti *et al* have termed, 'negative' international transmission of supply shocks. Here the terms of trade appreciate following a transitory increase in the home country's TFP. The phenomenon of negative transmission has recently and prominently been credited for solving a number of open economy puzzles. In this paper, I show that under incomplete financial markets, negative transmission is confined to an even narrower range of θ than the range that solves the terms of trade volatility puzzle or the Backus-Smith puzzle. Indeed, if firms display less home-bias in their choice of investment goods than consumers do in their choice of consumption goods, or if home and foreign investment goods are more substitutable than consumption goods, then it is possible that no positive value of θ will generate negative transmission.

A puzzling aspect of the model analysed in this paper is the fact that values of θ that solve the volatility puzzle or the Backus-Smith puzzle also tend to generate high levels of real exchange rate persistence. The persistence result holds even if the model is driven only by non-persistent white noise TFP shocks, but only in the incomplete markets model. Under financial autarky or under complete markets, the persistence of relative prices is largely invariant to the choice of θ.

As a critique of the literature that promotes low trade elasticities as a solution to a number of macro puzzles, the model presented in this paper is only partially suited. Low trade elasticities can be achieved not just through low values of θ, but also if home and foreign-produced intermediate goods have to be combined with locally produced non-traded goods in order to reach the final consumer. It could well be that in such a set up, the model's ability to capture key stylised facts of the international business cycle is somewhat more robust to changes in key parameters than is the case in the simple model presented here.

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A Log-linearized model

Home and foreign marginal utilities of consumption

$$-\rho \hat{C}_t = \hat{\mu}_t \tag{A1}$$

$$-\rho \hat{C}^*_t = \hat{\mu}^*_t \tag{A2}$$

Euler equations for home and foreign bonds

$$\hat{\mu}_t = \hat{\mu}_{t+1} + \hat{r}_t - (1-v)\left[\hat{T}_{t+1} - \hat{T}_t\right] + \beta_C \hat{C}_t + \beta_h \hat{h}_t \tag{A3}$$

$$\hat{\mu}^*_t = \hat{\mu}^*_{t+1} + \hat{r}_t - (1-v^*)\left[\hat{T}_{t+1} - \hat{T}_t\right] + \beta_C \hat{C}^*_t + \beta_h \hat{h}^*_t \tag{A4}$$

Euler equations for home and foreign labour supply

$$\hat{l}_t \frac{l}{(1-l)} \eta = \hat{\mu}_t + \hat{w}_t \tag{A5}$$

$$\hat{l}^*_t \frac{l^*}{(1-l^*)} \eta = \hat{\mu}^*_t + \hat{w}^*_t \tag{A6}$$

$$\beta \hat{b}_t - \hat{b}_{t-1} = (1-v)\left((\theta - 1)\hat{T}_t + \theta \widehat{RS}_t - \hat{C}_t + \hat{C}^*_t\right) \tag{A8}$$

Home and Foreign q equations

$$\begin{aligned} \hat{q}_t \;\; = \;\; & \hat{q}_{t+1}\beta(1-\delta) + \hat{\mu}_{t+1} - \hat{\mu}_t + (v-1)\hat{T}_{t+1}\left(1-\beta(1-\delta)\right) + \hat{\rho}_{t+1}\left(1-\beta(1-\delta)\right) \\ & + \beta_C \hat{C}_t + \beta_h \hat{h}_t \end{aligned} \tag{A9}$$

$$\begin{aligned} \hat{q}^*_t \;\; = \;\; & \hat{q}^*_{t+1}\beta(1-\delta) + \hat{\mu}^*_{t+1} - \hat{\mu}^*_t + \left[v\hat{T}_{t+1} - \widehat{RS}_{t+1}\right]\left(1-\beta(1-\delta)\right) + \hat{\rho}^*_{t+1}\left(1-\beta(1-\delta)\right) \\ & + \beta_C \hat{C}^*_t + \beta_h \hat{h}^*_t \end{aligned} \tag{A10}$$

Home and Foreign MPK equations

$$\hat{\rho}_t = \hat{A}_t - \alpha \hat{k}_{t-1} + \alpha \hat{l}_t \tag{A11}$$

$$\hat{\rho}_t^* = \hat{A}_t^* - \alpha \hat{k}_{t-1}^* + \alpha \hat{l}_t^* \tag{A12}$$

Optimal capital accumulation equations

$$\frac{\hat{q}_t}{(1+\beta)s''()} + \frac{1}{(1+\beta)}\hat{x}_{t-1} + \frac{\beta}{(1+\beta)}\hat{x}_{t+1} - (v-\varphi)\hat{T}_t \frac{1}{(1+\beta)s''()} = \hat{x}_t \tag{A13}$$

$$\frac{\hat{q}_t^*}{(1+\beta)s''()} + \frac{1}{(1+\beta)}\hat{x}_{t-1}^* + \frac{\beta}{(1+\beta)}\hat{x}_{t+1}^* - (v^*-\varphi^*)\hat{T}_t \frac{1}{(1+\beta)s''()} = \hat{x}_t^* \tag{A14}$$

Home and Foreign MPL equations

$$(v-1)\hat{T}_t + \hat{A}_t + (\alpha-1)\hat{l}_t + (1-\alpha)\hat{k}_{t-1} = \hat{w}_t \tag{A15}$$

$$v\hat{T}_t - \widehat{RS}_t + \hat{A}_t^* + (\alpha-1)\hat{l}_t^* + (1-\alpha)\hat{k}_{t-1}^* = \hat{w}_t^* \tag{A16}$$

Home and Foreign capital accumulation constraints

$$\hat{k}_t = \hat{k}_{t-1}(1-\delta) + \delta \hat{x}_t \tag{A17}$$

$$\hat{k}_t^* = \hat{k}_{t-1}^*(1-\delta) + \delta \hat{x}_t^* \tag{A18}$$

Home and Foreign production functions

$$\hat{y}_{H,t} = \hat{A}_t + (1-\alpha)\hat{k}_{t-1} + \alpha \hat{l}_t \tag{A19}$$

$$\hat{y}_{F^*,t} = \hat{A}_t^* + (1-\alpha)\hat{k}_{t-1}^* + \alpha \hat{l}_t^* \tag{A20}$$

Home and Foreign economy-wide constraints

$$\hat{y}_{H,t} = \frac{c_H}{y_H}\hat{c}_H + \frac{c_H^*}{y_H}\hat{c}_H^* + \frac{x_H}{y_H}\hat{x}_H + \frac{x_H^*}{y_H}\hat{x}_H^* \tag{A21}$$

$$\hat{y}_{F^*,t} = \frac{c_F}{y_{F^*}}\hat{c}_F + \frac{c_F^*}{y_{F^*}}\hat{c}_F^* + \frac{x_F^*}{y_{F^*}}\hat{x}_F^* + \frac{x_F}{y_{F^*}}\hat{x}_F \tag{A22}$$

Real exchange rate

$$\widehat{RS}_t = (v - v^*)\hat{T}_t \tag{A24}$$

Home and Foreign input demand functions

$$\hat{c}_H = -\theta(v-1)\hat{T}_t + \hat{C}_t \tag{A25}$$

$$\hat{c}_H^* = -\theta(v-1)\hat{T}_t + \theta\widehat{RS}_t + \hat{C}_t^* \tag{A26}$$

$$\hat{c}_F = -\theta v\hat{T}_t + \hat{C}_t \tag{A27}$$

$$\hat{c}_F^* = -\theta v\hat{T}_t + \theta\widehat{RS}_t + \hat{C}_t^* \tag{A28}$$

$$\hat{x}_{H,t} = -\tau(\varphi-1)\hat{T}_t + \hat{x}_t \tag{A29}$$

$$\hat{x}_{F,t} = -\tau\varphi\hat{T}_t + \hat{x}_t \tag{A30}$$

$$\hat{x}_{H^*,t} = -\tau(\varphi^*-1)\hat{T}_t + \hat{x}_t^* \tag{A31}$$

$$\hat{x}_{F^*,t} = -\tau\varphi^*\hat{T}_t + \hat{x}_t^* \tag{A32}$$

Relative price of investment based on CES price indexes

$$\widehat{\frac{Px_t}{P_t}} = (v-\varphi)\hat{T}_t \tag{A33}$$

$$\widehat{\frac{Px_t^*}{P_t^*}} = (v^*-\varphi^*)\hat{T}_t \tag{A34}$$

B Steady-state ratios

$$\frac{x_H}{y_H} = \frac{x}{k}\frac{k}{y} = \delta\left(\frac{i+\delta}{(1-\alpha)}\right)^{-1} \tag{B1}$$

$$\frac{x_H}{y_H} = \varphi \frac{x}{y_H} \tag{B2}$$

$$\frac{x_{H^*}}{y_H} = \varphi^* \frac{x}{y_H} \tag{B3}$$

$$\frac{y_H}{c_H} = \frac{1}{v}(1 - \frac{x_H}{y_H})^{-1} \tag{B4}$$

$$\frac{c_H^*}{y_H} = 1 - \frac{c_H}{y_H} - \frac{x_H}{y_H} - \frac{x_{H^*}}{y_H} \tag{B5}$$

$$\frac{x_F}{y_{F^*}} = \delta \left(\frac{i + \delta}{(1 - \alpha)} \right)^{-1} \tag{B6}$$

$$\frac{x_F^*}{y^*} = (1 - \varphi^*) \frac{x_F}{y_{F^*}} \tag{B7}$$

$$\frac{x_F}{y^*} = (1 - \varphi) \frac{x_F}{y_{F^*}} \tag{B8}$$

$$\frac{y_F}{c_{F^*}} = \frac{1}{(1 - v^*)} \left(1 - \frac{x_t^*}{y^*} \right)^{-1} \tag{B9}$$

$$\frac{c_F}{y^*} = 1 - \frac{c_{F^*}}{y^*} - \frac{x_F^*}{y^*} - \frac{x_F}{y^*} \tag{B10}$$

C The link between relative output and the terms of trade

To derive expression (35), I take expression (A21) and (A22), setting the share of investment goods in home-country to zero and substituting expressions (A25) -(A28), for convenience I also assume 'symmetric home bias' $v = 1 - v^*$:

$$\hat{y}_{H,t} = \theta v(1 - v)\hat{T}_t + v\hat{C}_t + \theta v(1 - v)\ \hat{T}_t + (1 - v)\hat{C}_t^*$$

$$\hat{y}_{F,t} = 2\theta v(1-v)\hat{T}_t + v\hat{C}_t + (1-v)\hat{C}_t^*$$

subtracting $\hat{y}_{F,t}$ from $\hat{y}_{H,t}$ yields:

$$\hat{y}_{H,t} - \hat{y}_{F,t} = 4\theta v(1-v)\hat{T} + (2v-1)\left[\hat{C}_t - \hat{C}_t^*\right] \tag{C1}$$

Financial autarky implies:

$$P_{H,t}y_{H,t} = P_tC_t \rightarrow y_{H,t} = \frac{P_t}{P_{H,t}}C_t \Rightarrow \hat{y}_{H,t} = (1-v)\hat{T}_t + \hat{C}_t \tag{C2}$$

$$P_{F,t}^*y_{F,t} = P_t^*C_t^* \rightarrow y_{F,t} = \frac{P_t^*}{P_{F,t}^*}C_t^* \Rightarrow \hat{y}_{F,t} = (v-1)\hat{T}_t + \hat{C}_t^* \tag{C3}$$

Substituting (C2) and (C3) into (C1) yields

$$\frac{[\hat{y}_{H,t} - \hat{y}_{F,t}]}{[1-2v(1-\theta)]} = \hat{T}_t$$

To derive expression (40) for complete financial markets, substitute the complete financial markets risk sharing condition

$$\widehat{RS}_t = (v - v^*)\hat{T}_t = \rho(\hat{C}_t - \hat{C}_t^*)$$

into expression (C1) to yield

$$\hat{T}_t = \frac{\hat{y}_t - \hat{y}_t^*}{4\theta v(1-v) + \frac{(2v-1)^2}{\rho}}$$

To derive expression (41), substitute the incomplete financial markets risk sharing condition

$$\begin{aligned} E_t\widehat{RS}_{t+1} - \widehat{RS}_t &= \rho E_t\left(C_{t+1} - C_t\right) - \rho E_t\left(C_{t+1}^* - C_t^*\right) \\ \frac{(v-v^*)}{\rho}E_t\left(\hat{T}_{t+1} - \hat{T}_t\right) &= E_t\left(C_{t+1} - C_{t+1}^*\right) - \left(C_t - C_t^*\right) \end{aligned}$$

into expression (C1) to yield

$$\hat{y}_t - \hat{y}_t^* = \left[4\theta v(1-v) + \frac{(2v-1)^2}{\rho}\right]\hat{T}_t - \frac{(2v-1)^2}{\rho}E_t\hat{T}_{t+1} + (2v-1)E_t\left(\hat{C}_{t+1} - \hat{C}_{t+1}^*\right)$$

D Data sources

1. US quarterly data on the consumption-based real exchange rate, the terms of trade and relative consumption are taken from table 2A in Corsetti *et al* (2006).

2. GDP refered to in table 2 real GDP per capita from BEA's NIPA table 7.1. 'Selected Per Capita Product and Income Series in Current and Chained Dollars', seasonally adjusted. The series was logged and H-P filtered.

3. Consumption referred to in table 2 is total consumption expenditures deflated by the relevant GDP deflator, both from BEA's NIPA tables 2.3.5 and 1.1.9.

4. Investment referred to in tables 2 is real fixed investment per capita from BEA's NIPA table 5.3.3. Real Private Fixed Investment by Type. Population is from NIPA table 7.1.

5. The estimated Solow residual is taken from Backus et al (1995) table 11.3.

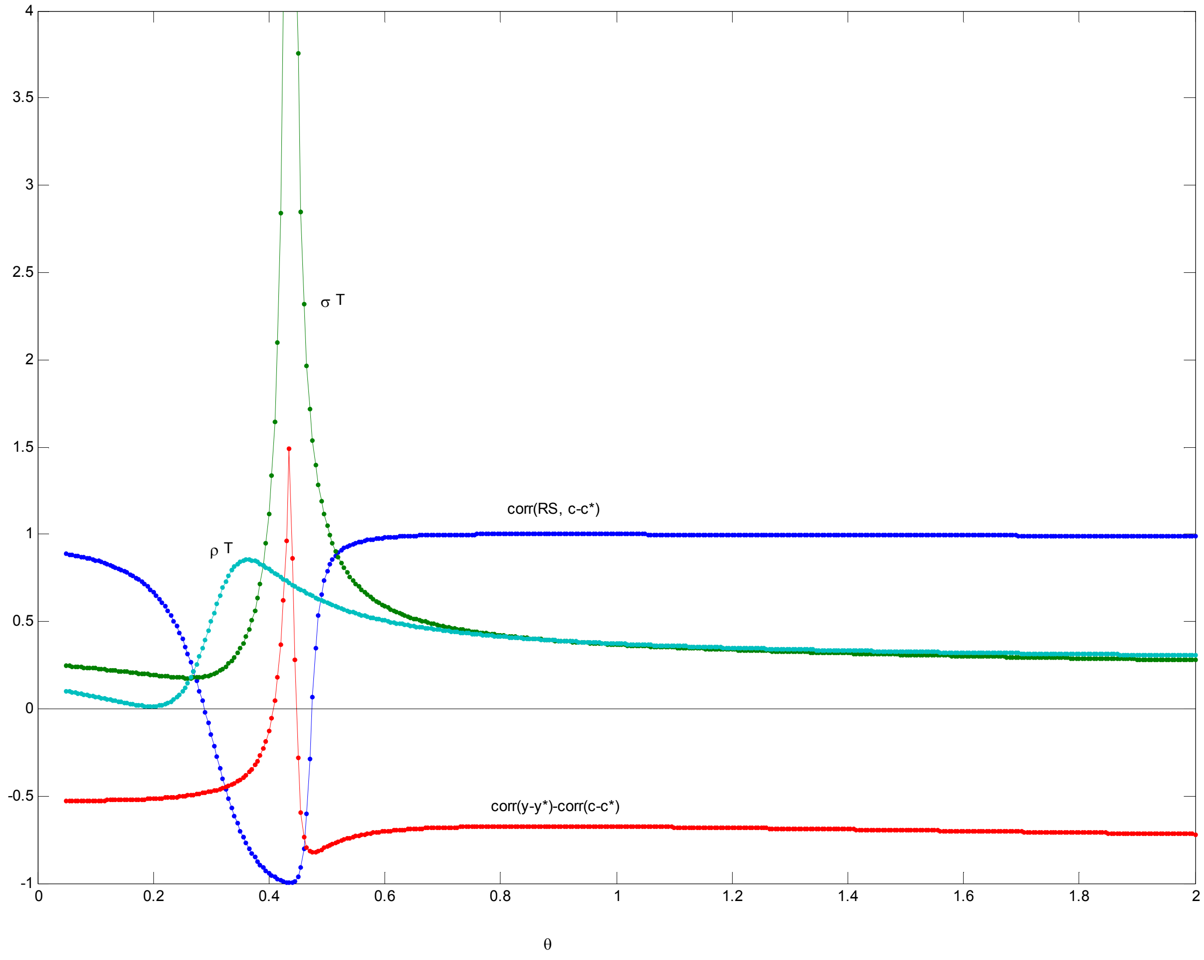

Figure 1 The volatility of the terms of trade relative to GDP, the correlation between the real exchange rate and relative consumption, the difference in the cross-country correlations between GDP and consumption, and the first-order autocorrelation coefficient of the terms of trade for various values of the elasticity of substitution, θ, from 0.05 to 2.00.

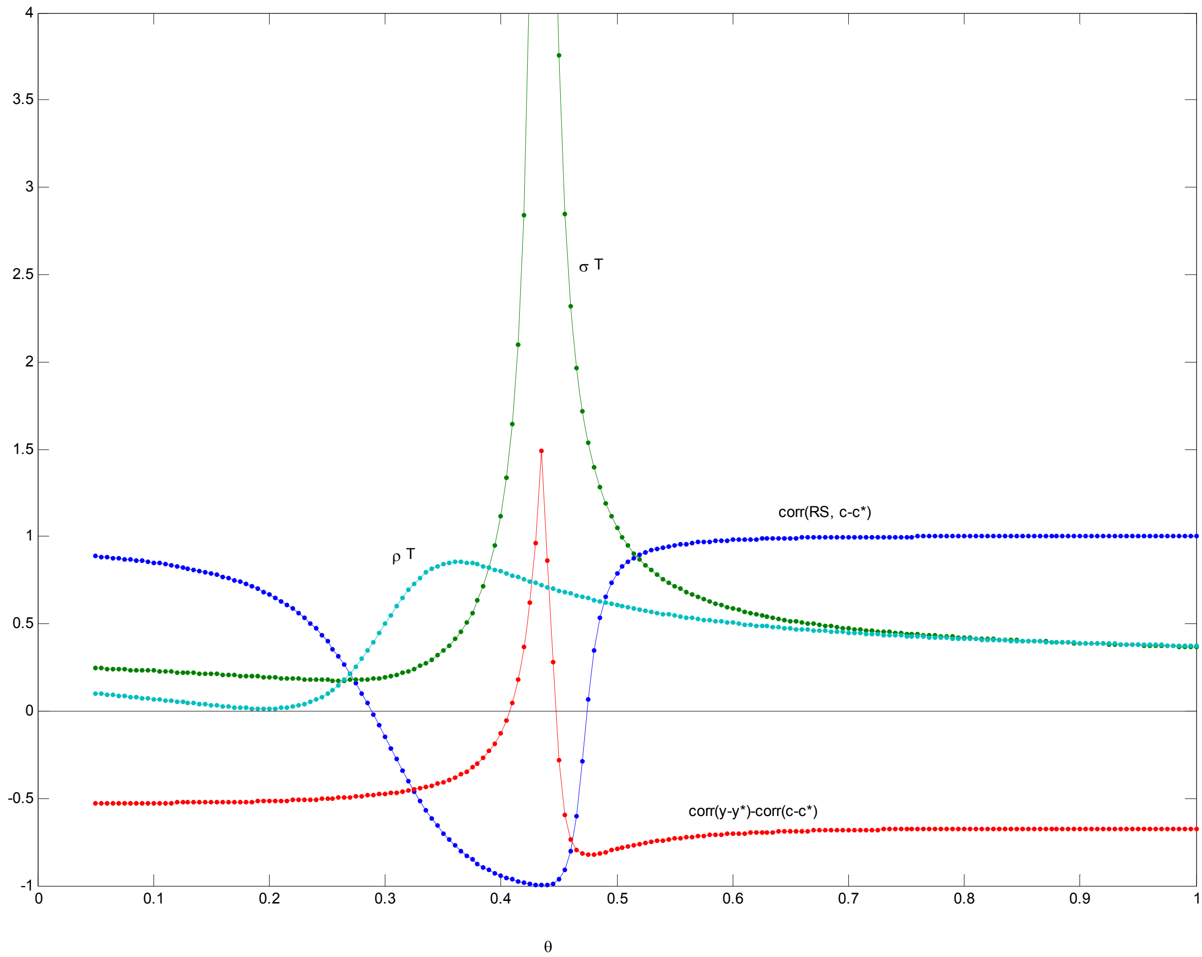

Figure 2 The volatility of the terms of trade relative to GDP, the correlation between the real exchange rate and relative consumption, the difference in the cross-country correlations between GDP and consumption, and the first-order autocorrelation coefficient of the terms of trade for various values of the elasticity of substitution, θ, from 0.05 to 1.00.

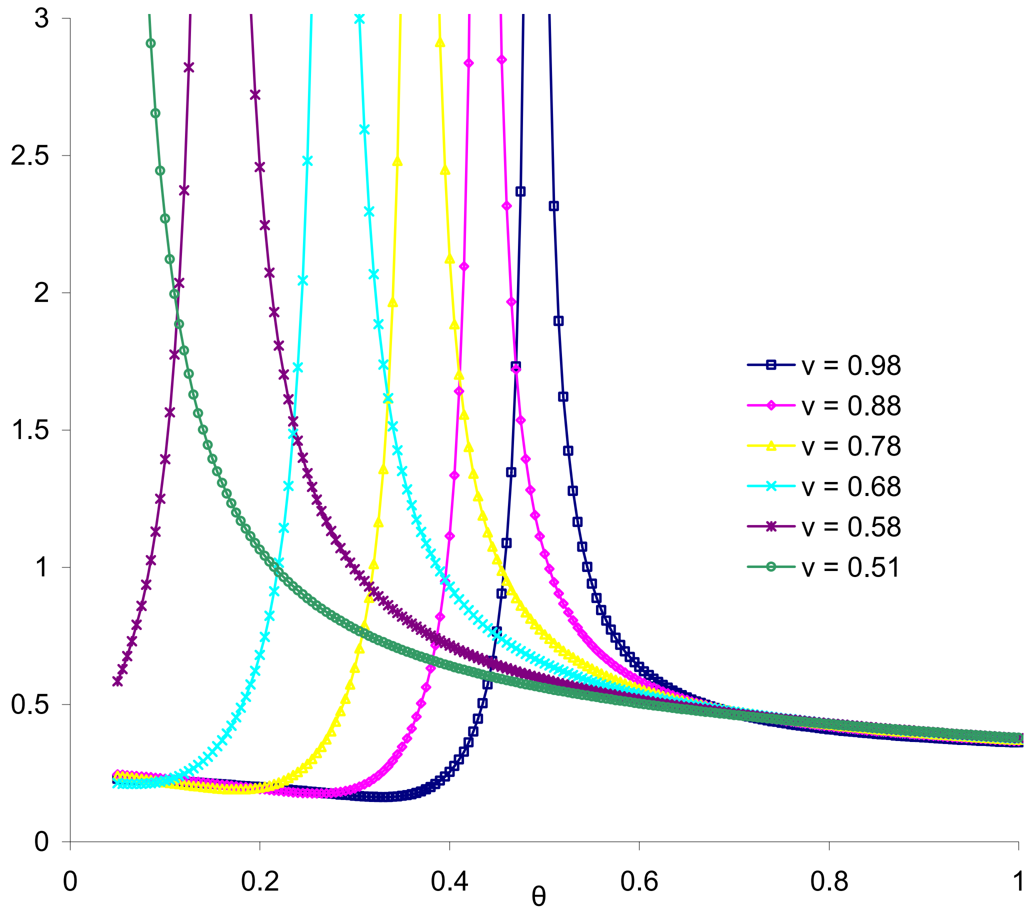

Figure 3 The relative volatility of the terms of trade for different values of consumption home-bias and the elasticity of substitution between home and foreign-produced goods.

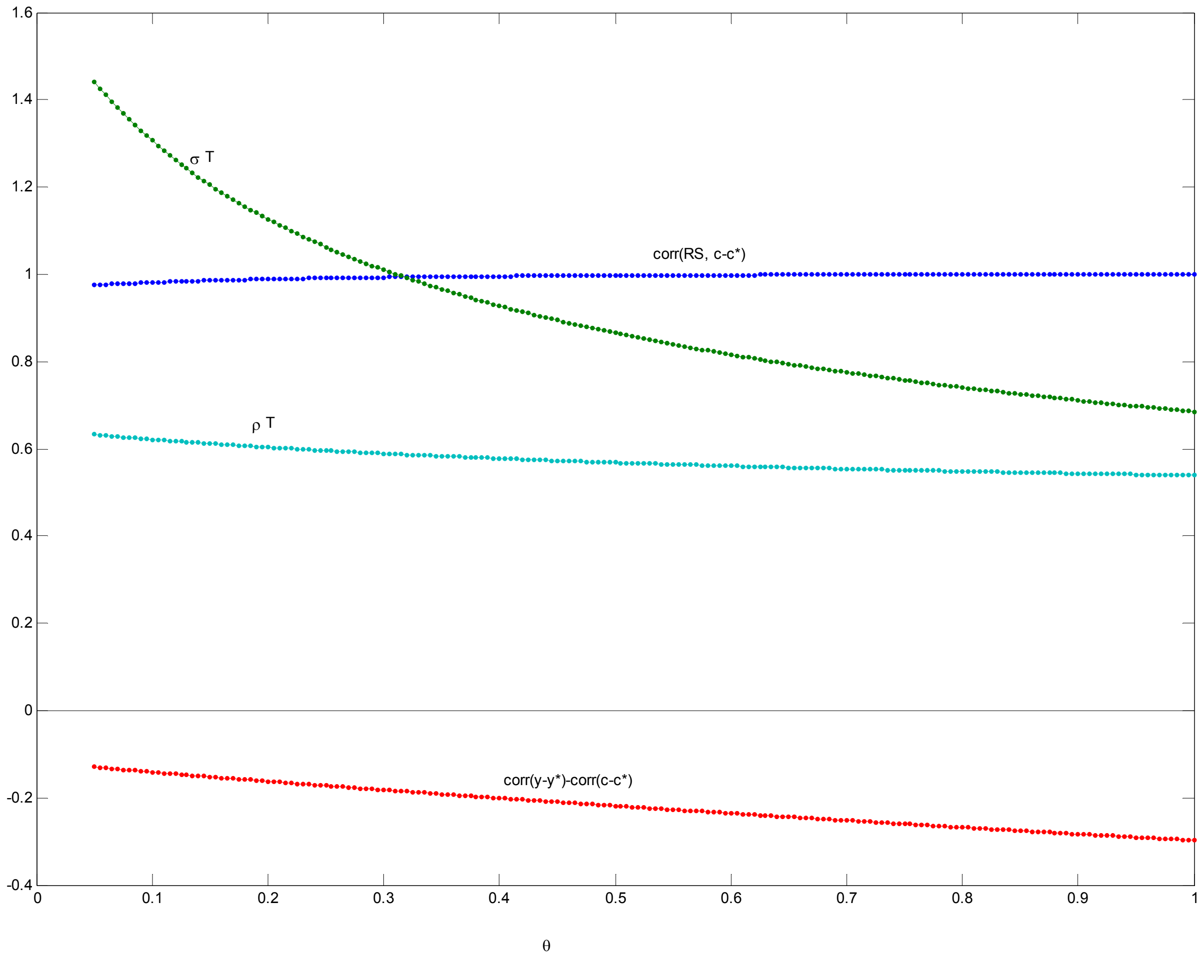

Figure 4 Same experiment as in figure 1, assuming domestic investment is 75% made up of home produced intermediate goods ψ = 0.75.

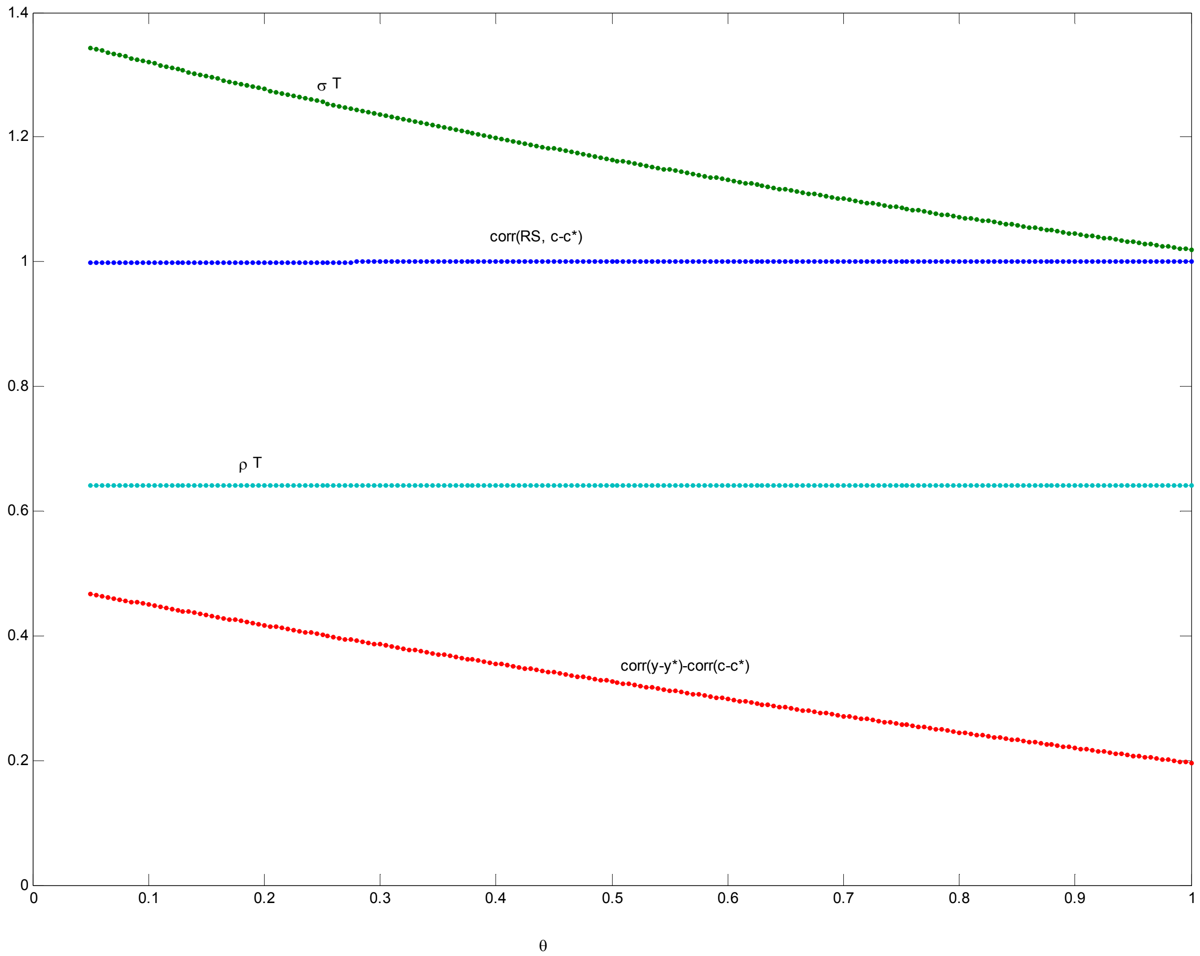

Figure 5 Same experiment as in figure 1, assuming no home-bias in investment goods.

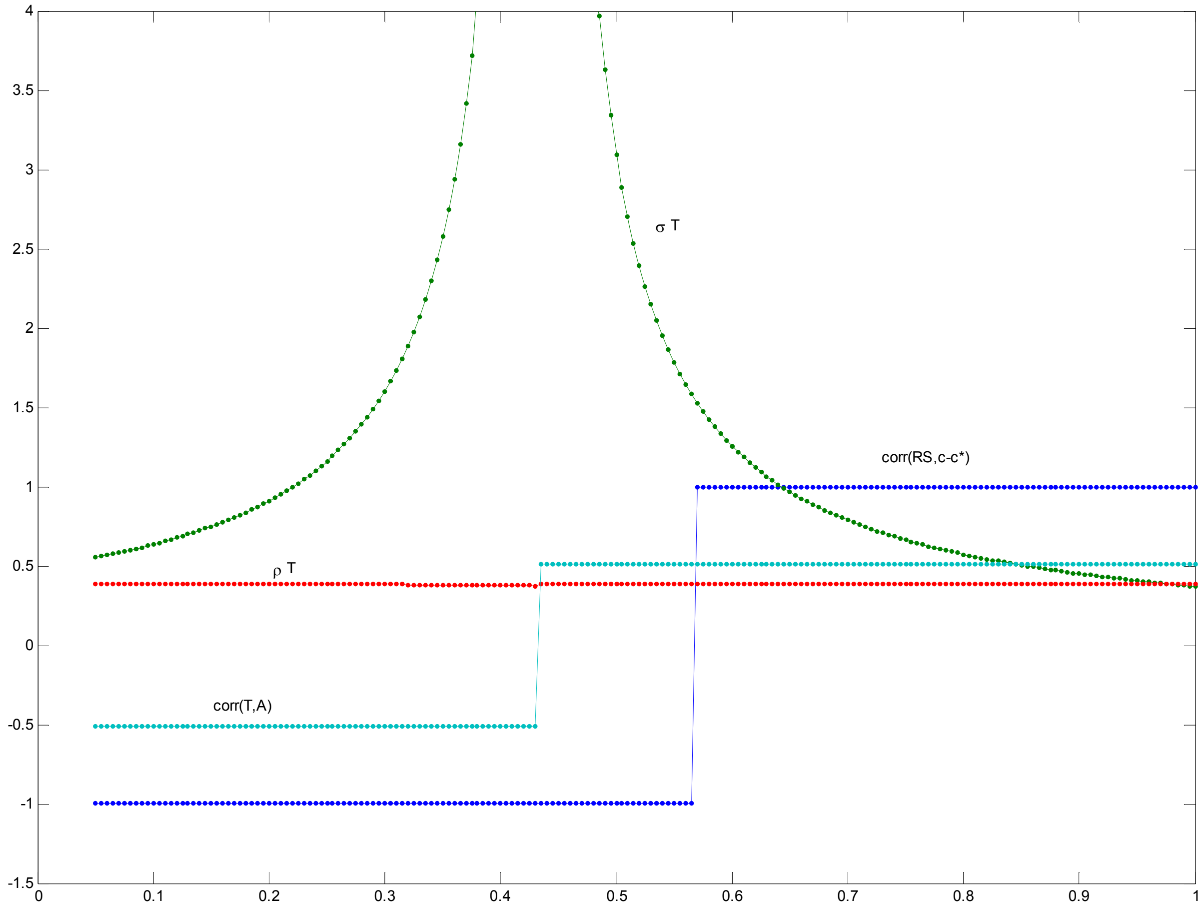

Figure 6 The volatility of the terms of trade relative to GDP, the correlation between the real exchange rate and relative consumption, the first-order autocorrelation coefficient of the terms of trade, and the correlation between home country TFP and the terms of trade (as a proxy for international shock transmission) for various values of θ in a model with financial autarky.

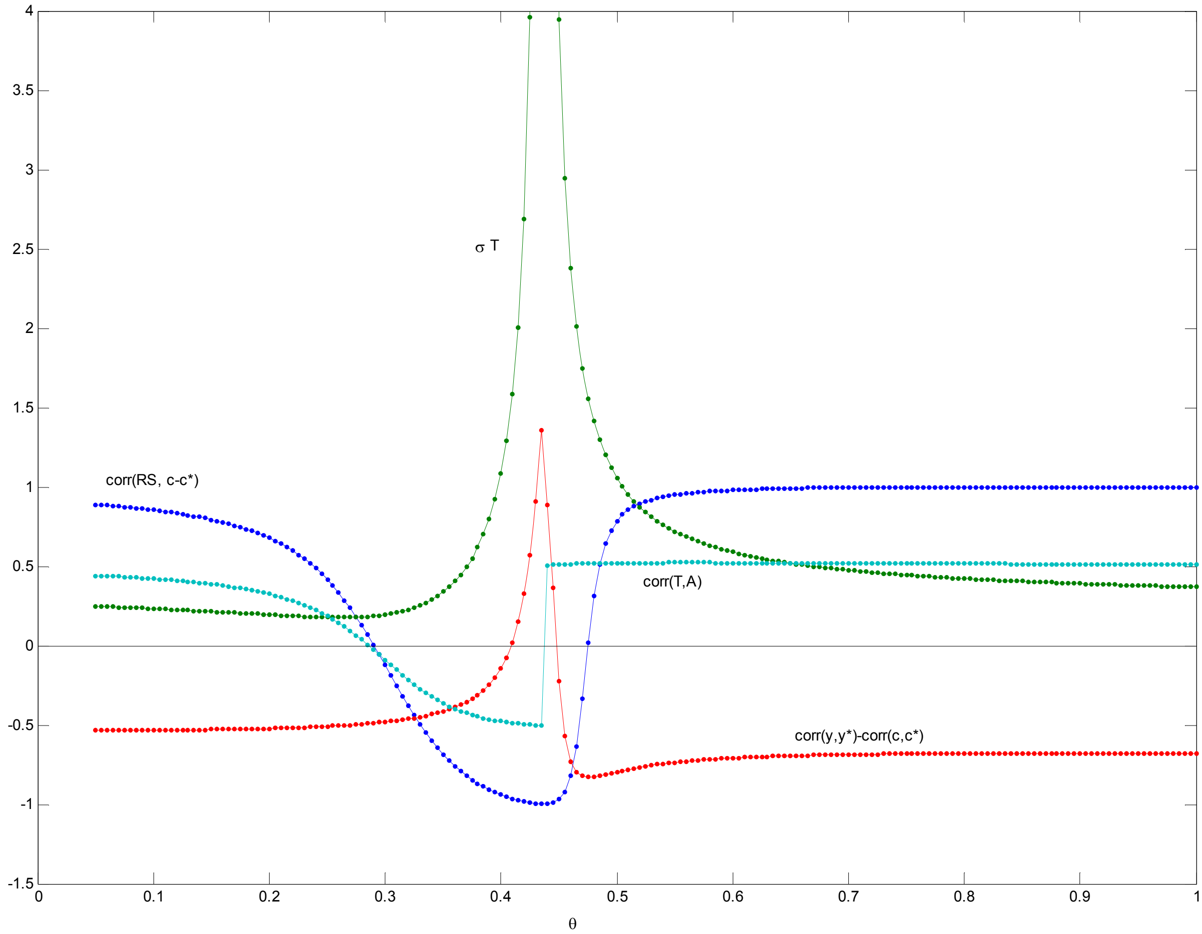

Figure 7 The volatility of the terms of trade relative to GDP, the correlation between the real exchange rate and relative consumption, the difference in the cross-country correlations between GDP and consumption, and the correlation between home country TFP and the terms of trade (as a proxy for international shock transmission) for various values of θ in the baseline model

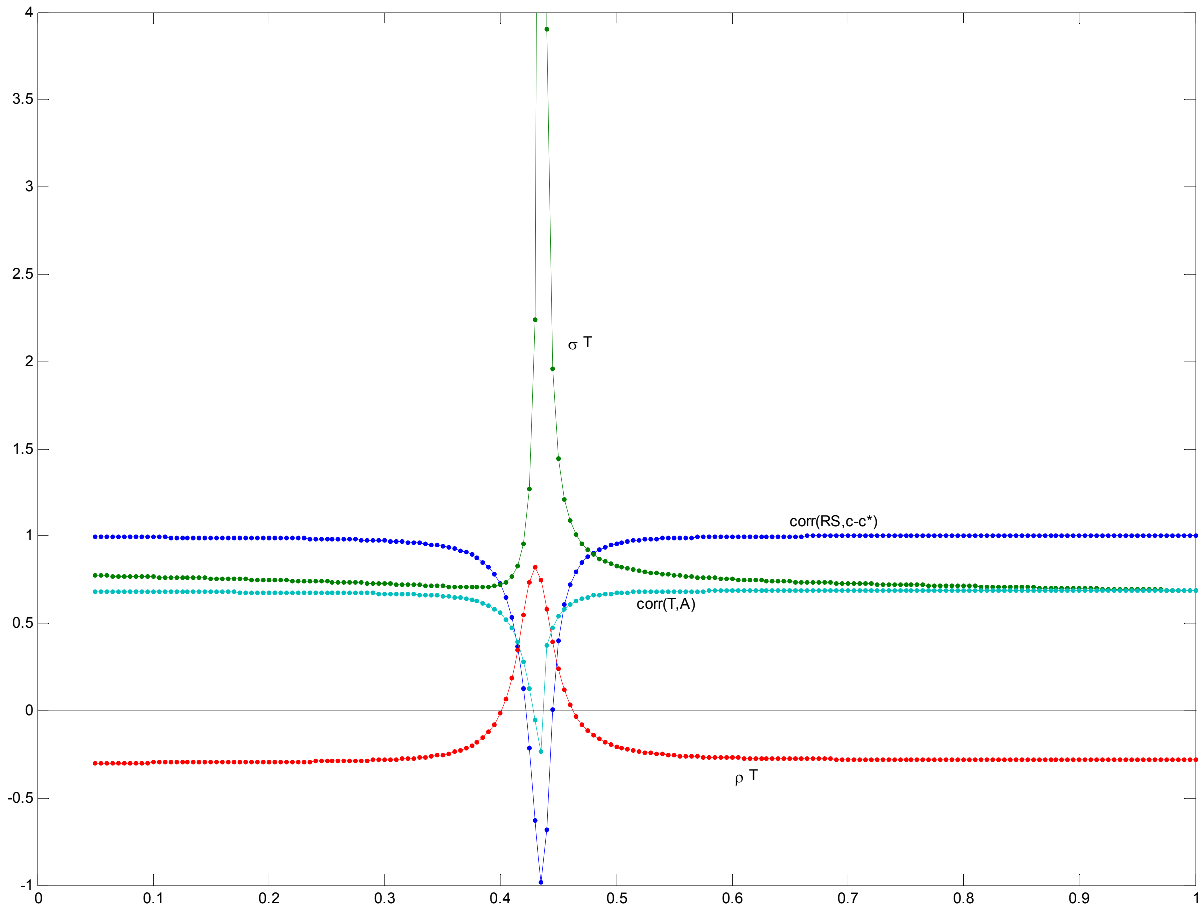

Figure 8 The volatility of the terms of trade relative to GDP, the correlation between the real exchange rate and relative consumption, the first-order autocorrelation coefficient of the terms of trade and the correlation between home country TFP and the terms of trade (as a proxy for international shock transmission) for various values of θ in the model driven by white noise TFP shocks.

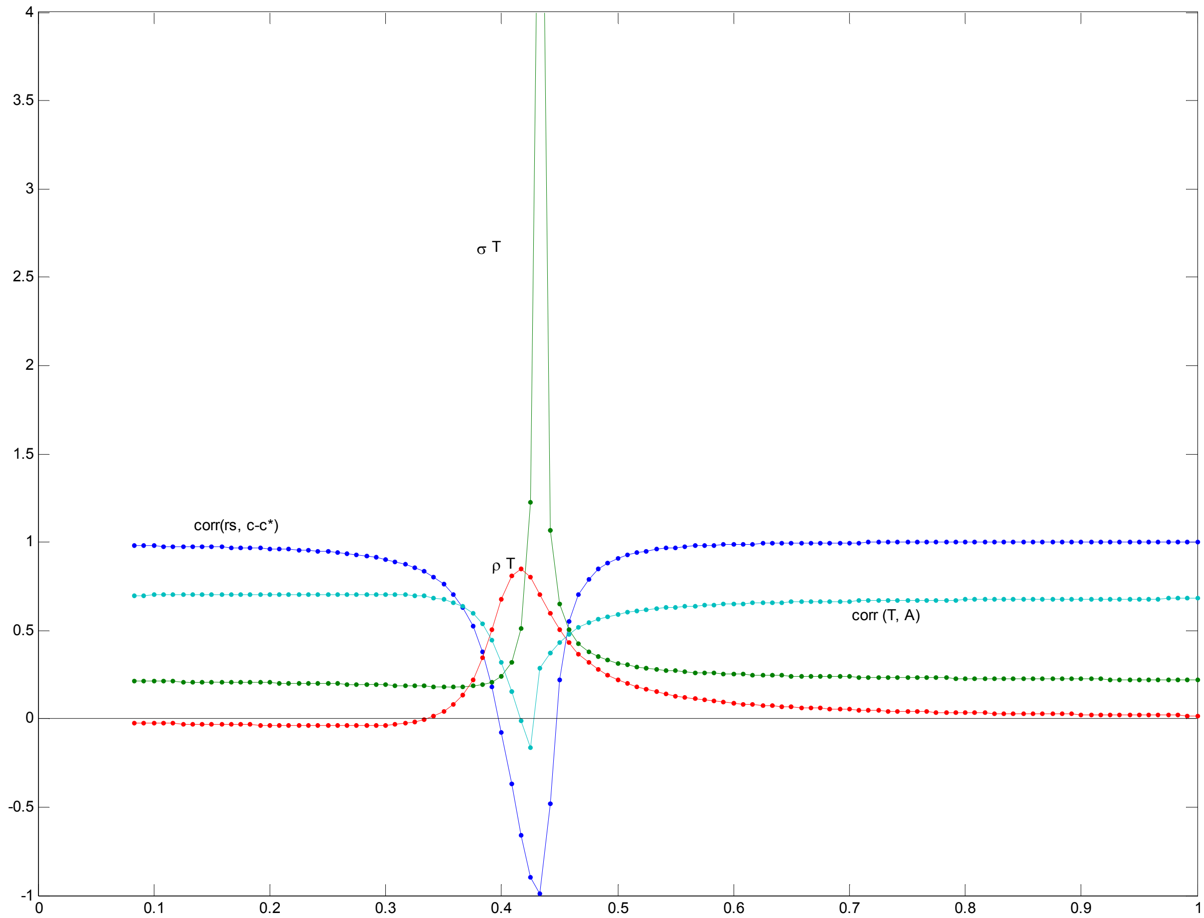

Figure 9a Sensitivity analysis: Same analysis as in Figure 8, but assuming Christiano et al's (2005) estimated value for s''(1) = 2.5.

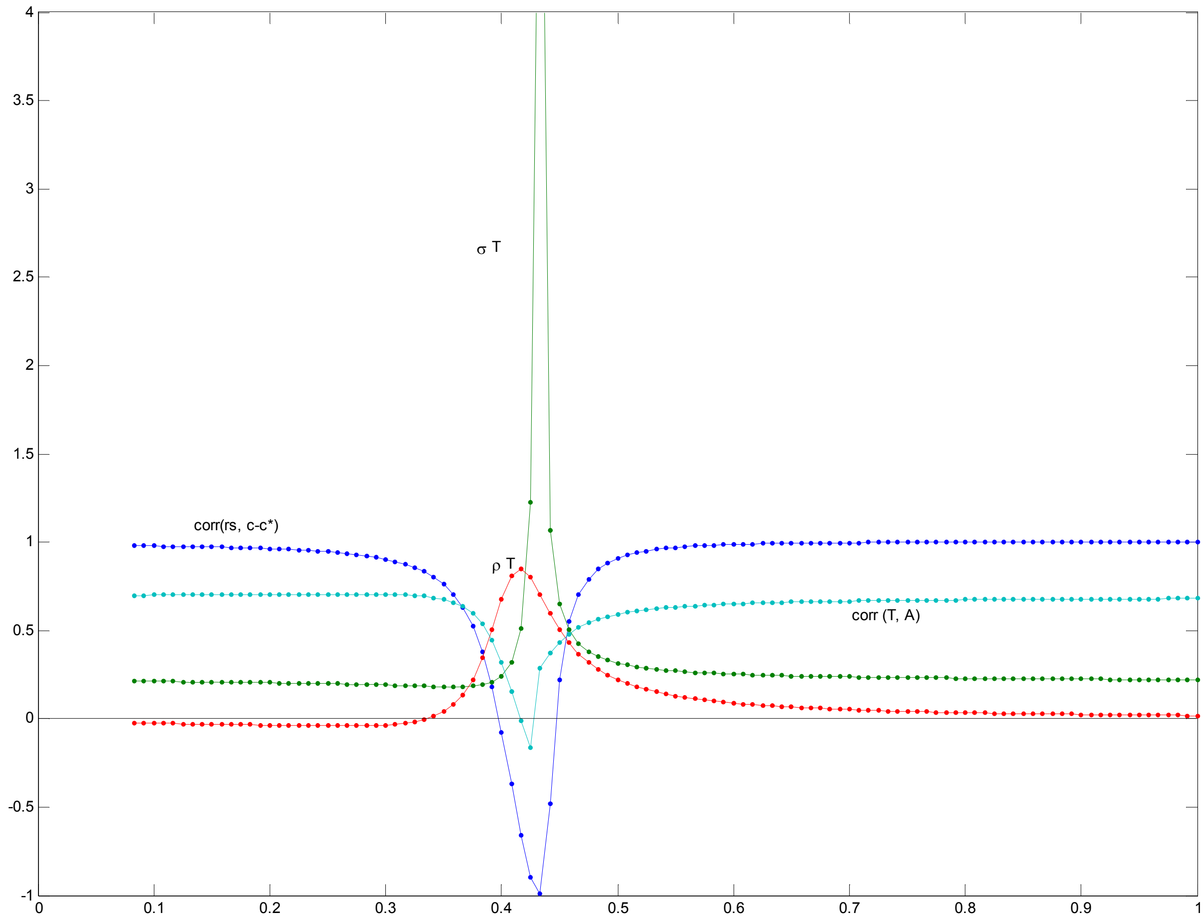

Figure 9b Sensitivity analysis: Same analysis as in Figure 8, but assuming Hayashi (1982) type capital adjustment costs.

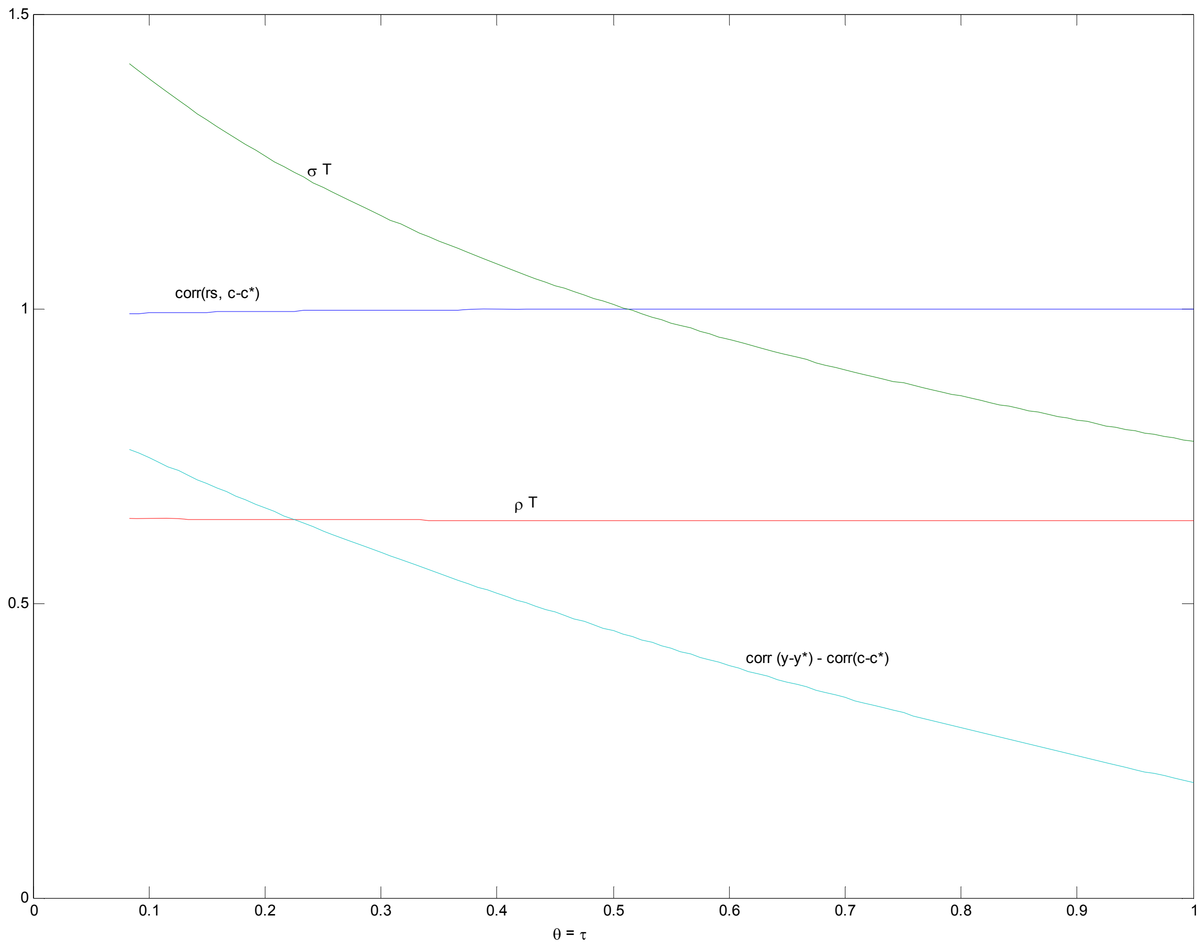

Figure 9c Sensitivity analysis: Same analysis as in figure 5, but here we let the elasticity of substitution between home and foreign intermediate inputs into investment vary along with that of final consumption goods, such that $\theta = \tau$.

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