Choice of optimum feedstock portfolio for a cellulosic ethanol plant – A dynamic linear programming solution

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Abstract

When the lignocellulosic biofuels industry reaches maturity and many types of biomass sources become economically viable, management of multiple feedstock supplies – that vary in their yields, density (tons per unit area), harvest window, storage and seasonal costs, storage losses, transport distance to the production plant – will become increasingly important for the success of individual enterprises. The manager's feedstock procurement problem is modeled as a multi-period sequence problem to account for dynamic management over time. The case is illustrated with a hypothetical 53 million annual US gallon cellulosic ethanol plant located in south west Kansas that requires approximately 700,000 metric dry tons of biomass. The problem is framed over 40 quarters (10 years), where the production manager minimizes cumulative costs by choosing the land acreage that has to be contracted with for corn stover collection, or dedicated energy production and the amount of biomass stored for off-season. The sensitivity of feedstock costs to changes in yield patterns, harvesting and transport costs, seasonal costs and the extent of area available for feedstock procurement are studied. The outputs of the model include expected feedstock cost and optimal mix of feedstocks used by the cellulosic ethanol plant every year. The problem is coded and solved using GAMS software. The analysis demonstrates how the feedstock choice affects the resulting raw material cost for cellulosic ethanol production, and how the optimal combination varies with two types of feedstocks (annual and perennial).

Keywords

Cellulosic ethanol, feedstock, switchgrass, corn stover, optimization

Introduction

Ligno-cellulosic (LC) biofuels– derived from agricultural and forestry wastes, dedicated energy crops, industrial and municipal solid wastes – are expected to be a key component in the future of transportation fuels mix (EERE, 2008a). Cellulosic ethanol, in particular, is being pursued aggressively to meet 16 billion gallon requirement of Energy Policy Act of 2008 (EISA, 2007, English, et al., 2006, HITEC, 2009, Perlack, et al., 2005).[1] More than two dozen pilot plants – some supported with federal grants – are being constructed as a first step toward achieving this goal (RFA, 2009). While the design of an individual biorefinery will depend on the feedstock it uses, all biorefineries will face a common issue of optimizing its feedstock procurement and logistics.[2]

The techno-economic studies of cellulosic ethanol production note that the cost of delivered feedstocks plays a major role in the overall economics of LC biofuels and will impact its competitiveness with fossil fuels (Aden, et al., 2002, Benemann, et al., 2006, Galbe, et al., 2007, Huang, et al., 2009, Wallace, et al., 2005). Other important factors identified are ethanol production plant's ability to handle multiple feedstocks with the same processing technology and an efficient supply chain which can ensure consistent supply of feedstocks over time to meet steady ethanol output during the life[3] of the ethanol plant. These studies also suggest that, at feedstock prices of $40 per dry ton, the raw material (biomass) cost can account for one-third to two third of total costs of LC ethanol production (Fales, et al., 2007, Kumar and Sokhansanj, 2007) – the costs rise

[1] EISA requires 21 billion gallons of cellulosic biofuels with 16 billion gallons of cellulosic ethanol and 5 billion gallons of other advanced biofuels. Various studies have estimated that 60-90 billion gallons of cellulosic ethanol can be produced from cellulosic sources. Still the goal of 21 billion gallons over 15 year time period is considerably higher than what has been achieved with corn ethanol during the same time frame in 1990s-2000s.

[2] Other types of liquid biofuels are butanol, biodiesel, Fishcer-Tropsch liquids

[3] LC biofuel plants are expected to operate for 10-20 years – here, we have assumed a life of 10 years for LC ethanol plant

primarily due to high transportation costs, low density of feedstock availability and seasonality of biomass harvesting.

For cellulosic ethanol to emerge as a reliable source of fuel, Fales et al (2007) suggest that the costs have to be reduced to less than 25 per cent of the total processing costs – attaining this cost goal will necessitate different strategies for cellulosic biofuel plants since their options vary with location, feedstocks and technology. Since the energy crops are not yet grown in a large scale,[4] their yields remain uncertain that might increase feedstock costs and affect the competitiveness of cellulosic ethanol plants. Many of these cellulosic ethanol plants are likely to source their raw material from a pool of major feedstocks such as agricultural residues and energy crops in US Midwest, or short rotation woody crops (trees) and corn stover in south-eastern US.[5] These biorefineries also heavily depend on feedstock produced within a 50-75 mile radius around the plant location to save on transport costs. Large scale storage near biorefineries or near the production sites (farm fields) is necessary to overcome the seasonality in biomass production and harvest raising the raw material costs for biofuel production.[6]

Ligno cellulosic feedstocks differ from each other on several key attributes: municipal solid wastes are available a little to no cost but require extensive pre-processing;, agricultural residue collection and transportation can be costlier than the material value of feedstock itself; forestry feedstock logistics may be much easier than that of agricultural crops since paper and pulp industry has been functioning for decades. Feedstock qualities, composition and properties can be very different resulting in a

[4] 1000ac trial plots in Oklahoma and switchgrass harvesting from 4000ac of Conservation Reserve Program lands in Iowa Chariton Valley Biomass Project are the only large scale field plantings of energy crops (OBC, 2009)
[5] Forestry, mill and logging residues in paper and pulp mills across the US
[6] Most of the feedstock is produced during the months of August – November

biofuel yield that is 60 – 90 per cent of theoretical maximum (EERE, 2009b, EERE, 2009a). Storage losses in quantity and quality can vary depending on the location and pre-processing technologies. Energy crops are not grown in large scale but they are preferred due to their ability to produce biomass with high output density (large volumes in unit area). Agricultural residues are already being produced but their removal may cause environmental problems (Blanco-Canqui and Lal, 2009 , Graham, et al., 2007); still they are economical due to wide availability. While perennial energy crops can be continually harvested from the same plot for many years, there is a need to shift the collection area for corn stover supplies since field crops are grown in rotations. There is also a yield delay for energy crops (full yield starts occurring after 2 years).

With the possibility of harvesting energy crops during early winter months, dependence on energy crops can help reduce storage costs over time. Alternatively annually replenished feedstocks such as corn stover or forestry residues could be contracted with flexibility to reduce costs over time (Cameron, et al., 2007, Kumar and Sokhansanj, 2007, Sokhansanj, et al., 2009). The interesting point is to find an optimal combination among multiple feedstocks – this paper aims to identify such an optimal composition for a hypothetical plant (described below) that depends on agricultural and energy crop biomass only. The sensitivity of feedstock costs to other factors (harvest, transport seasonal costs and others) mentioned above are studied as well.

Model plant:

This study evaluates the optimal combination of two major agricultural feedstocks for a cellulosic ethanol plant located in Kansas. The two primary feedstock types considered here are annual agricultural residues and perennial energy crops.[7] The manager's objective is to minimize the expected costs of feedstock supply over the life of the plant (10 years) subject to ethanol production requirements. The problem is formulated as a dynamic multi-period problem where dedicated energy crops would be planted in year 0 and feedstock production occur in years 1 through 10 years (Year 0 through 10 has is divided into 44 calendar year quarters (q)); ethanol production is assumed to start from q8 (8th quarter) after contracting with farmers in q1 (for energy crops) and construction of ethanol plant during quarters q5 through q7). The problem is designed to address how the plant manager would alter his feedstock procurement decisions with increases in technical conversion of feedstock to ethanol,[8] changes in material cost of biomass feedstock, harvest, transport and any seasonal costs that might occur during certain periods of the year.

The model plant considered here is representative of the proposed biorefineries for the mid-west, plains and south-eastern US states where there is potential to source more than one feedstock in the same region – perennial energy crops such as switchgrass, miscanthus, poplar and willow that can be planted exclusively for energy production and annual feedstocks such as corn stover, wheat and sorghum straw that are already under cultivation. Although feedstocks vary from each other, cellulosic ethanol production technology is being developed in such a manner to handle multiple feedstocks to reduce conversion costs (Mascoma, 2009). This study enables a study of relative importance of

[7] which differ from each other in yield, density, and quantity of ethanol produced per ton of feedstock
[8] Since most of cellulosic ethanol plants are pilot plants, their ethanol yield is expected to be low in the beginning time periods which would gradually increase over time due to efficiencies and learning-by-doing Waldman, D. E., and E. J. Jensen. 2006. *Industrial Organization: Theory and Practice*: Addison-Wesley

various issues faced by cellulosic ethanol operations in the future (that depend on multiple feedstocks including forestry feedstocks).[9]

As an illustration, consider Abengoa's proposed biorefinery in Hugoton, Kansas that plans to source perennial biomass (switchgrass) from Oklahoma Bioenergy Center's switchgrass plot[10] and corn stover from local farmers (Bickel, 2008, OBC, 2008).[11] In this case, the plant manager has the flexibility to choose and alter the acreage of corn stover catchment every year since it is replenished annually.[12] But the acreage under perennial feedstocks such as dedicated energy crops has to be decided at the start of plant operations to accommodate perennial feedstock establishment in the field and achieve full production after two years. Only one-fourth and one-half of the potential yield would be available from energy crops in the first two years suggesting a greater dependence on agricultural residues at least in the early stages of plant operations while waiting for a gradual ramp up in biomass production – the gap between planting and harvest of energy crops can be minimized by contracting with farmers one year earlier; the model plant Abengoa has already started entering into long term contracts ahead of time (Abengoa, 2007, Robb, 2007). In spite of such yield delays, perennial feedstock crops may be preferable due to their higher yields (and dense availability of biomass (tons per unit area)) that can be minimize harvest, baling and transport costs. The perennial crops are

[9] Forest residues or industrial biomass wastes are not considered for this hypothetical plant – being residues, forestry and agricultural residues are renewed every year but at different densities (tons per unit area) and at different costs. Forestry feedstock logistics are different from that of agricultural residues.
[10] Possibly from farmers or farm cooperative who are willing to grow energy crops
[11] All proposed cellulosic ethanol plants have listed more than one feedstock as their primary raw material comprising both annual and perennial crops (RFA, 2009); Abengoa's corn stover comes from irrigated crop rather than unirrigated crop as in some other areas.
[12] There is a compulsory need to shift across fields since annual field crops are grown in rotations – hence if a field is planted with corn this year, next year (or the one after) it could be planted with soybeans or other crop. This forces the cellulosic ethanol plant to source from a different set of corn stover farmers, in the same zone.

expected to have a greater ethanol conversion rate than agricultural residues because of their differences in physical and chemical composition which lead to more efficient pre-treatment and better conversion processes along with a chance to yield co-products (Bals, et al., 2005, Ceres, 2007, EERE, 2009a).[13] Since spot market and contractual arrangements are common in biomass procurement, a combination of short term (for annual feedstocks) and long term contracts (for perennial feedstocks) will be developed (Altman and Johnson, 2009, Robb, 2007).

The name plate capacity of the model plant is assumed to be 53 million US annual gallons (200 million liters per year), which requires about 700,000 dry metric tons annually in the earlier time periods[14] (exact volume depends on the type of feedstock and associated ethanol yield that changes over time). The biomass is required on a consistent (both quality[15] and quantity) basis over the entire year. This requirement differs from the harvest pattern and availability of biomass – which varies from season to season (see footnote 6, (USDA - NASS, 1997, USDA, 2009). The mismatch between uniform requirement and seasonal availability will necessitate an inventory of biomass feedstock. – this paper also identifies the optimal level of inventory to be maintained in addition to any preferences (constraints) of the plant manager, such as a minimum inventory requirements; we assume a minimum inventory of 20 per cent (of quarterly production capacity) for all the time periods, except the final time period when the cellulosic ethanol plant shuts down.

[13] Coproducts may not be produced with agricultural residues
[14] This quantity is derived based on the assumed conversion capacity of 75 gallons per dry metric ton – with improvements in the processes, this can reach 85-90 gallons per metric dry ton (learning-by-doing effect).
[15] Quality of biomass is akin to the gallons of ethanol produced using that biomass feedstock

Decision variables: The variables to be chosen for the plant manager are the number of acres that has to be contracted with for corn stover and grass harvesting. Since corn stover is annually reproduced, it is assumed that the ethanol plant will contract with these farmers on a short term basis – see footnote 12 – this provides flexibility to contract corn stover over all quarters q1 through q44 – suggesting that there are potentially 44 choice variables. Since there is no production in the first 7 quarters, the results of optimization would yield no procurement of corn stover, except possibly for inventory maintenance.

Contracting of perennial crop acreage is entirely different – the energy crops have to be planted in year 0 (q1) to allow crop establishment; the initial yields can be expected only from quarter 7 (q7). We allow establishment of perennial crops prior to ethanol plant's operations – but do not impose any acreage restrictions. So, if energy crops are not part of optimal solution, optimization would yield zero acreage under energy crops in the first year of operation.[16] The perennial energy crops grown exclusively for ethanol production have limited alternative uses (in limited cases, they can be used in bio-electricity production (Brummer, et al., 2002); also, the perennial nature of energy crops require the markets for these crops secure by long term contracting (Abengoa, 2007). The perennial crops are assumed to have a 10 year productive life coinciding with the life of cellulosic ethanol plant – for modeling simplicity. Since perennial crops produce biomass over the entire life of ethanol plant, farmers would find it profitable to keep them on the fields until year 10.[17] This assumption will be partially relaxed by limiting the yields to

[16] We assume that cellulosic ethanol plant starts its operations by the quarter 8 (q8) – although it seems long, such delays are common in pilot cellulosic ethanol plants.
[17] For simplicity, we assume that there are no subsequent plantings of energy crops in years 1 through 10; the energy crop yields are assumed to reach the maximum and remain the same starting quarter q15.

years 1 through 7 and allowing later planting of energy crops during years 1 through 5. For the base case, there is only one decision variable concerning perennial crop acreage – the acreage chosen in first quarter (q1) to be planted with energy crops.[18]

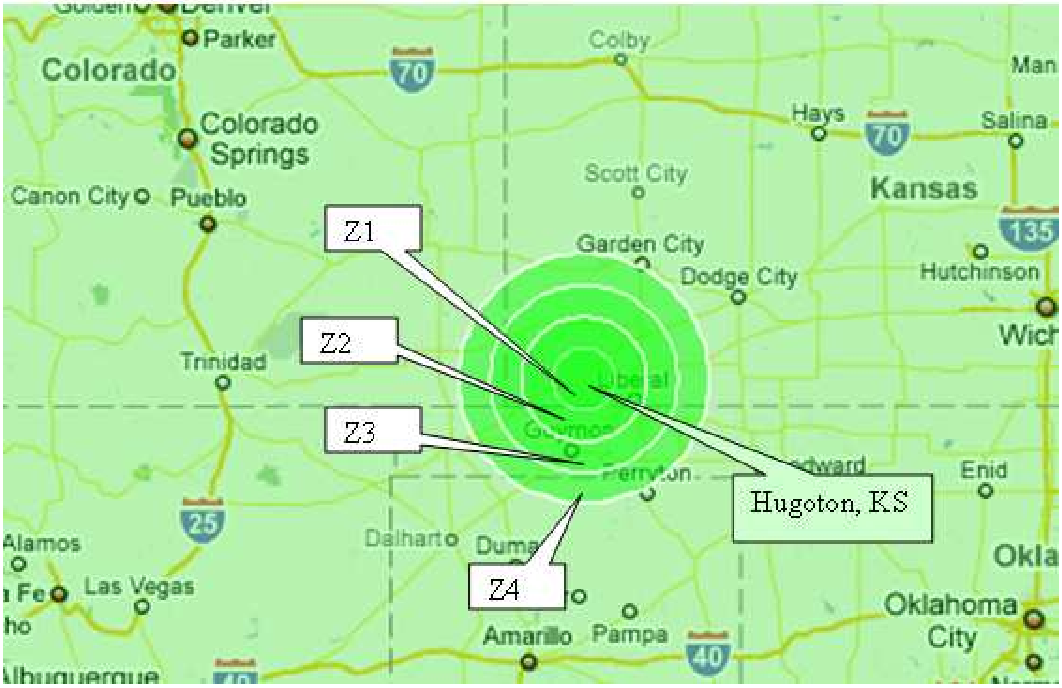
The third group of decision variables determines the amount of biomass inventoried in every period (all 44 quarters). There is an additional cost ($3/ton/quarter) associated with inventories which will be included in the cost function (objective function) that has to be minimized. Thus, there are a possible total of 44 (corn stover acreage selected every quarter) + 1 (energy crop acreage selected in q1) + 44 (choice of inventory levels during all quarters) = 89 choice variables in this multi-period problem. Note that all 89 choice variables will not necessarily be strictly positive; the optimized outcome would eliminate acreage or storage requirements during certain quarters (e.g. production starts only in q8 and no need for biomass until then). Flow of biomass from the fields into inventory for next quarter creates an inter-temporal optimization problem. The dynamic optimization is solved as a multi-period sequence problem where the objective function and constraints for every period are solved simultaneously to minimize the overall (cumulative) feedstock costs – that includes material, harvest, transport, seasonal and storage costs (explained below).

Geographic distribution of feedstock: Actual field trial operations suggest that it would be feasible to limit feedstock catchment area to 50-75 miles around the ethanol plant (Atchison and Hettenhaus, 2003). We restrict the collection region within 60 mile radius of cellulosic ethanol plant. Even within this radius of catchment area, the transport

[18] Farmers are assumed not to remove lands that are planted with energy crops – this is not a restricting assumption since the results identify the aggregate amount of land that has to be contracted with farmers for energy crop harvesting. It could be any set of farmers as long as the extents of acreage and yield levels are met.

costs can vary substantially: fields located within 15 miles of the cellulosic ethanol plant will only have a (two way) maximum distance[19] of 30 miles while the field at a distance of 60 miles will incur higher transportation costs equivalent to 120 miles of transport. To account for these differences, the 60 mile radius area is subdivided into 4 zones – Z1, Z2, Z3 and Z4 – in concentric circles of radius 15, 30, 45 and 60 miles respectively.[20] See figure 1.[21]

Figure 1: Graphical depiction of feedstock catchment area for model plant located in Hugoton, Kansas



Since, all these four zones include urban areas and lands under other non-agricultural purposes, all lands will not be available for feedstock cultivation. Available area for feedstock cultivation and harvest in every zone is restricted using flexibility

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[19] The additional costs due to road conditions or lack of straight roads to ethanol plants are ignored in this analysis; for a conversion of air distance to road distance, see French (1977, 1960)

[20] In the above illustration of Abengoa cellulosic ethanol plant, Oklahoma Bioenergy Center's switchgrass plot in Oklahoma panhandle area lies at a distance of 35 miles (in zone Z3) of ethanol processing plant.

[21] I will change the zone limits later - zone 1 (0-5 miles), zone 2 (5-10), zone 3 (10-15), zone 4 (15 to 25), zone 5 (25-50), zone 6 (50+). I would prefer uniform difference (10, 20, 30, 40 and 50 miles) anyhow.

constraints (De La Torre Ugarte and Ray, 2000, Johansson, et al., 2007, Ray, et al., 1998a, Ray, et al., 1998, Walsh, et al., 2003). These constraints – one for every zone – help model the fact that only a fraction of available land would be dedicated to any particular feedstock.[22] Although agriculture would be the major land use in a region, Robb (2007) reports that feedstocks could be collected from approximately 5 per cent of a geographic area in the region. For our base case scenario, we assume that feedstocks would be collected from 7% in all four zones and the sensitivity of costs to changes in amount of land area would be explored – the cellulosic ethanol plant could choose to contract either annually reproduced corn stover or perennially produced energy crops from these lands.

Table 1: Area distribution among the proposed zones:

Zone [1]	Maximum distance from ethanol plant (radius in miles) [2]	Geographic Area within every zone (thousand acres) [3]	Area available for biomass feedstock harvest at 7% land availability (thousand acres) [4]
Z1	15	452	31.64
Z2	30	1356	94.92
Z3	45	2261	158.27
Z4	60	3165	221.55

Note: More acreage will be available for feedstock harvest (Z4>Z3>Z2>Z1, see column [3]) as we move away from the cellulosic ethanol plant facility.

Objective function: The plant manager's objective is to minimize the expected total costs of production over all the years of operation. The costs incurred to procure feedstock vary by time and type of harvest, baling and transport options and inventory costs. The total delivered costs of bioamss ($/metric dry ton) is calculated as a sum of

[22] If the model plant (53 million gallon annual capacity) were supplied only with corn stover at an average yield of 1.5 tons per acre, then it would require approximately 0.47 million acres; if energy crops were the sole feedstock, then it would require only 0.07 million acres – without flexibility constraints, the LP formulation would limit feedstock catchment to the zones that are closest to the cellulosic ethanol plant (Z1 and Z2).

material costs (or opportunity costs), harvest costs, transportation costs and other seasonal costs are explained below: TC = OC + HC + TrnC + SC

Opportunity costs: The material cost (M) refers to the opportunity costs of biomass that is being removed from the agricultural fields. In case of annually reproduced agricultural wastes such as corn stover, it is equivalent to the soil nutrient value lost due to removal of corn stover, or through any other alternative use.[23]

Production costs: In case of perennial feedstock such as miscanthus or switchgrass, material cost refers to production costs incurred in establishment, and cultivation and maintenance of the crop itself. It includes the costs of fertilizers used for energy crop production and the opportunity costs such as the economic profits lost due to diverting a piece of land from row crops to energy crop.

Harvest cost: The harvest costs (HC) are assumed to be constant on a per ton basis and fixed at $1 per metric dry ton. It includes the fixed and variable costs of harvesting and collection equipment, and bundling materials (Atchison and Hettenhaus, 2003). The sensitivity of feedstock cost in cellulosic ethanol production to changes in harvest costs is studied (varying from $1/ton to $8/ton).[24]

Transportation costs (TrnC)**:** Since the fields in zone Z4 are far away from ethanol plant than the fields in zone Z1, the solution to the optimization problem will naturally favor production in the zones closer to cellulosic ethanol production plant. Irrespective of the

[23] When corn stover becomes valuable, the farmers are likely to apportion the total (corn) crop production costs between the grain output and stover output – in that case, material cost also refers to the value (cost of production) allocated to corn stover.

[24] The harvest costs decrease with an increase in biomass density (tons per unit area).

distance (zone) from the plant, the per unit transportation costs will be the same. It is assumed to be $0.28 per ton-mile (Brechbill and Tyner, 2008).[25]

Seasonal costs (SC): Corn harvests occur in the months of August through November. One-pass harvest where corn grains and stover are collected simultaneously would be an economical way to harvest feedstock for cellulosic ethanol production (EERE, 2008b). Abengoa plant operations in Kansas suggest that farmers would assemble feedstock bundles on the farms and the plant would collect these bundles as and when needed (provided the agreement is acceptable to farmers – (Robb, 2007). If the bundles are moved during off-season months (December – March), there may be extra costs due to road conditions or difficult access to the fields.[26] Or the ethanol plant can potentially move the feedstocks while it is available in abundance for a cheaper spot market price and incur the associated storage costs while maintaining its inventory (Robb, 2007). Hence the plant manager can choose either to transport and store feedstock early or collect it later at a higher cost (due to weather parameters). Here, we assume added seasonal costs during the first quarter (SC_1, January-March) to be $1.5 per metric dry ton.[27] Sensitivity analysis is conducted to study how an increase in (transport) costs during particular seasons would affect overall raw material costs for the cellulosic ethanol plant. The changes in these seasonal costs Note that this is an arbitrary assumption – it will be used to analyze the sensitivity of cumulative costs for a change in seasonal costs only during the first quarter (January-March) of every year. There will be no seasonal costs in the baseline case (explained later).

[25] All dollar values correspond to constant (2009) dollars.
[26] This is especially a problem with forestry logging residues in the northern states of US
[27] It could be uniformly distributed between $1 and $2 per metric dry ton

Harvest pattern of perennial grasses can be slightly different from that of corn residues. A delayed harvest of energy crops during early winter months (December – January) would increase desiccation[28] and make them suitable for processing – transport costs will also be lower since moist biomass costs more to transport. This is a seasonal gain since total costs (TC, dollars per dry metric ton) would be reduced in this special case. Note that all fields can be harvested in a staggered manner – some in fourth quarter of a calendar year (Oct-Dec) and the rest in the first quarter (Jan-Mar). For the baseline case, the seasonal costs are ignored (kept at zero).

Yield patterns: The yield of perennial energy crops can range from 4 to 12 tons per acre, depending on the feedstock crop and local growing conditions, especially temperature (Growing Degree Days, GDD) and precipitation (rainfall). See table 2 for the range of GDD and rainfall in south-western Kansas.

Table 2: Range of temperature and precipitation affecting perennial energy crop yields

	Minimum	**Maximum**
GDD	3528	4488
Rainfall (in inches)	18.2	35.23

For this range, the energy crop yields can vary from 4.5 dry metric tons per acre (switchgrass) to 11.43 dry metric tons per acre (miscanthus). Although switchgrass was considered to be a major bio-energy crop, the recent focus has shifted to miscanthus due to higher yield levels – various energy crop companies are involved in developing miscanthus varieties (Ceres, 2009b, Ceres, 2009a, Mendel Biotechnology, 2009). A normal random distribution (with mean yield of 10 dry metric tons per acre and standard

[28] Loss of moisture in leaves

deviation of 2.5 metric dry tons per acre is assumed), to capture the randomness in energy crop yields (Angelinia, et al., 2009).[29]

The corn crop is primarily an irrigated corn crop around the location of our model plant (in Hugoton, Kansas). The regional (multi-county) yield of corn grains is around 202 bushels per acre for irrigated corn and 42.5 bushels per acre for un-irrigated corn (USDA - NASS, 2009). To be representative of other states and regions, we consider the overall yield of 177 bushels per acre as corn grain yield in the region. Since corn stover is jointly produced with corn grains at a straw-to-grain ratio of 1:1, corn stover yield is 177 bushels of dry matter per acre as well (or 4.5 metric dry tons per acre). However, all of the crop residue cannot be removed because of soil erosion risk as well as loss of nutrients (Blanco-Canqui and Lal, 2009). Corn stover collection research suggest that about 25-30% of corn stover can be sustainably harvested while maintaining soil quality (Dreiling, 2009, Graham, et al., 2007). At these levels, about 1.1-1.35 dry metric tons per acre can be removed. Similar to our assumptions for perennial crop, a normal distribution is assumed for corn stover yield with mean 1.25 metric dry tons per acre and standard deviation of 0.3125 metric dry tons per acre.[29] Both feedstocks are expected to yield at their respective rates across all four zones.

Minimum inventory requirements: Since most of the biomass harvest occurs in a span of 4 or 5 months, cellulosic ethanol plant is likely to maintain inventory either in the farm fields or at the plant or as a combination of the two. During the harvest season there will be additional flexibility to keep low levels of inventory which help reduce storage costs.

[29] Standard deviation is assumed to be 25% of mean yield value – the sensitivity of costs to standard deviation of yield will be explored as well. Angelinia et al (2009) found two phases of yielding from years 3 to 8 and years 9 to 12 – for simplicity, we have ignored the gradual reduction in yield of energy crops over time but adjusted the overall mean value of yield.

To reduce uncertainty the plant manager might prefer to maintain a certain level of residues at all times in inventory for smooth operation of the plant. We assume the minimum quantity to be at least 20 per cent of residues during all quarters of operation (except the terminal period – last quarter – after which cellulosic ethanol plant shuts down its production operations). Note that this is not likely to be a limiting constraint during winter and spring (first and second quarters) since the plant operations would run entirely off biomass from the inventory. The sensitivity of raw material costs to changes in minimum required inventory is studied.

Other factors: A minimal quantity of biomass kept in storage will be lost which is assumed to be 3 per cent from one quarter to the next. Ethanol plant manager might be more concerned about the costs in the initial period since most of these operations are pilot plants and the importance of proof-of-concept for the supply chain would be crucial in the first few years; this is accounted for with a discount factor (δ) to discount the cumulative costs of feedstock procurement over 10 years. To ensure smooth production over all the quarters, we assume that the annual plant capacity of 53 million US gallons would be produced evenly across the quarters (i.e. 13.25 million US gallons every quarter).

Optimization problem: The above description of the hypothetical plant and assumptions are synthesized together in the following cost minimization problem.

Subscript notation:

f = Feedstocks Annual Stover (S) and Perennial Grasses (G)

z = Production Zones [Z1, Z2, Z3 and Z4 with radius Rds_z 15, 30, 45 and 60 miles respectively]

q = Time in quarters 1, 2, … 44 (terminal time, T = 44)

Parameters/constants:

Y = Yield

δ = Discount factor = 1/(1+r)

r = Annual discount rate of 4 per cent [or quarterly rate of 1 per cent]

d = Storage costs

Storage Loss = Quantity of biomass lost in storage

Plant Capacity_q = Quarterly ethanol processing capacity = 13.25 million gallons

K_{fq} = Feedstock to ethanol conversion efficiency

R_f = Compound quarterly growth rate in feedstock to ethanol conversion

MIR = Minimum Inventory Requirement

T = 44, Terminal time period [q44 for a 0-10 year period analysis]

P = Cost to transport one ton of biomass over one mile

Acreage_{fq} = Acreage contracted to harvest feedstock f in quarter q

X_{fq} = Amount of feedstock f stored in inventory at the end of quarter q

Accounting relationships:

(a) Total cost of feedstock

$$TC = M_{fzq} + HC + TrnC_z + SC_{fq}$$

(b) Feedstock availability constraint:

$$\text{Availability}_{Sq} = \sum_z Y_{Sq} * \text{Acreage}_{Szq}$$
$$\text{Availability}_{Gzq} = \sum_z Y_{Gq} * \text{Acreage}_{Gz}$$

(c) Increase in ethanol conversion over time ('learning-by-doing' factor):

$$K_{fq+1} = (1 + R_f)\, K_{fq}$$

(d) Transportation costs

$$TrnC_z = 2/3 * P * Rds_z$$

Objective function:

Minimize wrt $\mathbf{Acreage_{fzq}}$ and $\mathbf{X_{fq}}$

$$\sum_f \sum_z \sum_q [\delta^{\wedge q} * TC_{fzq} * Y_{fq} * Acreage_{fzq}] + \sum_f \sum_q [\delta^{\wedge q} * d_{fq} * X_{fq}]$$

subject to the following *constraints*:

- **Changes in stocks [expressed as equilibrium for every time period q]:**
  Availability (supply) = Used (demand)

$$\sum_f Availability_{fq} + (1 – Storage\ \ Loss) * X_{q-1} = Processed_q + X_q$$

- **Constraint to meet quarterly ethanol plant processing capacity:**

$$\sum_f K_{fq} * Processed_{fq} \geq Plant\ Capacity_q$$

- **Minimum inventory requirement (expressed in gallons of ethanol):**

$$\sum_f K_{fq} * X_{fq} \geq MIR * Plant\ Capacity_q$$

- **Terminal conditions:**

$$\sum_f \sum_z Availability_{fzT} + \sum_f X_{fT-1} – Use_T = 0$$

$$\sum_f X_{fT} = 0$$

Sample Results:

We coded and solved the problem using GAMS/CPLEX software (GAMS, 2009).[30] The underlying parametric assumptions for a base case model and results are presented in table 3 and 4 respectively (assuming fixed transportation costs and no seasonal costs).[31] The results are reported in terms of cumulative costs that the cellulosic ethanol plant would incur towards purchasing feedstock over a period of 10 years under the constraints

[30] The code for the base case scenario is attached in an appendix to this document; linear (CPLEX, BDMLP) and non-linear (MINOS, CONOPT, DyLP) solvers were tried to ensure consistency of results
[31] The impact of density of biomass availability (tons per unit area) on total costs (TC) is ruled out due to fixed transportation costs

given above. The results are also interpreted in terms of dollars per gallon of ethanol for easy interpretation. The sensitivity of results to changes in parametric values and how they compare with base case values are discussed in table 6 and figures 3 – 5.

Table 3: Parametric values for various scenarios:

Parameter	Level in base case scenario	Remarks/Sensitivity Analysis
Yield (metric dry tons per acre)		
Energy crop (miscanthus)	10	Harvested in every 3rd quarter, not random in base case scenario; sensitivity to yield variations is tested with normal distributions with 25% coefficient of variation;[32] staggered harvesting (in various quarters) is considered
Stover	1.25	
Discount factor, δ	1	Discount rate r_q = 0 in base case scenario; in other cases, a quarterly discount rate of r_q = 1% assumed
Costs of storage, d	$3/ton/quarter	
Storage Loss	3 % per quarter	
Land available for both grasses and stover	7%	Cost sensitivity to land availability factor is tested for the range of 5% to 15%
Minimum Inventory Requirement	20%	Cost sensitivity to changes in MIR for the range of 5% to 30%
Ethanol conversion efficiency (gallons per metric dry ton in q1)		
Energy crop	75	Conversion efficiency grows at a compound annual growth rate of 2% or compound quarterly growth rate of 0.5% reaching 86-93 gallons per ton by q44 – for all scenarios
Stover	70	
Cost components		
Material costs ($/metric dry ton) (Atchison and Hettenhaus, 2003);		As valued on field
Energy crop	$ 25	(Jensen, et al., 2005); increase material costs to $40 per dry metric ton
Stover	$ 18	Increase in economic costs (environmental costs) worth $4/ton
Harvesting costs	$ 13.96	Increase in harvesting costs by 30%
P, per mile transporting cost	$0.28/ton mile	Range of $0.20 to $0.40 per ton mile (Brechbill and Tyner, 2008)
Seasonal costs	none	Extra costs of $2.5 per dry metric ton when harvested in winter months (quarter 1)

[32] Coefficient of variation = standard deviation / mean value; for miscanthus, mean yield and standard deviation are assumed to be 10 and 2.5 metric dry tons per acre resulting in a coefficient of variation = 25%

Under the assumptions of base case scenario (see table 3), optimal sourcing and storage of biomass from energy crops (miscanthus) and corn stover would cost $280.70 million over 10 years (to produce 530 million gallons). This is equivalent to raw material cost of $0.53 per gallon of cellulosic ethanol. While this cost is close to the current industry estimates of $0.50 of raw material cost quoted in Robb(2007) for Abengoa plant in south west Kansas, it is considerably higher than raw material cost figure assumed in techno-economic studies that range from $0.26-$0.40 per gallon of cellulosic ethanol. Ten per cent of total biomass comes from perennial energy crops – much less than annually reproduced corn stover – in the base case scenario. Energy crops play a minor role due to higher material costs of $25 per dry metric ton higher than $18 per dry metric ton for corn stover. For related cost estimates of energy crop production see (Brummer, et al., 2002, Perrin, et al., 2008, Vadas, et al., 2008).[33] Feedstock procurement occurs only once in four quarters due to base case scenario assumption that all biomass is harvested and transported only during the third quarter (July-September) of every year. In other quarters (with no harvests), the supply is derived from inventory.

Table 4: Optimal acreage contracted to harvest corn stover and energy crop biomass - Base case scenario results

		Production Zones (thousand acres)			
Year	**Quarter**	**Z1**	**Z2**	**Z3**	**Z4**
Available Area at 7%					
All Years	All Quarters	31.7	95.0	158.3	221.6
Miscanthus Acreage					
Planting in Year 0	Retained in all quarters	6.9			
Corn Stover Acreage					
1	q5				
1	q6				
1	q7	24.8	95.0	158.3	213.3
1	q8				

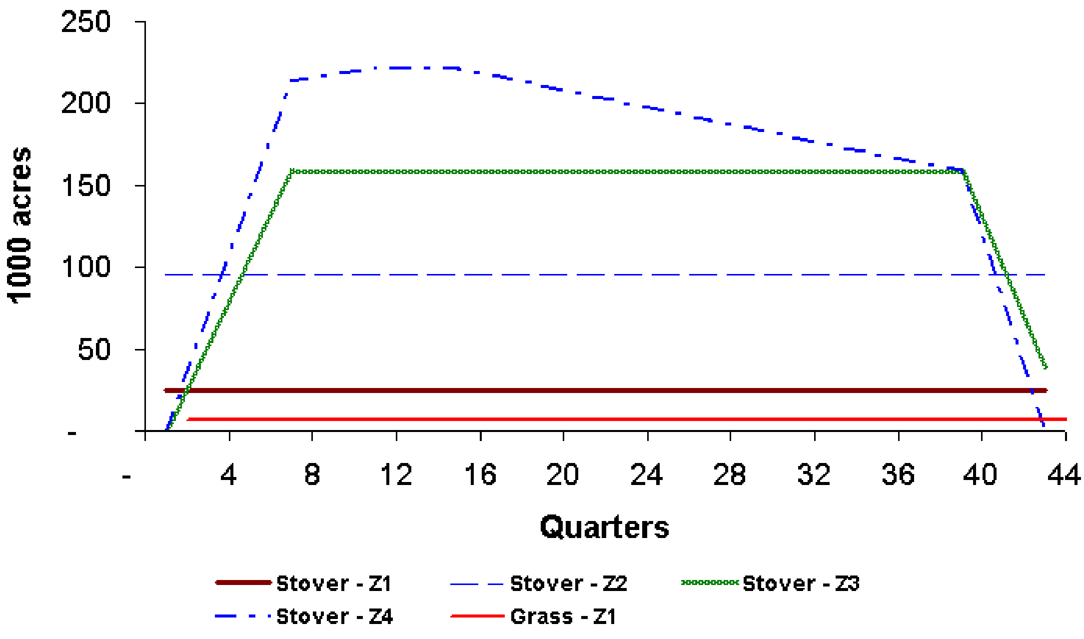
Continued…

[33] Energy crop material costs of $25 per dry metric ton is equivalent to annualized production costs of $250 per acre per year

		Production Zones (thousand acres)			
Year	Quarter	Z1	Z2	Z3	Z4
Corn Stover Acreage					
2	q9				
2	q10				
2	q11	24.8	95.0	158.3	221.6
2	q12				
3	q13				
3	q14				
3	q15	24.8	95.0	158.3	221.6
3	q16				
4	q17				
4	q18				
4	q19	24.8	95.0	158.3	210.7
4	q20				
5	q21				
5	q22				
5	q23	24.8	95.0	158.3	200.0
5	q24				
6	q25				
6	q26				
6	q27	24.8	95.0	158.3	189.6
6	q28				
7	q29				
7	q30				
7	q31	24.8	95.0	158.3	179.3
7	q32				
8	q33				
8	q34				
8	q35	24.8	95.0	158.3	169.3
8	q36				
9	q37				
9	q38				
9	q39	24.8	95.0	158.3	159.4
9	q40				
10	q41				
10	q42				
10	q43	24.8	95.0	39.0	-
10	q44				

As shown in table 4, all available acreage in production zones Z1 (31.7 thousand acres), Z2 (95 thousand acres) and Z3 (158.3 thousand acres) are contracted with for biomass harvests. Energy crops are grown in about 20 per cent of production zone Z1 (within 15 mile radius around the plant), and retained through out the entire period; In spite of their higher yields, energy crops are planted only in production zone Z1 due to higher material costs (as mentioned above) in the base case scenario. Corn stover is harvested from all production zones – they are grown in 24.8, 95, 153.8 thousand acres in production zones Z1, Z2 and Z3 and remain at these levels through production years 1 through 9; in year 10, less requirement of feedstock reduces the need to procure from production zone Z3 and the acreage contracted falls from 158.3 thousand acres to 39 thousand acres. Corn stover procurement from zone Z4 (fields that are 45-60 mile radius from the plant) reaches its maximum in years 2 and 3 but gradually reduces to reduce costs. See figure 2. The flexibility constraints (limiting acreage available in every production zone) are binding in all but Z4. Relaxing those constraints (and increasing the area available for harvest in lands closer to cellulosic ethanol plant) can help reduce raw material cost which is explored in sensitivity analysis below (figure 5).

Figure 2: Acreage to be contracted with corn stover and energy crops in four zones - Base case scenario



The trade off between energy crops and corn stover occurs across multiple production zones in the base case scenario. Energy crops are grown in zone Z1 in spite of their higher costs due to relative cost differences in procuring corn stover from farther production zones (Z4 in this case). The implication is that multiple feedstocks do not compete only within a zone but across the production zones as well – with the trade off occurring in the form of lower procurement of (low material cost but high transport cost) corn stover from production zone Z4 and higher procurement of (high material cost but low transport cost) energy crops available in production zone Z1 close to the cellulosic ethanol plant. In effect, cellulosic ethanol plant managers can easily incorporate such management decisions by comparing and minimizing delivered costs of feedstocks.

Table 5: Optimal amounts of feedstock use and storage – Base case scenario

	Use (thousand tons)		Storage (thousand tons)	
Quarter	Energy crops	Corn stover	Energy crops	Corn stover
q1				
q2				
q3				
q4				
q5				
q6				
q7			17.1	614.2
q8	16.6	155.1		440.7
q9		169.8		257.7
q10		168.9		81.0
q11	34.3	136.1		567.0
q12		167.2		382.7
q13		166.4		204.8
q14		165.6		33.1
q15	68.5	100.8		555.8
q16		163.9		375.2
q17		163.1		200.8
q18		162.3		32.5
q19	68.5	97.6		544.8
q20		160.7		367.8
q21		159.9		196.8
q22		159.1		31.8
q23	68.5	94.4		534.0

Continued in the next page

Table 5 Continued

Quarter	Use (thousand tons)		Storage (thousand tons)	
	Energy crops	Corn stover	Energy crops	Corn stover
q24		157.5		360.5
q25		156.7		192.9
q26		156.0		31.2
q27	68.5	91.2		523.5
q28		154.4		353.4
q29		153.6		189.1
q30		152.9		30.6
q31	68.5	88.2		513.1
q32		151.4		346.4
q33		150.6		185.4
q34		149.9		30.0
q35	68.5	85.2		503.0
q36		148.4		339.5
q37		147.6		181.7
q38		146.9		29.4
q39	68.5	82.2		493.1
q40		145.4		332.8
q41		144.7		178.1
q42		144.0		28.8
q43	68.5	79.3		147.0
q44		142.6		

Note: Cellulosic ethanol production is assumed to start in year 1 – quarter 8 (q8)

The inventory storage pattern for base case scenario is given in table 5. As shown in table 5, biomass harvest starts in q7 and stored for cellulosic ethanol production that starts in q8. There are two notable patterns – (i) under optimal conditions, energy crops will not be stored because the higher amounts of ethanol yield (higher conversion efficiency) compared to corn stover (table 3). Hence, it will be efficient and economical to process the feedstock that yields higher quantity of ethanol and reduce associated losses due to biomass loss in the storage; (ii) inventory levels of stover reaches a peak during the harvest season resulting in a jump in quarters q7, q11, q15, q19, q23, q27, q31, q35, q39, and q43 – stover in inventory is gradually depleted during the lean seasons.[34]

[34] The increase in inventory gradually declines over time

There is no storage in quarter q44 as imposed by the terminal conditions since there will be no production beyond the 10-year time frame.

Table 6 describes some representative scenarios of the impacts of material, harvest and transport costs as well as yield randomness on biomass raw material costs for the hypothetical cellulosic ethanol plant. In almost all cases, the biomass raw material costs range between $0.50 and $0.60 per gallon of cellulosic ethanol (except scenario D). As expected, an increase in the material costs of one feedstock favors increased procurement of the other feedstock.[35] With an increase of corn stover costs by $4 per metric ton compare to base scenario (at $22/dry ton, corn stover costs still costs less than $25/dry ton of energy crops) there is potential for energy crop acreage to quadruple (scenario B).

When both feedstocks vary in their average yield levels (due to randomness in yields), there is an increased reliance on energy crops (scenario D). The yield randomness favors energy crops which have potential to return very high levels of biomass and decrease raw material costs in the long run. In some cases, biomass raw material costs can reach (or exceed) $1.52 per gallon of cellulosic ethanol (scenario D – column [5]) also due to uncertain levels of yields. This may be the case if energy crop yields are low and more land (farther from the cellulosic ethanol plant) has to be harvested farther away from the plant; under uncertain yield scenario, energy crops become a primary source of feedstock and can be expected to supply at least 20% of feedstock for the cellulosic ethanol plant.

[35] Increase in direct production costs or indirect environmental or other economic costs as in scenario B

Table 6: Sample results derived for various scenarios using a simpler version of optimization model#:

Scenario [1]	Delivered cost ($/dry ton) Corn Stover [2]	Delivered cost ($/dry ton) Grasses [3]	Feedstock costs Cumulative costs ($ million over 10 years of production) [4]	Feedstock costs $ per gallon of cellulosic ethanol ^ [5]	Acreage under energy crops (Proportion of energy crops in feedstock consumption over 10 years) [6]	Remarks [7]
A. Base case	18	25	280.7	0.529	6.9 thousand acres (10.1%)	Grass yield = 10 t/ac, Stover yield = 1.25 t/ac (certain yield levels); all biomass harvested in the third quarter (Jul-Sept); harvest costs = $13.96/ton, transport costs = $0.28 per ton mile
B. Higher material cost for corn stover	22	25	301.0	0.570	28.1 thousand acres (40.9 %)	Corn stover removal may increase indirect costs (worth $3.20/ton) such as soil erosion and associated loss in soil fertility
C. Higher material cost for grasses	18	30	283.5	0.534	6.1 thousand acres (9.3%)	Energy crops have to be produced exclusively and their costs can be higher since they entail significant amount of inputs in the establishment years
D. Increase in feedstock yield variability*	18	25	284.4 – 806.6	0.526 – 1.52	12.8 – 237.5 thousand acres (20.4-98%)	Yield standard deviation: energy crops 2.5 tons per acre; corn stover 0.375 tons per acre
E. Higher harvesting costs	18	25	306.3	0.578	6.9 thousand acres (10.3%)	Harvest costs change from $13.96/ton to $18.15/ton
F. Higher transport costs	18	25	301.5	0.569	16.6 thousand acres (24.7%)	Transport cost increases from $0.28 to $0.40 per ton mile

Only a few selected scenarios are presented here; * Whenever feedstock yield is treated as a random variable, optimization results in different cost values for every iteration – a representative range of low and high values are presented here; ^ Raw material cost component in cellulosic ethanol production ($/gallon)

When harvest and transport costs go up, the costs of raw material feedstocks go up as well – the interesting result is that increase in harvesting costs does not change the feedstock portfolio much (energy crops are maintained at 10% of total biomass as in the base case scenario), but an increase in transport costs will favor energy crops and increase the share of in biomass portfolio. Another common feature in all these cases is that, the proportion of energy crop biomass seems to stabilize at 10 per cent. Provided that the energy crop and corn stover prices given in base case scenario (A) stay the same, we can expect that energy crops to play a nominal role in cellulosic ethanol production – if they can be planted and harvested from farm fields that are close to the cellulosic ethanol production plant.

Following two figures show how the raw material costs (expressed in terms of dollars per gallon of cellulosic ethanol) would change with increases in harvest costs (figure 3), and transport costs (figure 4).[36] Doubling of harvest costs can increase feedstock costs by $0.17 cents per gallon of ethanol (equivalent to an increase of 32 per cent in raw material costs compared to base case scenario). Doubling of transport costs causes a nominal increase in feedstock costs by $0.07 per gallon of ethanol (or an increase of 13 per cent compared to base case scenario).

[36] All other parameters are kept at the base case scenario level

Figure 3: Impact of harvest costs ($/ton) on feedstock costs

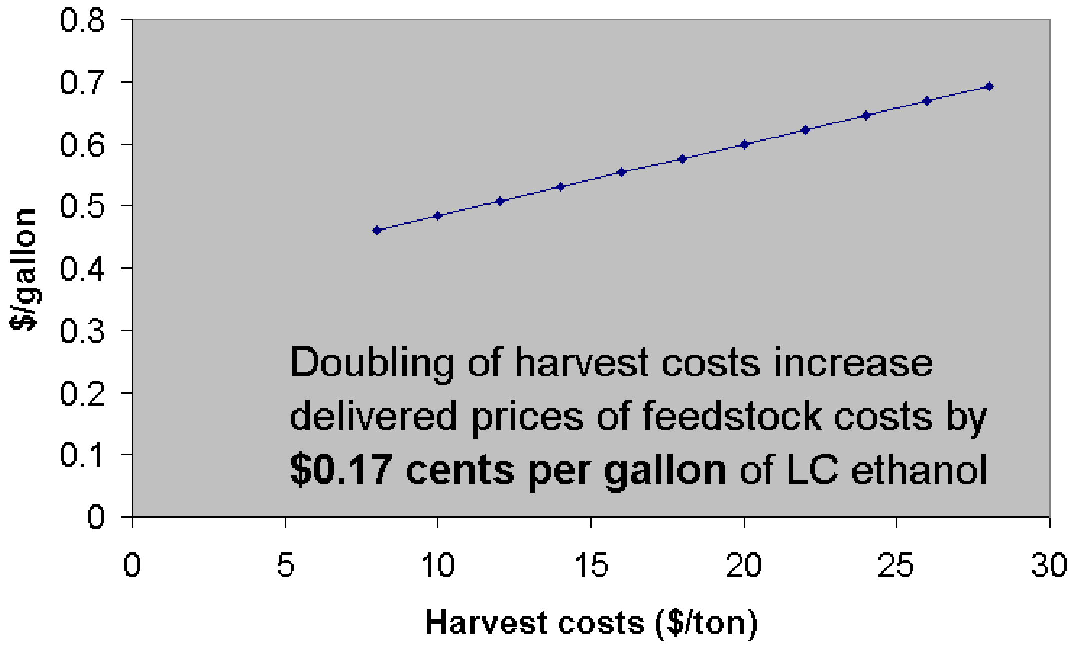


Figure 4: Impact of transport costs ($/ton-mile) on feedstock costs

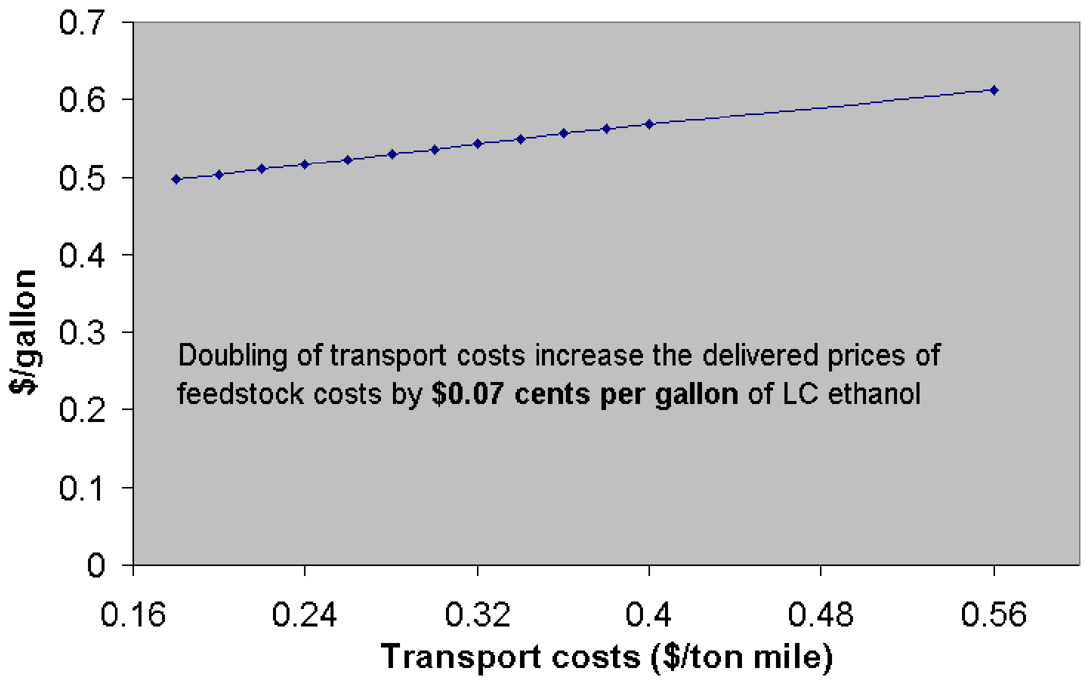
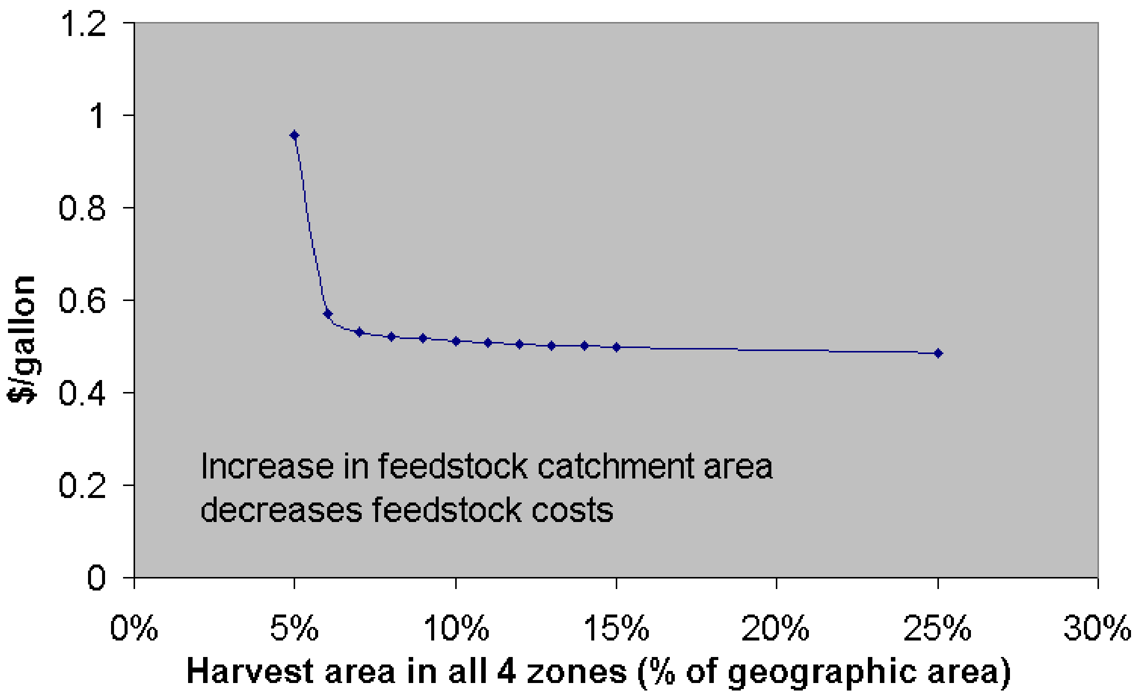


Figure 5 shows the decline in feedstock costs with an expansion in feedstock catchment area (harvests from a larger proportion of geographic area in every production zone, modeled using flexibility constraints). The increase in harvesting area seems to have a stabilizing effect around 7 per cent (scenario A) with raw material costs leveling at $0.50 per gallon of cellulosic ethanol. While, decline from this harvest area can result in rapid increase of raw material costs, expansion of area may not have much impact to reduce raw material costs further. Hence, the ethanol plant manager should concentrate on keeping the costs down by reducing the delivered costs of biomass rather than expanding the feedstock catchment area.[37]

Figure 5: Extent feedstock catchment area on feedstock costs



[37] There could also be an additional premium paid for the feedstock that is available in fields closer to cellulosic ethanol plant.

Limitations:

Many scenarios quoted above in table 3 are not discussed due to space limitations. The main idea of this manuscript is limited to the preliminary results, model features and how would those results vary depending upon assumptions.

Future Directions:

1. **Harvesting costs as a function of yield:** The density of biomass availability can be defined as the amount of biomass collected per acre (tons per unit area) in a production zone. The density of biomass availability can have a direct effect on harvest and transport costs depending on biomass yield, distance from the plant and other factors (French, 1977, French, 1960, Gallagher, et al., 2003). This function captures the higher (lower) harvesting costs (HC) when feedstocks are sparsely (densely) distributed. The presence of 'density' term introduces non-linearity in the decision variable 'acreage contracted for feedstock harvesting' resulting in a slightly complex optimization problem.

$$HC + TrnC = A_0 + A_1\, w\, \frac{2}{3} \left\{ \frac{\text{Plant Capacity}_q}{\Pi * \text{Density}_z * \text{Available biomass (tons/sq mile)}} \right\}^{\alpha}$$

$$\text{Where Density}_z = \frac{\text{Acreage contracted for harvesting feedstock f}}{\text{Geographic Area of the corresponding zone}}$$

Available biomass = Yield (tons/ac) * 640 (ac/sq mile)

w = a factor to convert distance from air distance to road distance

α = ½, the shape parameter (it may vary from 0 and 1)

A_0 = fixed costs of harvesting

A_1 = per mile transporting costs

2. **Staggered harvesting across fields:** Harvesting of biomass feedstocks was assumed to occur during the third quarter of every calendar year – if they can be harvested in a staggered manner (some fields in third quarter and the rest in fourth quarter), the harvest window gets expanded with a potential to reduce storage costs. There will be additional seasonal costs if they are harvested during different seasons of the year; the possible scenarios are given in table 7.

Table 7: Comparison of seasonal harvest scenarios

<table>
<tr><th></th><th>Scenario 1</th><th>Base case scenario</th></tr>
<tr><td>Annual corn stover</td><td>Harvest all biomass in 3rd quarter</td><td rowspan="2">Harvest all biomass in 3rd quarter</td></tr>
<tr><td>Perennial grasses</td><td>Harvest in 1st, 3rd and 4th quarter</td></tr>
</table>

3. **Strategic issues:** The above model can be expanded to answer certain strategic issues such as – will it be economical for the ethanol plant to take up biomass transport from the fields to cellulosic ethanol plant rather than letting farmers to bring it to the plant themselves? what are the impacts on costs if farmers store biomass feedstock in their fields rather than in the plant inventory? how biomass raw material costs would decline with additional plantings of energy crops during the years 1 through 5? and what are the implications of introducing another dedicated energy crop (e.g. energy cane or sweet sorghum) which can be re-planted annually?

4. An expanded analysis of scenarios B and C will help analyze the impacts of changes in feedstock relative prices and help understand the trade-off between annual and perennial energy crops.

References:

Abengoa. 2007. *Abengoa Bioenergy enters the Brazil market via the acquisition of Dedini Agro*. Available at http://www.abengoabioenergy.com/sites/bioenergy/en/acerca_de/sala_de_prensa/historico/2007/20070806_noticias.html (accessed April 2009)

Aden, A., et al. 2002. "Lignocellulosic Biomass to Ethanol Process Design and Economics Utilizing Co-Current Dilute Acid Prehydrolysis and Enzymatic Hydrolysis for Corn Stover." NREL Available at http://butanol.com/docs/2002_NREL_Lignin.pdf (accessed April 2009)

Altman, I., and T. Johnson. 2009. "Organization of the current U.S. biopower industry: A template for future bioenergy industries." *Biomass and Bioenergy* 33(5):779-784

Angelinia, L. G., et al. 2009. "Comparison of Arundo donax L. and Miscanthus x giganteus in a long-term field experiment in Central Italy: Analysis of productive characteristics and energy balance." *Biomass and Bioenergy* 33(4):635-643

Atchison, J. E., and J. R. Hettenhaus. 2003. "Innovative Methods for Corn Stover Collecting, Handling, Storing and Transporting." NREL Available at www.nrel.gov/docs/fy04osti/33893.pdf (accessed July 2006)

Bals, B. D., B. Dale, and B. Venkatesh 2005 *Separation of Protein from Switchgrass Using Aqueous Ammonia during Biomass Refining* in (ed.) 2005 AIChE Annual Meeting Cincinnati Convention Center, Cincinnati, Ohio Available at http://aiche.confex.com/aiche/2005/techprogram/P15338.HTM (accessed December 2007)

Benemann, J. R., et al. 2006 *Ethanol from lignocellulosic biomass - A techno-economic assessment* in (ed.) Biofuels and Bioenergy: Challenges and Opportunities Univ. British Columbia, Vancouver, Canada Available at http://www.mail-archive.com/sustainablelorgbiofuel@sustainablelists.org/msg70195.html (accessed March 31st, 2009)

The Hutchinson News (2008) 'Biomass' acres vital to project August 28, 2008; Available at http://www.hutchnews.com/Todaystop/bio2008-08-28T20-50-58

Blanco-Canqui, H., and R. Lal. 2009 "Corn stover removal for expanded uses reduces soil fertility and structural stability." *Soil Science Society of America Journal* 73(418-426

Brechbill, S. C., and W. E. Tyner. 2008. *The Economic of Biomass Collection, Transportation, and Supply to Indiana Cellulosic and Electric Utility Facilities - Dept of Agricultural Economics, Purdue University Working Paper #08-03*. Available at http://ageconsearch.umn.edu/bitstream/6148/2/wp080003.pdf (accessed April 2009)

Brummer, E. C., et al. 2002. "Switchgrass Production in Iowa: Economic analysis, soil suitability, and varietal performance." Bioenergy Feedstock Development Program, ORNL Available at http://www.iowaswitchgrass.com/__docs/pdf/Switchgrass%20in%20Iowa%202002.pdf (accessed

Cameron, J. B., A. Kumar, and P. C. Flynn. 2007. "The impact of feedstock cost on technology selection and optimum size." *Biomass and Bioenergy* 31(2-3):137-144

Ceres. 2007. *FAQ - How do biomass fuel yields compare to ethanol from corn starch?* Available at http://www.ceres.net/Products/Products-FAQ.html (accessed April 2009)

Ceres. 2009b. *Product Overview - Miscanthus*. Available at http://www.ceres.net/Products/Products-Miscanthus.html (accessed April 2009)

Ceres. 2009a. *Product Overview - Switchgrass*. Available at http://www.ceres.net/Products/Products-Switchgrass.html (accessed April 2009)

De La Torre Ugarte, D. G., and D. E. Ray. 2000. "Biomass and bioenergy applications of the POLYSYS modeling framework." *Biomass and Bioenergy* 18(4):291-308

High Plains Journal (2009) Study examines biomass harvesting, water quality March 16 2009; Available at http://www.hpj.com/archives/2009/mar09/mar16/Studyexaminesbiomassharvest.cfm

EERE. 2009b. *Biomass Feedstock Composition and Property Database*. Available at http://www1.eere.energy.gov/biomass/feedstock_databases.html (accessed January 2009)

EERE. 2008b. *Feedstock Logistics*. Available at http://www1.eere.energy.gov/biomass/feedstocks_logistics.html (accessed April 2009)

EERE. 2008a. *Feedstock Types*. Available at http://www1.eere.energy.gov/biomass/feedstocks_types.html (accessed January 2009)

EERE. 2009a. *Theoretical Ethanol Yield Calculator*. Available at http://www1.eere.energy.gov/biomass/ethanol_yield_calculator.html (accessed April 2009)

EISA. 2007. *Energy Independence and Security Act of 2007*. Available at http://frwebgate.access.gpo.gov/cgi-bin/getdoc.cgi?dbname=110_cong_bills&docid=f:h6enr.txt.pdf (accessed

English, B. C., et al. 2006. *25% Renewable Energy for the United States By 2025: Agricultural and Economic Impacts* Available at http://www.agpolicy.org/ppap/REPORT%2025x25.pdf (accessed March 2008)

Fales, S. L., J. R. Hess, and W. Wilhelm. 2007. *Convergence of Agriculture and Energy: II. Producing Cellulosic Biomass for Biofuels, CAST Commentary QTA2007-2*. Available at https://inlportal.inl.gov/portal/server.pt/gateway/PTARGS_0_1830_12165_0_0_18/cast_commentary_nov_2007_12145.pdf (accessed April 2009)

French, B. C. (1977) The Analysis of Productive Efficiency in Agricultural Marketing: Models, Methods, and Progress, ed. L. R. Martin, vol. I. Minneapolis, University of Minnesota Press, pp. 91-206.

French, B. C. 1960. "Some Considerations in Estimating Assembly Cost Functions for Agricultural Processing Operations." *Journal of Farm Economics* 42(4):767-778

Galbe, M., et al. (2007) Process Engineering Economics of Bioethanol Production, pp. 303-327.

Gallagher, P. W., et al. 2003. "Biomass from Crop Residues: Some Cost and Supply Estimates." Iowa State University, Department of Economics, Mar Available at http://ideas.repec.org/p/isu/genres/10240.html (accessed

GAMS. 2009. *Solver Descriptions*. Available at http://www.gams.com/solvers/solvers.htm (accessed February 2009)

Graham, R. L., et al. 2007. "Current and Potential U.S. Corn Stover Supplies." *Agron J* 99(1):1-11

HITEC. 2009. *Biofuels can provide viable, sustainable solution to reducing petroleum dependence, say Sandia researchers*. Available at http://hitectransportation.org/news/2009/Exec_Summary02-2009.pdf (accessed March 2009)

Huang, H. J., et al. 2009. "Effect of biomass species and plant size on cellulosic ethanol: A comparative process and economic analysis." *Biomass and Bioenergy* 33(2):234-246

Jensen, K., et al. 2005. "Tennessee Farmers' view of producing Switchgrass." Bio-Based Energy Analysis Group Available at www.beag.ag.utk.edu/pp/switchgrasssurvey.pdf (accessed November 2008)

Johansson, R., M. Peters, and R. House. 2007. *Regional Environment and Agriculture Programming Model (REAP)*. Available at http://www.ers.usda.gov/publications/tb1916/tb1916a.pdf (accessed April 2009)

Kumar, A., and S. Sokhansanj. 2007. "Switchgrass (Panicum vigratum, L.) delivery to a biorefinery using integrated biomass supply analysis and logistics (IBSAL) model." *Bioresource Technology* 98(5):1033-1044

Mascoma. 2009. *Consolidated Bio Processing*. Available at http://mascoma.com/technology/cbp.html (accessed April 2009)

Mendel Biotechnology. 2009. *Bioenergy Seeds - Miscanthus*. Available at http://mendelbio.com/bioenergy/index.php (accessed April 2009)

OBC. 2008. *Oklahoma set to plant first-ever 1,000 acre switchgrass field*. Available at http://okbioenergycenter.org/noble-foundation-to-plant-1000-acres-of-switchgrass-in-the-oklahoma-panhandle/ (accessed November 2008)

Perlack, R. D., et al. 2005. "Biomass as feedstock for a bioenergy and bioproducts industry: The technical feasibility of a billion-ton annual supply." USDOE, Office of Scientific & Technical Information, Oak Ridge, TN. Available at www.ornl.gov/~webworks/cppr/y2001/rpt/123021.pdf (accessed September 2008)

Perrin, R., et al. 2008. "Farm-Scale Production Cost of Switchgrass for Biomass." *Bioenergy Research* 1(1):91-97

Ray, D. E., et al. 1998a. *Chapter 6: Crop Supply Module - The POLYSYS Modeling Framework: A Documentation*. Available at http://apacweb.ag.utk.edu/poly/ch6.pdf (accessed November 30)

Ray, D. E., et al. 1998. *POLYSYS - Policy Analysis System*. Available at http://apacweb.ag.utk.edu/polysys.html (accessed January 15)

RFA. 2009. *US cellulosic ethanol projects under development and construction - January 2009*. Available at http://www.ethanolrfa.org/resource/cellulosic/documents/CurrentCellulosicEthanolProjects-January2009.pdf (accessed April 2009)

Robb, T. 2007. *Biomass derived energy Southwest KS Project*. Available at www.kcc.ks.gov/energy/kreeec/presentations/A2_Tom_Robb.ppt (accessed April 2009)

Sokhansanj, S., et al. 2009. "Large scale production, harvest and transport of switchgrass (Panicum vigratum, L.) - current technology and visioning a mature technology." *Biofuels, Bioproducts and Biorefining* 3(2):124-141
USDA - NASS. 2009. *U.S. & All States County Data - Crops*. Available at http://www.nass.usda.gov/ (accessed April 2009)
USDA - NASS. 1997. "Usual Planting and Harvesting Dates for U.S. Field Crops." Available at http://usda.mannlib.cornell.edu/usda/nass/planting/uph97.pdf (accessed April 2009)
USDA. 2009. *Crop Progress*. Available at http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1048 (accessed November 2007)
Vadas, P. A., K. H. Barnett, and D. J. Undersander. 2008. "Economics and Energy of Ethanol Production from Alfalfa, Corn, and Switchgrass in the Upper Midwest, USA." *Bioenergy Research* 1(1):44-55
Waldman, D. E., and E. J. Jensen. 2006. *Industrial Organization: Theory and Practice*: Addison-Wesley
Wallace, R., K. I. A. McAloon, and W. Yee. 2005. "Feasibility Study for Co-Locating and Integrating Ethanol Production Plants from Corn Starch and Lignocellulosic Feedstocks." USDA and US-DOE Available at http://www1.eere.energy.gov/biomass/pdfs/37092.pdf (accessed April 2009)
Walsh, M. E., et al. 2003. "The Economic Impacts of Bioenergy Crop Production on U.S. Agriculture (section 2.5)." Available at http://bioenergy.ornl.gov/papers/wagin/index.html (accessed March 2009)

Appendix – GAMS code

```
$OnText
Choice of optimum feedstock portfolio for a cellulosic ethanol plant
- A dynamic linear programming solution
Subbu Kumarappan and Rasto Ivanic

AAEA 09 Meetings
$OffText

******************************************************************************
* The decimal point is set to 1.
Option decimals=1;

$OnText
The following command ties the random number simulation to the computer clock to make it
random
* Source: SOME NOTES ON RANDOM NUMBER GENERATION WITH GAMS ERWIN
KALVELAGEN
$OffText

execseed = 1+gmillisec(jnow);

Set
F          Two feedstocks considered     /Grass, Stover/
q          time                    /q1*q44/
z          Zone the feedstock is located (within 15 30 45 and 60 miles) from the plant /z1*z4/
j(q)       subset of q                  /q1*q43/
;

Parameter
MnYld(F) Mean Yield of feedstock              /Grass 10, Stover 1.25/
*YldSD(F) Standard Deviation of feedstock yield   /Grass 2.5, Stover 0.3125/
YldSD(F) Standard Deviation of feedstock yield   /Grass 0, Stover 0/
SecHrvst Proportion of second grass harvest in the last quarter of every year /0.00/;

Parameter
Yield(F,q)  ;

$OnText
This command ensures that only the first harvest occurs in the third quarter
of every year q7, q11, q15, q19,... These commands are included within the FOR loop
to simulate a different set of yields for every iteration.

****************************************************************************
Yield('Grass',q) = Normal(MnYld('Grass'),YldSD('Grass'))$(Mod(Ord(q),4)=3);
Yield('Stover',q) = Normal(MnYld('Stover'),YldSD('Stover'))$(Mod(Ord(q),4)=3);

Yield(F,'q1')=0.00;
Yield(F,'q2')=00.00;
Yield(F,'q3')=00.00;
Yield(F,'q4')=0.000;
Yield(F,'q5')=00.00;
Yield(F,'q6')=00.00;
Yield('Grass','q7')= (1/4)* Normal(MnYld('Grass'), YldSD('Grass'));
Yield(F,'q8')=00.00;
```

```
Yield(F,'q9')=00.00;
Yield(F,'q10')=00.00;
Yield('Grass','q11') = (1/2)* Normal(MnYld('Grass'), YldSD('Grass'));

Yield('Grass',q)$(Yield('Grass',q-1) > 0) = 0.0*Yield('Grass',q-1);
*****************************************************************************


This last line adds a second harvest for Grasses in the last quarter of every year
The flexibility in harvesting a proportion of grasses in a few other months can be
important in deciding the land allocation between grasses and stover

$OffText

Scalars

Yr_BiomassReqt   Amount of biomass required per year                    /600000/


K_G           Initial quarter (q1) Grasses-to-ethanol conversion (gallons per thousand metric tons)
             /70/
r_G           Increase in ethanol yield due to future improvements (learning effect) - Expressed as
Compound "Annual" Growth Rate CAGR (can be set to zero)
             /0.005/

K_S           Grasses-to-ethanol conversion (gallons per thousand metric tons)
             /75/
r_S           Increase in ethanol yield due to learning and efficiency - C "Annual" GR (can be set
to zero)
             /0.005/


Loss          Percentage of biomass loss incurred in storage               /0.03/

InvProp       Inventory (in gallons) as percentage of plant capacity       /0.20/

pai           Value of pi                                   /3.14/

AcinSqMi      Number of Acres in one Sq Mile                         /640/

Shapefac      Non-linear Harvest Costs                            /0.5/
;


Scalar
D             Extra cost of storage in ($ per ton per "Quarter") in constant dollars /3/
;


Parameters
Quarter(q)      Assignment of counter for quarter
T             Final time period

Qu_PlntCap(q)    Quarterly Name plate capacity of plant  /q1*q7 0, q8*q44 13250000/

$OnText
```

```
Feedstock Plantation (especially Grasses) occurs in year 0 Ethanol plant construcion occurs in first three quarters
and production starts in quarter 4 of year 1 or q8 in this analysis
53 million gallons per year divided by 4 quarters = 13.25 million gallons per quarter
$OffText

rG          Rate of ethanol yield increase from Grass "over one Quarter"  derived by dividing r_G by 4 quarters
rS          Rate of ethanol yield increase from Stover "over one Quarter" derived by dividing r_S by 4 quarters

KG(q)        Ethanol yield from Grass in quarter q (gallons per ton)
KS(q)        Ethanol yield from Stover in quarter q (gallons per ton)
K_inv(q)     Average ethanol yield from feedstock in the inventory (gallons per ton)

;

Quarter(q) = Ord(q);

T = Card(q);

rG = r_G/4;
rS = r_S/4;

KG(q) = K_G * power((1+r_G),Quarter(q)-1);
KS(q) = K_S * power((1+r_S),Quarter(q)-1);
K_inv(q) = (KG(q) + KS(q)) /2;

Parameter
MinInv(q)     Minimum amount of feedstock (in terms of 'gallons of ethanol') to be kept in storage every quarter
;
MinInv(q) = InvProp*Qu_PlntCap(q);

$OnText
Above Command
Minimum Inventory in quarter q = (0.2 * 13.25 million gallon capacity)


*********************************************************************************
* Cost Calculations
*********************************************************************************

A. Material Cost Calculations
$OffText

Table
Mat_cost(F,z,q) Material cost by feedstock type and zone ($ per dry ton)
         z1.q1*q44     z2.q1*q44     z3.q1*q44     z4.q1*q44
Grass    25            25            25            25
Stover   18            18            18            18
;

* B. Seasonal Cost Calcuations

Parameter
```

```
Season_cost1(F,q) Extra costs incurred due to seasonal 1 Jan - Feb - Mar: Grass ~ U(0 0) Stover
~ U(0 0)
Season_cost2(F,q) Extra costs incurred due to seasonal 2 Apl - May - Jun: Grass ~ U(0 0) Stover
~ U(0 0)
Season_cost3(F,q) Extra costs incurred due to seasonal 3 Jul - Aug - Sep: Grass ~ U(1 2) Stover
~ U(3 4)
Season_cost4(F,q) Extra costs incurred due to seasonal 4 Oct - Nov - Dec: Grass ~ U(3 4)
Stover ~ U(5 6)

Season_cost(F,q)
;

$OnText
Grass Seasonal Costs
$OffText

Season_cost1('Grass',q) = uniform(0,0)$(Mod(Ord(q),4)=1);
Season_cost2('Grass',q) = uniform(0,0)$(Mod(Ord(q),4)=2);
Season_cost3('Grass',q) = uniform(0,0)$(Mod(Ord(q),4)=3);
Season_cost4('Grass',q) = uniform(0,0)$(Mod(Ord(q),4)=0);

$OnText
* Stover Seasonal Costs
$OffText

Season_cost1('Stover',q) = uniform(0,0)$(Mod(Ord(q),4)=1);
Season_cost2('Stover',q) = uniform(0,0)$(Mod(Ord(q),4)=2);
Season_cost3('Stover',q) = uniform(0,0)$(Mod(Ord(q),4)=3);
Season_cost4('Stover',q) = uniform(0,0)$(Mod(Ord(q),4)=0);

Season_cost(F,q) = Season_cost1(F,q) + Season_cost2(F,q) + Season_cost3(F,q) +
Season_cost4(F,q);

Display Season_cost;

$OnText
**************************************************************************
*Calculation of Harvest and Transport costs - Non Linearities enter here
**************************************************************************
*Zone related calculations
$OffText

Parameters
Radius(z)       Radius of collection zone from the plant /z1 15, z2 30, z3 45, z4 60/
Zlimit(z)       Proportion of land that could be harvested from each zone /z1*z4 .07/

$OnText
*This proportion of land is for both crops together -
*So, one crop can take up the whole area if it were dense as in case of energy crops
*or if it were cheapers as in case of stover
$OffText

CumArea(z)      Total (cumulative) land area within the radius limit
ZonalArea(z)    Area demarcated within the zone
AvaiZnAc(z)     Area available for harvesting within the zone
;
```

```
CumArea(z)  = pai * Radius(z) ** 2 * AcinSqMi;
ZonalArea(z) = CumArea(z) - CumArea(z-1);
AvaiZnAc(z) = Zlimit(z) * ZonalArea(z);

Display ZonalArea, AvaiZnAc;

Parameter
Hcst_fixed Fixed costs associated with harvesting biomass /13.96/

$OnText
* the following variable costs come from Purdue Transport cost study
* http://ageconsearch.umn.edu/bitstream/6148/2/wp080003.pdf - Wally Tyner's paper
$OffText

PerMileCst Transportation $ per mile (one-way) /0.28/

$OnText
*Trn_Cost(z) Cost of transportation
* The methods and formula from AER Report 819 will be used here
$OffText

H_T_cost(z) Total of Harvest and Transport costs

vcost(F,z,q);

$OnText
RadiusFactor(F,z,q) Factor that will be iterated - made constant to avoid non-linearity in the
optimization problem;

Table
StoverArea(z,q)    THIS HAS TO BE CHANGED ITERATIVELY
        z1          z2          z3          z4
q1*q44  10000       10000       100000      10000;

Parameter
GrassArea(z)    THIS HAS TO BE CHANGED ITERATIVELY
/z1 500, z2 250, z3 45, z4 40/;

RadiusFactor(F,z,q) = sqrt[Qu_PlntCap(q) / {(StoverArea(z,q) * Yield('Stover',q) + GrassArea(z)*
Yield('Grass',q))/ZonalArea(z)* pai}];
$OffText

H_T_cost(z) = Hcst_fixed + 2/3 * PerMileCst * Radius(z);

vcost(F,z,q) =  Mat_cost(F,z,q) + Season_cost(F,q) + H_T_cost(z);

Parameter FinalPerCost(F)  Cost of feedstock that is carried over beyond the horizon;

$OnText
*FinalCost(F).fx(q)$(ord(F,q) = card(F,q)) = Fdstkcst(F,q);
*Display Fdstkcst;
* Please ensure that my conceptualization is a good approximation by using "DISPLAY"
commands
$OffText
```

```
*****************************************************************************
Variables      vCumCost        The variable of biomass cost that has to be minimized

           vGrassAc(z)      Choice variable of Grass Acreage to be contracted in quarter 1 (year
0)
           vStoverAc(z,q)   Choice variable of Stover Acreage to be contracted in every
subsequent quarter
*          Note: the plant starts operation only in quarter 8 - so no biomass needed in the first 21
months)

           vStorage(F,q)   Choice variable of amount of biomass inventoried in every quarter

           vStoverS(q)     Supply of Stover at Quarter q
           vStoverD(q)     Demand for Stover at Quarter q
           vGrassS(q)      Supply of Grass at Quarter q
           vGrassD(q)      Demand for Grass at Quarter q


           vExcessStover_T
           vExcessGrass_T

           vTerminalcost(F,z)
;

Positive Variables

           vGrassAc(z)
           vStoverAc(z,q)

           vStorage(F,q)

           vStoverS(q)
           vStoverD(q)
           vGrassS(q)
           vGrassD(q)

           vExcessStover_T
           vExcessGrass_T
;

* Note: the prefix "e" refers to EQUATION

Equations
      eCostFunction        Function that has to be minimized - note there is no discounting as of
now
      eStoverSupply(q)      Supply Equation
      eGrassSupply(q)
      eStoverUse(q)        Demand Equation
      eGrassUse(q)

      eAvaiArea(z,q)

      eEthProdReqt(q)       Requirement to produce 53 Million gallons of ethanol per year
(except for year 1)

     eMinInventory(j)      Minimum inventory
```

```
*       eMinInventory(q)

       eTerminalcost(F,z)

*       ExcStover_T
*       ExcGrass_T

       ExcStov1
       ExcGrass1
*       eMinFdstckGrass(q)      Minimum amount of feedstocks to be derived from Stover and
Grasses
*       Terminal
;


eCostFunction..         vCumCost          =e=  Sum[(z,q), vcost("Grass",z,q) * Yield("Grass",q) *
vGrassAc(z)]
                                      + Sum[(z,q), vcost("Stover",z,q) * Yield("Stover",q) *
vStoverAc(z,q)]
                                        + Sum[(F,q),(D * vStorage(F,q))]
*                                           + [Sum(z,vTerminalcost('Stover',z))/Card(z)]*
vExcessStover_T
*                                           +  [Sum(z,vTerminalcost('Grass',z))/Card(z)]*
vExcessGrass_T
                                                                                  ;


*Supply of Stover and Grass in quarter q
eGrassSupply(q)..       vGrassS(q)                  =e= Sum(z,Yield("Grass",q) * vGrassAc(z))
;
eStoverSupply(q)..      vStoverS(q)                 =e= Sum(z,Yield("Stover",q) * vStoverAc(z,q))
;

$OnText
*Demand for Stover and Grass in quarter q
*Demand comes from two sources
*                 Current quarter use + Demand for storage EQUALS Current quarter supply +
Supply from inventory (adjusted for storage losses)
$OffText

eStoverUse(q)..         vStoverD(q)      + vStorage("Stover",q)       =e=  vStoverS(q)         +
vStorage("Stover",q-1) * (1-Loss)   ;
eGrassUse(q)..          vGrassD(q)       + vStorage("Grass",q)        =e=  vGrassS(q)          +
vStorage("Grass",q-1)  * (1-Loss)   ;

eAvaiArea(z,q)..          vGrassAc(z) + vStoverAc(z,q)              =l= AvaiZnAc(z);

$OnText
*Ethanol production capacity need to be met -- The actual amount of stover and grasses
multiplied by
*the technical coefficient on ethanol conversion (KS and KG) need to be greater than the
quarterly name plate capacity
$OffText

eEthProdReqt(q)..       Qu_PlntCap(q)    =l=         KS(q) * vStoverD(q)     +    KG(q) * vGrassD(q);
```

```
$OnText
*Note: MinInv(q) is measured in gallons of ethanol (above, 25% is assumed)
*RHS has variable amount of feedstock inventoried X in tons (by type of feedstock) which is
multiplied
*by K_inv(q) (gallons per ton) -- the average amount of ethanol yield from inventory feedstock.
* eMinInventory(q).. minimizes inventory over q1*q44 including the last period
* eMinInventory(j).. minimizes inventory over q1*q43 - the last period is said not to be inventoried.
* It seems that equation eMinInventory(q).. is better (results in a cheaper cost) than equation
eMinInventory(j)..
$OffText

 eMinInventory(j)..                    MinInv(j)     =l= vStorage('Grass',j) * KG(j) + vStorage('Stover',j) *
KS(j)                         ;
* eMinInventory(q)..                    MinInv(q)     =l= vStorage('Grass',q) * KG(q) +
vStorage('Stover',q) * KS(q)                          ;

$OnText
* Arbitrary assumption - may be dropped: a minimum of 20% biomass is forced to be derived from
Grasses
*eMinFdstckGrass(q)..     GrassProportion*Yr_BiomassReqt/4     =l=  vStoverD(q)  ;

*Terminal..       Fdstkcst('Stover',Card(q)) * [vStoverS(Card(q)) - vStoverD(Card(q))] +
Fdstkcst('Grass',Card(q))* [vGrassS(Card(q))-vGrassD(Card(q))] =l= vFinalValue ;
$OffText

*ExcStover_T..                      vExcessStover_T =e= vStoverS('q44') - vStoverD('q44') +
vStorage("Stover",'q43') ;
*ExcStover_T..                                     vStoverS('q44') - vStoverD('q44') +
vStorage("Stover",'q43') =e= 0;
*ExcGrass_T..                      vExcessGrass_T =e=  vGrassS('q44') - vGrassD('q44') +
vStorage("Grass",'q43')   ;
*ExcGrass_T..                                     vGrassS('q44') - vGrassD('q44') + vStorage("Grass",'q43')
=e= 0  ;

ExcStov1..      vStorage("Stover",'q44') =e=     0;
ExcGrass1..      vStorage("Grass",'q44') =e=     0;

eTerminalcost(F,z)..       vTerminalcost(F,z)=e= vcost(F,z,'q44')

Model SimpleModel /all/
;

Option nlp=minos;
Option lp = osi;

Parameter
GrassT          Total amount of grass consumed in the ethanol plant operations
StoverT         Total amount of stover consumed in the ethanol plant operations
GrassProp        Proportion of feedstock derived from grasses;

Scalar i ;

For (i = 1 to 1,

Yield('Grass',q) = Normal(MnYld('Grass'),YldSD('Grass'))$(Mod(Ord(q),4)=3);
Yield('Stover',q) = Normal(MnYld('Stover'),YldSD('Stover'))$(Mod(Ord(q),4)=3);
```

```
Yield(F,'q1')=0.00;
Yield(F,'q2')=00.00;
Yield(F,'q3')=00.00;
Yield(F,'q4')=0.000;
Yield(F,'q5')=00.00;
Yield(F,'q6')=00.00;
Yield('Grass','q7')= (1/4)* Normal(MnYld('Grass'), YldSD('Grass'));
Yield(F,'q8')=00.00;
Yield(F,'q9')=00.00;
Yield(F,'q10')=00.00;
Yield('Grass','q11') = (1/2)* Normal(MnYld('Grass'), YldSD('Grass'));

Solve SimpleModel minimizing vCumCost using nlp ;



GrassT = Sum[q,vGrassD.l(q)];
StoverT = Sum[q,vStoverD.l(q)];
GrassProp = 100*GrassT/(GrassT+StoverT);

Display   vGrassAc.l, vStoverAc.l, GrassProp;
) ;
```