# Measures of Brand Loyalty 

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#### Abstract

Though brand loyalty has been studied extensively in the marketing literature, the relationship between brand loyalty and equilibrium pricing strategies is not well understood. Designing sales pricing strategies involves two key decisions: the percentage reduction in price from the existing price point, and the number or frequency of promotions within a category or for a specific product. These decisions, in turn, are critically dependent upon how many consumers can be convinced to switch to a brand by temporarily reducing its price, and how many are instead brand loyal. Theoretical models of how the size and strength of brand loyalty influence optimal promotion strategies have been developed, but there are no rigorous tests of their hypotheses. We test how brand loyalty impacts promotion strategies for a frequently purchased consumer package good category. Our results largely confirm that retailers often promote many brands simultaneously and that depth and breadth can be complementary.


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## 1 Introduction

Recent estimates indicate that consumer packaged good manufacturers allocate fully $58 \%$ of their marketing expenditure toward sales promotion (Low and Mohr, 2000). Despite the popularity of retail price promotions, why supermarket retailers periodically offer products at discounted prices is not well understood, nor how promotions can exist in a competitive equilibrium given the law of one price. Explaining how price promotions work, therefore, is critically important in understanding competitive retail equilibria and the retailing function more generally. Price promotions are typically measured along two dimensions: their depth, or the percentage discount from a typical price, and their frequency, which is defined as the number of times a product is promoted during a given week. Consequently, this study seeks to explain the frequency and depth of retail price promotions using a frequently purchased food item as a case study. We find that some common assumptions of some theoretical models that explain retail price promotion do not hold within the food category we investigate.

There are a number of studies that try to explain why retailers use sales, each of which considers the competitive interactions of retailers or manufacturers. Varian (1980) develops a retailer model in which sales are the result of a mixed strategy equilibrium brought on by retailers competing over segments of the market that are relatively informed or uninformed. He argues that price promotions are used by retailers trying to capture the informed consumer's business. However, his study only considers a single product, while retailing is inherently multi-product. Pessendorfer (2002) argues that consumer demand increases over time as inventories are drawn down and, consequently, retailers may use a form of intertemporal price discrimination through retail promotions to generate more profits. He empirically investigates a consumer packaged goods category and concludes that the predicted probability of a sale increases in the time since the last one, thus confirming his central hypothesis. However, intertemporal demand is not an issue for most food products as they are at least somewhat perishable. Loss leadership has also been proposed as a possible rationale for price promotions (Hess and Gerstner, 1987; Walters, and McKenzie, 1988; Bliss, 1988; Epstein, 1998; and Hosken and Reiffen, 2001). These studies investigate the effect that promoting one product - even below cost - has on overall sales. However, these studies often do not include manufacturer competition within their model, neglecting the effect trade deals have
on a retailer's incentive to promote a product. Others extend this research to include the competitive interaction among retailers (Chintagunta, 2002; and Richards, 2006). Measuring promotion as both the depth and breadth of a price change, Richards (2006) finds that promotions will likely have their greatest impact on in-store product share, but promotions can increase store share if consumers regard the retailers has highly substitutable. However he looks at profits among categories, which may not extend to products within a category. Common among these studies is the notion that retailers can exploit some form of segmentation strategy, whether identifying informed and uninformed consumers, high valuation and low valuation groups, or brand loyalty and non-loyal buyers, to price discriminate across consumers. The specific mechanisms by which they do so, however, is a matter for further research.

Recognizing that some consumers have a preference for one brand over another, researchers in marketing tend to focus on brand loyalty to explain price promotion as an equilibrium outcome. Raju, Srinivasan, and Lal (1990) develop a theoretical model that explains how differences in loyalty leads to variations in the depth and frequency of price promotions offered by brands in the same product category. ${ }^{1}$ Agrawal (1996) extends this theoretical model to examine how media advertising and promotion interact for manufacturers of consumer packaged goods. His model suggests that a retailer would offer deeper discounts for brands with little brand loyalty and promote them less often compared to a brand with more loyal consumers. Similar to Agrawal (1996), Jing, and Wen (2008) develop a model that assumes consumers will switch to a preferred brand given a sufficient price discount, but also include the possibility of a consumer segment that is completely price sensitive. They find that the equilibrium promotional strategy depends critically on the brand strength and the number of price sensitive consumers. However, these studies do not consider competitive interactions among retailers. Lal and Villas-Boas (1998) extend these theoretical models further by including either loyalty to a retailer, a brand, both a retailer and brand, or neither. Their model suggests that when retailer loyalty is introduced, the promotional equilibrium allows for the possibility of promoting more than one brand. All of these studies assume, however, that loyalty is exogenous to the promotion strategy.

Among empirical studies, Huang, Perloff, and Villas-Boas (2006) challenge this assump-

[^1]tion and find that promotional frequency effects the share of loyal consumers so that, indeed, loyalty is endogenous. They investigate the relationship between retailer promotions and customer choice across brands within a specific consumer packaged goods category, concluding that very little brand loyalty exists. Their findings suggest, at least within the consumer packaged good category they investigated, that brand loyalty likely cannot constitute an explanation for equilibrium promotional strategies. However, their result depends critically on their definition of loyalty. Loyalty can be defined as either a time-series method of previous purchase patterns, or as the discount needed to persuade a loyal consumer to switch to a less preferred brand, often referred to as the 'money-metric' measure. The measure of repeat purchase assumes that past purchase decisions will be an indicator of future purchasing decision (Knox and Walker, 2001). Though this measure is able to capture the behavioral aspect of brand loyalty, it does not take into account the attitudinal aspects. Consequently our empirical analysis of brand loyalty follows Pessemier (1959) and operationalizes it as the price differential needed to persuade a consumer who prefers one brand, switch to a less preferred brand. This 'money metric' measure is able to capture both the attitudinal and behavioral elements and is commonly used in theoretical studies on equilibrium promotion to operationalize the measure of brand loyalty.

Most of the theoretical studies on equilibrium promotion revolve around a duopoly assumption, which only holds in a limited number of applications. This assumption does not allow for complementarity that can exist among products in a retail environment - complementary that can drive sales on multiple products. Furthermore, many models are built upon a duopoly assumption which may or may not extend to categories which have a large number of differentiated products. Our empirical study will relax the overly restrictive duopoly framework in developing a new empirical model of promotion strategy.

Though brand loyalty has been studied extensively in the marketing literature, the relationship between brand loyalty and equilibrium pricing strategies is not well understood. Retailer's promotional decisions are critically dependent upon how many consumers can be convinced to switch to a brand by temporarily reducing its price, and how many are instead brand loyal. Theoretical models of how the size and strength of brand loyalty influence optimal promotion strategies have been developed, but there are no rigorous tests of their hypotheses. As a result, the objective of this research is to empirically determine how brand
loyalty, measured by the size and strength of loyal cohorts, influences equilibrium promotional strategy for differentiated products by multi-product retailers.

Constructing an empirical test, however, first requires a clear theoretical framework to derive equilibrium promotional strategies. Building on theoretical studies of brand loyalty and promotion, we develop a theoretical model that describes the relationship between brand loyalty and promotion in multi-product retailer equilibria. Our empirical analysis focuses on the ready to eat (RTE) cereal category which is an ideal case study for a number of reasons. First, the RTE cereal industry is highly concentrated at the manufacturing level. Despite being highly concentrated, Nevo (2001) and Norman, Pepall, and Richards (2005) conclude that the high price cost margins are generally attributed to product differentiation and multi-product firm pricing. Second, cereal manufacturers often introduce new brands into a highly differentiated market (Hausman, 1997), attempting to win consumers loyal to a competing brand. High concentration and strong brands lead to fierce brand rivalry in both prices and product development. These attributes make the industry well-suited to empirically determining how brand loyalty influences equilibrium promotional strategies by retailers and manufacturers.

The remainder of the paper is organized as follows. In the next section, we describe the RTE cereal industry and why we focus on this category for the empirical analysis. The third section develops the theoretical and empirical models of brand loyalty. A fourth describes the econometric methods used to estimate the models developed in the third section, while the fifth section describes the data used to estimate the model. The results are presented in the sixth section, while the final section concludes, describes the implications for the study of the interaction among brand loyalty and promotional equilibrium, and provides suggestions for future work in the area.

## 2 The Ready to Eat Cereal Industry

The RTE cereal industry has many desirable characteristics that fit well with the purposes of this study. First, the industry has seen relatively stagnant growth in overall consumption for the past several years. With stagnant growth, firms will have to actively promote their products. Second, cereal manufacturers produce a wide variety of highly differentiated products. Third, RTE cereal products are frequently purchased during regular shopping
trips which allows for a larger role for loyal purchases. Finally, the RTE cereal industry is largely dominated by three manufacturers, with some private label products sold at the retail level.

The RTE cereal industry is characterized with a number of differentiated products characterized with fierce brand rivalry. The RTE cereal industry, like the food processing industry in general, has few manufacturers each selling multiple brands, making it a concentrated differentiated-products market. Within the U.S. Kellogg, General Mills, and Post Cereal accounted for $70.9,68.0$, and $68.1 \%$ of the RTE Cereal category sales for 2004, 2005, and 2006 respectively. Furthermore, private label sales account for $14.5,15.6$, and $15.4 \%$ of the cereal category sales for 2004, 2005, and 2006 respectively (Source: A.C. Nielson Homescan data). Despite being highly concentrated, Nevo (2001) and Norman, Pepall, and Richards (2005) conclude the high price cost margins are generally attributed to product differentiation and multi-product firm pricing. This suggests that brand loyalty likely exists with the industry.

Consumption levels of RTE cereal have remained fairly constant over the last several years causing the manufacturers to compete over marketshare, thereby making brand loyalty all the more important. A.C. Nielsen Homescan data suggests that the total consumption of RTE cereal in 2006 was just under 3.1 billion pounds for a total of $\$ 7.9$ billion in sales which is very similar to 1995 estimates. General Mills and Post have been able to maintain their marketshare since the early 1990's (Nevo, 2001), currently hovering around $22 \%$ and $12 \%$ respectively. During that time the private label producers were capturing about $7 \%$ of the market, but have since been able to gain twice as much market share coming in at about $15 \%$. This has been much to the disappointment of Kellogg who has seen their market share drop from the early 1990 levels of about $35 \%$ to a current level of $25 \%$. From 2004 to 2006 Quaker Oaks (owned by PepsiCo) and Malt-O-Meal had just over $5 \%$ of the market. All other cereal manufacturers account for less than half a percent of the marketshare. This high concentration of the industry in general, and fierce struggle for marketshare has caused prices to decrease over the years.

With consumption remaining relatively constant and new brands being continually introduced in an effort to gain marketshare, the industry has seen the retail price of RTE cereals decrease over the years. The average price of all RTE cereals dropped almost $20 \%$ since 2000 levels when adjusted for inflation. From table 1 we can see private label prices have
decreased $13 \%$ over the same time period while on average remaining almost $35 \%$ cheaper than the overall average. Such a large discount has probably contributed to the reduction in the price of all other cereal brands. We can also see from table 1 that though General Mill's brands have undergone a similar price decrease over time, they have still been able to maintain generally higher prices compared to the overall average. Except for Kellogg's Rice Krispy, Kellogg's brands are generally priced lower than the overall average. If we look closely at Kellogg's Frosted Mini-Wheat we can see that the brand was losing market share through 2003 until it lowered the price, at which point it began gaining marketshare back. However, a similar pricing strategy was undertaken by Kellogg's Frosted Flakes and it still lost market share through 2006. Lastly, we can see that though Post's brands have followed a similar decrease in price over the past several years when adjusted for inflation, and most of the brands have been able to maintain a steady market share.

Insert Marketshare Table Here

The downward pressure in prices is likely the result of the RTE cereal industry being dominated by three manufacturers each competing over marketshare. This competition for marketshare makes brand loyalty all the more important. Despite this downward pressure, the industry still has a vast range of prices among the different products and their manufacturers. This variety in prices along with being a frequently purchased product group allows for a better understanding of consumer's preferences. These attributes makes the industry ideal for empirically determining how brand loyalty changes retailer's equilibrium promotional strategy for differentiated products.

## 3 Theoretical Brand Loyalty Model

In this section, we develop a theoretical model of price promotions in which brand loyalty plays an important role. We then develop a model to empirically test the hypotheses implied by the theoretical model. In general, designing sales promotion strategies involves two key decisions: (1) the depth of the promotion, or the percentage reduction in price from the existing price point, and (2) the breadth, meaning the number or frequency of promotions within a category, or for a specific product. Several studies have sought to explain the promotional phenomenon, of which brand loyalty has played a significant role. However,
theoretical models of brand loyalty provide mixed results as to their role in framing retailers' promotional strategies. Furthermore, debate still remains as to the way in which brand loyalty can be defined, operationalized, and applied to the retail setting.

How a retailer defines brand loyalty, or loyalty more generally, can affect their decisions regarding the depth and frequency of promotions in a fundamental way. How they define loyalty is reflected in the measure they use to assess whether a consumer is likely to purchase again, or switch to another brand. Inaccurate measures of brand loyalty could result in promoting products either too deeply, or too frequently, or not deeply or frequently enough, which will erode a retailer's category and store profits. Studies of brand loyalty often follow Pessemier (1959) and define loyalty as the price differential needed to make a consumer who prefers one brand switch to a less preferred brand. This concept is able to capture both the strength and "size" of a loyal segment. Strength is often thought of as the intensity of a consumer's loyalty towards a brand, and size being the number of consumers in the loyal cohort. This measure also captures both the attitudinal and behavioral elements of brand loyalty as it assumes that the consumer has some preference for a brand (attitudinal) and then measures the degree of preference by observed purchasing incidents over time (behavioral). A price-based measure of loyalty is in contrast to a temporal measure of loyalty which uses repeat purchase behavior to measure brand loyalty. Repeat-purchase measures assume that past purchase decisions will be an indicator of future purchasing decision (Knox and Walker, 2001). Though these measures are able to capture the behavioral aspect of brand loyalty, they are not able to account for the attitudinal. As a result, in our theoretical model we operationalize brand loyalty as the discount needed to make a consumer who prefers one brand, switch to another. This measure is the same measure used by Agrawal (1996) and Villas-Boas and Lal (1998), while Guadagni and Little (1983) and Bucklin and Lattin (1991) are examples of studies that use a repeat-purchase measure.

Theoretical studies of brand loyalty generally model a duopoly market where brands compete for loyal cohorts of consumers. Assumptions regarding loyalty range from models that assume all consumers prefer a brand (Raju, Srinivasan, and Lal 1990; Agrawal 1996; and Lal and Villas-Boas 1998) to others that assume market shares are driven by the cheapest brand (Narasimhan 1988; Knox and Walker 2001; and Jing, and Wen 2008). Furthermore, studies vary on the level of competition assumed. In general, as the theoretical literature
has progressed researchers have built upon previous models to include deeper levels of competition, or relaxing some fundamental assumptions of the model. For example, Lal and Villas-Boas (1998) extend Agrawal (1996) by modeling both retailer and manufacturer competition. These studies find that market structure assumption plays an important role in the final promotional equilibrium of the retailer. However, rigorous empirical tests have yet to be conducted, even on the most basic models used in each theoretical study. As a result, we develop a theoretical model of brand loyalty following Raju, Srinivasan, and Lal (1990) and Agrawal (1996) that describes the influence brand loyalty has on price-promotions in a multi-product retailer equilibrium. Since our objective is to empirically determine how brand loyalty influences retailer's equilibrium promotional strategy for differentiated products, we focus on the hypotheses implied by these simpler models, and leave tests of more comprehensive models to future empirical work.

### 3.1 Model

Our model consists of two national brands: $s$ and $w$. We assume that the two brands are sold nationally in a market with $M$ consumers. We let the market consist of consumers who have a preference for one brand over the other, but can be persuaded to switch to the less preferred brand if a sufficient discount is given. We assume that the proportion of consumers loyal to brand $s$ is $m_{s}$ and the proportion loyal to brand $w$ is $m_{w}$, such that $m_{s}+m_{w}=1$. Furthermore, for those consumers loyal to brand $s$ we assume a reservation price $r$ for brand $s$ and a lower reservation price for brand $w$ of $\left(r-l_{s}\right)$. Similarly we assume a reservation price for those consumers loyal to brand $w$ of $r$ and a lower reservation price of $\left(r-l_{w}\right)$ for brand $s$. We assume that it takes a larger price differential to persuade a customer loyal to $s$ to switch to $w$ such that $0<l_{w} \leq l_{s}$. In general we assume that promotions only affect brand shares and not the category volume sold. Thus, each consumer is assumed to buy one unit of a brand per shopping trip such that utility is maximized. So the demand function for brand $s$ and $w$ can be seen below (Agrawal, 1996):

$$
\begin{aligned}
D_{s}\left(p_{s}, p_{w}\right) & = \begin{cases}M & \text { if } p_{s}<p_{w}-l_{w} \\
M m_{s} & \text { if } p_{s}-l_{s} \leq p_{w} \leq p_{s}-l_{w} \quad \text { and } \\
0 & \text { if } p_{w}<p_{s}-l_{s}\end{cases} \\
D_{w}\left(p_{s}, p_{w}\right) & = \begin{cases}M & \text { if } p_{w}<p_{s}-l_{s} \\
M m_{w} & \text { if } p_{w}-l_{w} \leq p_{s} \leq p_{w}-l_{s} \\
0 & \text { if } p_{s}<p_{w}-l_{w}\end{cases}
\end{aligned}
$$

where $p_{s}$ and $p_{w}$ represent the retail price of the brands $s$ and $w$ respectively. The above demand specification states that if brand $i \in\{s, w\}, i \neq j$ is discounted below the retail price of $j$ minus the loyalty factor $l_{j}$ then the consumers loyal to brand $i$ will switch to brand $j$. If however, the price of brand $i$ is lower than the price of brand $j$, but not lower than the price of brand $j$ minus the loyalty factor of $l_{j}$ then the consumers loyal to brand $i$ will stick with brand $i$.

While manufacturers simultaneously declare wholesale prices $w_{s}$ and $w_{w}$ for their respective brands, we assume they don't set wholesale prices strategically. After observing the wholesale prices, the retailer strategically sets retail prices $p_{s}$ and $p_{w}$ for the two brands so as to maximize category profits. We assume that the marginal cost of production and distribution for the two manufacturers, and the marginal cost of retailing are zero.

### 3.1.1 Retailer Pricing Strategy

The retailer's profit function is represented below:

$$
\Pi^{R}= \begin{cases}\left(p_{s}-w_{s}\right) M & \text { if } p_{s}<p_{w}-l_{w} \leq r \\ m_{s}\left(p_{s}-w_{s}\right) M+m_{w}\left(p_{w}-w_{w}\right) M & \text { if } p_{s}-l_{s} \leq p_{w} \leq p_{s}-l_{w} \leq r \\ \left(p_{w}-w_{w}\right) M & \text { if } p_{w}<p_{s}-l_{s} \leq r\end{cases}
$$

The retailer's profit function above allows for three distinct possibilities with which profits can be maximized. If the stronger brand is cheaper than the difference between the weaker brand price and the consumer's loyalty factor then the retailer will promote the weaker brand and sell it to both segments. If the weaker brand's wholesale price is not sufficiently lower than the strong brand then the retailer would simply sell both brands to the loyal segments. If however the weaker brand is sufficiently cheaper as to capture the loyal consumers of the stronger brand, then the retailer would discount the weaker brand and sell it the entire population segment. It can be shown that in equilibrium the optimal pricing strategy for the retail is as follows (Agrawal, 1996):

- if $w_{w}>w_{s}+\left(1 / m_{w}\right) l_{w}$ then charge $p_{s}=r-l_{w}-\epsilon, p_{w}=r$;
- if $w_{s}+\left(1 / m_{w}\right) l_{w} \leq w_{w} \leq w_{s}+\left(1 / m_{w}\right) l_{w}$ then charge $p_{s}=r$ and $p_{w}=r$; and finally
- if $w_{s}>w_{w}+\left(1 / m_{s}\right) l_{s}$ then charge $p_{w}=r-l_{s}-\epsilon, p_{s}=r$.

It is important to note that the retailer is maximizing profits for the overall category. As a result they always have the option to sell both brands at the reservation price to the corresponding loyal segments of the market. The retailer only discounts one of the brands if the wholesale price offered for that brand is low enough to warrant a higher profit by selling the one brand at a discounted rate to the entire market.

This equilibrium pricing strategy leads to several hypotheses about the frequency and depth of discounts offered by a retailer. First, deeper discounts are given by the retailer for the weaker brand in comparison to the stronger brand. This this result follows directly from the optimal pricing strategy above because the weaker brand has to offer a larger discount $l_{s}$ in order to attract the stronger brand's loyal customers and $0<l_{w} \leq l_{s}$. In other words,

- $\mathrm{H}_{1}$ : the average price discount at the retail level is negatively related to the strength of brand loyalty.

In equilibrium the retailer promotes the stronger brand with a probability: $\operatorname{Pr}\left[w_{w}>\right.$ $\left.w_{s}+\left(1 / m_{w}\right) l_{w}\right]$, and promotes the weaker brand with probability: $\operatorname{Pr}\left[w_{s}>w_{w}+\left(1 / m_{s}\right) l_{s}\right]$. Given the asymmetry in loyalties $\left(l_{w} \leq m_{w}^{2} r\right.$ and $\left.l_{s} \geq m_{s}\left(r-l_{w}\right)\left(1+m_{s} m_{w}\right)^{-1}\right)$ (Raju, Srinivasan, and Lal, 1990) it follows that the probability of the weaker brand obtaining the consumers loyal to the stronger brand is zero, which suggests that the retailer will promote the stronger brand more frequently. In other words,

- $\mathrm{H}_{2}$ : the frequency of price promotions at the retail level is positively related to the strength of brand loyalty.

Since these hypotheses follow directly from the model described above, their support or rejection will give valuable insight into the underlying assumptions of our theoretical model. For example $\mathrm{H}_{1}$ will be able to shed light on the validity of the underlying assumption of the measure of brand loyalty. If $\mathrm{H}_{1}$ is rejected then our definition and distinction of the stronger or weaker brand would be fundamentally flawed and our operationalization of brand loyalty
should be reconsidered. Similarly, rejection of $\mathrm{H}_{2}$ may indicate that retailers use promotions strategically to compete with other retailers even at the product level as suggested by Lal and Villas-Boas (1998). In the section that follows we develop an empirical model of consumer demand that provides household-level estimates of brand preference. These estimates will then be used to test $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$.

## 4 Empirical Model of Brand Loyalty

In this section we develop an empirical model of brand loyalty designed to test the hypotheses above. We estimate brand loyalty using a discrete choice approach, as opposed to a representative consumer demand model. Representative consumer demand models assume a consumer is, on the whole, average, and thus consumes a small amount of every brand in the category (Anderson, de Palma and Thisse, 1992). By assuming that the market consists of millions of representative, utility maximizing consumers, representative consumer demand models try to explain how demand is likely to change if prices, income, or some other dependent variable changes on the assumption that everyone is alike and has the same preferences. Examples of representative consumer demand models include the Constant Elasticity of Substitution (CES) (Dixit and Stiglitz, 1977), and the Almost Ideal Demand System (AIDS) (Deaton and Muellbauer, 1980).

Representative consumer models, however, are generally written as systems of demand equations, in which the interrelated nature of consumer demand is explicitly recognized. A systems approach to demand estimation provides consistently better empirical results that are more consistent with the underlying theory of constrained utility maximization elementary economic theory than single-equation alternatives. But, in a differentiated-product environment, the assumption that consumers buy every product is not tenable. Rather, when brands are differentiated, consumers prefer different brands given they perceive product attributes differently. Representative consumer demand models do not take into account this consumer heterogeneity.

As an alternative to representative consumer demand models, Luce (1959) and McFadden (1973) developed the logic that underlies discrete choice models. In a discrete choice framework, consumers are assumed to buy the one brand that yields the greatest utility over all other choices. Utility, however, is randomly distributed and depends upon the distribution
of unobserved consumer preferences, or unobserved heterogeneity. By specifying a functional form for the unobserved consumer heterogeneity, discrete choice models are able to more readily capture why consumers buy particular brands. However, the parameters in the utility function are assumed to be constant over consumers in the simple logit specification. This assumption is convenient as a closed form demand equations results. Nonetheless, the simple logit is subject to the independence of irrelevant alternatives (IIA) property, which means that substitution patterns are based on market share instead of fundamental attributes of the brand. In order to fully accommodate household heterogeneity we allow the parameters to vary randomly, which causes substitution between brands to be based on brand attributes instead of market share. This specification is not without cost, as the mixed logit no longer yields a closed form demand equation. However, estimation of the closed form demand equation is not necessary to empirically estimate the strength of loyalty for each product, nor the size of the loyal segment. We describe the specific model we use for this purpose in the next section.

### 4.1 Consumer Demand

Breakfast cereals are highly differentiated food products. Consequently, we model their demand using a discrete choice approach (Anderson, de Palma and Thisse, 1992; Jain, Vilcassim, and Chintagunta 1994; Nevo, 2001). Of all possible discrete choice models, we model consumer demand with a random coefficient logit model (RCL). Compared to the simple, and nested logit forms, the RCL has several advantages. First, the partial derivatives of the brand choice probability will not be determined by a single parameter as in the logit and nested logit models. Instead, each household in the sample will have a different price sensitivity, which is important to the objectives of this study. Namely, the price response parameter differs among brands, which provides an appropriate measure of the brand's strength. Second, the RCL model allows for flexible substitution patterns, unlike the simple and nested alternatives (within nests). Consumer heterogeneity is allowed to enter the model through a composite random shock term that is dependent on product and individual characteristics. Thus, if the price of a brand goes up, households are more likely to switch to a brand with similar characteristics, as opposed to the one that, on average, is most often purchased. Third, and because of this feature, households with similar characteristics will tend to have similar
purchasing patterns.
Formally, we assume that a sample of $H$ households $h \in\{1,2, \ldots, H\}$ make a purchase among $N$ brands $b \in\{1,2, \ldots, N\}$ on purchasing occasion $t \in\{1, \ldots, T\}$. Then the conditional indirect utility of household $h$ for alternative $b$ on purchasing occasion $t$ can be written as:

$$
\begin{equation*}
u_{h b t}=\gamma_{h b}+\boldsymbol{\eta}^{\prime} \mathbf{x}_{h b t}-\alpha_{h b} p_{h b t}+\xi_{h b t}+\epsilon_{h b t} \tag{1}
\end{equation*}
$$

where $\xi_{h b t}$ is an error term that accounts for all product-specific variations in demand that are unobserved by the econometrician. ${ }^{2}$ Brand $b$ 's price is represented by $p_{h b t}$, and $\gamma_{h b}$ captures the intrinsic preference of household $h$ for brand $b, \eta$ is a $k$-dimensional vector of parameters and $x_{h b t}$ is a vector of attributes of the household and the brand, which includes variables such as household income, an indicator of whether the product is offered on a temporary discount $\left(d c_{b t}\right)$, and an interaction term between the retail price and the discount ( $d c_{b t} p_{b t}$ ) (Chintagunta, 2002; Richards, 2006). By including an interaction term we allow for the possibility that promotion rotates the demand curve in addition to the expected demandshifting effect. In this way, we allow items on promotion to become less elastic if households perceive discounting as a means of differentiating otherwise similar products. These explanatory variables, and their distribution over households, govern the substitution patterns so that products with similar characteristics will be closer substitutes. Furthermore, we assume that the error term $\epsilon_{h b t}$ is an i.i.d. type I extreme value error term that accounts for household-specific heterogeneity in preferences. With this error assumption, the utility specification in (1) implies a discrete choice logit demand model.

It is well understood that a simple logit model suffers from the IIA property. The RCL model does not have this attribute because the correlation between random parameter variation and attributes included in the demand model introduces a degree of curvature that the simple logit lacks. In the RCL, we allow the product-preference and marginal utility of income parameters in (1) to vary over consumers in a random way (BLP, 1995; Nevo, 2001). Specifically, the marginal utility of income is normally distributed over households such that:

$$
\begin{equation*}
\alpha_{h b}=\alpha+\sigma_{\alpha} \mu_{h b}, \mu_{h b} \sim N(0,1) \tag{2}
\end{equation*}
$$

[^2]where $\alpha$ is the mean price response across all consumers and $\mu_{h b}$ is the household-specific variation in response with parameter $\sigma_{\alpha}$. We can decompose $\eta^{\prime}$ into $\left\{\eta_{1}, \ldots, \eta_{L}, \eta_{L+1} \ldots, \eta_{k}\right\}$ where the first $L$ parameters are the random parameters, and the $L+1$ parameters represent the non random parameters, such as the discount indicator and discount interaction term. We decompose $\mathbf{x}_{h b t}$ in a similar way. Households, therefore, are assumed to differ in their attribute preferences such that unobserved consumer heterogeneity is reflected in the distribution of each brand's marginal utility in the following way:
\[

$$
\begin{equation*}
\eta_{h l}=\eta_{l}+\rho_{\eta} z_{h l}+\sigma_{\eta} v_{h l}, v_{h l} \sim N(0,1) l=1, \ldots, L \tag{3}
\end{equation*}
$$

\]

where $\eta_{l}$ represents the fixed constant terms in the means of the distributions for the random parameters, $z_{h l}$ represents a set of observed variables which do not vary over time and enter the means of the random parameters, $\rho_{\eta}$ is the set of coefficients that form the observation specific term in the mean, $v_{h l}$ is the random term representing the household's unobserved heterogeneity and $\sigma_{\eta}$ is the coefficient of the household's unobserved heterogeneity. McFadden and Train (2000) interpret the elements of (3) in terms of an error-components model of attribute demand. In contrast to the IIA property of a single logit model, the heterogeneity assumption in (3) creates a general pattern of substitution over $N$ alternatives through the unobserved, random part of the utility function given in (1). As a result, the utility from different brands is correlated according to the set of attributes included in $x_{h b t}$.

The mixed logit model introduces a large number of parameters relative to the simple logit model. Therefore, we follow Nevo (2001), among others, and write the indirect utility function in terms of two sets of variables - those that are assumed to be random and those that are not:

$$
\begin{equation*}
u_{h b t}=\delta_{h b t}\left(p_{h b t}, \mathbf{x}_{h b t}, \xi_{h b t} ; \gamma_{h b}, \alpha_{h b}, \boldsymbol{\eta}_{h l}\right)+\tau_{h b t}\left(p_{h b t}, \mu_{h b}, v_{h l} ; \sigma_{\alpha}, \rho_{\eta}, \sigma_{\eta}\right)+\epsilon_{h b t} \tag{4}
\end{equation*}
$$

where $\delta_{h b t}$ is the mean utility level that varies over products, but not households, and $\tau_{h b t}$ is the idiosyncratic part that varies by household and product. For convenience let $\zeta_{h b t}=$ $\gamma_{h b}+\boldsymbol{\eta}^{\prime} \mathbf{x}_{h b t}-\alpha_{h b} p_{h b t}+\xi_{h b t}$. With this, the probability that household $h$ chooses brand $b$ at purchasing occasion $t$ conditional on $\boldsymbol{\eta}_{h l}$, and $\alpha_{h b}$ can be written:

$$
\begin{equation*}
\underset{h t}{\operatorname{Pr}}\left(b \mid \boldsymbol{\eta}_{h l}, \alpha_{h b}\right)=\prod_{t=1}^{T} \frac{e^{\zeta_{h b t}}}{1+\sum_{i=b}^{N-1} e^{\zeta_{h b t}}} . \tag{5}
\end{equation*}
$$

where the utility of purchasing brand $N$ has been normalized out.
The advantages of the RCL model do not come without a cost. Unlike the logit and nested logit models, there is no analytical closed form for (5). In order to overcome this difficulty, we integrate over the densities of the random parameters in the model. Define the densities of $\left\{v_{h 1}, \ldots, v_{h L}\right\}$ as $\left\{f\left(v_{h 1}\right), \ldots, f\left(v_{h L}\right)\right\}$ and $\mu_{h}$ as $g\left(\mu_{h}\right)$ so that the unconditional probability of household $h$ purchasing brand $b$ on purchasing occasion $t$ is obtained by integrating over (5) and the distributions reflecting consumer heterogeneity. Doing so we obtain:

$$
\begin{equation*}
\underset{h t}{\operatorname{Pr}}(b)=\int \ldots \int \prod_{t=1}^{T} \frac{e^{\zeta_{h b t}}}{1+\sum_{i=b}^{N-1} e^{\zeta_{h b t}}} f\left(v_{h 1}\right) \ldots f\left(v_{h L}\right) g\left(\mu_{h}\right) d v_{h 1} \ldots d v_{h L} d \mu_{h} \tag{6}
\end{equation*}
$$

which can then be estimated using simulated maximum likelihood.

### 4.2 Measure of Brand Strength

Our study of brand loyalty defines brand loyalty as the price differential needed to make a consumer who prefers one brand switch to a less preferred brand (Pessemier, 1959). Conveniently, this measure of brand loyalty is captured by the product-preference term (constant) in the RCL specification described above. By estimating the RCL over purchase occasion, time, and household we are able to recover unbiased estimates of each household's preference for each brand. Therefore, we use each household's preference $\left(\gamma_{h b}\right)$ as a measure of brand strength (Agrawal, 1996). We obtain the measure of brand strength for brand $b$ by taking the average over every household's preference for that brand which we'll denote $\gamma_{b}$. In this way, our measure of brand strength, $\gamma_{b}$, is able to capture the attitudinal and behavioral aspects of brand loyalty described above. Furthermore, we use this measure of brand strength to determine the number of consumers who prefer that brand, or the brand's "size." Our method differs from that of Agrawal (1996) in that our brand strength is estimated for household $h$ and brand $b$ which provides a richer set of data with which to estimate the relationship between brand strength and retailer's promotional strategies. Furthermore, our estimates of brand strength are computationally easier compared to the semiparametric alternatives he uses, while still accounting for household specific heterogeneity. Consequently, our method
guarantees strength and size estimates for each brand in the category, which is essential for testing the hypotheses of the theoretical brand loyalty model.

A precondition to explaining the frequency and depth of retail price promotions as they relate to brand loyalty is that brand loyalty indeed exists within the category (Huang, Perloff, and Villas-Boas, 2006). If the households within our sample have no preference for one brand over another then our empirical tests of $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$ cannot be carried out. As a result, our first hypothesis test will be

- $\mathrm{H}_{0}$ : no brand loyalty exists among the brands.

Since $\gamma_{b}$ measures the average strength of brand $b$ it follows that if $\gamma_{b}=0 \forall b$ then our model would suggest no brand loyalty exists among the brands, because on average the household wouldn't prefer any one brand over another. If we fail to reject $\mathrm{H}_{0}$, then the model wouldn't be suitable to test hypotheses $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$. If we reject $\mathrm{H}_{0}$ then we can empirically explain the frequency and depth of retail price promotions as they relate to brand strength.

In order to estimate the relationship between brand strength and price promotions, we specify a second-stage regression model in which the retailer's promotion strategy is a function of brand strength. Defining the discount offered by a retailer as the percentage change in prices from one week to the next, we use the actual percentage decrease as a measure of the depth of the retail promotion. If we regress brand $b$ 's strength on the measure of promotion depth, we are able to test the central hypothesis of the study, namely that the average price discount at the retail level is negatively related to the strength of brand loyalty. Therefore, the estimated model becomes:

$$
\begin{equation*}
d_{b t}=\phi_{d 0}+\phi_{d 1} p r e f_{b}+\phi_{d 2} s i z e_{b} \tag{7}
\end{equation*}
$$

where $p r e f_{b}$ is the strength of brand $b, d_{b t}$ is the depth of the discount offered on brand $b$ at week $t$, $\operatorname{size}_{b}$ is the percentage of households who prefer brand $b$, and $\phi_{d 0}$ is the intercept of the model that accounts for the depth of the discount when brand strength and size aren't present. ${ }^{3}$ This specification allows us to test $\mathrm{H}_{1}$, namely, if $\phi_{d 1}>0$ then the depth of the price discount at the retail level is negatively related to the strength of brand loyalty. Since

[^3]the discount is negative by definition, and brand strength is positive, it follows that $\phi_{d 1}>0$ implies an inverse, or negative, relationship. If however, $\phi_{d 1}<0$ then we would reject $\mathrm{H}_{1}$. Furthermore, this specification will allow us to investigate the relationship of the brand's marketshare and the depth of the retail discount.

We develop a similar test for the frequency of price promotions. We define promotion frequency as the average number of times the brand is discounted at least $10 \%$ from one day to the next each week. Similar to the approach described above, we then regress the strength and size of each brand on the measure of promotional frequency. We model frequency promotion as follows:

$$
\begin{equation*}
f_{b t}=\phi_{f 0}+\phi_{f 1} \text { pref }_{b}+\phi_{f 2} \text { size }_{b} \tag{8}
\end{equation*}
$$

where $f_{b t}$ is the promotional frequency for brand $b$ during week $t, p r e f_{b}$ and $s i z e_{b}$ are again the strength and size of brand $b$, and $\phi_{f 0}$ is the intercept of the model that accounts for the frequency of the discount when brand strength and size aren't present. Estimating this model allows us to test $\mathrm{H}_{2}$. More specifically, if $\phi_{f 1}>0$ then the frequency of retail price promotions is positively related to the strength of brand loyalty. Because $f_{b t}$, and pref $f_{b}$ are positive values, a positive coefficient estimate suggests that frequency rises as the strength of loyalty rises. This positive relationship comes from the asymmetry in loyalties derived by Raju, Srinivasan, and Lal, (1990), namely that $l_{w} \leq m_{w}^{2} r$ and $l_{s} \geq m_{s}\left(r-l_{w}\right)\left(1+m_{s} m_{w}\right)^{-1}$. From this it follows that the probability of the weaker brand obtaining the consumers loyal to the stronger brand is zero, which suggests that the retailer will promote the stronger brand more frequently. If, on the other hand, we find that $\phi_{f 1} \leq 0$ then we reject $\mathrm{H}_{2}$ and conclude that this asymmetry doesn't hold in the RTE cereal industry. If $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$ are supported then our empirical findings provide evidence that the theoretical model developed above is representative of the RTE cereal category and may serve as a basis for other theoretical derivations of brand loyalty. A summary of our tests for the frequency and depth of retail price promotions is as follows:

- $\mathrm{H}_{0}$ : No brand loyalty exists among any of the brands. $\left(\gamma_{b}=0 \forall b\right)$
- $\mathrm{H}_{1}$ : The average price discount at the retail level is negatively related to the strength of brand loyalty. $\left(\phi_{d 1}>0\right)$
- $\mathrm{H}_{2}$ : The frequency of price promotions at the retail level is positively related to the strength of brand loyalty. $\left(\phi_{f 1}>0\right)$


## 5 Estimation Method

In this section we explore the estimation method used to estimate the above models. Following Agrawal (1996) we estimate (7) and (8) using Joint Generalized Least Squares (SUR) estimation technique, which results in more efficient, though identical, estimates compared to OLS estimation. The SUR takes into account the correlation in error terms across the equations. There are several complications to address when estimating equation (6) above. First, the unobserved factors are correlated over time within each household. Maximum likelihood estimates will provide consistent estimates of the endogeneity of the households unobserved factors by allowing the coefficients on the decision to vary randomly. Second, the probability equation cannot be estimated using ordinary least squares because there is no closed form for the equation. Simulated maximum likelihood uses Monte Carlo simulation to solve the integral in (6) up to an approximation that is accurate to the number of random draws chosen, $R$. Finally, some of the unobserved factors may be correlated with the explanatory variables. As a result the demand model is estimated using the method of simulated maximum likelihood. This method provides consistent parameter estimates under general error assumptions and is readily able to accommodate complex structures regarding household heterogeneity. As models of this form can be quite computationally burdensome we implore a Halton draw sequence. This sequence significantly cuts down on the number of draws with no degradation to simulation performance (Bhat, 2003). We found that $R=200$ draws are sufficient to produce stable estimates without excessive estimation time.

## 6 Data Description

In this section we describe the data used for the econometric model developed in section 4. The empirical estimation requires data on prices, product characteristics, and household demographics. Wind and Lerner (1979) find that self-reported survey data is often unreliable as it is dependent on the recalling of purchase events from memory. Consequently this study uses a household panel data gathered by A.C. Nielson. A.C. Nielson, Inc.'s "HomeScan" data has participating households submit all food purchase information (price, quantity,
along with a product description) each time they visit any type of retail food outlet. The homescan database also includes a number of socioeconomic and demographic descriptors. We use all shopping trips from the beginning of 2006 to the end of 2006. This time period is long enough to observe several category purchases, while short enough to assume the household's preferences haven't changed over time. Using such a detailed household sample should provide sufficient empirical data to determine how a household's brand loyalty affects a retailer's equilibrium promotional strategy for differentiated products.

As mentioned above the RTE cereal industry fits well with the purposes of this study due to the high frequency of purchase, and fierce brand rivalry. We randomly select 1020 households who made at least two purchase within the RTE cereal industry. Selecting the households at random from the homescan data should give us a representative sample of US consumer buying habits. In table 2 we present the demographic information from the Current Population Survey (CPS) for 2006 and compare it to the surveyed households of the A.C. Nielson Homescan data. We can see from table 2 that the demographics of the A.C. Nielson households match those of U.S. households quite well. As a result, we suggest that our randomly sampled population is a representative sample of U.S. households

> Insert Demographic Table Here

The variety of brands available in the RTE cereal industry poses a bit of a problem because product specific attributes must be available on every brand the household might consider purchasing. As a result, we select only those brands within the RTE industry with the highest marketshare. By selecting the most popular brands in the category we increase the probability a randomly selected household made a purchase. We fill in the information on brands the household didn't purchase using purchases made by other households in the same geographic area, the same day, and if the same day isn't available, the same week. By selecting the most popular brands we increase the richness of the information on choices not selected by the household. The brands selected were General Mills' Cheerios, Kelloggs' Frosted Mini-Wheats, General Mills' Honey Nut Cheerios, Post's Honey Bunches of Oats, Kelloggs' Frosted Flakes, and General Mills' Cinnamon Toast Crunch. These brands tend to have the highest marketshare of all brands, which results in high incidences of purchase. Summary statistic on the brands used in the dataset can be found in table 3, including how
often the brands were purchased by the households throughout 2006 and the prices of each brand (in cents per ounce).

> Insert RTE summary stats. Table Here

### 6.1 Multiple Discreteness

It is well known that the choice set of discrete choice models must exhibit alternatives which are mutually exclusive from the decision maker's perspective (Train, 2003). In other words, choosing one alternative necessarily means another cannot be chosen. For many of the product categories found in retail grocery outlets consumers regularly purchase assortments of products and different brands within the product categories. The marketing literature provides several explanations as to the reason consumers might make several purchases within a particular category. McAlister (1982) found that consumers seek variety over time and in so doing switch their consumption of several different flavors. Consequently this form of variety seeking would induce a consumer to select an assortment of alternatives. Similarly, if households are unsure of their future tastes at the time of consumption they may purchase an assortment to ensure they have the right variety on hand (Hauser and Wernerfelt, 1990; Simonson, 1990; and Walsh, 1995). This could be particularly true if the household has several members and the purchasing member of the household is unsure of the other member's tastes.

This multiple discreteness violates the mutual exclusivity assumption of discrete choice models. This misspecification would produce incorrect measures of consumer response to marketing mix variables. Recognizing the difference between the time of purchase and the time of consumption, Dube (2005) accounts for multiple discreteness by modeling the number of consumption occasions the household faces during each shopping trip. Thereby suggesting that only one product is being consumed at a particular time. In this way they are able to develop a "vertical" variety-seeking model that accounts for multiple discreteness within category purchases. Hendel (1999) models the different tasks the consumer is going to use the different brands for. In so doing he is able to account for multiple discreteness within a RCL model. Bhat (2005) proposes a simple, parsimonious model to account for other choice occasions such as time allocation to different types of discretionary activities. He describes the decision process as "horizontal" variety-seeking, where the consumer selects an
assortment of alternatives due to diminishing marginal returns for each alternative. In so doing Dube (2005) and Bhat (2004) are able to take into account total demand elasticities.

The theoretical structure requirements proposed by Hendel (1999), Dube (2005), and Bhat (2005) impose too many restrictions on the underlying utility model to be entirely useful for our case. As a result we follow Train, McFadden, and Ben-Akiva, (1987) and expand the choice set such that each possible portfolio of choices is a distinct alternative. ${ }^{4}$ Expanding the choice set essentially ignores the complementarity between the two products being purchased since buying two together is fundamentally different from buying one and then the other. Since we are trying to identify the degree of brand loyalty for each product, we don't need to consider total demand elasticities. As we can see in table 3 the real gain from expanding the choice set doesn't come from the measure on the newly introduced choices, but rather from the increasing number of households and their corresponding brand purchases. For example, if a household bought Cheerios every time and once bought Cheerios and something else, their observations couldn't be used in our discrete choice model. Whereas now we can include those observations. Furthermore, this is still consistent with utility maximization because we're looking at households, the overall utility of the household is greater when buying two products together, as opposed to only one. Within the dataset if we had restricted the data to be only those households who purchased a single brand each trip we would have used 877 households ( $N=26,904$ ). By expanding the choice set to include households that made at least one purchase within the category, we are able to observe the purchasing habits of 1020 households in total $(N=76,024)$. By increasing the number of choices from 6 brands to 13 we have 5848 observations per choice. As a result, expanding the choice set allows us to include those households that make more than one brand purchase, adding valuable information to the analysis.

## 7 Empirical Results and Discussion

In this section, we present the results obtained from estimating the three equations that comprise the strategic price-promotion model as it pertains to brand loyalty and draw several implications for the further study. We first present and discuss the demand-system estimates

[^4]that empirically determine the level of preference each household has for each brand. With estimates of brand strength we first test $\mathrm{H}_{0}$ that states there is no brand strength among the products, and if rejected, we'll proceed to test the other hypotheses. Using the demand estimates we calculate the effect brand loyalty has on the depth and breadth of retailer promotions. In so doing we'll look at the hypothesis that the average price discount at the retail level is negatively related to the strength of brand loyalty $\left(\mathrm{H}_{1}\right)$ and the hypothesis that the frequency of price promotions at the retail level is positively related to the strength of brand loyalty $\left(\mathrm{H}_{2}\right)$.

As a first step to interpreting the demand model results we test the validity of the RCL model in general, and against the simpler logit specification as discussed in the estimation section. Several tests exist that allow us to investigate how well the model fit the data and whether or not a RCL specification is preferred over the simpler multinomial logit form in which the parameters are fixed. The likelihood ratio index is often used with discrete choice models to measure how well the model fit the data. If the estimated model is not any better than no model at all the we'll find that likelihood ratio index is very close to zero. We find that the log-likelihood ratio index for the RCL model is 0.49 which means that our RCL model fits the data better than a model containing no parameters. Furthermore in comparing the log-likelihood ratio index against the simpler logit model we find that the simpler model has a likelihood ratio index of 0.02 which implies that, given the same parameters and data, the RCL model fits the data better than the simple logit model. The main difference between the two models is their ability to handle household heterogeneity. If no heterogeneity exists in the model then the coefficients $\sigma_{\alpha}, \rho_{\eta}$, and $\sigma_{\eta}$ will all equal zero and the RCL will collapse into the simple logit model. As we can see in table 4 several of the coefficients are statistically different from zero. Consistent with Jain, Vilcassim, and Chintagunta (1994) this suggests that heterogeneity exists in the data and not accounting for it would provide biased estimates of brand strength. Therefore we conclude that the RCL model fits that data better than the simple logit model and we will use these estimates to interpret household demand.

In the demand model, the parameters of interest are the own-price effect, the discounteffect, the discount-price interaction term, and the average intrinsic preference for brand $b$. The own-price effect, measured by the marginal utility of income, is negative as expected. Consistent with the theoretical model this suggests that the households will have some reser-
vation price $r$, at which they will no longer make a purchase. We obtain the estimate of the marginal utility of income by averaging each household's estimate. In table 4 we can see that the coefficients on both the discount-effect, and the discount-price interaction term are not statistically significant. Within the context of the model this implies that discounting doesn't have a significant affect on which brand the household purchases. We'll now proceed to test $H_{0}$ which states that no brand strength exists among the brands ( $\gamma_{b}=0 \forall b$ ). The results in table 4 show that several of the brands are statistically different from zero, which suggests that on average, households have some preference for one brand over another. We therefore reject $\mathrm{H}_{0}$ and conclude that the households used for the sample exhibit brand loyalty.

Insert Demand Results Table Here

The parameter estimates in table 4 provide valuable information on the RTE cereal industry. Recall from section 4 we normalized brand $N$ out. As a result our estimates in table 4 are the average brand preference for brand $b$ relative to brand $N$. In our case brand $N$ happens to be the option of purchasing both Honey Bunches of Oats, and Cheerios in one shopping trip. From table 4 we can see Cheerios is the most preferred brand, which would be expected given it has the highest marketshare of any RTE cereal brand. Honey Bunches of Oats is the second highest preferred brand, and we find that the choice of purchasing both Honey Bunches of Oats and Cheerios is the preferred choice over any other choice combinations, which is consistent with the model's overall results. Furthermore, it seems that on average, households prefer to choose one brand at a time with the exception of Cinnamon Toast Crunch. Given the overall significance of the brand preference estimates, we can use these to estimate the strength of each brand and test $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$.

Using the estimates from the RCL model we calculate each household's preference for brand $b$. The results of the Joint Generalized Least Squares estimates of the discount model are presented in table 5. We'll first look at the overall fit of the model to the data. We find that the $R^{2}$ is 0.059 which indicates that the brand strength and size only account for $5.9 \%$ of the depth of the retailer discount. We can test the overall significance of the model using a F-test. We see in table 5 that the model's F-statistic is 4.50 . The corresponding test statistic at the $95 \%$ level of significance is 3.025 given a numerator of 2 , and a denominator
of 310 . Since our F-statistic is greater than 3.025 we can reject the null hypothesis that all of the parameters are equal to zero. Given the significance of the model overall we'll use these estimation results to test $\mathrm{H}_{1}$.

Insert Discount Regression Table Here

The primary parameters of interest in table 5 are $\phi_{d 1}$, and $\phi_{d 2}$. If $\phi_{d 1}>0$ then the depth of the price discount at the retail level is negatively related to the household's preference of brand $b$. From the discount model estimates we can see that $\phi_{d 1}>0$ and statistically significant at the $95 \%$ level of significance. As a result the estimate of $\phi_{d 1}$ provides evidence for $\mathrm{H}_{1}$, namely that the depth of the price discount at the retail level is negatively related to the strength of the brand preference. This suggests that the weaker brand will be promoted deeper than a stronger brand which provides evidence towards the promotional equilibrium of our theoretical model. This also provides evidence for the measurement of brand loyalty. Note that if $\mathrm{H}_{1}$ were rejected it would suggest that the stronger brand would be promoted deeper than the weaker in order to attract the loyal customers of the weaker. This would fundamentally contradict our definition and distinction of the stronger and weaker brands. As a result, by not rejecting $\mathrm{H}_{1}$ we provide evidence of Pessemiers' (1959) definition of brand loyalty and conclude that it is a reasonable assumption for the RTE cereal industry. Furthermore, it provides evidence to suggest that the duopoly model may extend to a multiproduct market. From table 5 we also see that the effect the size of the loyal cohort has on the depth of the discount is insignificant. Since $\phi_{d 2}$ is not statistically different from zero it follows that the size of brand $b$ 's loyal cohort doesn't have a significant effect on the depth of its promotion. We test our model further by investigating the relationship of brand strength and promotional frequency.

The demand estimates shown in table 4 form the key inputs to the second-stage promotional frequency model estimates shown in table 6. The results of the Joint Generalized Least Squares estimates of the frequency model are presented in table 6 . While looking at the fit of the model to the data we find that the $\mathrm{R}^{2}$ is only 0.02 , which indicates that the brand's strength and size account for very little in the variation of the promotional frequency. We can test the overall significance of the model using the F-statistic which we can see is 4.01 . The corresponding test statistic at the $95 \%$ level of significance is 3.025 given a numerator
of 2 and a denominator of 310 . Since our F-statistic is greater than 3.025 we can reject the null hypothesis that all of the parameters are equal to zero and proceed to test $\mathrm{H}_{2}$.

Insert Frequency Regression Table Here

In table 6 the primary parameters of interest are $\phi_{f 1}$, and $\phi_{f 2}$ which measure the effect the brand's strength and marketshare (size) has on the frequency of promotion. If $\phi_{f 1}>0$ then the frequency of price promotions at the retail level is positively related to the strength of brand loyalty. The results in table 6 indicate that $\phi_{f 1}<0$ and statistically significant at the $95 \%$ level of significance. As a result we reject $\mathrm{H}_{2}$ and conclude that the frequency of price promotions at the retail level is negatively related to the strength of brand loyalty. This is in contrast to our theoretical model developed in the 3rd section. Within the theoretical model's context this would suggest that the weaker brands are promoted more frequently than the stronger brand. This suggests that the probability of the weaker brand obtaining the loyal consumers of the stronger brand is not zero. Thus the asymmetry of loyalties derived by Raju, Srinivasan, and Lal's (1990) model does not hold within the RTE cereal category. Furthermore, we see from table 6 that the estimate of the brand's size is not statistically different from zero. This would suggest that the marketshare of a brand is not an indicator of the depth or frequency a brand will be promoted at the retail level.

Our result suggest that there is a high degree of brand loyalty within the RTE cereal industry and the data does not fully support the patterns suggested by our theoretical model. By rejecting $\mathrm{H}_{2}$ we conclude that the theoretical model developed above is not a representative model of brand loyalty in the RTE cereal category. However, our empirical results do confirm those of Agrawal (1996) and suggest that the promotional depth of a retailer's discount is negatively related to the strength of brand loyalty. In other words, deeper discounts are given for weaker brands if the wholesale price offered for that brand is low enough to warrant a higher profit by selling the weaker at the discounted rate to the entire market. This empirical result supports our definition, and distinction between the stronger and weaker brands, namely that $l_{w} \leq l_{s}$. Furthermore, our empirical results suggest that the brand's marketshare is insignificant in determining the depth and breadth of retail promotions.

Our results also suggest that the weaker brands are promoted more frequently than the stronger brands. In other words, the frequency of price promotions at the retail level is
negatively related to the strength of the brand. This suggests that the asymmetry of our theoretical model's loyalties, namely that $l_{w} \leq m_{w}^{2} r$ and $l_{s} \geq m_{s}\left(r-l_{w}\right)\left(1+m_{s} m_{w}\right)^{-1}$ as shown by Raju, Srinivasan, and Lal, (1990), does not hold for the RTE cereal category because the weaker brand is able to capture the marketshare of the stronger brand with a probability greater than zero. Otherwise the retailer would promote the stronger brand more frequently. Therefore it follows that this assumption is not suitable for brand loyalty models in the RTE cereal category and should be reconsidered in further research.

## 8 Conclusion and Implications

Though brand loyalty has been studied extensively in the marketing literature, the relationship between brand loyalty and equilibrium pricing strategies is not well understood. Retailer's promotional decisions are critically dependent upon how many consumers can be convinced to switch to a brand by temporarily reducing its price, and how many are instead brand loyal. Theoretical models of how the size and strength of brand loyalty influence optimal promotion strategies have been developed, but there are no rigorous tests of their hypotheses. As a result we empirically determine how brand loyalty changes retailer's equilibrium promotional strategy for differentiated products.

We develop and test a theoretical model of brand loyalty that resembles many of the fundamental characteristics of theoretical brand loyalty studies. In our empirical estimation we use a random coefficient logit model to estimate the intrinsic preference each household has for each brand. Estimating the random coefficient logit model using simulated maximum likelihood provides consistent estimates of the mean household preference while accounting for the heterogeneity among households. Using household panel data we applied the empirical test to the purchases made by the households throughout the US during 2006. By applying our random coefficient logit model to this rich panel data we were able to empirically investigate retail promotion strategies.

The result suggest that there is a high degree of brand loyalty within the RTE cereal category and the data does not fully support the patterns suggested by our theoretical model. We conclude that the theoretical model developed above is not a representative model of brand loyalty in the RTE cereal category. Our results suggest that the average price discount at the retail level is negatively related to the strength of brand loyalty as implied by the
theoretical model. In other words, deeper discounts are given for weaker brands if the wholesale price offered for that brand is low enough to warrant a higher profit by selling the weaker at the discounted rate to the entire market. These empirical result supports our definition, and distinction between the stronger and weaker brands. However, contrary to the maintained hypothesis of many studies in retailing our results also suggest that the weaker brands are promoted more frequently than the stronger brand. In other words, the frequency of price promotions at the retail level is negatively related to the strength of the brand. It therefore follows that assumptions of theoretical brand loyalty models that concluded that the frequency of price promotions at the retail level is positively related to the strength of the brand should be reconsidered in future research when being applied to the RTE cereal category.

Though our results have implications for future work with models of retail price promotion, there is much that remains for the study of both theoretical and empirical models. Future empirical research would benefit by empirically studying the effect a price sensitive segment would have on the retailer's promotional strategies. The size of a price sensitive segment may provide insight into the degree of brand loyalty within the market and the overall incentives of a retailer and their potential strategic interaction within that market. Future research would also benefit by considering, empirically, the effect trade deals have on a promotional equilibrium. Our empirical model wasn't able to estimate the retail pass through and trade deals as a result of the household specific data. However, methods have been proposed that a suggest ways to estimate the trade deals given to retailers. ${ }^{5}$ Incorporating this into empirical studies of brand loyalty would reveal valuable information on the retailer's strategic interaction within the market and their overall incentive to offer a price promotion. The empirical evidence from such a study would be able to distinguish between a retailer's incentive of a larger margin, and the competitive nature of the market in general.

Future theoretical research in the study of retailer promotional strategies may benefit by investigating the theoretical ramifications of a market that consists of more than two brands. Since most retail markets inherently have more than two products the applications certainly exist. In so doing one could investigate the possibility of complementary products and their effect on retailers promotional decisions in the marketplace. Future theoretical research may

[^5]also benefit from looking at the dynamic long run game theoretic aspects of promotional strategies. This would allow models to relax the assumption that the manufacturer sets prices simultaneously, and assume a sequential price setting. This would be a more realistic assumption and would allow theoretical applications to investigate the effect trade deals would have on the opposing manufacturer and the resulting equilibrium. Lastly, it would also be reasonable to assume that manufacturers are aware of the wholesale price of several products within the market. Since many highly concentrated industries have only a few manufacturers selling multiple products it follows that they would know the wholesale price of some portion of the products within the market. This would allow researchers to model the possibility that a manufacturer to promote several of their own brands at the same in order to attract the marketshare away from a competing brand.

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Table 1: Brand Market Share by Volume and Price summary Statistics.

| Price Index (2000 is base year) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Variable | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| CPI | 172.2 | 177.1 | 179.9 | 184 | 188.9 | 195.3 | 201.6 |
| All RTE Cereals | 304.6 | 288.3 | 285.8 | 283.3 | 267.6 | 255.0 | 245.6 |
| Gereal Mills Brands |  |  |  |  |  |  | 165.3 |
| Cheerios | $\begin{aligned} & 343.2 \\ & (9.8) \end{aligned}$ | $\begin{aligned} & 324.3 \\ & (9.4) \end{aligned}$ | $\begin{aligned} & 317.3 \\ & (8.4) \end{aligned}$ | $\begin{aligned} & 311.6 \\ & (8.5) \end{aligned}$ | $\begin{aligned} & 290.6 \\ & (7.4) \end{aligned}$ | $\begin{aligned} & 301.6 \\ & (6.5) \end{aligned}$ | $\begin{aligned} & 290.0 \\ & (6.9) \end{aligned}$ |
| Honey Nut Cheerios | $\begin{aligned} & 320.1 \\ & (0.0) \end{aligned}$ | $\begin{aligned} & 302.8 \\ & (2.9) \end{aligned}$ | $\begin{aligned} & 294.2 \\ & (4.3) \end{aligned}$ | $\begin{aligned} & 291.1 \\ & (3.9) \end{aligned}$ | $\begin{aligned} & 275.4 \\ & (4.0) \end{aligned}$ | $\begin{aligned} & 276.4 \\ & (3.4) \end{aligned}$ | $\begin{aligned} & 270.5 \\ & (4.0) \end{aligned}$ |
| Cinnamon Toast Crunch | $\begin{aligned} & 350.6 \\ & (1.5) \end{aligned}$ | $\begin{aligned} & 316.6 \\ & (1.7) \end{aligned}$ | $\begin{aligned} & 308.0 \\ & (1.8) \end{aligned}$ | $\begin{aligned} & 298.3 \\ & (1.8) \end{aligned}$ | $\begin{aligned} & 283.0 \\ & (1.3) \end{aligned}$ | $\begin{aligned} & 271.0 \\ & (0.9) \end{aligned}$ | $\begin{aligned} & 256.1 \\ & (0.9) \end{aligned}$ |
| Lucky Charms | $\begin{aligned} & 330.2 \\ & (0.0) \end{aligned}$ | $\begin{aligned} & 322.6 \\ & (1.1) \end{aligned}$ | $\begin{aligned} & 305.0 \\ & (1.1) \end{aligned}$ | $\begin{aligned} & 304.8 \\ & (1.1) \end{aligned}$ | $\begin{aligned} & 282.1 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & 278.0 \\ & (0.5) \end{aligned}$ | $\begin{aligned} & 264.2 \\ & (0.4) \end{aligned}$ |
| Yogurt Burst Cheerios | - | - | - | - | - | $\begin{aligned} & 356.8 \\ & (0.0) \end{aligned}$ | $\begin{aligned} & 326.8 \\ & (0.2) \end{aligned}$ |
| Total Whole Grain | $\begin{aligned} & 468.9 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 439.6 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 451.2 \\ & (0.1) \end{aligned}$ | $\begin{aligned} & 404.6 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 391.6 \\ & (0.1) \end{aligned}$ | $\begin{aligned} & 381.3 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 359.5 \\ & (0.2) \end{aligned}$ |
| Kellogg's Brands |  |  |  |  |  |  |  |
| Frosted Mini-Wheats | $\begin{aligned} & 234.1 \\ & (5.9) \end{aligned}$ | $\begin{aligned} & 223.9 \\ & (5.1) \end{aligned}$ | $\begin{aligned} & 225.2 \\ & (4.1) \end{aligned}$ | $\begin{aligned} & 232.5 \\ & (3.5) \end{aligned}$ | $\begin{aligned} & 219.6 \\ & (4.0) \end{aligned}$ | $\begin{aligned} & 215.7 \\ & (4.8) \end{aligned}$ | $\begin{aligned} & 211.7 \\ & (5.7) \end{aligned}$ |
| Frosted Flakes | $\begin{aligned} & 237.8 \\ & (7.4) \end{aligned}$ | $\begin{aligned} & 219.4 \\ & (8.1) \end{aligned}$ | $\begin{aligned} & 220.2 \\ & (6.7) \end{aligned}$ | $\begin{aligned} & 211.1 \\ & (6.1) \end{aligned}$ | $\begin{aligned} & 204.8 \\ & (3.9) \end{aligned}$ | $\begin{aligned} & 206.8 \\ & (3.0) \end{aligned}$ | $\begin{aligned} & 203.5 \\ & (2.6) \end{aligned}$ |
| Raisin Bran | $\begin{aligned} & 214.9 \\ & (4.0) \end{aligned}$ | $\begin{aligned} & 203.2 \\ & (4.2) \end{aligned}$ | $\begin{aligned} & 203.9 \\ & (2.8) \end{aligned}$ | $\begin{aligned} & 205.3 \\ & (2.1) \end{aligned}$ | $\begin{aligned} & 193.9 \\ & (1.5) \end{aligned}$ | $\begin{aligned} & 186.1 \\ & (1.5) \end{aligned}$ | $\begin{aligned} & 182.3 \\ & (1.4) \end{aligned}$ |
| Rice Krispies | $\begin{aligned} & 352.5 \\ & (1.6) \end{aligned}$ | $\begin{aligned} & 325.0 \\ & (1.6) \end{aligned}$ | $\begin{aligned} & 308.6 \\ & (1.2) \end{aligned}$ | $\begin{aligned} & 315.5 \\ & (1.3) \end{aligned}$ | $\begin{aligned} & 303.0 \\ & (0.9) \end{aligned}$ | $\begin{aligned} & 297.0 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & 300.0 \\ & (0.8) \end{aligned}$ |
| Special K Red Berry | - | $\begin{aligned} & 482.8 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 470.7 \\ & (0.6) \end{aligned}$ | $\begin{aligned} & 434.1 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & 389.7 \\ & (0.5) \end{aligned}$ | $\begin{aligned} & 352.3 \\ & (0.4) \end{aligned}$ | $\begin{aligned} & 339.2 \\ & (0.5) \end{aligned}$ |
| Fruit Loops | $\begin{aligned} & 288.9 \\ & (1.5) \end{aligned}$ | $\begin{aligned} & 276.8 \\ & (1.7) \end{aligned}$ | $\begin{aligned} & 266.0 \\ & (1.0) \end{aligned}$ | $\begin{aligned} & 347.2 \\ & (1.1) \end{aligned}$ | $\begin{aligned} & 273.8 \\ & (0.7) \end{aligned}$ | $\begin{aligned} & 268.8 \\ & (0.5) \end{aligned}$ | $\begin{aligned} & 259.5 \\ & (0.5) \end{aligned}$ |
| Post's Brands |  |  |  |  |  |  |  |
| Honey Bunches of Oats | $\begin{aligned} & 299.5 \\ & (1.6) \end{aligned}$ | $\begin{aligned} & 268.0 \\ & (2.1) \end{aligned}$ | $\begin{aligned} & 263.8 \\ & (4.6) \end{aligned}$ | $\begin{aligned} & 239.9 \\ & (4.4) \end{aligned}$ | $\begin{aligned} & 247.2 \\ & (4.5) \end{aligned}$ | $\begin{aligned} & 230.8 \\ & (4.8) \end{aligned}$ | $\begin{aligned} & 220.1 \\ & (5.5) \end{aligned}$ |
| Raisin Bran | - | $\begin{aligned} & 219.9 \\ & (0.0) \end{aligned}$ | $\begin{aligned} & 195.9 \\ & (0.9) \end{aligned}$ | $\begin{aligned} & 188.7 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & 163.4 \\ & (0.9) \end{aligned}$ | $\begin{aligned} & 158.2 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & 149.9 \\ & (0.9) \end{aligned}$ |
| Grape Nuts | - | $\begin{aligned} & 218.6 \\ & (0.0) \end{aligned}$ | $\begin{aligned} & 183.9 \\ & (0.4) \end{aligned}$ | $\begin{aligned} & 186.2 \\ & (0.4) \end{aligned}$ | $\begin{aligned} & 179.8 \\ & (0.4) \end{aligned}$ | $\begin{aligned} & 176.5 \\ & (0.4) \end{aligned}$ | $\begin{aligned} & 167.0 \\ & (0.3) \end{aligned}$ |
| Selects | $\begin{aligned} & 325.9 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 315.1 \\ & (0.3) \end{aligned}$ | $\begin{aligned} & 298.6 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 302.8 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 275.6 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 259.6 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & 247.9 \\ & (0.2) \end{aligned}$ |
| Trail Mix Crunch | - | - | - | - | - | $\begin{aligned} & 253.5 \\ & (0.0) \end{aligned}$ | $\begin{aligned} & 274.9 \\ & (0.2) \end{aligned}$ |
| Honey-Comb | $\begin{aligned} & 275.7 \\ & (0.6) \end{aligned}$ | $\begin{aligned} & 268.0 \\ & (0.5) \\ & \hline \end{aligned}$ | $\begin{aligned} & 254.7 \\ & (0.5) \\ & \hline \end{aligned}$ | $\begin{aligned} & 250.8 \\ & (0.4) \end{aligned}$ | $\begin{aligned} & 233.8 \\ & (0.2) \\ & \hline \end{aligned}$ | $\begin{aligned} & 230.0 \\ & (0.1) \end{aligned}$ | $\begin{aligned} & 230.1 \\ & (0.1) \end{aligned}$ |

All Price are in cents per pound.
Brand marketshare are in parathenses.

Table 2: Demographic distribution of surveyed households and US households.

| Variable | \% Households <br> Homescan | \% Households <br> CPS |
| :--- | ---: | ---: |
| Income Bracket |  |  |
| Less than 29,999 | 26.55 | 28.12 |
| 30,000 to 39,999 | 13.91 | 12.08 |
| 40,000 to 49,999 | 13.40 | 9.21 |
| 50,000 to 59,999 | 10.15 | 9.41 |
| 60,000 to 99,999 | 24.95 | 23.75 |
| 100,000 to 149,999 | 8.19 | 10.41 |
| 150,000 and over. | 2.85 | 7.02 |
| $\quad$ Marital Status |  |  |
| Married | 59.11 | 54.21 |
| Widowed | 10.12 | 6.14 |
| Divorced/Separated | 15.18 | 10.04 |
| Single | 15.59 | 29.61 |
| Residential Region |  |  |
| East | 16.05 | 20.57 |
| Central | 23.62 | 23.40 |
| South | 39.12 | 31.18 |
| West | 21.21 | 25.15 |

Table 3: RTE Cereal Summary Statistics.

| Variable | Mean | Std. Dev | Min | Max |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Percentage Purchased |  |  |  |  |  |
| Honey Bunches Oats | Overall | 0.077 | 0.266 | 0.0 | 1.000 |
| Cheerios | 0.235 | 0.424 | 0.0 | 1.000 |  |
| Cinnamon Toast Crunch | 0.239 | 0.426 | 0.0 | 1.000 |  |
| Honey Nut Cheerios | 0.058 | 0.234 | 0.0 | 1.000 |  |
| Frosted Flakes | 0.142 |  | 0.350 | 0.0 | 1.000 |
| Frosted Mini Wheats | 0.121 |  | 0.326 | 0.0 | 1.000 |
| Cheerios, and Honey Nut Cheerios | 0.168 |  | 0.374 | 0.0 | 1.000 |
| Frosted Flakes, and Frosted Mini Wheats | 0.007 |  | 0.082 | 0.0 | 1.000 |
| Honey Bunches of Oats, and Frosted Mini Wheats | 0.005 |  | 0.071 | 0.0 | 1.000 |
| Honey Nut Cheerios, and Frosted Mini Wheats | 0.007 |  | 0.070 | 0.0 | 1.000 |
| Cheerios, and Frosted Mini Wheats | 0.005 |  | 0.068 | 0.0 | 1.000 |
| Honey Bunches of Oats, and Honey Nut Cheerios | 0.004 |  | 0.067 | 0.0 | 1.000 |
| Honey Bunches of Oats, and Cheerios | 0.004 |  | 0.063 | 0.0 | 1.000 |
| Price |  |  |  |  |  |

[^6]Table 4: RTE Cereal Summary Statistics.

| Variable | Random Coef. Logit Model |  | Multinomial Logit Model |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Estimate | t-stat. | Estimate | t-stat. |
| Honey Bunches Oats | 0.010 | 0.229 | 0.032* | 0.881 |
| Cheerios | 0.016 | -0.367 | 0.036* | 1.007 |
| Cinnamon Toast Crunch | 0.040 | 0.925 | 0.043* | 1.166 |
| Honey Nut Cheerios | 0.053 | 1.345 | 0.054* | 1.489 |
| Frosted Flakes | 0.038 | 0.890 | 0.050* | 1.384 |
| Frosted Mini Wheats | 0.044 | 1.071 | 0.046* | 1.264 |
| Cheerios, and Honey Nut Cheerios | 0.073 | 1.351 | 0.040 | 0.877 |
| Frosted Flakes, and Frosted Mini Wheats | 0.110* | 2.322 | 0.102 | 2.033 |
| Honey Bunches of Oats, and Frosted Mini Wheats | 0.026 | 0.495 | 0.023 | 0.468 |
| Honey Nut Cheerios, and Frosted Mini Wheats | 0.086 | 1.430 | 0.062 | 1.339 |
| Cheerios, and Frosted Mini Wheats | 0.047 | 0.786 | 0.042 | 0.842 |
| Honey Bunches of Oats, and Honey Nut Cheerios | 0.031 | 0.576 | 0.027 | 0.539 |
| Price |  |  | -0.014* | -5.581 |
| Discount Dummy | -0.100 | -0.881 | -0.070 | -1.040 |
| Discount Price Interaction | -0.003 | -0.280 | 0.003 | 0.494 |
| Store Feature Dummy | 0.002 | 0.059 | -0.103* | -3.149 |
|  | $(\gamma)$ Means of the Random Parameters |  |  |  |
| Honey Bunches Oats | 2.863* | 3.557 | 3.446 | 4.933 |
| Cheerios | 3.522* | 4.222 | 3.418 | 4.893 |
| Cinnamon Toast Crunch | -0.033 | -0.039 | 1.847 | 2.566 |
| Honey Nut Cheerios | 1.826* | 2.445 | 2.524 | 3.584 |
| Frosted Flakes | 1.127 | 1.368 | 2.396 | 3.393 |
| Frosted Mini Wheats | 1.961* | 2.530 | 2.814 | 4.009 |
| Cheerios, and Honey Nut Cheerios | -2.880* | -2.700 | -0.197 | -0.219 |
| Frosted Flakes, and Frosted Mini Wheats | -2.074* | -2.160 | -1.816* | -1.746 |
| Honey Bunches of Oats, and Frosted Mini Wheats | -0.781* | -0.732 | -0.223 | -0.235 |
| Honey Nut Cheerios, and Frosted Mini Wheats | -1.850 | -1.540 | -0.692 | -0.747 |
| Cheerios, and Frosted Mini Wheats | -0.747 | -0.621 | -0.643 | -0.652 |
| Honey Bunches of Oats, and Honey Nut Cheerios | -2.483 | -1.608 | -0.394 | -0.403 |
| Price | -0.032* | -6.684 |  |  |
|  | $(\sigma)$ Std. Deviation of Random Parameter |  |  |  |
| Honey Bunches Oats | 2.542* | 19.617 |  |  |
| Cheerios | 2.907* | 22.200 |  |  |
| Cinnamon Toast Crunch | 2.795* | 19.693 |  |  |
| Honey Nut Cheerios | 2.079* | 18.301 |  |  |
| Frosted Flakes | 2.722* | 17.413 |  |  |
| Frosted Mini Wheats | 1.983* | 17.409 |  |  |
| Cheerios, and Honey Nut Cheerios | 2.218* | 9.645 |  |  |
| Frosted Flakes, and Frosted Mini Wheats | 0.316 | 0.673 |  |  |
| Honey Bunches of Oats, and Frosted Mini Wheats | 1.043* | 2.102 |  |  |
| Honey Nut Cheerios, and Frosted Mini Wheats | 1.258* | 4.332 |  |  |
| Cheerios, and Frosted Mini Wheats | 0.008 | 0.012 |  |  |
| Honey Bunches of Oats, and Honey Nut Cheerios | 2.111* | 4.264 |  |  |
| Price | 0.083* | 15.149 |  |  |
| Log-Likelihood Ratio | 0.491 |  | 0.002 |  |
| AIC | 2.624 |  | 3.744 |  |

Table 5: Relationship of Brand Strength and Discount.

| Variable | Estimate | T-ratio |
| :--- | ---: | ---: |
| $\phi_{d 0}$ | $-0.049^{*}$ | -7.731 |
| $\phi_{d 1}$ | $0.015^{*}$ | 2.092 |
| $\phi_{d 2}$ | -0.088 | -1.067 |
| $\mathrm{R}^{2}$ | 0.059 |  |
| F-statistic | 4.50 |  |

Table 6: Relationship of a Brand's Strength and Size on Frequency.

| Variable | Estimate | T-ratio |
| :--- | ---: | ---: |
| $\phi_{f 0}$ | $0.374^{*}$ | 24.981 |
| $\phi_{f 1}$ | $-0.035^{*}$ | -2.053 |
| $\phi_{f 2}$ | 0.209 | -1.109 |
| $\mathrm{R}^{2}$ | 0.02 |  |
| F-statistic | 4.01 |  |


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[^1]:    ${ }^{1}$ For example Guadagni and Little, (1983); and Ortmeyer, Lattin and Montgomery, (1987)

[^2]:    ${ }^{2}$ Because we use $t$ to denote the time period and T to denote the total number of time periods we use a ${ }^{\prime}$, , instead of the traditional poscript $T$, to denote the transpose of a vector or matrix.

[^3]:    ${ }^{3}$ Recall that we expanded the choice set to include all possible combinations of purchases within the category. For this stage of the model we will only use the actual brand estimates and not the expanded choice observations since they are aggregates of the other brands.

[^4]:    ${ }^{4}$ In fact we only had to expand the choice set by those combinations of brands that were actually purchased, which was signficantly less than all the possible combinations.

[^5]:    ${ }^{5}$ This includes Richards, Pofahl, and Hamilton (2007).

[^6]:    All Price are in cents per pound.

