

# The Volatility Spillover Effects and Optimal Hedging Strategy in the Corn Market

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## **Abstract**

This article examines the volatility spillovers from energy market to corn market. Using a volatility spillover model from the finance literature, we found significant spillovers from energy market to corn cash and futures markets, and the spillover effects are time-varying. The business cycle proxied by crude oil prices is shown to affect the magnitude of spillover effects over time. Based on the strong informational linkage between energy market and corn market, a cross hedge strategy is proposed and its performance studied. The simulation outcomes show that compared to alternative strategies of no hedge, constant hedge, and GARCH hedge, the cross hedge does not yield superior risk-reduction performance.

**Keywords:** Volatility Spillover, GARCH, Optimal Hedge Ratio, Energy Price, Corn Price

The rising corn-based ethanol production has established strong price links between energy and corn markets; the volatile energy market has a spillover effect on corn markets. The spillover is not just in price levels, but also in price volatilities of corns. Zulauf and Roberts (2008) measured corn historical and expected volatilities in the period of 1989 to 2007 and found substantial volatility transmission from energy to corn markets. Increased price volatility of the corn market has added to worry on U.S. farms (New York Times 2008) because it results in greater costs for managing risks, such as more costly crop insurance premiums, higher option premiums, and greater margins for hedging commodities. Therefore, it is crucial for decision maker to be aware of the behavior and sources of volatility so as to adopt the appropriate risk management method, especially, hedging strategy and portfolio, to reduce risk.

This article has two objectives. The first objective is to examine the volatility spillover effects of energy market, represented by crude oil market, on corn market. More specifically, we focus on to what extent volatility in corn market is impacted by external shocks from energy markets. In the finance literature, volatility spillover effects have been extensively studied (Bekaert and Harvey 1997, Ng 2000, and Bekaert, Harvey and Lumsdaine 2002). Ng, for example, analyzes the sources of the return volatility of stock markets in the pacific-basin and finds transmission from a world factor and a regional factor. In this article, we construct a volatility spillover model, similar to the one in Ng (2000), assuming that there is only one foreign source of shocks - crude oil futures price - to corn cash and futures prices. Of particular interest is the impact of energy act and subsequent financial crises on volatility spillovers. Introducing the Renewable Fuel Standard, the 2005 Energy Policy Act aims to increase use of renewable energy through providing economic incentives and regulations. Subsequent tax incentives, federal and state mandates, and the progressive elimination of MTBE as an additive

in many states have quickly increased the demand for biofuels, particularly corn-based ethanol. The record high energy prices further pushed up the demand for corn which is the major feedstock of ethanol. The linkage between energy market and corn market is thus built and strengthened. One would expect stronger spillover effects from energy markets following the energy act and in the period of high and volatile crude oil prices. This spillover is potentially made more prominent by the subsequent financial crisis. The financial crisis and the deep economic recession have caused a substantial drop in crude oil price, which is making biofuel less attractive as a substitute and may be changing the spillover effects on corn market. This study further investigates whether the ongoing crisis and recession impacts the size and pattern of volatility spillovers. With the elasticity of substitution between biofuel and fossil fuel changing over time, volatility spillover effects may also be time-varying. In this study, we examine the potentially time-varying spillover effect from energy market, incorporating both energy act and crude oil price factors into the underlying model.

Given evidences of strong informational linkages between energy market and corn market, this study further develops a cross hedge strategy to help reduce price risk for corn store merchants. This research examines, for the first time, the effectiveness of allocating assets among corn cash, corn futures and crude oil futures under the expected utility maximization framework. A considerable amount of research has focused on the optimal futures hedge strategies and optimal hedge ratio (Baillie and Myers 1991, Myers 1991, Moschine and Myers 2002, and Haigh and Holt 2002). However, previous research has not considered the possibility of hedging corn cash, corn futures and crude oil futures simultaneously in a time-varying setting. Strategies proposed in the literature only limit asset portfolio in corn cash and corn futures and calculate the time-varying or constant futures contract holdings. We compare this strategy to

non-cross hedge alternatives and estimate a series of bivariate GARCH models that link corn cash and futures without accounting for the volatility spillover effect from energy market and examine their hedge performance both in sample and out of sample. We also take account of constant hedge outcomes obtained using conventional regression techniques and evaluate its performance relative to complex portfolio outcome. Our results show that cross hedge strategy only performs marginally better than limited asset portfolio.

## Data Analysis

We use weekly data in the empirical analysis, covering the period beginning 20 February 1992 to 25 March 2009 for corn cash price, corn futures price, and crude oil futures price. Corn cash price  $p_t$  is the average of low and high bid level for #2 yellow corn from mid-states Terminals, Toledo, Ohio. Corn futures price  $f_{c,t}$  is for the nearest expiration contract on CBOT and sourced from the Econstats website. Crude oil futures price  $f_{o,t}$  is for the nearest expiration contract on NYMEX and sourced from Energy Information Administrative (EIA). All data are the mid-week closing price (Thursday) and include 870 observations.

<TABLE 1>

Table 1 presents summary statistics for the weekly prices for the three sub-periods and whole period. The data set are broken into three subsets separated by two important dates, the first being July 29,2005 when the energy act was passed by the congress, and the second being July 3, 2008 when the crude oil climbed to the historical record. The correlation matrixes show that the linkage between corn market and energy market changes from weakly negative

relationship into strong positive relationship. The measures of skewness and excess kurtosis show that the price distributions are asymmetric and fat-tailed. The formal test rejects the null hypothesis of unconditional normality at the 5% level of significance.

As with most asset price data, the unit root tests of Augmented Dickey-Fuller test and Phillips-Perron test performed to three price series could not reject the unit root hypothesis. However, cointegration between corn cash and futures price could not be rejected. An error correction model is necessary for the mean equations of corn prices.

### **Volatility Spillover Model**

Ng (2000) develops a two-factor spillover model in which unexpected stock returns on any pacific-basin market are influenced not only by news originating from home but also by two foreign shocks. We follow the similar approach but take account of two markets—corn cash and corn futures market—simultaneously with a foreign shock from the crude oil futures price. The spillover effect is tested based on the ARCH family of models developed by Engle (1982) and generalized by Bollerslev (1986). These models have been shown empirically to provide a good fit for many financial return series and commodity price. First A GARCH(1,1) model is developed to model crude oil futures price and then the parsimonious BEKK GARCH model is presented for corn cash and futures markets.

The following univariate GARCH(1,1) model is proposed for crude oil futures price, denoted by  $f_{o,t}$ :

$$(1) \Delta f_{o,t} = e_{o,t}$$

$$(2) e_{o,t} | I_{t-1} \sim N(0, \sigma_t^2)$$

$$(3) \sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 + a_2 e_{o,t-1}^2$$

where  $\Delta$  denotes a first-difference operator. In the model, the expected return conditional on information available at time t-1 from holding futures is zero. Despite its simplicity, the evidence of efficiency in futures market supports this model for commodity futures data. Its prediction error is assumed to have time-varying variance. With a sample of T observations of the futures series, the parameters of the model can be estimated with the maximum likelihood method. The non-linear optimization technique based on the Berndt-Hall-Hall-Hausman (BHHH) algorithm is used to calculate it. Bollerslev-Wooldridge robust quasi-maximum likelihood covariance matrix are reported which are robust to misspecification of the distribution of the error term. In order to test the model specification, the LM test is performed to the residuals for the univariate distribution.

Based on the results from the unit root and cointegration analysis, the following error correction models for corn cash price and futures price are specified:

$$(4) \Delta p_t = \beta_0 + \sum_{i=1}^6 \beta_i \Delta p_{t-i} + \sum_{i=1}^6 \beta_{i+6} \Delta f_{c,t-i} + \tau_1 ECT_{t-1} + \varepsilon_{p,t}$$

$$(5) \Delta f_{c,t} = \theta_0 + \sum_{i=1}^6 \theta_i \Delta p_{t-i} + \sum_{i=1}^6 \theta_{i+6} \Delta f_{c,t-i} + \tau_2 ECT_{t-1} + \varepsilon_{f,t}$$

where ECT denotes error correction term to capture the cointegration relationships. Both equations were estimated by employing an AR(6) processes, which render all residuals white noise. Seasonality proxied by dummy variables was initially accounted for in both equations. We exclude the seasonal variables as they are statistically insignificant and small in magnitude.

Similar in spirit to Bekaert and Harvey (1997), Ng(2000), and Baele (2005), shocks in corn cash price and corn futures price are – apart from a purely local component- allowed to be driven by an innovation in energy market.

$$(6) \varepsilon_{p,t} = e_{p,t} + \varphi e_{o,t}$$

$$(7) \varepsilon_{f,t} = e_{f,t} + \omega e_{o,t}$$

$$(8) e_t = (e_{p,t}, e_{f,t})' | I_{t-1} \sim N(0, H_t)$$

$$(9) H_t = C'C + A'e_t e_t' A + B'H_{t-1} B$$

where  $e_t$  is a purely idiosyncratic shock vector which is assumed to be uncorrelated with shock  $e_{o,t}$  from energy market and follow a conditional normal distribution with mean zero and time-varying covariance matrix  $H_t$ . There are several possible parameterizations of multivariate GARCH process: the VEC model of Bollerslev et al. (1988), the constant correlation model of Bollerslev (1990), the factor ARCH model of Engle et al. (1990), and the BEKK of Engle and Kroner (1995). Here a positive definite BEKK parameterization is used in eq.9. C, A, and B are symmetric (2×2) parameter matrices in order that the model is parsimonious.

If  $\varphi$  and  $\omega$  are assumed constant, the above model assumes that spillover effects stay constant over time and investigates whether there are significant volatility spillovers from energy market. However, a number of studies have found that the spillovers may be time-varying in response to changes, especially to legislative events and business cycles. As discussed earlier, the energy act has effectively integrated the agricultural commodity market and energy market into a unified one. Volatility transmission would lead to a more volatile agricultural product market. We further allow legislative events and the business cycle to have an impact on the spillover effects:



$$(10) \quad \varphi_t = \varphi_0 + \varphi_1 D_t + \varphi_2 f_{o,t}$$

$$(11) \quad \omega_t = \omega_0 + \omega_1 D_t + \omega_2 f_{o,t}$$

where  $D_t$  are energy act dummy variable which equal 1 for the period after the act is passed (29 July 2005) and 0 otherwise. Crude oil price  $f_{o,t}$  is used to proxy the business cycle.

We use a two-step method to estimate the parameters in the system<sup>1</sup>. In the first step, we estimate a univariate crude oil model and the mean equations of corn prices. In the second step, conditional on the estimates, the bivariate spillover models (eq.6-9) are estimated by maximizing the log-likelihood function. In the time-varying spillover model, eqs.11-12 are substituted into the eqs. 6-7 and the similar estimate procedure is implemented. The estimation results are reported in Tables 2, 3, 4 and 5.

<TABLE 2, 3, 4, 5>

### **Spillover Effects Results**

To test for spillover effects from energy market, we first constrain all spillovers to be constant over time. The estimation results are presented in Table 4. The parameter estimates show that  $\omega$  is almost equal to  $\varphi$ , indicating that cash price and futures price are comparably influenced by crude oil price. Meanwhile, the spillover effects are significant at the conventional level. The positive sign shows evidence of transmission from energy market to corn market. The further model examines the effects of the business cycle and energy act events on spillovers from energy. The estimation results are presented in Table 5. As above, the estimated coefficients in corn cash and futures spillover models are very close. As expected, the business cycle proxied by crude oil

prices has a positive effect on the spillover from energy market. Higher crude oil price will make biofuel a more competitive substitute for fossil fuel. Surprisingly, the introduction of energy act appears to have negative effects on volatility spillovers from energy market, although it is statistically insignificant. The reason behind it may be due to collinearity between the crude oil price and the dummy variable of energy act. Overall, important events do have an impact on the spillover effects from energy market, but the magnitude and significance differ from event to event.

### **Optimal Hedge Strategy**

When storage traders participate in both cash and futures markets they must choose a hedging instruments and ratio to maximize his utility or minimize asset risk. Informational linkage between corn market and energy market provides an opportunity for them to expand their asset portfolio and improve their risk management strategies. Consider an investor with a fixed corn cash position who wishes to hedge some proportion of this cash position in not only corn futures market, but also crude oil futures market. Following Myers (1991), we use one-period portfolio selection framework and the initial wealth is allocated into a risky asset and a risk-free bond. The investor has a utility function, defined over the end-of-period wealth. Utility is assumed to be increasing, strictly concave and twice differentiable. The objective of this investor is to:

$$\max_{q_{t-1}, b_{t-1}, a_{t-1}} E[U(W_t)|I_{t-1}]$$

Subject to

$$W_t = (1+r)(W_{t-1} - p_{t-1}q_{t-1}) + p_t q_{t-1} + b_{t-1}(f_{c,t} - f_{c,t-1}) + a_{t-1}(f_{o,t} - f_{o,t-1})$$

where  $W_t$  is end-of-period wealth;  $W_{t-1}$  is initial wealth;  $r$  is the risk-free interest rate;  $p_{t-1}$  is the initial corn cash price;  $p_t$  is the end-of-period corn cash price;  $q_{t-1}$  is the quantity of cash corn purchased;  $f_{c,t-1}$  and  $f_{o,t-1}$  are the initial corn and crude oil futures prices;  $f_{c,t}$  and  $f_{o,t}$  are the (stochastic) end-of-period futures prices; and  $b_{t-1}$  and  $a_{t-1}$  are the quantity of corn and crude oil futures purchased (sold if negative).

The three first order conditions can therefore be written as

$$E[U'(W_t)|I_{t-1}]\mu_t^p + Cov[U'(W_t), p_t|I_{t-1}] = 0$$

$$E[U'(W_t)|I_{t-1}]\mu_t^{f_c} + Cov[U'(W_t), f_{c,t}|I_{t-1}] = 0$$

$$E[U'(W_t)|I_{t-1}]\mu_t^{f_o} + Cov[U'(W_t), f_{o,t}|I_{t-1}] = 0$$

where  $\mu_t^p$  and  $\mu_t^f$  are the conditional expected returns from holding cash and futures positions, respectively:

$$\mu_t^p = E(p_t|I_{t-1}) - (1+r)p_{t-1}$$

$$\mu_t^{f_c} = E(f_{c,t}|I_{t-1}) - f_{c,t-1}$$

$$\mu_t^{f_o} = E(f_{o,t}|I_{t-1}) - f_{o,t-1}$$

Assuming the joint distribution of  $\{W_t, p_t, f_{c,t}, f_{o,t}\}$  conditional on  $I_{t-1}$  is multivariate normal, then first order conditions can be written as<sup>ii</sup>

$$\mu_t^p + \frac{U''}{U'} Cov(W_t, p_t|I_{t-1}) = 0$$

$$\mu_t^{f_c} + \frac{U''}{U'} Cov(W_t, f_{c,t}|I_{t-1}) = 0$$

$$\mu_t^{f_o} + \frac{U''}{U'} Cov(W_t, f_{o,t}|I_{t-1}) = 0$$

This leads to

$$M_t \begin{bmatrix} q_{t-1} \\ b_{t-1} \\ a_{t-1} \end{bmatrix} = -\frac{U'}{U''} \begin{bmatrix} \mu_t^p \\ \mu_t^{f_c} \\ \mu_t^{f_o} \end{bmatrix}$$

where  $M_t$  is the covariance matrix of  $(p_t, f_{c,t}, f_{o,t})'$  conditional on the information set.

$$\begin{bmatrix} q_{t-1} \\ b_{t-1} \\ a_{t-1} \end{bmatrix} = -\frac{U'}{U''} \frac{1}{|M_t|} \begin{pmatrix} m_t^{22}m_t^{33} - m_t^{23}m_t^{32} & m_t^{23}m_t^{31} - m_t^{33}m_t^{21} & m_t^{21}m_t^{32} - m_t^{31}m_t^{22} \\ m_t^{13}m_t^{32} - m_t^{12}m_t^{33} & m_t^{11}m_t^{33} - m_t^{13}m_t^{31} & m_t^{12}m_t^{31} - m_t^{11}m_t^{32} \\ m_t^{12}m_t^{23} - m_t^{13}m_t^{22} & m_t^{21}m_t^{13} - m_t^{11}m_t^{32} & m_t^{22}m_t^{11} - m_t^{21}m_t^{12} \end{pmatrix} \begin{bmatrix} \mu_t^p \\ \mu_t^{f_c} \\ \mu_t^{f_o} \end{bmatrix}$$

where  $m_t^{ij}$  is the element in the  $i$ -th row and the  $j$ -th column of  $M_t$ .

The optimal hedge ratio is the proportion of the long cash position which should be covered by corn and crude oil futures selling. It is often assumed that the expected return to trading futures is approximately zero ( $\mu_t^{f_c} = \mu_t^{f_o} = 0$ ). So the optimal hedge ratio gives

$$-a_{t-1} : -b_{t-1} : q_{t-1} = (m_t^{13}m_t^{22} - m_t^{12}m_t^{23}) : (m_t^{12}m_t^{33} - m_t^{13}m_t^{32}) : (m_t^{22}m_t^{33} - m_t^{23}m_t^{32})$$

According to constant spillover models, we know that

$$m_t^{11} = E(\varepsilon_{p,t}^2 | I_{t-1}) = H_t^{11} + \varphi^2 \sigma_t^2$$

$$m_t^{12} = E(\varepsilon_{p,t} \varepsilon_{f,t} | I_{t-1}) = H_t^{12} + \varphi \omega \sigma_t^2$$

$$m_t^{22} = E(\varepsilon_{f,t}^2 | I_{t-1}) = H_t^{22} + \omega^2 \sigma_t^2$$

$$m_t^{13} = E(\varepsilon_{p,t} e_{o,t} | I_{t-1}) = \varphi \sigma_t^2$$

$$m_t^{23} = m_t^{32} = E(\varepsilon_{f,t} e_{o,t} | I_{t-1}) = \omega \sigma_t^2$$

$$m_t^{33} = E(e_{o,t}^2 | I_{t-1}) = \sigma_t^2$$

## Comparisons of Hedging Performance

In order to compare different hedge strategies, first we break the whole data into two parts to implement two-horizon analysis: in-sample and out-of-sample. In-sample is for the period from 2 January 1992 to 22 March 2007. Out-of-sample is for 29 March 2007 to 19 March 2009, yielding additional 100 observations. Assume an investor holds one bushel of cash corn continuously over the sample period. The investor hedges fluctuations in his or her wealth (caused by fluctuating cash prices) by selling nearest expiration futures contracts of corn and crude oil. An increase in the cash value of a bushel may be offset by a loss on futures, or vice versa. The futures position can be adjusted on a weekly basis conditional on all past information. As the futures contract matures, futures positions are rolled over into the next expiration contract<sup>iii</sup>. Performance is evaluated in terms of effects on the mean and variance of the investor's wealth position of each strategy. The performance comparisons are conducted under four different hedging rules: no hedge, constant hedge, GARCH hedging without spillover effects and cross hedging taking account of spillover effects. For out-of-sample performance evaluation, we implement dynamic forecasts. That is, the model is re-estimated with the new observation included and the optimal hedge ratio is re-calculated. This process continues until 100 forecasts have been generated.

<TABLE 6 >

Results in the left panel of Table 6 illustrate the average in-sample hedge ratios and their respective standard deviations generated from each strategy. Interestingly, two time-varying hedging models recommend holding fairly similar magnitude of corn futures, which is less than the constant hedge ratio. Furthermore, two standard deviations are also similar. Different from expectation, the cross hedging strategy suggests holding long crude oil futures position, although the quantity is fairly small. However, out-of-sample hedge ratios differ substantially between different hedge rules. The conventional time-varying hedge ratio is close to constant hedge ratio,

while the cross hedge ratio declines with a small proportion of holding long crude oil futures contract.

<TABLE 7>

Evaluations of out-of-sample hedge ratio performance in terms of effects on the mean and variance of the investor's wealth position are reported in Table 7. The results are consistent with Myers (1991): the constant hedge, the conventional GARCH hedge, and the cross hedge all provide a remarkably similar hedging performance. From the standpoint of wealth standard deviation, no-hedge strategy in the in-sample horizon performs best, while the GARCH hedge is the best in the out-of-sample horizon. These results do not show overwhelming support for cross hedge strategy over alternative strategies. As an additional issue, the cross hedging strategy may incur extra commission charges (because of the involvement in multiple markets) and difficult to estimate. It may not be a sufficiently competitive strategy in reducing asset risk.

## **Conclusions**

The article investigates the changing nature and the magnitude of volatility spillover effects from energy market to corn market. We follow the approach adopted by Ng (2000) is followed. It is found that volatility spillover effect on corn cash and futures markets from energy market are almost equal and both are significant. The relative importance of spillover effect of energy market is influenced by the business cycle (proxied by crude oil price). In the period of much higher crude oil price, more substantial volatility spillover occurs. However, the proportion of corn market volatility influenced by energy market is generally small indicated by the fairly small coefficients, which suggests that corn market is more agriculture-relevant than energy-

relevant . It is also found that the cross hedge strategy does not provide superior hedging performance than either the constant hedge ratio model or the GARCH model. Both within sample and out-of-sample evaluations demonstrates no potential rewards of applying such cross market hedge strategy even if their price are closely linked together.

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<sup>i</sup> The two-part method is not so efficient as the simultaneous method, but it brings about convenience and still generates consistent coefficient estimates.

<sup>ii</sup> The derivation is based on the relation  $Cov[g(x), y] = E[g'(x)]Cov(x, y)$ , if random variables  $x$  and  $y$  are joint normally distributed, and  $g$  is a differentiable function.

<sup>iii</sup> Rolling cost is not considered in the performance evaluation because each strategy is involved in contract rolling.



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Table 1. Descriptive Statistics and Correlation Analysis on Corn Cash and Futures Price and Crude Oil Futures Price

	Descriptive Statistics			Correlation		
	$p_t$	$f_{c,t}$	$f_{o,t}$	$p_t$	$f_{c,t}$	$f_{o,t}$
2 January 1992-28 July 2005(686 obs.)						
Mean	2.399	2.470	24.806	1.000		
Std. dev.	0.595	0.537	9.576	0.981	1.000	
Min	1.580	1.763	10.720	-0.190	-0.205	1.000
Max	5.430	5.400	60.730			
Skewness	-0.644	-0.614	1.471			
Kurtosis	10.995	13.256	5.061			
28 July 2005-3 July 2008(150 obs.)						
Mean	3.311	3.481	75.883	1.000		
Std. dev.	1.345	1.341	20.270	0.997	1.000	
Min	1.525	1.883	50.480	0.801	0.808	1.000
Max	7.195	7.538	145.290			
Skewness	-0.108	0.220	1.471			
Kurtosis	5.573	4.953	4.478			
3 July 2008-19 March 2009(34 obs.)						
Mean	4.315	4.479	75.059	1.000		
Std. dev.	0.881	1.001	34.899	0.994	1.000	
Min	3.180	3.183	33.980	0.937	0.956	1.000
Max	6.395	6.753	141.650			
Skewness	-0.680	-0.604	0.370			
Kurtosis	3.624	3.178	1.566			
All Sample (870 obs.)						
Mean	2.631	2.722	35.577	1.000		
Std. dev.	0.923	0.919	24.939	0.990	1.000	
Min	1.525	1.763	10.720	0.577	0.626	1.000
Max	7.195	7.538	145.290			
Skewness	-1.020	-0.937	1.823			
Kurtosis	13.238	13.264	6.198			

Notes: skewness and kurtosis are presented for the weekly change in the price series.

**Table 2. Univariate GARCH (1, 1) Models for Crude Oil Futures Price**

$$\Delta f_{o,t} = e_{o,t}$$

$$e_{o,t} | I_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 + a_2 e_{o,t-1}^2$$

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Variance Equation				
$a_0$	0.006058	0.006203	0.976533	0.3288
$a_1$	0.103268	0.019440	5.312128	0.0000
$a_2$	0.906954	0.017860	50.78092	0.0000
Log likelihood	-1510.539			

**Table 3. Error Correction Model for Corn Cash and Futures Prices**

$$\Delta p_t = \beta_0 + \sum_{i=1}^6 \beta_i \Delta p_{t-i} + \sum_{i=1}^6 \beta_{i+6} \Delta f_{c,t-i} + \tau_1 ECT_{t-1} + \varepsilon_{p,t}$$

$$\Delta f_{c,t} = \theta_0 + \sum_{i=1}^6 \theta_i \Delta p_{t-i} + \sum_{i=1}^6 \theta_{i+6} \Delta f_{c,t-i} + \tau_2 ECT_{t-1} + \varepsilon_{f,t}$$

	$\Delta p_t$	St. De.		$\Delta f_{c,t}$	St. De.
$\tau_1$	-0.104695	0.04094	$\tau_2$	-0.024347	0.04118
$\beta_1$	0.236160	0.09057	$\theta_1$	0.138394	0.09110
$\beta_2$	0.092332	0.08908	$\theta_2$	-0.007653	0.08960
$\beta_3$	0.108516	0.08878	$\theta_3$	0.187536	0.08930
$\beta_4$	0.186942	0.08797	$\theta_4$	0.225801	0.08848
$\beta_5$	-0.053193	0.08825	$\theta_5$	0.064351	0.08877
$\beta_6$	0.000903	0.08850	$\theta_6$	0.095078	0.08902
$\beta_7$	-0.211326	0.09070	$\theta_7$	-0.067820	0.09123
$\beta_8$	-0.109380	0.08990	$\theta_8$	0.006105	0.09043
$\beta_9$	-0.083495	0.08949	$\theta_9$	-0.168465	0.09002
$\beta_{10}$	-0.181423	0.08853	$\theta_{10}$	-0.241287	0.08905
$\beta_{11}$	0.149414	0.08892	$\theta_{11}$	0.021162	0.08944
$\beta_{12}$	-0.042896	0.08932	$\theta_{12}$	-0.162686	0.08985
$\beta_0$	0.001323	0.00445	$\theta_0$	0.001350	0.00447

**Table 4. Estimation of Constant Volatility Spillover Effect and BEKK GARCH Models**

$$\begin{aligned} \varepsilon_{p,t} &= e_{p,t} + \varphi_t e_{o,t} \\ \varepsilon_{f,t} &= e_{f,t} + \omega_t e_{o,t} \\ e_t &= (e_{p,t}, e_{f,t})' | I_{t-1} \sim N(0, H_t) \\ H_t &= C'C + A'e_t e_t' A + B'H_{t-1} B \end{aligned}$$

	Coefficient	Std. Error	z-Statistic	Prob.
$\varphi$	0.005683	0.002713	2.094735	0.0362
$\omega$	0.004162	0.002489	1.671838	0.0946

Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1,1)	0.000295	9.58E-05	3.075834	0.0021
C(1,2)	0.000267	7.64E-05	3.486814	0.0005
C(2,2)	0.000319	8.42E-05	3.793901	0.0001
A(1,1)	0.100830	0.016492	6.113973	0.0000
A(1,2)	0.090501	0.017013	5.319423	0.0000
A(2,2)	0.097414	0.019924	4.889276	0.0000
B(1,1)	0.858349	0.025727	33.36317	0.0000
B(1,2)	0.864216	0.022879	37.77310	0.0000
B(2,2)	0.857826	0.023901	35.89033	0.0000

**Table 5. Time-Varying Spillover Effects and BEKK GARCH Models**

$$\begin{aligned} \varepsilon_{p,t} &= e_{p,t} + \varphi_t e_{o,t} \\ \varepsilon_{f,t} &= e_{f,t} + \omega_t e_{o,t} \\ \varphi_t &= \varphi_0 + \varphi_1 f_{o,t} + \varphi_2 D_t \\ \omega_t &= \omega_0 + \omega_1 f_{o,t} + \omega_2 D_t \\ e_t &= (e_{p,t}, e_{f,t})' | I_{t-1} \sim N(0, H_t) \\ H_t &= C' C + A' e_t e_t' A + B' H_{t-1} B \end{aligned}$$

	Coefficient	Std. Error	z-Statistic	Prob.
$\varphi_0$	-0.012027	0.004246	-2.832581	0.0046
$\varphi_1$	0.000458	0.000114	4.010414	0.0001
$\varphi_2$	-0.006817	0.005981	-1.139707	0.2544
$\omega_0$	-0.014270	0.004285	-3.330231	0.0009
$\omega_1$	0.000542	0.000115	4.702868	0.0000
$\omega_2$	-0.015902	0.006091	-2.610785	0.0090

Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1,1)	0.000368	0.000107	3.452975	0.0006
C(1,2)	0.000347	9.73E-05	3.569985	0.0004
C(2,2)	0.000398	0.000105	3.774355	0.0002
A(1,1)	0.118002	0.018664	6.322569	0.0000
A(1,2)	0.111134	0.017982	6.180248	0.0000
A(2,2)	0.117209	0.020176	5.809270	0.0000

B(1,1)	0.831309	0.028068	29.61724	0.0000
B(1,2)	0.830481	0.028016	29.64356	0.0000
B(2,2)	0.827500	0.027854	29.70884	0.0000

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**Table 6. Average Hedge Ratio**

	In-sample		Out-of-sample	
	Corn futures	Oil futures	Corn futures	Oil futures
No hedge	0	0	0	0
Constant Hedge	0.9706	0	0.9704(0.0011)	0
BGARCH	.8665(0.0754)	0	0.9421(0.0505)	0
MGARCH	0.8673(0.0730)	-0.0119(0.0004)	0.7769(0.0235)	-0.0075(0.0045)

Note: figures in parentheses are standard deviations.

**Table 7. In-and Out-of-sample Hedging Performance**

	In-sample		Out of-sample	
	Mean	St. De.	Mean	St. De
No hedge	2.4052	0.6140	4.3703	1.0480
Constant Hedge	2.4031	0.6141	4.3716	1.0426
BGARCH	2.4023	0.6160	4.3693	1.0400
MGARCH	2.4030	0.6164	4.3709	1.0471