Assessing the Uncertainty of Land Based Carbon Sequestration: A Parameter

Uncertainty Analysis with a Global Land Use Model

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Yoon Hyung Kim
AED Economics
Ohio State University
kim.1933@osu.edu

Brent Sohngen
AED Economics
Ohio State University
sohngen.1@osu.edu

Abstract

Several studies have suggested that it may be efficient to reduce net greenhouse gas (GHG) emissions through carbon sequestration in forests or through reductions in deforestation (e.g., Sohngen & Mendelsohn, 2003; Tavoni et al., 2007; Kindermann et al., 2008). These studies, however, are based on large-scale deterministic models of global land uses. As such, the modelers make numerous assumptions about parameters that could have important effects on the projected costs. For example, the model used by Sohngen and Mendelsohn (2003) and Tavoni et al. (2007) assumes that land supply elasticity is 0.25. Although there is some empirical support for this in the United States, land supply elasticity has not been estimated globally. It would be useful to assess the implications of uncertainty in this parameter on the global supply of carbon, particularly since carbon from the projected land use activities can have potentially large effects on the costs of economy-wide carbon constraints.

This paper develops an uncertainty analysis of a number of important parameters in the global land use model of Sohngen and Mendelsohn (2003). Uncertainty analysis can assist policy decisions by providing information about the distribution of potential outcomes, and it can help identify factors that have the biggest impacts on uncertain outcomes. Sensitivity analysis, while useful, cannot express the results of model uncertainty due to the voluntary and discrete definition of the range of uncertainty relative to input parameters. Furthermore, sensitivity analysis cannot provide explanations about the relative relationships that exist among the possible combinations of input parameters since it analyzes the effects of change based on a single parameter for model results when the

remaining parameters are fixed.

This paper analyzes the effect of uncertainty in several key parameters on the marginal costs of carbon sequestration in forests. These parameters include the land supply elasticity, which governs the conversion of land from agriculture to forests and vice versa; parameters of the forest biomass yield function; parameters of the forest carbon density function; and parameters of the costs functions for accessing inaccessible land. Monte Carlo techniques are thus used to turn the global forest model with no probability (e.g., Sohngen & Mendelsohn, 2003; 2007) into a proper probability model through Latin hypercube sampling.

For this paper, we have restricted our analysis to consideration of probability distributions for only two of the parameters described above. Specifically, these are the parameters of the forest biomass yield function and the land supply elasticity. The importance index and the least square linearization are used to determine the relative contribution of input parameters to the model results. Five hundred model runs in one simulation were performed with covariability among the parameters. The Monte Carlo simulations indicated that most of the uncertainty in forest area in developed countries relates to uncertainty in parameters of the biomass function while in developing countries, where deforestation is more important (e.g., Brazil), the simulation showed the parameters of land supply elasticity to have the most important implications for carbon supply. These results are perhaps not too surprising but they do point to the need to empirically estimate land supply elasticities in regions like Brazil, where such estimates are not currently available in the literature. The results also provide information that can be used to estimate uncertainty intervals for carbon sequestration cost functions.

Introduction

Between 1970 and 2004, carbon dioxide (CO_2) emissions increased about 80%
from 21 to 38 gigatonnes (Gt). As of 2004, they accounted for 77% of total Green House
Gas (GHG) emissions (IPCC 2007). Of the six GHGs identified by the Kyoto Protocol to
the United Nations Framework Convention on Climate Change (UNFCCC), carbon dioxide
received the most scrutiny. The biggest cause of such increases to GHG emissions is
energy consumption by industry and transportation; however, one cannot afford to ignore
the increase in carbon dioxide emissions from land-use change and deforestation. As of
2004, the emission of GHGs due to forestland conversion accounted for about 17.4% of the
total. Given these situations, the policy of carbon sequestration that uses forests appears to
be a relatively low cost means of removing carbon dioxide from the atmosphere (US
Environmental Protection Agency, 1995), and can be an effective long-term option for
carbon abatement when combined with other policies (Sohngen & Mendelsohn 2007). The
Kyoto Protocol even recognizes land use, land-use change, and forestry activities as a
potential means for reducing GHGs in the atmosphere. Among those activities, those
affiliated with forests include afforestation, reforestation and deforestation, activities that
can be used to meet the emission reduction targets in developed countries (Annex I). Given
the significance of forests as carbon sinks, there has been a great deal of research
conducted on forest carbon sequestration. Some researchers point out critical aspects as
being the inevitable existence of uncertainty in the process of modeling natural

environments, such as forests, and the usefulness in policy decisions of providing appropriate information about descriptions of uncertainty as well as the causes of such uncertainty. For the most part, uncertainty anlaysis of carbon sequestration has only been conducted with ecological models to date.

For example, Smith and Health (2001) analyzed the uncertainty of carbon budgets within forests located in the United States by using the forest carbon budget model, FORCARB. This model adds up carbon budgets estimated from various carbon pools, including above-ground portions of hardwood and softwood trees, understory species, forest floor and soil carbon, in an effort to measure the entire carbon budget. Several estimations are generally made to explain the relationship between those carbon pools and carbon budgets, and this process allows one to calculate each carbon pool's budget. The FORCARB model consists of many parameters identified through these estimations. The authors analyzed the influence of model parameter uncertainties on overall model results by using the Monte Carlo simulation technique, and they employed the importance and contribution index to compare and analyze the relative effects of each parameter uncertainty on the model results. In addition, they examined covariability among the model parameters as well as the impacts of the distribution's shape.

Verbeeck et al. (2006) analyzed NEE (net ecosystem exchange) for the Hesse forest in France by using the FORUG model. Like FORCARB, this model is comprised of many parameters that are intended to model CO_2 and H_2O conversions between the forest and air. The authors analyzed the uncertainty via the Monte Carlo technique and used the LSL (least square linearization) technique to analyze the relative contribution of each parameter to the uncertainty of all the model results.

While regional studies such as the two analyses described above are useful, they are susceptible to some important flaws. First, carbon policy is likely to be carried out globally and carbon credits are likely to be traded from country to country. It is thus important to understand the potential supply of carbon credits in different regions, and it useful to assess how "certain" these supply functions may be (Richards & Stokes 2004; Sohngen & Mendelsohn 2007). Second, many regional analyses use static models (e.g., Lubowski et al., 2006) that do not account for the dynamics of forest carbon accumulation over time (e.g., Sohngen & Mendelsohn 2003Dynamic optimization models of land use do account for important dynamic adjustments within timber inventories, but these models have only been used with limited sensitivity analysis to date (Sohngen and Mendelsohn , 2007). Sensitivity analysis is helpful, but it does not provide a fully characterization of the potential uncertainties associated with carbon credits from forests, and it cannot be used to describe relative relationships that exist among the possible combinations of input parameters since it analyzes the effects of change based on a single parameter on model results when the remaining parameters are fixed. As policy makers increasingly rely on these credits to hold down carbon prices (Tavoni et al., 2007), it will be useful to more carefully illustrate this uncertainty to them more carefully.

This study extends earlier work of Sohngen and Mendelsohn (2007) by analyzing uncertainty in forest carbon sequestration. The objective of this study is twofold. First, we use the Monte Carlo analyses to analyze the effects of uncertainty in land-use change between agriculture and forestland as well as the uncertainty of forest yield, which were not dealt with in previous sensitivity analyses, on the uncertainty of carbon sequestration. The importance index and the least square linearization are used to determine the relative

contribution of input parameters to the model results. Second, we estimate uncertainty

intervals for carbon sequestration cost functions.

Definition of 'Uncertainty'

One of the most fundamental and critical procedures in uncertainty analysis

concerns the definition of 'uncertainty'. Models used in uncertainty analysis generally rely

on a definition of uncertainty that relates to the derivations of a model from reality,

derivations which manifest themselves in the process of modeling a complex environment.

Such uncertainty takes a variety of forms due to its variety of causes. The most basic cause

of uncertainty happens when a mathematical model fails to express accurately a complex

environment of reality. This kind of problem cannot be solved by simply setting a

complicated model with which to explain the environment in more detail. In practice, it is

nearly impossible to build a model that explains a natural environment perfectly.

Furthermore, it is not feasible to gather accurate data for the many parameters used to

implement such a model. Another source of uncertainty is that of the model parameters.

Such uncertainty is divided into two types—that which comes from a lack-of-knowledge

(Morgan & Henrion 1990; Smith & Heath 2001; Verbeeck at al 2006; Gottschalk at al.

2007), and that which is associated with variability (Cullen & Frey 1999). In a

deterministic model where a single value is used for each model parameter, only the results

of that value are estimated. Thus, when the parameter value is not the optimal value for

representing the system, it will provide estimates that differ from reality. Uncertainty that

stems from lack-of-knowledge is similar to the deterministic model in that it assumes the

existence of the optimal parameter value. At the same time, though, there is a difference between the two. The former assumes that there is too little information or knowledge to choose a single value and uses PDFs most often to quantify uncertainty about input parameter values. Meanwhile, the uncertainty associated with variability assumes that parameter values keep changing according to time or the situation, instead of assuming one single optimal parameter value. This study assumes the existence of uncertainty derived from lack-of-knowledge, instead of uncertainty associated with variability. That is, it uses PDFs to quantify parameter uncertainty and takes no consideration of parameter changes according to time.

One needs expected values and uncertainty ranges in order to quantify uncertainty using PDFs. When analyzing the influences of parameters on the model uncertainty, it is critical to apply the common criteria for ranges of uncertainty because the wider the range of uncertainty, the bigger the influence it has on the model results. For that purpose, we use the 95% confidence interval of each distribution, using expected values and standard deviation. We base the uncertainty ranges of forest yield on the estimation of parameters of yield function, using data from the Forest Inventory and the analysis program of the United States Forest Service. To operationalize these concepts, we refer to the econometric work of Choi al.(2006) for the elasticity of substitution between land uses.

Estimation in Forest Yield Function using Random Coefficient Model

The yield function plays an important role in the agricultural and forestry model because it determines the yield per hectare, the optimal rotation period and the area of

forest (Sedjo & Lyon, 1990). Researchers commonly use yield functions to quantify the growth of a forest, but it is difficult to design a generic forest model because forest themselves are diverse with many characteristics. Included among these characteristics are climate, soil quality, and fertilizer. All of these serve to substantially influence the yield of forest volume. However, in this study, the yield of merchantable volume in cubic meter per hectare is a function of the age of the stand and the management intensity applied to the stand. Even though the management intensity substantially influences the merchantable stock volume at the time of harvest and there is uncertainty associated with the parameters used in the management intensity, we deal with parameter uncertainty relating only to yield.

For that purpose, we assume that the yield volume in cubic meters per hectare is a function of the age of the stand. There are consequently only two parameters to be estimated in the yield per hectare function. The yield function can be expressed as follows:

(1) $\ln Y_j = \alpha - \beta / T_j + \varepsilon_j,$

where Y_j is the average yield per hectare and T_j is time interval, and α and β are unknown parameters.

Combining all of the measurements into one simple regression is not appropriate here, since data in the forest are structured hierarchically and fitting regression models that ignore this hierarchical structure can lead to false inferences being drawn. Multi-level regression, or the Random coefficient model, is a mixed model of the simple linear regression (complete pooling) and estimates separate models within each group (no pooling) (Gelman & Hill, 2006). It also differs from the usual multiple regression model in that the equation defining the hierarchical linear model contains more than one error term,

one of which is the group level in this study (Snijders & Bosker 1999). The basic idea of multilevel modeling is that the parameters are actually different in each species group and we do not assume they are all the same. Instead, this approach states that β's follow a random distribution. The yield function that expresses the different regression lines for each species is represented as:

(2) $\ln Y_{ij} = \alpha_j - \beta_j / T_{ij} + \varepsilon_{ij}$,

Where Y_{ij} is the average yield per hectare at the jth species of the ith state at age T_{ij}. The intercepts α_j and slope β_j are group-dependent. The group-dependent coefficients can be split into an average coefficient and a group-dependent deviation:

$\alpha_j = \alpha + U_{\alpha j}$

$\beta_j = \beta + U_{\beta j}$

Substitution leads to the model:

(3) $\ln Y_{ij} = \alpha - \beta / T_{ij} + U_{\alpha j} + U_{\beta j} / T_{ij} + \varepsilon_{ij}$.

$\alpha - \beta / T_{ij}$ is called the fixed part of the model and $U_{\alpha j} + U_{\beta j} / T_{ij} + \varepsilon_{ij}$ is called the random part. It is assumed that the random effects $U_{\alpha j}$, $U_{\beta j}$ and ε_{ij} have means 0. Also, given the value of the variable T, these random effects are mutually independent and identically distributed. The variance of ε_{ij} is denoted σ^2 and the variance of $U_{\alpha j}$ and $U_{\beta j}$ are denoted as follows:

$Var(U_{\alpha j}) = \sigma_\alpha^2$, $Var(U_{\beta j}) = \sigma_\beta^2$

Then, the expected value of the regression model is

$E(\ln Y_{ij}) = \alpha - \beta / T_{ij}$,

and the variance is

$Var(\ln Y_{ij}) = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma^2$.

Six states and three major species[1] are selected from each state, based on the size of the area (Forest Resources of the United States, 2002). The major species inventory and areas of each species were obtained from the Forest Inventory and Analysis Program of the United States Forest Service. With the assumption of normality, a regression is implemented using the STATA software package by employing the xtmixed option. Results from the regression are reported in table1.

Table1

Coefficient	α	β
Average	7.63613	-14.14969
Std. Err.	.1450898	1.485786
Z	52.63	-9.52
95% Conf. Interval	(7.351759, 7.920501)	(-17.06178, -11.2376)

The confidence interval is a range of values within which we are confident that the true value lies. It quantifies the degree of uncertainty and gives some information about uncertain parameters. A wide confidence interval suggests great uncertainty, while a narrow confidence interval suggests less uncertainty. The regression results show that there is little uncertainty in α, but considerably more uncertainty in β. Indeed uncertainty for

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[1] See Appendix Table 1

β amount to 21% of its median. Since we have little information on yields obtained in other countries, we apply the same percentage change throughout the simulation.

The Elasticity of Substitution between land uses

The parameter for the elasticity of substitution between land uses is based on the econometric work of Choi at al. (2006). They explored the factors influencing land use change between agriculture, forestry, and urban uses in the midwestern states using the multinomial logit framework.

The form of multinomial logit model is:

$\text{Prob}[\,y_i = j\,] = \exp(\beta'_j X_i) / \sum_j \exp(\beta'_j X_i)$,

Where j denotes specific land use choices, X_i is a vector of the independent variables and

β_j is a vector of coefficients.

The estimation function for the choice of land use and the elasticity of land supply can be formulated as follows:

$\ln(P_j / P_0) = X\beta_j$

$$e_{ij} = f(\beta) = \frac{\partial P_j}{\partial X_i} \frac{\overline{X}_i}{\overline{P}_j} = P_j[\beta_j - \sum_{k=0}^{J} P_k \beta_k] \frac{\overline{X}_i}{\overline{P}_j}.$$

The standard error and confidence interval of the elasticity can be obtained by delta method (Green 2003).

$Var(e_{ij}) = G'VG$

Where G is the gradient vector with the partial derivative of $f(\beta)$ and V is the variance-covariance matrix of β's. The elasticity of forestland supply is -0.516 and the standard deviation of the elasticity is 0.04587. Therefore, the 95% confidence interval for the elasticity of forestland is 0.426 to 0.606. However, the elasticity of forestland supply can be approximated by the elasticity of substitution[2]. We use the elasticity of substitution to reflect the uncertainty in land-use change between agriculture and forestland.

Monte Carlo Simulation

One of the most popular methods of analyzing uncertainty, the Monte Carlo techniques (Smith & Heath 2000, 2001; Ogle at al. 2003; Verbeeck at al 2006; Gottschalk at al. 2007), turns a deterministic model with no probability into a proper probability model. Applying computer-based statistical sampling, this method allows for problem solution. It is especially popular when there is a mathematically complex question, or many input parameters. While sensitivity analysis analyzes the impacts of a parameter's changes on the overall results after fixing all other parameters, monte carlo simulation (MCS) allows several inputs to be used at the same time to create the probability distribution of one or more outputs. Also, MCS analyzes the impacts of possible combinations among input parameters on the overall results of the model by assuming the probability distribution of each parameter. Thus, one can say that MCS is complementary to the usual way of scenario and sensitivity analyses. MCS is usually comprised of three stages (New & Hulme 2000):

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[2] See Appendix

(a) Definition of prior probability distribution for input parameters;

(b) Repetitive model simulation through random sampling of input parameters according to the probability density functions (PDFs) defined earlier; and

(c) Understanding the relationships between the uncertainty of input parameters and model results.

Once the probability density functions of model input parameters are defined, one can easily sample using the computer. One of the simplest sampling methods is Simple Random Sampling (SRS). Random numbers are sampled from the uniform U(0,1) distribution, which can be transformed into drawings from other distributions. For example, the researcher can generate random numbers from $N(\mu, \sigma^2)$ using the fact that if η is distributed as U(0,1), then the inverse cumulative density function (CDF) of normal distribution is distributed as N(0,1); however, the method requires a large number of samples in order to fully express the characteristics of the PDFs. This requirement exists because a clustering problem may arise when a small number of iterations are performed and this clustering problem does not represent low probability outcome. However, low probability outcome could have a major impact on model results and a large number of simulations must be run to include this low probability outcome. Therefore, there rises a need to include this low probability outcome using a parsimonious approach. The stratified sampling method divides distributions into equal probability intervals , whose numbers correspond to the number of samples and then samples are drawn from these intervals. Using this method, one can express characteristics of distributions with even a small number of samples. As a result, a smaller number of samples are needed when compared to

simple random sampling. Latin Hypercube sampling (LHS), as suggested by McKay et al. (1979), is considered the foremost stratified sampling method.

In LHS, one divides each probability distribution into n non-overlapping intervals of equal probability. This allows one to sample values evenly in the sample space. The process is that one samples n values one by one in each probability distribution, randomly selecting values without overlapping and the n values for a uncertain input are paired randomly with n values of the other inputs. In the process of random pairing, there is a possibility of spurious correlation, which is a false relationship between samples in the Latin Hypercube Sampling. This probability increases when the number of samples is relatively smaller than the number of inputs. Such a false relationship can be avoided by using the technique developed by Iman and Conover (1982). Their technique sets all the pair-wise rank correlations at 0 among the input parameters of the model, while maintaining the basic characteristics of LHS. The technique can also be used to induce certain rank correlation among model variables.

Uncertainty and Factor Importance

Input parameters with uncertainty have different impacts on model results. Although it is expected that those parameters with high uncertainty will have a large effect on uncertainty in results, even large uncertainties in some parameters may have negligible effects on uncertainty in results. In essence, the impacts on model results depend on the correlation between parameters and the model structure, but it is difficult to explain the causes and routes of a complex model. Thus, the simplest way to understand the impacts of

input parameters with uncertainty on the results is to investigate the relationship between the input parameters and model results. For that purpose, the importance index and the least square linearization (LSL) that are usually used in uncertainty analysis are employed in this study. The importance index is the rank order correlation coefficient developed by Karl Spearman and used to measure the relative influences of input parameters on the uncertainty of the model results (Cullen & Frey 1999; Vose 2000). In the importance index, one first arranges the data from minimum to maximum value and then uses their ranks instead of the actual data values. When calculated in this way, the index is in no way influenced by the distribution shape of the data set. In other words, only the ranks of the data sets to be analyzed affect the importance index. The index value will not change simply because the distribution shapes of the data sets are altered. The importance index is calculated as:

$$\text{Importance index} = 1 - \left(\frac{6 \sum (\Delta R)^2}{n(n^2 - 1)} \right),$$

Where n is the number of data pairs and ΔR is the difference in the ranks between data values in the same pair. The importance index ranges between +1 and -1 and represents the strength and direction of correlation between each parameter and model result simultaneously. When the rank between two parameters is perfectly positively correlated, for example, the index value becomes 1 ($\sum (\Delta R)^2 = 0$); and if it is perfectly negatively correlated, the value becomes -1 ($\sum (\Delta R)^2 = n(n^2 - 1)/3$).

The other method used in the study is LSL, which is a regression analysis between parameter changes and model results and helps to calculate the effects of each parameter on model results with ease (Lei & Schilling1996; Verbeeck et al. 2006). LSL can be briefly described as follows: Consider a model with n independent parameters $(x_1, x_2, \ldots x_n)$. When one does Monte Carlo simulations m times, one can calculate the means of parameters and model results. Then differences between model results and the mean can be linearized at mean values of parameters $(m_{x1}, m_{x2,} \ldots m_{xn})$. The regression model can be expressed as follows.

$$Y \approx \omega_1 \times (x_1 - m_{x1}) + \omega_2 \times (x_2 - m_{x2}) + \cdots + \omega_n \times (x_n - m_{n2}) + m_y$$

Where Y is model output and is its mean value. Next, the method of least squares (OLS) is used to estimate the coefficients of regression equation. If the deviation from the means can be defined as uncertainty, each coefficient from regression analysis represents the linear relations between the uncertainty of parameters and that of model results. The standard deviation of model output can be calculates as:

$$\delta_Y^2 = \sum_{i=1}^{n} \omega_i^2 \times \delta_{xi}^2$$

Where is the standard deviation of parameter x_i.

The standard deviation of mode output represents the overall uncertainty and illustrates the uncertainty of the entire input parameters on the model results. The contribution of each

parameter to overall uncertainty can be expressed as contribution index as changes of normalized percentages. While high percentage indicates a big degree of influence on the uncertainty of model results, a low percentage indicates a small degree of influence. The contribution of each parameter can be defined as:

$$S_{xi} = \frac{\omega_i^2 \times \delta_{xi}^2}{\delta_Y^2} \times 100\%$$

Results

In an attempt to determine the relative contribution of the forest biomass yield function and the land supply elasticity parameters on the overall model results, we used the important index and LSL in the study. Five hundred model runs in one simulation were performed after we set a 95% confidence interval to prevent unrealistic samples from being extracted, and sampled at that interval. Table 2 shows the results of the important index that indicate the impacts of the input parameters on cropland and livestock land on the baseline (2015). In almost every country, the important index of land supply elasticity is marked 0.9 or higher, which means that land supply elasticity had the biggest influences on the uncertainty of cropland and livestock land. Meanwhile, was evident that the yield parameter had few impacts. As for each parameter's contribution to the uncertainty of the model results calculated with LSL, land supply elasticity made the greatest contribution as shown with the important index.

Table 3 contains two indexes that reveal the impacts of each parameter's uncertainty on forest area, and these findings are rather interesting. In developed countries like the US, the uncertainty in parameters of the biomass function had overwhelming effects on the uncertainty of forest area, which matches the current reality that there is not much land use change but a heavy focus on forest management in developed nations. However, the parameters of land supply elasticity had bigger impacts in Brazil, which correspond to the fact that large amounts of forest are converted to cropland and livestock land in the country and explains the land use change in countries where deforestation is occurring. Since deforestation following land use change produces carbon in the air, land supply elasticity has a significant effect on carbon sequestration in those nations. Using such findings, we can understand the impacts of each parameter on carbon sequestration costs, estimate uncertainty intervals for carbon sequestration cost functions, and gather useful information about the influences of deforestation and the policies needed to prevent them.

Conclusion

The purpose of this paper was to develop an uncertainty analysis of several key parameters in the global land use model of Sohngen and Mendelsohn (2003) and to analyze the effect of uncertainty in input parameters on the marginal costs of carbon sequestration in forests. In this paper, we considered two parameters for analyzing the uncertainty in the model results, including the parameters of the forest biomass yield function and the land supply elasticity. Monte Carlo simulation techniques and Latin Hypercube sampling (LHS)

were employed to generate the samples of input parameters and to analyze model uncertainty.

This paper aims to make several contributions to research in forest carbon sequestration. First, we considered the dynamic global model (Sohngen & Mendelsohn 2007) to better understand the potential supply of carbon credits in different regions by compensating for previous regional studies. Second, we extended the earlier sensitivity analysis of Sohngen and Mendelsohn (2007) by providing a full characterization of the potential uncertainties associated with carbon credits from the forest.

The simulation results indicated the important dependency of input parameters and uncertainty in model output. Most of the uncertainty in the forest area in developed countries related to uncertainty in the parameters of the yield function, while in Brazil, where deforestation is occurring, the simulation showed the parameters of land supply elasticity to have the most significant impact on carbon supply. These results corresponded to the fact that large amounts of forest are converted to agricultural land, and this land use change has a significant effect on carbon credit in that region. The results also provided information about uncertainty intervals for carbon sequestration cost functions in each region, and this information can be useful to policy makers who make decisions related to assessing carbon prices.

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APPENDIX

$$X_F = \frac{XE \cdot (\frac{\alpha_F^\tau}{R_F^\tau})}{(\alpha_C^\tau R_C^{1-\tau} + \alpha_L^\tau R_L^{1-\tau} + \alpha_F^\tau R_F^{1-\tau})^{(\frac{\tau}{\tau-1})}} = \frac{XE \cdot (\frac{\alpha_F^\tau}{R_F^\tau})}{S^{(\frac{\tau}{\tau-1})}}$$

(Let $\alpha_C^\tau R_C^{1-\tau} + \alpha_L^\tau R_L^{1-\tau} + \alpha_F^\tau R_F^{1-\tau}$ be S)

$$\frac{\partial X_F}{\partial R_F} = XE \cdot \left(\frac{-\tau \frac{\alpha_F^\tau}{R_F^{\tau+1}} S^{\frac{\tau}{\tau-1}} + \tau \frac{\alpha_F^\tau}{R_F^\tau} \frac{\alpha_F^\tau}{R_F^\tau} S^{\frac{1}{\tau-1}}}{S^{\frac{\tau}{\tau-1}} S^{\frac{\tau}{\tau-1}}}\right) = XE \cdot \left(\frac{-\tau \frac{\alpha_F^\tau}{R_F^{\tau+1}} + \tau \frac{\alpha_F^\tau}{R_F^\tau} \frac{\alpha_F^\tau}{R_F^\tau} S^{-1}}{S^{\frac{\tau}{\tau-1}}}\right)$$

$$= \frac{XE \cdot \frac{\alpha_F^\tau}{R_F^\tau}}{S^{\frac{\tau}{\tau-1}}} \cdot \left(\frac{-\tau}{R_F} + \tau \frac{\alpha_F^\tau}{R_F^\tau S}\right) = X_F \cdot \left(\frac{-\tau}{R_F} + \tau \frac{\alpha_F^\tau}{R_F^\tau S}\right) = \frac{X_F}{R_F} \cdot \left(-\tau + \tau \frac{\alpha_F^\tau}{S}\right)$$

The elasticity of land supply is

$$= \frac{\partial X_F}{\partial R_F} \frac{R_F}{X_F} = \frac{X_F}{R_F} \cdot \left(-\tau + \tau \frac{\alpha_F^\tau}{S}\right) \cdot \frac{R_F}{X_F} = \left(-\tau + \tau \frac{\alpha_F^\tau}{S}\right) \approx -\tau$$

Table 1

State	Species
California	California mixed conifer: Canyon live oak / interior live oak: Tanoak
Alabama	Loblolly pine: Loblolly pine / hardwood: Mixed upland hardwoods
Maine	Sugar maple / beech / yellow birch: Balsam fir: Paper birch
Michigan	Sugar maple / beech / yellow birch: Aspen: Northern white cedar
Montana	Douglas-fir: Lodgepole pine: Ponderosa pine
Oregon	Douglas-fir: Ponderosa pine: Lodgepole pine

Table2.The impacts of the parameters on cropland and livestock land (2015): Importance Index of supply elasticity parameters and forest biomass yield parameters.

	Cropland		Livestock land	
	Supply elasticity parameters	Forest biomass yield parameter	Supply elasticity parameters	Forest biomass yield parameter
US	- 0.999	0.028	1.000	-0.027
CHINA	0.891	-0.041	1.000	-0.032
BRAZIL	1.000	-0.027	-0.999	0.027
CANADA	1.000	-0.027	-1.003	0.024
RUSSIA	1.000	-0.027	1.000	-0.029
EU ANNEX I	1.000	-0.027	-1.002	0.026
EU NON-ANNEX	1.000	-0.027	0.019	0.070
SOUTH ASIA	-1.000	0.027	1.000	-0.026
CENT AMERICA	-1.001	0.025	-0.995	0.028
REST SOUTH AM	-0.118	- 0.831	-0.970	-0.183
SUB SAHARAN AF	-0.827	0.036	0.818	-0.041
SOUTHEAST ASIA	-1.000	0.027	0.997	-0.060
OCEANIA	0.996	-0.073	-0.919	0.044
JAPAN	0.994	-0.039	-0.966	0.033
EAST ASIA	-1.003	0.013	-0.861	-0.002

Table3.The impacts of the parameters on forestland (2015): Importance Index and LSL contribution of supply elasticity parameters and forest biomass yield parameters.

	Important Index		LSL Contribution to overall uncertainty (%)	
	Supply elasticity parameters	Forest biomass yield parameter	Supply elasticity parameters	Forest biomass yield parameter
US	-0.0514	0.95789	0.1%	99.9%
CHINA	0.01750	-0.44857	0.2%	99.8%
BRAZIL	-0.7975	0.44562	74.9%	25.1%
CANADA	0.21042	0.85323	8.2%	91.8%
RUSSIA	0.32133	0.25933	55.4%	44.6%
EU ANNEX I	0.4559	-0.64399	28.6%	71.4%
EU NON-ANNEX	0.13672	-0.66207	3.7%	96.3%
SOUTH ASIA	-0.22079	-0.53880	16.3%	83.7%
CENT AMERICA	-0.17339	-0.21765	39.4%	60.6%
REST SOUTH AM	0.23129	-0.63954	9.9%	90.1%
SUB SAHARAN AF	0.57780	-0.66728	41.9%	58.1%
SOUTHEAST ASIA	-0.32766	-0.81782	15.2%	84.8%
OCEANIA	0.05279	0.51583	0.9%	99.1%
JAPAN	-0.11202	0.92215	0.9%	99.1%
EAST ASIA	-0.1909	0.77988	0.2%	99.8%