Mathematical Programming Modeling of Agricultural Supply Response

by:

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Introduction

Mathematical programming models have been used widely for simulating decision making at farm level, regional level or sectoral level (Takayama and Judge 1971; McCarl and Spreen 1980; Hazell and Norton 1986). When costs and returns per unit production activity (input and output prices) and input requirements (production functions/relations) are assumed to be constant, producer's resource allocation decisions can be modeled using linear programming. Due to its computational efficiency, linear programming has been used in numerous studies at farm level and regional level. At a more aggregate or sectoral level, the assumption of constant output and input prices may not be valid and price responsive demands and supply responses need to be incorporated. Various large-scale nonlinear (typically quadratic) programming models have been used for this purpose where market equilibrium prices and quantities are determined endogenously in a unifying framework by maximizing the sum of producers' and consumers' surplus (Takayama and Judge, 1971; McCarl and Spreen, 1980; Norton and Schiefer 1980).

When working at a large scale regional level or sectoral level it is inevitable to use some sort of aggregation in the model and work with aggregate representative producers instead of numerous individual producers. There are two major reasons for this: 1) disaggregate data for individual producers, including resource availability, technology parameters (input uses per unit activity), and crop budgets may not be available; 2) incorporating the resource allocation decisions of all producers in a unified framework would lead to an unmanageably large programming model. To avoid computational difficulties and work with a smaller model that may serve the purposes of the original simulation model individual firms/decision makers need to be aggregated into a small number of firms each of which is assumed to be endowed with the resources available to all firms that comprise the aggregate firm and utilize a production technology that characterizes the ‘average’ technology used by the individual firms. Various approaches have been used for this purpose, such as grouping of firms with similar characteristics (size, etc.) and averaging the data over those firms in the same geographical area.

Whether the input and output prices are constant or determined endogenously, a common problem when using mathematical programming models for farm-level or sectoral analysis is the possibility of extreme specialization in supply responses. This is particularly crucial when working with aggregate representative producers since aggregation bias may distort the data and the decision space underlying the original decision problem. For instance, a region may be assigned a small subset (even just one) of all possible production activities in the model solution simply because the cost benefit parameters for the aggregate problem may make those activities the most profitable ones and resources owned by individual firms are assumed to be traded freely within the aggregate unit, which is unrealistic and therefore would not be allowed in the original formulation (before aggregation). Such extreme solutions may be dramatically different from the observed supply responses, therefore they would be useless for practical purposes. A widely used approach to prevent extreme specialization and obtain diversified solutions is to impose upper/lower bounds (flexibility constraints) in the model. However, this approach is usually based on subjective judgment and may not have a solid justification, therefore the validity of such bounds and the representativeness of the model solutions are often questioned. Another widely used approach to prevent extreme specialization is consideration of multi-output production activities (e.g. crop rotations) instead of simple (one output) production activities. This is consistent with reality since most producers prefer multi-output production alternatives because of agronomic reasons (such as pest problems and resulting yield losses) and also because of hedging against market risks (price uncertainty). Therefore, unlike imposing bounds on planting decisions, this approach is justifiable and reflects the actual decision making behavior. Incorporating multi-output planting decisions allows expanding the production of any given crop only by simultaneous expansion of other crops included in the rotation practices that appear in the optimal solution, therefore the possibility of extreme specialization will be reduced if not totally eliminated. However, this may not be an ultimate solution the problem. Due to data inaccuracies and aggregation bias, still unrealistic crop patterns may come up (which may not be as extreme as the solutions that could be obtained otherwise –i.e. without rotation activities) if the model selects a set of most profitable rotation practices that may not reflect the observed behavior. Altering the flexibility constraints (bounds) or composition of the rotation activities considered in the aggregate model may result in dramatically different solutions than that would be obtained by aggregating the true optimum solutions of the individual firm models.

Responding to the needs described above, McCarl (1982) introduced the 'historical crop-mix approach' as a methodological alternative in programming models of supply response. Instead of detailed micro-level data, this approach relies on historical observations of farmers' aggregate responses, which can be easily obtained from publicly available statistics (such as NASS) and other data sources. Assuming that the 'feasible' solutions must lie within the convex hull (weighted averages) of historical planting decisions the model finds the best combination of those solutions that optimizes the objective function under the prevailing market conditions that may be different from the market conditions that have led to the observed responses. This is done by imposing a constraint which restricts the solution to that range and determining optimal values of the weights assigned to individual crop mixes which are treated as endogenous variables.

The historical crop mix approach described above has a theoretical foundation related to linear programming. The optimum solutions of linear programs (e.g. crop production decisions at firm level) occur at corner solutions (extreme points). Önal and McCarl (1989, 1991) show that the optimum solutions of an aggregate linear program including all firms as independent decision makers are in a one-to-one correspondence with the optimum solutions of the individual firm models. More precisely, the aggregate solution (an extreme point of the aggregate model – assuming linear constraints) is formed by stacking the optimum solutions (extreme points) of the firm level models. Thus, an observed historical crop pattern (mix) reflects the aggregates of the optimum responses of individual farms if we assume that farmers make their resource allocation decisions in an optimization framework. Another important theoretical result in linear programming is that a weighted average (convex combination) of two optimal solutions is again optimal. Therefore, the observed crop mixes can be considered as corner solutions of the decision space of the aggregate producer and an optimum solution would be a convex combination of those extreme points. This eliminates the need for full information about micro-level input/output data and extreme points of the firm problems. Rather, we only need a set of observed aggregate supply responses, namely historical crop mixes that characterize the decision space of the aggregate producer. The model assigns a non-negative weight variable to each historical crop mix that will be determined endogenously. Once the weight variables are determined the optimal aggregate supply response will be determined as a weighted sum of the corresponding historical mixes. Therefore, this approach is computationally simple, incorporates easily available data, and replaces subjective planting flexibility constraints with a constraint set that is theoretically justified. This approach can also be combined with other modeling approaches aiming at limited flexibility, such as crop rotation activities. For a detailed theoretical discussion see Önal and McCarl (1991). Several empirical applications have employed the historical crop mix approach in various contexts (e.g., Adams et al. 1985; Schneider and McCarl 2005; Butt et al. 2005).

Shortcoming of historical mixes

Under 'normal' market conditions the set of historical crop mixes would be adequate to produce satisfactory results with a mathematical programming model. When production possibilities are expected to fall far outside the historical ranges, however, we may encounter a problem that is opposite of extreme specialization, namely the historical crop mixes may become too restrictive. Since, by construct, the optimal responses obtained from the model have to be within the historical ranges they would not allow large deviations from the observed supply responses, but such deviations might occur under market conditions that are very different from the past market conditions due to a supply or demand shift. The unprecedented increase in US ethanol production from corn is a typical example of this and it is in fact the main motivation of the present paper. In the past decade the crop acreage trends have been altered dramatically, both at regional and national levels, as a result of the substantial shift in corn demand driven by the demand for fuel ethanol. This trend is likely to continue in the next decade given the ambitious Renewable Fuels Standards (RFS) and biofuels production targets (mandates) stated by the Energy Independence and Security Act (EISA) of 2007. Annual ethanol production in the US increased from about 1.6 billion gallons in 2000 to 6.5 billion gallons in 2007 (RFA). This increased the competition between energy crops and commercial row crops on existing agricultural lands beyond expectations and resulted in dramatic price volatility, which was particularly noteworthy in the past few years. The EISA targets a 36-billion gallon ethanol production capacity by 2022, of which 15 billion will be corn ethanol and the remaining 21 billion will be comprised by advanced biofuels, mostly cellulosic ethanol derived from crop residues (corn stover, wheat straw), woody biomass and perennial grasses (Figure 1). This will further increase the competition between energy crops and commercial row crops on existing agricultural lands, and may also increase the pressure on degraded or marginal lands and lands set aside for conservation (CRP lands). The recent economic crisis and reduced oil prices have slowed down the expansion in ethanol production capacity and some ethanol plants have curtailed or entirely halted their operations, but if implemented the renewable energy policy would reverse the recent developments and put the ethanol industry back on track and stimulate further expansion. Consequently, producers' acreage responses and crop pattern are likely to be dramatically different from the observed patterns, thus the historically observed crop mixes would be inadequate and too restrictive when simulating supply responses influenced by bioenergy demands and the prospect of planting bioenergy crops on a substantial amount of land. In this paper we introduce a method to address this issue by expanding the crop mixes synthetically based on historical data.

Synthetic crop mixes

The restrictiveness of the historical crop mix approach has been acknowledged by other researchers also. To address this issue, historical mixes are appended by crop mixes that have been generated either by using expert opinion or by using an auxiliary farm level model that allows increased planting flexibility (as in Adams et al., 1985). This paper introduces an alternative approach for enlarging the set of historical mixes in a systematic way relying again on historically observed supply responses.

The method proposed here is quite simple and practical. Suppose using historical acreages and market prices a set of own and cross price acreage elasticities are estimated. We generate 'hypothetical' or 'synthetic' those mixes by considering prospective market conditions (commodity prices). By systematically varying the commodity prices, we generate a number of new 'columns' (vectors of crop acreages) each representing the supply response under a given hypothetical price vector. Specifically, we consider *n* crops and assume that the acreage response of an aggregate producer (representative farmer at regional or sectoral level) is a function of the vec tor of prices of all crops, including the own price and the prices of competitor crops. Using the estimated acreage response elasticities, we can express this functional relationship by:

$$\ln(A_i) = \sum_{j \in I} \varepsilon_{ij} \ln(P_j)\text{, for } i \in I$$

where I is the set of crops; $i, j \in I$; A_i denotes the acreage of crop i; P_j denotes the price of crop j; and ε_{ij} denotes the elasticity of acreage of crop *i* with respect to the price of crop *j*. We then consider N arbitrarily specified probable crop price vectors, denoted by $(\wp_i)_n, n = 1,..,N$. By plugging each of these N prospective price vectors into the above equation, we determine N 'hypothetical' or 'synthetic' acreage responses denoted by $(\mathbb{S}_i)_n, n = 1,..,N$. These mixes are appended to a set of observed mixes denoted by $(A_i)_m, m = 1,..M$. The supply response (crop pattern) is then restricted to be a weighted average (convex combination) of the mixes in the resulting set $\{(A_i)_m, (\mathbb{S}_i)_n\}, n = 1,..,N, m = 1,..M$. Since the elasticities are assumed to be estimated using historical acreage responses to price variations this approach is 'objective' and relies on the observed behavior of producers rather than 'subjective' judgment of experts.

In the next section we explain how the synthetic mixes are used in a prototype price endogenous model along with the historical mixes to determine the market equilibrium. We then present an empirical application of the proposed approach to determine the acreage responses that are consistent with the ethanol mandates and the derived demand for corn under the RFS standards. We compare the results of the model with and without incorporating the synthetic mixes to demonstrate the merits of the proposed method.

Supply responses under unusual market conditions

In this section we present the empirical results of a price endogenous mathematical programming model that utilizes the expanded crop mix approach described above and determines the market equilibrium and acreage responses under market conditions that are substantially different from the market conditions observed in the past. We also determine the market equilibrium without employing synthetic mixes and using only the historical crop mixes and compare the empirical results of the model obtained with both approaches to illustrate the merits of using synthetic mixes.

Given the scope of the paper, we present only a simplified version of the actual mathematical programming model used in the analysis and illustrate the use of expanded crop mixes. Suppose the demand function for crop $i \in I$ is given by $p_i = f_i(q_i)$, where p_i, q_i denote the price and consumption of crop i. Let Q_i denote the equilibrium demand level; c_i, y_i, a_{ik} denote the cost, crop yield and use of input *k* per unit acreage of crop *i*; and b_k denote the availability of input *k*. The model below determines the market equilibrium endogenously:

$$
\begin{aligned}
& Max \quad \sum_i \int_0^{Q_i} f_i(q_i)dq_i - \sum_i c_i X_i \\
& \text{s.t.:} \\
& \qquad Q_i \le y_i X_i \quad \text{for all i} \\
& \qquad X_i = \sum_{m=1}^{M} \lambda_m A_{i\ m} + \sum_{n=1}^{N} \beta_n \mathbb{S}_{i\ n} \quad \text{for all i} \\
& \qquad \sum_i a_{ik} X_i \le b_k \quad \text{for all k} \\
& \qquad \sum_{m=1}^{M} \lambda_m + \sum_{n=1}^{N} \beta_n \le 1 \\
& \qquad Q_i, X_i, \lambda_m, \beta_n \ge 0
\end{aligned}
$$

where the weights λ_m and β_n are to be determined by the mathematical program as endogenous variables. This restricts the acreage responses generated by the model to a weighted average of historical and synthetic crop mixes, thus prevents extreme crop specialization and allows some flexibility beyond the observed acreage responses but within the limits of the synthetic mixes. When only the historical mixes are to be used, the β_n variables and the summation terms involving those variables are eliminated.

In the actual implementation of the model, whose results are reported below, we consider annual and perennial bioenergy crops, a multiperiod planning horizon and dynamic relationships, various tillage practices and rotation activities for producing individual crops, multiple regions and regional crop mixes (both historical and sytnthetic). We used the Illinois data and incorporated the RFS mandates by assuming a proportional share for Illinois in the total ethanol production (both corn and cellulosic ethanol) based on the State’s share in the current ethanol production. The purpose here is to demonstrate the merits of the synthetic mixes rather than providing an indepth analysis of the biofuel mandates.

Table 1 displays some of the key results obtained from the model with and without incorporating synthetic crop mixes (in the latter case the model uses the historical mixes only). As can be seen in the table, incorporating synthetic mixes has a significant impact on the acreage of two major crops, corn and soybeans, produced in Illinois. While corn acreage in year-16 is underestimated only slightly, 3.4%, in the case when synthetic mixes are not included in the model, the impact on soybean acreage is somewhat larger, 6.3%. These results may be considered insignificant, but it should be noted that the limited rotation practices in Illinois (50-50 corn soybeans) play a significant role. Applications to other regions may show dramatic effects on crop pattern. At national level the impacts on supply responses and market prices may be more pronounced than the results reported here.

Table 1: Selected results of the model with and without expanded crop mixes (for Illinois)

	Historical crop mixes only			Historical and synthetic mixes		
	Year-1	Year-10	Year-16	Year-1	Year-10	Year-16
Corn acreage (1000 acres)	11,530	12,714	11,906	11,556	13,026	12,312
Soybean acreage (1000	10,145	8,415	7,019	10,073	8,201	6,581
Biomass acreage (1000	-	710	3,153	-	589	3,123
Corn price ($/bu)	4.70	5.81	6.24	4.70	5.72	6.09
Soybean price ($/bu)	10.57	10.91	11.19	10.58	10.98	11.35
Corn consumption (mil.bu)	1,382	921	811	1,385	957	852
Corn use for ethanol (mil.	464	1,071	1,071	464	1,071	1,071
Soybean consumption	454	367	313	451	355	285

Conclusions

This paper presented an extension of the crop mix approach introduced by McCarl (1982) by incorporating synthetic mixes generated by acreage elasticities (estimated by using historically observed data) and prospective crop prices that may prevail under market conditions that differ significantly from the past. Empirical results of a price endogenous mathematical programming model show that incorporating synthetic mixes reduces the inflexibility of using historical crop mixes alone.

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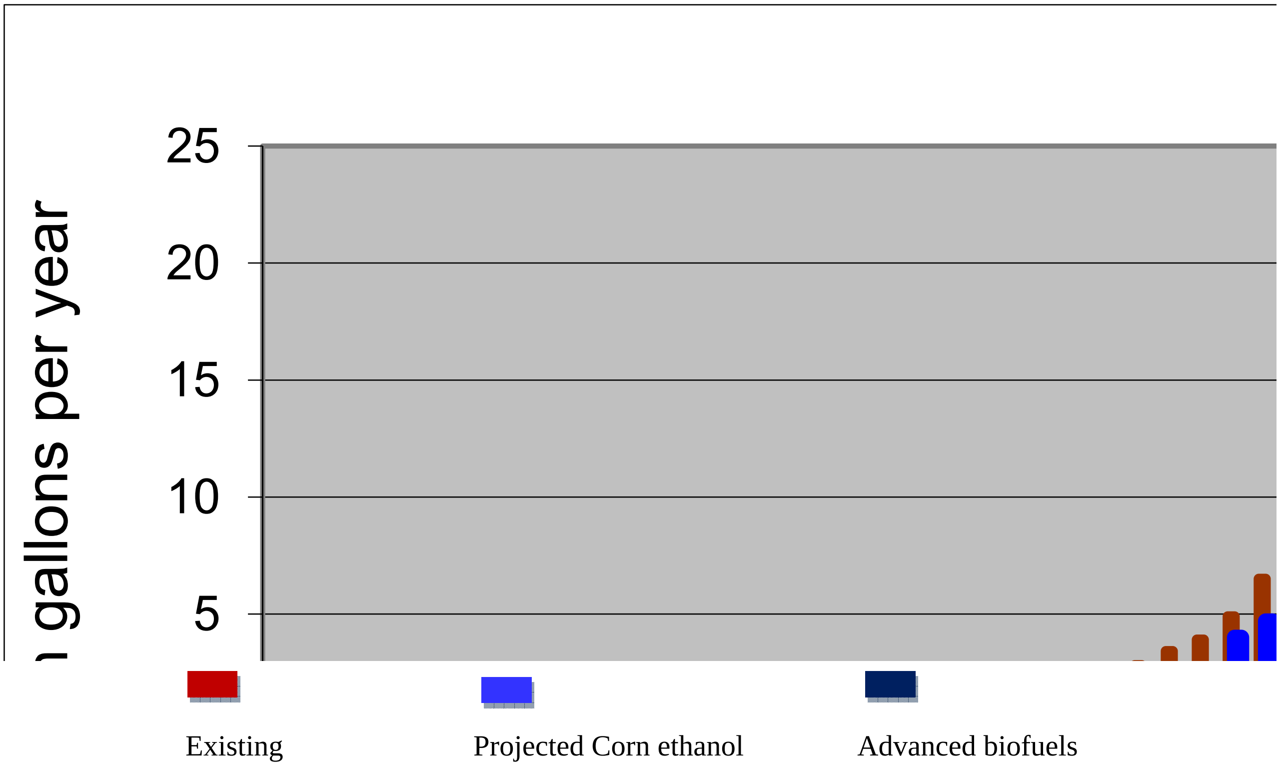

Figure 1: Biofuels production in the US and EISA Mandates.

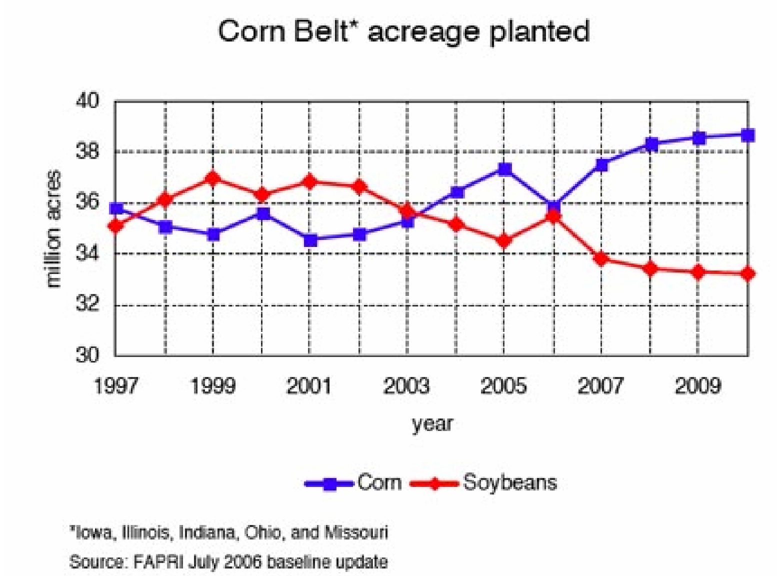

Figure 2: Competition for land between corn and soybeans

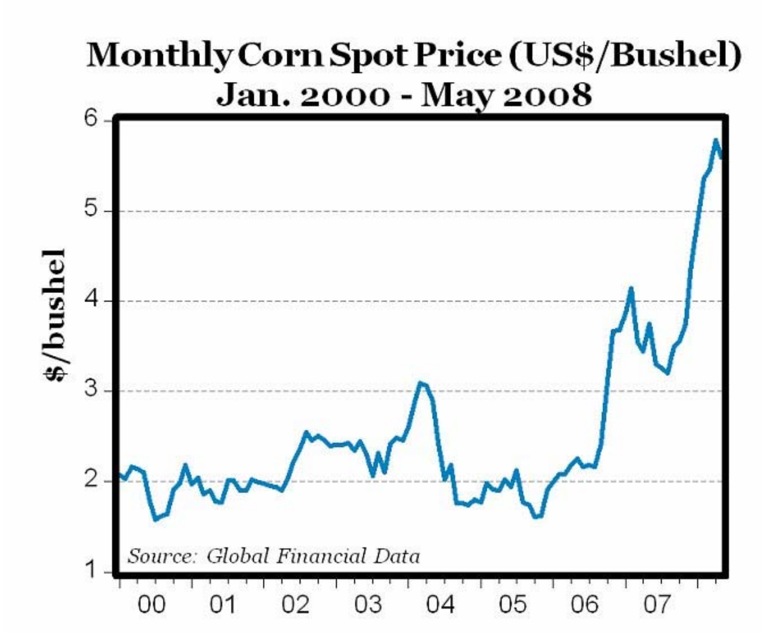

Figure 3: Trend in corn prices