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Estimating age-status-specific demographic rates that are consistent with the projected summary measures in family households projection

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ESTIMATING AGE-STATUS-SPECIFIC DEMOGRAPHIC RATES THAT ARE CONSISTENT WITH THE PROJECTED SUMMARY MEASURES IN FAMILY HOUSEHOLDS PROJECTION[1]

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Abstract

This paper proposes procedures for estimating age-status-specific demographic rates to ensure that the projected summary measures of marriage/union formation and dissolution and marital and non-marital fertility in the future years are achieved consistently. The procedures proposed in this paper can be applied in both macro and micro models for family household or actuarial/welfare projections and simulations that need the time-varying age-status-specific demographic rates as input.

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ESTIMATING AGE-STATUS-SPECIFIC DEMOGRAPHIC RATES THAT ARE CONSISTENT WITH THE PROJECTED SUMMARY MEASURES IN FAMILY HOUSEHOLDS PROJECTION

Introduction

Family households projections with changing age-status-specific demographic rates as input are useful in socio-economic, actuarial and welfare planning, policy analysis, and market trend studies. For example, several welfare programs in the United States restrict eligibility to single-parent families (Yelowitz 1998). As a result, projecting the costs of such programs depends heavily upon projections of the numbers, types and sizes of single-parent family households in the future (Moffitt 2000). What would happen to Chinese family households structure and family support for the elderly in the next decades, if the fertility and mortality rates continue to decrease to a very low level, but divorce rates increase substantially? Family households projections with changing age-status-specific demographic rates are highly responsive to this kind of policy analysis concern (Hammel et al. 1991; Zeng, Vaupel, and Wang 1997; 1998). Another example illustrating the usefulness of family households projection using changing age-status-specific demographic rates as input is that creating a new household, e.g., by divorce or union dissolution, generates an immediate increase in energy consumption. A divorce, by creating a new household, may cause more CO_2 emissions than an additional birth would (Mackellar, Lutz, Prinz, and Goujon 1995). The consumption and market analysis for housing and consumer durables, such as appliances, furniture, health care, cars, water, gas, electricity, the development of household related public utilities and services, needs family households projections, which are among statistical offices' best sellers (George 1999: 8-9).

The practical usefulness explains why family households projection models have received considerable attention from demographers (e.g. Hammel, McDaniel, and Wachter 1981; Van Imhoff and Keilman 1992; Wolf 1994; Wachter 1997; Zeng, Vaupel, and Wang 1997; Zeng, Vaupel, and Wang 1998; Wachter 1998; Tomassini and Wolf 2000).

Yet a technical problem remains to be resolved: how do we estimate demographic rates including age-sex-specific marital status transitions and age-parity-specific marital and non-marital fertility for projecting or simulating family households in the future years? As Keyfitz (1972) pointed out, projection with trend extrapolation of each age-specific rate can result in an excessive concession to flexibility and readily produce erratic results. We, therefore, focus on projection of the demographic summary measures of propensities of marital status transitions, Total Fertility Rate (TFR), and the ratio of the non-marital general fertility rate to the marital general fertility rate, based on trend extrapolation (or expert opinion). We also use a set of age-specific standard schedules to define the age patterns of the demographic processes.

The propensities of marital status transitions are defined as the total number of events of transition from marital status i to j divided by the total number of events that lead to entering marital status i in the context of multi-state marital status life tables (Schoen 1988:95)[1]. For example, the propensity of divorce in a period or a cohort is defined as the total number of divorces divided by the sum of total numbers of first marriages and remarriages during the whole life course of a hypothetical cohort or a real cohort. The period propensities refer to the probabilities of marriage/union formation and dissolution of a hypothetical cohort of persons if they experience the period age-specific rates of the marital status transitions in their life.

The propensities of marital status transitions are clearly defined and easily understandable demographic summary measures that represent the overall average intensity of occurrence of the events of the marital status changes among the at-risk population in the life table context. For example, that the propensity of divorce in year t is 0.4 means that 40% of all marriages would eventually end in divorce, if a hypothetical cohort experienced the observed (or projected) rates of the marital status transitions in year t. TFR and the ratio of the non-marital general fertility rate to the marital general fertility rate are clearly defined and easily understandable summary measures that represent the overall fertility level and fertility differentials between non-married and married women. The projected summary measures that reflect the anticipated changes in marital status and marital and non-marital fertility may be the results of time series analysis or forecasting based mainly on experts options[2].

The needed standard schedules of age-specific rates of marital status transitions and marital and non-marital fertility rates are derived from the recent demographic data resources. We need only one set of the age-specific standard schedules based on recent data from the population under study or from another population that has similar age patterns of the demographic rates as compared to the study population. The age-specific standard schedules define the age pattern of marital status changes and marital and non-marital fertility based on the empirical data and serve as a baseline for estimating the time-varying age-status-specific demographic rates to be consistent with the projected summary measures in the future years. The age-specific standard schedules can either be fixed or include systematic changes in timing and shapes in the projection years[3].

The projected summary measures and the needed standard schedules in the family households projection are similar to the requirement of standard schedules of age-specific fertility, mortality, and migration rates and the projected future years' TFR, life expectancies at birth, and Total Migration Rates in the classical population projection.

The basic strategy for estimating future years' age-specific rates adopted in the family households projection is similar to the one used in the classical population projections, but the procedures differ substantially.

The classical population projection forecasts age and sex distributions. It includes births, deaths, and migration only, but disregards marital, parity and family status changes. In this case, one may follow either non-parametric or parametric approaches (e.g. Lee and Carter 1992; Rogers 1989) to estimate the needed time-varying age-specific fertility, mortality and migration rates independently. The simplest non-parametric approach inflates or deflates the standard age-specific schedules of fertility, mortality, and migration to get age-specific rates that are in consistent with the projected TFR, life expectations at birth, and Total Migration Rates in the future years. For example, if the projected TFR increases by 10%, one may simply inflate all age-specific fertility rates by 10%, independent of changes in age-specific mortality and migration rates.

But the estimation of the age-specific marital status transition rates and age-parity-marital status-specific fertility rates in the family households projection cannot be carried out by simply inflating or deflating each set of age-specific standard schedules of the demographic rates independently. This is because interrelations and consistencies of changes in transitions among various marital statuses and the fertility differentials of married and non-married women must be considered. For example, changes in the propensity of one marital status transition affect the at-risk population and the number of events of other marital status transitions. Changes in the propensities of first marriage and remarriages cause changes in the at-risk population and the number of events of divorce; changes in the propensity of divorce affect the at-risk population and the number of events of remarriage. This interrelation between the events and at-risk populations of various marital status changes is the reason why the projected x% of changes in the propensity of the marital status change cannot be achieved through simply inflating or deflating the corresponding age-specific transition rates by x%.

As an example for purposes of illustration, we consider the observed age-specific probabilities of marital status transitions in 1990-1994, based on the U.S. National Survey of Family Households and National Survey of Family Growth, as the standard schedules. Five marital statuses (single, cohabiting, married, widowed, divorced[4]) are distinguished; nine sets of possible age-specific probabilities of transitions among the five marital statuses are involved (denoted as $p_{ij}(x,s)$, i, j = 1,2,3,4,5; x is from 15 to 99; "s" refers to the standard schedule). We use these standard schedules to construct a multi-state life table to get a initial set of propensities of marital status transitions ($A^1(i,j,s)$, "s" refers to the standard schedule). Suppose that the absolute values of the differences between the projected propensities (A(i,j,t), "t" refers to the projection year) and the initial ones ($A^1(i,j,s)$) are 2-10%. We use $X^1(i,j,t) = \frac{A(i,j,t+1)}{A^1(i,j,s)}$ to get the adjusted $p^1_{ij}(x,t)$ ($p^1_{ij}(x,t) = X^1(i,j,t)\, p_{ij}(x,s)$). We then use the adjusted $p^1_{ij}(x,t)$ to construct a new multi-state life table, but the resulted propensities of the marital status transitions based on $p^1_{ij}(x,t)$ differ from the projected ones by 2-9%. Such descriptions (see Appendix for detailed numerical results) demonstrates clearly why the projected x% of changes in propensity of the marital status changes cannot be achieved through simply inflating or deflating the corresponding age-specific probabilities of the marital status transitions by x%. This is entirely different from the case in the classical population projection that x% changes in TFR can be achieved through simply inflating or deflating the age-specific fertility rates by x%. Thus, estimations of age-specific rates of marital status transitions and non-marital and marital fertility in the future years need procedures that ensure that the projected propensities of marital status changes and fertility will be achieved consistently. We have not seen publications that deal with such estimation procedures. We aim to investigate a systematic approach in this paper to estimate age-status-specific demographic rates to be in consistent with the projected summary measures of demographic changes for enhancing the family households projection.

One may consider projecting the summary measures and age-status-specific demographic rates for each cohort and following the cohort approach to project family households. This, however, is often impractical, since the cohort data on changes in marital status, fertility, mortality, and migration, on which the extrapolation into future trends of each cohort are based, are usually unavailable or incomplete. In contrast, the period data, which can be based on cross-sectional and retrospective survey data or vital statistics, are much easier to obtain. We, therefore, practically follow the period approach to estimate the age-status-specific demographic rates for the family households projection[5], although the procedures proposed in this paper are in principle applicable to both period and cohort approaches. We will present the procedures for estimating age-specific probabilities of marital status transitions and age-parity-marital status-specific probabilities of fertility in the following two sections[6].

Estimating Age-Specific Probabilities of Marital Status Transitions that are Consistent with the Projected Propensities of Marriage/Union Formation and Dissolution

The marital status life table is constructed based on the age-specific probabilities, which are translated from the occurrence/exposure rates of marital status transitions. Instead of using the general term "rates", as was used in the Introduction for simplicity, we will use the more precisely defined term "probabilities" in the rest of this paper. We will establish a set of general simultaneous equations, that yield a consistent set of projected propensities of marital status transitions. Based on the general simultaneous equations, we will then discuss an iterative procedure for estimating the age-specific probabilities of marital status transitions that are consistent with the projected propensities of marriage/union formation and dissolution.

Let A(i,j,t+1) (i,j=1, 2 , ... M, , where M is the number of marital statuses distinguished) denote the projected propensity of transition from marital status i to j in year t+1, i.e., the life table proportion of events of transition from marital status i to j among all events of transitions leading to marital status i in year t+1; l_i(x,t+1), the number of persons aged x with marital status i in the life table population in year t+1; p_{ij}(x,t+1), the probability of transition from marital status i to marital status j between age x and x+1 in year t+1. Thus,

$$\frac{\sum_{x=\alpha}^{\omega} l_i(x,t+1)\, p_{ij}(x,t+1)}{\sum_{x=\alpha}^{\omega} \sum_{k=1,2\ldots M,k\neq i} l_k(x,t+1)\, p_{ki}(x,t+1)} = A(i,j,t+1) \qquad (1)$$

Where α and ω are the lowest and the highest ages at which the marital status transition may occur. Our task is to estimate the unknown p_{ij}(x,t+1) based on the known p_{ij}(x,t), and to estimate p_{ij}(x,t+2) based on p_{ij}(x,t+1), …, and so forth. We start with t=T_0, the starting year of the projection, and let p_{ij}(x,T_0-1) be equal to the standard schedules of probabilities of marital status transitions, which are estimated based on the recent demographic data. The question now is: how can one estimate an unknown p_{ij}(x,t+1) that are in consistent with the projected A(i,j,t+1) based on the known p_{ij}(x,t)?

Let X(i,j,t) denote the factor for adjusting p_{ij}(x,t) in order to estimate p_{ij}(x,t+1). We replace p_{ij}(x,t+1) by p_{ij}(x,t) X(i,j,t) in the simultaneous Eqs. (1):

$$\frac{\sum_{x=\alpha}^{\omega} l_i(x,t+1)\, p_{ij}(x,t)\, X(i,j,t)}{\sum_{x=\alpha}^{\omega} \sum_{k=1,2,\ldots M, k\neq i} l_k(x,t+1)\, p_{ki}(x,t)\, X(k,i,t)} = A(i,j,t+1) \qquad (2)$$

If we knew $l_i(x,t+1)$, the simultaneous Eqs. (2) would be analytically solvable and the unknown adjusting factors X(i,j,t) would be derived directly and analytically, because the number of equations is equal to the number of unknowns in the simultaneous Eqs. (2). Yet $l_i(x,t+1)$, which can be derived only through construction of a multi-state marital status life table based on the age-specific schedule of $p_{ij}(x,t+1)$ (see, e.g., Willekens 1987; Schoen 1988), are, however, unknown. Thus, we cannot directly and analytically solve Eqs. (2) to derive the adjusting factors X(i,j,t). Since $l_k(x,t+1)$ and $l_k(x,t)$ in the two adjunct years are generally close to each other, we replace $l_k(x,t+1)$ by $l_k(x,t)$ as an approximation in Eqs. (2):

$$\frac{\sum_{x=\alpha}^{\omega} l_i(x,t)\, p_{ij}(x,t)\, X(i,j,t)}{\sum_{x=\alpha}^{\omega} \sum_{k=1,2,\ldots M, k\neq i} l_k(x,t)\, p_{ki}(x,t)\, X(k,i,t)} \approx A(i,j,t+1) \qquad \text{(i, j, k=1,2,…M)} \qquad (3)$$

Note that $l_i(x,t)$ and $p_{ij}(x,t)$ are known, X(i,j,t) are unknown, and the left side and right side are approximately equal in Eqs. (3). Here we adopt an iterative procedure to drive $p_{ij}(x,t+1)$ that are in consistent with the projected A(i,j,t+1).

We first use $p_{ij}(x,t)$ to construct a multi-state life table to get the first set of propensities of marital status transitions in year t ($A^1(i,j,t)$), which are not equal to A(i,j,t+1), when the propensities are changing. We then use $X^1(i,j,t) = \frac{A(i,j,t+1)}{A^1(i,j,t)}$ as the first approximation of X(i,j,t) to get the first adjusted $p^1_{ij}(x,t) = X^1(i,j,t)\, p_{ij}(x,t)$. We use the adjusted $p^1_{ij}(x,t)$ to construct a new multi-state life table to get a new set of approximations $A^2(i,j,t)$, which are not equal, but closer to A(i,j,t+1), as compared to

A(i,j,t). We then get $X^2(i,j,t) = \frac{A(i,j,t+1)}{A^2(i,j,t)}$, where X^2(i,j,t) is the second approximation of X(i,j,t). We get $p^2_{ij}(x,t) = X^2(i,j,t)\ p^1_{ij}(x,t)$. We use the adjusted $p^2_{ij}(x,t)$ to construct another new multi-state life table to get another new set of approximations A^3(i,j,t), which are closer to A(i,j,t+1). We repeat this iterative process for n times, until all of the A^n(i,j,t) are almost exactly equal to A(i,j,t+1). For example, the absolute value of the relative discrepancy rate is less than 0.001, namely,

$\left|\frac{A^n(i,j,t+1) - A(i,j,t+1)}{A(i,j,t+1)}\right| < 0.001$. The numerical applications have shown that the above described procedure converges to the true estimates of $p_{ij}(x,t+1)$ that are precisely consistent with the projected propensities of marriage/union formation and dissolution (A(i,j,t+1)). The number of the iterations depends on the number of marital statuses distinguished in the model and the magnitude of the changes in the propensities between year t and t+1.

We present in the Appendix an illustrative numerical application to verify the convergence including nine sets of age-specific marital status transition probabilities among five marital statuses (Single, Married, Widowed, Divorced, and Cohabiting). After 46 iterations (n=46) in this illustrative application, each X^n(i,j,t) is equal to 1.000; each A^n(i,j,t) is extremely close or equal to project A(i,j,t+1); all absolute values of the discrepancy rate between A^n(i,j,t) and A(i,j,t+1) are less than 0.001. The discrepancy rate can be reduced further with additional iterations. For example, the discrepancy rate is less than 0.0001with 91 iterations. We therefore consider that that convergence is achieved, and the goal of estimating $p_{ij}(x,t)$ to be consistent with the projected A(i,j,t+1) has been achieved.

In theory, there are M x M equations with M x M unknowns (X(i,j,t)) in simultaneous Eqs. (3). The M x M equations include some equations containing zeros that represent impossible direct transitions, such as from single to divorced and from married to single. In practice, there are 4 equations and 4 unknowns, 9 equations and 9 unknowns, and 12 equations and 12 unknowns in the models of 4, 5, and 6 marital statuses, respectively[7]. The number of unknowns is equal to the number of equations, so the simultaneous Eqs. (3) are solvable. In our preliminary work, we derived the solutions of the simultaneous Eqs. (3) for the models of 4, 5, and 6 marital statuses. Solving the simultaneous Eqs. (3) involves extremely complicated algebra; the solutions consist of several pages of mathematical expressions (Zeng and Wang 1998). Furthermore, the solutions of Eqs. (3) are the approximations of X(i,j,t) only, since we replace $l_k(x,t+1)$ by $l_k(x,t)$ as proxies in deriving Eqs. (3), and we have to follow basically the same iterative procedure as the one discussed above to derive $p_{ij}(x,t+1)$. The solutions of Eqs. (3) and the iterative procedure led to almost the same results of convergence with A(i,j,t+1) as the ones obtained based on the simpler procedure described above. Such consistency demonstrates that the simpler iterative procedure described above is valid. The simpler procedure is adopted since it avoids unnecessary complications while it produces equally accurate results. We do not present the extremely complicated mathematical expressions of the solutions of Eqs. (3) in this paper (they are available upon request), because they do not make any additional contributions.

Estimating Age-Parity-Marital Status-Specific Probabilities of Births that are Consistent with the Projected Summary Measure of Fertility in the Future Years

Let m denote marital status (e.g. m=1,2,3,4,5, referring to never-married, married, widowed, divorced, and cohabiting);

p, parity status (p=0,1,2,3,…C, where C is the highest parity considered);

fs(x,m,p), standard schedules of age-parity-marital status-specific probabilities of birth

$(p \geq 1)$;

$w(x,m,p,t)$, number of women of age x, marital status m, and parity p in year t before parity status changes are calculated[8];

$f(x,m,p,t)$, age-parity-marital status specific probabilities of birth in the projection year t $(p \geq 1)$;

$TFR(p,t)$, parity-specific total fertility rates for women of all marital statuses combined $(p \geq 1)$; $r(m,t)$, ratios of non-marital fertility to marital fertility; $r(m,t)$ is defined as the ratio of the general fertility rate of women with non-married status m to the general fertility rate of currently married women[9]; the marital status specific general fertility rate is defined as the number of births given by women with marital status m divided by the total number of women with reproductive ages (15 to 49) and marital status m.

The projected or assumed $TFR(p,t)$ and $r(m,t)$ are known. $fs(x,2,p)$, the standard schedule of fertility for married women, is required and known. The standard schedule of fertility for non-married women ($fs(x,m,p)$, $m \neq 2$) is optional, and is either known or can be estimated. $w(x,m,p,t)$ in year t is known from the projection of the preceding year t-1. When t is the starting year (T_0), $w(x,m,p,T_0)$ is derived from the census sample (or 100%) data file. $f(x,m,p,t)$ is unknown, and will be estimated to be consistent with the projected summary measures of fertility ($TFR(p,t)$ and $r(m,t)$) using the following procedures.

Step 1. Initial estimates of f(x,m,p,t)

We let the known age-parity-marital status specific probabilities of birth in year t-1 ($f(x,m,p,t-1)$) be the initial estimate of the unknown age-parity-marital status specific probabilities of birth in year t, denoted as $f'(x,m,p,t)$, namely, $f'(x,m,p,t)=f(x,m,p,t-1)$. In the starting year ($t=T_0$) of the projection, $f'(x,m,p,T_0)$ is equal to $fs(x,m,p)$. If $fs(x,m,p)$ for non-married women ($m \neq 2$) are not available, the $f'(x,m,p,T_0)$ in the starting year is estimated as f'(x,m,p,T_0)=fs(x,2,p) r(m,t), where fs(x,2,p) are standard schedule of age-parity specific probabilities of birth for currently married women. In this case, one assumes that the age trajectory (rather than level) of fertility of unmarried women is the same as that for married women.

Step 2. Estimate B(•,t)

Using the known age-parity-marital status-specific number of women (w(x,m,p,t)) at the beginning of year t and the initial estimate of f'(x,m,p,t), we get initial estimates of TFR'(p,t):

$$\text{TFR'}(p,t)=\sum_{x=\alpha}^{\beta} \frac{\sum_{m} w(x,m,p-1,t)\, f'(x,m,p,t)}{\sum_{p}\sum_{m} w(x,m,p,t)} \qquad (p\geq 1) \qquad (3)$$

where α and β are the lowest and highest ages at childbearing. TFR'(p,t) are not equal to the projected TFR(p,t) if TFR(p,t) change over time. Using [TFR(p,t)/TFR'(p,t)] as an adjusting factor, we get the second estimate of f''(x,m,p,t):

f''(x,m,p,t) = f'(x,m,p,t) [TFR(p,t)/TFR'(p,t)] (4)

where f''(x,m,p,t) are consistent with the projected TFR(p,t). Using w(x,m,p,t) and f''(x,m,p,t), we can get the first estimate of the projected total numbers of births given by women of marital status m (denoted as B'(m,t)) and the total number of births given by all women of all marital statuses combined in year t (denoted as B(•,t)):

$$B'(m,t) = \sum_{x}\sum_{p} [w(x,m,p-1,t)\, f''(x,m,p,t)]$$

$$B(\bullet,t) = \sum_{m} B'(m,t).$$

B(•,t) are consistent with the projected TFR(p,t) and the age-parity-marital status distribution of women (w(x,m,p,t)). B(•,t) is not marital status specific, so it is not affected by changes in r(m,t) in year t as compared to t-1, and is the right estimate of the total number of births for year t. But B'(m,t) and f''(x,m,p,t) are marital status specific, and may not be consistent with the projected ratio of non-marital fertility to marital fertility (r(m,t)) in year t, if r(m,t) changes over time. In other words, although the total number of births given by all women is correctly projected now for year t (i.e. B(•,t)), its marital status specific composition may differ from the right ones due to possible changes in the ratio of non-marital fertility to marital fertility in year t (i.e. r(m,t)). Therefore, we need to further perform the following computations.

Step 3. Estimate B(m,t)

With the estimated B(•,t) and the projected (or assumed) r(m,t), and the total number of women of reproductive ages with marital status m in the year t (i.e. tw(m,t) = $\sum_x \sum_p$ w(x,m,p,t)), we can estimate the projected general fertility rate by the marital status of women in year t, denoted as gf(m,t). gf(m,t) is defined as the number of births with mother's marital status m divided by the total number of women with marital status m and reproductive ages from 15 to 49 in year t. This implies that

$$B(\bullet,t) = \sum_m tw(m,t)\, gf(m,t) = \sum_m [tw(m,t)\, gf(2,t)\, r(m,t)],$$

Therefore,

$$gf(2,t)=B(\bullet,t) / \sum_m [tw(m,t)\, r(m,t)] \quad (6)$$

$$gf(m,t)=gf(2,t)\, r(m,t) \quad (7)$$

$$B(m,t) = tw(m,t)\, gf(m,t)$$

B(m,t) are consistent with the projected TFR(p,t) and the age-parity-marital status distribution of women, as well as the projected ratio of non-marital fertility to marital fertility (r(m,t)) in year t.

Step 4. The final estimate of f(x,m,p,t)

Let $f(x,m,p,t) = f''(x,m,p,t)\, [B(m,t)/B'(m,t)]$ (8)

then f(x,m,p,t) are consistent with the projected TFR(p,t) and the age-parity-marital status distribution of women, as well as the projected ratio of non-marital fertility to marital fertility (r(m,t)) in year t.

Note that r(m,t) are not parity-specific to avoid unmanageable complications. This implies that we implicitly assume that the pattern of parity differentials in ratios of marital fertility and non-marital fertility in the projection years are the same as what were observed in the recent past, which were reflected by the age-parity-specific standard schedules of marital and non-marital fertility. But the ratio of non-marital fertility to marital fertility may change over time. One may wish to consider projecting parity-marital status-specific total fertility rates (TFR(m,p,t)) in the future years. If so, there would be 5x5=25 values of TFR(m,p,t) to be projected for each year of the projection period assuming 5 marital statuses and a highest parity of 5. It would be too difficult to project (or assume) so many parameters of TFR(m,p,t), because of the lack of observed TFR(m,p,t) in the past and the difficulty in keeping them consistent with the overall projected (or assumed) TFR. Thus, we use TFR(p,t) rather than TFR(m,p,t) because we want model to be manageable and the combination of parity-specific TFR(p,t) and marital status specific r(m,t) works reasonably well in presenting the parity and marital status differentials of fertility.

As shown in Eqs. (4)-(8), we use the projected age-parity-marital status-specific number of women in year t before fertility is computed plus other related information to estimate the age-parity-marital status-specific probabilities of birth in year t that are consistent with the projected TFR(p,t). Although we extended our projection to marital status and parity, this is basically similar to what is done in the classical population projections that use projected age-specific number of women in year t and age-specific fertility rates in year t-1 to achieve the projected TFR in year t.

The age-specific probabilities of marital status transitions in year t that are consistent with the projected propensities in year t cannot be estimated based on the age-marital status-specific number of persons at beginning of year t and age-specific probabilities of marital status transitions in year t-1 (or the standard schedules when $t=T_0$). This is because the propensity is defined as the total number of events of transition from marital status i to j divided by the total number of events that lead to entering marital status i during the entire life course in the context of marital status life table (Schoen 1988:95). It makes no sense to compute the ratio of total number of events of transitions from i to j to the total number of events leading to status i in the same year t. For example, we cannot compute propensity of divorce in year t through dividing total number of divorce in year t by total number of marriages in year t, because almost all divorces in year t are dissolution of marriages occurred before year t. We, therefore, need to follow the period life table approach plus the iterative procedure to estimate the age-specific probabilities of marital status transitions to achieve the projected propensities in year t as described in previous section[10]. This is basically similar to and an extension of the life table approach for estimating age-specific probabilities of death that are consistent with the projected life expectancy at birth in year t in the classical population projection.

In short, the procedures proposed in this paper for estimating age-status-specific demographic probabilities that are consistent with the projected summary measures in family households projections are extensions of the approaches of estimating age specific mortality and fertility rates in the classical population projections.

Concluding Remarks

Family households projection/simulation or other relevant projection/simulation (e.g. actuarial forecasting) needs to estimate the age-status-specific demographic rates to achieve the projected (or assumed) summary measures of demographic changes in the future years. The estimation cannot be done by simply inflating or deflating each set of age-specific probabilities of marital status transitions independently, as in estimating fertility rates in the classical population projections. This is because the changes in the propensity of one status transition affect the at-risk population and the number of events of other status transitions. This paper presents procedures for estimating age-status-specific demographic probabilities to ensure that the projected summary measures are achieved consistently. The method proposed in this paper can be applied in both macro and micro models for family households or actuarial/welfare projections and simulations that need the time-varying age-status-specific demographic probabilities as input.

[1] Note that the measurement "propensity of marital status transitions" used in this paper is equivalent to the "probability of marital status transitions" used in Schoen's book (1988: 95). Schoen defines "probability of marital status transitions" as the total number of events of transitions from marital status i to j divided by the total number of events leading to marital status i, which is the same as the definition of "propensity" in this paper. We prefer to use the word "propensity", rather than "probability", in order to distinguish it from the age-specific probabilities of marital status transitions, which are frequently used in multi-state marital status life table construction and family household projection.

[2] One may fit the observed age-specific rates of divorce (or other marital status transitions) to a parametric model (Rogers 1986) with parameters α, β, and γ or more; one (or more than one) of these parameters represents the intensities of divorce (or other marital status transitions). Such parametric modeling is useful in theoretical analysis, but less attractive in the practical application of family household projections because the main parameters α, β and γ are most likely demographically un-interpretable (Rogers 1986: 60), and more difficult for policy makers and the public to understand. It is thus difficult to formulate assumptions about the future trends of these parameters for estimating the age-status-specific demographic rates needed in the family household projections aiming at policy analysis or planning. Furthermore, linking α, β and γ with socio-economic and human behavior variables to better understand or forecast demographic rates is even more implausible. Therefore, we follow the nonparametric approach and use the demographically interpretable and easily understandable summary measures plus the age-specific standard schedules to estimate the age-status-specific demographic rates in the future years.

[3] For cases in which fertility is delaying or advancing, one may simply shift the age-specific standard schedule of fertility to the right or left by the anticipated amount of increase or decrease in the mean age at childbearing, while the shape of the fertility schedule remains unchanged. Or one may assume that fertility would be delayed or advanced while the curve becomes more spread or more concentrated, or, more specifically, assume that young people delay birth more than older people do, or vice versa, through parametric modelling (Zeng et al. 2000).

[4] "Divorced" also includes "legally separated".

[5] The classical population projection also follows the period approach. It projects the age and sex distribution in year t +1 (or t+5 if 5-year age data and a 5-year projection interval are employed) based on the age and sex distribution in year t. The age-specific demographic rates in year t+1 (or t+5) are estimated based on the age-specific demographic rates in year t.

[6] All the rates and propensities referred to in this article are also sex-specific, but we omit the sex dimension in the presentations for simplicity.

[7] The 4 marital statuses model includes single, married, widowed, and divorced; The 5 marital statuses model includes single, married, widowed, divorced, and cohabiting; The 6 marital statuses model includes single, married, widowed, divorced, cohabiting, and separated.

[8] In order to consider the impacts of marital status changes on marital and non-marital fertility, one may take w(x,m,p,t) as the average of w'(x,m,p,t) at beginning of the year before marital status changes in year t are computed and the w''(x,m,p,t) after the marital status changes in year t are computed; both w'(x,m,p,t) and w''(x,m,p,t) are before fertility in year t are computed. The strategy of computing marital status changes and fertility at different points of time in the single year age interval in year t

was first proposed by Bongaarts (1987), and mathematically and numerically verified by Zeng (1991: 61-63 and 81-84).

[9] Obviously, r(2,t) = 1.0, when m=2 (currently married).

[10] One may consider using general rates of marital status transitions as summary measures of marital status changes and age-marital status-specific number of women at beginning of year t to estimate the age-specific probabilities of marital status transitions in year t. The general rate of marital status transitions is defined as total number of events of transition from marital status i to j divided by the total number of persons of marital status i in year t. Such approach is, however, not recommendable, because the general rates of marital status transitions whose denominators include all persons of marital status i may be heavily distorted by age distribution of the population.

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Appendix: An Illustrative Numerical Application to Verify the Convergence

Codes of marital statuses and the marital status transitions:
i,j=1, 2, 3, 4, 5
1. Single; 2. Married; 3. Widowed; 4. Divorced; 5. Cohabiting

Let $p_{ij}(x,s)$ denote the standard schedule of age-specific probabilities of transition from marital status i to marital status j;
$p_{ij}(x,t+1)$, to-be-estimated age-specific probabilities of transition from marital status i to marital status j in year t+1;
A(i,j,t+1), projected propensities of transition from marital status i to marital status j in year t+1;
$X^n(i,j,t)$, n^{th} factor for adjusting $p_{ij}(x,t)$ to estimate $p_{ij}(x,t+1)$.

Standard schedules estimated based on the U.S. National Survey of Family Growth and National Survey of Family Households

x	$p_{12}(x,s)$	$p_{15}(x,s)$	$p_{24}(x,s)$	$p_{32}(x,s)$	$p_{35}(x,s)$	$p_{42}(x,s)$	$p_{45}(x,s)$	$p_{51}(x,s)$	$p_{52}(x,s)$
15	0.0269	0.0380	0.0566	0.0737	0.0636	0.0921	0.3906	0.0830	0.2188
16	0.0310	0.0439	0.0565	0.0737	0.0636	0.0913	0.3864	0.0844	0.2171
17	0.0350	0.0495	0.0564	0.0737	0.0636	0.0904	0.3822	0.0858	0.2154
18	0.0400	0.0567	0.0563	0.0737	0.0636	0.0896	0.3780	0.0873	0.2138
19	0.0467	0.0663	0.0562	0.0737	0.0636	0.0888	0.3740	0.0888	0.2121
20	0.0533	0.0758	0.0562	0.0737	0.0636	0.0880	0.3699	0.0903	0.2105
21	0.0599	0.0852	0.0561	0.0737	0.0636	0.0872	0.3658	0.0918	0.2089
22	0.0665	0.0945	0.0560	0.0737	0.0636	0.0864	0.3616	0.0933	0.2072
23	0.0673	0.0994	0.0539	0.0737	0.0636	0.0883	0.3482	0.0935	0.2047
24	0.0623	0.1000	0.0498	0.0737	0.0636	0.0930	0.3250	0.0922	0.2012
25	0.0574	0.1006	0.0457	0.0737	0.0636	0.0976	0.3009	0.0910	0.1978
26	0.0524	0.1011	0.0416	0.0737	0.0636	0.1022	0.2759	0.0898	0.1943
27	0.0474	0.1017	0.0375	0.0737	0.0636	0.1068	0.2501	0.0886	0.1908
28	0.0443	0.0987	0.0347	0.0737	0.0636	0.1069	0.2278	0.0870	0.1836
29	0.0431	0.0921	0.0332	0.0737	0.0636	0.1024	0.2095	0.0849	0.1726
30	0.0419	0.0854	0.0318	0.0737	0.0636	0.0980	0.1907	0.0828	0.1614
31	0.0407	0.0787	0.0303	0.0737	0.0636	0.0935	0.1714	0.0806	0.1501
32	0.0395	0.0719	0.0289	0.0737	0.0636	0.0890	0.1517	0.0785	0.1386
33	0.0376	0.0672	0.0277	0.0680	0.0632	0.0848	0.1377	0.0763	0.1313
34	0.0351	0.0645	0.0267	0.0565	0.0623	0.0809	0.1296	0.0740	0.1282
35	0.0325	0.0619	0.0258	0.0449	0.0614	0.0769	0.1214	0.0717	0.1251
36	0.0299	0.0592	0.0248	0.0332	0.0606	0.0730	0.1132	0.0693	0.1221
37	0.0273	0.0566	0.0239	0.0213	0.0597	0.0690	0.1048	0.0670	0.1190
38	0.0262	0.0540	0.0228	0.0153	0.0593	0.0643	0.0967	0.0637	0.1156
39	0.0265	0.0516	0.0216	0.0153	0.0593	0.0587	0.0890	0.0596	0.1119
40	0.0268	0.0491	0.0204	0.0153	0.0593	0.0532	0.0811	0.0554	0.1082
41	0.0272	0.0466	0.0192	0.0153	0.0593	0.0476	0.0732	0.0512	0.1045
42	0.0275	0.0441	0.0179	0.0153	0.0593	0.0419	0.0652	0.0470	0.1008
43	0.0262	0.0414	0.0169	0.0149	0.0547	0.0374	0.0586	0.0465	0.0998
44	0.0234	0.0383	0.0159	0.0141	0.0454	0.0341	0.0533	0.0496	0.1016
45	0.0206	0.0352	0.0149	0.0133	0.0361	0.0308	0.0480	0.0527	0.1034
46	0.0178	0.0322	0.0139	0.0125	0.0267	0.0274	0.0427	0.0558	0.1051

47	0.0150	0.0291	0.0129	0.0116	0.0171	0.0241	0.0373	0.0589	0.1069
48	0.0138	0.0253	0.0124	0.0112	0.0124	0.0220	0.0325	0.0612	0.1045
49	0.0142	0.0208	0.0122	0.0112	0.0124	0.0210	0.0281	0.0625	0.0980
50	0.0000	0.0163	0.0120	0.0112	0.0124	0.0201	0.0237	0.0638	0.0914
51	0.0000	0.0118	0.0118	0.0112	0.0124	0.0192	0.0193	0.0652	0.0848
52	0.0000	0.0072	0.0116	0.0112	0.0124	0.0182	0.0149	0.0665	0.0782
53	0.0000	0.0050	0.0107	0.0109	0.0115	0.0178	0.0127	0.0671	0.0748
54	0.0000	0.0050	0.0091	0.0104	0.0099	0.0178	0.0127	0.0671	0.0748
55	0.0000	0.0050	0.0074	0.0099	0.0082	0.0178	0.0127	0.0671	0.0748
56	0.0000	0.0050	0.0058	0.0093	0.0065	0.0178	0.0127	0.0671	0.0748
57	0.0000	0.0050	0.0041	0.0088	0.0049	0.0178	0.0127	0.0671	0.0748
58	0.0000	0.0045	0.0033	0.0085	0.0040	0.0168	0.0116	0.0654	0.0719
59	0.0000	0.0035	0.0034	0.0085	0.0040	0.0149	0.0095	0.0619	0.0661
60	0.0000	0.0025	0.0036	0.0085	0.0040	0.0129	0.0074	0.0584	0.0602
61	0.0000	0.0015	0.0037	0.0085	0.0040	0.0110	0.0053	0.0549	0.0542
62	0.0000	0.0005	0.0037	0.0085	0.0040	0.0090	0.0032	0.0514	0.0483
63	0.0000	0.0000	0.0036	0.0082	0.0037	0.0080	0.0021	0.0496	0.0453
64	0.0000	0.0000	0.0030	0.0076	0.0031	0.0080	0.0021	0.0496	0.0453
65	0.0000	0.0000	0.0025	0.0070	0.0025	0.0080	0.0021	0.0496	0.0453
66	0.0000	0.0000	0.0020	0.0064	0.0018	0.0080	0.0021	0.0496	0.0453
67	0.0000	0.0000	0.0015	0.0058	0.0012	0.0080	0.0021	0.0496	0.0453
68	0.0000	0.0000	0.0012	0.0055	0.0009	0.0077	0.0019	0.0496	0.0453
69	0.0000	0.0000	0.0010	0.0055	0.0009	0.0072	0.0015	0.0496	0.0453
70	0.0000	0.0000	0.0007	0.0055	0.0009	0.0066	0.0011	0.0496	0.0453
71	0.0000	0.0000	0.0005	0.0055	0.0009	0.0060	0.0006	0.0496	0.0453
72	0.0000	0.0000	0.0003	0.0055	0.0009	0.0054	0.0002	0.0496	0.0453
73	0.0000	0.0000	0.0005	0.0050	0.0008	0.0051	0.0000	0.0496	0.0453
74	0.0000	0.0000	0.0010	0.0041	0.0006	0.0051	0.0000	0.0496	0.0453
75	0.0000	0.0000	0.0016	0.0032	0.0004	0.0051	0.0000	0.0496	0.0453
76	0.0000	0.0000	0.0022	0.0023	0.0003	0.0051	0.0000	0.0496	0.0453
77	0.0000	0.0000	0.0027	0.0014	0.0001	0.0051	0.0000	0.0496	0.0453
78	0.0000	0.0000	0.0028	0.0009	0.0000	0.0051	0.0000	0.0448	0.0408
79	0.0000	0.0000	0.0025	0.0009	0.0000	0.0051	0.0000	0.0350	0.0319
80	0.0000	0.0000	0.0021	0.0009	0.0000	0.0051	0.0000	0.0251	0.0229
81	0.0000	0.0000	0.0018	0.0009	0.0000	0.0051	0.0000	0.0152	0.0138
82	0.0000	0.0000	0.0014	0.0009	0.0000	0.0051	0.0000	0.0051	0.0046
83	0.0000	0.0000	0.0012	0.0008	0.0000	0.0051	0.0000	0.0000	0.0000
84	0.0000	0.0000	0.0009	0.0007	0.0000	0.0051	0.0000	0.0000	0.0000
85	0.0000	0.0000	0.0006	0.0005	0.0000	0.0051	0.0000	0.0000	0.0000
86	0.0000	0.0000	0.0004	0.0003	0.0000	0.0051	0.0000	0.0000	0.0000
87	0.0000	0.0000	0.0001	0.0001	0.0000	0.0051	0.0000	0.0000	0.0000
88	0.0000	0.0000	0.0000	0.0000	0.0000	0.0046	0.0000	0.0000	0.0000
89	0.0000	0.0000	0.0000	0.0000	0.0000	0.0036	0.0000	0.0000	0.0000
90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0026	0.0000	0.0000	0.0000
91	0.0000	0.0000	0.0000	0.0000	0.0000	0.0015	0.0000	0.0000	0.0000
92	0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0000	0.0000	0.0000
93	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
94	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
95	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

96	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
97	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
98	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

In the starting year T_0, let $p_{ij}(x,t) = p_{ij}(x,s)$. We first use the $p_{ij}(x,t)$ to construct a multi-state life table to get the first set of propensities of marital status transitions in year t ($A^1(i,j,t)$), which are not equal to A(i,j,t+1), when the propensities are changing. We then use $X^1(i,j,t) = \frac{A(i,j,t+1)}{A^1(i,j,t)}$ to get the first adjusted $p^1_{ij}(x,t)$ ($p^1_{ij}(x,t) = X^1(i,j,t)\ p_{ij}(x,t)$). We use the adjusted $p^1_{ij}(x,t)$ to construct a new multi-state life table to get a new set of approximations $A^2(i,j,t)$.

<u>Number of iterations = 1 (n=1)</u>

I→j	1→2	1→5	2→4	3→2	3→5	4→2	4→5	5→1	5→2
$X^1(i,j,t)$	1.08	1.07	1.10	1.08	0.98	1.08	1.07	1.06	0.97
A(i,j,t+1)	0.4973	0.5895	0.4502	0.0505	0.0295	0.3288	0.5137	0.3593	0.6356
$A^2(i,j,t)$	0.4605	0.5509	0.4093	0.0468	0.0301	0.3044	0.4801	0.3390	0.6553
% diff.	-7.4	-6.5	-9.1	-7.4	2.0	-7.4	-6.5	-5.7	3.1

% diff. = 100 x [$A^2(i,j,t)$-A(i,j,t+1)] / A(i,j,t+1)

The absolute values of “% diff.” are 2-9%. This demonstrates why the projected x% of changes in propensity of the marital status change cannot be achieved through simply inflating or deflating the corresponding age-specific probabilities of the marital status transitions by x%.

After 46 iterations (n=46), the $p^n_{ij}(x,t)$ which match the projected A(i,j,t+1) are estimated and listed as follows. The $X^n(i,j,t)$, $A^n(i,j,t)$ and the % diff. are listed and discussed at the end of the following table.

<u>Number of iterations = 47 (n=47)</u>

x	$P_{12}(x,t+1)$	$p_{15}(x,t+1)$	$p_{24}(x,t+1)$	$p_{32}(x,t+1)$	$p_{35}(x,t+1)$	$p_{42}(x,t+1)$	$p_{45}(x,t+1)$	$p_{51}(x,t+1)$	$p_{52}(x,t+1)$
15	0.0450	0.0698	0.0604	0.0773	0.0587	0.1212	0.4784	0.0892	0.2100
16	0.0518	0.0807	0.0603	0.0773	0.0587	0.1201	0.4733	0.0907	0.2084
17	0.0584	0.0910	0.0603	0.0773	0.0587	0.1190	0.4681	0.0922	0.2068
18	0.0669	0.1044	0.0602	0.0773	0.0587	0.1180	0.4630	0.0938	0.2052
19	0.0780	0.1220	0.0601	0.0773	0.0587	0.1169	0.4580	0.0954	0.2036
20	0.0891	0.1394	0.0600	0.0773	0.0587	0.1159	0.4530	0.0970	0.2021
21	0.1001	0.1567	0.0599	0.0773	0.0587	0.1148	0.4480	0.0987	0.2005
22	0.1110	0.1738	0.0598	0.0773	0.0587	0.1137	0.4429	0.1003	0.1989
23	0.1124	0.1829	0.0576	0.0773	0.0587	0.1163	0.4265	0.1004	0.1965
24	0.1041	0.1839	0.0533	0.0773	0.0587	0.1224	0.3980	0.0991	0.1932
25	0.0958	0.1850	0.0489	0.0773	0.0587	0.1285	0.3685	0.0978	0.1898
26	0.0875	0.1860	0.0445	0.0773	0.0587	0.1346	0.3380	0.0965	0.1865
27	0.0791	0.1871	0.0400	0.0773	0.0587	0.1406	0.3063	0.0952	0.1832
28	0.0739	0.1815	0.0370	0.0773	0.0587	0.1407	0.2790	0.0935	0.1762
29	0.0719	0.1693	0.0355	0.0773	0.0587	0.1348	0.2565	0.0912	0.1657
30	0.0700	0.1571	0.0340	0.0773	0.0587	0.1290	0.2335	0.0889	0.1549
31	0.0680	0.1447	0.0324	0.0773	0.0587	0.1231	0.2100	0.0867	0.1441

32	0.0660	0.1322	0.0309	0.0773	0.0587	0.1171	0.1858	0.0844	0.1331
33	0.0628	0.1235	0.0296	0.0714	0.0583	0.1116	0.1686	0.0820	0.1260
34	0.0585	0.1187	0.0286	0.0594	0.0575	0.1065	0.1587	0.0795	0.1231
35	0.0543	0.1138	0.0276	0.0472	0.0567	0.1013	0.1487	0.0770	0.1201
36	0.0499	0.1090	0.0265	0.0349	0.0559	0.0961	0.1386	0.0745	0.1172
37	0.0456	0.1041	0.0255	0.0224	0.0551	0.0909	0.1284	0.0720	0.1142
38	0.0437	0.0994	0.0244	0.0161	0.0547	0.0847	0.1185	0.0685	0.1109
39	0.0443	0.0948	0.0231	0.0161	0.0547	0.0774	0.1090	0.0640	0.1074
40	0.0448	0.0903	0.0218	0.0161	0.0547	0.0700	0.0993	0.0595	0.1039
41	0.0454	0.0857	0.0205	0.0161	0.0547	0.0626	0.0896	0.0550	0.1003
42	0.0459	0.0812	0.0192	0.0161	0.0547	0.0552	0.0799	0.0505	0.0968
43	0.0438	0.0761	0.0180	0.0157	0.0505	0.0493	0.0717	0.0499	0.0958
44	0.0392	0.0704	0.0170	0.0148	0.0419	0.0449	0.0653	0.0533	0.0975
45	0.0345	0.0648	0.0159	0.0139	0.0333	0.0405	0.0588	0.0567	0.0992
46	0.0297	0.0591	0.0149	0.0131	0.0246	0.0361	0.0523	0.0600	0.1009
47	0.0250	0.0535	0.0138	0.0122	0.0158	0.0317	0.0457	0.0633	0.1026
48	0.0230	0.0465	0.0132	0.0118	0.0114	0.0289	0.0398	0.0657	0.1003
49	0.0237	0.0383	0.0130	0.0118	0.0114	0.0277	0.0344	0.0672	0.0941
50	0.0000	0.0300	0.0128	0.0118	0.0114	0.0265	0.0291	0.0686	0.0878
51	0.0000	0.0217	0.0126	0.0118	0.0114	0.0252	0.0237	0.0700	0.0814
52	0.0000	0.0133	0.0124	0.0118	0.0114	0.0240	0.0183	0.0715	0.0750
53	0.0000	0.0091	0.0114	0.0115	0.0106	0.0234	0.0156	0.0722	0.0718
54	0.0000	0.0091	0.0097	0.0109	0.0091	0.0234	0.0156	0.0722	0.0718
55	0.0000	0.0091	0.0079	0.0104	0.0076	0.0234	0.0156	0.0722	0.0718
56	0.0000	0.0091	0.0062	0.0098	0.0060	0.0234	0.0156	0.0722	0.0718
57	0.0000	0.0091	0.0044	0.0092	0.0045	0.0234	0.0156	0.0722	0.0718
58	0.0000	0.0082	0.0036	0.0090	0.0037	0.0221	0.0143	0.0703	0.0690
59	0.0000	0.0064	0.0037	0.0090	0.0037	0.0196	0.0117	0.0666	0.0634
60	0.0000	0.0046	0.0038	0.0090	0.0037	0.0170	0.0091	0.0628	0.0578
61	0.0000	0.0027	0.0039	0.0090	0.0037	0.0144	0.0065	0.0590	0.0521
62	0.0000	0.0009	0.0040	0.0090	0.0037	0.0119	0.0039	0.0552	0.0464
63	0.0000	0.0000	0.0038	0.0087	0.0034	0.0106	0.0026	0.0533	0.0435
64	0.0000	0.0000	0.0033	0.0080	0.0029	0.0106	0.0026	0.0533	0.0435
65	0.0000	0.0000	0.0027	0.0074	0.0023	0.0106	0.0026	0.0533	0.0435
66	0.0000	0.0000	0.0022	0.0067	0.0017	0.0106	0.0026	0.0533	0.0435
67	0.0000	0.0000	0.0016	0.0061	0.0011	0.0106	0.0026	0.0533	0.0435
68	0.0000	0.0000	0.0012	0.0058	0.0008	0.0102	0.0023	0.0533	0.0435
69	0.0000	0.0000	0.0010	0.0058	0.0008	0.0094	0.0018	0.0533	0.0435
70	0.0000	0.0000	0.0008	0.0058	0.0008	0.0087	0.0013	0.0533	0.0435
71	0.0000	0.0000	0.0006	0.0058	0.0008	0.0079	0.0008	0.0533	0.0435
72	0.0000	0.0000	0.0003	0.0058	0.0008	0.0071	0.0003	0.0533	0.0435
73	0.0000	0.0000	0.0005	0.0053	0.0008	0.0068	0.0000	0.0533	0.0435
74	0.0000	0.0000	0.0011	0.0043	0.0006	0.0068	0.0000	0.0533	0.0435
75	0.0000	0.0000	0.0017	0.0034	0.0004	0.0068	0.0000	0.0533	0.0435
76	0.0000	0.0000	0.0023	0.0024	0.0002	0.0068	0.0000	0.0533	0.0435
77	0.0000	0.0000	0.0029	0.0015	0.0001	0.0068	0.0000	0.0533	0.0435
78	0.0000	0.0000	0.0030	0.0010	0.0000	0.0068	0.0000	0.0481	0.0392
79	0.0000	0.0000	0.0026	0.0010	0.0000	0.0068	0.0000	0.0376	0.0306
80	0.0000	0.0000	0.0023	0.0010	0.0000	0.0068	0.0000	0.0270	0.0220

81	0.0000	0.0000	0.0019	0.0010	0.0000	0.0068	0.0000	0.0163	0.0133
82	0.0000	0.0000	0.0015	0.0010	0.0000	0.0068	0.0000	0.0055	0.0044
83	0.0000	0.0000	0.0012	0.0009	0.0000	0.0068	0.0000	0.0000	0.0000
84	0.0000	0.0000	0.0010	0.0007	0.0000	0.0068	0.0000	0.0000	0.0000
85	0.0000	0.0000	0.0007	0.0005	0.0000	0.0068	0.0000	0.0000	0.0000
86	0.0000	0.0000	0.0004	0.0003	0.0000	0.0068	0.0000	0.0000	0.0000
87	0.0000	0.0000	0.0001	0.0001	0.0000	0.0068	0.0000	0.0000	0.0000
88	0.0000	0.0000	0.0000	0.0000	0.0000	0.0061	0.0000	0.0000	0.0000
89	0.0000	0.0000	0.0000	0.0000	0.0000	0.0047	0.0000	0.0000	0.0000
90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0034	0.0000	0.0000	0.0000
91	0.0000	0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000
92	0.0000	0.0000	0.0000	0.0000	0.0000	0.0007	0.0000	0.0000	0.0000
93	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
94	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
95	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
96	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
97	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
98	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Number of iterations = 46 (n=46)

i→j	1→2	1→5	2→4	3→2	3→5	4→2	4→5	5→1	5→2
X^n(i,j,t)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
A(i,j,t+)	0.4973	0.5895	0.4502	0.0505	0.0295	0.3288	0.5137	0.3593	0.6356
A^{n+1}(i,j,t)	0.4969	0.5889	0.4503	0.0505	0.0295	0.3288	0.5139	0.3593	0.6357
% diff.	-0.09	0.097	0.02	0.01	0.01	0.03	0.04	0.01	0.02

% diff. = 100 x [A^ni,j,t)-A(i,j,t+1)] / A(i,j,t+1)

After 46 iterations, each X^n(i,j,t) is equal to 1.000; each A^n(i,j,t) is extremely close or equal to A(i,j,t+1); all absolute values of the discrepancy rate (% diff.) are less than 0.1%. The discrepancy rate can be reduced further with additional iterations. For example, the discrepancy rate is less than 0.01% with 91 iterations. We therefore consider that that convergence is achieved, and the goal of estimating p_{ij}(x,t) to be consistent with the projected A(i,j,t+1) has been achieved.