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MPIDR WORKING PAPER WP 2003-024
JULY 2003

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ESTIMATING TIME-VARYING SEX-AGE-SPECIFIC O/E RATES OF MARITAL STATUS TRANSITIONS IN FAMILY HOUSEHOLD PROJECTION OR SIMULATION[1]

Zeng Yi, Eric Stallard, and Zhenglian Wang[2]

Abstract

This article presents a procedure for estimating time-varying sex-age-specific occurrence/exposure (o/e) rates of marital status transitions to ensure that the projected life course propensities of marriage/union formation and dissolution are achieved consistently in the one-sex family status life table model. Procedures for estimating time-varying sex-age-specific marital status transition o/e rates that are consistent with the two-sex constraints and projected summary measures of marriage/union formation and dissolution in the future years in the two-sex family household projection model is proposed. The procedures proposed in this article are practically useful and can be applied in both macro and micro models for family household projections or simulations that need time-varying sex-age-specific o/e rates of marital status transitions.

1. Introduction

[1] The preliminary version of this article was part of the first author's presentation at the NIA sponsored workshop on "Future Seniors and Their Kin" in honor of Professor E.A. Hammel's retirement with outstanding career contributions to demography, April 5-7, 2002, Berkeley. Research reported in this article was supported by research grants (1 R03 AG 18647 and 1P01 AG17937) from the National Institute on Aging and institutional support from the Center for Demographic Studies and the Max Planck Institute for Demographic Research.

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Family household projections or simulations with time-varying age-status-specific demographic rates are useful in socio-economic, actuarial and welfare planning, policy analysis, and market trend studies. For example, several welfare programs in the United States restrict eligibility to single-parent families (Yelowitz, 1998). As a result, projecting the costs of such programs depends heavily upon projections of the numbers, types and sizes of single-parent family households in the future (Moffitt, 2000). What would happen to Chinese family household structure and family support for the elderly in the next decades if fertility and mortality rates continue to decrease to very low levels, but divorce rates increase substantially? Family household projections with time-varying age-status-specific demographic rates are highly responsive to this kind of policy analysis concerns (Hammel et al., 1991; Zeng, Vaupel, and Wang, 1997; 1998). Another example illustrating the usefulness of family household projections with changing demographic rates is that creating a new household, e.g., by divorce or union dissolution, generates a greater and more immediate increase in energy consumption than an additional birth would (Mackellar, Lutz, Prinz, and Goujon, 1995). Two recent articles published in *Nature* show that a rapid increase in households of smaller size, which results in higher per capita resource consumption, implies a threat of larger demand for resources (Keilman, 2003) and poses serious challenges to biodiversity conservation (Liu et al., 2003). The consumption and market analysis for housing and consumer durables (such as appliances, furniture, automobiles, water, gas, and electricity), the development of household related public utilities and services, and the determination of long-term care needs for the elderly, require family household projections. Household projections are among statistical offices' best sellers (George, 1999: 8-9). Their practical usefulness explains why family household projection models have received considerable attention from demographers (e.g., Hammel, McDaniel, and Wachter, 1981; Van Imhoff and Keilman, 1992; Wolf, 1994; Wachter ,1997; Wachter, 1998; Tomassini and Wolf, 2000).

Yet a technical problem remains to be resolved: how do we estimate time-varying sex-age-specific marital status transition rates for projecting or simulating family households in the future years? As Keyfitz (1972) pointed out, projection with trend extrapolation of each age-specific rate can result in an excessive concession to flexibility and readily produce erratic results. Proportionally inflating or deflating the age-specific rates without projection of the summary measures is a simplistic option but it cannot provide concise and meaningful indices of demographic changes, which are necessary for informing policy makers and the public. Thus, in the classical population projection, demographers focus on projecting the summary measures of Total Fertility Rates (TFR) and life expectancies, and then proportionally inflate or deflate the age-specific fertility and death rates or apply more sophisticated parametric models for estimating the age-specific rates to achieve the projected TFR and life expectancy.

The basic strategy for estimating future years' time-varying age-specific rates adopted in the family household projection is similar to the one used in the classical population projections, but it requires additional technical procedures. We project the demographic summary measures of marriage/union formation and dissolution. The projection of these demographic summary measures can be based on trend extrapolation or expert opinion. In addition to the demographic summary measures, we also use the age-specific standard schedules of marital status transitions to define the age patterns of the demographic processes. The standard schedules of the age-specific demographic rates are derived from recent data resources. Ideally, the age-specific standard schedules should be based on data from the population under study. They can be taken from another population that has age patterns of demographic rates similar to the study population if the needed data are not available.

Basically, we proportionally inflate or deflate the age-specific standard schedules to estimate time-varying age-status-specific demographic rates that are consistent with the projected summary measures. The age-specific standard schedules can either be assumed to be stable or include systematic changes in timing and shapes in the projection years[1].

The classical population projection forecasts age and sex distributions. It includes births, deaths, and migration only, but disregards changes in marital status; one may follow either non-parametric or parametric approaches (e.g., Lee and Carter, 1992; Rogers, 1986) to estimate the needed time-varying age-specific fertility, mortality, and migration rates independently. The simplest non-parametric approach inflates or deflates the standard age-specific schedules of fertility and mortality to get time-varying age-specific rates that are consistent with the projected TFR and life expectations at birth in the future years. For example, if the projected TFR increases by 10%, one may simply inflate all age-specific fertility rates by 10%.

Estimations of the age-specific occurrence/exposure (o/e) rates of marital status transitions in the family household projection or simulation are, however, not so simple. This is because interrelations and consistencies of changes in transitions among various marital statuses and between males and females (i.e., the two-sex constraint) must be considered.

One important conceptual note must be clarified – we adjust the initial standard schedules of age-specific o/e rates rather than probabilities to achieve consistency with the projected summary measures and the two-sex constraints. The age-specific o/e rate is estimated as the number of events that occurred in the age interval divided by the number of person-years lived at risk of experiencing the event. The age-specific o/e rates can be analytically translated to the age-specific probabilities using the matrix formula in the context of multiple increment-decrement models (see, for example, Willekens et al., 1982; Schoen, 1988; Preston et al., 2001). This approach adequately handles the issues of competing risks. Furthermore, adjusting probabilities directly may result in an inadmissible value that is greater than one. Adjusting age-specific o/e rates would never yield such inadmissible estimates of the probability, however.

In the next section, we outline a model including marital statuses and cohabitation, which is more and more popular in many countries. We then propose procedures for estimating the sex-age-specific o/e rates of marital status transitions in the one-sex family status life table model (Section 3) and the two-sex family household projection model (Sections 4).

2. A Model Including Marital Statuses and Cohabitation

The classic four marital statuses model includes 1. Never-married; 2. Married; 3. Widowed; 4. Divorced (Willekens et al., 1982; Schoen, 1988; see Figure 1). It does not include cohabiting, which is increasingly popular in modern societies. In this study, we employ a model including the four classic marital statuses, cohabitation, and their combinations (see Figure 2): 1. Never-married & not-cohabiting; 2. Married; 3.Widowed & not-cohabiting; 4.Divorced & not cohabiting; 5.Never-married & cohabiting; 6.Widowed & cohabiting; 7. Divorced & cohabiting[2]. If cohabitation is negligible (or data concerning cohabitation are not available) in the population under study, one may simply employ the classic four marital statuses model; the procedures proposed in this article are still applicable when variables concerning cohabitation are zero.

Let $m_{ij}(x,s,t)$ denote the age-specific o/e rate of transition from marital status i to marital status j between age x and x+1 among persons of sex s in year t; i,j=1, 2, 3, ... 7

(or 4 if cohabitation is neglected); s=1,2, referring to females and males, respectively; $P_{ij}(x,s,t)$, the sex-age-specific probability of transition from marital status i at age x to marital status j at x+1 among persons of sex s in year t.

The relationship between $m_{ij}(x,s,t)$ and $P_{ij}(x,s,t)$ in multi-state (i.e., multiple increment-decrement) models is expressed by matrix formulas. The definitions and structures of elements of the o/e rates matrix $\mathbf{M}(x)$ and transition probabilities matrix $\mathbf{P}(x)$ can be found in published articles or standard text books (see, e.g., Willekens, 1982; Schoen, 1988; Preston et al., 2001). Based on the assumption of constant intensities of marital status transitions within the age interval, the matrix formula for translating the o/e rates into the probabilities between age x and x+1 in year t is, as follows (see, e.g., Willekens, 1982; Schoen, 1988; Preston et al., 2001):

$\mathbf{P}(x,s,t)=$

$$e^{-\mathbf{M}(x,s,t)} = \mathbf{I} - \mathbf{M}(x,s,t) + \frac{1}{2!}\mathbf{M}^2(x,s,t) - \frac{1}{3!}\mathbf{M}^3(x,s,t) + \frac{1}{4!}\mathbf{M}^4(x,s,t) - \frac{1}{5!}\mathbf{M}^5(x,s,t) + \ldots \qquad (1)$$

Where $\mathbf{I}$ is an identity matrix with 0 values everywhere except for the diagonal elements which are all equal to 1. Based on the assumption of uniform event distribution within the age interval[3], the matrix formula for translating the o/e rates into the probabilities between age x and x+1 in year t is, as follows (see, e.g., Willekens, 1982; Schoen, 1988; Preston et al., 2001):

$$\mathbf{P}(x,s,t) = [\mathbf{I} + \frac{1}{2}\mathbf{m}(x,s,t)]^{-1}[[\mathbf{I} - \frac{1}{2}\mathbf{m}(x,s,t)]$$

3. Estimating Age-Specific o/e Rates of Marital Status Transitions that are Consistent with the Projected Propensities in the One-Sex Life Table Model

The propensity of marital status transition is defined as the total number of events of transition from marital status i to j divided by the total number of events that lead to entering marital status i (Schoen, 1988:95)[4]. For example, the propensity of divorce is defined as the total number of divorces divided by the sum of the total numbers of marriages during the whole life course of a hypothetical cohort or a real cohort. The period propensities summarize the massive amount of information of age-specific rates of marital status transitions and reflect the period intensities of marriage/union formation and dissolution through the life course of a hypothetical life table cohort experiencing the observed period rates.

Changes in the propensity of one marital status transition affect the at-risk population and the number of events of other marital status transitions. For example, changes in the propensities of first marriage and remarriage cause changes in the at-risk population and the number of events of divorce. Changes in the propensity of divorce affect the at-risk population and the number of events of remarriage. This interrelation between the events and at-risk populations is the reason why the projected x% of changes in the propensity of the marital status change cannot be achieved through simply inflating or deflating the corresponding age-specific transition rates by x%.

We will establish a set of simultaneous equations that yield a consistent set of projected propensities of marital status transitions. Based on the simultaneous equations, we will then discuss an iterative procedure for estimating age-specific o/e rates of marital status transitions that are consistent with the projected or assumed propensities of marriage/union formation and dissolution.

Note that the index “t” in all variables in this section refers to the period hypothetical cohort life table or the real cohort life table of the one-sex model. The index “t” in all variables in the fourth section refers to calendar year because we deal with two-sex family household projection models in Section 4.

Let $m_{ij}^{s}(x,s)$ denote sex-age-specific standard schedules of o/e rates of transition from marital status i to marital status j between age x and x+1;

$A^s(i,j,s)$, the life table sex-specific propensity of transition from marital status i to j, implied by the standard schedules.

A(i,j,s,t), the projected or assumed sex-specific propensity of transition from marital status i to j in life table cohort t;

$l_i(x,s,t)$, the life table function of the number of persons aged x with marital status i and sex s of the period hypothetical cohort or real cohort t; $l_i(x,s,t)$ is a function of $P_{ij}(x,s,t)$.

It follows that:

$$\frac{\sum_{x=\alpha}^{\omega} l_i(x,s,t)P_{ij}(x,s,t)}{\sum_{x=\alpha}^{\omega}\sum_{k} l_k(x,s,t)P_{ki}(x,s,t)} = A(i,j,s,t), \qquad k\neq i;\ (i\rightarrow j) \neq (2\rightarrow 3)^{5} \tag{2}$$

where α is the lowest age at marriage; ω is the highest age considered in the life table model (e.g., 85). When i=1,

$$\frac{\sum_{x=\alpha}^{\omega} l_1(x,s,t)P_{1j}(x,s,t)}{100{,}000} = A(1,j,s,t)$$

How can one estimate the unknown $m_{ij}(x,s,t)$ that are consistent with the projected A(i,j,s,t) based on the known $m_{ij}^{s}(x,s)$? We first use $m_{ij}^{s}(x,s)$ to estimate $P_{ij}^{s}(x,s)$ using the matrix formula (1) discussed earlier, and construct a multi-state life table to get the propensities of marital status transitions implied by the standard schedules ($A^s(i,j,s)$). We then use $m_{ij}^{1}(x,s,t) = \frac{A(i,j,s,t)}{A^{s}(i,j,s)} m_{ij}^{s}(x,s)$ as the first approximation of $m_{ij}(x,s,t)$. We use the adjusted $m_{ij}^{1}(x,s,t)$ to estimate $P_{ij}^{1}(x,s,t)$, and construct a new multi-state life table to get a new set of approximations $A^1(i,j,s,t)$, which are not equal, but are closer to A(i,j,s,t), as compared to A^s(i,j,s). We then get $m_{ij}^2(x,s,t) = \frac{A(i,j,s,t)}{A^1(i,j,s,t)} m_{ij}^1(x,s,t)$, the second approximation of m_{ij}(x,s,t). We then use $m_{ij}^2(x,s,t)$ to estimate $P_{ij}^2(x,s,t)$, and construct another new multi-state life table to get another new set of approximations A^2(i,j,s,t), which may not be equal, but are closer to A(i,j,s,t) as compared to A^1(i,j,s,t). We repeat this iterative process for n times, until all of the A^n(i,j,s,t) are almost exactly equal to A(i,j,s,t). For example, the model generally is sufficiently accurate when the absolute value of the largest relative discrepancy rate is less than 0.001, namely, $\left|\frac{A^n(i,j,s,t) - A(i,j,s,t)}{A(i,j,s,t)}\right| < 0.001$ for all combinations of i, j, s, and t.

Based on the illustrative numerical applications of the four and seven marital statuses models using the U.S. 1990-96 observed age-specific o/e rates of marital status transitions, we conclude that convergence of the iterative procedure can be achieved and the goal of estimating m_{ij}(x,s,t) consistently with the projected A(i,j,s,t) can be achieved. The number of iterations required depends on the number of marital statuses distinguished in the model and the magnitude of the changes in the propensities. The illustrative applications are presented in the Appendix.

It is crucial to indicate that the procedure described above is applicable only in the one-sex family status life table models. Bongaarts (1987) developed a one-sex nuclear family status life table model, and presented illustrative applications to three hypothetical life table populations at three different points in the demographic transition (pre-transitional, transitional, and post-transitional). Watkins, Menken, and Bongaarts (1987) applied Bongaarts’s model to the U.S. population, using the U.S. demographic data in 1800, 1900, 1960 and 1980, to estimate the length of life spent in various family statuses. Applying Bongaarts's model, Lee and Palloni (1992) estimated cohort family status life tables for women born in 1890-1894, 1910-1914, 1930-1934, 1950-1954, and 1970-1974, and conducted cohort and cross-sectional analyses of changes in the family status of elderly women in Korea. Zeng (1986, 1988, 1991) extended Bongaarts's model into a general family status life table model that includes both nuclear and three-generation family households, and applied the extended model to the Chinese data of 1950-70 and 1981.

The family status life table models by Bongaarts (1987) and Zeng (1986; 1988; 1991) are female-dominant one-sex models. Their applications assume that age-specific demographic rates are as constant as those observed in a particular period in the past or directly use the rates of a real cohort, and thus there is no need to estimate the time-varying age-specific o/e rates in the future years. Using the procedure proposed in this section, one can construct different one-sex family status life tables for future years using sex-age-specific standard schedules observed in the recent past and the projected or assumed propensities of marital status transitions in the future years. Such exercises are useful to understand the implications of demographic changes on life course in the one-sex dominant model. For example, one can address the demographic impacts on the length of life of elderly living alone, children living with a single parent, and adults living with old parents and young children, if various demographic rates change in the future years to different extents and in different combinations. The procedure presented in this section can, therefore, be regarded as an extension of the applications of the female dominant one-sex family-status life table model originally developed by Bongaarts (1987) and extended by Zeng (1986; 1988; 1991).

The procedure proposed in this section, however, does not ensure the consistency across sexes required in the two-sex model of family household projection for monogamous societies. More specifically, in any given projection period, the total numbers of newly married (or cohabiting) men and women should be equal. Similar requirements must be formulated for the total numbers of divorces (or union dissolutions), numbers of new widows compared to married men who die, and vice versa. The age-specific o/e rates obtained by the adjustment procedure described above will almost surely not satisfy these two-sex constraints, because it is very hard to ensure that the projected (or assumed) sex-specific propensities are consistent with the two-sex constraints given the unknown future years' sex-age-marital status distributions. If the o/e rates are adjusted one more time to make them consistent across the sexes, the projected (or assumed) propensities of marital status transitions may very likely be violated. Therefore, an appropriate solution would be to develop a model to satisfy the constraints across sexes and achieve the projected summary measures simultaneously. We present and discuss such a solution in the next section.

4. Estimating Time-Varying Sex-Age-Specific o/e Rates of Marital Status Transitions that are Consistent with the Two-Sex Constraints and Projected Summary Measures in the Family Household Projection Model

As stated earlier, the estimated time-varying sex-age-specific o/e rates of marital status transitions must ensure the consistency across sexes required in the two-sex model of family household projection for monogamous societies. Step 1 of the procedures to be described in Section 4.2 ensures such consistency across sexes.

The estimated time-varying sex-age-specific o/e rates of marital status transitions must also be consistent with the summary measures of marriage/union formation and dissolution in future years. In defining the summary measures of marriage/union formation and dissolution for family household projection, we need to consider (1) whether the summary measures are appropriate for both measuring the overall level and ensuring the two-sex constraints; (2) whether the summary measures are demographically interpretable, measurable, and predictable as well as easily understandable from the public's and policy makers' points of view; (3) whether the number of summary measures to be projected is small enough that the model and applications will be manageable. With these considerations, we define and discuss in the following subsection two alternative groups of summary measures: overall propensities and standardized general rates. Applicants of the model will choose a group based on data availability and the purpose of their research. We first present the definitions and formulas for computing the two alternative groups of summary measures in subsections 4.1.1 and 4.1.2, respectively. We then discuss the rationale, assumptions, and implications associated with these summary measures in subsection 4.1.3.

4.1. Two alternative groups of summary measures of marriage/union formation and dissolution

4.1.1. Overall propensities of marriage/union formation and dissolution in the context of the period life table

Let $\overline{P}_{ij}(x,t)$ denote the age-specific probability of transition from marital status i at age x to marital status j at x+1 of males and females combined in year t; $\overline{P}_{ij}(x,t)$ is estimated based on the average of the male and female age-specific o/e rates of marital status transitions ($m_{ij}(x,s,t)$), using the matrix formula (1).

$l_i(x,t)$, the life table function of the number of persons aged x with marital status i for males and females combined in the period life table in year t; $l_i(x,t)$ is a function of $\overline{P}_{ij}(x,t)$.

Let PM(t) denote the overall propensity of marriages in the context of the period life table;

PM1(t), the probability of eventually becoming ever-married (i.e., average number of 1^{st} marriages per average person) during the life course if the sex-age-specific o/e rates of marriage/union formation and dissolution in year t are applied to a hypothetical cohort of males and females combined.

PM2(t), the average number of remarriages per average person during the life course if the sex-age-specific o/e rates of marriage/union formation and dissolution in year t are applied to a hypothetical cohort of males and females combined.

PM(t) = PM1(t) + PM2(t)

$$\frac{\sum_{x=\alpha}^{\omega}[l_1(x,t)\overline{P}_{12}(x,t)+l_5(x,t)\overline{P}_{52}(x,t)]}{100{,}000}+\frac{\sum_{x=\partial}^{\omega}[l_3(x,t)\overline{P}_{32}(x,t)+l_4(x,t)\overline{P}_{42}(x,t)+l_6(x,t)\overline{P}_{62}(x,t)+l_7(x,t)\overline{P}_{72}(x,t)]}{100{,}000} \quad (3)$$

where α is the lowest age at marriage; ω is the highest age considered in the life table.

Let PD(t) denote the overall propensity of divorce of an average person during the life course if the sex-age-specific o/e rates of marriage/union formation and dissolution in year t are applied to a hypothetical cohort of males and females combined;

PD(t)=

$$\frac{\sum_{x=\alpha}^{\omega} l_2(x,t)\overline{P}_{24}(x,t)}{\sum_{x=\alpha}^{\omega}[l_1(x,t)\overline{P}_{12}(x,t)++l_3(x,t)\overline{P}_{32}(x,t)+l_4(x,t)\overline{P}_{42}(x,t)+l_5(x,t)\overline{P}_{52}(x,t)+l_6(x,t)\overline{P}_{62}(x,t)+l_7(x,t)\overline{P}_{72}(x,t)]} \quad (4)$$

Let PC(t) denote the overall propensity of cohabitation of never-married and ever-married males and females combined;

PC1(t), the average number of cohabitations before first marriage per average person during the life course if the sex-age-specific o/e rates of marriage/union formation and dissolution in year t are applied to a hypothetical cohort of males and females combined;

PC2(t), the average number of cohabitations after first marriage dissolution per average person during the life course if the sex-age-specific o/e rates of marriage/union formation and dissolution in year t are applied to a hypothetical cohort of males and females combined;

$$PC(t) = PC1(t) + PC2(t) = \frac{\sum_{x=\alpha}^{\omega}[l_1(x,t)\overline{P}_{15}(x,t)]}{100{,}000} + \frac{\sum_{x=\partial}^{\omega}[l_3(x,t)\overline{P}_{36}(x,t) + l_4(x,t)\overline{P}_{47}(x,t)]}{100{,}000} \tag{5}$$

Let PCS(t) denote the overall propensity of cohabitation union dissolution of an average person during the life course if the sex-age-specific o/e rates of marriage/union formation and dissolution in year t are applied to a hypothetical cohort of males and females combined.

$$PCS(t) = \frac{\sum_{x=\alpha}^{\omega}[l_5(x,t)\overline{P}_{51}(x,t) + l_6(x,t)\overline{P}_{63}(x,t) + l_7(x,t)\overline{P}_{74}(x,t)]}{\sum_{x=\alpha}^{\omega}[l_1(x,t)\overline{P}_{15}(x,t) + l_3(x,t)\overline{P}_{36}(x,t) + +l_4(x,t)\overline{P}_{47}(x,t)]} \tag{6}$$

4.1.2. Standardized general rates of marriage/union formation and dissolution

The standardized general rate of marriage/union formation and dissolution in the year t[6] is defined as the total number of events that would occur if the sex-age-specific o/e rates of occurrence of the events in the year t were applied to the most recent census-counted sex-age-marital status distribution of males and females divided by the census-counted total number of males and females who are at risk of experiencing the events. Following the language used in Preston et al. (2001: 24), the standardized general rate in year t is the estimated general rate in year t if it retained its sex-age-specific o/e rates but had the age distribution of the most recent census year.

Let $N_i(x,s,T0)$ denote the number of persons of age x, marital status i and sex s counted in the most recent census (i.e., the starting population of our family household projection)[7];

$m_{ij}(x,s,t)$, the sex-age-specific o/e rates of transition from marital i to j in year t.

Let GM(t) denote the standardized general rate of marriages including 1st marriage and remarriage for males and females combined.

$$GM(t) = \frac{\sum_{x=\alpha}^{\beta}\sum_{s=1,2}\sum_{i} N_i(x,s,T0)m_{i2}(x,s,t)}{\sum_{x=\alpha}^{\beta}\sum_{s=1,2}\sum_{i} N_i(x,s,T0)}, \qquad i = 1,3,4,5,6,7 \qquad (7)$$

Where α is the lowest age at marriage; β is the higher boundary of the age range in which the standardized general rate of marriage/union formation and dissolution is defined.

Let GD(t) denote the standardized general divorce rate for males and females combined.

$$GD(t) = \frac{\sum_{x=\alpha}^{\beta}\sum_{s=1,2} N_2(x,s,T0)m_{24}(x,s,t)}{\sum_{x=\alpha}^{\beta}\sum_{s=1,2} N_2(x,s,T0)} \qquad (8)$$

Let GC(t) denote the standardized general rate of cohabiting of never-married and ever-married males and females combined.

$$GC(t) = \frac{\sum_{x=\alpha}^{\beta}\sum_{s=1,2}[N_1(x,s,T0)m_{15}(x,s,t) + N_3(x,s,T0)m_{36}(x,s,t) + N_4(x,s,T0)m_{47}(x,s,t)]}{\sum_{x=\alpha}^{\beta}\sum_{s=1,2}[N_1(x,s,T0) + N_3(x,s,T0) + N_4(x,s,T0)]} \qquad (9)$$

Let GCS(t) denote the standardized general union dissolution rate for males and females combined.

GCS(t) =

$$\frac{\sum_{x=\alpha}^{\beta}\sum_{s=1,2}[N_5(x,s,T0)m_{51}(x,s,t)+N_6(x,s,T0)m_{63}(x,s,t)+N_7(x,s,T0)m_{74}(x,s,t)]}{\sum_{x=\alpha}^{\beta}\sum_{s=1,2}[N_5(x,s,T0)+N_6(x,s,T0)+N_7(x,s,T0)]} \quad (10)$$

4.1.3. Rationale, assumptions, and implications of the summary measures

It is important to clarify several points concerning the rationale, assumptions, and implications of the two alternative groups of the summary measures of marriage/union formation and dissolution. *First, the summary measures are defined for males and females combined.* We cannot employ the summary measures of marriage/union formation and dissolution in future years for males and females separately to estimate the sex-age-specific o/e rates since it would be extremely hard (or impossible) to ensure that the projected sex-specific summary measures are consistent with the two-sex constraints. This is because the two-sex constraints also depend on the unknown (to-be-projected) sex-age-marital status distributions in the future years. While employing the summary measures of marriage/union formation and dissolution for males and females combined[8], we estimate and use the sex-age-specific o/e rates of marital status transitions (see Section 4.2) to compute changes in the marital status of individuals of the population. Such a strategy of employing the summary measures for the two sexes combined while using the sex-age-specific o/e rates in the projection computation plus the ensured two-sex constraints adequately model the overall level and gender differentials of marriage/union formation and dissolution.

Second, standardization. The summary measures in different years must be age and marital status structure standardized, in order to eliminate the possible bias in measuring changes in the levels of marriage/union formation and dissolution due to cross-temporal changes in the age and marital status structures of the population. For example, the not-age-standardized general marriage (or divorce) rate, which is defined as the total number of marriages (or divorces) divided by the total number of not-married (or currently married) persons in year t, would decrease/increase purely due to the structural growth/decline of the numbers of elderly even if the level of marriage (or divorce) does not change. This is because the risks of marriage (or divorce) of the elderly are substantially lower than those of younger people. One may consider the simple summary measures of Total First Marriage Rate (TFMR) and Total Divorce Rate (TDR), which are defined by summing up the age-specific frequencies of first marriage and divorce, respectively. The denominators of the age-specific frequencies of first marriage and divorce are the total number of persons at each age regardless of the distribution of the risk and non-risk populations. TFMR and TDR are age-standardized, but subject to bias due to cross-temporal changes in marital status distributions; the TFMR or TDR would increase/decrease purely due to the increase/decrease of the not-married or currently married persons even if the age-specific probabilities of marriage or divorce remain constant. Furthermore, TFMR would produce an inadmissible value due to temporal changes to be discussed below.

We also follow the multi-state period life table approach in defining the overall propensities of marriage/union formation and dissolution based on $m_{ij}(x,s,t)$, the sex-age-specific o/e rates. The $m_{ij}(x,s,t)$ are standardized for age and marital status structures. Thus, the life table propensities based on $m_{ij}(x,s,t)$ adequately measure the intensities of marital status transitions. For example, the proportion of eventually marrying based on a period life table using the age-specific o/e rates indicates the period propensity of 1^{st} marriage; suppose if it is 0.95 for one population in one year, one can confidently inform the public that the current age-specific o/e rates of 1^{st} marriage imply that 95% of persons would eventually marry if the current age-specific 1st marriage rates are applied to a hypothetical cohort. But the period TFMR based on age-specific frequencies of first marriage for the same population in the same year could be greater than 1.0 if the age at 1st marriage is substantially declining and/or a lot of women who had postponed marriage in previous years decided to marry in the current year. In such a case, due to temporal changes TFMR is a biased inadmissible index in the sense of measuring the probability of eventually marrying[9], and thus is inappropriate for informing the public. The proportion of eventually marrying based on a period life table using the age-specific o/e rates can, however, never be unreasonably greater than 1.0. The life table analysis and the "age-standardization" for the general rates are widely applied methods in demography (see, e.g., Preston et al., 2001: 24-28).

Third, overall summary measures of marriage/union formation. Never-married, widowed, and divorced men and women may marry each other; a cohabiting couple whose legal marital statuses may be different may marry, or a cohabiting person may leave his or her partner to marry another not-cohabiting person. Similarly, never-married, widowed, and divorced persons may form a cohabitation union with each other. As implied by the equations to be presented in Step 2a and Step 2b in Section 4.2, employing separate summary measures of marriage/cohabitation for never-married, widowed, and divorced persons would make it impossible to ensure the two-sex constraints because of the cross-marriage/union-formation among people with different non-marital statuses. Thus, we define the overall summary measures of marriage/union formation (PM(t), GM(t), PC(t) and GC(t)) to include relevant events with different marital statuses before the onset of marriage or cohabitation. This implies that the changes of the overall intensities of various marriages and cohabitations are proportional to the changes in the overall summary measures. This assumption is generally reasonable since different kinds of marriages/cohabitations are all related to the general social attitudes towards marriage and cohabitation. If one is not satisfied with such an assumption, one may simply inflate or deflate the standard schedules (estimated from survey data) of sex-age-specific o/e rates of marriage/cohabitation of never-married, widowed, and divorced persons differently according to one's assumptions, to reflect the speculated differentials in future years, while the overall summary measures reveal the general level of marriage/union formation.

On the other hand, as shown in Section 4.2, the sex-age-specific o/e rates of marriage/union formation are estimated for persons with different marital statuses before the onset of marriage and cohabitation, respectively. Combining the detailed sex-age-marital status-specific o/e rates with the overall summary measures of marriage/union formation seems a reasonable approach for modelling differentials in marriage/cohabitation among different types of not-married persons, while meeting the two-sex constraints. Furthermore, one can easily estimate the more detailed sex-specific summary measures of first marriage, remarriage, cohabitation of never-married and ever-married persons in year t once the sex-age-specific o/e rates of marital status transitions in year t ($m_{ij}(x,s,t)$) have been estimated.

Fourth, a practical consideration concerning the age range in computing the summary measures. We can, in theory, define the summary measures for the age range from the lowest age (e.g., 15) to the highest age (e.g., 100 or higher) at which marriage/union formation and dissolution can possibly occur. In practice, however, we restrict the high boundary of the age range of the summary measures to a certain age (e.g., 85). This is because the sex-age-specific o/e rates (especially the rates of cohabitation union formation and dissolution) at older ages in the recent past for estimating the trends of the summary measures may not be reliable, given the subsample size problems in the surveys that collect marriage/union history data. Consequently, we assume that the changes in intensities of marriage/union formation and dissolution among males and females over the high boundary age are proportional to the changes in the summary measures below that age; we also use the estimated standard schedules of the sex-age-specific o/e rates over the high boundary age (through averaging over a longer period or using model fittings) to determine the age pattern of marriage/union formation and dissolution at older ages.

4.2. Estimation procedures to ensure the consistency of the two-sex constraints and the projected summary measures

The estimation procedures to ensure the consistency of the two-sex constraints and the projected summary measures consist of two steps.

Step 1. Adjustment to comply with the two-sex constraints, following the harmonic mean approach

We use the harmonic mean approach to ensure two-sex consistency in family household projection for monogamous societies. The harmonic mean satisfies most of the theoretical requirements and practical considerations for handling consistency problems in a two-sex model (Pollard, 1977; Schoen, 1981; Keilman, 1985).

In order to compute the number of events that occurred in year t, we need to estimate the mid-year population ($\overline{N}'(x,s,t)$), classified by age, sex, and marital status. $\overline{N}'(x,s,t)$ is the average of the populations at the beginning and the end of the year t and can be considered as an approximation of the person-years lived in status i (i.e., at risk of experiencing the event of transition from status i to j).

Let $N_i(x,s,t)$ denote the number of persons of age x, marital status i and sex s at the beginning of year t. The sex-age-specific o/e rates $m_{ij}(x,s,t)$ and sex-age-specific probabilities $P_{ij}(x,s,t)$ were defined earlier and their relationship is expressed in the matrix formula (1). We wish to estimate $m_{ij}(x,s,t)$ through adjusting $m_{ij}(x,s,t-1)$. The estimated $m_{ij}(x,s,t)$ must be consistent with the two-sex constraints and the projected summary measures of the marriage/union formation and dissolution in year t.

$\overline{N}'(x,s,t)$ = 0.5 [$N_i(x,s,t)$ and $N'_i(x+1,s,t+1)$]

where the $N_i(x,s,t)$ and $N'_i(x+1,s,t+1)$ are the elements of the vectors ***N***(x,s,t) and ***N'***(x+1,s,t+1); ***N'***(x+1,s,t+1) is estimated through the following matrix calculation[10]:

$\mathbf{P'}(x,s,t) = e^{-\mathbf{M}(x,s,t-1)}$;

$\boldsymbol{N'}(x+1,s,t+1) = \mathbf{P'}(x,s,t)\, \boldsymbol{N}(x,s,,t)$

Keep in mind for later consideration that $\overline{N}'_i\,(x,s,t)$ (the average of $N_i(x,s,t)$ and $N'_i(x+1,s,t+1)$) is only a first approximation, since $N'_i(x+1,s,t+1)$ is based on **M**(x,s,t-1), which is not the final estimate for year t. When t is the starting year of the projection, $m_{ij}(x,s,t-1)$ is the standard schedule of age-specific o/e rates of marital status transitions and $N_i(x,s,t)$ is derived from the census data file.

The total number of new marriages of persons of sex s (s=1, 2, referring to females and males, respectively) who were not cohabiting before marriage in year t (TM(s,t)) is estimated as follows:

$$TM(s,t) = \sum_i \sum_{x=\alpha}^{\omega} \overline{N}'_i\,(x,s,t) m_{i2}(x,s,t-1)\,, \qquad i=1,3,4$$

Where ω is the highest age considered in the family household projection; α is the lowest age at marriage. To meet the two-sex constraint, the sex-age-specific o/e rates of marriage among persons who were not cohabiting before marriage need to be adjusted:

$$m'_{i2}(x,s,t) = m_{i2}(x,s,t-1)\left[\frac{2(TM(1,t)TM(2,t))}{TM(1,t)+TM(2,t)} / TM(s,t)\right], \qquad i=1,3,4 \qquad (11)$$

The estimated total number of new divorces of persons of sex s in year t (TD(s,t)) is

$$TD(s,t) = \sum_{x=\alpha}^{\omega} \overline{N}'_2(x,s,t) m_{24}(x,s,t-1)$$

To meet the two-sex constraint, the sex-age-specific o/e rates of divorce need to be adjusted:

$$m'_{24}(x,s,t) = m_{24}(x,s,t-1) \left[\frac{2(TD(1,t)TD(2,t))}{TD(1,t)+TD(2,t)} / TD(s,t) \right] \quad (12)$$

The o/e rates of widowhood depend on spouses' death rates, which are estimated before the two-sex constraints adjustments, based on the standard mortality schedules and the projected life expectancy at birth in year t. The already projected spouses' death rates should not be adjusted again; they must be used as a "standard". Thus, instead of employing the harmonic mean approach, we simply adjust the o/e rates of widowhood to be consistent with the total number of spouses who die in year t. The total number of persons (i.e., spouses) of sex s who died in year t with an intact marriage before death (TDM(s,t)) based on already projected sex-age-specific death rates is

$$TDM(s,t) = \sum_{x=\alpha}^{\omega} \overline{N}'_2(x,s,t) d_2(x,s,t)$$

where $d_2(x,s,t)$ is the already projected death rate of married persons of age x and sex s in year t.

The estimated total number of newly widowed persons of sex s in year t (TW(s,t)) is

$$TW(s,t) = \sum_{x=\alpha}^{\omega} \overline{N}'_2(x,s,t) m_{23}(x,s,t-1)$$

To meet the two-sex constraint, the sex-age-specific o/e rates of widowhood need to be adjusted using TDM(s,t) as a "standard":

$$m'_{23}(x,s,t) = m_{23}(x,s,t-1) \left[\frac{TDM(s^{-1},t)}{TW(s,t)} \right] \quad (13)$$

Where "s^{-1}" indicates the opposite sex of "s".

The estimated total number of newly cohabiting persons of sex s in year t (TC(s,t)) is

TC(s,t) =

$$\sum_{x=\alpha}^{\omega} \overline{N}'_1(x,s,t)m_{15}(x,s,t-1) + \sum_{x=\alpha}^{\omega} \overline{N}'_3(x,s,t)m_{36}(x,s,t-1) + \sum_{x=\alpha}^{\omega} \overline{N}'_4(x,s,t)m_{47}(x,s,t-1)$$

To meet the two-sex constraint, the sex-age-specific o/e rates of cohabiting need to be adjusted:

$$m'_{15}(x,s,t) = m_{15}(x,s,t-1)\left[\frac{2(TC(1,t)TC(2,t))}{TC(1,t)+TC(2,t)}/TC(s,t)\right] \quad (14)$$

$$m'_{36}(x,s,t) = m_{36}(x,s,t-1)\left[\frac{2(TC(1,t)TC(2,t))}{TC(1,t)+TC(2,t)}/TC(s,t)\right] \quad (15)$$

$$m'_{47}(x,s,t) = m_{47}(x,s,t-1)\left[\frac{2(TC(1,t)TC(2,t))}{TC(1,t)+TC(2,t)}/TC(s,t)\right] \quad (16)$$

The estimated total number of new marriages of persons of sex s who were cohabiting before marriage in year t (TCM(s,t)) is

$$TCM(s,t) = \sum_i \sum_{x=\alpha}^{\omega} \overline{N}'_i(x,s,t)m_{i2}(x,s,t-1), \qquad i = 5, 6, 7$$

To meet the two-sex constraint, the sex-age-specific o/e rates of marriage of persons who were cohabiting before marriage need to be adjusted:

$$m'_{i2}(x,s,t) = m_{i2}(x,s,t-1)\left[\frac{2(TCM(1,t)TCM(2,t))}{TCM(1,t)+TCM(2,t)}/TCM(s,t)\right], \qquad i=5,6,7 \quad (17)$$

The estimated total number of events of cohabitation union dissolution of persons of sex s in year t (TCD(s,t)) is

TCD(s,t) =

$$\sum_{x=\alpha}^{\omega} \overline{N}'_5(x,s,t)m_{51}(x,s,t-1) + \sum_{x=\alpha}^{\omega} \overline{N}'_6(x,s,t)m_{63}(x,s,t-1) + \sum_{x=\alpha}^{\omega} \overline{N}'_7(x,s,t)m_{74}(x,s,t-1)$$

To meet the two-sex constraint, the sex-age-specific o/e rates of cohabitation union dissolution need to be adjusted:

$$m'_{51}(x,s,t) = m_{51}(x,s,t-1)\left[\frac{2(TCD(1,t)TCD(2,t))}{TCD(1,t)+TCD(2,t)}/TCD(s,t)\right] \quad (18)$$

$$m'_{63}(x,s,t) = m_{63}(x,s,t-1)\left[\frac{2(TCD(1,t)TCD(2,t))}{TCD(1,t)+TCD(2,t)}/TCD(s,t)\right] \quad (19)$$

$$m'_{74}(x,s,t) = m_{74}(x,s,t-1)\left[\frac{2(TCD(1,t)TCD(2,t))}{TCD(1,t)+TCD(2,t)}/TCD(s,t)\right] \quad (20)$$

The sex-age-specific o/e rates, $m'_{ij}(x,s,t)$, are adjusted for consistency with the two-sex constraint as described above, but they need to be further adjusted to be consistent with the projected summary measures of marriage/union formation and dissolution in year t. To achieve such consistency, as discussed in Section 4.1, one may choose either the overall propensities (PM(t), PD(t), PC(t), PCS(t)) or the standardized general rates (GM(t), GD(t), GC(t), GCS(t)) as the summary measures. The alternatives Step 2a and Step 2b to be discussed below present estimation procedures associated with these two alternative groups of the summary measures, respectively, and one should choose one of them according to data availability and research purpose.

Alternative Step 2a. Adjustment for consistency with the projected overall propensities of marriage/union formation and dissolution in year t

We first use the sex-age-specific $m'_{ij}(x,s,t)$, which satisfy the two-sex constraints estimated in Step 1, to estimate the average $\overline{m}'_{ij}(x,t)$ for males and females combined: $\overline{m}'_{ij}(x,t) = 0.5\,[m'_{ij}(x,1,t)+ m'_{ij}(x,2,t)]$.

We use $\overline{m}'_{ij}(x,t)$ to estimate $\overline{P}'_{ij}(x,t)$ using the matrix formula (1) discussed earlier, and construct a multi-state life table for males and females combined to obtain the overall propensities of $PM'(t)$, $PD'(t)$, $PC'(t)$, and $PCS'(t)$ implied by the $\overline{m}'_{ij}(x,t)$. We then get the approximation of $m_{ij}(x,s,t)$:

$$m''_{i2}(x,s,t) = \frac{PM(t)}{PM'(t)} m'_{i2}(x,s,t), \qquad i=1, 3, 4, 5, 6, 7 \quad (21)$$

Note that we use the same factor $\frac{PM(t)}{PM'(t)}$ to adjust sex-age-specific $m'_{i2}(x,s,t)$ (i=1,3,4,5,6,7), o/e rates of 1st marriage and remarriages of not-cohabiting and cohabiting males and females in order to sustain the two-sex consistency obtained in Step 1.

$$m''_{24}(x,s,t) = \frac{PD(t)}{PD'(t)} m'_{24}(x,s,t) \tag{22}$$

$$m''_{15}(x,s,t) = \frac{PC(t)}{PC'(t)} m'_{15}(x,s,t) \tag{23}$$

$$m''_{36}(x,s,t) = \frac{PC(t)}{PC'(t)} m'_{36}(x,s,t) \tag{24}$$

$$m''_{47}(x,s,t) = \frac{PC(t)}{PC'(t)} m'_{47}(x,s,t) \tag{25}$$

$$m''_{51}(x,s,t) = \frac{PCS(t)}{PCS'(t)} m'_{51}(x,s,t) \tag{26}$$

$$m''_{63}(x,s,t) = \frac{PCS(t)}{PCS'(t)} m'_{63}(x,s,t) \tag{27}$$

$$m''_{74}(x,s,t) = \frac{PCS(t)}{PCS'(t)} m'_{74}(x,s,t) \tag{28}$$

Note that the adjustments described in Step 1 are related to $\overline{N}'_i(x,s,t)$, which are approximations, since they are not based on the final estimates of the sex-age-specific o/e rates. Although we use the same adjustment factors for males and females, the o/e rates adjusted so far may not be exactly consistent with the two-sex constraints because of the approximation of $\overline{N}'_i(x,s,t)$. We, therefore, need to use the $m''_{ij}(x,s,t)$ estimated so far to compute **P**''(x,s,t), **N**''(x,s,t+1), and $\overline{N}''_i(x,s,t)$.

$$\mathbf{P''}(x,s,t) = e^{-\mathbf{M''}(x,s,t-1)} \tag{29}$$

$$\boldsymbol{N''}(x+1,s,t+1) = \mathbf{P''}(x,s,t)\, \boldsymbol{N}(x,s,t) \tag{30}$$

$\overline{N}''_i(x,s,t)$ = 0.5 [N_i(x,s,t) + N''_i(x+1,s,t+1)] (31)

We then use $\overline{N}''_i(x,s,t)$ and $m''_{ij}(x,s,t)$ to replace $\overline{N}'_i(x,s,t)$ and m_{ij}(x,s,t-1) in all formulas in Step 1 and repeat the two-sex consistency adjustment procedure described in Step 1 to get the new estimates $m'''_{ij}(x,s,t)$, which satisfy the two-sex constraints. We then use the new estimates of $m'''_{ij}(x,s,t)$ to compute the adjusted average $\overline{m}'''_{ij}(x,t)$[=0.5 $(m'''_{ij}(x,1,t)+m'''_{ij}(x,2,t))$] for males and females combined to estimate $\overline{P}'''_{ij}(x,t)$, and construct a new multi-state life table to get a new set of approximations PM''(t), PD''(t), PC''(t), and PCS''(t). If the absolute values of the relative difference between the new estimates of the overall propensities and the corresponding projected overall propensities are all less than a selected criterion (e.g., 0.01 or 0.001), we have completed Step 2a. Otherwise, we have to repeat Step 1 and Step 2a until the selected criterion is achieved. By the end of Step 2a, we have estimated m_{ij}(x,s,t), which are consistent with the two-sex constraints and the projected overall propensities of marriage/union formation and dissolution.

As an illustrative numerical example, we used PM(t), PD(t), PC(t), and PCS(t) as summary measures and the procedures described above in Steps 1 and Step 2a to estimate the time-varying sex-age-specific o/e rates of marital status transitions in the projection years. The standard schedules are based on the estimates of the U.S. sex-age-specific o/e rates of marriage/union formation and dissolution in 1990-1996 (Zeng, Yang, Wang and Morgan, 2002). We estimated models with seven marital statuses including cohabitation (Figure 2) for four race groups: White & non-Hispanic, Black & non-Hispanic, Hispanic, Asian/others & non-Hispanic. The required number of repetitions of Steps 1 and Step 2a using different criterion are between 2 and 5, as indicated in Table 1. This demonstrates that the iterative procedures expressed in Steps

1 and 2a are valid for practical applications. Note that the number of iterations required is much smaller than that for the one-sex family status life table model procedure, presented in Section 3 with the illustrative numerical example in the Appendix, which involves much more detailed propensities of marital status transitions.

Table 1. Number of repetitions of Step 1 and Step 2a (PM(t) decrease by 4%, PD(t) increase by 5%; PC(t) increase by 8%; PCS(t) increase by 6%)

Criterion (relative difference): 0.01		Criterion (relative difference): 0.001	
All races combined	Four race groups	All races combined	Four race groups
2	4	2	5

Alternative Step 2b. Adjustment for consistency with the projected standardized general rates of marriage/union formation and dissolution in year t

To estimate the unknown $m_{ij}(x,s,t)$[11] to be consistent with GM(t), GD(t), GC(t) and GCS(t), one needs to use the following procedure based on $m'_{ij}(x,s,t)$, which meet the two-sex constraints estimated in Step 1. Again, we use the same adjustment factor for adjusting male and female o/e rates because we wish to maintain the two-sex consistency achieved in Step 1.

$$m''_{i2}(x,s,t) = \frac{GM(t)}{GM'(t)} m'_{i2}(x,s,t), \qquad \text{i=1,3,4,5,6,7} \tag{32}$$

where,

$$GM'(t) = \frac{\sum_{s=1,2}\sum_{x=\alpha}^{\beta}\sum_{i} N_i(x,s,T0) m'_{i2}(x,s,t)}{\sum_{s=1,2}\sum_{x=\alpha}^{\beta}\sum_{i} N_i(x,s,T0)}, \qquad \text{i=1,3,4,5,6,7}$$

$$m''_{24}(x,s,t) = \frac{GD(t)}{GD'(t)} m'_{24}(x,s,t) \tag{33}$$

where,

$$GD'(t) = \frac{\sum_{s=1,2}\sum_{x=\alpha}^{\beta} N_2(x,s,T0) m'_{24}(x,s,t)}{\sum_{s=1,2}\sum_{x=\alpha}^{\beta} N_2(x,s,T0)}$$

$$m''_{15}(x,s,t) = \frac{GC(t)}{GC'(t)} m'_{15}(x,s,t) \tag{34}$$

$$m''_{36}(x,s,t) = \frac{GC(t)}{GC'(t)} m'_{36}(x,s,t) \tag{35}$$

$$m''_{47}(x,s,t) = \frac{GC(t)}{GC'(t)} m'_{47}(x,s,t) \tag{36}$$

$$m''_{51}(x,s,t) = \frac{GCS(t)}{GCS'(t)} m'_{51}(x,s,t) \tag{37}$$

$$m''_{63}(x,s,t) = \frac{GCS(t)}{GCS'(t)} m'_{63}(x,s,t) \tag{38}$$

$$m''_{74}(x,s,t) = \frac{GCS(t)}{GCS'(t)} m'_{74}(x,s,t) \tag{39}$$

The adjustments described in Step 1 are related to $\overline{N}'_i(x,s,t)$, which are approximations, since they are not based on the final estimates of the sex-age-specific o/e rates. Although we use the same adjustment factors for males and females, the o/e rates adjusted in Step 2b may not be exactly consistent with the two-sex constraints because $\overline{N}'_i(x,s,t)$ are not the final estimates. We, therefore, need to use the $m''_{ij}(x,s,t)$ estimated in Step 2b to compute **P**''(x,s,t), **N**''(x,s,t+1), and $\overline{N}''_i(x,s,t)$, using the formulas (29)-(31).

We then use $\overline{N}''_i(x,s,t)$ and $m''_{ij}(x,s,t)$ to replace $\overline{N}'_i(x,s,t)$ and m_{ij}(x,s,t-1) in all formulas in Step 1 and repeat the adjustment procedure described in Step 1 to get the new estimates m'''_{ij}(x,s,t), which satisfy the two-sex constraints. We then use the new estimates of m'''_{ij}(x,s,t) to compute the estimates of the new standardized general rates of the marital status transitions: GM''(t), GD''(t), GC''(t), GCS''(t). If the absolute values of the relative difference between the new estimates of the standardized general rates and the corresponding projected standardized general rates are all less than a selected criterion (e.g., 0.01 or 0.001), we have achieved our goals for estimating the sex-age-specific o/e rates in year t. Otherwise, we need to repeat the adjustment procedures described in Step 1 and Step 2b until the selected criterion is met.

As an illustrative numerical example, we used GM(t), GD(t), GC(t), and GCS(t) as summary measures and the procedures described above in Steps 1 and Step 2b to estimate the time-varying sex-age-specific o/e rates of marital status transitions in the projection years. Again, the standard schedules are based on the estimates of the U.S. sex-age-specific o/e rates of marriage/union formation and dissolution in 1990-1996 (Zeng et al., 2002). The sex-age-marital status distributions of the starting year of the projection were derived from the U.S. 2000 census micro sample data file. We estimated models with seven marital statuses including cohabitation (Figure 2) for the four race groups. The required number of repetitions of Steps 1 and Step 2b using different criterion are between 2 and 4, as indicated in Table 2. This demonstrates that the iterative procedures expressed in Steps 1 and 2b are valid for practical applications[12].

Table 2. Number of repetitions of Step 1 and Step 2b (GM(t) decrease by 4%, GD(t) increase by 5%; GC(t) increase by 8%; GCS(t) increase by 6%)

Criterion (relative difference): 0.01		Criterion (relative difference): 0.001	
All races combined	Four race groups	All races combined	Four race groups
2	3	3	4

Based on the final estimates of $m_{ij}(x,s,t)$ through either Step 2a or Step 2b, one can also construct the multi-state life tables for males and females separately and compute the detailed sex-specific period life table propensities of transitions from marital status i to j ($PP_{ij}(s,t)$) in the year t; or one can compute the sex-specific standardized general rates of transitions from marital status i to j ($GG_{ij}(s,t)$) in year t. $PP_{ij}(s,t)$ and $GG_{ij}(s,t)$

are informative to reflect the gender differentials of the intensities of transitions among various marital statuses, including cohabitation, in the projection years.

5. Concluding Remarks

Family household projection/simulation and other relevant projections/simulations (e.g., actuarial and welfare forecasting) often need to estimate the time-varying sex-age-o/e rates of marital status transitions to achieve the projected (or assumed) summary measures of marriage/union formation and dissolution in the future years. In the applications of family household projection/simulation for monogamous populations, consistency across the sexes must also be ensured. The estimation cannot be done by simply inflating or deflating each set of age-specific o/e rates independently as in estimating fertility rates in the classical population projections. This is because changes in the propensity of one status transition affect the at-risk population and the number of events of other status transitions as well as the two-sex constraints must be met.

This article proposes a procedure for estimating sex-age-specific o/e rates of marital status transitions to ensure that the projected propensities of transitions from marital status i to j are achieved consistently in the one-sex family status life table model. Another more practically useful procedure for estimating sex-age-specific o/e rates of marital status transitions that are consistent with the two-sex constraints and the projected summary measures of marriage/union formation and dissolution in the future years in the two-sex family household projection model is also proposed.

We define, discuss, and employ two alternative groups of the summary measures for males and females combined in the two-sex family household projection model: overall propensities -- PM(t), PD(t), PC(t), and PCS(t); standardized general rates -- G(t), GD(t), GC(t), and GCS(t). The two alternative groups of the summary measures are appropriate for both measuring the overall level of marriage/union formation/dissolution and ensuring the two-sex constraints; they are all demographically interpretable, measurable, and predictable; each group has only four summary parameters to be projected, which makes the model and applications manageable.

As compared to the standardized general rates, the overall propensities are relatively easier to understand from the public's and policy makers' points of view. For example, to report that the divorce level in year t implies that 40 percent of the marriages would eventually end in divorce is more understandable to the public and policy makers than reporting the standardized general rate of divorce. This is useful in policy analysis. On the other hand, time series of GM(t) and GD(t) are likely available from vital statistics. The trends of GC(t) and GCS(t) with shorter average periods (e.g., three-year average) can be estimated from retrospective surveys that collect cohabitation information. Thus, projecting GM(t), GD(t), GC(t), and GCS(t) based on time series extrapolation would be relatively easy, which is useful in short-term forecasting for market analysis and business planning purposes. Estimations of the multi-state life table based PM(t), PD(t), PC(t), and PCS(t) in the past, however, need intensive sex-age-specific o/e rates of marital status transitions, which may be difficult to obtain, especially for time series estimates with short period intervals. Thus, the overall propensities PM(t), PD(t), PC(t), and PCS(t) may be more informative in policy analysis/scenarios, which do not necessarily need time series extrapolation. The standardized general rates GM(t), GD(t), GC(t), and GCS(t), which are more likely to be projected based on time series extrapolation, would be recommended for short-term family household forecasts for market analysis and business planning purposes.

The procedures proposed in this article are useful and can be applied in both macro and micro models for family household projections or simulations that need the time-varying sex-age-specific o/e rates of marital status transitions. Of course, the accuracy of family household projections depends on how well the summary measures are projected based on time series trend extrapolation or expert opinion approaches. It also depends on how well the sex-age-specific standard schedules are estimated from the demographic data and how well the sex-age-marital status-specific population distributions at the starting year of the projection are derived from the census data file (Zeng et al., 2003). Family household projection also needs to reasonably estimate the time-varying age-parity-marital status-specific o/e rates of birth and other time-varying sex-age-specific rates such as mortality, migration, and leaving the parental home, which are beyond the scope of this article, but certainly deserve further research.

Appendix: Illustrative Numerical Applications to Verify the Convergence of the Iterative Procedure for Estimating Age-Specific o/e Rates of Marital Status Transitions that are Consistent with the Projected Propensities in the One-Sex Life Table Model

The sex-age-specific o/e rates of marital status transitions in 1990-1996 used in these illustrative applications as the standard schedules are part of the results of our research project on U.S. family household projection (Zeng, Yang, Wang, and Morgan, 2002). These o/e rates are based on the pooled data from four U.S. national surveys (NSFH, NSFG, CPS, and SIPP) conducted in 1990-1996. We present the main results of the illustrative numerical applications that verify the convergence of the iterative procedure in the four tables presented in this Appendix. We do not present the massive amount of age-specific o/e rates of marital status transitions due to space limitations, but they are available from the authors upon request.

(a) Classic model of four marital statuses *(see Figure 1):*

As discussed earlier in the text, we first use $m_{ij}^{s}(x,s)$ to estimate $P_{ij}^{s}(x,s,t)$ using the matrix formula (1), and construct a multi-state life table to get the propensities of marital status transitions implied by the standard schedules ($A^s(i,j,s)$). The projected propensities of first marriage, divorce, remarriage of widows, and remarriage of divorcees (A(i,j,s,t)) are arbitrarily set as 8%, 10%, 8%, and 8% higher than those implied by the standard schedules ($A^s(i,j,s)$). This indicates the projected changes in the propensities of the marital status transition (as compared to the standard schedules) are 8-10%, and the first adjusting factors ($X^1(i,j,s,t) = \frac{A(i,j,s,t)}{A^s(i,j,s)}$) are 1.08, 1.10, 1.08, and 1.08 (see second row of Table A-1a). We use $m_{ij}^1(x,s,t) = X^1(i,j,s,t)m_{ij}^s(x,s) = \frac{A(i,j,s,t)}{A^s(i,j,s)}m_{ij}^s(x,s)$ as the first approximation of $m_{ij}(x,s,t)$. We then use the first approximation $m_{ij}^1(x,s,t)$ to estimate $P_{ij}^1(x,s,t)$, and use $P_{ij}^1(x,s,t)$ to construct a new multi-state life table to get a new set of approximations $A^1(i,j,s,t)$, as listed in the fourth row of Table A-1a. The absolute values of the relative difference between $A^1(i,j,s,t)$ and the projected A(i,j,s,t) are 7-9% (see the last row of Table A-1a), while the projected changes in the propensities are 8-10% (see second row of Table A-1a). This demonstrates why the projected x% of changes in propensity of the marital status change cannot be achieved through simply inflating or deflating the corresponding age-specific o/e rates of the marital status transitions by x%.

After 39 iterations (n=39) in this illustrative application of the four marital statuses model, each $A^n(i,j,s,t)$ is equal or extremely close to the corresponding projected A(i,j,s,t) (see the third and fourth row of Table A-1b); all absolute values of the relative discrepancy rate between $A^n(i,j,s,t)$ and the projected A(i,j,s,t) are less than 0.1% (see last row of Table A-1b). The relative discrepancy rate can be reduced further with additional iterations. For example, the relative discrepancy rate is less than 0.01% with 61 iterations.

Table A-1a. Four marital statuses model (4 sets of i→j marital statuses transitions, see Figure 1)

Number of iterations = 1 (n=1)

I→j	1→2	2→4	4→2	3→2
X^1(i,j,st)	1.0800	1.1000	1.0800	1.0800
A(i,j,s,t+1)	0.9254	0.6037	0.6839	0.0969
A^1(i,j,s,t)	0.8569	0.5489	0.6333	0.0897
% diff.	-7.4074	-9.0909	-7.4074	-7.4074

% diff. = 100 x [A^1(i,j,s,t)- A(i,j,s,t+1)] / A(i,j,s,t+1)

Table A-1b. Four marital statuses model (4 sets of i→j marital statuses transitions, see Figure 1)

Number of iterations = 39 (n=39)

I→j	1→2	2→4	4→2	3→2
X^1(i,j,t)	1.0000	1.0000	1.0000	1.0004
A(i,j,t+1)	0.9254	0.6037	0.6839	0.0969
A^n(i,j,s,t)	0.9254	0.6037	0.6839	0.0969
% diff.	-0.0034	0.0018	0.0014	-0.0433

% diff. = 100 x [A^n(i,j,s,t)- A(i,j,s,t+1)] / A(i,j,s,t+1)

(b) A model of seven marital statuses including cohabitation *(see Figure 2):*

The projected changes in the propensities of the marriage/union formation and dissolution (as compared to the standard schedules) are between –3% to 10%, and the first adjusting factors ($X^1(i,j,s,t)$) are between 0.97 and 1.10 (see second row of Table A-3a). We use $m^1_{ij}(x,s,t) = X^1(i,j,s,t)m^s_{ij}(x,s)$ as the first approximation of $m_{ij}(x,s,t)$. We then use the first approximation $m^1_{ij}(x,s,t)$ to estimate $P^1_{ij}(x,s,t)$, and use $P^1_{ij}(x,s,t)$ to construct a new multi-state life table to get a new set of approximations A^1(i,j,s,t), as listed in the fourth row of Table A-2a. The absolute values of the relative difference between A^1(i,j,s,t) and the projected A(i,j,s,t) are 2-9% (see the last row of Table A-2a), while the projected changes in the propensities are between –3% and 10% (see second row of Table A-2a).

After 174 iterations (n=174) in the illustrative application of the seven marital statuses model, each A^n(i,j,s,t) is equal or extremely close to the corresponding projected A(i,j,s,t) (see third and fourth row of Table A-2b); all absolute values of the relative discrepancy rate between $A^n(i,j,s,t)$ and the projected A(i,j,s,t) are less than 0.1% (see last row of Table A-2b). The relative discrepancy rate can be reduced further with additional iterations. For example, the relative discrepancy rate is less than 0.01% with 277 iterations.

Table A-2a. Seven marital statuses model (13 sets of i→j marriage/union statuses transitions, see Figure 2)

Number of iterations = 1 (n=1)

i→j	1->2	1->5	2->4	3->2	3->6	4->2	4->7	5->1	6->3	7->4	5->2	6->2	7->2
$X^1(i,j,s,t)$	1.0800	1.0700	1.1000	1.0800	0.9800	1.0800	1.0700	1.0200	1.0600	1.0600	0.9700	0.9700	0.9700
A(i,j,s,t+1)	0.3424	0.6060	0.5092	0.0890	0.0752	0.2254	0.5160	0.7286	0.5992	0.4310	0.2492	0.2247	0.6138
$A^1(i,j,s,t)$	0.3171	0.5663	0.4629	0.0824	0.0768	0.2087	0.4822	0.7143	0.5653	0.4066	0.2569	0.2316	0.6327
% diff.	-7.4074	-6.5421	-9.0909	-7.4074	2.0408	-7.4074	-6.5421	-1.9608	-5.6604	-5.6604	3.0928	3.0928	3.0928

% diff. = 100 x $[A^1(i,j,s,t) - A(i,j,s,t+1)] / A(i,j,s,t+1)$

Table A-2b. Seven marital statuses model (13 sets of i→j marriage/union statuses transitions, see Figure 2)

Number of iterations = 174 (n=174)

i→j	1->2	1->5	2->4	3->2	3->6	4->2	4->7	5->1	6->3	7->4	5->2	6->2	7->2
$X^1(i,j,s,t)$	1.0001	1.0000	1.0000	1.0007	0.9991	1.0000	1.0000	0.9998	0.9998	1.0000	0.9998	1.0005	1.0000
A(i,j,s,t+1)	0.3424	0.6060	0.5092	0.0890	0.0752	0.2254	0.5160	0.7286	0.5992	0.4310	0.2492	0.2247	0.6138
$A^n(i,j,s,t)$	0.3424	0.6060	0.5092	0.0889	0.0753	0.2254	0.5160	0.7287	0.5993	0.4310	0.2492	0.2246	0.6138
% diff.	-0.0086	0.0009	0.0003	-0.0662	0.0921	0.0004	-0.0005	0.0222	0.0171	-0.0006	0.0237	-0.0477	0.0011

% diff. = 100 x $[A^n(i,j,s,t) - A(i,j,s,t+1)] / A(i,j,s,t+1)$

13

Notes

[1] When 1st marriage and/or fertility are delayed or advancing, for example, one may shift the age-specific standard schedules of 1st marriage and/or fertility to the right or left by the amount of increase or decrease in the mean age at 1st marriage and/or fertility, while the shape of the schedules remains unchanged. One may also assume that 1st marriage and/or fertility would be delayed or advanced while the curves become more spread or more concentrated through parametric modelling (Zeng et al., 2000).

Zeng, Yang, Wang, and Morgan (2002) recently estimated the U.S. race-sex-age-specific o/e rates of marital status transitions and the race-age-parity-marital status-specific o/e rates of fertility in the 1970s, 1980s, and 1990s. This work is based on pooled data from 10 waves of four major national surveys conducted from 1980 to 1996 (with a total sample size of 394,791 women and men). The estimates show empirically that the basic shapes of the demographic schedules remain reasonably stable from the 1970s to the 1990s, while the timing was changing. We, thus, may reasonably assume that in normal circumstances the basic shapes of the standard schedules remain stable, while the changes in timing are modelled through the changing mean age at marriage and fertility in the family household projection.

[2] If one lumps never-married & cohabiting, widowed & cohabiting, and divorced & cohabiting into one status of "cohabiting", the model is simpler and contains five statuses only. However, in such a five-status model, the three kinds of cohabiting people with different legal marital statuses (never-married, widowed, and divorced) are not distinguishable and they are all mixed into "single" once their union is broken. Such mixture of the never-married, widowed, and divorced into one "single" status is not appropriate because these three kinds of people may likely to behave differently.

[3] Many scholars believe that the formula based on the constant intensity assumption is better than the formula based on the uniform distribution assumption, since it is better suited for non-linear modeling. Furthermore, when the o/e rate $m_{ij}(x)$ is greater than 2/n (n is the length of the age interval), the formula based on the uniform distribution assumption will produce an unreasonable probability that is greater than one. The formula based on the constant intensity assumption will never produce such an unreasonable probability. After complex mathematical derivation, Gill and Keilman (1990) concluded that the numerical difference in the estimates based on the uniform and constant assumptions is $n^2\, m_{ij}(x)^3 / 12$. Obviously, when the age interval is one year

(n=1) and the value of $m_{ij}(x)$ is not too large, the numerical difference is negligible. In this study, we use the formula (1) that is based on the constant intensity assumption.

[4] The measurement "propensity of marital status transitions" used in this article is equivalent to the "probability of marital status transitions" used by Schoen (1988: 95). We prefer to use the word "propensity", rather than "probability", in order to distinguish it from the age-specific probabilities of marital status transitions, which are frequently used.

[5] Note that the widowhood rates entirely depend on spouses' death rates, which are projected independently based on the standard mortality schedules and the projected life expectancy at birth in year t.

[6] "t" refers to both future years of projection and years in the recent past. We estimate the standardized general rates of marriage/union formation and dissolution in the recent past and in the future projection years using the same "standard" of the sex-age distribution of the population observed from the most recent census and the time-varying sex-age-specific o/e rates to eliminate the possible distortion caused by changes in the age structure of the population.

[7] If the population in the household projection model is classified by race or by rural-urban sectors, the "standard" $N_i(x,s,T0)$ are sex-age-marital status distribution for all races combined or for rural-urban sectors combined counted in the most recent census. This is to standardize the age distributions not only across time but also across race groups or rural-urban sectors, in order to eliminate possible distortions in measuring levels of marriage/union formation and dissolution due to changes in age structures in different years and between race groups or rural-urban sectors.

[8] In monogamy societies in a given year, the number of currently married (or cohabiting) males is equal to the number of currently married (or cohabiting) females;

the total numbers of newly divorced (or union broken) men and women should be equal. Thus, the overall probabilities of divorce (or union break) of men and women should be equal and defining summary measures of divorce (or union break) for men and women combined is naturally reasonable. Although the numbers of newly marrying (or newly cohabiting) men and women are equal, the overall probabilities of marriage/cohabitation of men and women are not necessarily the same because males and females who are eligible to newly marry (or newly cohabit) may not be equal. Thus, the summary measures of marriage/cohabitation for men and women combined indicate the average intensity of marriage/union formation across sexes.

[9] The distortions of the total rates of the non-repeatable events (such as first marriage and first birth) due to changes in tempo are discussed intensively in the literature (See, for example, Ryder 1964; 1980; 1983; Keilman, 1995; Van Imhoff and Keilman, 1995; Bongaarts and Feeney, 1998; Kim and Schoen, 2000; Kohler and Philipov, 2001; Van Imhoff and Keilman, 2000; Zeng and Land, 2001; 2002).

[10] The format and structure of the vector $\boldsymbol{N}(x,s,t)$ and the matrices $\boldsymbol{M}(x,s,t)$ and $\boldsymbol{P}(x,s,t)$ are referred to the standard text book (Schoen, 1988; Preston et al., 2001).

[11] The symbols of some variables (such as $m_{ij}(x,s,t)$, $m''_{ij}(x,s,t)$, $m'''_{ij}(x,s,t)$) used in the Step 2b are the same as those in the Step 2a to simplify the notations, given the similar principles/structure of the procedures; but their numerical values are different because we use different summary measures in Step 2b and Step 2a.

[12] We do not present the massive amount of time-varying sex-age-specific o/e rates of marital status transitions in future projection years following the different approaches of Step 1 – Step 2b and Step 1 – Step 2a due to space limitations, but they are available from the authors upon request.

[13]

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Figure 1. A model of four marital statuses

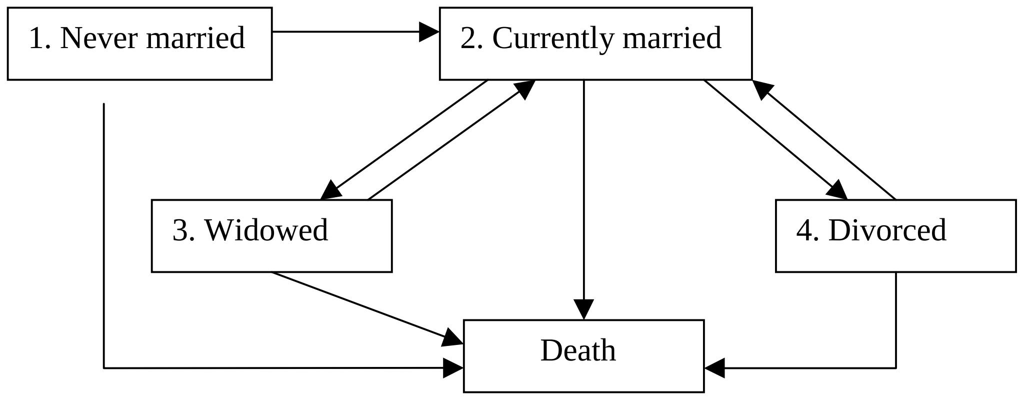

Figure 2. A model of seven marital statuses (including cohabitation)