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Research Article

The modal age at death and the shifting mortality hypothesis

Vladimir Canudas-Romo

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Vladimir Canudas-Romo[1]

Abstract

The modal age at death is used to study the shifting mortality scenario experienced by low mortality countries. The relations of the life table functions at the modal age are analyzed using mortality models. In the models the modal age increases over time, but there is an asymptotic approximation towards a constant number of deaths and standard deviation from the mode. The findings are compared to changes observed in populations with historical mortality data. As shown here the shifting mortality scenario is a process that might be expected if the current mortality changes maintain their pace. By focusing on the modal age at death, a new perspective on the analysis of human longevity is revealed.

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1. Introduction

Lexis (1878) considered that the distribution of deaths consisted of three parts: a decrease in the high number of deaths with age after birth to account for infant mortality; deaths centred around the late modal age at death (referred to hereafter as modal age at death), accounting for senescent mortality; and premature deaths that occur infrequently at young ages between the high infant mortality and senescent deaths. Life expectancy, or the mean of the life table distribution of deaths, is the indicator most frequently used to describe this distribution. In a regime with a high level of infant mortality, life expectancy will be within the age range of premature deaths, even when most deaths occur around ages zero and the modal age at death. The early stages of the epidemiological transition (Omran 1971) are characterized by a reduction in infant mortality. These changes in infant mortality have been captured very accurately with the rapid increase in life expectancy over time. However, an alternative perspective is to study the age where most of the deaths are occurring, that is the modal age at death.

Currently, mortality is concentrated at older ages in most countries. Life expectancy has slowed down its rapid increase and is now moving at a similar pace as the late modal age at death. Studying the modal age at death provides an opportunity to have a different perspective of the changes in the distribution of deaths and to explain the change in mortality at older ages (Kannisto 2000, Kannisto 2001, Robine 2001, Cheung et al. 2005, Canudas-Romo 2006, Cheung and Robine 2007, Canudas-Romo and Wilmoth 2007). This study describes the important role of the modal age at death in populations experiencing mortality decline.

The aim of this article is to study the shifting mortality hypothesis by assessing the changes in the late modal age at death. The shifting mortality hypothesis suggests a shifting force of mortality schedule which retains its shape over time as mortality falls (Bongaarts and Feeney 2002, 2003). Bongaarts (2005) furthers this idea by describing the shifting mortality regime as one where adult mortality is assumed to shift to higher ages over time. These characteristics of the shifting hypothesis can be observed in any of the life table functions: the hazard function, the survival function and the density function describing the distribution of deaths. The interrelations of these three curves imply that changes in one will be carried on to the others (Wilmoth 1997). Under the shifting mortality regime a shift in the density function describing the distribution of deaths implies that the hazard function declines but retains its shape, and that the survival function increases as the curve moves further to the right. In the present study, these three functions are analyzed together with the changes that occur in the modal age at death. The debate on "How long do we live?" initiated by Bongaarts and Feeney has been focused on the study of life expectancy at birth (Barbi et al. 2008, Feeney 2006, Goldstein 2006, Guillot 2006, Horiuchi 2005, Rodriguez 2006, Schoen and Canudas-Romo 2005, Vaupel 2005, Wachter

2005, Wilmoth 2005). However, as shown in this manuscript the modal age at death gives an appealing alternative perspective to investigate the shifting mortality hypothesis.

The modal age at death, the number of survivors and deaths at the modal age, and the concentration of deaths around this measure are studied under simple dynamic models. Analytical expressions for the increase in longevity, seen as the change over time in modal age at death, and variability of age at death around the modal age are found using mathematical models whose parameters change over time and age. The use of models to study the modal age at death complements the studies that have addressed research in this measure from the empirical point of view (Kannisto 2000, Kannisto 2001, Robine 2001, Cheung et al. 2005, Cheung and Robine 2007). Furthermore, as pointed out by Wilmoth and Horiuchi (1999), expressions of this type can be used in criticism of the idea that the human survival curve becomes more rectangular with decreasing levels of mortality, i.e. hypothesis of rectangularization proposed by Fries (1980). Nusselder and Mackenbach (1996) define the rectangularization as a trend toward a more rectangular shape of the survival curve due to increased survival and concentration of deaths around the mean age at death. The latter idea is used here to contrast results from a shifting mortality and the rectangularization hypothesis by looking at the variability of deaths around the modal age at death instead of the mean.

The paper is divided into seven parts. The second section introduces the reader to the formal definition of modal age at death, denoted by the letter M. The third section shows the constancy of survivors and number of deaths under four mortality models. The fourth part contains an examination of the concentration of deaths around the modal age. Applications to human populations with historical data are presented in section five. Finally, the discussion and conclusion constitute the last sections of the paper.

2. Modal age

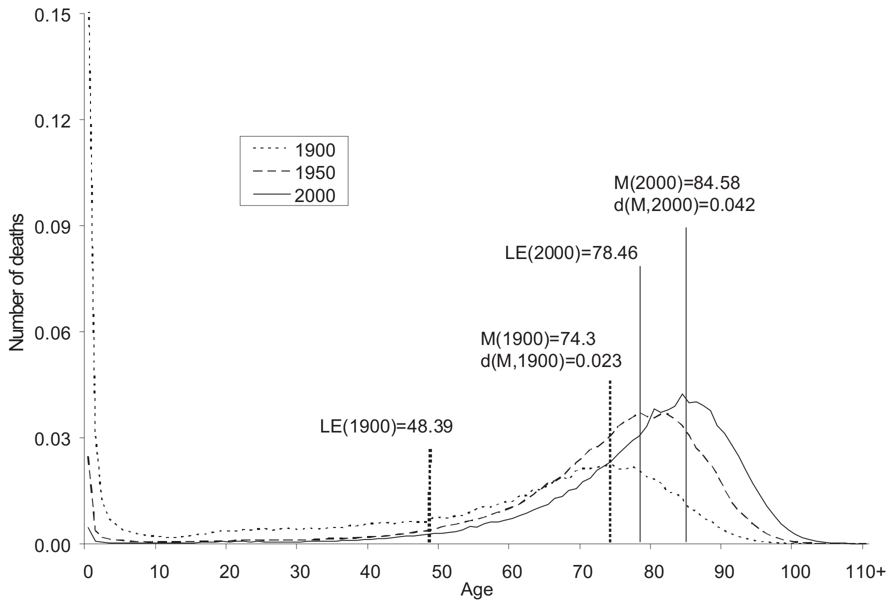
Let the force of mortality at age a and time t be denoted as $\mu(a,t)$. If the radix of the life table is one, i.e. $l(0,t) = 1$, then the life table survivorship function at age a under the death rates at time t is defined as

$$l(a,t) = e^{-\int_0^a \mu(x,t)\mathrm{d}x}. \tag{1}$$

Let $d(a,t)$ be the density function describing the distribution of deaths (i.e., life spans) in the life table population at age a and time t. The distribution of deaths for a given age is found as the product of the survival function up to that age multiplied by its force of mortality, $d(a,t) = \mu(a,t)l(a,t)$.

Figure 1 shows the distribution of life table deaths for the Netherlands at the beginning, middle, and end of the twentieth century.

Figure 1: Life expectancy, LE, modal age after age 5, M, and modal number of deaths, $d(M)$, in the life table distribution of deaths for the Netherlands total population in 1900, 1950 and 2000



Source: Author's calculations of M and $d(M)$ as explained in the appendix based on the Human Mortality Database (2008).

Also included in Figure 1 are the life expectancy, modal age after age 5, and modal number of deaths in 1900 and 2000. Here it is possible to appreciate how life expectancy is found at age 48.4 in 1900, while most of the deaths in this year are concentrated at ages below 5 and around the late modal age of 74.3. In 2000, life expectancy reached a value of 78.5 years, and reduced markedly its distance to the modal age, which moved to 84.6. The modal number of deaths changed from 2.3% of all deaths in 1900 to be 4.2% in 2000.

In industrialized countries where infant mortality has decreased dramatically, the modal age of the distribution of deaths is found at older ages. In some countries the increase in young adult mortality has created unexpected excess number of deaths at these young ages, e.g. the reversal in mortality in Russia in the 1990s. Nevertheless, these local modal ages have not passed the higher level of mortality concentration at older ages around age

80. Assuming continuity over age in the life table functions, the modal age at death after age 5 can be calculated as the age at which the derivative with respect to age of $d(a,t)$ is equal to zero. From the relation in equation (1) and the product $d(a,t) = \mu(a,t)l(a,t)$, the partial derivative of the distribution of deaths with respect to age is

$$\frac{\partial d(a,t)}{\partial a} = d(a,t)\left[\frac{\partial \ln\left[\mu(a,t)\right]}{\partial a} - \mu(a,t)\right]. \tag{2}$$

Equation (2) is equal to zero when $d(a,t)$ or $\left[\frac{\partial \ln[\mu(a,t)]}{\partial a} - \mu(a,t)\right]$ are equal to zero. In the first case there are no deaths, and therefore also no modal age. The second case implies that at the modal age M the force of mortality equals its relative derivative with respect to age,

$$\frac{\partial \ln\left[\mu(a,t)\right]}{\partial a} = \mu(a,t). \tag{3}$$

This relation at the modal age proved to be interesting under the Gompertz model of mortality (Pollard 1991, Pollard and Valkovics 1992). To further add to this special age in the distribution of deaths, Wilmoth and Horiuchi (1999) showed that the modal age is also the inflection point in the survival curve.

Model populations provide a useful way to examine changes in mortality. In the next section the trend over time in modal age at death, the number of deaths and survivors at this age under four types of mortality models are assessed. The selected models change over age and time, and have been adopted by demographers as good approximations for the force of mortality (Thatcher et al. 1998).

3. Mortality models

Four models of mortality are studied in this section, two only have information on adult mortality, and the other two models also include child mortality. A general version of the Gompertz mortality change model is presented in section 3.1. To account for the over-estimation of mortality at older ages the Logistic model is used in section 3.2. To study the effect of changes in infant mortality in the modal age at death, the Siler mortality change model is employed in section 3.3. The latter section also includes a model that combines the Logistic and Siler models to assess the combined effect of infant and senescent mortality in section 3.3.2. The results of the models are presented over 150 years (units of change in models). The first hundred years allow comparison with the twentieth century experience in human populations with mortality data presented in section 5. The remaining 50 years could be a likely scenario for the first half of the twenty-first century.

3.1 Gompertz mortality change model

Bongaarts and Feeney (2002, 2003) have stimulated a new debate about how to interpret period summary measures of mortality when rates of death vary over time. The parallel shift in adult mortality analyzed by Bongaarts and Feeney can be characterized by a Gompertz mortality change model (Vaupel 1986, Vaupel and Canudas-Romo 2003). Their formulation is an extension of the Gompertz (1825) model of mortality, which has a changing force of mortality component,

$$\mu(a,t) = \mu(0,t)e^{\beta a}, \tag{4}$$

where $\mu(0,t)$ reflects the value of the rate of mortality decrease over time and parameter $\beta > 0$ is the fixed rate of mortality increase over age.

Substituting the Gompertz mortality change model in equation (3) and solving for a gives us the modal age at time t, by letting $M = a$,

$$M(t) = \frac{\ln[\beta] - \ln[\mu(0,t)]}{\beta}. \tag{5}$$

The survival function for this model is obtained by substituting the force of mortality of equation (4) in equation (1). At the modal age, (5), the survival function can then be simplified to

$$l(M,t) = e^{\left[\frac{\mu(0,t)}{\beta} - 1\right]}, \tag{6}$$

with a maximum number of deaths of

$$d(M,t) = l(M,t)\mu(M,t) = \beta e^{\left[\frac{\mu(0,t)}{\beta} - 1\right]}. \tag{7}$$

Bongaarts and Feeney (2002, 2003) showed that the value of $\mu(0,t)$ declines over time. When the reduction in mortality is almost negligible, and the value of $\mu(0,t)$ approaches zero, equations (6) and (7) decrease to a constant number of survivors

$$\lim_{\mu(0,t)\to 0} l(M,t) = e^{-1} \approx 0.37, \tag{8}$$

and thus the number of deaths is $d(M,t) = \beta e^{-1}$ (Pollard 1991, Pollard and Valkovics 1992). However, the modal age at death increases to infinity. Therefore, under this model the rectangularization process of the survival curve has stopped completely and no more concentration is observed in $l(M,t)$. Instead, a shift occurs in the modal age towards advanced ages.

A particular case of equation (4) is where the $\mu(0,t)$ is parameterized, and the force of mortality at age 0 and time t is $\mu(0,t) = e^{\alpha-\rho t}$ (Vaupel 1986, Schoen et al. 2004). The force of mortality at age a and time t is defined as

$$\mu(a,t) = e^{\alpha-\rho t+\beta a}, \tag{9}$$

where α is a constant that reflects the value of the force of mortality at age zero and time zero, $\mu(0,0)$, and parameter ρ is the rate of mortality decrease over time.

In this model of continuous mortality decline, we set $\beta = 0.11$ as the conventional value for the pace of mortality increase over age (see the average row in Table 1). Reasonable values for contemporary Western low mortality populations are $\alpha = -10.5$ and $\rho = 0.01$ (Bongaarts 2005, Schoen et al. 2004). Using these values and equations (5) and (9), we obtain a modal age for time t of $M(t) = 75.4 + \left(\frac{\rho}{\beta}\right)t$, i.e., increasing one year of age every eleven calendar years, $\frac{\rho}{\beta} = 0.909$. However, the survivors and number of deaths in (6) and (7) change very modestly over time, reaching their limit values of $l(M) = e^{-1}$ and $d(M) = \beta e^{-1} = 0.04$, respectively.

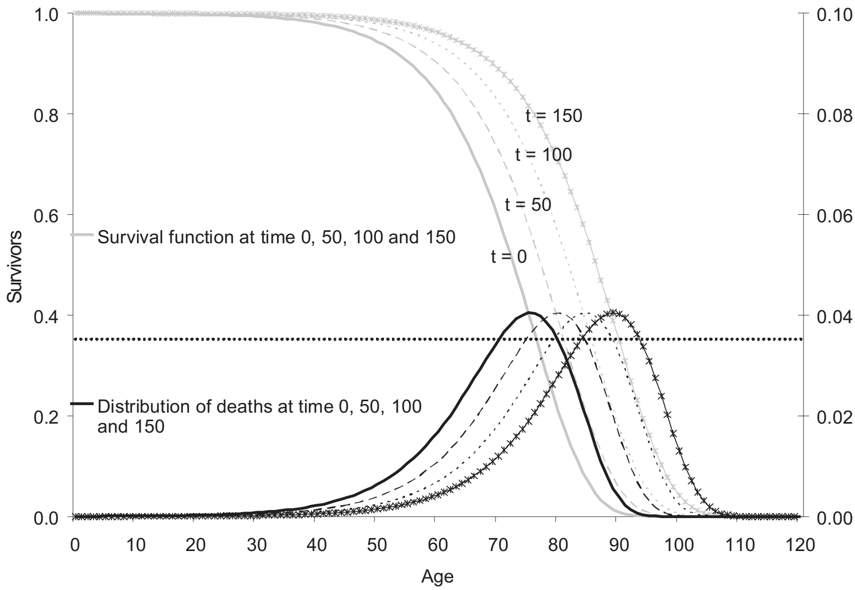
Figure 2 shows the survival function and distribution of deaths in the continuous declining mortality model (9) over 150 years.

As observed in Figure 2, the modal age increases every 50 years by 4.5 years of age. However, the number of survivors and deaths at the modal age remain constant (values underlying Figure 2 are shown in Table 2).

The shifting mortality model observed by Bongaarts and Feeney (2002) shows many implications for the survival function and the distribution of deaths. These authors have advanced some possible implications for this type of shift in mortality, e.g. the need to find alternative summary measures of mortality that take this shift into account. As shown here, for populations where mortality is concentrated at adult ages the modal age at death is a good candidate to asses the question: how long do we live? This model is not unique, however. These results are tested in the next subsections under alternative mortality models.

It is not possible to have an analytical expression of the modal age at death for all models of mortality, e.g. the Makeham model. However, it is possible to moderate this limitation by studying the models by means of data simulation. In the following sections the simulation procedure is employed for the models that have complex analytical expressions.

Figure 2: Survival function and distribution of deaths from a life table of a Gompertz mortality change model with parameters: $\alpha = -10.5, \beta = 0.11$ and $\rho = 0.01$



3.2 Logistic model

The logistic model has been used in place of the Gompertz model to account for the overestimation of mortality at older ages (Thatcher et al. 1998, Thatcher 1999). Bongaarts (2005) studied separately the two components of the logistic model, senescent and background mortality. Here only the senescent component is examined, because the background component does not vary over age. The force of mortality is here expressed as

$$\mu(a,t) = \frac{e^{\alpha(t)+\beta(t)a}}{1+e^{\alpha(t)+\beta(t)a}}, \tag{10}$$

where the parameters $\alpha(t)$ for the level of mortality and $\beta(t)$ for the rate of increase in mortality change over time. The corresponding modal age for the logistic equation (10) can be found by applying the relation in equation (3) as

$$M(t) = \frac{\ln[\beta(t)] - \alpha(t)}{\beta(t)}. \tag{11}$$

and the number of deaths at this modal age is

$$d(M,t) = l(M,t)\mu(M,t) = \left[\frac{1 + e^{\alpha(t)}}{1 + \beta(t)}\right]^{\frac{1}{\beta(t)}} \frac{\beta(t)}{1 + \beta(t)}. \tag{12}$$

Bongaarts (2005) examined change over time for $e^{\alpha(t)}$ and $\beta(t)$ parameters in several countries in the second half of the twentieth century. As noted by Bongaarts, the first parameter decreased to levels around $\alpha(t) = -10.5$, while $\beta(t)$ has remained almost constant around values of 0.11. According to these values, the number of deaths at the modal age is 0.038, just slightly less than the results for the Gompertz mortality change model.

If further decline in mortality is observed the parameter $e^{\alpha(t)}$ will also continue to decrease. In this case equation (12) depends only on the measure of the rate of increase in mortality with age. The survivors and distribution of deaths follow similar shifting patterns over time to those observed in Figure 2, because modal age at death continues to increase with the change in $\alpha(t)$. As shown by Thatcher et al. (1998) and Thatcher (1999), most mortality models fall between the overestimation of the Gompertz model and the logistic curve. Therefore, the constant value of deaths at the modal age is likely to appear in those models as well.

Table 1 presents the modal age and modal number of deaths using equations (11) and (12) and the logistic parameters presented by Bongaarts (2005). As observed in Table 1, females have higher modal age and higher modal number of deaths than their male counterparts. The largest difference between males' and females' modal ages is found in Finland, while the smallest is in Japan. The largest difference between the sexes in modal number of deaths is seen in Finland, while the smallest is in England and Wales. The modal number of deaths in the United States for females and males stand as the lowest of all examined countries although their modal ages are not at the end of the list of countries included. This suggests that in the United States deaths are more dispersed over age than in the other countries.

Table 1: Parameters of the logistic model for adult mortality, the modal age and the modal number of deaths (modal value) in 14 countries

	Females			
	α	β	Modal Age	Modal Value
Austria	-11.7	0.117	81.3	0.0407
Canada	-11.1	0.106	83.3	0.0371
Denmark	-11.1	0.108	82.1	0.0377
England and Wales	-11.2	0.109	82.1	0.0380
Finland	-11.8	0.119	81.3	0.0413
France	-11.7	0.115	82.7	0.0400
Italy	-11.8	0.118	82.1	0.0410
Japan	-11.8	0.118	81.8	0.0410
Netherlands	-11.8	0.116	83.0	0.0404
Norway	-11.9	0.117	83.7	0.0407
Sweden	-11.9	0.117	83.2	0.0407
Switzerland	-12.0	0.120	82.3	0.0417
United States	-10.7	0.101	83.6	0.0354
West Germany	-11.7	0.116	82.1	0.0404
Average	-11.5	0.114	81.9	0.0397
	Males			
	α	β	Modal Age	Modal Value
Austria	-10.4	0.106	77.1	0.0371
Canada	-10.1	0.100	78.3	0.0351
Denmark	-10.5	0.106	78.2	0.0371
England and Wales	-10.5	0.107	77.0	0.0374
Finland	-9.8	0.099	75.2	0.0347
France	-10.1	0.101	77.1	0.0354
Italy	-10.6	0.107	78.0	0.0374
Japan	-10.7	0.108	78.6	0.0377
Netherlands	-10.8	0.109	79.0	0.0381
Norway	-10.8	0.109	79.1	0.0381
Sweden	-11.1	0.112	79.7	0.0390
Switzerland	-10.9	0.111	78.6	0.0387
United States	-9.7	0.094	77.6	0.0331
West Germany	-10.4	0.105	78.0	0.0367
Average	-10.4	0.105	77.3	0.0367

Source: The logistic parameters come from the average of annual estimates for all available years from 1950 to 2000 from Bongaarts (2005).
Modal age and modal values calculated as indicated in equations (11) and (12) in the text.

3.3 Siler mortality change model

3.3.1 Siler model

The mortality models presented above assume that infant mortality has already declined and the distribution of deaths is only composed of deaths at senescent ages. However, the first stages of the epidemiological transition were characterized by a decline in infant mortality, which was followed later by declines at advanced ages (Omran 1971). Therefore, to have a complete understanding of the change over time in modal age at death and the modal number of deaths, it is necessary to include infant mortality and the premature component of the distribution of deaths.

The Gompertz model with a continuous rate of decline of equation (9), can be extended to include these two additional components. A proposal by Canudas-Romo and Schoen (2005) combines the mortality model used by Siler (1979) and parameters that account for improvement in mortality over time

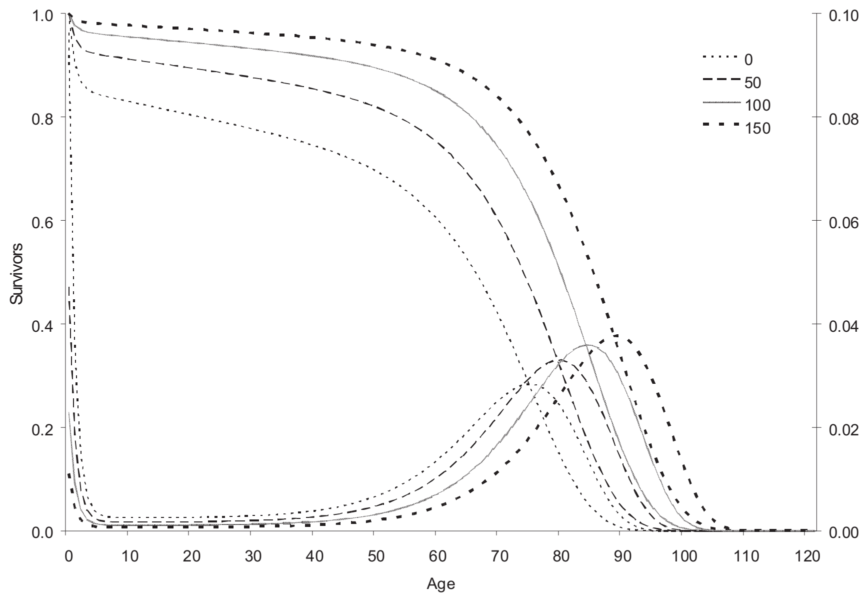
$$\mu(a,t) = e^{\alpha_1 - \rho_1 t - \beta_1 a} + e^{\alpha_2 - \rho_2 t} + e^{\alpha_3 - \rho_2 t + \beta_3 a}, \tag{13}$$

where three constant terms reflect the value of $\mu(0,0) = e^{\alpha_1} + e^{\alpha_2} + e^{\alpha_3}$; the parameters β_1 and β_3 are fixed rates of mortality decline and increase over age, respectively, and account for infant and senescent mortality; the parameters ρ_1 and ρ_2 are constant rates of mortality decrease over time. Parameters αs and βs come from the Siler model, while the ρs are used in Gompertz models with a continuous rate of decline (Vaupel 1986, Schoen et al. 2004). In the remaining text I refer to equation (13) as the Siler mortality change model.

In the model we begin with a fairly high infant mortality (103 per thousand), resulting from the values of $e^{\alpha_1} = 0.1$, $e^{\alpha_2} = 0.003$ and $e^{\alpha_3} = 0.00002$. The early decline over age proceeds at a pace of $\beta_1 = 1$ with an overall increase with age at a rate of $\beta_3 = 0.11$. At time 0, the modal age at death at advanced ages is 75.5, and the modal number of deaths at this age is 0.028 deaths. These values approach those observed in populations with historical data. For example, in Sweden in the year 1900 the late modal age at death was 76.9 and the number of deaths at this age 0.024. For the pace of mortality improvement we have chosen $\rho_1 = 0.015$ and $\rho_2 = 0.01$. These values correspond to a 1.5% decline at younger ages and mortality improvement of one percent per year at older ages. More details on the values for parameters α, β and ρ are discussed in Canudas-Romo and Schoen (2005).

Figures 3a and 3b show the change in survivors, distribution of deaths, modal age and modal number of deaths over time under the Siler mortality change model. As observed in Figures 3a and 3b, the modal age at death at advanced ages increases linearly, while the modal number of deaths increases asymptotically to its maximum value. Prevented deaths in infancy have very little probability of occurring during the ages of the premature deaths before the modal age. Therefore, the new survivors add to the distribution of deaths of senescent mortality and increase the modal number of deaths (values underlying Figures 3a and 3b are shown in Table 2).

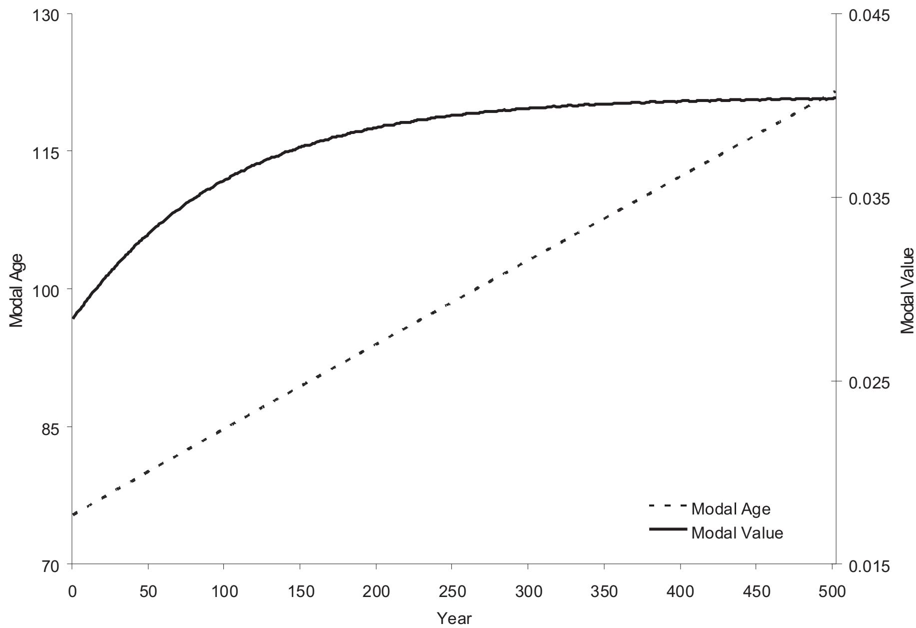
Figure 3a: Survival function and distribution of deaths from a life table of a Siler mortality change model of decline over time: $\alpha_1 = -2.3, \beta_1 = 1, \rho_1 = 0.015, \alpha_2 = -5.8, \alpha_3 = -10.5, \beta_3 = 0.11$ and $\rho_3 = 0.01$



3.3.2 Log-Siler model

A final model analyzed here is a combination of the Siler mortality change model and a logistic mortality change model. This is carried out by substituting the logistic model in equation (10) for the third term of equation (13) which accounts for the senescent part. The log-Siler model takes into account the infant mortality share and does not overestimate mortality at old ages. To better understand the dynamics of the modal age at death, under the log-Siler model, it is interesting to look at the central relation that occurs at the modal age. At this age the force of mortality and the rate of change with respect to age are exactly the same, mathematically this is expressed in equation (3). Figure 3c shows this crossover point for the log-Siler model at time 0, 50, 100 and 150.

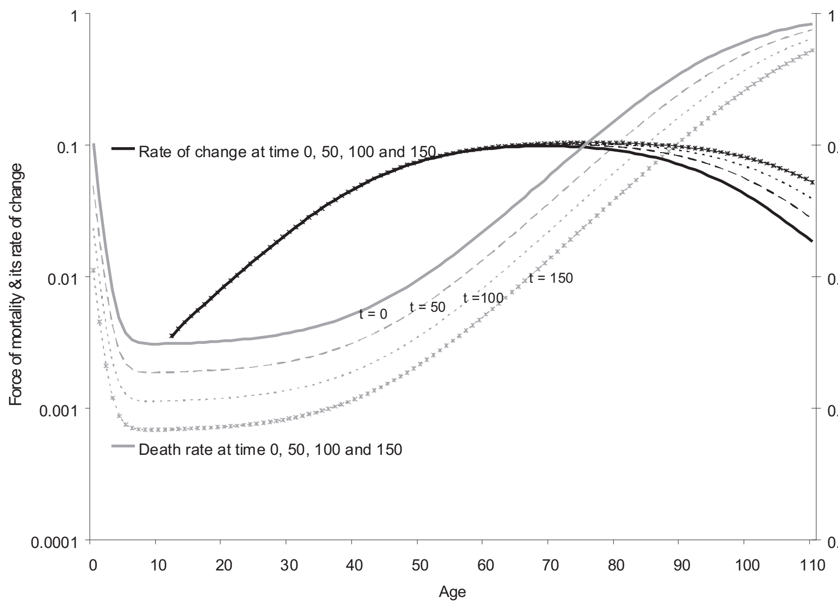
Figure 3b: Change over time in the modal age and modal number of deaths (modal value) under a Siler mortality change model of decline over time: $\alpha_1 = -2.3, \beta_1 = 1, \rho_1 = 0.015, \alpha_2 = -5.8, \alpha_3 = -10.5, \beta_3 = 0.11$ and $\rho_3 = 0.01$



In Figure 3c, the force of mortality in the log-Siler model has the characteristic U-shape of the age-pattern of mortality for human populations. The rate of change of the force of mortality increases with age after reaching its minimum around age 10. However, the increase slows down with age and at older ages a decline in the rate is observed. The latter effect comes from the logistic model. The crossover age between the force of mortality and its rate of change occurs at the modal age at death and it is found at age 75.3 at time zero, and increases to 80.1, 84.8, and 89.4 in the following 50, 100 and 150 years, respectively. Two points can be mentioned from Figures 3a, 3b and 3c:

1. The modal age at death is strongly dependent on the force of mortality and its rate of change over age prevailing at older ages.
2. Changes in infant mortality are indirectly related to the modal age at death, by having an effect in the modal number of deaths.

Figure 3c: Force of mortality and its rate of change over age under a log-Siler mortality change model of decline over time: $\alpha_1 = -2.3, \beta_1 = 1, \rho_1 = 0.015, \alpha_2 = -5.8, \alpha_3 = -10.5, \beta_3 = 0.11$ and $\rho_3 = 0.01$



The increase in the late modal age at death over the twentieth century has been observed in several countries (Kannisto 2001, Cheung et al. 2005, Cheung and Robine 2007, Canudas-Romo and Wilmoth 2007). For example, in figures for France reported by Robine (2001), there are almost linear trends in this measure for the entire century. However, the number of deaths at this age or, as studied by Robine, the verticalization of the survival curve, began a slower pace of increase in the 1950s. This discontinuity during the second half of the century is in fact noted by Robine. Nevertheless, findings from the analysis reported here provide important information about how infant mortality changes influence the modal number of deaths. To further support the results from the above models, the concentration of deaths around the modal age is analyzed in the next section.

4. Concentration of the number of deaths around the modal age at death

Standard deviations around the late modal age at death have been used to study the dispersion of deaths centred at the modal age at death (Kannisto 2001, Cheung et al. 2005, Cheung and Robine 2007). To prove the constancy of the concentration of deaths around the mode, the standard deviation from the modal age at death, SDM, is studied. It is defined as

$$SDM(t) = \sqrt{\int_0^{\omega} [a - M(t)]^2 d(a,t) da}, \tag{14}$$

where ω is the highest age attained in the population and the denominator of this measure is equal to one, i.e. $\int_0^{\omega} d(a,t)da = 1$. SDM is parallel to the standard deviation around the mean, and both measures are expressed in units of years of age (Wilmoth and Horiuchi 1999), although the centre of the observations is changed. SDM helps at quantifying the bulk of deaths around the mode. This is different than the root mean square deviations from M, which are positive, and used by Kannisto (2001), Cheung et al. (2005) and Cheung and Robine (2007). The latter measure is a hypothetical number of deaths around the mode by considering only the experience of deaths above the mode and reflecting it with respect to the mode. Opposed to this, SDM includes all the deaths below and above the mode. The limitation of this measure is that it does not differentiate from child, premature and senescent mortality. However, for our purposes of measuring the constancy of the concentration of deaths around the mode it is the most appealing measure.

Table 2 presents the modal age, modal number of deaths, and the standard deviation from the mode for the Gompertz mortality change model of Figure 2 and equation (9), and the Siler mortality change model of Figures 3a and 3b, and equation (13).

As observed in Table 2 under both models the modal age at death increases linearly. The modal number of deaths and SDM are nearly constants at values of 0.040 and 13, respectively, for the Gompertz mortality change model. The modal age and modal number of deaths for the Siler mortality change model increase over time reaching the Gompertz's values of 112.2 and 0.040, respectively, after 400 years. However, for the Siler mortality change model the standard deviation never reaches the value of 13 as its Gompertz counterpart.

Table 2: Modal age, modal number of deaths (modal value), and standard deviation from the mode, SDM, over 600 years (units of time) for a Gompertz mortality change model with parameters $\alpha = -10.5$, $\beta = 0.11$ and $\rho = 0.01$ and a Siler mortality change model with parameters $\alpha_1 = -2.3, \beta_1 = 1, \rho_1 = 0.015, \alpha_2 = -5.8$, $\alpha_3 = -10.5, \beta_3 = 0.11$ and $\rho_3 = 0.01$

	Gompertz Mortality Change Model			Siler Mortality Change Model		
Year	Modal Age	Modal Value	Standard Deviation	Modal Age	Modal Value	Standard Deviation
0	75.9	0.040	12.9	75.4	0.028	35.8
50	80.4	0.040	12.9	80.1	0.033	29.5
100	85.0	0.040	13.0	84.8	0.036	24.7
150	89.5	0.040	13.0	89.4	0.038	21.1
200	94.1	0.040	13.0	94.0	0.039	18.5
400	112.2	0.040	13.0	112.2	0.040	14.1
600	130.4	0.040	13.0	130.4	0.040	13.2

Under the mortality models analysed here we confirm that the rectangularization process of mortality compression dramatically decreases once infant mortality has become a minor factor. However, our results further the rectangularization debate by suggesting that the current situation might be the beginning of a shifting trend in mortality (Wilmoth and Horiuchi 1999, Kannisto 2000).

To this point we have analyzed the distribution of deaths and modal age at death under mortality model assumptions. In the next section we contrast the results presented above with those for changes in human populations with available time series of mortality.

5. Six industrialized countries: An illustration

The Human Mortality Database (2008) provides detailed mortality and population data for industrialized countries. For our purpose, period life table data for available years in the twentieth century and first years in the new century for England and Wales (1900-2003), France (1900-2005), Italy (1900-2004), Japan (1947-2005), Sweden (1900-2005) and the United States (1948-2003) were retrieved from this database. The results presented here are based on life tables for the total population, however further analysis by sex showed results similar to those shown below. For France and England and Wales only the civilian populations were included in the analysis to avoid the abrupt disruption in the time patterns caused by wars. The years of the influenza pandemic of 1918-1919 were taken out of the analysis because our interest is on overall time trends and not the year to year fluctuations.

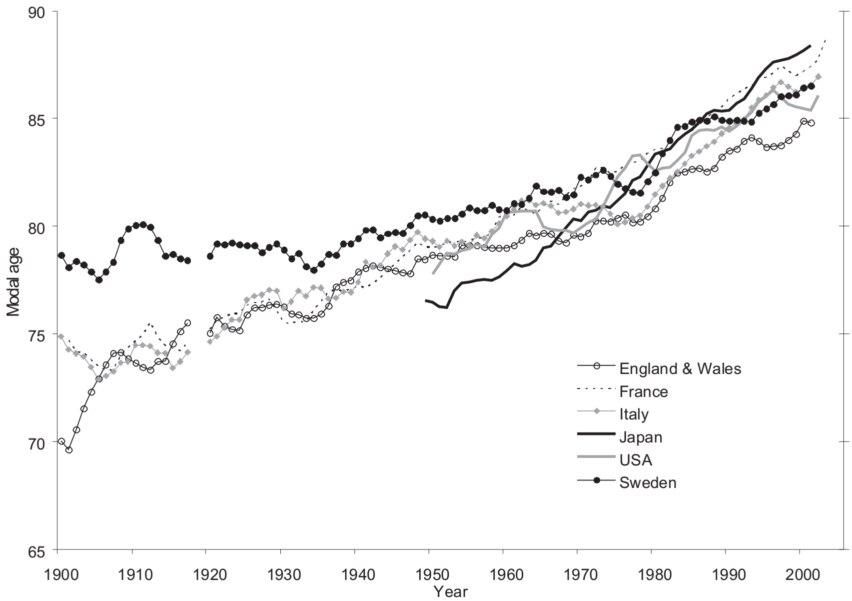
Figure 4a, 4b and 4c show the change over time in the modal age at death, the modal number of deaths and the standard deviation respect to the mode (SDM) in England and Wales, France, Japan, Italy, Sweden, and the United States. The modal age at death and the modal number of deaths were calculated by fitting a quadratic curve as explained in the appendix.

Figure 4a shows an increasing linear trend in the modal age at death for the six countries examined, although at different increasing pace from country to country. At the beginning of the Japanese series low values for modal age at death are observed, but this country rapidly changed to become the country with the highest value, followed closely by France. The modal age in England and Wales changed at a modest pace, continuing to exhibit the lowest values for this group of countries in the most recent years together with the US. Figure 4b displays the logistic trend in the number of deaths at the modal age for the six selected countries. The trend over time is less clear than for the modal age at death. However, we find an increase from low to high values and levelling off in the second half of the century. The last decade of the century reveals unexpected changes, which include a decline for Japan and an increase for the other countries. These changes have not been followed by any sudden expansion in the distribution of deaths in Japan and concentration in this distribution for the other countries (Figure 4c). As seen in Figure 4c, the common trend for all the countries is a decline in SDM over time, although at different levels from country to country. In the most recent years, Sweden has the strongest compression, as measured by the SDM, and the values of SDM in the United States are much higher than for the other countries.

6. Discussion

Life expectancy remains the most familiar measure of longevity among demographers. The life expectancy associated with the Gompertz mortality change model in equation (3) moves at a similar pace as the modal age at death, because only old age mortality is included. In the models with a child-mortality component, life expectancy increases initially at a faster pace than the modal age at death. However, this pace of increase reduces to the levels of the modal age at death. The change from a dominance of child mortality reductions to a dominance of adult mortality reductions causes the change in the life expectancy increase over time (Canudas-Romo and Wilmoth 2007). In low mortality countries the modal age at death can be an important reference point to study deviations in mortality not perceived in the life expectancy change.

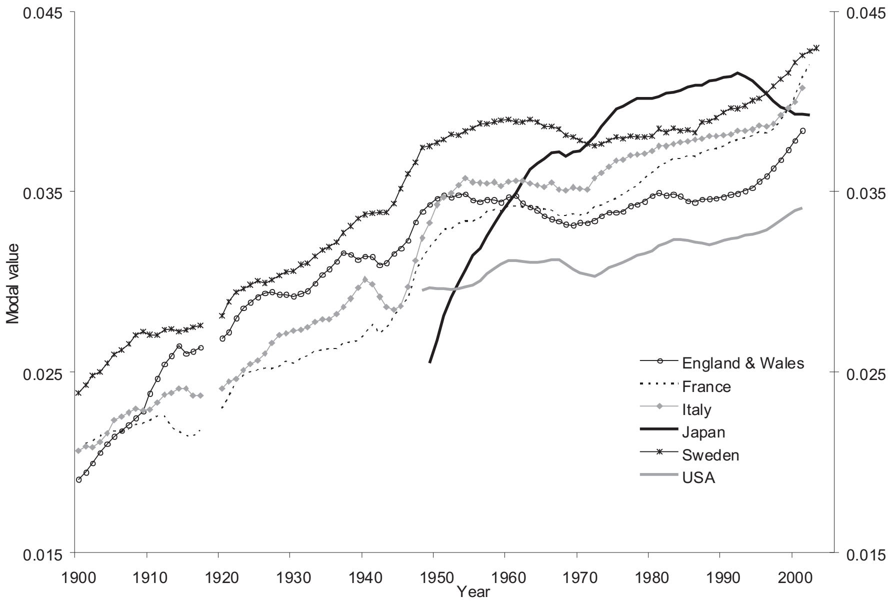
Figure 4a: Five year moving average of the modal age at death for England and Wales, France, Italy, Japan, Sweden and the United States, for available years between 1900 and 2005



*S*ource: Authors' calculations based on Human Mortality Database (2008). The years of the influenza epidemic 1918-1919 have been excluded.

The empirical results of the modal age at death, and the SDM in Figures 4a and 4c resemble those presented in the models in section three. However, the changes in the number of deaths at the modal age in the last decades of the studied period are unexpected. A tentative explanation for the decline in the modal number of deaths in Japan and the increase in the other countries (see Figure 4b) could be that those years constitute the transition period from rectangularization to shifting mortality regime. During such transitional period, population heterogeneity plays a mayor impact. Part of this heterogeneity arises from the change over time in the rate of progress in reducing death rates (Vaupel and Canudas-Romo 2003). The mortality models used in section three assume a constant reduction in death rates over time, although different from age to age. This problem is not unique to these models, it is also found in projection models, such as the Lee-Carter (1992) model (Bongaarts 2005, Janssen and Kunst 2007). However, in actual populations at each age there is great variation in the rate of improvement of mortality over time (Vaupel and Canudas-Romo 2003).

Figure 4b: Five year moving average of the modal number of deaths for England and Wales, France, Italy, Japan, Sweden and the United States, for available years between 1900 and 2005

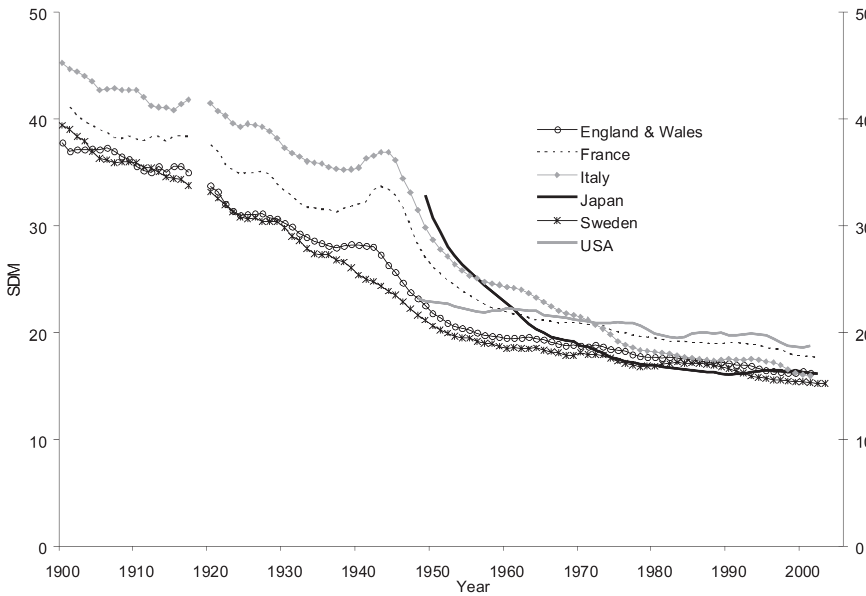


*S*ource: Authors' calculations based on Human Mortality Database (2008). The years of the influenza epidemic 1918-1919 have been excluded.

7. Conclusion

Modal age at death is an alternative measure for examining changes over time in mortality. It has been shown that the increasing modal age at death illustrates changes from a dominance of child mortality reductions to a dominance of adult mortality reductions. This process has been described as a shifting mortality process where the bulk of deaths around the modal age at death move toward older ages. The shifting mortality scenario, where the compression of mortality has stopped, may be a realistic description of the current situation in low mortality countries.

Figure 4c: Five year moving average of the standard deviation from the modal age at death for England and Wales, France, Italy, Japan, Sweden and the United States, for available years between 1900 and 2005



*S*ource: Authors' calculations based on Human Mortality Database (2008). The years of the influenza epidemic 1918-1919 have been excluded.

Mortality models, adopted as good approximations for the force of mortality, show that the modal age at death increases over time. However, the number of survivors and deaths at the modal age move towards constant levels. The distribution of deaths around the modal age and standard deviation from the mode also show a constant concentration -almost a reallocation- of the characteristic distribution around the modal age at death towards more advanced ages.

In the analyzed populations, the time trends in modal age and standard deviation around the mode are similar to those observed in the models. However, in more recent years the number of deaths at the modal age among the countries included in this analysis reveals deviations from the pattern suggested by the models. This could be resultant of a transition period to a shifting mortality scenario. The interest of the present research is to show overall patterns of mortality over time, but it cannot be discarded the possibility of reversals or accelerations in mortality during the transitional period. Future studies of the shape of distribution of deaths by cause of death could improve our understanding of the dynamics of mortality and their distribution.

Finally, the rectangularization of the survival curve is characterized by a change from a wide dispersion of deaths to a concentration in the number of deaths. Although it describes the mortality changes observed in the last half of the century well, it might only be a temporary phenomenon. The shifting mortality scenario studied in this article might also be transitory, yet it brings light to alternative processes that might be expected if the current mortality changes maintain their pace.

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Appendix

This appendix includes some specific calculations used in this study.

1. For countries with available data, the modal age at death was calculated by using Kannisto's (2001) proposal of modal age with decimal precision. To calculate the modal number of deaths Kannisto's quadratic function assumption has been applied. Let x be the age with the highest number of deaths in the life table at time t, $d(x,t)$. The number of deaths at ages x, $x-1$ and $x+1$ are used to fit a quadratic polynomial to the function describing the death distribution, $d(a,t)$. The quadratic function has parameters $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ that change over time,

$$d(a,t) = \alpha(t)a^2 + \beta(t)a + \gamma(t). \tag{A1}$$

The modal age at death with decimal point precision is found at the age $M(t) = -\frac{\beta(t)}{2\alpha(t)}$. In terms of the ages and distribution of deaths $d(a,t)$ the expression of the modal age at death is

$$M(t) = x - 0.5 + \frac{[d(x,t) - d(x-1,t)]}{[d(x,t) - d(x-1,t)] + [d(x,t) - d(x+1,t)]}, \tag{A2}$$

where x is the age with the highest number of deaths in the life table at time t, $d(x,t)$. This calculation is correct for a life table that changes continuously over age in which case $d(x,t)$ represents the exact number of deaths at age x, $d(x,t) = \mu(x,t)l(x,t)$. For life tables of one year age-groups the function describing the distribution of deaths should be put at the middle of the interval at age $x + 0.5$, which changes equation (A2) to be exactly Kannisto's proposal. The modal number of deaths is calculated by substituting the value of the mode in equation (A1) as

$$d(M(t),t) = \frac{-\beta(t)^2}{4\alpha(t)} + \gamma(t). \tag{A3}$$

2. Equation (3) includes the force of mortality and its relative derivative with respect to age. The force or mortality is calculated as the relative derivative of the life table survival function as

$$\mu(a,t) = -\frac{\partial \ln[l(a,t)]}{\partial a}. \tag{A4}$$

When data were available for two ages a and $a+k$ the following approximation for the relative derivative with respect to time of the function $l(a,t)$ was used:

$$\frac{\partial \ln[l(a,t)]}{\partial a} \approx \frac{\ln\left[\frac{l(a+k,t)}{l(a,t)}\right]}{k}. \tag{A5}$$

Similarly the partial derivative with respect to age for the function $\mu(a,t)$ was calculated as

$$\frac{\partial \ln[\mu(a,t)]}{\partial a} \approx \frac{\ln\left[\frac{\mu(a+k,t)}{\mu(a,t)}\right]}{k}. \tag{A6}$$

also called the life-table aging rate by Horiuchi and Wilmoth (1998).