A General Analysis of Sequential Merger Games, with an Application to Cross-Border Mergers

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Abstract

This paper seeks to uncover why the pattern of equilibria in sequential merger games of a certain type is similar across a fairly wide class of models much studied in the literature. By developing general conditions characterising each element of the set of possible equilibria, I show that the solution to models that satisfy a certain sufficient condition will be restricted to the same subset of equilibria. This result is of empirical relevance in that the pattern of equilibria obtained for this wide class of models is associated with mergers not happening in isolation but rather bunching together. I extend the results to the analysis of cross-border mergers, studying two standard models that satisfy the sufficient condition -- Sutton's (1991) vertically-differentiated oligopoly and Perry and Porter's (1985) fixed-supply-of-capital model.

Keywords: Mergers; sequential mergers; cross-border investment; technology transfer.

JEL Nos.: D43, F23, G34, L13

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1 Introduction

Several papers have studied the interdependence of horizontal mergers by modelling a particular type of sequential game of merger formation (Fauli-Oller 2000, Matsushima 2001, Motta and Vasconcelos 2003). In this type of game, merger decisions are made in sequence by different and exogenous groups of firms, ending in a product market competition stage. Despite employing different setups, such as different demand or cost conditions, or different merger technologies, the pattern of equilibria in the sequential game is similar across these models. As parameter values vary, only elements from the same subset of the set of possible equilibria is obtained. The equilibrium outcomes corresponding to the equilibria obtained in all of these papers is such that either *all* groups of firms that are allowed to merge choose to do so or that *none* of the groups of firms choose to merge. In particular, outcomes where only some groups of firms choose to merge while other groups choose not to are not supported in equilibrium. As will be noted, this "all-or-none" finding may be of empirical relevance in that mergers are commonly observed to "bunch" together, where periods of high merger activity are followed by periods of low activity.

Fauli-Oller (2000) considers cost-asymmetric firms facing linear demand under Cournot competition. In the merger stages, prior to the final market competition stage, low-cost firms are allowed to bid in sequence for their high-cost counterparts. In a spatial context, Matsushima (2001) studies sequential mergers by placing firms symmetrically around a unit circle, with firms facing linear demand at each consumer location and engaging in Cournot competition. Sequential pairwise mergers between firms located diametrically opposite one another are then allowed. Motta and Vasconcelos (2003) consider an industry with a fixed supply of capital distributed among its firms, with firms' marginal costs being a decreasing function of the capital they own[1]. Exogenous groups of firms decide in sequence on whether to merge, leading up to a market competition stage. What is of interest to me in the current paper is that, despite the different setups of each model, the equilibria obtained are of underlying similarity, in the sense that only a similar subset of the set of possible equilibria of the sequential merger game is obtained.

This paper makes two contributions. The first aim is to uncover this underlying similarity in the pattern of equilibria across these models studied in the literature. By writing conditions on firms' reduced-form profit functions, I characterise each possible equilibrium of the sequential merger game. I explain why only a similar subset of the set of possible equilibria is obtained in these models by reference to a sufficient condition which holds across these models. When this condition is satisfied, a common subset of the set of possible equilibria is ruled out. I motivate this sufficient condition by resorting to the way non-participating firms react to the merger of rival firms, as is well understood in the literature (see Salant, Switzer and Reynolds 1983 in the case of Cournot competition, and Deneckere and Davidson 1985 in the case of Bertrand competition).

The second aim is to illustrate that the class of models for which this similar pattern of equilibria is obtained is fairly broad. To this end, I develop two other standard examples – Sutton's (1991) vertically-differentiated Cournot oligopoly and Perry and Porter's (1985) quadratic-cost (fixed-supply-of-capital) model – in which the sufficient

[1] This feature of their model is thus similar to the second example presented in this paper, based on Perry and Porter (1985).

condition holds. As such, the same pattern of equilibria as that in, say, Fauli-Oller (2000) obtains. By introducing an additional parameter (namely a trade cost between two countries), I frame the sequential merger game in both examples in a cross-border context. I thus extend the results to the analysis of cross-border mergers.

Adapting a vertically-differentiated Cournot oligopoly model due to Sutton (1991) and later applied in a trade context by Motta (1992, 1994), I allow high-quality firms located in one country to merge with low-quality firms located in the second country. The merger technology is such that if a merger is undertaken, the level of quality offered by the merged (multinational) firm is the higher of the qualities offered by its constituent firms. Further, goods produced in one country but sold in another (exports) incur (linear) trade costs. Thus cross-border mergers enable firms to transfer technology.

The second example is an adaptation of Perry and Porter (1985)'s quadratic-cost model. This model is of particular interest for the following reason. The majority of models (with single-product firms) in the merger literature considers a merged firm to be "about" the same size (bar the presence of merger synergies) as each of its constituent firms and the non-participating firms; there is no notion of assets or firm size. In contrast, in the Perry and Porter model, a merger results in a "new firm that is 'larger' than the others" (pp. 219). They model this by assuming a fixed factor of production (say capital) whose total supply is fixed to the industry; what distinguishes firms is the amount they own of this factor[2]. In the adaptation, I again allow mergers between firms located in the different countries, where the capital stock of the merged (multinational) firm is the sum of the capital stock of its constituents. Thus, the "larger" multinational firm has a lower marginal cost than either of its constituents at a given level of output.

This all-or-none merger result is of empirical relevance in that the pattern of equilibria found for this fairly wide – as I claim – class of models is associated with mergers not happening in isolation but rather bunching together. Empirical studies have tested for bunching, or wave-like behaviour, using aggregate industry data (see, for example, Town 1992, Golbe and White 1993, and Barkoulas et al 2001) while other authors have argued that this phenomenon is also observed within individual industries (Mueller 1989). Casual observation suggests, moreover, that this kind of bunching phenomenon appears to be of relevance also in the case of cross-border merger activity (UNCTAD 2000, Knickerbocker 1973). Caves (1991) offers two lines of reasoning to explain why mergers may bunch together. One line attributes this to an unprecedented shift in an exogenous industry- or economy-wide parameter suddenly making mergers profitable[3]. The other line looks at the strategic interaction between firms' merger decisions within an industry: under certain conditions firms will merge only if rivals merge. It is the latter line of reasoning which sequential merger games have sought to address.

Using similar notation and structure to the sequential merger game laid out in Nilssen and Sørgard (1998), in the following section I spell out general conditions – in terms

[2]In the Perry and Porter (1985) model, on which my example is based, marginal cost is linear in output; by increasing its capital stock, the firm reduces the slope of its marginal cost curve. In Motta and Vasconcelos (2003), marginal cost is flat in output; a higher capital stock lowers marginal cost, regardless of output.

[3]On this first line of reasoning, Stigler (1950) and Bittlingmayer (1985) analyse the effects of changes in competition policy. Van Wegberg (1994) studies how the business cycle affects the profitability of mergers.

of firms' reduced-form profit functions – characterising each possible equilibrium. I show that if a certain sufficient condition holds then a subset of the possible equilibria can be ruled out. I briefly illustrate the general analysis by reference to Fauli-Oller (2000). I then turn to this paper's application, developing two further standard examples in which the sufficient condition holds, and showing that the general analysis can be extended to the case of cross-border mergers. Section 3 lays out and solves the sequential merger game in the vertically-differentiated Cournot oligopoly. Results are discussed in light of the international trade and investment literature, where foreign investment has been modelled mostly in the form of greenfield investment, as opposed to mergers and acquisitions. I argue that modelling foreign investment in the form of cross-border mergers suggests just how widespread such activity may be. Considerations are also made on the robustness of results to the number of merger stages and on the specification of the extensive form of the merger game (i.e. sequential moves in counterpoint to merger decisions being made simultaneously). Section 4 considers the fixed-capital-stock model of Perry and Porter (1984), to show that similar results hold. In Section 5 I revisit the vertically-differentiated oligopoly model to introduce fixed costs associated with implementing a merger. In so doing I attempt to address an inherent weakness in the results of Sections 3 and 4 as to their usefulness in explaining the typically-observed flurry of merger activity as countries integrate their markets: if, as the results in these earlier sections suggest, cross-border mergers occur along the equilibrium path for a "large" region of parameter space, then why were these mergers not undertaken prior to market intergration? Section 6 briefly concludes.

2 General conditions for equilibria in sequential merger games

I begin by defining the structure of the sequential merger game, following Nilssen and Sørgard (1998)[4]. In brief, disjoint and exogenously-given groups of firms make sequential merger decisions, leading up to a final market competition stage, where payoffs of the merged and independent firms are conditional on market structure resulting from the earlier merger stages. Bearing in mind the later aim of extending the results to a cross-border context, I adapt the notation and structure to a two-country setting. (I also point out how the analysis easily collapses to the single-country case.)

Consider two countries, $l \in \{A, B\}$, each initially with n^l firms, where $n^l \geq 2$. Denote the set of firms located in country A as $\mathcal{A}$ and the set of B-country firms as $\mathcal{B}$; thus $n^A = |\mathcal{A}|$, $n^B = |\mathcal{B}|$ and the global industry consists of $n := n^A + n^B \geq 4$ firms. Now consider the following form of sequential (cross-border) merger games. Assume that two cross-border mergers[5] can take place in sequence in this industry: label the set of firms that first decide whether to merge $\mathcal{M}_1$, and the set of firms that subsequently decide whether to merge $\mathcal{M}_2$. Label the merged firms possibly arising from the first and second

[4]Where convenient, I use similar notation and structure to that in Nilssen and Sørgard (1998)'s general discussion of sequential merger decisions. This is done for the benefit of the reader who is acquainted with their paper and also to point out how this section builds on their work. Notice that there is no repeated product market interaction as in Kamien and Zang (1990, 1993).

[5]The choice of *two* mergers is done for ease of exposition, as in Nilssen and Sørgard (1998). In the application of Section 3 I consider a set-up where three mergers can take place, in addition to the case with two mergers, and discuss the implications of allowing greater numbers of mergers to take place.

merger decisions M_1 and M_2 respectively. The extensive form of the merger game in consideration satisfies the following assumptions:

- At least two firms take part in each merger decision, i.e. $|\mathcal{M}_i| \geq 2$, $i \in \{1, 2\}$.
- At least one firm from each country takes part in each merger decision, i.e. $\mathcal{A} \cap \mathcal{M}_i \neq \emptyset$ and $\mathcal{B} \cap \mathcal{M}_i \neq \emptyset$, $i \in \{1, 2\}$.
- Each firm participates in at most one merger decision, $\mathcal{M}_1 \cap \mathcal{M}_2 = \emptyset$, so that $|\mathcal{M}_1| + |\mathcal{M}_2| \leq n$.

An example is in order: country A has three firms, labelled 1, 2 and 3, while country B has two firms, labelled 4 and 5. The sequential merger game given by firms 1 and 4 first deciding whether to merge, followed by firms 2 and 5 deciding whether to merge, would satisfy the above assumptions. If both mergers are carried through, the market structure is given by two (multinational) merged firms M_1 and M_2, where $\mathcal{M}_1 = \{1, 4\}$ and $\mathcal{M}_2 = \{2, 5\}$, and an independent firm located in country A, firm 3.

Coming out of the merger stages, four situations (market structures) are possible (see Figure 1). Denote the situation where neither merger is undertaken by s_0. If the firms in $\mathcal{M}_1$ do not merge but those in $\mathcal{M}_2$ do, call this situation s_1; in contrast, let situation s_2 depict a favourable merger decision by firms in $\mathcal{M}_1$ followed by a no-merger decision by firms in $\mathcal{M}_2$. Finally, situation s_3 denotes the market structure arising from two favourable merger decisions[6].

Finally, the firms which have merged or remained independent compete on the world market; goods produced in one country can be exported for consumption in the other country. This completes the description of the type of sequential cross-border merger games being considered[7].

2.1 Reduced-form profit functions and conditions

Denote the equilibrium payoff to firm i under market structure s by $\Pi_{i,s}$, where $s \in \{s_0, s_1, s_2, s_3\}$ and $i \in \mathcal{S} \subset \mathcal{A} \cup \mathcal{B} \cup \{M_1, M_2\}$, with $\mathcal{S}$ being the set of (merged and/or independent) firms competing under market structure s. Note that $\Pi_{i,s}$ is in reduced form, indexed by a vector of "primitives" of the specific model in question, such as demand and supply-side parameters[8]. The merger surplus functions are then defined in terms of these reduced-form profit functions.

[6] Note that situations s_1 and s_2 are reversed as compared to the use of notation by Nilssen and Sørgard (1998).

[7] As I comment shortly, if there is no geographic dimension to the game and a single-country setting is being considered, the second assumption, namely that at least one firm from each country takes part in each merger decision, is dropped. There is no longer a distinction between firms located in one country or the other and the relevant set of firms is $\mathcal{I} := \mathcal{A} \cup \mathcal{B}$.

[8] Formally, the reduced-form payoff function is $\Pi_{i,s}(\theta)$, where $\theta \in \mathcal{P}$ is the vector of parameters capturing relevant features of the model and $\mathcal{P}$ is the space of parameters. For ease of exposition, I suppress the argument θ of the functions.

Definition 1 *(Merger surplus functions) Define six Φ functions*[9] *as follows:*

$\Phi_I := \Pi_{M_2,s_1} - \sum_{i\in\mathcal{M}_2} \Pi_{i,s_0}$ $\quad$ $\Phi_{IV} := \Pi_{M_1,s_2} - \sum_{i\in\mathcal{M}_1} \Pi_{i,s_1}$

$\Phi_{II} := \Pi_{M_2,s_3} - \sum_{i\in\mathcal{M}_2} \Pi_{i,s_2}$ $\quad$ $\Phi_V := \Pi_{M_1,s_2} - \sum_{i\in\mathcal{M}_1} \Pi_{i,s_0}$

$\Phi_{III} := \Pi_{M_1,s_3} - \sum_{i\in\mathcal{M}_1} \Pi_{i,s_0}$ $\quad$ $\Phi_{VI} := \Pi_{M_1,s_3} - \sum_{i\in\mathcal{M}_1} \Pi_{i,s_1}$

Figure 1 illustrates the six merger surplus functions. To exemplify, Φ_{III} is the difference between the profit of merged firm M_1 under market structure s_3 and the sum of profits of its constituent (independent) firms under market structure s_0. As shown below, these six functions capture the surplus behind all the relevant merger decisions by either firms in $\mathcal{M}_1$, in the first stage, or firms in $\mathcal{M}_2$, in the second stage.

It follows from the consideration of two sequential merger stages that there are eight possible sets of Nash strategies by the players. These are illustrated in Figure 2[10]. In sets (a), (b), (g) and (h), the early firms' merger decision has no effect on the subsequent firms' merger decision. By way of example, for set (a) to be an equilibrium, firms in $\mathcal{M}_2$ need to find it profitable to merge irrespective of $\mathcal{M}_1$-firms' decision whether to merge or not. Also, firms in $\mathcal{M}_1$ need to find it profitable to merge in anticipation of $\mathcal{M}_2$-firms' subsequent merger decision.

In sets (of strategies) (c), (d), (e) and (f), on the other hand, the subsequent firms' merger decision does depend on the early firms' merger decision. If set (c) characterises an equilibrium, then firms in $\mathcal{M}_2$ find it profitable to merge *only if* firms in $\mathcal{M}_1$ merge. (To complete the characterisation of this equilibrium, firms in $\mathcal{M}_1$ must find it profitable to merge, anticipating how the subsequent merger decision depends on their own decision.) As in set (a), mergers occur along the equilibrium path of the game yet here $\mathcal{M}_1$-firms' merger decision has a bearing on (and takes place anticipating) $\mathcal{M}_2$-firms' merger decision[11].

Proposition 1 states necessary and sufficient conditions for each of these sets of strategies to form an equilibrium (unique for a given combination of parameters)[12].

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[9] The notation here departs from that in Nilssen and Sørgard (1998) for the following reason. While I define six functions, for the purposes of their paper they define eight "profitability of merger" functions for merger participants, in addition to four such functions for non-participating parties to a merger. For the sake of comparability, note that Φ_I, Φ_{II}, Φ_{III}, Φ_{IV}, Φ_V and Φ_{VI} in my paper are equivalent to Δ_2^1, Δ_2^2, Δ_1^3, Δ_1^4, Δ_1^1 and Δ_1^2 respectively, in their paper.

[10] To avoid clutter, I rely on Figure 2 to define each of the eight sets of Nash strategies, rather than doing so in words. In any case, this is standard. For example, set (c) would be defined as follows. $\mathcal{M}_1$-firms: 'merger'; $\mathcal{M}_2$-firms: 'merger' only if $\mathcal{M}_1$-firms merge, otherwise 'no merger'.

[11] Some authors, such as Fauli-Oller (2000), have related results (c) and (d) to the merger-wave phenomenon, in the sense that an early decision to merge (either along the equilibrium path of the game, as in (c), or as a deviation from it, as in (d)) triggers subsequent mergers.

[12] Proposition 1 is consistent with Proposition 1 in Nilssen and Sørgard (1998; pp.1689). One must be careful, however, in comparing the conditions in their Propostion 2 and Table 1 (pp.1690-1) with mine. For example, they state that "the occurrence of a Fat Cat strategy implies a sequence of two decisions to merge" (pp.1691) yet this stands at odds with my conditions (and their Proposition 1) for a sequence of two mergers to be supported as an equilibrium outcome. They refer to a "Fat Cat" strategy as $\Delta_2^1 < 0 < \Delta_2^2$ and $\Delta_{-2}^2 > 0$. Yet these same conditions could also lead to a sequence of two decisions *not* to merge as long as Δ_1^1 is sufficiently negative such that $\Delta_1^3 = \Delta_1^1 + \Delta_{-2}^2 < 0$. If $\Delta_2^1 < 0 < \Delta_2^2$ (i.e. $\Phi_I < 0 < \Phi_{II}$ in my Proposition 1) then $\Delta_1^3 > 0$ (i.e. $\Phi_{III} > 0$) is the necessary and sufficient condition supporting a "sequence of two decisions to merge".

Proposition 1 *(NSC for equilibria) Necessary and sufficient conditions for each of Nash-strategy sets (a) to (h), as depicted in Figure 2, to be supported as the unique equilibrium (for a given combination of parameters) are:*

Set (a) $\Phi_I \geq 0$, $\Phi_{II} \geq 0$ *and* $\Phi_{VI} \geq 0$
Set (b) $\Phi_I \geq 0$, $\Phi_{II} \geq 0$ *and* $\Phi_{VI} < 0$
Set (c) $\Phi_I < 0$, $\Phi_{II} \geq 0$ *and* $\Phi_{III} \geq 0$
Set (d) $\Phi_I < 0$, $\Phi_{II} \geq 0$ *and* $\Phi_{III} < 0$
Set (e) $\Phi_I \geq 0$, $\Phi_{II} < 0$ *and* $\Phi_{IV} \geq 0$
Set (f) $\Phi_I \geq 0$, $\Phi_{II} < 0$ *and* $\Phi_{IV} < 0$
Set (g) $\Phi_I < 0$, $\Phi_{II} < 0$ *and* $\Phi_V \geq 0$
Set (h) $\Phi_I < 0$, $\Phi_{II} < 0$ *and* $\Phi_V < 0$

Proof. This follows by backward induction. Begin by the conditions for set (a) to be the equilibrium. In the second stage, following a decision by $\mathcal{M}_1$-firms not to merge, $\mathcal{M}_2$-firms will find it profitable to merge if and only if $\Phi_I = \Pi_{M_2,s_1} - \sum_{i\in\mathcal{M}_2} \Pi_{i,s_0} \geq 0$. Conditional on a favourable merger decision by $\mathcal{M}_1$-firms, that $\mathcal{M}_2$-firms find it profitable to merge requires $\Phi_{II} = \Pi_{M_2,s_3} - \sum_{i\in\mathcal{M}_2} \Pi_{i,s_2} \geq 0$. In the first stage, anticipating $\mathcal{M}_2$-firms' subsequent merger regardless of its merger decision, $\mathcal{M}_1$-firms will find it profitable to merge if and only if $\Phi_{VI} = \Pi_{M_1,s_3} - \sum_{i\in\mathcal{M}_1} \Pi_{i,s_1} \geq 0$. Turn to the conditions for (b). As in (a), $\mathcal{M}_2$-firms' strategy is optimal if and only if $\Phi_I \geq 0$ and $\Phi_{II} \geq 0$. Yet for firms in $\mathcal{M}_1$ choosing not to merge to be optimal implies $\Phi_{VI} < 0$. For (c) to be the equilibrium, $\mathcal{M}_2$-firms finding it profitable to merge in the second stage only if $\mathcal{M}_1$-firms choose to merge earlier requires $\Phi_I < 0$ and $\Phi_{II} \geq 0$. In the first stage, anticipating how $\mathcal{M}_2$-firms' merger decision depends on its own decision, $\mathcal{M}_1$-firms will merge if and only if $\Phi_{III} = \Pi_{M_1,s_3} - \sum_{i\in\mathcal{M}_1} \Pi_{i,s_0} \geq 0$. In (d), as in (c), $\mathcal{M}_2$-firms' strategy is optimal if and only if $\Phi_I < 0$ and $\Phi_{II} \geq 0$. Yet firms in $\mathcal{M}_1$ will find it optimal not to merge if and only if $\Phi_{III} < 0$. Proofs of conditions for sets of strategies (e) through (h) to be equilibria follow similarly and are omitted. ■

2.2 Special case: Symmetry

When the sets of firms $\mathcal{M}_1$ and $\mathcal{M}_2$ are "symmetric", a common case in models studied in the literature, the general conditions characterising each possible equilibrium can be simplified. What I mean by symmetry here needs to be made precise:

Definition 2 *(Symmetry) There is symmetry about the sets of firms $\mathcal{M}_1$ and $\mathcal{M}_2$ when for all admissible parameter values:*

- $\sum_{i\in\mathcal{M}_1} \Pi_{i,s_1} = \sum_{i\in\mathcal{M}_2} \Pi_{i,s_2}$
- $\Pi_{M_1,s_2} = \Pi_{M_2,s_1}$
- $\Pi_{M_1,s_3} = \Pi_{M_2,s_3}$
- $\sum_{i\in\mathcal{M}_1} \Pi_{i,s_0} = \sum_{i\in\mathcal{M}_2} \Pi_{i,s_0}$

In words, the sum of payoffs of (independent) firms in $\mathcal{M}_1$ under market structure s_1 (where only $\mathcal{M}_2$-firms have merged) equals the sum of payoffs of (independent) firms in $\mathcal{M}_2$ under market structure s_2 (where only $\mathcal{M}_1$-firms have merged). The payoff to (merged) firm M_1 under market structure s_2 equals the payoff to (merged) firm M_2 under market structure s_1. The payoff to (merged) firm M_1 equals the payoff to (merged) firm M_2 when the two mergers have taken place (market structure s_3). The sum of payoffs of (independent) firms in $\mathcal{M}_1$ equals the sum of payoffs of (independent) firms in $\mathcal{M}_2$ when no merger has been carried through (market structure s_0).

Recall the above example, where firms 1, 2 and 3 are located in country A, firms 4 and 5 are located in country B and the sequential merger game specifies firms 1 and 4 first deciding whether to merge (if so forming M_1), followed by firms 2 and 5 deciding whether to merge (if so forming M_2). Symmetry would require that: (i) the sum of payoffs to firms 1 and 4 under market structure $s_1 = \{M_2, 1, 3, 4\}$ equals the sum of payoffs to firms 2 and 5 under market structure $s_2 = \{M_1, 2, 3, 5\}$, (ii) the payoff to M_1 under market structure $s_2 = \{M_1, 2, 3, 5\}$ equals the payoff to M_2 under market structure $s_1 = \{M_2, 1, 3, 4\}$, (iii) the payoff to M_1 equals the payoff to M_2 under market structure $s_3 = \{M_1, M_2, 3\}$, and finally, (iv) the sum of payoffs to firms 1 and 4 equals the sum of payoffs to firms 2 and 5 under fragmented market structure $s_0 = \{1, 2, 3, 4, 5\}$.

By symmetry, it follows from Definition 1 that $\Phi_I = \Phi_V$ and $\Phi_{II} = \Phi_{VI}$. Only four merger surplus functions Φ need now be computed. The number of possible sets of strategies by the players that can be supported in equilibrium collapses to six : sets (b) and (g) are no longer possible[13]. The conditions for each to form an equilibrium are simplified somewhat, as stated in the following proposition (see Figure 3):

Proposition 2 *(NSC for equilibria in the case of symmetry) In the special case of symmetry, Nash-strategy sets (b) and (g) cannot be supported in equilibrium. Necessary and sufficient conditions for each of the six remaining sets to be supported as the unique equilibrium (for a given combination of parameters) are:*

Set (a) $\Phi_I \geq 0$ *and* $\Phi_{II} \geq 0$
Set (b) This cannot be supported in equilibrium
Set (c) $\Phi_I < 0$, $\Phi_{II} \geq 0$ *and* $\Phi_{III} \geq 0$
Set (d) $\Phi_I < 0$, $\Phi_{II} \geq 0$ *and* $\Phi_{III} < 0$
Set (e) $\Phi_I \geq 0$, $\Phi_{II} < 0$ *and* $\Phi_{IV} \geq 0$
Set (f) $\Phi_I \geq 0$, $\Phi_{II} < 0$ *and* $\Phi_{IV} < 0$
Set (g) This cannot be supported in equilibrium
Set (h) $\Phi_I < 0$ *and* $\Phi_{II} < 0$

Proof. This follows from the definition of symmetry, the definition of the Φ functions and Proposition 3. ■

A simple corollary to Proposition 2 states a sufficient condition that rules out each of sets (d), (e) and (f) as candidate equilibria. A model that satisfies this sufficient

[13] This is intuitive. For example, consider set (b). If it is optimal for $\mathcal{M}_2$-firms to merge regardless of $\mathcal{M}_1$-firms' earlier decision, including when $\mathcal{M}_1$-firms decide to merge (this requires $\Phi_{II} \geq 0$), then it cannot be optimal for $\mathcal{M}_1$-firms to decide not to merge, knowing that $\mathcal{M}_2$-firms will subsequently decide to merge regardless of their decision (by symmetry, this would require $\Phi_{VI} = \Phi_{II} < 0$, in contradiction).

condition for every admissible combination of parameter values can only have sets (a), (c) and (h) among its equilibria. That is, of the eight possible equilibria in Figure 2, only the three equilibria highlighted in Figure 3 remain in the case of any symmetric model that satisfies the corollary's sufficient condition. This is the case for several models considered in the literature, in addition to the two models considered in this paper's subsequent application.

Corollary 1 *(Sufficient condition to rule out sets (d), (e) and (f) as candidate equilibria, in the case of symmetry) Consider that symmetry holds in the sense defined above. If for all admissible parameter values, it holds that (i) whenever $\Phi_I \geq 0$ it happens that $\Phi_{II} \geq 0$, and (ii) whenever $\Phi_{II} \geq 0$ it happens that $\Phi_{III} \geq 0$, then Nash-strategy sets (d), (e) and (f) cannot be supported in equilibrium.*

Formally, this sufficient condition can be stated as

$$\{\theta \mid \Phi_I(\theta) \geq 0\} \subseteq \{\theta \mid \Phi_{II}(\theta) \geq 0\} \subseteq \{\theta \mid \Phi_{III}(\theta) \geq 0\}$$

where θ is a vector of admissible parameter values (see earlier footnote).

Proof of the corollary follows by inspection of Proposition 2[14]. A comment is in order. The corollary states only a *sufficient* condition that rules out sets (d), (e) and (f) as candidate equilibria for symmetric models. As I argue, this condition is satisfied for a fairly wide class of models. However, other sufficient conditions can be written that rule out each of sets (d), (e) and (f), as I show in Appendix A.

An illustration: Fauli-Oller (2000)[15] I briefly illustrate the sufficient condition of Corollary 1 by reference to Fauli-Oller's (2000) (symmetric) model. There are four firms in a homogeneous-good Cournot industry: firms 1 and 2 have zero marginal cost, while firms 3 and 4 have constant marginal cost $c > 0$. Demand is linear, where the price intercept is given by α (and the slope is normalised to -1). As such, the parameters θ of the model are given by α and c, and Fauli-Oller admits parameters in the range given by $\alpha > 3c$. A slightly simplified version of the sequential merger game considered by the author has firms 1 and 3 first deciding whether to merge (i.e. $\mathcal{M}_1 = \{1, 3\}$), followed by firms 2 and 4 who then decide whether to merge (i.e. $\mathcal{M}_2 = \{2, 4\}$); a final market competition stage ensues. The merger technology is such that a merged entity produces with zero cost (the cost of its lower-cost constituent). From the reduced-form payoff functions for this standard Cournot model (stated by the author in equation 1), the

[14] See Appendix A. The sufficient condition of the corollary can be phrased as follows: (i) whenever it is profitable to merge in isolation, it happens that it is profitable to merge conditional on the merger of the rival set of firms; (ii) whenever it is profitable to merge conditional on the merger of the rival set of firms, it happens that each set of firms (i.e. $\mathcal{M}_1$ and $\mathcal{M}_2$) prefers that the two mergers are undertaken over no merger being undertaken.

[15] When there is no geographic dimension to the sequential merger game – as in Nilssen and Sørgard (1998) or in this illustration – the setup of the general analysis just presented requires one minor simplification. Since there is no longer a distinction between firms located in one country or the other, the assumption that at least one firm from each country takes part in each merger decision no longer holds. The two other assumptions remain, that at least two firms take part in each merger decision and that each firm participates in at most one merger decision. The set $\mathcal{A} \cup \mathcal{B}$ can be replaced by $\mathcal{I}$, defined as the initial set of (independent) firms, containing n elements. The necessary and sufficient conditions for equilibria are as in Proposition 1 and, in the special case of symmetry, Proposition 2.

relevant merger surplus functions of the corollary can then be computed: $\Phi_I = \left(\frac{\alpha+c}{4}\right)^2 - \left(\frac{\alpha+2c}{5}\right)^2 - \left(\frac{\alpha-3c}{5}\right)^2$, $\Phi_{II} = \left(\frac{\alpha}{3}\right)^2 - \left(\frac{\alpha+c}{4}\right)^2 - \left(\frac{\alpha-3c}{4}\right)^2$ and $\Phi_{III} = \left(\frac{\alpha}{3}\right)^2 - \left(\frac{\alpha+2c}{5}\right)^2 - \left(\frac{\alpha-3c}{5}\right)^2$. It is easy to show that the sufficient condition of the corollary holds: (i) for all (α, c) such that $\Phi_I \geq 0$ it happens that $\Phi_{II} \geq 0$, and (ii) for all (α, c) such that $\Phi_{II} \geq 0$ it happens that $\Phi_{III} \geq 0$. Consequently, only Nash-strategy sets (a), (c) and (h) *can* obtain in equilibrium as parameter values vary. (It is easy to show from these three merger surplus functions that all three sets indeed *are* supported as unique equilibria at different ranges of parameters.) Notice from Figure 3 that the outcomes associated with these equilibria are such that mergers do not occur in isolation but rather bunch together. In other words, whenever a merger happens along the equilibrium path of the game it is undertaken alongside other mergers. I postpone discussion of this result – which hinges on the way non-participating firms react to the merger of rival firms, as is well understood in the literature – to the application.

3 Example 1: Cross-border mergers in Sutton's (1991) vertically-differentiated industry

The first example explores the profitability of sequential cross-border mergers in a Cournot oligopoly with vertical product differentiation. I use a partial equilibrium model with differentiated goods due to Sutton (1991) and later applied in a trade context by Motta (1992, 1994).

Before carefully laying out the setup, a brief description is in order. I embed the two countries, a large country and a small country, each with three firms, where firms in the large country offer a product of quality at least as high as that offered by firms in the small country. While product quality varies between the firms located in different countries, it does not (initially) vary across firms located within the same country. The structure of the sequential cross-border merger game in Section 2, where two cross-border mergers were allowed to take place in sequence, is extended here to three merger stages: in each stage a large-country high-quality firm and a small-country low-quality firm decide whether to merge or not. If the merger takes place, the level of quality offered by the merged (and multinational) firm is the higher of the qualities offered by its constituent firms (i.e. the quality offered previously by the large-country constituent). Thus cross-border mergers enable firms to transfer technology. Consistent with the type of sequential cross-border merger game spelled out in Section 2, in a final stage, firms which have merged or remained independent engage in Cournot competition on the world market.

Demand setup Consider two countries, $l \in \{A, B\}$, with m_l consumers each. Consumers in both countries have identical preferences defined over a quality (differentiated) good and an outside good, indexed by quantities x and y respectively, where u denotes the quality level of the quality good:

$$U = (ux)^\beta y^{1-\beta} \qquad 0 < \beta < 1$$

Total expenditure on the quality good in economy l, S_l, is:

$$S_l = \sum_{i=1}^{n_l} p_i x_i = \sum_{k=1}^{m_l} \beta z_k \tag{1}$$

where n_l is the number of firms selling in country l ($n_l \geq 2$), x_i and p_i are respectively the quantity and price of the (single) quality good sold by firm i (referred to as variety i), and z_k is the income of consumer k. Assume $S_A \geq S_B$ due either to a larger population or a higher per capita income in (large) country A relative to (small) country B.

Given any vector of qualities and associated prices, the consumer chooses a variety i that maximises the quality/price ratio $\frac{u_i}{p_i}$[16]. All varieties that command positive sales at equilibrium must therefore have prices proportional to qualities:

$$\frac{p_i}{u_i} = \lambda \quad \forall i$$

where λ is a constant. From equation (1), we can then write $S = \sum_{i=1}^{n} p_i x_i = \lambda \sum_{i=1}^{n} u_i x_i$ (momentarily dropping the country subscript l for simplicity), expressing the price-to-quality ratio λ as:

$$\lambda = \frac{S}{\sum_{i=1}^{n} u_i x_i} \tag{2}$$

The inverse demand function for variety i is also obtained:

$$p_i = \frac{S}{\sum_{j=1}^{n} \frac{p_j}{p_i} x_j} = \frac{S}{\sum_{j=1}^{n} \frac{u_j}{u_i} x_j} \tag{3}$$

By assumption each variety i is sold to the nth part of the population.

Supply and historical motivation for entry and quality Firms are assumed to have the same constant marginal cost of production $c > 0$. Goods produced in one country but sold in another (exports) incur an additional unit trade cost $t \geq 0$.

As mentioned above, each country is initially embedded with three (independent) firms, where a firm from country l offers a good of quality $u_l \geq 1$. By construction, $u_A \geq u_B$ and define the quality ratio (or gap) v as the ratio of the quality offered by large-country firms to that offered by their small-country counterparts, $v := \frac{u_A}{u_B} (\geq 1)$.

The number of firms in each country and their respective (asymmetric) qualities can be motivated by considering the following long-term entry and investment game (Motta 1992). Countries are initially closed to foreign trade and investment and in this autarkic setting, without forseeing any changes, firms make entry and investment decisions. In each country, the following two-stage game is played. In a first stage, firms simultaneously decide whether to enter and, if so, with which quality. In a second stage, they engage in Cournot competition. In order to offer a good of quality u, firms must incur a fixed and sunk cost $F(u) = u^\gamma$, $u \geq 1$.

Consistent with models of this type, in equilibrium the same number of firms enters in each country, with the large-country firms making larger investments in quality than

[16] Consumer k chooses to consume quantity x_{ik} of variety i such that $u_i x_{ik}$ is maximised subject to his budget constraint $p_i x_{ik} = \beta z_k$; i.e. he solves $\max_i \beta z_k \frac{u_i}{p_i}$ by selecting a variety i such that $\frac{u_i}{p_i}$ is maximised across i.

the small-country firms, $u_A \geq u_B$. The convexity of the fixed cost function is chosen so that three firms enter in each country[17].

By this historical motivation, market integration then unexpectedly occurs. Firms are "locked in" with their previous (autarky-based) quality levels, reflecting the long-term nature of their investment decisions (capability-building) as opposed to the short-run process of market integration and market competition. Firms located in each country are now allowed to export to the other market (due to trade integration) and/or can merge with a (now) rival firm located in the other market (due to investment integration), as set out below.

The experience of some Latin American and Southeast Asian countries, which during the 1990s underwent a relatively unplanned process of trade and investment liberalisation, with the ensuing high volume of cross-border mergers and acquisitions, comes to mind. It is in this historical setting that the cross-border merger game is set.

Sequential cross-border merger game In sequence, each of the three firms located in country A – producing with quality u_A – is allowed to merge with each of the three firms located in country B – producing with quality u_B. With no loss of generality, the first A-country firm and the first B-country firm which decide whether to merge (stage 1) are labelled 1 and 4 respectively, the second A-country firm and the second B-country firm which decide whether to merge (stage 2) are labelled 2 and 5 respectively, and the third A-country firm and the third B-country firm which decide whether to merge (stage 3) are labelled 3 and 6 respectively. Keeping with the notation of Section 2, $\mathcal{M}_1 = \{1, 4\}$, $\mathcal{M}_2 = \{2, 5\}$ and $\mathcal{M}_3 = \{3, 6\}$ and the merged (multinational) firms which may come to form are labelled M_1, M_2 and M_3. As mentioned above, the merger-technology assumption is that full transfer of technology takes place, the merged firm being able to produce at the high level of quality u_A not only at its A-country facilities but also at its B-country facilities[18]. In the fourth and final stage, merged or independent firms compete à la Cournot on the world market.

The sequence of merger decisions is depicted in Figure 4, as are the possible market structures coming out of the merger stages. The market structure where no cross-border merger occurs, $\{1, 2, 3, 4, 5, 6\}$, is denoted r_0, while that where three cross-border mergers are carried through, $\{M_1, M_2, M_3\}$, is labelled r_3. Given that symmetry holds (in

[17] See Appendix B for a derivation. The number of firms (in each country), $n = 3$, depends on the choice of parameter, $\gamma = 3$, of the fixed cost function, $F(u) = u^{\gamma}$. By making the cost function more convex, i.e. making (R&D, say) spending less effective in raising quality, the larger is the number of firms entering in equilibrium. For example, for $\gamma = 5$, $n = 4$ firms enter in equilibrium. We may write $n = n(\gamma)$, where $n' > 0$. Notice that n does not depend on market size: this "non-convergence" result is consistent with the finiteness property of many vertical product differentiation models (Shaked and Sutton 1983, Sutton 1991). For $\gamma = 3$, the three firms entering in country l offer a common quality given by $u_l = \frac{2}{3}\sqrt[3]{\frac{S_l}{3}}$. It follows from the assumption that $S_A \geq S_B$ that the quality offering of a large-country firm exceeds that of a small-country firm: $u_A \geq u_B$.

[18] This full-transfer-of-technology assumption is also made in Fauli-Oller (2000), for example. It is one extreme case of a more general assumption where, by merging with a high-quality foreign firm, a B-country firm can produce at an "average" level of quality given by, for example, the convex combination $\delta u_A + (1 - \delta)u_B$, where $0 \leq \delta \leq 1$ is the technology transfer coefficient. That a low-quality firm located in a developing country will not always achieve "world class" quality by merging with a high-quality developed-country counterpart is a fair comment, yet this assumption allows us to examine sequential mergers in the context of international trade and quality asymmetries in a simple way.

the sense defined in Section 2), market structures where one cross-border merger is undertaken are denoted r_1 and those where two cross-border mergers occur are denoted r_2. For example, r_2 comprises structures $\{M_1, M_2, 3, 6\}$, $\{M_1, M_3, 2, 5\}$ and $\{M_2, M_3, 1, 4\}$, where there are two multinational firms and one independent firm producing in each country[19].

3.1 Equilibria as a function of the quality gap and the trade cost

The reduced-form profits for each firm, as a function of the parameters in the model, for every possible market structure are derived in Appendix B. The notation introduced in Section 2 needs to be complemented: let subscript a denote an independent A-country firm, subscript b denote an independent B-country firm and subscript m denote a merged (multinational) firm. Thus Π_{a,r_2}, for instance, denotes the payoff to an independent firm located in country A under market structure r_2. Reduced-form profits are a function of the quality gap v, the trade cost normalised by the marginal cost of production $\tilde{t} := \frac{t}{c}$ and the market sizes S_A and S_B.

I assume that $(1 \leq) v \leq \frac{3}{2}$ (the quality ratio is low enough) and $(0 \leq) \tilde{t} \leq \frac{3-2v}{2v}$ (the trade cost is low enough) so that in equilibrium low-quality firms command positive sales in both countries, i.e. there is two-way (intra-industry) trade between countries under any market structure where at least two firms remain independent. This space of parameter values is labelled $\mathcal{P}$. (Such an assumption is made for simplicity of exposition; extending the space of parameter values outside $\mathcal{P}$, i.e. to include $\{(v, \tilde{t}) \in \mathcal{R}^2 \mid v \geq 1$ and $\tilde{t} \geq 0\} \setminus \mathcal{P}$, does not add to the results, as discussed in Appendix B.)

Given that there are three merger stages, merger surplus functions are now denoted Ψ (cf Φ in Section 2). Coupled with the fact that symmetry holds, eight merger surplus functions are relevant[20], defined as follows (see Figure 4 for an illustration):

Definition 3 *(Merger surplus functions) Define eight Ψ functions as follows:*

$\Psi_I := \Pi_{m,r_1} - \Pi_{a,r_0} - \Pi_{b,r_0}$	$\Psi_V := \Pi_{m,r_3} - \Pi_{a,r_1} - \Pi_{b,r_1}$
$\Psi_{II} := \Pi_{m,r_2} - \Pi_{a,r_1} - \Pi_{b,r_1}$	$\Psi_{VI} := \Pi_{m,r_3} - \Pi_{a,r_0} - \Pi_{b,r_0}$
$\Psi_{III} := \Pi_{m,r_3} - \Pi_{a,r_2} - \Pi_{b,r_2}$	$\Psi_{VII} := \Pi_{m,r_1} - \Pi_{a,r_1} - \Pi_{b,r_1}$
$\Psi_{IV} := \Pi_{m,r_2} - \Pi_{a,r_0} - \Pi_{b,r_0}$	$\Psi_{VIII} := \Pi_{m,r_2} - \Pi_{a,r_2} - \Pi_{b,r_2}$

[19] For the sake of simplicity, now that there are three merger stages, the r-notation here takes into account that symmetry holds (cf the s-notation in Section 2, introduced for the general case, as in Nilssen and Sørgard 1998).

[20] While in Section 2, with two merger stages and symmetry the number of relevant merger surplus functions was four, here it is eight. In general, if there are T merger stages, $T \geq 1$, and there is symmetry, the number of relevant merger surplus functions is given by $T - 1 + \sum_{i=1}^{T} i$. If there are T merger stages but symmetry does not hold, then the number of relevant merger surplus functions increases to $\sum_{i=1}^{2^T-1} i$.

Parameters v, $\tilde{t}$, S_A and S_B are suppressed as arguments of the functions here for simplicity. When I do include arguments, subsequently, I ignore S_A and S_B since Proposition 3 holds irrespective of market sizes (these enter the profit functions multiplicatively).

The sufficient condition of Corollary 1 holds Along the lines of Proposition 2 and Figure 3, how the signs of these merger surplus functions change as parameter values vary pins down the equilibria of the merger game. One can show that in general there are twenty possible equilibria in a symmetric three-stage game, and conditions supporting each can be written. However, the verification that the model satisfies the sufficient condition of Corollary 1 simplifies matters considerably: the set of candidate equilibria is reduced from twenty to four elements. This is captured in the following lemma, which is the natural extension of the sufficient condition of Corollary 1 to a game with three merger stages.

Lemma 1 *For* $(v, \tilde{t}) \in \mathcal{P}$*:*
(i) $\Psi_I(v, \tilde{t}) \geq 0 \Longrightarrow \Psi_{II}(v, \tilde{t}) \geq 0 \Longrightarrow \Psi_{III}(v, \tilde{t}) \geq 0$
(ii) $\Psi_{II}(v, \tilde{t}) \geq 0 \Longrightarrow \Psi_{IV}(v, \tilde{t}) \geq 0$
$\Psi_{III}(v, \tilde{t}) \geq 0 \Longrightarrow \Psi_{V}(v, \tilde{t}) \geq 0 \Longrightarrow \Psi_{VI}(v, \tilde{t}) \geq 0$

While proof that the lemma holds is provided in Appendix B, the intuition is straightforward. Consider part (i). A merged firm produces less than the pre-merger sum of outputs of its constituents since each constituent now internalises the externality it confers upon the other constituent when making its output decision[21]. Since quantities are strategic substitutes, firms not participating in the merger ("outsiders") respond by increasing output, which may render a merger unprofitable for its participants ("insiders"). (Such "accommodation" was highlighted by Salant, Switzer and Reynolds 1983 in a Cournot model with symmetric firms.) Now, whenever firms in $\mathcal{M}_i$ do not find it profitable to merge when two other cross-border mergers are taking place (i.e. $\Psi_{III} < 0$) and consequently there are less outsiders to free ride on the merger, then they would not find it profitable to merge were only one other merger (i.e. $\Psi_{II} < 0$), let alone none (i.e. $\Psi_I < 0$), taking place, as there would be more free-riders to expand output following $\mathcal{M}_i$-firms' merger[22]. As for part (ii) of the lemma, consider $\Psi_{II} \geq 0 \Longrightarrow \Psi_{IV} \geq 0$, for example. The decision to merge captured by merger surplus function Ψ_{II} entails an opportunity cost given by the sum of the profits of an independent A-country firm and an independent B-country firm under market structure r_1. This opportunity cost is larger than the opportunity cost in the decision to merge captured by Ψ_{IV}, which amounts to the profits of the same two independent firms but under r_0, a more "fragmented" market structure (in the sense that while in r_1 there are five firms selling into each country, in r_0 there are six firms)[23].

[21] In this model, for $(v, \tilde{t}) \in \mathcal{P}$, this occurs despite the quality jump from u_B to u_A enjoyed by the B-country constituent. More generally, note that if a quality increase (or, isomorphously, a marginal cost reduction) through merger were very large, insiders to the merger could actually *expand* output. A merger, by lowering prices, would then be detrimental to outsiders, who would respond by lowering output (under quantity competition).

[22] Fauli-Oller (2000) interprets a similar result as "previous takeovers stimulate takeover profitability" (pp.197).

[23] In this model, in the space $\mathcal{P}$ of parameter values, it can be shown that despite merging parties having the advantage of producing with high quality in both countries and no longer cross-hauling product between them, non-participants always gain in the event of a merger, be they multinational firms or independent firms. Part (ii) of the lemma follows from noting this. Outside $\mathcal{P}$, for example, for $\tilde{t}$ high enough (and $v > 1$) that not even high-quality A-country firms command positive sales in country B (i.e. trade flows in neither direction), a merger has a detrimental effect on outside (independent B-country) firms.

The set of remaining candidate equilibria, containing four elements, is illustrated in Figure 5, alongside the conditions on the reduced-form functions in the left-hand margin. The equilibria are labelled (a), (c1), (c2) and (h), in a manner consistent with the labelling of equilibria in Section 2. In view of Lemma 1, only three merger surplus functions need be computed in parameter space $\mathcal{P}$: Ψ_I, Ψ_{II} and Ψ_{III}. By inspecting the signs of these functions in $\mathcal{P}$, the game can be solved.

The solution to the game is stated in the next proposition. It states that all four remaining candidate equilibria occur in space $\mathcal{P}$. Figure 6 partitions the space into four disjoint zones, each labelled after the equilibrium which can be supported for the combinations of parameters defining the zone. Thus, for low enough values of both the quality gap v and the (normalised) trade cost $\tilde{t}$, in zone (h), the "unprofitability of mergers" result common to models with quantity competition and in the absence of merger "synergies" (Salant, Switzer and Reynolds, 1983) is obtained. Here, along the equilibrium path, as well as in subgames hanging from nodes off the equilibrium path, no cross-border merger occurs. At the other end, for high enough values of v and/or $\tilde{t}$, in zone (a), mergers are always profitable: along the equilibrium path, as well as off it, cross-border mergers always occur. For intermediate values of v and/or $\tilde{t}$, in zones (c1) and (c2), mergers again occur along the equilibrium path. Here, however, the merger decisions in the early stages of the merger game have a bearing on the merger decisions in the later stages.

Proposition 3 *Space $\mathcal{P}$ can be partitioned into four disjoint zones, as depicted in Figure 6. A straight line segment drawn from $(v, \tilde{t}) = (1, 0)$ to any point on $\tilde{t} = \frac{3-2v}{2v}$ begins in zone (h), crossing into zone (c2), followed by zone (c1) and ending in zone (a). In each zone, the unique equilibrium is given in Figure 5, labelled after the zone. The boundaries between zones (h) and (c2), between zones (c2) and (c1), and between zones (c1) and (a), are continuous and downward-sloping, joining a point $(v, 0)$, $1 < v < \frac{3}{2}$, on the v-axis to a point $(1, \tilde{t})$, $0 < \tilde{t} < \frac{1}{2}$, on the $\tilde{t}$-axis. The boundary between zones (h) and (c2) lies strictly below the boundary between zones (c2) and (c1), which in turn lies strictly below the boundary between zones (c1) and (a).*

Intuition and decomposition analysis (A decomposition analysis of the profitability of a merger provides intuition on the results of Proposition 3. The reader may wish to skip to Section 3.2 without loss of continuity. Proof of the proposition is set out in Appendix B.) How the total effect of a single cross-border merger (say, a stand-alone first merger that shifts market structure from r_0 to r_1[24]) on the profit of insiders changes as v and $\tilde{t}$ vary can be broken down in a simple way into three effects. The first effect, labelled the "insider output contraction" effect, is calculated assuming a merger technology where there is *no* quality upgrade in the production facilities located in country B as a result of the merger, and further that outsiders do *not* react to the merger by increasing output, but rather maintain their pre-merger output levels. This profit effect

[24] By "stand-alone" I am considering the profitability of a first merger in isolation, abstracting from whether it is followed by further mergers and therefore from whether r_1 is an equilibrium market structure.

is unambiguously positive[25] for $(v, \tilde{t}) \in \mathcal{P}$. A merger internalises the externality each constituent insider confers upon the other when making an output decision (the insiders' reaction function shifts inward); as a result, the (quality-adjusted[26]) combined output of insiders falls, prices of high and low-quality goods rise in both countries and insider profits rise.

The second profit effect, labelled the "technology transfer" effect, still assumes that outsiders' output remains unchanged upon merger, but now the merger technology assumed in the model, that quality of B-country production facilities jumps from u_B to u_A through merger, is re-established. This effect is also unambiguously positive: insider profits on sales in country B rise as a result of the transfer of technology[27]. By this effect, insiders expand the (quality-adjusted) quantity sold in country B (with prices in country B falling), yet this increase in sales in country B does not offset the output contraction in country B as a result of the first effect (the net effect on country B prices is positive).

The positive "insider output contraction" and "technology transfer" effects on the profit of insiders are countervailed by a negative "outsider output expansion" effect. This third effect captures the impact on the profit of insiders of allowing outsiders to respond to the merger. Since quantities are strategic substitutes, outsiders to a merger free ride on insiders' output contraction (the equilibrium moves out along the outsiders' reaction function as the insiders' reaction function shifts inward). As outsiders' (quality-adjusted) output increases, insiders react by further contracting output, to which outsiders further expand output, and so on, until equilibrium is reached. This third effect on insiders' profit is unambiguously negative.

The overall impact of the cross-border merger on the profit of insiders turns on the relative magnitude of the countervailing effects. Intuitively, the "outsider output expansion" effect is increasingly negative (outsiders' output expansion increasingly high) as v and/or $\tilde{t}$ falls: for $(v, \tilde{t})$ sufficiently close to $(1, 0)$, this negative effect on the profit of insiders dominates the two positive effects and the stand-alone merger is unprofitable. To see this, note that at $(v, \tilde{t}) = (1, 0)$, with no quality asymmetries and no trade costs, with six firms competing in each country on an equal footing[28], output expansion by outsiders is at its most severe, rendering a merger at its most unprofitable for insiders.

[25] Insider profits rise on sales in both countries. Under this modified merger technology, it can be shown that when $v > 1 + \tilde{t}$ the merged firm discontinues production operations in country B, shipping high-quality product produced in country A to country B. When $v < 1 + \tilde{t}$, the merged firm no longer cross-ships, producing low-quality product in country B for domestic consumption.

[26] Quality-adjusted output is defined as output multiplied by the quality gap $v = \frac{u_A}{u_B}$ for product of high quality u_A *when* in competition with product of low quality u_B, and output multiplied by 1 otherwise. This measure of output is natural in view of the inverse demand functions (e.g. see equation (3)).

[27] Clearly, there is no effect on sales, prices and profit on sales in country A. As in the model, the merged firm now produces high-quality product in country B for domestic consumption. This effect could alternatively be called the "(insider) trade cost elimination" effect. The upgrade in quality of B-country facilities has an equivalent effect to the elimination of trade costs on cross-hauling by insiders: by eliminating trade costs insiders would produce only high-quality product in country A, which is equivalent to producing high-quality product in both countries for domestic consumption when trade costs are present. Indeed this is an example of the symmetry of the model with respect to v and $\tilde{t}$.

[28] Recall that I am analysing the profitability of a stand-alone *first* cross-border merger for the purpose of motivating intuition.

The equilibrium outcomes of the sequential game at different parameter values can then be understood by reference to these effects. In zone (h), where v and $\tilde{t}$ are both low, the output expansion by outsiders in response to a merger would be significant, while the advantage enjoyed by the multinational of producing with high quality in both countries and no longer cross-hauling product between them would be limited. Being an outsider in the event of a merger is highly profitable. As a result, no cross-border merger occurs along the equilibrium path.

At the other end, for high enough values of v and/or $\tilde{t}$, in zone (a), mergers are always profitable: along the equilibrium path, as well as off it, cross-border mergers always occur. Under high v and/or $\tilde{t}$, the output expansion by outsiders to a merger is limited. To see why this is so for sufficiently high v even when $\tilde{t}$ is close to zero, note that starting from a market structure with no cross-border merger, there are effectively only three (high-quality, A-country) firms selling in each country. If only one cross-border merger is now allowed to occur, the number of firms effectively selling in each country remains unchanged, hence the limited output expansion by outsiders. To see why there is limited output expansion by outsiders for sufficiently high $\tilde{t}$ even when v is close to 1, note that again a cross-border merger leaves the number of firms effectively selling in each market unchanged (three A-country firms selling in country A and three B-country firms selling in country B). When v and $\tilde{t}$ are greater than 1 and 0 respectively, the technology transfer effect is present: there is the added gain from merging arising from upgrading the production facilities located in country B to the higher level of quality.

For intermediate values of v and/or $\tilde{t}$, mergers again occur along the equilibrium path. Here, however, early mergers induce subsequent mergers. In zone (c2), along the equilibrium path, firms 1 and 4's merger is decisive for firms 2 and 5 to merge, and both mergers are in turn decisive for firms 3 and 6 to merge. Stated loosely, more mergers mean less free riding by outsiders in the market competition stage. Notice that if, say, firms 1 and 4 were not to anticipate the effect their merger has on subsequent merger decisions, but viewed their merger in isolation, they would not merge since $\Psi_I < 0$ in zone (c2)[29]. In zone (c1), earlier mergers are still decisive for later mergers. However, due to higher v and/or $\tilde{t}$ compared to zone (c2), and hence lower output expansion by outsiders, only one previous decision to merge suffices for firms 3 and 6 to find it profitable to choose likewise.

3.2 Discussion: Foreign investment literature and robustness

In this subsection results are discussed in light of the international trade and investment literature. Considerations are also made on the specification of the extensive form of the merger game and the number of merger stages.

The trade literature has modelled foreign direct investment (FDI) mostly in the form of *greenfield* investment, where firms choose between exporting to a market and setting up production facilities in that market; see for example Motta (1994), who studies greenfield investment in a similar vertically-differentiated global oligopoly. The literature

[29] In an international oligopoly in the presence of technological innovation, Graham (1985) suggests that a firm's motivation for foreign direct investment may be "to respond to or anticipate the action of a rival... creating a clustering of DFI (foreign direct investment) in the industry" (pp. 69; parenthesis added).

has emphasised the tariff-jumping (more precisely, trade-cost-jumping) rationale behind FDI: unless trade costs are sufficiently low, runs the argument, it may be rational for a firm to set up foreign production operations. On the one hand, greenfield investment requires a fixed setup cost but, on the other hand, it enables firms to reduce variable (trade) cost.

Yet empirically, the major channel by which foreign investment is made is that of mergers and acquisitions (UNCTAD 2000). By analysing investment in this mode, in counterpoint to greenfield investment, my model points out that cross-border mergers allow firms not only to save on trade costs and transfer technology but also, by reducing the number of rivals, to enjoy muted competition. It explores a different mechanism to the tariff-jumping rationale of why foreign investment may not occur when trade costs (and the quality gap) are low: one where the profitability of investment turns on the response of non-participating firms in a Cournot oligopoly. In such situations where there is low product differentiation along both horizontal and vertical dimensions, any cross-border merger would be met with a fierce output response by those producers not party to the merger.

An example is in order. Analysing the substitution-of-exports-for-greenfield-investment decision, Motta (1994) finds that a decrease in the value of the exporting cost, ceteris paribus, results "in an enlargement of the regions (in parameter space) where exports, rather than investments, prevail" (pp.191). This is supported in my model of acquisition investment (in Figure 6 the boundary between zones (h) and (c2) is downward-sloping), yet owing to a different mechanism. Motta's result derives from tariff-jumping: the lower is the trade cost, ceteris paribus, the lower are the gains from (greenfield) investment. In my model, the lower is the trade cost, ceteris paribus, the greater is the intensity of competition and the *higher* are the gains to the industry from (acquisition) investment (see below). Yet it is precisely when the trade cost is low that free riding on other mergers is so profitable that in equilibrium no (acquisition) investment occurs.

Clearly, for parameter values in all zones including zone (h), the "grand cross-border coalition" – where three cross-border mergers occur and there are no independent firms – is the structure under which industry profits are maximal[30]. In fact, it is in zone (h), where both v and $\tilde{t}$ are low enough, that the industry gains from merging[31] are greatest: intuitively, the lower is v or the lower is $\tilde{t}$, the greater is the intensity of competition resulting in lower industry profits under r_0, and therefore the higher are the gains to be reaped from eliminating competitors through merger. Despite this, in zone (h) free riding on a merger is "far too" profitable and intense. Due to this prisoner's dilemma character of the game in this zone, the three-merger outcome cannot be supported in equilibrium.

Modelling foreign investment in the form of cross-border mergers suggests just how widespread such activity may be. While it is difficult to rationalise greenfield investment when there are no trade costs ($\tilde{t} = 0$), since location of production does not matter, cross-border mergers will occur when the quality gap is *high enough:* there exist $(v, \tilde{t}) \in \mathcal{P}$

[30] When $(v, \tilde{t}) = (\frac{3}{2}, 0)$ or $(1, \frac{1}{2})$, the grand cross-border coalition is weakly dominant. Otherwise it is strictly dominant for $(v, \tilde{t}) \in \mathcal{P}$.

[31] Note that the industry gains from merging is the difference between profits under market structure r_3 and profits under r_0, i.e. $3\Psi_{VI}(v, \tilde{t})$. That these are monotonically decreasing in $\tilde{t}$, for a given v, and monotonically decreasing in v, for a given $\tilde{t}$, is intuitive and can be shown.

in zones (a), (c1) and (c2) when $\tilde{t} = 0$ and v exceeds certain thresholds. Thus the existence of a (large enough) quality gap is a sufficient condition for mergers to occur, unlike the case of a simple greenfield investment model, where $\tilde{t} > 0$ is also necessary. As with mergers in general, via cross-border mergers firms can dampen competition. My model further predicts the phenomenon of investment-bunching. When cross-border mergers occur they do not happen in isolation: (Merger, Merger, Merger)[32] is the unique equilibrium outcome for most parameter values with the exception of low values for the quality gap and the trade cost.

It is interesting to compare the equilibrium outcomes of the sequential game with those of a different game, where the three merger decisions are made *simultaneously* in a first stage followed by a market competition stage, as before. As seen above, in the sequential-move game, (Merger, Merger, Merger) is the unique subgame perfect Nash equilibrium (SPNE) outcome for parameter values in zones (a), (c1) and (c2), while (No Merger, No Merger, No Merger) is the unique SPNE outcome in zone (h). Consider now the simultaneous-move game. In each zone the same outcome as that in the sequential game can be supported in equilibrium. Yet in zones (c1) and (c2), this equilibrium outcome is no longer unique: in addition to (Merger, Merger, Merger), (No Merger, No Merger, No Merger) is also a pure-strategy Nash equilibrium outcome.

The set of equilibrium outcomes in the simultaneous-move game is thus a (strict, in zones (c1) and (c2)) superset of the set of equilibrium outcomes in the sequential game. By modelling the merger decisions sequentially, the no-merger outcome which could be supported as a second equilibrium outcome in zones (c1) and (c2) were the merger decisions to be modelled simultaneously, no longer survives. The sequential nature of moves acts as a coordination device, and may help explain the empirical observation of widespread, and bunched together in time, cross-border mergers[33].

Some final remarks pertaining to the number of merger stages. One may ask how robust is the result of widespread cross-border mergers in equilibrium to the number of merger stages (for a game with T merger stages, modifying the setup by, say, embedding each country initially with T independent firms). Consider first the game with two merger stages (and two firms in each country), i.e. $T = 2$. Similar results to those of Figure 5 (where $T = 3$) obtain, with one exception. A zone like (h) in Figure 5, where mergers are unprofitable and do not occur along the equilibrium path, is no longer obtained. For v and $\tilde{t}$ as low as 1 and 0 respectively, mergers already occur along the equilibrium path; specifically, for $(v, \tilde{t}) = (1, 0)$, a merger wave (in the sense of set (c) in Figure 3) obtains[34]. Intuitively, there are at most two non-participating firms to each

[32] By this representation I mean that all three cross-border mergers are carried out: firms 1 and 4 merge, firms 2 and 5 merge and firms 3 and 6 merge.

[33] In the specification of the sequential game, I could have explicitly modelled each merger stage as two separate stages, a bidding stage by one of the two constituent firms, the "bidder", followed by an acceptance stage by the other constituent firm, the "target". (This is done by Fauli-Oller (2000), for example, who equips low-cost firms with the ability to place "take-it-or-leave-it" bids to merge with high-cost firms.) Had I done this, the same merger outcomes would be obtained in equilibrium, with the observation that the surplus from merger would be captured by the bidder. In this case, one can show that the sequential nature of the merger game, for parameter values in zone (c2), lowers the average bid accepted in equilibrium by the target firms vis-à-vis the game where all three merger decisions are made simultaneously (see also Kamien and Zang 1990, 1993). Merging sequentially can thus lower the cost of setting up multinational firms and reducing competition.

[34] This contrasts with the result in Fauli-Oller (2000) that a "takeover wave" occurs only for a sufficiently large (cost) difference among firms.

merger (as opposed to four when $T = 3$) and thus the "outsider output expansion" effect is never negative enough to "undo" the profitability of merging for participating parties.

As for solving the game when the number of merger stages $T \geq 4$, Appendix B provides expressions for the reduced-form profit functions of the different firms, under the different possible market structures, in a game with T merger stages: from these the merger surplus functions are computed. As T increases, the space $\mathcal{P}$ of parameters $(v, \tilde{t})$ shrinks: the downward-sloping boundary at which two-way trade is just feasible shifts inward towards the "origin" $(1, 0)$. One can show that the unprofitability-of-mergers result for parameter values close enough to, and including, $(1, 0)$ (similar to equilibrium (h) in Figure 5), and the profitability-of-mergers result for parameter values close enough to, and including, the two-way-trade boundary (similar to equilibrium (a) in Figure 5) carry through for this general case. As with the case where $T = 3$, it can further be shown that for all parameter combinations outside $\mathcal{P}$ (where trade may flow only from country A to country B, or not at all), mergers are profitable; a similar equilibrium to (a) in Figure 5 obtains. In sum, the main results of the particular $T = 3$ game analysed carry through more generally, namely that (i) cross-border mergers may be unprofitable only when *both* the quality gap and the trade cost are low, and (ii) the profitability of cross-border mergers is widespread, with such mergers bunching together.

4 Example 2: Cross-border mergers in the Perry and Porter (1985) model

I now analyse cross-border merger profitability in another standard model that satisfies the sufficient condition of Corollary 1, yet where the motive for merger is different in kind. Perry and Porter (1985) introduce a notion of "firm size" by considering a fixed factor of production (say capital) whose total supply is fixed to the industry; what distinguishes one firm from another is the stock of capital it owns. Thus, a "larger" firm arising from the merger of two firms owning the same amount of capital has a lower marginal cost than either of its constituents at a given level of output (to be made precise below).

Perry and Porter (1985) setup Let $k > 0$ denote the amount of the fixed factor of production whose total supply is fixed to an industry producing a homogeneous good. Firm i's cost function is given by $C(x_i, k_i)$, where x_i denotes its output of this good and k_i is the amount of the fixed factor it owns. The cost function is taken to be homogeneous of degree one in output and capital, implying constant returns to scale and that the marginal cost function, $C_1(x, k) := \frac{\partial C(x,k)}{\partial x}$, is homogeneous of degree zero in x and k. Because of the presence of a fixed factor of production, it is assumed that marginal costs are decreasing in k, $C_{12}(x, k) := \frac{\partial^2 C(x,k)}{\partial k \partial x} < 0$ and hence, by Euler's theorem, marginal costs are increasing in output, $C_{11}(x, k) := \frac{\partial^2 C(x,k)}{\partial x^2} > 0$. Firms compete à la Cournot.

Perry and Porter (1985) specify particular functional forms for demand and cost. Both price and marginal cost are assumed to be linear functions of output. The industry inverse demand function is given by $P(X) = a - X$, where P is price and $X := \sum_i x_i$ is industry output. (I adapt this to the two-country setting by taking $P^l(X^l) = a - X^l$, where P^l and X^l are respectively the price and sales in country $l \in \{A, B\}$.) A firm's cost function is quadratic (and convex) in output, $C(x, k) = gk + dx + \frac{e}{2k}x^2$, where industry fixed costs $g\sum_i k_i$ are distributed in proportion to capital ownership. Coefficients d, e and g are (weakly) positive and $a > d$. (In this two-country setting, I again assume a linear trade technology: exports incur a unit trade cost $t \geq 0$.)

Sequential cross-border merger game I take the simplest structure of the sequential merger game considered in Section 2. The (global) industry, with capital stock $4k$, consists of four symmetric firms, each owning one-quarter of this stock, $k_i = k$, each country hosting two firms, $n^A = n^B = 2$. With no loss of generality, firms in country A are labelled 1 and 2, their B-country rivals are labelled 3 and 4, the first set of firms that decide whether to merge is $\mathcal{M}_1 = \{1, 3\}$ and the set of firms that subsequently decide whether to merge is $\mathcal{M}_2 = \{2, 4\}$. The merger technology is such that the capital stock of a merged (multinational) firm is the sum of the capital stock of its constituent firms. Notice that the assumptions rule out economies of scale as a motive for merger.

4.1 Equilibria as a function of the rate of change of marginal cost and the trade cost

In view of the symmetry (in the sense of Definition 2), I resort to Proposition 2 in order to solve the game. The reduced-form profit functions for an independent firm and merged firm under every possible market structure, necessary to compute the merger surplus functions, are derived in Appendix C. The merger surplus functions Φ are a function of (i) the demand intercept less the marginal cost when output is zero, $(a - d)$, (ii) the trade cost t, and (iii) the rate of change of the marginal cost of a firm with capital stock k, $C_{11}(x, k) = \frac{e}{k}$. Nevertheless, the Φ-functions can be unambiguously signed, pinning down the equilibria of the game, by $\tilde{e} := \frac{e}{k}$ and $\tilde{t} := \frac{t}{a-d}$, which I will refer to, respectively, as the rate of change of the marginal cost and the (normalised) trade cost.

Clearly, the space of parameter values of interest is that where trade between countries is feasible. (Otherwise there is zero surplus from merger.) Again I label this space $\mathcal{P}$, defined by $\tilde{e} \geq 0$ and $(0 \leq)\tilde{t} \leq \frac{1}{3+\tilde{e}}$.

The sufficient condition of Corollary 1 holds It is easy to show that the model satisfies the sufficient condition of Corollary 1, implying that of the six possible equilibria depicted in Figure 3 for a symmetric game with two merger stages, only three equilibria can obtain: sets of strategies (a), (c) and (h). Inspection of (the sign of) the merger surplus functions Φ_I and Φ_{II} in parameter space establishes that all three equilibria do obtain. Further, the way in which the equilibrium changes as parameters vary is similar to Example 1 (Section 3). When the rate of change of the marginal cost $\tilde{e}$ and the trade cost $\tilde{t}$ are *both* low, no cross-border merger occurs along the equilibrium path of the game, as well as off it: set (h) is the equilibrium. At the other end, for high enough values of $\tilde{e}$ and/or $\tilde{t}$, mergers are profitable, along the equilibrium path as well as off it: set (a) is the equilibrium. For intermediate values of $\tilde{e}$ and/or $\tilde{t}$, mergers occur along the path yet the early merger by $\mathcal{M}_1$-firms induces the subsequent merger by $\mathcal{M}_2$-firms:

set (c) is the equilibrium. The zones in which the different strategies are supported as equilibria are labelled accordingly as zones (a), (c) and (h). The complementary sets of strategies in Figure 3, namely (d), (e) and (f), cannot be supported as equilibria for any combination of parameters. The following proposition summarises the result.

Proposition 4 *Space $\mathcal{P}$ can be partitioned into three disjoint zones, as depicted in Figure 7. A straight line segment drawn from $(\tilde{e}, \tilde{t}) = (0, 0)$ to any point on $\tilde{t} = \frac{1}{3+\tilde{e}}$ begins in zone (h), crossing into zone (c) and ending in zone (a). In each zone, the unique equilibrium is given in Figure 3, labelled after the zone. The boundaries between zones (h) and (c), and between zones (c) and (a), are continuous and downward-sloping, joining a point $(\tilde{e}, 0)$, $0 < \tilde{e} < \infty$, on the $\tilde{e}$-axis to a point $(0, \tilde{t})$, $0 < \tilde{t} < \frac{1}{3}$, on the $\tilde{t}$-axis. The boundary between zones (h) and (c) lies strictly below the boundary between zones (c) and (a).*

The parallel to Proposition 3 is clear, both results being driven by a similar underlying mechanism (the reaction of non-participating firms), despite the motive for merger being different in kind: "growth in size" presently as opposed to "technology transfer" in Section 3, in addition to dampening competition and jumping tariffs, which both models share. That the response of non-participating firms again takes centre-stage in explaining the profitability of cross-border mergers is intuitive. When *both* the rate of increase of marginal cost and the trade cost are low, non-participating firms respond to a merger by significantly expanding output, rendering the merger unprofitable to its participants. As either the rate of increase of marginal cost or the trade cost increase from these low levels, the "outsider output expansion" effect (see the decomposition analysis in Section 3) falls in magnitude and merger profitability increases. As in Example 1, cross-border mergers are widespread and do not occur in isolation, but rather bunch together.

5 Revisiting the vertically-differentiated-industry model: Investment integration and fixed costs of merger

The solutions to the two standard models above share the common phenomenon that cross-border mergers are undertaken and bunch together for a "large" region of parameter space. These results, however, beg the question: if the all-merger outcome indeed occurs *along* the equilibrium path for these parameter values, why is it that cross-border mergers are often empirically observed to occur from a sudden point in time onwards (and usually concentrated over a short period)? In other words, why were they not undertaken earlier?[35]

I revisit the vertically-differentiated-industry model in Section 3 to introduce the notion of a fixed cost G associated with implementing a cross-border merger. Borrowing from the historical motivation for the quality asymmetry in the model, I ask why do

[35] In particular, one setting in which this flurry of investment activity has been seen to happen is that of countries undergoing integration, such as the European Union (EEC) from the 1980s, or several newly-industrialised countries undergoing trade and investment reform during the 1990s.

cross-border mergers not take place even before countries undergo unexpected trade and investment integration[36]?

I argue that initially, before countries open up their industries to foreign investment, or lift curbs on the right of foreign firms to acquire shares in domestic firms, this fixed cost G of implementing a cross-border merger is very high, to the extent that cross-border mergers are not profitable and no merger occurs along the equilibrium path. Then, as countries open up to foreign investment, with the relaxation of these curbs on the right of foreign firms to merge with (or acquire) domestic firms, and the cost of doing business (such as acquiring information, or communicating) in a new environment declining as barriers are pulled down, G falls gradually[37]. In the historical context, the point in time at which G begins to fall corresponds to the moment at which unexpected integration takes place.

I investigate how the solution to the model, for the different zones in $(v, \tilde{t}) \in \mathcal{P}$, changes as the fixed cost G associated with implementing a cross-border merger declines from an initial high level G_0 (at which mergers do not occur). Figure 8 illustrates the solution for $(v, \tilde{t})$ in each of zones (a), (c1) and (c2) as G falls[38]. When G reaches a threshold given by $\Psi_{III}(v, \tilde{t})$, the equilibrium outcome (Merger, Merger, Merger) is reestablished for $(v, \tilde{t})$ in each of zones (a), (c1) and (c2). Interestingly, when this threshold $G = \Psi_{III}(v, \tilde{t})$ is reached, the equilibrium for $(v, \tilde{t})$ in *each* of zones (a), (c1) and (c2) (the second game tree from top to bottom in Figure 8) replicates the equilibrium for zone (c2) in the absence of fixed costs, depicted in Figure 5, which was referred to as a "merger wave"[39] [40].

[36] By the results of that section, mergers are profitable in autarky (outside $\mathcal{P}$ for $\tilde{t}$ high enough) in the presence of a quality gap ($v > 1$).

[37] According to UNCTAD (2000), "over the period 1991-1999, 94% of the 1,035 changes worldwide in the laws governing FDI created a more favourable framework for FDI. Complementing the more welcoming national FDI regimes, the number of bilateral investment treaties – concluded increasingly also between developing countries – has risen from 181 at the end of 1980 to 1,856 at the end of 1999. Double taxation treaties have also increased, from 719 in 1980 to 1,982 at the end of 1999. At the regional and interregional levels, an increasing number of agreements (most recently between the EC and Mexico) are helping create an investment environment more conducive to international investment flows" (pp.xv). See also Caves (1991).

[38] The proof is in Appendix B. Clearly, in zone (h), no cross-border mergers occur along the equilibrium path for all values of $G \geq 0$: if this was the case in the absence of fixed costs of implementation ($G = 0$), it remains the case when these are introduced ($G > 0$).

[39] A relevant question at this point is: if firms forsee that G will fall further, why not wait for this to occur and thus guarantee a larger surplus from merger? Though this can be ruled out by making firms (sufficiently) impatient in a dynamic setting, or unable to forsee the future trajectory of G, the relevance of the question remains.

[40] As G falls further, assuming no mergers have occurred, for $\Psi_I(v, \tilde{t}) < G \leq \Psi_{II}(v, \tilde{t})$, the equilibria in zones (a) and (c1) (see the third tree in Figure 8) replicate the equilibrium for zone (c1) in the absence of fixed costs (the equilibrium in zone (c2) still replicates the equilibrium for zone (c2) in the absence of fixed costs). For $0 \leq G \leq \Psi_I(v, \tilde{t})$, assuming no mergers have occurred, the equilibrium in zone (a) (see the fourth tree in Figure 8) replicates the equilibrium for zone (a) in the absence of fixed costs (the equilibria in zones (c1) and (c2) still replicate the equilibria for zones (c1) and (c2) respectively in the absence of fixed costs).

6 Concluding remarks

This paper has sought to explain why the pattern of equilibria in sequential (horizontal) merger games of a certain type studied in the literature is similar across a fairly wide class of models. In this class of models typically only a similar subset of the possible set of equilibria is obtained. By developing general conditions characterising each possible equilibrium, I have shown that the solution to models that exhibit symmetry (in the sense defined earlier) and satisfy a certain sufficient condition will be restricted to this subset of equilibria. These models capture two (commonly) alternative types of situation: (i) one where firms are better off under market structures where their rivals have merged as against market structures where rivals remain independent, or (ii) one where by merging firms may gain a competitive edge over their rivals. This result is of empirical relevance in that the pattern of equilibria found for this class of models is associated with mergers not happening in isolation but rather bunching together.

I have then employed the general analysis to solve two other standard models where the sufficient condition holds, illustrating that despite the motive for merger being different in kind – one where mergers are a means of technology transfer, the other where mergers enable firms to grow their holdings of capital – the pattern of equilibria is again similar. In so doing, I have utilised a two-country setting to show that the results can easily be extended to cross-border mergers. I argued that sequential games can be used to explore the the profitability of cross-border mergers and can cast light on the phenomenon of bunching. Cross-border mergers are widespread and bunching occurs for "most" combinations of parameter values. The occurrence of mergers hinges critically on the magnitude of the reaction by non-participating firms being limited (e.g. when there is sufficient differentiation along either vertical or horizontal dimensions, as in the first model).

Much research to date on foreign investment has studied the export versus greenfield investment decision by firms as countries undergo market integration. Such models assume that as borders open up and there is increased competition, firms decide whether to enter new markets via exports or by setting up production operations, concentrate on existing markets or exit altogether. In reality, oligopolistic firms have a larger action set which includes merging with one another. Modelling foreign investment in the form of cross-border mergers – in counterpoint to greenfield investment – suggests just how widespread such activity may be. By merging or buying their way into foreign markets firms can dampen competition, a feature which is absent in simple exports-versus-greenfield investment studies. This helps explain the empirical predominance of mergers and acquisitions as a channel of investment by firms in foreign markets.

Finally, modelling mergers in the form of a sequential game as opposed to a game with simultaneous moves, captures the interdependence result – where earlier mergers trigger subsequent mergers – holding for certain regions in parameter space. In these regions the sequential nature of moves acts as a coordination device ensuring that the all-merger equilibrium outcome is unique. This may further help explain the empirical predominance and bunching of cross-border mergers.

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A Appendix: Proof of Corollary 1 and other sufficient conditions that rule out Nash-strategy sets (d), (e) and (f) as candidate equilibria, in the case of symmetry

I begin by proving Corollary 1. Consider the first part of the sufficient condition, namely that $\{\theta \mid \Phi_I(\theta) \geq 0\} \subseteq \{\theta \mid \Phi_{II}(\theta) \geq 0\}$. It rules out sets (e) and (f) as candidate equilibria. To see this, notice that if either set (e) or set (f) forms an equilibrium, necessary conditions from Proposition 2 are that $\Phi_I \geq 0$ and $\Phi_{II} < 0$. Hence if whenever $\Phi_I \geq 0$ it happens that $\Phi_{II} \geq 0$, i.e. if $\Phi_{II} \geq 0 \mid \Phi_I \geq 0$, then neither set (e) nor set (f) can form an equilibrium. (Recall that, from the symmetry property, $\Phi_I = \Phi_V$ and $\Phi_{II} = \Phi_{VI}$. Φ_I is thus the surplus from any merger in isolation. Φ_{II} is thus the surplus from any merger conditional on the merger of the rival set of firms.) Now consider the second part of the sufficient condition, namely that $\{\theta \mid \Phi_{II}(\theta) \geq 0\} \subseteq \{\theta \mid \Phi_{III}(\theta) \geq 0\}$. It rules out set (d) as a candidate equilibrium. To see this, notice that if set (d) forms an equilibrium, necessary conditions are that $\Phi_{II} \geq 0$ and $\Phi_{III} < 0$. Hence if whenever $\Phi_{II} \geq 0$ it happens that $\Phi_{III} \geq 0$, i.e. if $\Phi_{III} \geq 0 \mid \Phi_{II} \geq 0$, then set (d) cannot form an equilibrium. (Recall that, from the symmetry property, $\Pi_{M_1,s_3} = \Pi_{M_2,s_3}$ and $\sum_{i \in \mathcal{M}_1} \Pi_{i,s_0} = \sum_{i \in \mathcal{M}_2} \Pi_{i,s_0}$, so that $\Phi_{III} = \Pi_{M_1,s_3} - \sum_{i \in \mathcal{M}_1} \Pi_{i,s_0} = \Pi_{M_2,s_3} - \sum_{i \in \mathcal{M}_2} \Pi_{i,s_0}$. Φ_{III} is thus $\mathcal{M}_1$-firms' or $\mathcal{M}_2$-firms' difference in profits when two mergers are undertaken over no merger being undertaken.) *Q.E.D.*

Inspection of Proposition 2 indicates other sufficient conditions that rule out Nash-strategy sets (d), (e) and/or (f) as candidate equilibria. I provide three.

1. Set (d) is ruled out as an equilibrium if the sum of payoffs of independent firms in $\mathcal{M}_1$ ($\mathcal{M}_2$) is always lower under the fragmented market structure s_0 (s_0) than under the more concentrated market structure s_1 (s_2), where the rival set of firms has merged.

   Proof. If set (d) forms an equilibrium, necessary conditions from Proposition 2 are that $\Phi_{II} \geq 0$ and $\Phi_{III} < 0$, i.e. $\Pi_{M_2,s_3} - \sum_{i \in \mathcal{M}_2} \Pi_{i,s_2} \geq 0$ and $\Pi_{M_1,s_3} - \sum_{i \in \mathcal{M}_1} \Pi_{i,s_0} < 0$. From the definition of symmetry, these two conditions can be combined into $\sum_{i \in \mathcal{M}_1} \Pi_{i,s_0} = \sum_{i \in \mathcal{M}_2} \Pi_{i,s_0} > \Pi_{M_1,s_3} = \Pi_{M_2,s_3} \geq \sum_{i \in \mathcal{M}_2} \Pi_{i,s_2} = \sum_{i \in \mathcal{M}_1} \Pi_{i,s_1}$. Hence if $\sum_{i \in \mathcal{M}_1} \Pi_{i,s_0} = \sum_{i \in \mathcal{M}_2} \Pi_{i,s_0} \leq \sum_{i \in \mathcal{M}_1} \Pi_{i,s_1} = \sum_{i \in \mathcal{M}_2} \Pi_{i,s_2}$ (note that this occurs iff $\Phi_{II} \leq \Phi_{III}$) then set (d) cannot form an equilibrium. Note that this condition and the second part of the sufficient condition of Corollary 1 are based on the same necessary conditions from Proposition 2; they are not, however, equivalent. ■

2. Set (e) is ruled out as an equilibrium if the payoff to merged firm M_1 (M_2) is always lower under market structure s_2 (s_1), where the rival set of firms has not merged, than under the more concentrated market structure s_3 (s_3), where the two mergers have been undertaken.

   Proof. If set (e) forms an equilibrium, necessary conditions are that $\Phi_{II} < 0$ and $\Phi_{IV} \geq 0$, i.e. $\Pi_{M_2,s_3} - \sum_{i \in \mathcal{M}_2} \Pi_{i,s_2} < 0$ and $\Pi_{M_1,s_2} - \sum_{i \in \mathcal{M}_1} \Pi_{i,s_1} \geq 0$. Again, from the definition of symmetry, these are jointly equivalent to $\Pi_{M_1,s_2} = \Pi_{M_2,s_1} \geq \sum_{i \in \mathcal{M}_1} \Pi_{i,s_1} = \sum_{i \in \mathcal{M}_2} \Pi_{i,s_2} > \Pi_{M_2,s_3} = \Pi_{M_1,s_3}$. Hence if $\Pi_{M_1,s_2} = \Pi_{M_2,s_1} \leq$

$\Pi_{M_1,s_3} = \Pi_{M_2,s_3}$ (note that this occurs iff $\Phi_{IV} \leq \Phi_{II}$) then set (e) cannot form an equilibrium. ■

3. Both set (e) and set (f) are ruled out as equilibria if the surplus from merging in isolation, when positive, is always lower than when the rival set of firms also merges.

   Proof. If either set (e) or set (f) forms an equilibrium, necessary conditions are that $\Phi_I \geq 0$ and $\Phi_{II} < 0$. Hence if whenever $\Phi_I \geq 0$ it happens that $\Phi_I \leq \Phi_{II}$, i.e. if $\Phi_I \leq \Phi_{II} \mid \Phi_I \geq 0$, then neither set (e) nor set (f) can form an equilibrium. (Recall that, from the symmetry property, $\Phi_I = \Phi_V$ and $\Phi_{II} = \Phi_{VI}$. Φ_I is thus the surplus from any merger in isolation, i.e. when the rival set of firms does not merge. Φ_{II} is thus the surplus from any merger conditional on the merger of the rival set of firms.) Notice that when this condition is satisfied, then the first part of the sufficient condition of Corollary 1 is necessarily satisfied (but the reverse is not true). ■

In addition to satisfying the sufficient condition of Corollary 1, which alone already rules out sets (d), (e) and (f), it can be shown that the models of Examples 1 and 2 also satisfy sufficient conditions 1-3 just stated. That is, for all admissible parameter values, it holds that (1) $\Phi_{II} \leq \Phi_{III}$, (2) $\Phi_{IV} \leq \Phi_{II}$, and (3) whenever $\Phi_I \geq 0$ it happens that $\Phi_I \leq \Phi_{II}$. For instance, consider the Perry and Porter (1985) model of Example 2 and sufficient conditions 1 and 2 just stated. For sets (d) and (e) respectively to be supported in equilibrium, for some parameter values firms would need to be better off under less concentrated market structures as against more concentrated ones: firm M_1 would need to be better off under $\{M_1, 2, 4\}$ than under $\{M_1, M_2\}$, or firm 1 would need to be better off under $\{1, 2, 3, 4\}$ than under $\{M_2, 1, 3\}$. This is not the case for any combination of parameters and hence sets (d) and (e) can be ruled out as candidate equilibria.

Another example, not mentioned elsewhere in this paper, is afforded by Deneckere and Davidson (1985)'s model of symmetrically differentiated goods with *Bertrand competition.* It is straightforward to solve the simple sequential merger game with two merger stages for an industry initially with four homogeneous firms and, say, $\mathcal{M}_1 = \{1, 3\}$ and $\mathcal{M}_2 = \{2, 4\}$. For the entire range of the substitutability parameter, it can be shown that $\Phi_{IV} < 0 < \Phi_I < \Phi_{II} < \Phi_{III}$. Therefore, not only is the sufficient condition of Corollary 1 satisfied, but the other sufficient conditions 1-3 just stated also hold. As such, sets (d), (e) and (f) cannot be supported in equilibrium. Indeed, the Bertrand assumption ensures that only set (a) obtains in equilibrium, with mergers taking place from each node along the equilibrium path of the game and also from each node lying outside the equilibrium path. Again, this result hinges on the response of non-participating firms to a merger. As Deneckere and Davidson (1985) have shown, reaction schedules under competition in prices are typically upward-sloping, with non-participants to a merger reacting to the participants' price increase by themselves raising prices. (See also Levy and Reitzes 1992.)

B Appendix: Cross-border mergers in a vertically-differentiated industry

B.1 Historical motivation: Derivation of the autarky equilibrium

In order to motivate the number of firms in each country (three) and their respective qualities (high quality u_A for firms located in country A and low quality u_B for firms located in country B, such that $v = \frac{u_A}{u_B} \geq 1$), the equilibrium to the autarkic (long-term) entry and investment game is derived[41]. Recall the two-stage game played in each country: in a first stage, firms simultaneously make entry and quality investment decisions while, in a second stage, they engage in Cournot competition.

In the second (market) stage, given that n firms have entered in the first (entry and investment) stage with qualities $u = (u_j)$, $j = 1, ..., n$, the gross profit of firm i (recalling the price-to-quality ratio λ) is

$$\Pi_i = p_i x_i - c x_i = \lambda u_i x_i - c x_i$$

Firm i maximises Π_i taking the vector of qualities u from the earlier stage and x_j, $j \neq i$, as given. The first order condition is $\lambda u_i + u_i x_i \frac{d\lambda}{dx_i} - c = 0$, where from (2) we have $\frac{d\lambda}{dx_i} = -\frac{S}{(\sum_{j=1}^n u_j x_j)^2} u_i = -\frac{u_i}{S}\lambda^2$, or

$$u_i x_i = \frac{c - \lambda u_i}{\frac{d\lambda}{dx_i}} = \frac{S}{\lambda} - \frac{cS}{\lambda^2}\frac{1}{u_i} \tag{4}$$

Summing over j, we obtain $\sum_{j=1}^n u_j x_j = n\frac{S}{\lambda} - \frac{cS}{\lambda^2}\sum_{j=1}^n(\frac{1}{u_j})$. Using expression (2), I can solve for the price-to-quality ratio, $\lambda = \frac{c}{n-1}\sum_{j=1}^n(\frac{1}{u_j})$. Substituting for λ in FOC (4), I solve for the output of firm (variety) i:

$$x_i = \frac{S}{\lambda u_i}(1 - \frac{c}{\lambda}\frac{1}{u_i}) = \frac{S}{c}\frac{n-1}{u_i \sum_{j=1}^n(\frac{1}{u_j})}(1 - \frac{n-1}{u_i \sum_{j=1}^n(\frac{1}{u_j})}) \tag{5}$$

Note that by labelling the firm offering the lowest quality as firm 1, a necessary and sufficient condition for all n firms to command positive sales in equilibrium is $u_1 \sum_{j=1}^n(\frac{1}{u_j}) > n - 1$[42]. I can further solve for the price and gross profit:

$$p_i = \lambda u_i = c\frac{u_i}{n-1}\sum_{j=1}^n(\frac{1}{u_j}) \tag{6}$$

$$\Pi_i = (p_i - c)x_i = (1 - \frac{n-1}{u_i \sum_{j=1}^n(\frac{1}{u_j})})^2 S \tag{7}$$

Note that price does not depend on market size and that profit is increasing in quality and does not depend on marginal cost at equilibrium.

[41] The specification here closely resembles Motta (1992) and the derivation follows Sutton (1998, Appendix 15.1).

[42] That is, the lowest quality offering with positive sales has a quality that exceeds the harmonic mean of the qualities of all offerings multiplied by $\frac{n}{n-1}$ (the latter approximates 1 for large n).

I now turn to the entry and investment stage. Sutton (1991, c.3) proves that at the unique Nash equilibrium, firms choose the same quality level u. In this case, $u_i \sum_{j=1}^{n} (\frac{1}{u_j}) = n$, and for every firm choosing to enter, output, price and gross profit collapse to:

$$x_i = x = \frac{S}{c} \frac{n-1}{n^2} \tag{8}$$

$$p_i = p = c\frac{n}{n-1} \tag{9}$$

$$\Pi_i = \Pi = \frac{S}{n^2} \tag{10}$$

Recall from Section 3 that $F(u) = u^3$, $u \geq 1$. (The convexity of the fixed cost function is chosen so that, in equilibrium, three firms find it profitable to enter each country.) Net profit per firm is given by $\pi(n,u) = \Pi(n) - F(u) = \frac{S}{n^2} - u^3$. The industry equilibrium, where n firms enter with quality u, is characterised by two conditions (Motta 1992):

(I) (Free entry) $\pi(n,u) \geq 0$ (viability) and $\pi(n+1,u) < 0$ (stability)

(II) (Optimal quality[43]) $\frac{d\Pi}{du} = \frac{dF}{du}$

By considering a deviant firm i offering quality u_i when all its rivals $j \neq i$ offer a common quality u, condition (II) becomes[44]:

$$\frac{d\pi_i}{du_i} \mid_{u_i=u} = \frac{d\Pi_i}{du_i} \mid_{u_i=u} - \frac{dF_i}{du_i} \mid_{u_i=u} = \frac{2S(n-1)^2}{un^3} - 3u^2 = 0$$

which can be rearranged to

$$u^3 = \frac{2S(n-1)^2}{3n^3} \tag{11}$$

Given $u = u(n)$ by condition (II), and that condition (I) can be expressed as $\sqrt{\frac{S}{[u(n+1)]^3}} - 1 < n \leq \sqrt{\frac{S}{[u(n)]^3}}$, condition (I) may then be rewritten as

$$\sqrt{\frac{3(n+1)^3}{2n^2}} - 1 < n \leq \sqrt{\frac{3n^3}{2(n-1)^2}}$$

The only possible solution to this inequality is $n = 3$ and this does not depend on S[45].

The autarky equilibrium is then given by equations (8) to (11), where $n = n_l = 3$ firms enter in each country $l \in \{A, B\}$ and the common quality, price, firm output and firm (gross and net) profits are given by:

$$\begin{aligned} u_l &= \frac{2}{3}\sqrt[3]{\frac{S_l}{3}}, \quad p_l = \frac{3}{2}c, \quad x_l = \frac{2}{9}\frac{S_l}{c}, \\ \Pi_l &= \frac{S_l}{9}, \quad \pi_l = \frac{S_l}{9} - (u_l)^3 = \frac{S_l}{81} \end{aligned} \tag{12}$$

[43] As in Motta (1992), I consider only internal solutions.

[44] From equation (7), when all firm i's rivals offer a common quality u, I obtain $\Pi_i = (1 - \frac{1}{\frac{1}{n-1} + \frac{u_i}{u}})^2 S$. Then $\frac{d\Pi_i}{du_i} = \frac{2S}{u} \frac{\frac{1}{n-1} + \frac{u_i}{u} - 1}{(\frac{1}{n-1} + \frac{u_i}{u})^3}$, and by evaluating this expression at $u_i = u$ the marginal benefit for the deviant firm i of increasing quality when all firms choose a common quality u is $\frac{d\Pi_i}{du_i} \mid_{u_i=u} = \frac{2S(n-1)^2}{un^3}$. The SOC is satisfied at the solution $n = 3$ (see below).

[45] This "non-convergence" result is consistent with the finiteness property of many vertical product differentiation models. The symmetry of the quality chosen by firms at equilibrium is, however, less common in the literature and hinges on the symmetry of consumer preferences and the assumption of quantity as opposed to price competition (Shaked and Sutton 1983, Sutton 1991, Motta 1992).

Given the assumption $u_l \geq 1$, I further assume that $S_l > \frac{81}{8}$. Quality is an increasing function of market size so in view of the assumption that $S_A \geq S_B (> \frac{81}{8})$, quality offered by firms in country A is at least equal to that offered by their counterparts in country B: $u_A \geq u_B (\geq 1)$.

B.2 Reduced-form profit functions in game with T merger stages and (initially) T independent firms in each country

Consider the sequential cross-border merger game spelled out at the beginning of Section 3 yet embed each country initially with T independent firms, $T \geq 2$, and extend the game to T merger stages. (The setup in Section 3 refers to the case $T = 3$. See B.2.1 below.) Thus, the firms located in country A (producing with quality u_A) are labelled $1, 2, 3, ..., T$ and the firms located in country B (quality u_B) are labelled $T+1, T+2, T+3, ..., 2T$. In the same vein, in the first stage firms 1 and $T+1$ decide whether to merge (if formed, merged firm is labelled M_1), in the second stage firms 2 and $T+2$ decide whether to merge (if formed, merged firm is labelled M_2), and so on, in sequence until stage T, where firms T and $2T$ are the last pair of firms to undertake a merger decision (if formed, merged firm is labelled M_T). The game ends at stage $T+1$, the market competition stage. Given the symmetry, the $T+1$ possible market structures coming out of the T merger stages are labelled $r_0, r_1, r_2, ..., r_T$ where, as in Section 3, the subscript denotes the number of cross-border mergers undertaken.

To solve for market competition equilibrium outcomes as a function of market structure, begin by considering market structure r_i, where $i = 1, 2, 3, ..., T-1$, i.e. both independent and merged (multinational) firms exist (I will return to the "corner" structures r_0 and r_T shortly). Specifically, under market structure r_i there are i multinational firms, $T-i$ independent A-country firms and $T-i$ independent B-country firms.

By the merger-technology assumption, a multinational firm produces at quality level $\max(u_A, u_B) = u_A$ not only in country A but also in country B. Clearly, given the unit trade cost $t \geq 0$, it will no longer trade between countries, meeting the demand for its (high-quality) product in each country through domestic production[46].

From the demand setup, in equilibrium consumer prices in country $l \in \{A, B\}$ are such that $\frac{p_m^l}{u_A} = \frac{p_a^l}{u_A} = \frac{p_b^l}{u_B}$, where p_j^l denotes the price of the good produced by firm j and sold in country l. (Recall that the subscripts m, a and b denote a multinational firm, an independent A-country firm and an independent B-country firm, respectively.) This may be written in terms of the quality ratio $v = \frac{u_A}{u_B} \geq 1$:

$$p_m^l = p_a^l = v p_b^l \qquad l \in \{A, B\} \tag{13}$$

[46]This would not necessarily be the case were a merger *not* to lead to a quality upgrade in the production facilities in country B. For a large enough quality ratio relative to the trade cost, it can be shown that a multinational firm would continue exporting high-quality product produced by its A-country plant to country B, discontinuing production operations in country B. For example, when $T = 3$ under market structure r_1 (one multinational firm and two independent firms in each country), this would happen if $v > 1 + \frac{t}{c}$. Otherwise, for $v \leq 1 + \frac{t}{c}$, the multinational firm would produce high-quality product in country A for consumption in country A and produce low-quality product in country B for consumption in country B.

Thus, to illustrate, consumer prices in country B are given by p_m^B, p_a^B and p_b^B, where high-quality goods (produced by multinational firms and independent A-country firms) command a price premium relative to low-quality goods (produced by independent B-country firms).

With no loss of generality (solely for the purpose of labelling firms), assume that the i multinational firms in market structure r_i were formed in the first i merger stages. Similar to equation (3), the inverse demand functions for a low-quality product in country A and country B are, respectively:

$$p_b^A = \frac{S_A}{v\sum_{j=M_1}^{M_i} x_j^A + v\sum_{j=i+1}^{T} x_j^A + \sum_{j=T+i+1}^{2T} x_j^A}$$

$$p_b^B = \frac{S_B}{v\sum_{j=M_1}^{M_i} x_j^B + v\sum_{j=i+1}^{T} x_j^B + \sum_{j=T+i+1}^{2T} x_j^B}$$

The first term in the denominator corresponding to each market is the quality-adjusted sales of the i multinational firms in that market; the second and third terms respectively refer to the quality-adjusted sales of the $T-i$ independent A-country firms and the $T-i$ independent B-country firms in that market. The inverse demand functions for high-quality product in both countries, $p_m^l = p_a^l$, $l \in \{A, B\}$, can then be obtained from equation (13).

Each firm's optimisation problem can now be written. A merged multinational firm m, $m = M_1, M_2, ..., M_i$, maximises profits by setting x_m^A and x_m^B to solve

$$\max_{x_m^A \geq 0, x_m^B \geq 0} \; p_m^A x_m^A + p_m^B x_m^B - c(x_m^A + x_m^B)$$

taking all other firms' outputs as given. As in equation (2), writing the price-to-quality ratio in country A as $\lambda^A = \frac{S_A}{v\sum_{j=M_1}^{M_i} x_j^A + v\sum_{j=i+1}^{T} x_j^A + \sum_{j=T+i+1}^{2T} x_j^A}$, and the price-to-quality ratio in country B as $\lambda^B = \frac{S_B}{v\sum_{j=M_1}^{M_i} x_j^B + v\sum_{j=i+1}^{T} x_j^B + \sum_{j=T+i+1}^{2T} x_j^B}$, the optimisation problem for the multinational firm may be rewritten

$$\max_{x_m^A \geq 0, x_m^B \geq 0} \; v(\lambda^A x_m^A + \lambda^B x_m^B) - c(x_m^A + x_m^B)$$

Since $\frac{d\lambda^l}{dx_m^l} = -v\frac{(\lambda^l)^2}{S_l}$, $l \in \{A, B\}$, the two FOCs are

$$v x_m^A = \frac{S_A}{\lambda^A}\left(1 - \frac{1}{\lambda^A}\frac{c}{v}\right) \tag{14}$$

$$v x_m^B = \frac{S_B}{\lambda^B}\left(1 - \frac{1}{\lambda^B}\frac{c}{v}\right) \tag{15}$$

An independent A-country firm a, $a = i+1, i+2, ..., T$, located in country A, sets x_a^A and x_a^B to solve

$$\max_{x_a^A \geq 0, x_a^B \geq 0} \; p_a^A x_a^A + (p_a^B - t)x_a^B - c(x_a^A + x_a^B)$$

where sales in country B are subject to the unit trade cost. This may be rewritten in terms of the price-to-quality ratios in each market:

$$\max_{x_a^A \geq 0, x_a^B \geq 0} \quad v(\lambda^A x_a^A + \lambda^B x_a^B) - c x_a^A - (c+t) x_a^B$$

Since $\frac{d\lambda^l}{dx_a^l} = -v\frac{(\lambda^l)^2}{S_l}$, $l \in \{A, B\}$, the two FOCs are

$$v x_a^A = \frac{S_A}{\lambda^A}\left(1 - \frac{1}{\lambda^A}\frac{c}{v}\right) \tag{16}$$

$$v x_a^B = \frac{S_B}{\lambda^B}\left(1 - \frac{1}{\lambda^B}\frac{c+t}{v}\right) \tag{17}$$

Similarly, an independent B-country firm b, $b = T+i+1, T+i+2, ..., 2T$, located in country B, solves

$$\max_{x_b^A \geq 0, x_b^B \geq 0} \quad (p_b^A - t)x_b^A + p_b^B x_b^B - c(x_b^A + x_b^B)$$

or,

$$\max_{x_b^A \geq 0, x_b^B \geq 0} \quad \lambda^A x_b^A + \lambda^B x_b^B - (c+t)x_b^A - c x_b^B$$

Now $\frac{d\lambda^l}{dx_b^l} = -\frac{(\lambda^l)^2}{S_l}$, $l \in \{A, B\}$, and the two FOCs are

$$x_b^A = \frac{S_A}{\lambda^A}\left(1 - \frac{1}{\lambda^A}(c+t)\right) \tag{18}$$

$$x_b^B = \frac{S_B}{\lambda^B}\left(1 - \frac{1}{\lambda^B}c\right) \tag{19}$$

Adding across all FOCs pertaining to market A (i FOCs (14) for the multinational firms, $T-i$ FOCs (16) for the independent A-country firms and $T-i$ FOCs (18) for the independent B-country firms), I obtain:

$$v\sum_{j=M_1}^{M_i} x_j^A + v\sum_{j=i+1}^{T} x_j^A + \sum_{j=T+i+1}^{2T} x_j^A = \frac{S_A}{\lambda^A}\left(2T - i - \frac{1}{\lambda^A}\left(c\frac{T + (T-i)v}{v} + (T-i)t\right)\right)$$

and noting that the LHS is simply $\frac{S_A}{\lambda^A}$ I can solve for λ^A:

$$\lambda^A = \frac{1}{2T - (i+1)}\left(\frac{c(T + (T-i)v) + (T-i)tv}{v}\right)$$

Similarly adding across all FOCs pertaining to market B (i FOCs (15), $T-i$ FOCs (17) and $T-i$ FOCs (19), λ^B can be obtained:

$$\lambda^B = \frac{1}{2T - (i+1)}\left(\frac{c(T + (T-i)v) + (T-i)t}{v}\right)$$

Substituting for the price-to-quality ratios in FOCs (14) through (19), sales per firm in each market are obtained:

$$
\begin{array}{ll}
x_m^A = \left(2T-(i+1)\right) S_A \frac{c((T-i)v-(T-i-1))+(T-i)tv}{(c(T+(T-i)v)+(T-i)tv)^2} & x_m^B = \left(2T-(i+1)\right) S_B \frac{c((T-i)v-(T-i-1))+(T-i)t}{(c(T+(T-i)v)+(T-i)t)^2} \\
x_a^A = x_m^A & x_a^B = \left(2T-(i+1)\right) S_B \frac{c((T-i)v-(T-i-1))-(T-1)t}{(c(T+(T-i)v)+(T-i)t)^2} \\
x_b^A = \left(2T-(i+1)\right) S_A v \frac{c(T-(T-1)v)-(T-1)tv}{(c(T+(T-i)v)+(T-i)tv)^2} & x_b^B = \left(2T-(i+1)\right) S_B v \frac{c(T-(T-1)v)+(T-i)t}{(c(T+(T-i)v)+(T-i)t)^2}
\end{array}
$$

Non-negativity constraints – ensuring that despite trade costs $t \geq 0$ and quality asymmetries $v \geq 1$, all firms command positive sales in both countries – can be summarised as two parameter restrictions. The "low-enough-quality-ratio" restriction follows from requiring that low-quality B-country firms are still able to sell in their home country when imports are most competitive ($t = 0$): $x_b^B \geq 0 \iff v \leq \frac{T}{T-1} + \frac{T-i}{T-1}\tilde{t}$ which implies $v \leq \frac{T}{T-1}$ when $\tilde{t} = 0$ (recall $\tilde{t} = \frac{t}{c}$). The "low-enough-trade-cost" restriction ensures that low-quality B-country firms' exports to country A are not priced out of the market: $x_b^A \geq 0 \iff \tilde{t} \leq \frac{T-(T-1)v}{(T-1)v}$. Denote by $\mathcal{P}$ the set of parameter values $(v, \tilde{t})$ satisfying these two conditions[47].

Prices are obtained noting that $p_b^l = \lambda^l$, and from equation (13), $p_m^l = p_a^l = v\lambda^l$, $l \in \{A, B\}$. Finally, evaluating the objective functions at these sales and prices, profits per firm in each market are obtained:

$$
\begin{array}{ll}
\Pi_{m,r_i}^A = \left(\frac{(T-i)v-(T-i-1)+(T-i)\tilde{t}v}{T+(T-i)v+(T-i)\tilde{t}v}\right)^2 S_A & \Pi_{m,r_i}^B = \left(\frac{(T-i)v-(T-i-1)+(T-i)\tilde{t}}{T+(T-i)v+(T-i)\tilde{t}}\right)^2 S_B \\
\Pi_{a,r_i}^A = x_{m,r_i}^A & \Pi_{a,r_i}^B = \left(\frac{(T-i)v-(T-i-1)-(T-1)\tilde{t}}{T+(T-i)v+(T-i)\tilde{t}}\right)^2 S_B \\
\Pi_{b,r_i}^A = \left(\frac{T-(T-1)v-(T-1)\tilde{t}v}{T+(T-i)v+(T-i)\tilde{t}v}\right)^2 S_A & \Pi_{b,r_i}^B = \left(\frac{T-(T-1)v+(T-i)\tilde{t}}{T+(T-i)v+(T-i)\tilde{t}}\right)^2 S_B
\end{array}
\tag{20}
$$

The reduced-form profit functions per firm are thus the sum of the profit components in the two markets, e.g. for a multinational firm, $\Pi_{m,r_i} = \Pi_{m,r_i}^A + \Pi_{m,r_i}^B$.

While the derivation above was carried out for "intermediate" market structures where both independent and multinational firms exist, it similarly applies to structures r_0 (where there are no multinational firms) and r_T (where there are no independent firms). It is easy to see that the reduced-form profit functions (20) – as well as the outputs and prices derived above) – also hold where applicable. In other words, Π_{m,r_i} calculated from (20) holds for $i = 1, 2, 3, ..., T$, while Π_{a,r_i} and Π_{b,r_i} hold for $i = 0, 1, 2, ..., T-1$.

---

[47] Formally, space $\mathcal{P}$ is defined as $\{(v, \tilde{t}) \in \Re^2 \mid 1 \leq v \leq \frac{T}{T-1}$ and $0 \leq \tilde{t} \leq \frac{T-(T-1)v}{(T-1)v}\}$. Alternatively, $\mathcal{P}$ can be defined by the restrictions $0 \leq \tilde{t} \leq \frac{1}{T-1}$ and $1 \leq v \leq \frac{T}{T-1}\frac{1}{1+\tilde{t}}$.

The reason why the analysis in Section 3 is confined to $\mathcal{P}$ is simplicity. Lifting these parameter restrictions adds little insight: extending the space of parameter values to $\{(v, \tilde{t}) \in \Re^2 \mid v \geq 1$ and $\tilde{t} \geq 0\}$ would enlarge zone (a), the zone where mergers are always profitable, both along and off the equilibrium path of the game.

Proof of this claim is available from the author, but headway can be made by noting that, say, for the case considered in Section 3 (i.e. $T = 3$): (i) independent B-country firms do not command positive sales *even* in their home country, $x_b^A = x_b^B = 0$, when $v \geq \frac{3}{2} + \frac{3}{2}\tilde{t}$ and $\tilde{t} \geq 0$ under market structure r_0, or when $v \geq \frac{3}{2} + \tilde{t}$ and $\tilde{t} \geq 0$ under r_1, or when $v \geq \frac{3}{2} + \frac{1}{2}\tilde{t}$ and $\tilde{t} \geq 0$ under r_2; (ii) trade is too expensive *even* for A-country firms, $x_a^B = x_a^A = 0$, when $1 \leq v \leq \frac{2}{3} + \frac{2}{3}\tilde{t}$ under r_0, or when $1 \leq v \leq \frac{1}{2} + \tilde{t}$ under r_1, or when $1 \leq v \leq 2\tilde{t}$ under r_2. Note that for $\frac{3}{2} + \frac{1}{2}\tilde{t} \leq v \leq 2\tilde{t}$ under r_2, only two (multinational) firms command positive sales in country B, unlike all other parameter combinations and market structures where the number of firms selling into each country is at least three.

B.2.1 Market competition equilibrium outcomes as a function of market structure (case $T = 3$)

The reduced-form profit functions used to compute the merger surplus functions (Definition 3) follow from plugging $T = 3$ in equations (20) for the general case considered above (T merger stages and initially T independent firms in each country). The space of parameter values $\mathcal{P}$ follows similarly from the space derived for the general case.

In view of Lemma 1, I reproduce the three merger surplus functions which need to be signed in order to solve the three-merger-stage game in Section 3:

$$\Psi_I(v,\tilde{t}) = \left[\left(\frac{2v-1+2\tilde{t}v}{3+2v+2\tilde{t}v}\right)^2 - \frac{\left(3v-2+3\tilde{t}v\right)^2+\left(3-2v-2\tilde{t}v\right)^2}{9\left(1+v+\tilde{t}v\right)^2}\right] S_A + \left[\left(\frac{2v-1+2\tilde{t}}{3+2v+2\tilde{t}}\right)^2 - \frac{\left(3v-2-2\tilde{t}\right)^2+\left(3-2v+3\tilde{t}\right)^2}{9\left(1+v+\tilde{t}\right)^2}\right] S_B$$

$$\Psi_{II}(v,\tilde{t}) = \left[\left(\frac{v+\tilde{t}v}{3+v+\tilde{t}v}\right)^2 - \frac{\left(2v-1+2\tilde{t}v\right)^2+\left(3-2v-2\tilde{t}v\right)^2}{\left(3+2v+2\tilde{t}v\right)^2}\right] S_A + \left[\left(\frac{v+\tilde{t}}{3+v+\tilde{t}}\right)^2 - \frac{\left(2v-1-2\tilde{t}\right)^2+\left(3-2v+2\tilde{t}\right)^2}{\left(3+2v+2\tilde{t}\right)^2}\right] S_B$$

$$\Psi_{III}(v,\tilde{t}) = \left[\frac{1}{9} - \frac{\left(v+\tilde{t}v\right)^2+\left(3-2v-2\tilde{t}v\right)^2}{\left(3+v+\tilde{t}v\right)^2}\right] S_A + \left[\frac{1}{9} - \frac{\left(v-2\tilde{t}\right)^2+\left(3-2v+\tilde{t}\right)^2}{\left(3+v+\tilde{t}\right)^2}\right] S_B$$

B.3 Proof of Lemma 1

Part (i) This proof amounts to verifying that for all $(v,\tilde{t}) \in \mathcal{P}$:

- $\Psi_{II}(v,\tilde{t}) \geq 0$ whenever $\Psi_I(v,\tilde{t}) \geq 0$,
- $\Psi_{III}(v,\tilde{t}) \geq 0$ whenever $\Psi_{II}(v,\tilde{t}) \geq 0$.

This is done separately for the merger surplus emanating from each market: a second subscript is added to denote the market. Thus, for example, $\Psi_{I,A}(v,\tilde{t}) = \Pi^A_{m,r_1} - \Pi^A_{a,r_0} - \Pi^A_{b,r_0}$ denotes the terms in $\Psi_I(v,\tilde{t})$ corresponding to market A, obtained from equations (20), for $T = 3$, and reproduced in B.2.1.

I begin with $\Psi_{I,A}(v,\tilde{t})$, $\Psi_{II,A}(v,\tilde{t})$ and $\Psi_{III,A}(v,\tilde{t})$. Consider a straight line segment going from $(v,\tilde{t}) = (1,0)$ to any point on $\tilde{t} = \frac{3-2v}{2v}$, the boundary of $\mathcal{P}$ where the "low-enough-trade-cost" restriction binds (i.e. $x^A_b = 0$). By writing this line segment as $\tilde{t} = \rho(v-1)$, where $0 \leq \rho \leq \infty$ and $0 \leq \rho(v-1) \leq \frac{3-2v}{2v}$, changes in v and $\tilde{t}$ along this line segment parameterised by ρ may be referred to simply as changes in v (for $\rho = 0$ the line segment lies on the v-axis; for $\rho \to \infty$ the line segment lies on the $\tilde{t}$-axis). I seek to uncover how the functions[48] $\Psi_{I,A}$, $\Psi_{II,A}$ and $\Psi_{III,A}$ change as I increase v along line segment ρ (i.e. jointly increasing $\tilde{t}$ such that $\tilde{t} = \rho(v-1)$) from the lower end $v = 1$ (i.e. $(v,\tilde{t}) = (1,0)$) to the upper end defined implicitly by $\rho(v-1) = \frac{3-2v}{2v}$ (label this value $v = \bar{v}$; formally this label should carry the parameter ρ, omitted for simplicity). The following may be verified. At the lower end of the line segment, when $v = 1$, all three functions are negative. Intuitively, merging firms' surplus arising from any stand-alone cross-border merger (be this the first, the second or the third) is negative: since both v and $\tilde{t}$ are low, non-participating firms respond to the merger by increasing output considerably. At the upper end of the line segment, when $v = \bar{v}$, all three functions are equal to zero. Intuitively, since $x^A_b = 0$, there is no surplus to be enjoyed on sales in country A from cross-border mergers when B-country firms' exports to country A are (just) priced out of the market. Prior to any merger, there are three firms offering

[48]For simplicity we drop the arguments of the functions.

quality v commanding positive sales in country A; after a merger this is unchanged. Now, starting at $v = 1$ and increasing v along the line segment, $\Psi_{I,A}$, $\Psi_{II,A}$ and $\Psi_{III,A}$ each increase continuously from negative values toward positive values, reaching a maximum, then decreasing continuously toward zero when $v = \bar{v}$. Label the first value of v at which $\Psi_{I,A}$ is zero as v'_A, the first value of v at which $\Psi_{II,A}$ is zero as v''_A, and the first value of v at which $\Psi_{III,A}$ is zero as v'''_A (again the labels omit the reference to ρ for simplicity). One can verify that $0 < v'''_A < v''_A < v'_A < \bar{v}$. Since this is true along any line segment parameterised by ρ, $0 \leq \rho \leq \infty$, the following result holds:

$$\begin{aligned}\left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{I,A}(v,\tilde{t}) \geq 0\right\} &\subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II,A}(v,\tilde{t}) \geq 0\right\} \\ &\subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{III,A}(v,\tilde{t}) \geq 0\right\} \subset \mathcal{P} \end{aligned} \tag{21}$$

It may also be verified that, in addition to intersecting at $v = \bar{v}$, $\Psi_{II,A}$ and $\Psi_{I,A}$ cross as they slope upwards at a point, labelled $v''^{-\prime}_A$, which lies between 0 and v''_A. In other words, $\Psi_{II,A} - \Psi_{I,A} = 0$ at $v = v''^{-\prime}_A$, where $0 < v''^{-\prime}_A < v''_A$. To the right of this point, for $v''^{-\prime}_A < v < \bar{v}$, $\Psi_{II,A} - \Psi_{I,A} > 0$, whereas to its left, for $0 \leq v < v''^{-\prime}_A$, $\Psi_{II,A} - \Psi_{I,A} < 0$. Since $0 < v''^{-\prime}_A < v''_A < v'_A < \bar{v}$, the following (stronger) result holds:

$$\left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{I,A}(v,\tilde{t}) \geq 0\right\} \subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II,A}(v,\tilde{t}) - \Psi_{I,A}(v,\tilde{t}) \geq 0\right\} \tag{22}$$

It can further be verified that $\Psi_{III,A} - \Psi_{II,A} > 0$ for $0 \leq v < \bar{v}$ (recall $\Psi_{III,A} = \Psi_{II,A}$ when $v = \bar{v}$) and thus

$$\forall\ (v,\tilde{t}) \in \mathcal{P},\ \Psi_{III,A}(v,\tilde{t}) - \Psi_{II,A}(v,\tilde{t}) \geq 0 \tag{23}$$

Turning now to the merger surplus terms in market B, the same results come through, despite some differences in the corresponding functions which are detailed next. The values of the functions $\Psi_{I,B}$, $\Psi_{II,B}$ and $\Psi_{III,B}$ are no longer zero for *all* $(v,\tilde{t})$ along the border of $\mathcal{P}$ for which the "low-enough-trade-cost" restriction binds. For these parameter values, unlike in country A, B-country firms still command positive sales in their home country and hence there is surplus from merger to be made on sales in country B (i.e. the functions are strictly positive), with two exceptions. One is when imports are at their most competitive, $(v,\tilde{t}) = (\frac{3}{2}, 0)$, and B-country firms command zero sales in their home country (as they do abroad), in which case $\Psi_{I,B}$, $\Psi_{II,B}$ and $\Psi_{III,B}$ again equal zero. The other situation along this border of $\mathcal{P}$ where these functions equal zero occurs when $(v,\tilde{t}) = (1, \frac{1}{2})$. Here, quality is symmetric and markets are effectively autarkic: cross-border merger does not change the (effective) number of competitors in each market. Now, starting at $v = 1$ and increasing v along any line segment parameterised by ρ, $\Psi_{I,B}$, $\Psi_{II,B}$ and $\Psi_{III,B}$ each increase continuously from negative values toward positive values; whether the functions increase monotonically or whether they reach a (positive) maximum before decreasing to a lower albeit positive value or zero when $v = \bar{v}$ depends on the line segment parameterised by ρ. Labelling the lower and possibly only value of v at which $\Psi_{I,B}$ ($\Psi_{II,B}$, $\Psi_{III,B}$) is zero as v'_B (v''_B, v'''_B respectively), one verifies that $0 < v'''_B < v''_B < v'_B < \bar{v}$. Since this is true along any line segment parameterised by ρ, $0 \leq \rho \leq \infty$, a result analogous to (21) holds:

$$\begin{aligned}\left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{I,B}(v,\tilde{t}) \geq 0\right\} &\subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II,B}(v,\tilde{t}) \geq 0\right\} \\ &\subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{III,B}(v,\tilde{t}) \geq 0\right\} \subset \mathcal{P} \end{aligned} \tag{24}$$

Similar to their country A counterparts, $\Psi_{II,B}$ and $\Psi_{I,B}$ cross as they slope upwards at a point, labelled $v_B''^{-\prime}$, which lies between 0 and v_B''. To the right of this point, for $v_B''^{-\prime} < v < \bar{v}$, $\Psi_{II,B} - \Psi_{I,B} > 0$[49], whereas to its left, for $0 \leq v < v_B''^{-\prime}$, $\Psi_{II,B} - \Psi_{I,B} < 0$. Since $0 < v_B''^{-\prime} < v_B'' < v_B' < \bar{v}$, the following (stronger) result holds:

$$\left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{I,B}(v,\tilde{t}) \geq 0\right\} \subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II,B}(v,\tilde{t}) - \Psi_{I,B}(v,\tilde{t}) \geq 0\right\} \tag{25}$$

It can further be verified that $\Psi_{III,B} - \Psi_{II,B} > 0$ for $0 \leq v < \bar{v}$[50] and thus

$$\forall\ (v,\tilde{t}) \in \mathcal{P},\ \Psi_{III,B}(v,\tilde{t}) - \Psi_{II,B}(v,\tilde{t}) \geq 0 \tag{26}$$

Finally, it can be verified that along all line segments parameterised by ρ, $0 \leq \rho \leq \infty$, $v_B''^{-\prime} < v_A'$ and $v_A''^{-\prime} < v_B'$.

The results for both markets are now combined to conclude the proof. I wish to show that for $(v,\tilde{t})$ such that $\Psi_I(v,\tilde{t}) = \Psi_{I,A}(v,\tilde{t}) + \Psi_{I,B}(v,\tilde{t}) \geq 0$, then $\Psi_{II}(v,\tilde{t}) = \Psi_{II,A}(v,\tilde{t}) + \Psi_{II,B}(v,\tilde{t}) \geq 0$. (I also need to show that for $(v,\tilde{t})$ such that $\Psi_{II}(v,\tilde{t}) = \Psi_{II,A}(v,\tilde{t}) + \Psi_{II,B}(v,\tilde{t}) \geq 0$, then $\Psi_{III}(v,\tilde{t}) = \Psi_{III,A}(v,\tilde{t}) + \Psi_{III,B}(v,\tilde{t}) \geq 0$. This is postponed briefly.) For $(v,\tilde{t})$ such that $\Psi_I \geq 0$ this may be due to either of three possibilities:

- $\Psi_{I,A} \geq 0$ and $\Psi_{I,B} \geq 0$. From (21), $\Psi_{I,A} \geq 0 \Longrightarrow \Psi_{II,A} \geq 0$, while from (24), $\Psi_{I,B} \geq 0 \Longrightarrow \Psi_{II,B} \geq 0$. Hence

$$\left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_I(v,\tilde{t}) \geq 0\right\} \subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II}(v,\tilde{t}) \geq 0\right\} \tag{27}$$

  from which the first half of Lemma 1, part (i) follows.

- $\Psi_{I,A} > 0$, $\Psi_{I,B} < 0$ but $\Psi_{I,A} + \Psi_{I,B} \geq 0$. Here, since $\Psi_{I,A} > 0$ and $\Psi_{I,B} < 0$, v must be greater than v_A' but less than v_B', i.e. $v_A' < v < v_B'$. As verified earlier, $v_B''^{-\prime} < v_A'$, whence it follows that $v_B''^{-\prime} < v$ and thus $\Psi_{II,B} - \Psi_{I,B} > 0$. From (22), $\Psi_{I,A} \geq 0$ implies that $\Psi_{II,A} - \Psi_{I,A} \geq 0$. Since $\Psi_{I,A} + \Psi_{I,B} \geq 0$ it must be that $\Psi_{II,A} + \Psi_{II,B} > 0$ and result (27) follows.

- $\Psi_{I,A} < 0$, $\Psi_{I,B} > 0$ but $\Psi_{I,A} + \Psi_{I,B} \geq 0$. Here, since $\Psi_{I,A} < 0$ and $\Psi_{I,B} > 0$, v must be greater than v_B' but less than v_A', i.e. $v_B' < v < v_A'$. As verified earlier, $v_A''^{-\prime} < v_B'$, whence it follows that $v_A''^{-\prime} < v$ and thus $\Psi_{II,A} - \Psi_{I,A} > 0$. Again, since $\Psi_{I,A} + \Psi_{I,B} \geq 0$ it must be that $\Psi_{II,A} + \Psi_{II,B} > 0$ and result (27) follows.

I now show that for $(v,\tilde{t})$ such that $\Psi_{II} = \Psi_{II,A} + \Psi_{II,B} \geq 0$, it must be that $\Psi_{III} = \Psi_{III,A} + \Psi_{III,B} \geq 0$. The proof is a simplified version of the previous one, and follows from noting by (23) and (26) that $\forall\ (v,\tilde{t}) \in \mathcal{P}$, $\Psi_{III,A} - \Psi_{II,A} \geq 0$ and $\Psi_{III,B} - \Psi_{II,B} \geq 0$. Thus the counterpart to result (27) for the second half of Lemma 1 is obtained:

$$\left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II}(v,\tilde{t}) \geq 0\right\} \subset \left\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{III}(v,\tilde{t}) \geq 0\right\} \tag{28}$$

[49] $\Phi_{II,B} - \Phi_{I,B} > 0$ for $v_B''^{-\prime} < v \leq \bar{v}$ when $0 < \rho < \infty$; i.e. only when $\rho = 0$ or $\rho \to \infty$ does $\Phi_{II,B} - \Phi_{I,B} = 0$ when $v = \bar{v}$.

[50] The previous footnote similarly applies: only when $\rho = 0$ or $\rho \to \infty$ does $\Phi_{III,B} - \Phi_{II,B} = 0$ when $v = \bar{v}$.

Summarising results (27) and (28),

$$\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_I(v,\tilde{t}) \geq 0\} \subset \{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{II}(v,\tilde{t}) \geq 0\} \subset \{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_{III}(v,\tilde{t}) \geq 0\} \subset \mathcal{P} \tag{29}$$

from which Lemma 1, part (i) follows. *Q.E.D.*

Part (ii) Recall from Definition 3 that $\Psi_{II}(v,\tilde{t}) \geq 0$ is equivalent to $\Pi_{m,r_2} \geq \Pi_{a,r_1} + \Pi_{b,r_1}$. It can be verified that (for $(v,\tilde{t}) \in \mathcal{P}$) $\Pi_{a,r_1} + \Pi_{b,r_1} \geq \Pi_{a,r_0} + \Pi_{b,r_0}$ (i.e. non-participating independent firms gain from a first stand-alone merger). Hence if $\Psi_{II} \geq 0$, it follows that $\Pi_{m,r_2} \geq \Pi_{a,r_0} + \Pi_{b,r_0}$, which is equivalent to $\Psi_{IV} \geq 0$.

Similarly, $\Psi_{III} \geq 0$ is equivalent to $\Pi_{m,r_3} \geq \Pi_{a,r_2} + \Pi_{b,r_2}$. It can be verified that $\Pi_{a,r_2} + \Pi_{b,r_2} \geq \Pi_{a,r_1} + \Pi_{b,r_1}$ (i.e. non-participating independent firms gain from a second stand-alone merger). Hence if $\Psi_{III} \geq 0$, it follows that $\Pi_{m,r_3} \geq \Pi_{a,r_1} + \Pi_{b,r_1}$, which is equivalent to $\Psi_V \geq 0$. From $\Pi_{a,r_1} + \Pi_{b,r_1} \geq \Pi_{a,r_0} + \Pi_{b,r_0}$, it follows from $\Psi_V \geq 0$ that $\Pi_{m,r_3} \geq \Pi_{a,r_0} + \Pi_{b,r_0}$, which is equivalent to $\Psi_{VI} \geq 0$. *Q.E.D.*

B.4 (Sketch of) Proof of Proposition 3

This proof follows largely from the proof of Lemma 1, noting the following definitions:

- Zone (a):= $\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_I(v,\tilde{t}) \geq 0,\ \Psi_{II}(v,\tilde{t}) \geq 0,\ \Psi_{III}(v,\tilde{t}) \geq 0\}$
- Zone (c1):= $\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_I(v,\tilde{t}) < 0,\ \Psi_{II}(v,\tilde{t}) \geq 0,\ \Psi_{III}(v,\tilde{t}) \geq 0\}$
- Zone (c2):= $\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_I(v,\tilde{t}) < 0,\ \Psi_{II}(v,\tilde{t}) < 0,\ \Psi_{III}(v,\tilde{t}) \geq 0\}$
- Zone (h):= $\{(v,\tilde{t}) \in \mathcal{P} \mid \Psi_I(v,\tilde{t}) < 0,\ \Psi_{II}(v,\tilde{t}) < 0,\ \Psi_{III}(v,\tilde{t}) < 0\}$

That the zones and the boundaries between them are as stated (and depicted in Figure 6) follows additionally from verifying that:

- $\bar{v}$, v'_A, v''_A, v'''_A, v'_B, v''_B and v'''_B are decreasing in ρ, the parameter of the straight line segment, as this increases from 0 (line segment lies on top of the v-axis) to ∞ (line segment lies on top of the $\tilde{t}$-axis).
- $v'''_B \leq v'''_A < v''_B \leq v''_A$ for $0 \leq \rho \leq \infty$.
- $v''_B \leq v''_A \leq v'_B \leq v'_A$ for $0 \leq \rho \leq \check{\rho} < \infty$ (line segment steep enough) and $0 < \hat{\rho} \leq \rho \leq \infty$ (line segment flat enough), where $\check{\rho} < \hat{\rho}$, while $v''_B < v'_B < v''_A < v'_A$ for $\check{\rho} < \rho < \hat{\rho}$.
- $v''^{-\prime}_A < v''_B$ and $v''^{-\prime}_B < v''_B$ so that $\Psi_{II,A} > \Psi_{I,A}$ and $\Psi_{II,B} > \Psi_{I,B}$ for $v''_B < v < v'_A$ along any line segment ρ.

  These elements suffice to prove Proposition 3. *Q.E.D.*

B.5 Proof of results depicted in Figure 8

I consider the equilibrium in each of zones (a), (c1) and (c2) of parameter space $\mathcal{P}$ in turn, starting from a fixed cost G associated with implementing a cross-border merger equal to zero (equilibria as in Figure 5, in the absence of fixed costs). I analyse how these equilibria change as G increases from zero.

Notice that the fixed cost G of implementing a merger changes the conditions for any merger to be profitable: whereas in the absence of the fixed cost this was given by $\Psi_X(v, \tilde{t}) \geq 0$, for $X \in \{I, II, III, IV, V, VI\}$, where X represents the relevant merger surplus function, the introduction of the fixed cost changes this condition to $\Psi_X(v, \tilde{t}) - G \geq 0$. (Refer to Definition 3. Recall that, for example, $\Psi_V(v, \tilde{t})$ reflects the profitability of a merger which if carried through would induce a change in market structure from r_1 (one cross-border merger) to r_3 (three cross-border mergers).

Note from the Proof of Lemma 1, part (ii) above[51] that for all $(v, \tilde{t}) \in \mathcal{P}$, $\Psi_{IV} \geq \Psi_{II}$ and $\Psi_{VI} \geq \Psi_V \geq \Psi_{III}$.

I begin with $(v, \tilde{t})$ in zone (a). From the Proof of Lemma 1, part (i), in this zone $\Psi_{III} \geq \Psi_{II} \geq \Psi_I \geq 0$. For $G \leq \Psi_I$, the equilibrium (solved-out game tree) replicates the equilibrium for zone (a) in the absence of fixed costs (Figure 5), where firms choose to merge from all nodes in the game tree. For $\Psi_I < G \leq \Psi_{II}$, we have that $\Psi_I - G < 0$ and thus firms 3 and 6 will now choose not to merge conditional on $\overline{\mathbf{14}}$ and $\overline{\mathbf{25}}$. By $\Psi_{II} - G \geq 0$, $\Psi_{III} \geq \Psi_{II}$ and $\Psi_{IV} \geq \Psi_{II}$, firms continue choosing to merge from all other nodes in the game tree. For this range of values of G, the equilibrium then replicates the equilibrium for zone (c1) in the absence of fixed costs (Figure 5). For $\Psi_{II} < G \leq \Psi_{III}$, $\Psi_{II} - G < 0$ implies that firms 3 and 6 will now choose not to merge conditional on either **14** and $\overline{\mathbf{25}}$, or $\overline{\mathbf{14}}$ and **25**. Since $\Psi_I - G < 0$, firms 3 and 6 will still choose not to merge conditional on $\overline{\mathbf{14}}$ and $\overline{\mathbf{25}}$, and firms 2 and 5 will now choose not to merge conditional on $\overline{\mathbf{14}}$. By $\Psi_{III} - G \geq 0$, $\Psi_V \geq \Psi_{III}$ and $\Psi_{VI} \geq \Psi_{III}$, firms continue choosing to merge from all other nodes in the game tree. For this range of values of G, the equilibrium then replicates the equilibrium for zone (c2) in the absence of fixed costs (Figure 5). For $G > \Psi_{III}$, firms 3 and 6 will now choose not to merge conditional on **14** and **25**. Since $\Psi_{II} - G < 0$, firms 3 and 6 will still choose not to merge conditional on either **14** and $\overline{\mathbf{25}}$, or $\overline{\mathbf{14}}$ and **25**, and firms 2 and 5 will now choose not to merge conditional on **14**. Since $\Psi_I - G < 0$, firms 3 and 6 will still choose not to merge conditional on $\overline{\mathbf{14}}$ and $\overline{\mathbf{25}}$, firms 2 and 5 will still choose not to merge conditional on $\overline{\mathbf{14}}$, and firms 1 and 4 will now choose not to merge. For this range of values of G, along the equilibrium path no mergers occur and the equilibrium then replicates the equilibrium for zone (h) in the absence of fixed costs (Figure 5).

The proofs of the equilibria for $(v, \tilde{t})$ in zones (c1) and (c2) follow from that of zone (a). In zone (c1), by definition, $\Psi_I < 0 \leq \Psi_{II}$, and only the equilibria for zones (b), (c2) and (h) in the absence of fixed costs can be replicated as G increases from zero, as analysed for zone (a). In zone (c2), by definition, $\Psi_{III} \geq 0$ and $\Psi_I, \Psi_{II} < 0$, and only the equilibria for zones (c2) and (h) in the absence of fixed costs can be replicated as G increases from zero, as analysed for zone (a). *Q.E.D.*

[51] For ease of exposition, I again suppress the arguments $(v, \tilde{t})$ of the merger surplus functions. I also introduce the notation **ij** to depict the outcome where firms i and j merge and $\overline{\mathbf{ij}}$ as the complementary (no-merger) outcome.

C Appendix: Cross-border mergers in the Perry and Porter (1985) model

C.1 Derivation of reduced-form profit functions

Coming out of the (two) merger stages, there are three possible market structures: (i) r_0, where no cross-border is undertaken; (ii) r_1, where one merger decision is favourable but the other is not; and (iii) r_2, where both mergers take place. I now turn to each.

(i) Under r_0, independent A-country firm 1 (say), owning capital stock k, sets outputs in both countries A and B, respectively x_1^A and x_1^B, to solve

$$\max_{x_1^A \geq 0, x_1^B \geq 0} P^A(X^A)x_1^A + \left(P^B(X^B) - t\right)x_1^B - C(x_1^A + x_1^B, k)$$

where its exports to country B are subject to the unit trade cost t. (Recall that $P^l(.)$ denotes the inverse demand function for the homogeneous good in country l, $l \in \{A, B\}$, $X^l = \sum_{i=1}^4 x_i^l$, and $C(x, k)$ denotes the firm's cost function.) Given the functional forms laid out in Section 4, the FOCs may be written:

$$a - X^A - x_1^A - \left(d + \frac{e}{k}(x_1^A + x_1^B)\right) = 0 \tag{30}$$

$$a - X^B - t - x_1^B - \left(d + \frac{e}{k}(x_1^A + x_1^B)\right) = 0 \tag{31}$$

The FOCs for the other independent firms, namely A-country firm 2 and B-country firms 3 and 4, can be similarly written, adjusting for the trade cost being incurred by the latter two on their exports to country A. Solving the system of FOCs, and recalling $\tilde{e} = \frac{e}{k}$ (the rate of change of marginal cost) and $\tilde{t} = \frac{t}{a-d}$ (the normalised trade cost), one obtains $X^A = X^B = \frac{(a-d)(4-2\tilde{t})}{5+2\tilde{e}}$. From FOCs (30) and (31), it follows that $\left|x_i^A - x_i^B\right| = t$ (firm i's sales in its home market exceed its foreign-market sales by t, where $i = 1, ..., 4$) and each firm's foreign-market sales are $x_a^B = x_b^A = \frac{(a-d)\left(1-\tilde{t}(3+\tilde{e})\right)}{5+2\tilde{e}}$. (As before, despite the slight abuse of notation, subscripts a, b and m denote an independent A-country firm, an independent B-country firm and a multinational firm, respectively.) Clearly, for trade between countries to be feasible, the parameter restriction $\tilde{t} \leq \frac{1}{3+\tilde{e}}$ must be satisfied (this condition along with $\tilde{e} \geq 0$ and $\tilde{t} \geq 0$ define space $\mathcal{P}$). Equilibrium prices in both countries are $p^A = p^B = \frac{a(1+2\tilde{e})+4d+2t}{5+2\tilde{e}}$. The reduced-form profit function follows from evaluating each firm's objective function:

$$\Pi_{a,r_0} = \Pi_{b,r_0} = \frac{(a-d)^2}{(5+2\tilde{e})^2}\left(2(1+\tilde{e})(1-\tilde{t}) + \frac{\tilde{t}^2}{2}(2+\tilde{e})(13+4\tilde{e})\right) - gk$$

(ii) Under r_1, each of the two independent firms a and b solves the same problem as in (i). The multinational firm m, formed from the merger of an independent A-country firm and an independent B-country firm, owns capital stock $2k$ and is twice as "large" as either of its independent rivals. Clearly, given the unit trade cost $t \geq 0$ and the cost function $C(x_m^A + x_m^B, 2k)$, it will no longer trade between countries, supplying each country through domestic production; it solves

$$\max_{x_m^A \geq 0, x_m^B \geq 0} P^A(X^A)x_m^A + P^B(X^B)x_m^B - C(x_m^A + x_m^B, 2k)$$

The FOCs become

$$a - X^A - x_m^A - \left(d + \frac{e}{2k}(x_m^A + x_m^B)\right) = 0 \tag{32}$$

$$a - X^B - x_m^B - \left(d + \frac{e}{2k}(x_m^A + x_m^B)\right) = 0 \tag{33}$$

Solving the system of FOCs, one obtains sales in each country, $X^A = X^B = \frac{(a-d)(3+4\tilde{e})-\tilde{t}(1+\tilde{e})}{4+7\tilde{e}+2\tilde{e}^2}$. The multinational firm's sales in each country are $x_m^A = x_m^B = \frac{(a-d)(1+2\tilde{e})+\tilde{t}}{4+7\tilde{e}+2\tilde{e}^2}$. Independent firms' home-market sales exceed foreign-market sales by t, i.e. $\left|x_i^A - x_i^B\right| = t$, $i \in \{a, b\}$, where foreign-market sales $x_a^B = x_b^A = \frac{(a-d)(1+\tilde{e})\left(1-\tilde{t}(3+\tilde{e})\right)}{4+7\tilde{e}+2\tilde{e}^2}$. Equilibrium prices are $p^A = p^B = \frac{a(1+\tilde{e})(1+2\tilde{e})+d(3+4\tilde{e})+t(1+\tilde{e})}{4+7\tilde{e}+2\tilde{e}^2}$. Finally, the reduced-form profit functions are given by:

$$\Pi_{m,r_1} = \frac{(a-d)^2}{(4+7\tilde{e}+2\tilde{e}^2)^2}\left((1+2\tilde{e})^2(2+\tilde{e}) + 2\tilde{t}(1+2\tilde{e})(2+\tilde{e}) + \tilde{t}^2(2+\tilde{e})\right) - 2gk$$

$$\begin{aligned}\Pi_{a,r_1} &= \Pi_{b,r_1} \\ &= \frac{(a-d)^2}{(4+7\tilde{e}+2\tilde{e}^2)^2}\left(2(1+\tilde{e})^3 - 2\tilde{t}(1+\tilde{e})^2(2+\tilde{e}) + \tilde{t}^2(10+32\tilde{e}+35\tilde{e}^2+\frac{29}{2}\tilde{e}^3+2\tilde{e}^4)\right) - gk\end{aligned}$$

(iii) Under r_2, there are no shipments across countries, each multinational firm solving the same problem as in (ii) (FOCs given by (32) and (33)). Equilibrium sales in each country are $X^A = X^B = \frac{2(a-d)}{3+\tilde{e}}$, while $x_m^A = x_m^B = \frac{a-d}{3+\tilde{e}}$; prices are $p^A = p^B = \frac{a(1+\tilde{e})+2d}{3+\tilde{e}}$. The reduced-form profit function for each multinational firm is

$$\Pi_{m,r_2} = \frac{(a-d)^2(2+\tilde{e})}{(3+\tilde{e})^2} - 2gk$$

By Proposition 2, the four merger surplus functions necessary to solve the game can then be computed:

$$\begin{aligned}\Phi_I &= \Pi_{m,r_1} - \Pi_{a,r_0} - \Pi_{b,r_0} \\ &= \frac{(a-d)^2(1-3\tilde{t}-\tilde{t}\tilde{e})\left(16\tilde{t}\tilde{e}^5+148\tilde{t}\tilde{e}^4+508\tilde{t}\tilde{e}^3+4\tilde{e}^3+4\tilde{e}^2+789\tilde{t}\tilde{e}^2+535\tilde{t}\tilde{e}-23\tilde{e}+122\tilde{t}-14\right)}{(5+2\tilde{e})^2(4+7\tilde{e}+2\tilde{e}^2)^2}\end{aligned}$$

$$\begin{aligned}\Phi_{II} &= \Pi_{m,r_2} - \Pi_{a,r_1} - \Pi_{b,r_1} \\ &= \frac{(a-d)^2(1-3\tilde{t}-\tilde{t}\tilde{e})\left(4\tilde{t}\tilde{e}^5+41\tilde{t}\tilde{e}^4+157\tilde{t}\tilde{e}^3+\tilde{e}^3+2\tilde{e}^2+274\tilde{t}\tilde{e}^2+212\tilde{t}\tilde{e}-4\tilde{e}+60\tilde{t}-4\right)}{(3+\tilde{e})^2(4+7\tilde{e}+2\tilde{e}^2)^2}\end{aligned}$$

$$\begin{aligned}\Phi_{III} &= \Pi_{m,r_2} - \Pi_{a,r_0} - \Pi_{b,r_0} \\ &= \frac{(a-d)^2(1-3\tilde{t}-\tilde{t}\tilde{e})\left(4\tilde{t}\tilde{e}^3+33\tilde{t}\tilde{e}^2+89\tilde{t}\tilde{e}+5\tilde{e}+78\tilde{t}+14\right)}{(3+\tilde{e})^2(5+2\tilde{e})^2}\end{aligned}$$

$$\begin{aligned}\Phi_{IV} &= \Pi_{m,r_1} - \Pi_{a,r_1} - \Pi_{b,r_1} \\ &= \frac{(a-d)^2(1-3\tilde{t}-\tilde{t}\tilde{e})\left(4\tilde{t}\tilde{e}^3+17\tilde{t}\tilde{e}^2+19\tilde{t}\tilde{e}-3\tilde{e}+6\tilde{t}-2\right)}{(4+7\tilde{e}+2\tilde{e}^2)^2}\end{aligned}$$

Note that the sign of each Φ-function corresponds to the sign of the polynomial in $\tilde{t}$ and $\tilde{e}$ in the last bracket in the numerator (since $1-3\tilde{t}-\tilde{t}\tilde{e} \geq 0$ in space $\mathcal{P}$ and $1-3\tilde{t}-\tilde{t}\tilde{e} = 0$ only along the boundary $\tilde{t} = \frac{1}{3+\tilde{e}}$).

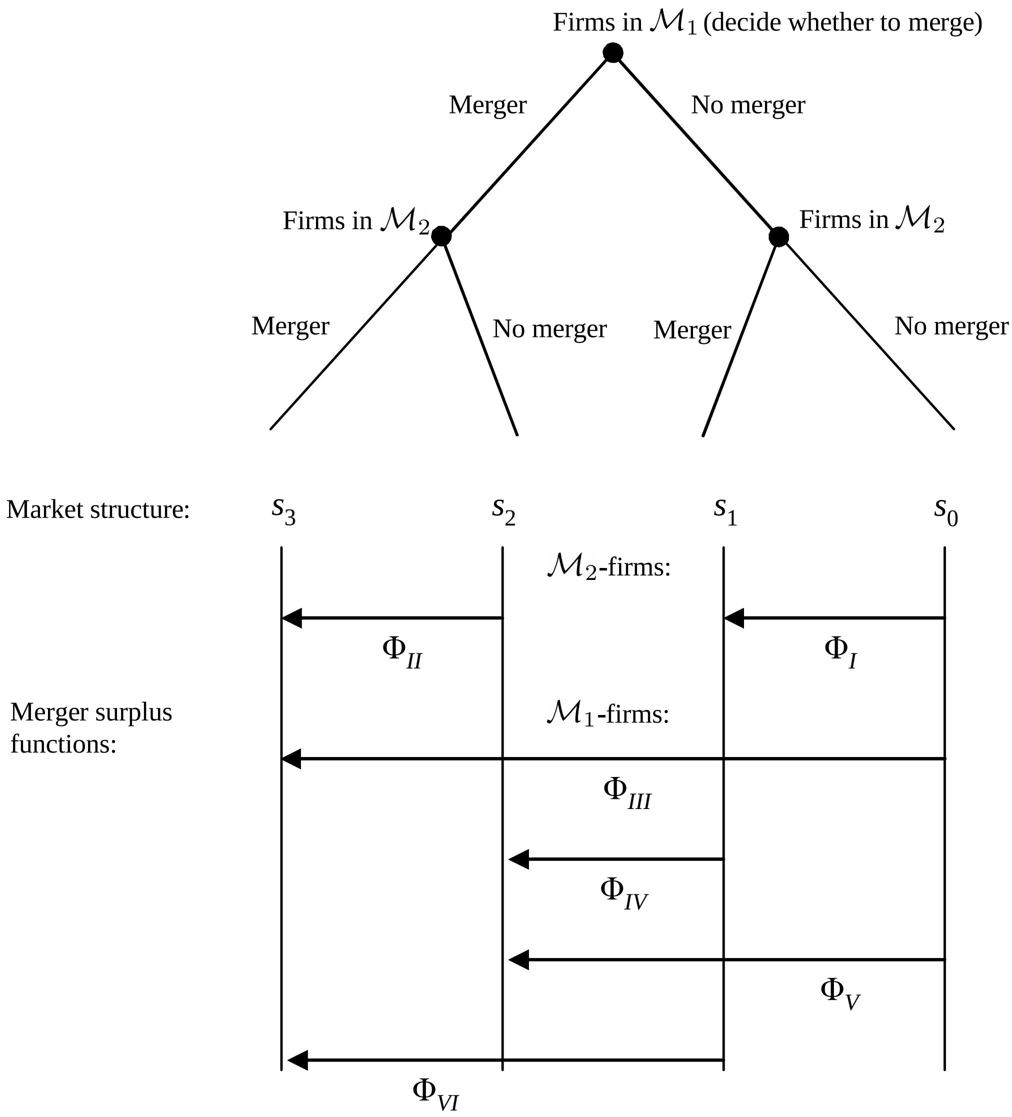

Figure 1: Sequential merger game and merger surplus functions

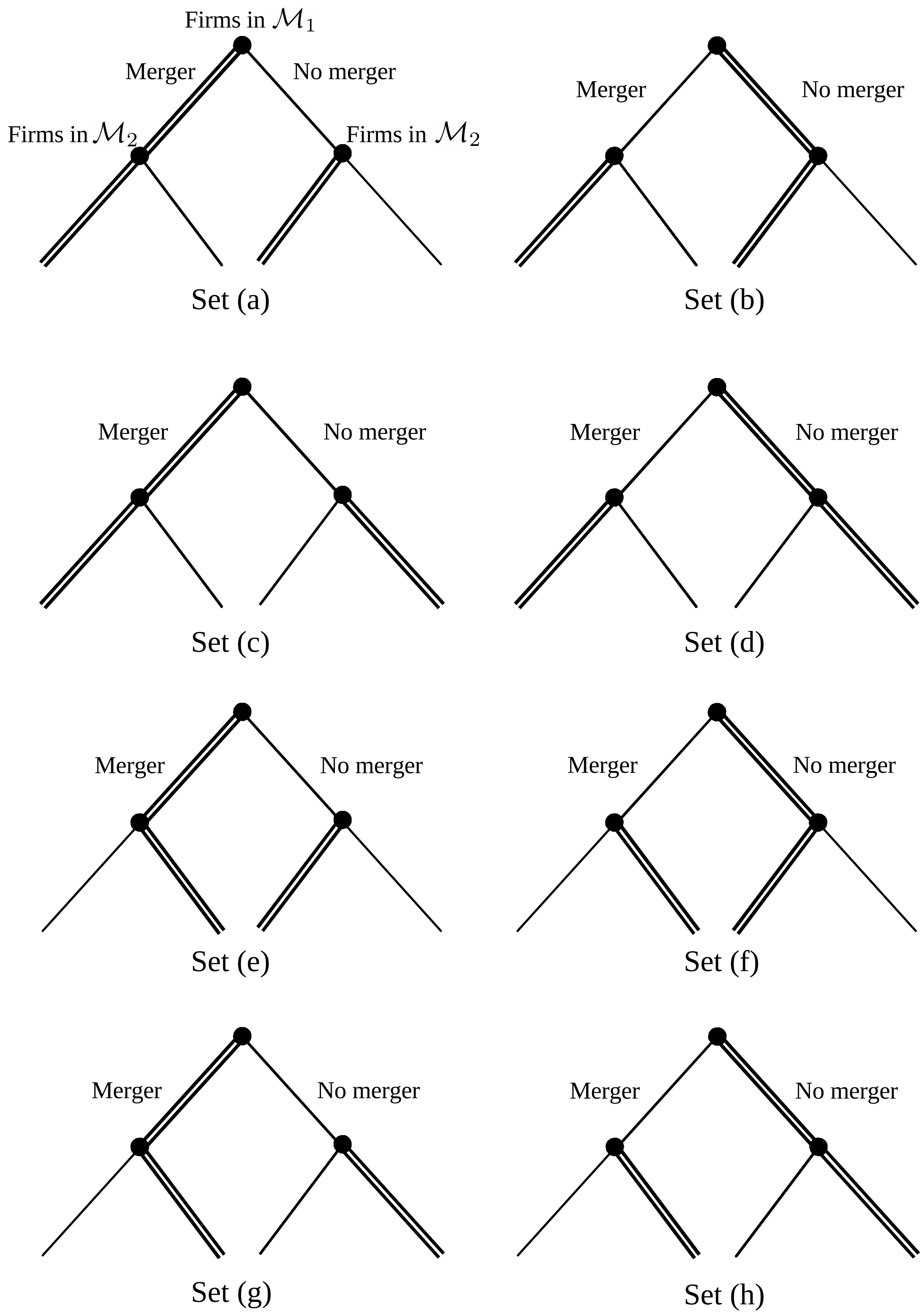

Figure 2: Possible equilibrium sets of Nash strategies. From each node, left depicts "merger" and right depicts "no merger"

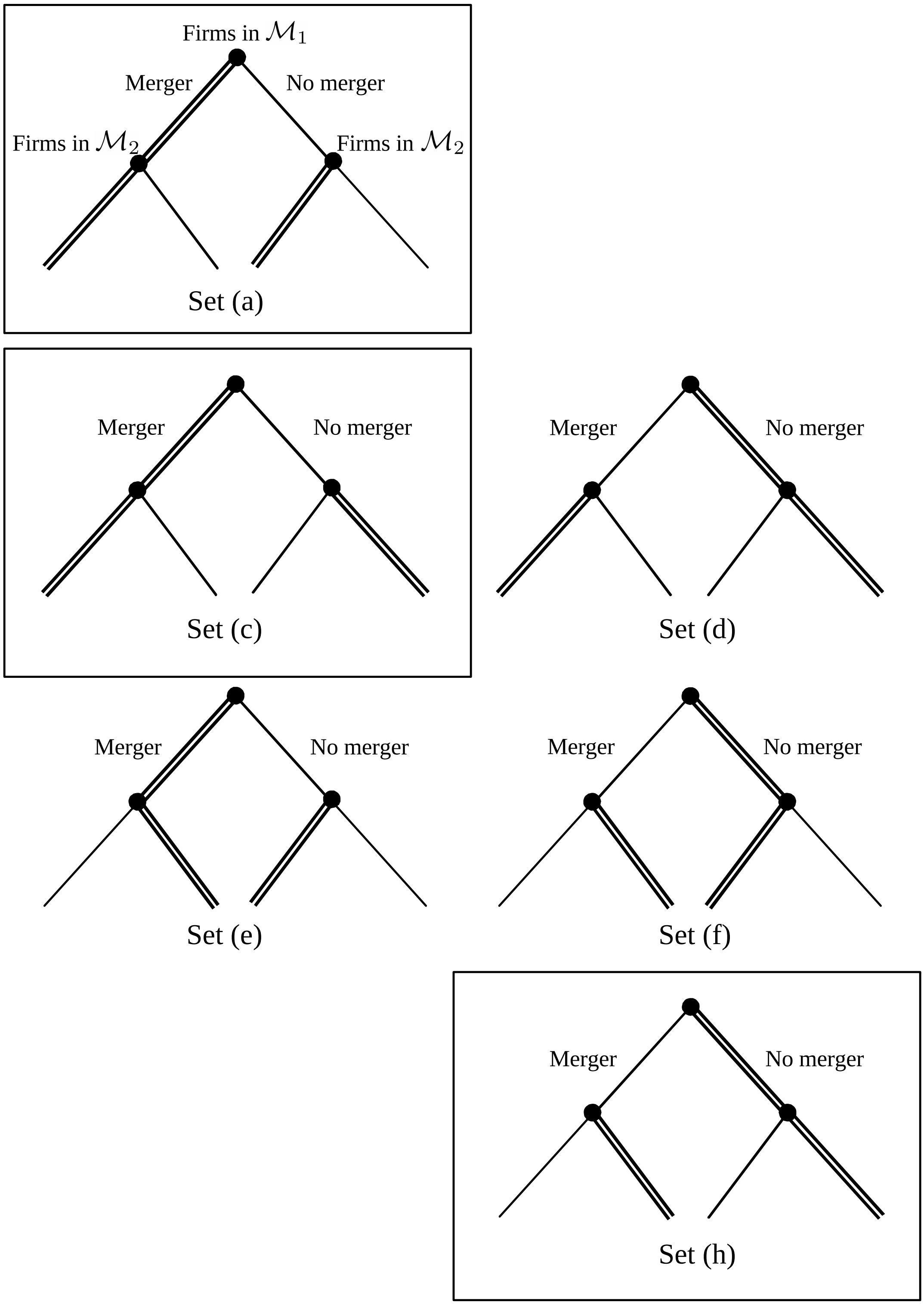

Figure 3: Possible equilibrium sets of strategies in the case of symmetry: all six are possible, three of which survive after the imposition of the Sufficient Condition of Corollary 1 (and are highlighted inside boxes)

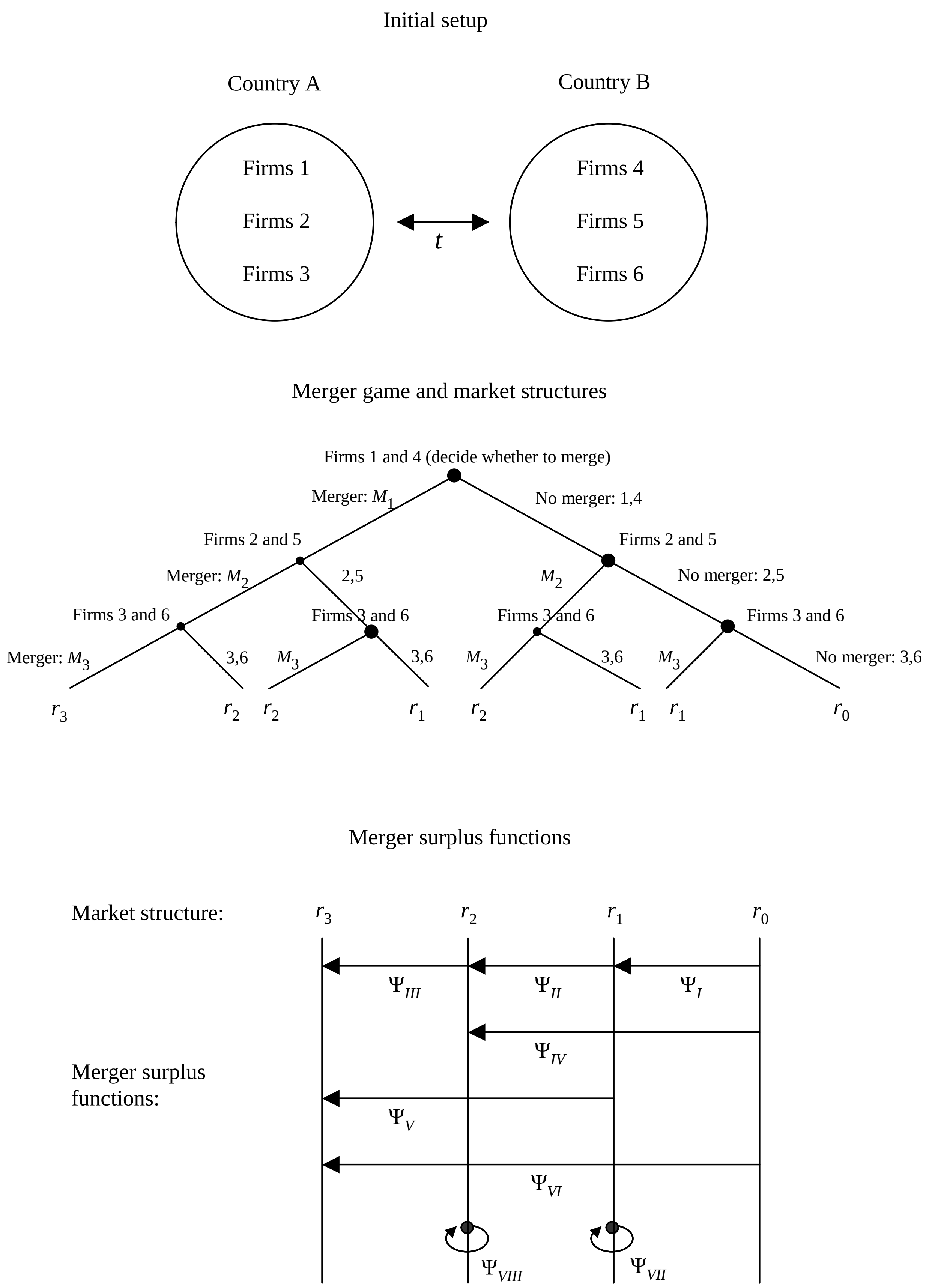

Figure 4: (Example 1) Sequential cross-border merger game and merger surplus functions. (The notation is adapted to reflect three merger stages and considers the symmetry of the model.)

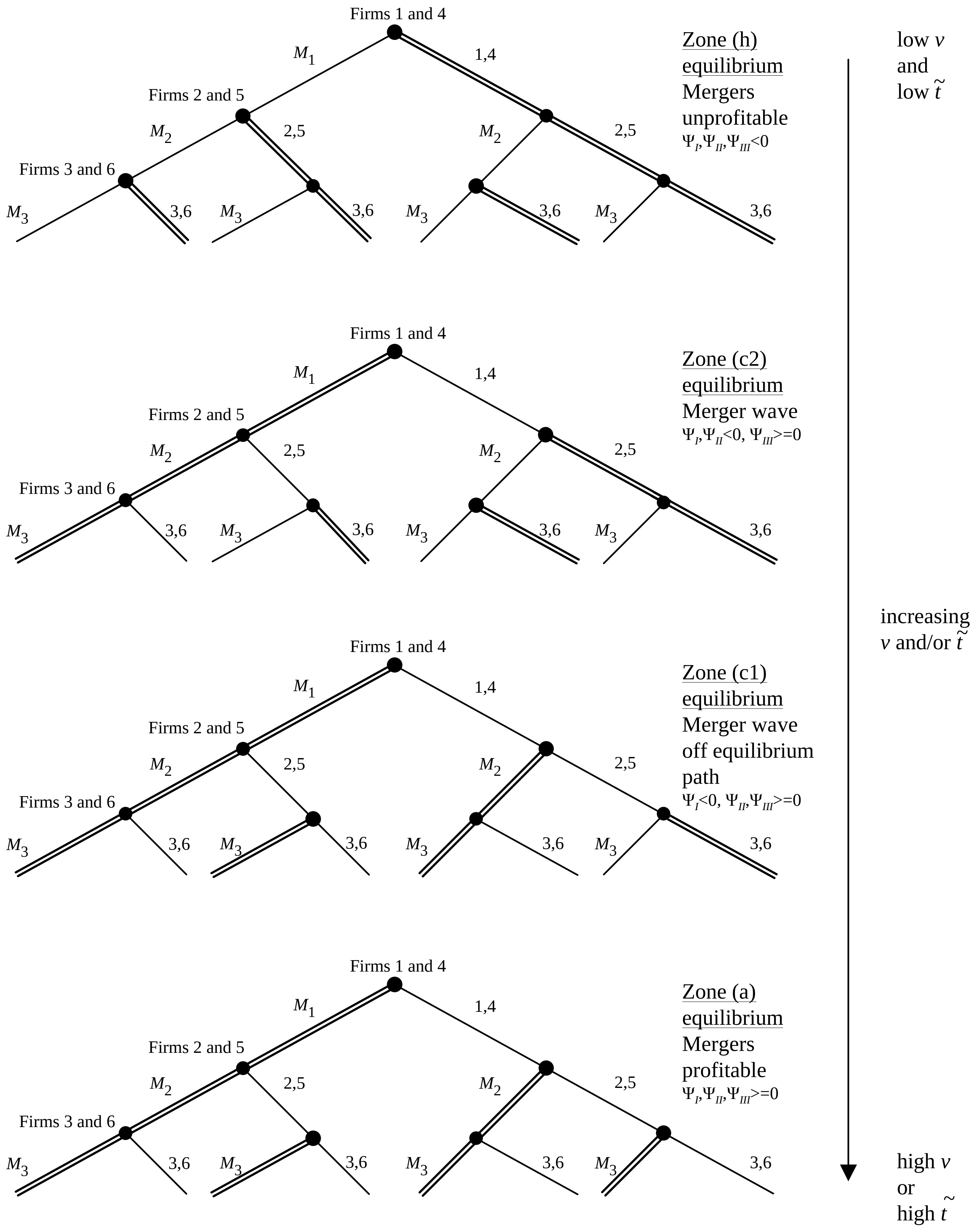

Figure 5: (Example 1) Equilibrium of sequential merger game in each zone; v quality gap, $\tilde{t}$ (normalised) trade cost

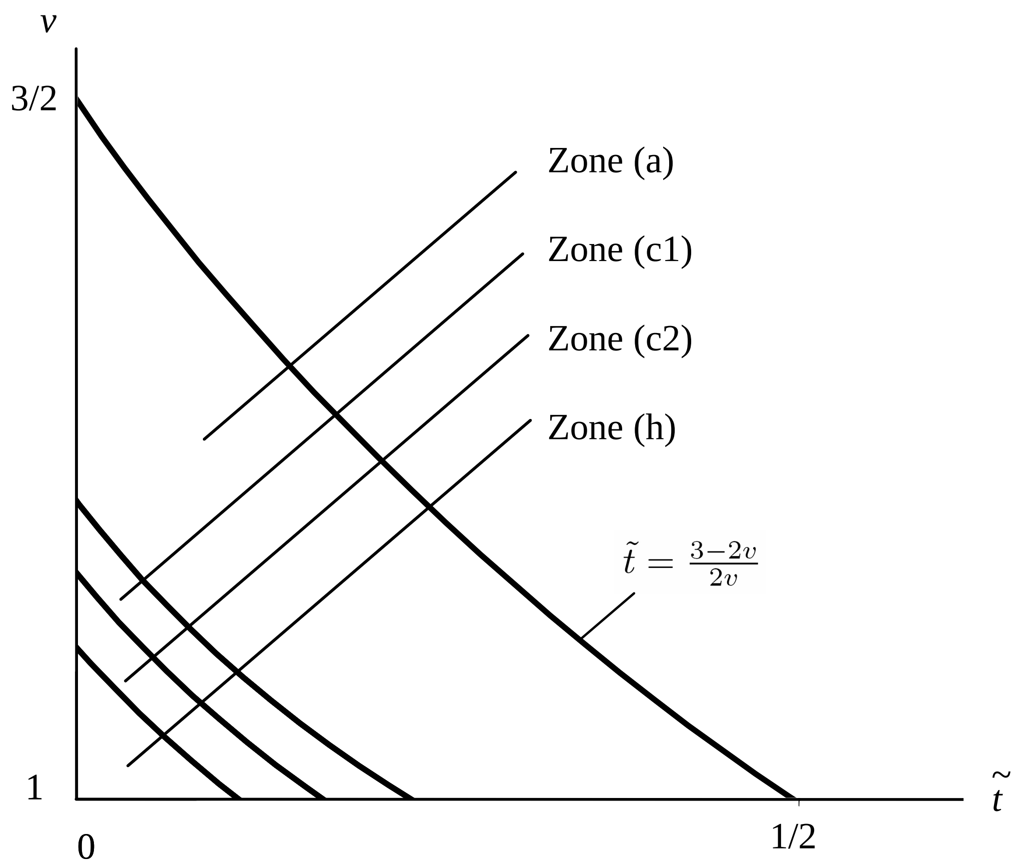

Figure 6: (Example 1) Zones in $\mathcal{P}$ space

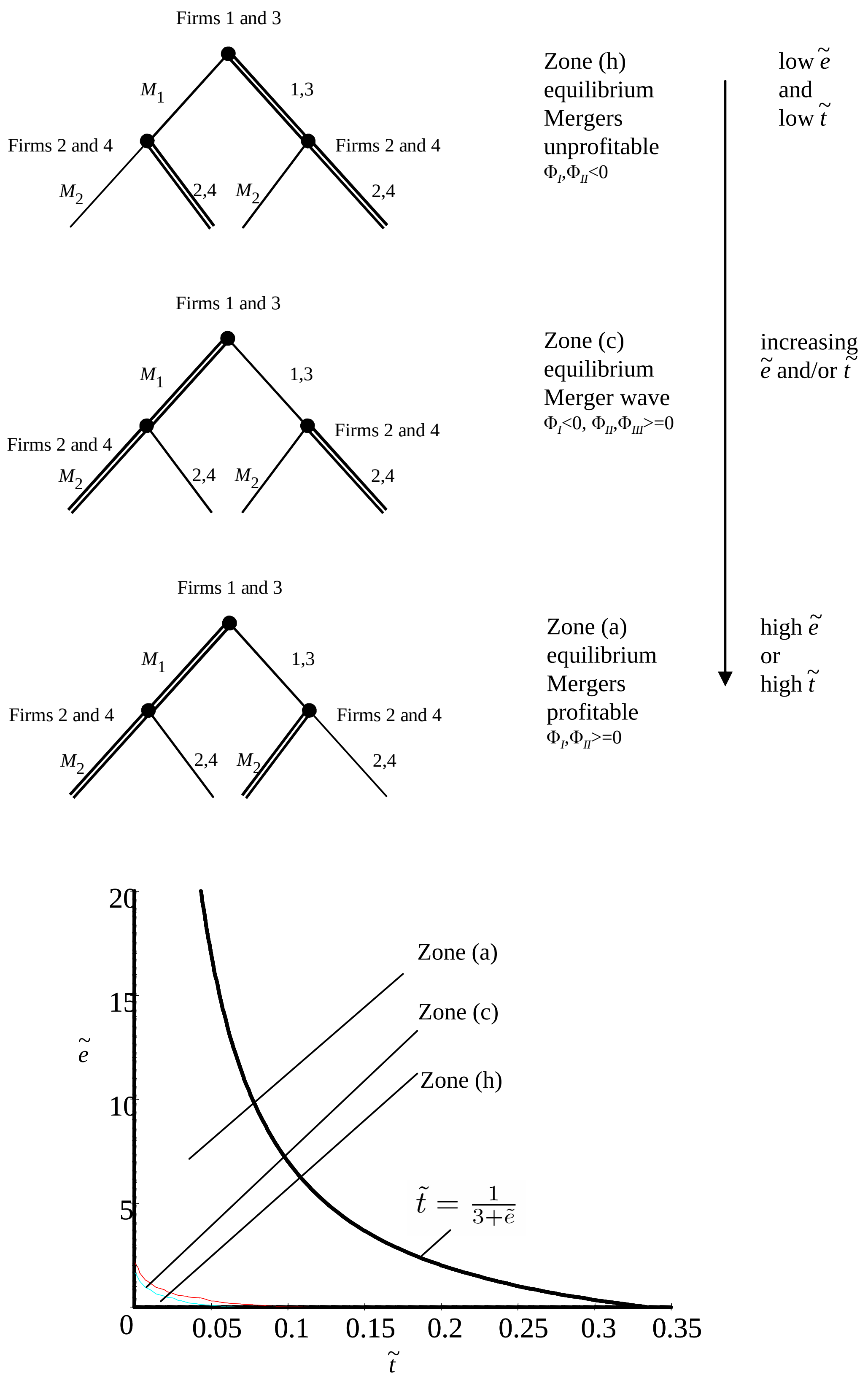

Figure 7: (Example 2) Equilibrium of sequential merger game in each zone and zones in $\mathcal{P}$ space; $\tilde{e}$ rate of change of marginal cost, $\tilde{t}$ trade cost

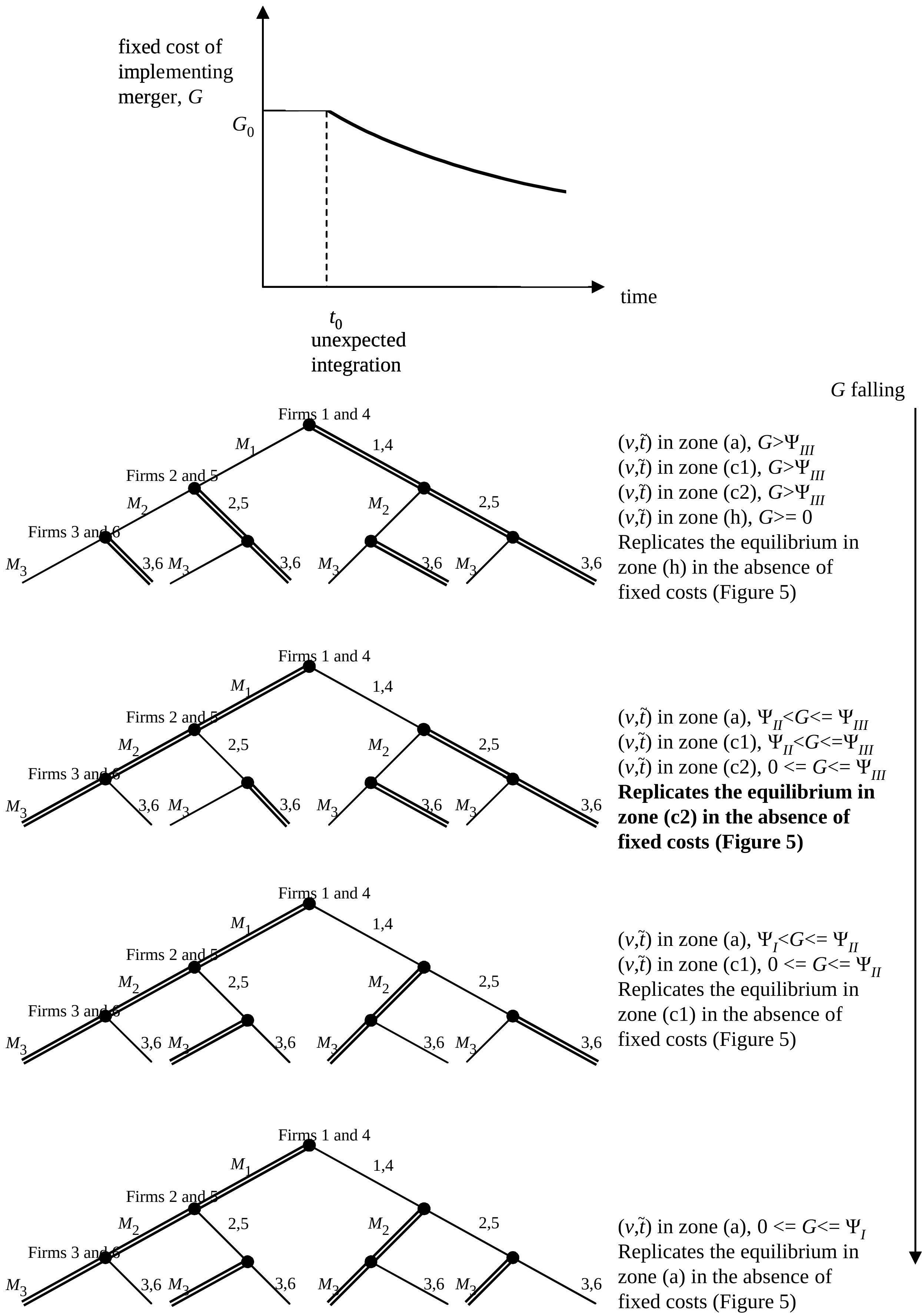

Figure 8: (Example 1) Investment integration and sequential cross-border mergers