Inequality and Envy*

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Abstract

Using a simple axiomatic structure we characterise two classes of inequality indices - absolute and relative - that take into account “envy” in the income distribution. The concept of envy incorporated here concerns the distance of each person’s income from his or her immediately richer neighbour. This is shown to be similar to justice concepts based on income relativities.

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1 Introduction

There is considerable interest in a possible relationship between inequality and envy. However there has been little attempt to incorporate the concept of individual envy directly into the formal analysis of inequality measurement. Of course there is a substantial economics literature that models envy in terms of individual utility – see for example Arnsperger (1994) – but our focus here is different in that we concentrate directly on incomes rather than on utility and commodities. We seek an alternative way of characterising envy broadly within the literature that has formalised related concepts such as deprivation and individual complaints about income distribution. Indeed there is an aspect of envy that can be considered as akin to the notion of a "complaint" that has been used as a basic building block of inequality analysis (Temkin 1993).[1]

A further motivation for our analytical approach can be found in the work of social scientists who have sought to characterise issues of distributive justice in terms of relative rewards. This is sometimes based on a model of individual utility that has as arguments not only one's own income, consumption or performance, but also that of others in the community. A recent example of this approach is the model in Falk and Knell (2004) where a person's utility is increasing in his own income and decreasing in some reference income; Falk and Knell (2004) raise the key question as to what constitutes reference income. Should it be the same for all or relative to each person's income? Should it be upward looking, as in the case of envy? Here we address these issues without explicitly introducing individual utility. Our approach has a connection with the seminal contribution of Merton (1957) who focuses on a proportionate relationship between an individual's income and a reference income characterised in terms of justice: we will show that there is a close relationship between some of the inequality measures developed below and Merton's work.

The paper is organised as follows: section 2 outlines the basic framework within which we develop our analysis; section 3 characterises and examines the properties of a class of absolute inequality measures; section 4 analyses the corresponding class of relative indices.

2 The approach

We assume that the problem is one of evaluating and comparing income distributions in a finite fixed-sized population of at least two members where individual "income" has been defined as a real number, not necessarily positive. Throughout the following we will work with vectors of ordered incomes.

[1] Our methodology is similar to that used in the analysis of poverty (Ebert and Moyes 2002), individual deprivation (Bossert and D'Ambrosio 2006, Yitzhaki 1982) and complaint-inequality (Cowell and Ebert 2004).

2.1 Notation and definitions

Let $\mathbb{D}$ be the set of all logically possible values of income. For different parts of the analysis we will need two different versions of this set, namely $\mathbb{D} = \mathbb{R}$ and $\mathbb{D} = \mathbb{R}_{++}$. An income distribution is a vector

$$\mathbf{x} := (x_1, x_2, ..., x_n) \in \mathbb{D}^n$$

where the components are arranged in ascending order and $n \geq 2$. So the space of all possible income distributions is given by

$$\mathfrak{X}(\mathbb{D}) := \{\mathbf{x} \mid \mathbf{x} \in \mathbb{D}^n \; ; \; x_1 \leq x_2 \leq ... \leq x_n\}.$$

Write $\mathbf{1}_k$ for the k-vector $(1, 1, ..., 1)$ and let $\mathbf{x}(k, \delta)$ denote the vector $\mathbf{x}$ modified by increasing the kth component by δ and decreasing the component $k+1$ by δ:

$$\mathbf{x}(k, \delta) := (x_1, x_2, ..., x_k + \delta, x_{k+1} - \delta, ..., x_n)$$

where $1 < k < n$ and

$$0 < \delta \leq \frac{1}{2}[x_{k+1} - x_k] \tag{1}$$

Definition 1 *An* inequality measure *is a function* J*:* $\mathfrak{X}(\mathbb{D}) \to \mathbb{R}_+$*.*

Definition 2 *For any* k *such that* $1 < k < n$ *and any* $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$ *such that* $x_k < x_{k+1}$*, a* progressive transfer at position k *is a transformation* $\mathbf{x} \mapsto \mathbf{x}(k, \delta)$ *such that (1) is satisfied.*

Note that definition 2 applies the concept of Dalton (1920) to transfers between neighbours. It should also be noted that we describe our envy-related index everywhere as an inequality measure even where we do not insist on the application of the principle of progressive transfers. As is common in the inequality literature we will deal with both absolute and relative approaches to inequality measurement.

2.2 Basic axioms

Our main ethical principle is captured in the following two axioms.

Axiom 1 (Decomposability: nonoverlapping subgroups) *For all* $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$ *and* $1 \leq k \leq n-1$*:*

$$J(\mathbf{x}) = J(x_1, ..., x_k, x_k\mathbf{1}_{n-k}) + J(x_{k+1}\mathbf{1}_k, x_{k+1}, ..., x_n) + J(x_k\mathbf{1}_k, x_{k+1}\mathbf{1}_{n-k}) \tag{2}$$

Axiom 2 (Monotonicity) *For all* $x, y \in \mathbb{D}$ *such that* $x \leq y$ *and for all* $1 \leq k \leq n-1$ *inequality* $J(x\mathbf{1}_k, y\mathbf{1}_{n-k})$ *is increasing in* y*.*

Axiom 1 is fundamental in that it captures the aspect of the income distribution that matters in terms of envy at any position k. It might be seen as analogous to a standard decomposition – total inequality (on the left of 2) equals the sum of inequality in the lower and upper subgroups defined by position k (the first two terms on the right of 2) and a between-group component (last term on the right). However the analogy with conventional decomposition by subgroups is not exact – note for example that the first two terms on the right are not true subgroup-inequality expressions (which would have to have population sizes k and $n-k$ respectively) but are instead modified forms of the whole distribution. Indeed, a better analogy is with the focus axiom in poverty analysis: in the breakdown depicted in (2) we have first the information in the right-censored distribution then that in the left-censored distribution then the information about pure envy at position k.

Axiom 2 has the interpretation that an increase in the pure envy component in (2) must always increase inequality and that this increase is independent of the rest of the income distribution.

To make progress we also need some assumptions that impose further structure on comparisons of income distributions. We will first consider the following two axioms:

Axiom 3 (Translatability) *For all* $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$ *and* $\varepsilon \in \mathbb{R}$*:*

$$J(\mathbf{x} + \varepsilon \mathbf{1}_n) = J(\mathbf{x})$$

Axiom 4 (Linear Homogeneity) *For all* $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$ *and* $\lambda \in \mathbb{R}_{++}$*:*

$$J(\lambda \mathbf{x}) = \lambda J(\mathbf{x})$$

Axioms 3 and 4 are standard in the literature; however, in section 4 we will examine the possibility of replacing these with an alternative structure.

3 Absolute measures

We begin with results for the most general definition of the space of incomes. Here incomes can have any value, positive, zero or negative; i.e. $\mathbb{D} = \mathbb{R}$. We will first characterise the class of measures that is implied by the parsimonious axiomatic structure set out in section 2 and then we will examine this class in the light of the conventional properties with which inequality measures are conventionally endowed.

3.1 Characterisation

To start with let us note that the decomposability assumption implies that J has a convenient property for a distribution displaying perfect equality:

Proposition 1 *Axiom 1 implies that, for all* $x \in \mathbb{D}$*:*

$$J\left(x\mathbf{1}_n\right) = 0 \tag{3}$$

Proof. For an arbitrary integer k such that $1 \leq k \leq n-1$ Axiom 1 implies

$$J\left(x\mathbf{1}_n\right) = 3J\left(x\mathbf{1}_k, x\mathbf{1}_{n-k}\right) = 3J\left(x\mathbf{1}_n\right)$$

But this is only true if (3) holds. ∎

We use this property in the proof of the main result, Proposition 3 below.

Proposition 2 *Axioms 1 and 2 imply that,*

1. *for all* $\mathbf{x} \in \mathfrak{X}\left(\mathbb{D}\right)$ *such that not all components of* $\mathbf{x}$ *are equal:*

$$J\left(\mathbf{x}\right) > 0; \tag{4}$$

2. *for all* $\mathbf{x} \in \mathfrak{X}\left(\mathbb{D}\right)$

$$J\left(\mathbf{x}\right) = \sum_{i=1}^{n-1} K_i\left(x_i, x_{i+1}\right), \tag{5}$$

where each K_i *satisfies the property* $K_i\left(x_i, x_{i+1}\right) > 0$ *if* $x_{i+1} > x_i$*.*

Proof. Applying Axiom 1 in the case $k = 1$ we have

$$J\left(\mathbf{x}\right) = J\left(x_1, x_1\mathbf{1}_{n-1}\right) + J\left(x_2, x_2, x_3, ..., x_n\right) + J\left(x_1, x_2\mathbf{1}_{n-1}\right).$$

So, by Proposition 1, we have

$$J\left(\mathbf{x}\right) = J\left(x_2, x_2, x_3, ..., x_n\right) + J\left(x_1, x_2\mathbf{1}_{n-1}\right) \tag{6}$$

Applying Axiom 1 again to the first term in (6) we obtain

$$J\left(\mathbf{x}\right) = \left[J\left(x_3, x_3, x_3, x_4, ..., x_n\right) + J\left(x_2\mathbf{1}_2, x_3\mathbf{1}_{n-2}\right)\right] + J\left(x_1, x_2\mathbf{1}_{n-1}\right)$$

Repeated application of the same argument gives us

$$\begin{aligned} J\left(\mathbf{x}\right) &= \sum_{i=1}^{n-1} J\left(x_i\mathbf{1}_i, x_{i+1}\mathbf{1}_{n-i}\right) \\ &= \sum_{i=1}^{n-1} K_i\left(x_i, x_{i+1}\right), \end{aligned} \tag{7}$$

where the function K_i is defined such that

$$K_i\left(x, y\right) = J\left(x\mathbf{1}_i, y\mathbf{1}_{n-i}\right). \tag{8}$$

Also note that, for any $x, y \in \mathbb{D}$ such that $y > x$ and $1 \leq k \leq n-1$ Axiom 2 and Proposition 1 together imply

$$J\left(x\mathbf{1}_k, y\mathbf{1}_{n-k}\right) > J\left(x\mathbf{1}_k, x\mathbf{1}_{n-k}\right) = 0 \tag{9}$$

Equations (7)-(9) are sufficient to show that $K_i(x,y) > 0$ if $y > x$. ■

Part 1 of Proposition 2 shows that J satisfies a minimal inequality property; part 2 demonstrates that Axioms 1 and 2 are sufficient to induce an appealing decomposability property.

Proposition 3 *For $\mathbb{D} = \mathbb{R}$ the inequality measure J satisfies axioms 1 to 4*[2] *if and only if there exist weights $\alpha_1, ..., \alpha_{n-1} \in \mathbb{R}_{++}$ such that, for all $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$:*

$$J(\mathbf{x}) = \sum_{i=1}^{n-1} \alpha_i [x_{i+1} - x_i] \tag{10}$$

Proof. The "if" part is immediate.

From the proof of Proposition 2 we know that J must have the form (7). Using Axiom 3 for the distribution $(x\mathbf{1}_i, y\mathbf{1}_{n-i})$ we have

$$\begin{aligned} K_i(x,y) &= J(x\mathbf{1}_i, y\mathbf{1}_{n-i}) \\ &= J(0\times\mathbf{1}_i, [y-x]\mathbf{1}_{n-i}) \\ &= K_i(0, y-x). \end{aligned} \tag{11}$$

Putting $x = 0$ in equation (11) and using axiom 4 it is clear that

$$\begin{aligned} K_i(0,y) &= J(0\times\mathbf{1}_i, y\mathbf{1}_{n-i}) \\ &= yJ(0\times\mathbf{1}_i, \mathbf{1}_{n-i}) \\ &= \alpha_i y, \end{aligned} \tag{12}$$

where $\alpha_i := K_i(0,1)$. Applying axiom 2 to equation (7) we have $\alpha_i > 0$. ■

Let us note that, by rearrangement, of (10) we have the convenient form

$$J(\mathbf{x}) = \alpha_{n-1}x_n + \sum_{i=2}^{n-1} [\alpha_{i-1} - \alpha_i] x_i - \alpha_1 x_1 \tag{13}$$

This weighted-additive structure is useful for clarifying the distributive properties of the index J.

3.2 Properties of the J-class

To give shape to the class of measures found in Proposition 3 we need to introduce some extra distributive principles. The following axiom may be stated in weak or strict form for each position k where $1 \leq k \leq n-1$:

Axiom 5 (Position-k Monotonicity) *For all $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$ such that $x_k < x_{k+1}$ inequality $J(\mathbf{x})$ is decreasing or constant in x_k.*

[2] If $n = 2$ then Axiom 1 is not required.

However, the progressive-transfers axiom requires a slightly tighter choice of k:

Axiom 6 (Progressive transfers) *For any k such that $1 < k < n$ and any $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$ such that $x_k < x_{k+1}$, a progressive transfer at position k implies*

$$J(\mathbf{x}(k, \delta)) \leq J(\mathbf{x}). \tag{14}$$

These two axioms have some interesting implications for the structure of the inequality measure J. However their properties are independent and in each case there is an argument for considering the axiom in strong or a weak form. In the light of this there are a number of special cases that may appear to be ethically attractive, including the following:

- Strong position-k monotonicity only – inequality is strictly decreasing in x_k for all positions k.
- Strong progressive transfers – the "$<$" part in (14) is true for all positions k.
- Indifference across positions – inequality is constant in x_k for all positions k in the statement of Axiom 5. Indifference clearly implies that the "$=$" part in (14) is true.

Imposition of one or other form of Axioms 5 and 6 will have implications for the structure of the positional weights $\{\alpha_k\}$. First, it is clear from (13) that Axiom 5 implies

$$0 < \alpha_1 \leq ... \leq \alpha_k \leq \alpha_{k+1} \leq ... \leq \alpha_{n-1}. \tag{15}$$

since we have $\alpha_1 > 0$ in view of Proposition 3. So, increasing the poorest person's income always reduces J-inequality. Second if we adopt the position of indifference across positions in Axiom 5 then

$$0 < \alpha_1 = ... = \alpha_k = \alpha_{k+1} = ... = \alpha_{n-1}.$$

In this case it is clear from (13) that the inequality measure becomes just a multiple of the range.[3] Third, an important property follows directly from Axiom 6 alone:

Proposition 4 *Given the conditions of proposition 3, imposition of the principle of progressive transfers at each position k, $k = 2, ..., n-1$ implies that the weights α_k in (10) can be written*

$$\alpha_k = \varphi(k)$$

where φ is a concave function.

[3] We have to say "a multiple of" because we have not introduced a normalisation axiom to fix, say, $\alpha_1 = 1$.

Proof. Applying Axiom 6 to equation (13) we get

$$J(\mathbf{x}) - J(\mathbf{x}(k,\delta)) = [\alpha_{k-1} - \alpha_k]\,\delta - [\alpha_k - \alpha_{k+1}]\,\delta \geq 0$$

Given that $\delta > 0$ this implies

$$\alpha_k \geq \frac{\alpha_{k-1} + \alpha_{k+1}}{2}$$

which establishes concavity. ■

4 Relative measures

The discussion in section 3 is essentially "absolutist" in nature – the translatability property (Axiom 3) ensures this. Here we look at the possibility of a "relativist" approach to characterising an envy-regarding index.

4.1 Characterisation

In this case we have to deal with a restricted domain: $\mathbb{D}$ can consist only of positive numbers ($\mathbb{D} = \mathbb{R}_{++}$) and we impose the following axioms:

Axiom 7 (Zero Homogeneity) *For all* $\mathbf{x} \in \mathfrak{X}(\mathbb{R}_{++})$ *and* $\lambda \in \mathbb{R}_{++}$*:*

$$J(\lambda\mathbf{x}) = J(\mathbf{x})$$

Axiom 8 (Transformation) *For all* $\mathbf{x} \in \mathfrak{X}(\mathbb{R}_{++})$ *and* $\varepsilon \in \mathbb{R}_{++}$*:*

$$J(\mathbf{x}^{\varepsilon}) = \varepsilon J(\mathbf{x})$$

where

$$\mathbf{x}^{\varepsilon} := (x_1^{\varepsilon}, x_2^{\varepsilon}, ..., x_n^{\varepsilon})$$

Axioms 7 and 8 replace axioms 3 and 4 now that the definition of $\mathbb{D}$ is changed from $\mathbb{R}$ to $\mathbb{R}_{++}$. This enables us to introduce a modified characterisation result:

Proposition 5 *For* $\mathbb{D} = \mathbb{R}_{++}$ *the inequality measure* J *satisfies axioms 1, 2, 7 and 8 if and only if there exist weights* $\alpha_1, ..., \alpha_{n-1} \in \mathbb{R}_{++}$ *such that, for all* $\mathbf{x} \in \mathfrak{X}(\mathbb{D})$*:*

$$J(\mathbf{x}) = \sum_{i=1}^{n-1} \alpha_i [\ln x_{i+1} - \ln x_i] \tag{16}$$

Proof. For any $\mathbf{y} \in \mathfrak{X}(\mathbb{R})$ let

$$\mathbf{x} := (\mathrm{e}^{y_1}, \mathrm{e}^{y_2}, ..., \mathrm{e}^{y_n}) \tag{17}$$

and consider a function $\hat{J}$ defined as

$$\hat{J}(\mathbf{y}) := J(\mathbf{x})$$

with $\mathbf{x}$ given by (17). Clearly $\hat{J}$ satisfies decomposition and monotonicity (axioms 1 and 2). Also, given (17), for any $\lambda \in \mathbb{R}_{++}$ we have

$$\mathbf{x}^{\lambda} = \left(\mathrm{e}^{\lambda y_1}, \mathrm{e}^{\lambda y_2}, ..., \mathrm{e}^{\lambda y_n}\right) \tag{18}$$

and so

$$\hat{J}(\lambda\mathbf{y}) = J\left(\mathbf{x}^{\lambda}\right) = \lambda J(\mathbf{x})$$

by axiom 8; hence $\hat{J}(\lambda\mathbf{y}) = \lambda\hat{J}(\mathbf{y})$. Furthermore, given (17), for any $\varepsilon \in \mathbb{R}$ we have

$$\mathrm{e}^{\varepsilon}\mathbf{x} = \left(\mathrm{e}^{y_1+\varepsilon}, \mathrm{e}^{y_2+\varepsilon}, ..., \mathrm{e}^{y_n+\varepsilon}\right) \tag{19}$$

and so

$$\hat{J}(\mathbf{y} + \varepsilon\mathbf{1}_n) = J(\mathrm{e}^{\varepsilon}\mathbf{x}) = J(\mathbf{x})$$

by axiom 7; hence $\hat{J}(\mathbf{y} + \varepsilon\mathbf{1}_n) = \hat{J}(\mathbf{y})$. Therefore if J satisfies axioms 1, 2, 7 and 8 then $\hat{J}$ satisfies axioms 1 to 4 on $\mathfrak{X}(\mathbb{D})$ for $\mathbb{D} = \mathbb{R}$. Using proposition 3 we have therefore

$$\hat{J}(\mathbf{y}) = \sum_{i=1}^{n-1} \alpha_i \left[y_{i+1} - y_i\right] \tag{20}$$

with $\alpha_1, ..., \alpha_{n-1} \in \mathbb{R}_{++}$. Using the transformation (17) in equation (20) gives the result. ■

4.2 Properties

The properties of J are similar to those established in section 3.2. Clearly

$$J(\mathbf{x}) = \alpha_{n-1}\ln x_n + \sum_{i=2}^{n-1}\left[\alpha_{i-1} - \alpha_i\right]\ln x_i - \alpha_1 \ln x_1 \tag{21}$$

and position-k monotonicity (Axiom 5) again implies condition (15) and the corollaries of this condition still apply. The counterpart of Proposition 4 is as follows.

Proposition 6 *Given the conditions of proposition 5, imposition of the principle of progressive transfers at each position k, $k = 2, ..., n-1$ implies that the weights α_k in (16) can be written*

$$\alpha_k = \varphi(k)$$

where φ is a concave function.

Proof. If we have a progressive transfer at position k then, from equation (21) the reduction in inequality is given by

$$\begin{aligned} J(\mathbf{x}) - J(\mathbf{x}(k,\delta)) &= [\alpha_k - \alpha_{k+1}]\ln x_{k+1} + [\alpha_{k-1} - \alpha_k]\ln x_k \\ &\quad - [\alpha_k - \alpha_{k+1}]\ln(x_{k+1} - \delta) - [\alpha_{k-1} - \alpha_k]\ln(x_k + \delta) \\ &= [\alpha_{k+1} - \alpha_k]\ln\left(1 - \frac{\delta}{x_{k+1}}\right) + [\alpha_k - \alpha_{k-1}]\ln\left(1 + \frac{\delta}{x_k}\right) \end{aligned}$$

Expanding the last line of this expression we get

$$[\alpha_{k+1} - \alpha_k] \left[-\frac{\delta}{x_{k+1}} - \frac{1}{2} \left[\frac{\delta}{x_{k+1}} \right]^2 - ... \right] + [\alpha_k - \alpha_{k-1}] \left[\frac{\delta}{x_k} - \frac{1}{2} \left[\frac{\delta}{x_k} \right]^2 + ... \right]$$

which, neglecting second order and higher terms for small δ, gives

$$J(\mathbf{x}) - J(\mathbf{x}(k,\delta)) \simeq [\alpha_k - \alpha_{k-1}] \frac{\delta}{x_k} - [\alpha_{k+1} - \alpha_k] \frac{\delta}{x_{k+1}} \tag{22}$$

Applying Axiom 6 expression (22) must be non-negative which, given that $\delta > 0$, implies

$$\alpha_k [x_{k+1} + x_k] \geq \alpha_{k-1} x_{k+1} + \alpha_{k+1} x_k \tag{23}$$

Defining

$$\theta := \frac{x_{k+1}}{x_{k+1} + x_k},$$

condition (23) becomes

$$\alpha_k \geq \theta \alpha_{k-1} + [1 - \theta] \alpha_{k+1}$$

where $\frac{1}{2} \leq \theta < 1$, which is sufficient to establish concavity. ■

5 Discussion

As we noted in the introduction, an important application of the relative indices developed here is the formalisation of Merton's index, which is based on a sum of "justice evaluations." An individual's justice evaluation is given by

$$\ln \left(\frac{A}{C} \right) \tag{24}$$

where A is the actual amount reward and C is the just reward – see also Jasso (2000), page 338. Since we are concerned with inequality (and its counterpart distributive injustice) it makes sense to consider the inverse of $\frac{A}{C}$. If the just reward for individual i is an immediately upward-looking concept then $A = x_i$ and $C = x_{i+1}$ and we should focus on

$$\ln \left(\frac{C}{A} \right) = \ln x_{i+1} - \ln x_i, \tag{25}$$

which is exactly the form that we have in (16).

Finally note that the indices derived here, although based on a set of axioms that might appear similar to those used in conventional inequality analysis are fundamentally different from those associated with conventional non-overlapping decomposable inequality indices (Ebert 1988). Instead the measures (10) and (16) capture a type of "keeping-up-with-the-Joneses" form of envy.

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