A PRACTICAL SHORT-RUN APPROACH TO MARKET EQUILIBRIUM

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Abstract

The "short-run approach" calculates long-run producer optima and general equilibria by building on short-run solutions to the producer's profit maximization problem and on profit-based valuation of the fixed inputs. We outline this method and illustrate it on an example of peak-load pricing.

Keywords: general equilibrium, fixed-input valuation, nondifferentiable joint costs, Wong-Viner Envelope Theorem, peak-load pricing.

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1 Introduction

The "short-run approach" is a scheme for calculating long-run producer optima and general equilibria by building on short-run solutions to the producer's profit maximization problem, in which the capital inputs and natural resources are treated as fixed. These fixed inputs are valued at their marginal contributions to the operating profits and, where possible, their levels are then adjusted accordingly. Since short-run profit is a concave but generally nondifferentiable function of the fixed inputs, their marginal values are defined as nonunique supergradient vectors.[1] Also, they usually have to be obtained as solutions to the dual programme of fixed-input valuation because there is rarely an explicit formula for the operating profit: for examples of this use of duality, see [4] and [7].[2] Therefore, a general framework for the short-run approach requires the use of subdifferential calculus and duality of convex programmes. This is fully set up in [6]. Here, we illustrate the short-run approach on an example in which the operating profit function can be calculated and differentiated directly.

The short-run approach starts with fixing the producer's capacities k and optimizing the variable quantities, viz., the outputs y and the variable inputs v. For a competitive, price-taking producer, the optimum quantities, $\hat{y}$ and $\hat{v}$, depend on their given prices, p and w, as well as on k. The primal solution ($\hat{y}$ and $\hat{v}$) is associated with the dual solution $\hat{r}$, which gives the unit values of the fixed inputs (with $\hat{r} \cdot k$ as their total value); the optima are, for the moment, taken to be unique for simplicity. When the goal is limited to finding the producer's long-run profit maxima, it can be achieved by part-inverting the short-run solution map of (p, k, w) to $(y, -v; r)$ so that the prices (p, r, w) are mapped to the quantities $(y, -k, -v)$. This is done by solving the equation $\hat{r}(p, k, w) = r$ for k and substituting any solution into $\hat{y}(p, k, w)$ and $\hat{v}(p, k, w)$ to complete a long-run profit-maximizing input-output bundle. Such a bundle may be unique but only up to scale if the returns to scale are constant (making $\hat{r}$ homogeneous of degree zero in k).

Even within the confines of the producer problem, this approach saves effort by building on the short-run solutions that have to be found anyway: the problems of plant operation and plant valuation are of central practical interest and always have to be tackled by producers. But the short-run approach is even more important as a practical method for calculating market equilibria. For this, with the input prices r and w taken as fixed for simplicity, the short-run profit-maximizing supply $\hat{y}(p, k, w)$ is equated to the demand for the products $\hat{x}(p)$ to determine the short-run equilibrium output prices $p^{\star}_{\mathrm{SR}}(k, w)$. The capacity values $\hat{r}$, calculated at the equilibrium prices $p^{\star}_{\mathrm{SR}}(k, w)$ with the given k and w,

[1] The set of all supergradients, a.k.a. the superdifferential, of a concave function Π is defined by $\widehat{\partial}\Pi := -\partial(-\Pi)$, where ∂ denotes the subdifferential of a convex function. For subdifferential calculus, see, e.g., [9].

[2] In the two examples, the short-run profit *is* differentiable with respect to the fixed inputs (k) if the output price for the time-differentiated output good (p) is a continuous function of time. This condition is verified in [5] for the market equilibrium price.

are only then equated to the given capacity prices r to determine the long-run equilibrium capacities $k^{\star}(r,w)$, and hence also the long-run equilibrium output prices and quantities (by substituting $k^{\star}$ in the short-run equilibrium solution).[3] In other words, the determination of investment is postponed until *after* the equilibrium in the product markets has been found: the producer's long-run problem is split into two—that of operation and that of investment—and the equilibrium problem is "inserted" in-between. Since the operating solutions usually have relatively simple forms, doing things in this order can greatly ease the fixed-point problem of solving for equilibrium: in our example the problem is elementary if approached in this way. Furthermore, unlike the optimal investment of the pure producer problem, the equilibrium investment $k^{\star}$ has a definite scale (determined by demand for the products). Put another way: $\hat{r}\left(p^{\star}_{\mathrm{SR}}(k,w),k,w\right)$, the value to be equated to r, is *not* homogeneous of degree zero in k like $\hat{r}(p,k,w)$. Thus one can keep mostly to single-valued maps and avoid dealing with multi-valued correspondences. And finally, like the short-run producer optimum, the short-run market equilibrium is of interest in itself.

This approach to long-run market equilibrium is illustrated in Figure 1, on the assumption that iterative methods are used to solve the demand-supply equation for p and the price-value equation for k (which correspond to the inner and the outer loops in Figure 1).

As an easy but instructive introduction to this method, we rehearse Boiteux's treatment of the simplest peak-load pricing problem, viz., the problem of pricing the services of a homogeneous capacity that produces a nonstorable good with cyclic demands, such as electricity. A direct calculation of the long-run equilibrium poses a fixed-point problem but, with cross-price independent demands, the short-run equilibrium can be obtained by intersecting the supply and demand curves for each time instant separately, and the long-run equilibrium is found by adjusting the capacity so that its unit cost equals the unit operating profit (which is the total capacity charge over the cycle). With discretized time, this solution is given in [1, 3.2–3.3];[4] we give its continuous-time version. Elsewhere, in [6], we develop this idea into a frame for the analysis of investment and pricing by any industry that supplies a range of commodities—such as a good differentiated over time, locations or events—and we apply it to augment the rudimentary one-station model to a full-blown continuous-time equilibrium model of electricity pricing with a diverse technology, including energy storage and hydro as well as thermal generation, and with a

[3] The short-run approach to equilibrium might also be based on short-run cost minimisation, in which not only the capital inputs (k) but also the outputs (y) are kept fixed and are shadow-priced in the dual problem, but such cost-based calculations are usually much more complicated than those using profit maximisation: see [6] for a comparison.

[4] Boiteux's work is also presented by Drèze [2, pp. 10–16], but the short-run character of the approach is more evident in the original [1, 3.2–3.3] because Boiteux discusses the short-run equilibrium first, before using it as part of the long-run equilibrium system. Drèze mentions the short-run equilibrium on its own only as an afterthought [2, p. 16].

general, cross-price dependent demand.

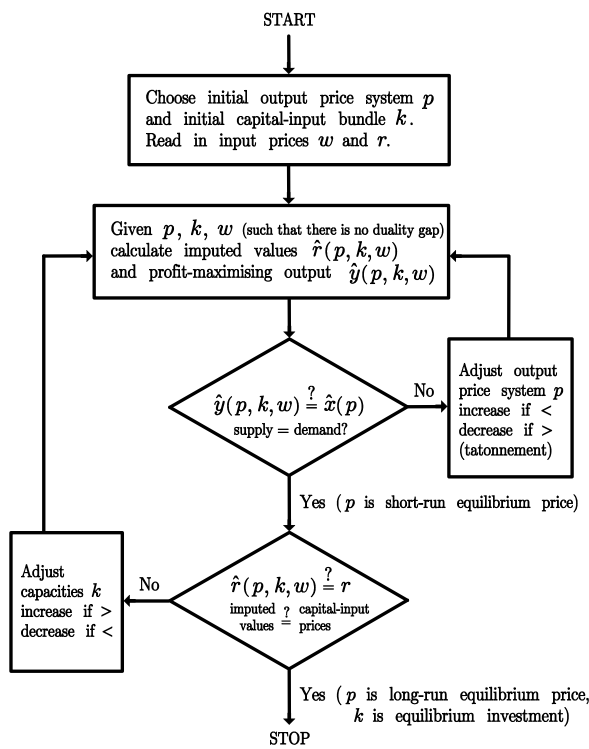

Figure 1: Flow chart for iterative implementation of the short-run approach to long-run market equilibrium. For simplicity, all input prices are assumed to be fixed (in terms of the numeraire).

2 Peak-load pricing with cross-price independent demands

Consider the problem of pricing, over the demand cycle, the services of a homogeneous productive capacity with a unit capital cost r and a unit running cost w. The technology can be interpreted as, e.g., electricity generation from a single type of thermal station with a fuel cost w (in \$/kWh) and a capacity cost r (in \$/kW) per period. The cycle is represented by a continuous time interval $[0, T]$. Demand for the time-differentiated, nonstorable product, $D_t(\mathsf{p})$, is assumed to depend only on the time t and the current price p. As a result, the short-run equilibrium can be found separately at each instant t, by intersecting the demand and supply curves in the price-quantity plane. This is because, with this technology, short-run supply is cross-price independent: given a capacity k, the supply is

$$S(\mathsf{p},k,w)=\begin{cases}0 & \text{for } \mathsf{p}<w\\ [0,k] & \text{for } \mathsf{p}=w\\ k & \text{for } \mathsf{p}>w\end{cases} \tag{1}$$

where p is the current price. That is, given a time-of-use (TOU) tariff p (i.e., given a price $p(t)$ at each time t), a profit-maximizing output trajectory is any selection from the correspondence $t \mapsto S(p(t),k,w)$. When $D_t(w) > k$, the short-run equilibrium TOU price, $p^\star_{\mathrm{SR}}(t,k,w)$, exceeds w by whatever is required to bring the demand down to k (Figure 2a). The total premium over the cycle is the unit operating profit, which in the long run should equal the unit capacity cost r—i.e., the long-run equilibrium capacity, $k^\star(r,w)$, can be determined by solving for k the equation

$$r=\int_0^T \left(p^\star_{\mathrm{SR}}(t,k,w)-w\right)^+ \mathrm{d}t \tag{2}$$

where $\pi^+=\max\{\pi,0\}$ is the nonnegative part of π (i.e., by equating to r the shaded area in Figure 2b). Put into the short-run equilibrium price function, the equilibrium capacity gives the long-run equilibrium price

$$p^\star_{\mathrm{LR}}(t;r,w)=p^\star_{\mathrm{SR}}(t,k^\star(r,w),w). \tag{3}$$

For comparison, to calculate the long-run equilibrium directly requires timing the capacity charges so that they are borne entirely by the resulting demand peaks—i.e., it requires finding a density function $\gamma \geq 0$ such that

$$\int_0^T \gamma(t)\,\mathrm{d}t=1 \quad \text{and if } \gamma(t)>0 \text{ then } y(t)=\sup_\tau y(\tau) \tag{4}$$
$$\text{where:}\quad y(t)=D_t(p(t)) \text{ and } \quad p(t)=w+r\gamma(t).$$

This poses a fixed-point problem that, unlike the short-run approach, is not much simplified by cross-price independence of demands.[5]

Since the operating profit is $\Pi_{\mathrm{SR}}(p,k,w)=k\int_0^T(p(t)-w)^+\mathrm{d}t$, the break-even condition (2) can be rewritten as $r=\partial\Pi_{\mathrm{SR}}/\partial k$, i.e., it can be viewed as equating the capital input's price to its profit-imputed marginal value. This is the first-order necessary and sufficient condition for a profit-maximizing choice of the investment k: together with a choice of output y that maximizes the short-run profit (SRP), such a choice of k maximizes the long-run profit (LRP), and thus turns the short-run equilibrium into the long-run equilibrium.

[5] In terms of the subdifferential, ∂C, of the long-run cost (5) as a function of output, the fixed-point problem is to find a function p such that $p \in \partial C_{\mathrm{LR}}(D(p))$, where $D(p)(t)=D_t(p(t))$ if demands are cross-price independent.

When the producer is a public utility, competitive profit maximization takes usually the form of marginal-cost pricing. In this context, the equality $r = \partial \Pi_{\mathrm{SR}} / \partial k$, or $r = \nabla_k \Pi_{\mathrm{SR}}$ when there is more than one type of capacity, guarantees that an SRMC price system is actually an LRMC. The result applies to any convex technology—even when the costs are nondifferentiable, and the marginal cost has to be defined by using the subdifferential as a generalized, multi-valued derivative. This is so in the above example of capacity pricing, since the long-run cost

$$C_{\mathrm{LR}}(y(\cdot), r, w) = w \int_0^T y(t)\, \mathrm{d}t + r \sup_{t \in [0,T]} y(t) \tag{5}$$

is nondifferentiable if the output y has multiple peaks: indeed, for every γ satisfying (4), the function $p = w + r\gamma$ represents a subgradient of C_{LR} with respect to y. The function γ is the density of a nonunique distribution of the total capacity charge (r) over the multiple peaks. And multiple peaks are more of a rule than an exception in equilibrium (note the peak output plateau in Figure 2d here, and see [3] for an extension to the case of cross-price dependent demands). Similarly, the short-run cost

$$C_{\mathrm{SR}}(y(\cdot), k, w) = \begin{cases} w \int_0^T y(t)\, \mathrm{d}t & \text{if } 0 \le y \le k \\ +\infty & \text{otherwise} \end{cases} \tag{6}$$

is nondifferentiable if $\sup_t y(t) = k$. At a time t when $y(t) = k$, an instantaneous SRMC is the sum of the unit operating cost (w) and an indeterminate capacity premium. In Figure 2a, the nondifferentiability shows in the (infinite) vertical interval $[w, +\infty)$ that represents the multi-valued instantaneous SRMC at $\mathsf{y} = k$.[6] In Figure 2c, it shows as a kink, at $\mathsf{y} = k$, in the graph of the instantaneous cost function

$$c_{\mathrm{SR}}(\mathsf{y}) = \begin{cases} w\mathsf{y} & \text{if } 0 \le \mathsf{y} \le k \\ +\infty & \text{otherwise} \end{cases} \tag{7}$$

(which gives $C_{\mathrm{SR}}(y)$ as $\int_0^T c_{\mathrm{SR}}(y(t))\, \mathrm{d}t$, so that a TOU price p is an SRMC at y if and only if $p(t)$ is an instantaneous SRMC at $y(t)$ for each t). With this technology, C_{SR} is therefore nondifferentiable whenever k is the cost-minimizing capital input for the required output y: cost-optimality of k means merely that it provides just enough capacity, i.e., that $k = \mathrm{Sup}(y)$. This condition, being quite unrelated to the input prices r and w, obviously cannot ensure that an SRMC price system is an LRMC. To guarantee this, one must strengthen it to the condition that $r = \int_0^T (p - w)^+ \mathrm{d}t$ in this example or, generally, that $r = \nabla_k \Pi_{\mathrm{SR}}$ (or that r belongs to the supergradient set $\widehat{\partial}_k \Pi_{\mathrm{SR}}$, should Π_{SR} be nondifferentiable in k).[7] The capital's cost-optimality would suffice for

[6] The SRMC and the short-run supply correspondences are inverse to each other, i.e., have the same graph: in Figure 2a, the broken line is both the supply curve and the SRMC curve.

[7] This condition ($r = \nabla_k \Pi_{\mathrm{SR}}$) is stronger than cost-optimality of the fixed inputs when p is an SRMC.

the SRMC to be the LRMC if the costs were differentiable; this is the Wong-Viner Envelope Theorem. The preceding remarks show how to reformulate it to free it from differentiability assumptions. This is detailed in [6] and [8].

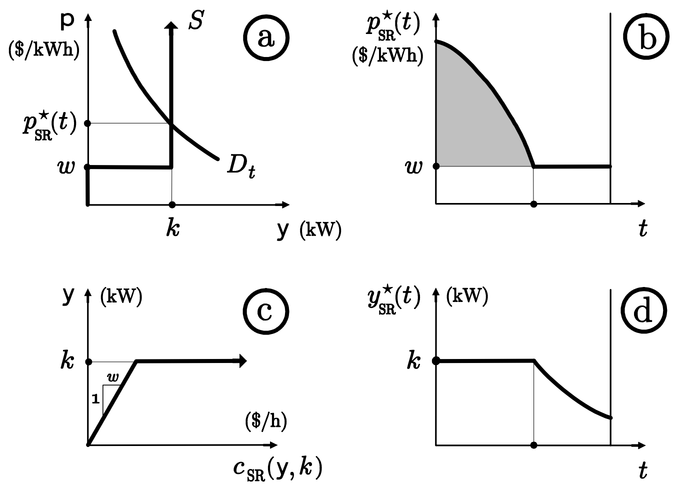

Figure 2: Short-run approach to long-run equilibrium of supply and (cross-price independent) demand for thermally generated electricity: (a) determination of the SR equilibrium price and output for each instant t, given a capacity k; (b) and (d) trajectories of the SR price and output; (c) the SR cost curve. When k is such that the shaded area in (b) equals r, the SR equilibrium is the LR equilibrium.

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