### Abstract

This paper aims to examine unemployment persistence in Spain by the so-called "ladder" effect. This arises when highly-skilled workers who do not find a job matching their skills accept jobs which previously were occupied by less qualified staff. We develop a dynamic general equilibrium model, in which two types of workers – characterised by their level of formal education – coexist on the labour market. Highly educated workers are then assumed to compete with low-skilled workers, generating a ladder effect. The model is then calibrated on the Spanish economy. Our results replicate the observed decline in the ratio of high- to low-skilled vacancies, and explains how firms substitute high- for low-skilled employment. The results also suggest that the Spanish ladder effect may reflect increases in the training costs as a result of a biased-shock against low-skilled workers.

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# Spanish Unemployment Persistence and the Ladder Effect

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### 1 Introduction

During the two last decades, there has been a reduction in the demand for low-skilled workers relative to highly-skilled in many industrialised countries (see OECD, (1994)). This phenomenon is usually accompanied by a deterioration of economic conditions of low-skilled workers. In Europe, this problem has manifested itself in a larger increase in the unemployment rate together with a longer duration of unemployment for low-skilled workers relative to highly-skilled (Drèze and Sneessens, (1997)). In the US and UK, this phenomenon has been associated with greater wage inequality across skill groups (see Krugman, (1994)). Meanwhile, many workers have attained higher levels of education. This latter phenomenon has led to a structural change in the composition of the labour force, which has essentially taken place for new entrants in the labour market (see Robinson and Manacorda, (1997)).

The large increases in low-skilled unemployment rates in the EU countries may be explained in di¤erent ways. One standard explanation is "skill mismatch" resulting from relative wage rigidities in the face of biased technological shock. Another explanation arises from job competition between highly and low-skilled workers: by applying to low-skilled jobs, highly skilled workers manage to increase their probability to get a job displacing low-skilled ones- the so-called "ladder e¤ect". Firms may wish to hire highly skilled workers for low-skilled jobs, for instance, to avoid training cost (see Thurow (1975)), or because they have a high productivity (see Gautier (1999)). The fact that highly skilled workers may occupy low-skilled positions has been documented in several countries (see Van Ours and Ridder (1995), and Muysken and Ter Weel (1998), for the Netherlands, and Green et al. (1999), for the UK).

This explanation is particularly relevant for Spain where the proportion of educated workers has sharply increased after 1980 (see García Montalvo (1995), and Blanco (1997)). Speci...cally it increases more from middle of the 1980's than before. For instance, EPA (Spanish Labour Population Survey) data shows that in 1988 only 17.3% of the labour force attained upper–secondary schooling and 28.4% in 1996.<sup>1</sup> In Table 1, we categorise employment and unemployment rates into educational and occupational categories. The Table shows that the ratio of the highly to low–skilled labour force averages about 30% and that it rose considerably during the period 1988-1996. At the same time, the unemployment rate of the low–skilled workers rose too. For instance, illiterate workers and those with primary and low-secondary schooling have experienced a marked increase in unemployment rates (from 19% in 1988 up to 23% in 1996). In the same period, the proportion of low-skilled jobs ...Iled by highly skilled workers rose from 9.8% in 1988 to 15.1% in 1996.<sup>2</sup>

Alba-Ramirez (1993), ...nds that Spanish highly skilled workers on low-skilled jobs are mainly young skilled workers, without experience, who need a ...rst job in order to obtain on-the-job training. After some time, they become the main job turnover group, which

<sup>&</sup>lt;sup>1</sup>More information about composition of the labour force by education in appendix B.

<sup>&</sup>lt;sup>2</sup>We have analysed these proportions for the whole of the economy. Although the ladder e<sup>x</sup>ect is relevant in all groups of workers, it seems to have a stronger importance in women and young workers than in among older men.

implies they move into jobs that require higher educational levels than their original ones. García-Serrano and Malo (1995), analyse the substitutability between education and on-the-job training. They conclude that Spanish ...rms hire highly skilled workers to reduce investment in speci...c human capital formation. Beneito et al. (1996), also conclude that in Spain education is a substitute for on-the-job training, and suggest that this fact is a source of ine¢ciency in the allocation of resources by generating 'overeducation'.<sup>3</sup>

The aim of this paper is to examine to what extent a ladder exect may contribute to explain changes in the unemployment rate and unemployment dixerences across skill groups in Spain. To this aim, we develop an intertemporal general equilibrium model with two types of workers (highly and low-skilled)) and two types of jobs. We distinguish two kinds of shocks: (i) a demand shock and (ii) a general training cost. The second shock can be interpreted as a skill-biased technological shock against low-skilled workers as well as a supply shock. Production technology is such that highly skilled jobs can be ...Iled only by educated workers, while low-skilled jobs may be ...Iled by both types of workers. Following Pissarides (2000), we assume that trade in the labour market is represented by a matching process with a Nash wage bargain. There are three matching functions <sup>4</sup> for each sort of employment. Highly skilled workers look for highly skilled jobs, but if they do not ...nd them, they look for low-skilled jobs as a temporary stop gap.

We ...nd that the "ladder exect" increases after each shock. However, employment and vacancies variables have not the same importance and evolution facing each shock. The model can replicate the observed decline in the ratio of highly to low–skilled vacancy rates. This decline is best explained by a decrease in both types of vacancies, as produced by a training cost change. Moreover, the evolution of the decrease in low-skilled employment is better reproduced by introducing training cost changes than highly skilled labour force changes.

The second section presents the model, showing the speci...c circumstances that generate a ladder exect. The third section describes the data and our calibration procedure. The forth section analyses the response of some key variables to (i) the introduction of a training cost for low-skilled workers and (ii) an increase in the relative size of the highly skilled labour force. We evaluate the implications of these shocks on the model steady state, thereby enabling us to perform comparative static exercises. We study transitional dynamics. The last section oxers some concluding remarks.

<sup>&</sup>lt;sup>3</sup>These studies use di¤erent data surveys and estimation methods.

<sup>&</sup>lt;sup>4</sup>Dolado et al. (2000), build a matching model closely related to our approach. Their results focus on the youth overeducation in Spain. Their model exogenously set the wages mechanism whereas our model sets wages in a Nash bargaining process. This is important in terms of posting vacancies and in the evolution of endogenous probabilities.

### 2 A model of the ladder exect

#### 2.1 Trade in the labour market

We consider an economy with of two types of workers: highly skilled and low-skilled. Highly skilled workers can perform low-skilled jobs, whereas low-skilled workers are unproductive in highly skilled jobs. In each period there are L<sup>h</sup> highly skilled workers and L<sup>1</sup> low-skilled workers. L<sup>h</sup> and L<sup>1</sup> are exogenous. Because ...rms observe the workers' skill level, low-skilled workers can only apply to low-skilled jobs. Highly skilled workers can apply to both types of jobs. This creates an asymmetry among workers pertaining to the competition between highly and low-skilled workers on the low-skilled job markets, which is the source of the ladder e<sup>x</sup>ect. Although there are no di<sup>x</sup>erences in terms of productivity between highly and low-skilled workers when working on a low-skilled job, ...rms can incur into a general training cost whenever they hire low-skilled workers.

All variables  $V_{j;t}^{i}$ ,  $U_{j;t}^{i}$ ,  $N_{j;t}^{i}$  and  $H_{j;t}^{i}$  where j;i 2 fh; Ig are ratios divided by their corresponding labour force. Let  $N_{h;t}^{h}$  denote the proportion of highly skilled workers working as highly skilled,  $N_{l;t}^{h}$  denote the proportion of highly skilled workers working as low-skilled, and  $U_{t}^{h}$  denote the highly skilled unemployment rate. We have

$$N_{h:t}^{h} + N_{l:t}^{h} + U_{t}^{h} = 1:$$

Likewise, denote  $N_t^{\,\text{I}}$  and  $U_t^{\,\text{I}}$  the proportion of low–skilled working and unemployed. We have

$$N_{t}^{I} + U_{t}^{I} = 1$$

Following Pissarides (2000), we assume that trade in the labour market is an uncoordinated and costly activity. Whenever a ...rm posts vacancies only a fraction of each of them will be ...lled  $-V_{j;t}^{h}$  and  $V_{j;t}^{l}$  vacancies rate highly skilled and low-skilled jobs respectively. We take a constant returns to scale the Cobb-Douglas matching function relating the number of matches to the number of vacancies and the number of job seekers.

We assume that high-skilled workers. This group can ...II highly skilled jobs or lowskilled jobs. And low-skill educated workers can ...II only the rest of low-jobs not ...Iled by highly skilled workers.

We distinguish two matching functions to ...II low-skilled jobs: workers with more education take ...rst low-skilled vacancies. We assume that the e $\oplus$ ciency matching factors of highly skilled workers will be larger than the e $\oplus$ ciency matching factor of low-skilled workers.

Let us now be more precise on the timing of events, and …rst consider highly skilled workers. A given highly skilled worker …rst looks for a highly skilled job, such that  $H_{h;t}^{h} \stackrel{\frown}{} H_{h}^{h} V_{t}^{h}$ ;  $U_{t}^{h} + N_{l;t}^{h}$  are formed in period t. Implicit in this formulation is the fact that job seekers are composed of highly skilled workers who do not work in highly skilled jobs in period t.

When the highly skilled job seeker does not match with a ...rm in period t, she goes on the low-skilled labour market and attempts to get a match as a low-skilled worker. Then,  $H_{l;t}^{h} \cap H_{l}^{h} V_{t}^{l} L^{l} = L^{h}; 1_{i} N_{h;t+1}^{h}$  are formed on this market. The level of low–skilled vacancies posted by ...rms has to be adjusted for the relative size of the two populations, in order to preserve the absence of size exects in the matching process.

Finally, low-skilled workers can be employed on a low-skilled job — not already occupied by highly skilled workers — such that the level of hirings for the low-skilled is  $H_t^{I} \stackrel{\frown}{} H_{l;t}^{I} \stackrel{\frown}{} H_{l;t}^{h} L^{h} = L^{I}; U_t^{I}$ , where only those vacancies not …lled by the highly skilled workers remain available to the low-skilled workers. Like the previous case, the ratio  $L^h = L^I$  adjusts for the di¤erent sizes of each group.

It is worth noting that, as in Pissarides (2000), each matching function only depends on aggregate quantities, thus retecting the fact that ...rms and job seekers have no control on the matching process. This assumption retects the existence of the traditional positive trade externalities and congestion exects, associated with the matching process. The evolution of the level of each type of employment is therefore given by:

$$N_{h;t+1}^{n} = H_{h;t}^{h} + (1_{i} s)N_{h;t}^{n}$$
(1)

$$N_{l;t+1}^{h} = H_{l;t}^{h}$$
 (2)

$$N_{t+1}^{I} = H_{t}^{I} + (1_{i} \ ^{1})N_{t}^{I}$$
(3)

where s; <sup>1</sup> 2 (0; 1) denote the constant exogenous separation rates for each type of employment. The second law of motion (equation (2)) represents the fact that highly skilled workers do not occupy a low-skilled job for more than one period, but rather go back on the search. We can interpret it as a temporary job for highly skilled workers, which is consistent with the high turnover rate for this group of workers. Since 1984, in Spain one third of the level of unemployment comes from temporary jobs.<sup>5</sup>

 $p_{h;t}^h$  is the probability that a highly skilled unemployed worker will be employed in a highly skilled job in the next period;  $p_{l;t}^h$  is the probability that a highly skilled unemployed worker will be employed in a low-skilled job; and  $p_t^l$  is the probability that a low-skilled unemployed worker will be employed in a low-skilled job in the next period. Thus:

$$p_{h;t}^{h} = \frac{H_{h;t}^{h}}{U_{t}^{h} + N_{l;t}^{h}}, \ p_{l;t}^{h} = \frac{H_{l;t}^{h}}{1 \ i \ H_{h;t}^{h} \ i \ (1 \ i \ s)N_{h;t}^{h}} \ \text{and} \ p_{t}^{l} = \frac{H_{t}^{l}}{U_{t}^{l}}.$$
(4)

In Figure (1) we can observe ‡ows in and out of employment. Below we express the unemployment dynamic to explain the evolution in both labour markets:

$$U_{t+1}^{h} = 1_{i} H_{h;t}^{h} i H_{l;t}^{h} i (1_{i} s) N_{h;t}^{h}$$
  
$$U_{t+1}^{l} = 1_{i} H_{t}^{l} i (1_{i} ) N_{t}^{l}$$

<sup>&</sup>lt;sup>5</sup>We have checked that the total destruction rate is consistent with our calibration of separation rate of highly skilled workers in a low-skilled job. For more details see Section 3.2.

#### 2.2 Firms

The economy is comprised of a continuum of ...rms with measure one. In each period, a ...rm j has access to a constant returns to scale technology represented by the following production function:

$$Y_{j;t} = AK_{j;t}^{*} (L^{h}N_{h;j;t}^{h})^{\mu} L^{h}N_{l;j;t}^{h} + L^{l}N_{j;t}^{l}^{{}^{\psi}1_{i} *}$$
(5)

where  $K_{j;t}$ ,  $N_{h;j;t}^{h}$ ,  $N_{l;j;t}^{h}$  and  $N_{j;t}^{l}$  respectively denote the level of physical capital, the highly-skilled employment rate, and the low-skilled employment rate respectively occupied by high and low skilled workers. We suppose there is perfect substitution in terms of productivity between low-skilled workers and highly-skilled workers in order to ...II a low-skilled job. The value of A is a positive constant that represents the level of total factor productivity. Finally, ®;  $\mu 2$  (0; 1) respectively denote the elasticity of output with regards to physical capital and highly-skilled employment.

Each period, the ...rm invests a level  $I_{j;t}$  to form capital that standardly accumulates as:

$$K_{j;t+1} = I_{j;t} + (1_{j} \pm)K_{j;t}$$
(6)

where  $\pm 2$  (0; 1) is the constant depreciation rate. It also posts vacancies.  $V_{j;t}^{h}$  and  $V_{j;t}^{l}$  are vacancies rates for respectively high and low-skilled jobs, and the ...rm incurs a linear cost ! h and ! per posted vacancy. These vacancies determine the employment that will be used in the following period. The law of motion of each type of employment are given by:

$$N_{h;j;t+1}^{h} = q_{h;t}^{h}V_{j;t}^{h} + (1_{i} s)N_{h;j;t}^{h}$$
(7)

$$N_{l;j;t+1}^{h} = q_{l;t}^{h} V_{j;t}^{l} \frac{L'}{L^{h}}$$
(8)

$$N_{j;t+1}^{I} = q_{t}^{I} 1_{i} q_{l;t}^{h} V_{j;t}^{I} + (1_{i} 1) N_{j;t}^{I}$$
(9)

where  $q_{h;t}^h$ ; is the probability of ...lling highly-skilled vacancies,  $q_{l;t}^h$ ; is the probability of ...lling a low-skilled vacancy with a highly-skilled worker and  $q_t^l$ , is the probability of ...lling a low-skilled vacancy with a low-skilled worker. These probabilities are thus given by

$$q_{h;t}^{h} = \frac{H_{h;t}^{h}}{V_{t}^{h}}, \ q_{l;t}^{h} = \frac{H_{l;t}^{h}}{V_{t}^{l}} \frac{L^{h}}{L^{l}} \text{ and } q_{t}^{l} = \frac{H_{t}^{l}}{V_{t}^{l}} \frac{H_{t}^{l}}{L^{h}} = \frac{H_{t}^{l}}{V_{t}^{l}} \frac{L^{h}}{L^{h}}$$
(10)

It is worth noting that these probabilities are determined by aggregate quantities and thus re‡ect the trade externalities implied by the search process.  $w_{h;t}^h$ ;  $w_{h;t}^h$  and  $w_t^l$  are the bargained wages. And ...nally, when hiring a low–skilled worker, the ...rm has to train her and therefore incurs a proportional cost { per hiring. We suppose that a low-skilled worker needs training when she is hired in a new ...rm, (i.e. it can lead to a change in technology). We also suppose that a highly-skilled worker has an exogenous general skill from her education, which avoids this training cost.

The period t instantaneous pro...t can be expressed as<sup>6</sup>

$$|_{j;t} = Y_{j;t} |_{i} w_{h;t}^{h} N_{h;j;t}^{h} L^{h} |_{i} w_{l;t}^{h} N_{l;j;t}^{h} L^{h} |_{i} w_{l}^{l} N_{j;t}^{l} L^{l}$$

$$|_{j;t} |_{i} v_{l;t}^{h} L^{h} |_{i} |_{i} V_{l;t}^{l} L^{l} |_{i} \{H_{t}^{l} L^{l} \}$$

$$(11)$$

Each ...rm j determines its factor demand and investment plan— both on the good and the labour market — maximising its market value:<sup>7</sup>

$$(S_{j;0}^{F}) = X_{t=0} R_{t} |_{j;t}$$

subject to (5)–(11) where " $(S_{j;0}^{F})$  denotes the value of …rm  $R_t = \frac{Q_t}{\lambda = 0}(1 + r_{\lambda})^{i-1}$  and  $g_t$  is the real interest rate. Finally, the set of each …rm's state variables is  $S_{j;t}^{F} = K_{j;t}; N_{h;j;t}^{h}; N_{h;j;t}^{l}; N_{j;t}^{l}$ . Hereafter  $X_{j;t}^{k}, X_{h;j;t}^{h}, X_{l;j;t}^{h}$  and  $X_{j;t}^{l}$  will denote the Lagrange multipliers associated to the capital and employment laws of motion respectively. The …rst order conditions associated with the control variables investment,  $I_{j;t}$  and vacancies,  $V_{i;t}^{h}$  and  $V_{i;t}^{l}$ ; are given by

$$X_{i:t}^{k} = 1 \tag{12}$$

$$q_{h;t}^{h} X_{h;j;t}^{h} = !_{h}$$

$$\tag{13}$$

$$q_{l;t}^{h} X_{l;j;t}^{h} + q_{t}^{l} (1_{j} q_{l;t}^{h}) X_{j;t}^{l} = !_{l} + \{ q_{t}^{l} (1_{j} q_{l;t}^{h}) :$$
(14)

Equation (12) represents the optimal level of investment, the marginal cost of capital goods for the ...rm that is one. Equation (13) represents the optimal level of highly-skilled vacancies posted by a ...rm. It states that the ...rm will post highly-skilled vacancies up to the point where the expected marginal value of ...lling an additional highly-skilled job  $(q_{h;t}^h X_{h;j;t}^h)$ , is just compensated by the marginal cost to post a highly-skilled vacancy  $(!_h)$ . The ...rst order condition (14), the marginal value for the ...rm to ...II a low-skilled job receives a similar interpretation, up to the point the marginal cost of posting a vacancy is complemented by an additional cost of training to hire a low-skilled worker. If we suppose no training cost { then  $q_{i;t}^h X_{1;j;t}^h + q_t^l (1_i q_{1;t}^h) X_{j;t}^l = !_1$ . A part of the left side is the expected marginal value corresponding to ...II the vacancy with a highly-skilled worker and the other part the expected marginal value corresponding to ...II the vacancy with a low-skilled worker. Both depending on their di¤erent probabilities to ...II this vacancy.

The parameters will be chosen in a such a way that the ...rm prefers highly–skilled workers to low–skilled ones to ...II a low-skilled vacancy<sup>8</sup>:

$$X_{1;j;t}^{h} > X_{j;t}^{l}$$
: (15)

<sup>&</sup>lt;sup>6</sup>After rearranging,  $\{ H_t^I L^I = \{ q_t^I V_t^I L^I \}_i \{ q_t^I H_{I;t}^h L^h = \{ q_t^I (1_i q_{I;t}^h) V_t^I L^I \}$ 

<sup>&</sup>lt;sup>7</sup>The interested reader is lead to Appendix A.1 for the optimality conditions associated to the ...rm's problem.

<sup>&</sup>lt;sup>8</sup>We have checked that assumption 15 is validated ex-post.

### 2.3 Households

We now present the behaviour of each type of household — highly-skilled and low-skilled, indexed by i. Following Andolfatto (1996), households of the same type are assumed to be identical ex ante. The random matchings and separations in the labour market induce di¤erent states in the labour market which can lead to ex post heterogenous wealth positions, which would then make the problem intractable as we would have to keep track of each individual story. For the sake of simplicity, we assume that there exists a perfect insurance market, which allows risk averse households to fully insure against the di¤erent income ‡uctuations and labour market transitions. We assume that the labour force is randomly assigned across jobs at the beginning of each period. Thus, the representative household assumption can be made and the probability of employment status. A detailed description of the household problem with full insurance is provided in Appendix A.2.

#### 2.3.1 Low-skilled households

In each period, a low-skilled household i can be in two alternative states in the labour market — either employed with probability  $N_t^1$  or unemployed with probability  $U_t^1 = 1_i N_t^1$ . Depending on the state, the instantaneous utility function of the low-skilled household i is given by:

$$\begin{array}{rcl} u_{i,t}^{l} &=& log(C_{i,t}^{l} i i^{l}) & \text{ if employed} \\ u_{i,t}^{l^{2}} &=& log(C_{i,t}^{l^{2}} i i^{l^{n}}) & \text{ if unemployed} \end{array}$$

where  $C_{i;t}^{I}$  and  $C_{i;t}^{I^{a}}$  respectively denote the level of consumption of an employed and unemployed low-skilled household.  $i^{I}$  and  $i^{I^{a}}$  represent a utility cost — expressed in terms of physical goods — associated with the state in the labour market. This cost is assumed to be constant over the business cycle.

The household enters the period with a level of assets  $B_{i;t}^{I}$  carried over from the previous period, from which she gets interest revenues. When employed, the household receives the real wage,  $w_{ijt}^{I}$ , bargained with the ...rm. When unemployed, she receives an unemployment insurance, associated with the insurance contract signed with an insurance company.<sup>9</sup> These revenues are then used to consume  $C_{i;t}^{I}$  and  $C_{i;t}^{I^{?}}$  with probabilities  $N_{t}^{I}$  and 1 i  $N_{t}^{I}$ , and to buy new assets. Therefore, the consolidated budget constraint after the insurance contract faced by the low-skilled household<sup>10</sup> is

$$N_{t}^{I}C_{i;t}^{I} + (1_{i} N_{t}^{I})C_{i;t}^{I^{2}} + B_{i;t+1}^{I} N_{t}^{I}w_{ijt}^{I} + (1 + r_{t})B_{i;t}^{I}$$
(16)

Let the probability of being employed be  $N_t^I$ . The problem of the representative household i is therefore to maximise the expectation of the discounted sum of its instantaneous

<sup>&</sup>lt;sup>9</sup>Appendix A.2 proves that households will choose to be fully insured against the risk of unemployment. <sup>10</sup>Implicit in this formulation is that the insurance problem has already been solved.

utility with respect to the consumption and the assets she holds:

$$\mathbf{X}_{t=0} = \mathbf{v}_{t=0}^{-t} \mathbf{v}_{t}^{T} \mathbf{u}_{i;t}^{T} + (1_{i} \mathbf{v}_{t}^{T}) \mathbf{u}_{i;t}^{T}$$

subject to equation (16).

#### 2.3.2 Highly-skilled households

Like low-skilled households, a highly-skilled household faces di¤erent states in the labour market. She can be either employed as a highly-skilled worker with probability  $N_{h;t}^h$ , employed as a low-skilled worker with probability  $N_{l;t}^h$ , or unemployed with probability  $U_t^h = 1_i N_{h;t}^h i N_{l;t}^h$ . For each state, the instantaneous utility function of the highly-skilled household i is given by:

where  $C_{h;i;t}^{h}$ ;  $C_{l;i;t}^{h}$  and  $C_{i;t}^{h^{a}}$  respectively denote the employed household's consumption when she works as a highly–skilled or low–skilled worker and her consumption when unemployed.  $i^{h}$  represents a cost, in terms of physical goods, associated with the highly– skilled working activity.  $i^{h}_{l}$  and  $i^{h^{a}}$  are the same costs previously de…ned for low-skilled households. These costs are assumed to be constant over the business cycle.

The consolidated budget constraint after the insurance contract faced by the highlyskilled worker is similar to that faced by the low-skilled. Now however, the household may be in three alternative states

$$N_{h;t}^{h}C_{h;i;t}^{h} + N_{l;t}^{h}C_{l;i;t}^{h} + U_{t}^{h}C_{i;t}^{h'} + B_{i;t+1}^{h}$$
(17)  
$$N_{h;t}^{h}w_{h;iit}^{h} + N_{l;t}^{h}w_{l;iit}^{h} + (1 + r_{t})B_{i;t}^{h}$$

The highly-skilled worker solves the same problem as the low-skilled worker. She maximises the discounted sum of its instantaneous utility with respect to consumption and the assets she holds:

$$\mathbf{X} = \mathbf{V}_{t=0}^{-t} \mathbf{N}_{h;t}^{h} u_{h;i;t}^{h} + \mathbf{N}_{l;t}^{h} u_{l;i;t}^{h} + \mathbf{U}_{t}^{h} u_{i;t}^{h^{2}}$$

subject to equation (17).

#### 2.4 Wage determination

Following Pissarides (2000), we assume that wages are determined by a Nash bargaining process between the ...rm and the household. The rent is shared according to the Nash

solution of the bargaining problem. Because of the coexistence of dixerent types of workers, there are dimerent wage bargaining processes which give rise to dimerent levels of wages. At the beginning of every period; there is a re-negotiation between ...rms and workers.

Let  $*_{h}^{h}$ ,  $*_{h}^{l}$  and  $*_{h}^{l}$  denote the exogenous parameters, which measure the bargaining power of highly-skilled households applying for highly-skilled and low-skilled jobs and low-skilled workers, the level of wages in a symmetric equilibrium can be shown to be:<sup>11</sup>.

$$w_{ht}^{h} = w_{h}^{h} \mu \frac{Y_{t}}{L^{h}N_{ht}^{h}} + p_{ht}^{h}X_{ht}^{h} + (1_{i} w_{h}^{h}) + \frac{N_{lt}^{h}}{1_{i} N_{ht}^{h}} (w_{lt}^{h} + 1_{i})$$
(18)

$$w_{lt}^{h} = w_{l}^{h} (1_{i} \otimes i_{j} \mu) \frac{Y_{jt}}{L^{h}N_{l;jt}^{h} + L^{l}N_{jt}^{l}} + (1_{i} \otimes_{h}^{h})_{i}^{l}$$
(19)  
$$\tilde{A}$$

ļ

$$w_{t}^{I} = *^{I} (1_{i} \otimes_{i} \mu) \frac{Y_{jt}}{L^{h}N_{l;jt}^{h} + L^{I}N_{jt}^{I}} + p_{t}^{I}X_{t}^{I} + (1_{i} *^{I})_{i}^{I}$$
(20)

which just amounts to the standard total rent sharing rule between the participant of the bargaining process. The ...rm acquires the gain in marginal labour productivity and the expected marginal value of a newly created job,  $X_{h;t}^h$  and  $X_t^l$ : When a highly-skilled worker is in a low-skilled vacancy, the separation rate is 1, and the gain for the ...rm is only the marginal labour productivity. The share accrued by the worker is given by the di¤erential in the disutility of work. It is worth noting that for the highly-skilled worker, who ... Is a highly-skilled vacancy takes into account the disutility of work and of working in a low-skilled vacancy.<sup>12</sup>

#### Equilibrium 2.5

A perfect foresight equilibrium of this economy is a sequence of prices  $fP_tg_{t=0}^1 = fw_{h;t}^h; w_{h;t}^h;$  $w_t^l$ ;  $r_tg_{t=0}^1$  and a sequence of quantities  $fQ_tg_{t=0}^1 = ffQ_t^Hg_{t=0}^1$ ;  $fQ_t^Fg_{t=0}^Hg_{t=0}^1$ .  $fQ_t^Hg_{t=0}^1 = fC_{h;t}^h$ ;  $C_t^l$ ;  $B_t^h$ ;  $B_t^Ig_{t=0}^1$  and  $fQ_t^Fg_{t=0}^1 = fY_t$ ;  $I_t$ ;  $K_t$ ;  $N_{h;t}^h$ ;  $N_{t}^h$ ;  $V_t^h$ ;  $V_t^h$ ;  $V_t^Ig_{t=0}^1$  such that:

- (i) given a sequence of prices  $fP_tg_{t=0}^1$ ,  $fQ_t^Hg_{t=0}^1$  is a solution to the representative household's problem;
- (ii) given a sequence of prices  $fP_tg_{t=0}^1$ ,  $fQ_t^Fg_{t=0}^1$  is a solution to the representative ... rm's problem;
- (iii) given a sequence of quantities  $fQ_tg_{t=0}^1$ ,  $fP_tg_{t=0}^1$  clears the goods markets in the sense

$$Y_{t} = C_{t} + I_{t} + I_{h}V_{t}^{h}L^{h} + I_{l}V_{t}^{l}L^{l} + \{H_{t}^{l}V_{t}^{l}\}$$

Ã

<sup>&</sup>lt;sup>11</sup>See appendix A.3 for a detailed exposition of the bargaining process <sup>12</sup>Notice as well that  $i^{I^{\mu}} = i^{h^{\mu}} = 0$  and  $i^{I} = i^{h}_{I}$ :

and the capital markets.

- (iv) wages are set according to the rent sharing mechanism.
- (v) labour market ‡ows are determined by hiring functions,  $H^h_{h;t}; H^h_{l;t}$  and  $H^1_t;$

### 3 Data and calibration

We are now to analyse the response of some key variables following two alternative types of shocks: a labour demand and a labour supply shock, which will be embodied in changes in the training cost, { , and the highly–skilled labour force.

Since the model does not admit any analytical solution, We rely on numerical simulations of the model. This is achieved using the software dynare developed by Juillard (1996)<sup>13</sup>. This implies that numerical values have to be assigned parameters. Such values are obtained from the Spanish economy. Before proceeding to the calibration, We describe the data we will rely on.

#### 3.1 The data

The model is calibrated on the Spanish economy using quarterly data. Macroeconomic time series are borrowed from Puch and Licandro (1997), who elaborated on the National Accounts of the Spanish Economy (Contabilidad Nacional de España). Aggregate consumption is given by the sum of non-durable consumption and government expenditures, and investment is the sum of durable consumption and ...xed investment.

In order to obtain data consistent with the measure of the "ladder e¤ect" problem, we rely on the Linked Labour Population Survey (Encuesta de Población Activa enlazada, EPA hereafter). This quarterly survey collects panel data of individuals during six consecutive quarters periods. The sample runs from 1987:1 to 1996:4. It covers a large number of individuals and characteristics, such as formal education attainment, occupation, employment status, age and gender. It de...nes 5 groups of education and 10 groups of occupation (range 0–9). High–skilled individuals are de...ned, for our purpose, as those with a level of education greater or equal to upper-secondary education. Therefore, low–skilled individuals essentially consist of illiterate and uneducated persons, primary or low–secondary educated people. Highly–skilled occupations are taken to be managers, professionals and technicians and support professionals (range 1–3 in EPA classi...cation), the rest (range 0,4–9 in EPA are armed forces, clerks, service, skilled agricultural/...shing, Craft-Transport, Plant-Manufacture and unskilled group) is taken to de...ne low–skilled occupations.<sup>14</sup>. Ratios are reported in Table 2.

<sup>&</sup>lt;sup>13</sup>More about dynare and the underlying relaxation algorithm can be found in La¤argue (1990) and Boucekkine (1995).

<sup>&</sup>lt;sup>14</sup>More detailed employment data in Appendix B.

The "ladder exect" problem is measured as the percentage of highly educated people who occupy a job with low-skilled requisites ( $N_1^h$  in our model). Finally, those highly-educated workers in a highly-skilled occupation and those low educated workers in a low-skilled occupation are represented as  $N_h^h$  and  $N_1^l$ , respectively. The highly-skilled (low-skilled) population,  $L^h$  ( $L^1$ ), is given by the sum of highly (low) educated employed and unemployed individuals.

Table 2 reports the probability of ...nding a job for each type of worker. For our estimations and stylised facts we have used the de...nitions of education and occupation in the linked EPA survey.<sup>15</sup> For instance,  $p_h^h$  (or  $p_h^l$ ) — the probability that a highly skilled unemployed worker ...nds a highly skilled (or a low-skilled job) — is measured as the proportion of highly educated unemployed workers who ...nd a highly skilled (low-skilled) job from one period to the other. Likewise, the probability that a low-skilled unemployed ...nds a (low-skilled) job,  $p^l$  is measured as the proportion of low-skilled unemployed in a (low-skilled) occupation from one period to other, corrected by total population.

Vacancies are measured by the number of vacancies not ...Iled at the end of period as reported by the National Employment O¢ce (INEM). These data also range from 1987:1 to 1996:4 and are categorised in terms of occupations following the same de...nition as EPA data for employment.<sup>16</sup> They are taken in terms of proportion of the high-skilled or low-skilled total population.

#### 3.2 Calibration

Table 3 reports the calibrated value of behavioural parameters. These values are obtained from the model to their empirical counterpart. For instance, the di¤erent elasticities of output are set in steady state to be a constant returns to scale Cobb-Douglas. The discount rate, -, is set such that, using the Euler equation associated to the capital accumulation decision, the model matches a capital/output ratio of 9.8 — obtained from Spanish Quarterly data. Likewise, the depreciation rate, ±, is 0.0292.

Cost of vacancies  $!_{i}$  and  $!_{h}$  are set in steady state. We suppose that the cost of being unemployment is  $i^{h^{n}} = i^{l^{n}} = 0$ : We also suppose that the cost of working in a low-skilled job is the same for both high and low skilled worker. Those together with the cost of working as highly-skilled worked in a highly-skilled job  $i^{h}$  are set in steady state. It is supposed that  $i^{h} > i^{h}_{l} = i^{l} > i^{h^{n}} = i^{l^{n}}$ :

Training costs are set to zero in our benchmark case, but a sensitivity analysis to changes in this parameter will also be considered. The total factor productivity is set such that, in steady state, output is equal to 1.

°, the elasticity of the matching process with regards to the number of vacancies is assumed to be the same in each function and is set to  $^{\circ} = 0.5$ , which lies within the range of estimated values for the Spanish economy. The bargaining power of households

<sup>&</sup>lt;sup>15</sup>The proportions and probabilities data are that we have used to solve the model (except to  $p_{l;t}^h$  value set in steady state, see calibration section).

<sup>&</sup>lt;sup>16</sup>National de...nitions according with R.D. 2240/79 from 14 of August. Using by INEM and EPA.

 $(f_{s_i^j}g_{i;j_{2}fh;lg})$  is also set to 0.5, such that it is equal to the elasticity of matching with respect to vacancies. We therefore recover the so-called Hosios' condition.

First of all, the three matching functions are speci...ed as

$$\begin{split} H_{h;t}^{h} &= \overline{H}_{h}^{h}(V_{h;t}^{h})^{\circ}(1_{j}N_{h;t}^{h})^{1_{j}}^{\circ} \\ \mu \\ H_{l;t}^{h} &= \overline{H}_{l}^{h} V_{h;t}^{h} \frac{L^{l}}{L^{h}} (1_{j}N_{h;t+1}^{h})^{1_{j}}^{\circ} \\ H_{l;t}^{h} &= \overline{H}_{l}^{l} V_{t}^{h}_{ij} \frac{L^{h}}{L^{l}} H_{l;t}^{h} (1_{j}N_{t}^{l})^{1_{j}}^{\circ} : \end{split}$$

In the steady state, we set  $H_{l;t}^h = N_{l;t}^h$ : Given probabilities of ...nding a job, and the level of the employment rates, we are able to compute the average number of hirings as

$$H_{h}^{h} = p_{h}^{h}(1 \mid N_{h}^{h})$$
, and and  $H^{I} = p^{I}(1 \mid N^{I})$ :

In order to solve the model, we get the value of  $p_l^h$  at steady state using the probability of ...nding a low-skilled job by a highly skilled workers  $p_{l;t}^h = \frac{H_{l;t}^h}{1_i H_{h;t}^h (1_i s) N_{h;t}^h}$ . These numbers can then be used to calibrate  $\overline{H}_{h}^h$ ,  $\overline{H}_{l}^h$  and  $\overline{H}^l$  in steady state respectively. As we have remarked in the description of the model, we suppose that ...rms prefer to hire a highly-skilled worker in a low-skilled job, that is the e¢ciency factor of the matching function  $\overline{H}_{l}^h$  is higher than the other parameters and its intuence can be seen in that their wages become more competitive.

The exogenous quit rates are calibrated using equations (7)–(10) evaluated in steady state. Therefore, we obtain

$$s = \frac{H_h^h}{N_h^h}$$
 and  ${}^1 = \frac{H^I}{N^I}$ .

The separation rate for highly skilled workers in a highly skilled job s, is larger than the separation rate for low-skilled workers, <sup>1</sup>. This is consistent as a high turnover rate in the skilled workers. We set the separation rate for highly skilled workers in a low-skilled job to the value of one. In our economy, the total destruction is about 11%. This is consistent with the average of destruction rate for Spanish ...rms in Díaz-Moreno and Galdón (2000).<sup>17</sup>

Skilled and unskilled vacancies, V<sup>h</sup> and V<sup>1</sup>, are de...ned as the sum of high and lowskilled hirings and un...lled high and low-skilled vacancies. This permits obtaining the probability that a ...rm ...lls a highly-skilled vacancy with a highly-skilled worker,  $q_h^h$  and the probability of ...lling a low-skilled vacancy with a high or low-skilled worker,  $q_h^h$  and  $q^I$ , as de...ned in equation (10). Wage variables are set in steady state. Notice that highlyskilled wages are highest. And slightly low-skilled wages larger that highly-skilled wages when the worker is highly-educated in a low-skilled job. These numbers are reported in Table 4.

<sup>&</sup>lt;sup>17</sup>They estimate quaterly job ‡ows for Spanish economy for the period 93:11 to 95:1.

### 4 Analysis of results

This section proposes an analysis of the response of some key variables characteristic of the labour market to exogenous permanent shocks to (i) the training cost of low-skilled workers and (ii) the relative size of the highly–skilled labour force. We …rst perform a static exercise assessing the steady state implications of such changes in the model. We then study the transitional dynamics delivered by the model in the face of such shocks. This comparative analysis enables us to o¤er some insights on the role played by the ladder e¤ect in Spanish unemployment dynamics.

#### 4.1 Demand side: training cost

We model the relative demand shock as a change in the cost of training new low-skilled employed.<sup>18</sup> This change in the training cost may be seen as a consequence of a technological change. We assume that highly skilled workers do not require training: Given their education level, we assume that they have exogenous general skills. In our model the increase of this training cost can be interpreted as a skilled-biased technological shock against low-skilled workers. Highly skilled workers are willing to accept low-skilled vacancies temporarily. Despite their high separation rates, ...rms hire highly skilled workers to avoid general training costs inherent to the hiring of low-skilled workers. This training cost a¤ects employment asymmetrically. An increase in this training cost decreases total employment but more largely low-skilled employment.

Thus, we now analyse the exects of a permanent increase in the training cost paid by the ...rm when hiring a low-skilled worker, { : Figures 2–3 report the exects of a permanent shock in { ranging from values 0.0 to 0.2, on the steady state of some key variables characterising the labour market.

An increase in { induces ...rms to reduce low-skilled employment. It therefore increases the marginal value of low-skilled jobs, and decreases the marginal values of highly skilled ones (see Figures 2 and 3). As the wage bargaining process implies that wages are strongly correlated with the marginal productivity of employment, the real wage paid to highly skilled workers when employed on highly skilled jobs, reduces more than that received by low-skilled workers on low-skilled occupation (see Figure 2), reducing the wage gap among them.

The increase in the value of the shock has two opposite exects for the highly skilled workers. On the one hand, the cost of low-skilled employment rises and ...rms will reduce hirings of both low and highly skilled workers. On the other hand, ...rms will partially substitute highly skilled workers for low-skilled ones to ...II low-skilled vacancies as the former do not require any training. Overall, the low-skilled employment decreases most (see Figure 2). This substitution exect explain the huge drop in N<sup>1</sup>: Besides, noteworthy

<sup>&</sup>lt;sup>18</sup>We have also considered a permanent non-biased technological shock. As in literature the increase in the productivity of labour increases employment and decreases unemployment. However the e¤ect in the equilibrium unemployment rates will be neutral. Interesting further research will be to keep in temporary technological shocks in order to experience if there exists some relationship between technological shock and "ladder e¤ect".

is the fact that N<sup>1</sup> drops dramatically more than both N<sub>1</sub><sup>h</sup> and N<sub>h</sub><sup>h</sup> (see upper–left panel of Figure 2), illustrating the substitution  $e^{x}$  ect between highly and low–skilled workers in low–skilled occupations due to the increase in the training cost.

The reduction in the marginal value and the increase in the training cost therefore discourages posting vacancies, whatever their type, as illustrated by Figure 2. For values of { larger than 0.1, the reduction in the number of highly skilled vacancies is larger than the reduction in low-skilled vacancies. This reduction in both types of vacancies and the increase in unemployment reverses the congestion e¤ect.<sup>19</sup> Therefore, the probability that a ...rm ...Ils a vacancy rises whatever the type of posted vacancy. But, as the increase in unemployment is higher for low-skilled workers than for highly skilled ones, the rise in this probability is much higher in the case of low-skilled workers, q<sup>1</sup> (by an order of about 5 when { takes value 0.2) . Likewise, the implied negative trade externality explains the larger decrease in the probability to ...nd a job for low-skilled workers. The investment behaviour of ...rms leads to employment reduction for these types of occupation.

Our results are consistent with Van Ours and Ridder (1995), which give evidence that low- and highly skilled unemployment are strongly correlated and that low-skilled unemployment ‡uctuates more strongly. For instance, in face of a value of  $\{ = 0:1, N_h^h \text{ and } N_l^h \text{ decrease by 5\%, while } N^l \text{ drops by an amount of 10\%.}$  This shows up in the measure of the ladder exect reported in Figure 4.

The ladder exect, measured in terms of stocks,

$$\frac{L^{h}N_{l}^{h}}{L^{h}N_{l}^{h} + L^{l}N^{l}} \neq 100;$$

illustrates the previous analysis. As the ladder exect increases by 3% when  $\{ = 0:1 \text{ to } 11\% \text{ in face of } \{ = 0:2, \text{ once again re‡ecting the expected substitution exect between highly and low-skilled workers that is at work in the face of a training cost applied to low-skilled workers.$ 

This exect is far from proportional, and the higher the training cost, the larger the ladder exect becomes. It could be interpreted that in a society with rapid technological progress, those less quali...ed will need more training to be able to work even in less quali...ed vacancies. Stated otherwise, the low-skilled vacancies will require larger skills and the cost of qualifying low-skilled workers will rise more than proportionally.

Then, highly skilled workers take priority over low-skilled workers to ...II these vacancies. This leads to an increase in low-skilled unemployment. The exect is an increase in the "ladder" exect indicator.

#### 4.2 Supply Side: skilled labour force

There is a large amount of literature about the increase of skills in the labour force (see Green et al. (1999)). This section analyses the exects of a permanent increase in the

<sup>&</sup>lt;sup>19</sup>Under the presence of coordination failures on the labour market, an increase of competition among ...rms creates a congestion exect that lowers the probability of ...Iling up a vacant job.

relative size of the highly skilled labour force — a positive shift in  $L^{h}=L^{I}$  — which might be interpreted as an increase in the aggregate education attainments of workers.

Figures 5–6 report the exects of a shock ranging from 1 to 19% on the steady state of some key variables characterising the dynamics in the labour market.

The ...rst direct implication of a permanent increase in the relative availability of highly skilled workers is to shift downward the probability for this type of worker to ...nd a job. Indeed, the higher relative availability of this type of labour creates a congestion e<sup>a</sup>ect on the supply side which, via the matching process, makes it much harder for highly skilled workers to ...nd a job. Thus, as can be seen from Figure 6, both  $p_h^h$  and  $p_l^h$  drop. For instance, the probability for a highly skilled worker to ...nd a highly skilled job diminishes by 4% in face of a 5% permanent increase in  $L^h=L^1$ . On the contrary, it exerts a positive trade externality that bene...ts the ...rms and makes the probability of ...lling a vacancy higher. Noteworthy is that this e<sup>a</sup>ect does exist both for highly skilled jobs ( $q_h^h$ ) and low–skilled jobs ( $q_h^h$ ), as skilled workers may apply to such jobs. Nevertheless, the e<sup>a</sup>ect is more pronounced on highly skilled jobs. Conversely, the probability to ...II a low–skilled job with a low–skilled worker ( $q^1$ ) is lowered, as low–skilled workers are proportionally scarcer, thus increasing the competition among ...rms and creating a relative congestion e<sup>a</sup>ect.

The marginal values of employment that also depend on the labour market tightness through the wage setting mechanism do not all increase. Indeed, only the marginal value of low-skilled jobs increase while that of highly—skilled occupations decrease. This implies that ...rms post a higher number of low-skilled vacancies whereas the number of highly skilled decreases, as Figure 5 shows. This together with the evolution of the probability to ...II vacancies implies that employment of low-skilled workers and highly skilled workers employed as low-skilled increase, whereas that of highly skilled decreases. Therefore low-skilled unemployment diminishes while highly skilled unemployment increases, the overall exect on total unemployment being slightly positive.

The overall exect on wages is easily understood in the light of previous results as increases in the marginal productivity of all types of employment exerts an upward pressure on wages, which is countered by the decrease in the tightness of the highly skilled labour market. Thus, while the real wages paid in compensation to low-skilled job increase, those paid to highly skilled job decreases.

Beside these aggregate e¤ects, the measure of the ladder e¤ect — as reported in Figure 7 — re‡ects the earlier story, as increases in the relative size of the highly skilled labour force lead to an increase in the ladder, implying that more extensive use of low-skilled workers in low-skilled jobs is at work. This is explained as the increase in the probability to ...II a low-skilled vacancy with a highly skilled worker makes it worth increasing the use of this type of labour. In others an increase in the availability of highly skilled workers implies that ...rms substitute low-skilled workers for highly skilled ones in low-skilled occupations. However, this is not enough to compensate for the rise in the highly skilled labour supply, implying a larger level of highly skilled unemployment. It seems to provide a pretty good explanation to the positive correlation between the rise in the number of educated workers and in the highly skilled unemployment rate in the Spanish economy during the eighties.

Figures from 8 to 9 ...nally report the transitional dynamics of employment, unemployment and vacancies that follows a 5% rise in the relative availability of highly skilled labour force. As expected, ...rms instantaneously post a larger number of low-skilled vacancies whereas the number of highly skilled vacancies drops by a larger amount (7.5%). Therefore, the probability of ...lling a highly skilled job increases due to the negative congestion e<sup>x</sup>ect and that associated with low-skilled vacancies whatever its utilisation while low-skilled employment increases, explaining the behaviour of unemployment rates.

Nevertheless, the decrease in highly skilled employment yields an increase in the probability of ...Iling a highly skilled vacancy, pushing upward the marginal value of highly skilled employment, such that after their initial decline highly skilled vacancies go upward. However, this is not enough to push highly skilled employment back to the initial steady state.

#### 4.3 Comparative analysis

This analysis deserves additional comments that may shed light on the recent Spanish experiment. We have introduced both demand and supply changes. Both shocks are signi...cant to explain evolution of labour market variables, but the explicative amplitude of these changes is di¤erent.

#### Ladder exect and relative wages

From the last two sections, we observe that both training cost and relative labour force changes are important to explain the "ladder e<sup>x</sup>ect" and reproduce the same relative wages evolution. They have similar e<sup>x</sup>ects in both features, such a ladder e<sup>x</sup>ect indicator increases. Both explain that highly skilled workers take low-skilled jobs leading to a low-skilled unemployment persistence. When we introduce a change in the training cost, both highly skilled employment and low-skilled employment proportions decrease (see Figure 2). Since the former drops more than the latter, the ladder e<sup>x</sup>ect indicator increases very signi...cantly. Nevertheless, when a change in the relative highly skilled labour force occurs the ladder e<sup>x</sup>ect is due to both an increase in highly skilled employment working in low-skilled jobs and the fact that low-skilled employment increases at a slower rate (see Figure 5). In both shocks real wages paid to low-skilled job increase, while paid to highly skilled job decreases.

#### Employment and vacancies

Their exects on unemployment and vacancies rates are totally dixerent. Both changes reproduce the increase in highly skilled unemployment observed in Spanish data during the period. However, the training cost change only reproduces the employment data. Low-skilled unemployment only increases with the introduction of a training cost. In the other shock, the low-skilled unemployment rate decreases. Hence, the training cost can better explain the evolution of the "ladder exect".

Figures 10-12 report actual vacancies for the Spanish economy for our benchmark sample. The ratio of highly to low–skilled vacancies diminishes within this period. Indeed, as predicted by the model under any shock the ratio of highly skilled vacancies to low-skilled vacancies reduces (see Figure 10). In the face of a labour supply shock, ...rms post

a lower number of highly skilled vacancies. This reduction is even larger in the case of a greater training cost. Decreases in the number of highly skilled vacancies would rather be explained by both labour force and the training cost shocks as shown in Figure 11. When there was a change in the number of highly skilled workers, low-skilled vacancies increased in the model. While the decrease in the number of the low-skilled vacancies posted observed from Figure 12 provides empirical support to the training cost shock.

These results are understood in our framework of temporary jobs for highly skilled employees who work in low-skilled jobs. The model is built with a exogenous separation rate of 1 for this group. It leads to highly skilled employees to accept low-skilled vacancies temporarily, given that a highly skilled worker really wants to ...II a highly skilled job. It is consistent with the Spanish context where ...xed-term employment contracts are more ‡exible from 1984. The number of temporary employment has growth from 15.6% in 1985 to 33.6% in 1996 (see Dolado et al., 2001). From our results, ...rms prefer to take a highly skilled worker for a low-skilled job to avoid general training costs of low-skilled workers, which is interpreted as a technological change. This leads to an increase in low-skilled unemployment. It is also consistent with the idea that general skills of highly skilled workers allow more ‡exibility to change the job. However, the importance of the increase of the number of highly skilled workers is not negligible at all as we have commented in this section. The change in the labour supply can also reproduce the increase in the indicator of ladder e¤ect in Spain. These results indicate further research should be done to study interrelations between demand and supply.

### 5 Conclusion

This paper attempts to shed light on alternative explanations for the "ladder exect" phenomenon, which is a signi...cant source of Spanish unemployment persistence.

We have used a calibrated version of the model to assess the implications of labour demand and supply shocks. The labour demand shocks are related to the increase of the training cost in low-skilled vacancies and are seen as a biased technological progress against low–skilled workers. Labour supply shocks, often associated with increases in the number of highly-skilled workers, are introduced as increases in the relative availability of a highly–skilled labour force. Our results indicate that the ladder e¤ect generated by the model may account for the recent Spanish experience. The model can replicate the observed decline in the ratio of high to low–skilled vacancies, and shows how ...rms substitute high for low–skilled employment. We argue that the Spanish ladder e¤ect re‡ected by an increase in the training cost as a result of a biased shock against low–skilled workers. The positive change in the training cost of the low-skilled workers better reproduce the evolution on employment and in particular the decrease of low-skilled employment, as well as the evolution of the vacancy data.

What remains to be investigated is the extent to which the ladder-exect merely retects the role of education as an entry screening device by ...rms, which in turn would imply that o¢cial job descriptions may have little relation to job-content. Tables

	1988	1990	1992	1994	1996
high-sk. emp. as high-skilled. (Nh)	36.9	39.7	40.2	41.5	45.3
high-sk. emp. as low-skilled.(N <sup>h</sup> )	42.3	45.0	43.6	36.5	34.6
high-sk. unemployment (U <sup>h</sup> )	20.6	15.2	16.1	21.9	20.0
low-sk. emp. as low-skilled.(N <sup>1</sup> )	80.7	83.4	80.9	75.0	76.9
low-sk. unemployment (U <sup>1</sup> )	19.2	16.5	19.0	24.9	23.0
rat. high ov. low-skilled. popul. (L <sup>h</sup> =L <sup>1</sup> )	20.8	24.6	28.3	33.5	39.5
ladder e <sup>x</sup> ec. ind. $(N_{L}^{h}L^{h}=(N_{L}^{h}L^{h}+N^{T}L^{T}))$	9.8	11.7	13.2	14.0	15.1

<u>Source:</u> Own calculations from EPA data. The superscript denotes the education: high-skilled (h) and low-skilled (l). The subscript indicates the occupation. Education:h=superior and upper secondary and I=low secondary, primary and without studies. Occupation h=range 1-3 in EPA classi...cation (see appendix B) and I=range 0 and range 4 to 9 from EPA classi...cation.

Table 1: Labour rates by education-occupation (%)

Aggregated Ratios and Probabilities

capital/output ratio	k=y	9.8458
investment/output ratio	i=y	0.2877
consumption/output ratio	с=у	0.6923
high-skilled workers in high-skilled occupations	N <sup>h</sup>	0.4027
high-skilled workers in low-skilled occupations	N <sup>h</sup>	0.4102
low-skilled workers in low-skilled occupations	NI	0.7958
prob. for a high-skilled tond a high-skilled job	թ <mark>հ</mark>	0.0526
prob. for a high-skilled tond a low-skilled job	թ <mark>հ</mark>	0.0915
prob. for a low-skilled tond a low-skilled job	р <sup>і</sup>	0.1384
ratio high over low-skilled population	L <sup>h</sup> =L <sup>I</sup>	0.2847
Not Iled high-skilled vacancies over high-sk. labour force	V <sub>h</sub> ?	0.0093
Not Iled low-skilled vacancies over low-sk. labour force	V <sub>I</sub> ?	0.0082

Table 2: Aggregated Ratios and Probabilities (average)

Behavioural parameters

elasticity of output wrt capital	®	0.3471
elasticity of output wrt high-skilled labour	μ	0.1143
depreciation rate	±	0.0292
discount factor	-	0.9940
cost of posting low-vacancies	ļ <sub>1</sub>	0.8626
cost of posting high-vacancies	! <sub>h</sub>	0.0652
cost of working in a high-job	i <sup>h</sup>	0.7314
cost of working in a low-job	i	0.5363
cost of no working	$\mathbf{i}^{\mathbf{h}^{\mathbf{m}}} = \mathbf{i}^{\mathbf{I}^{\mathbf{m}}}$	0.0000
training costs	{	0.0000
total factor productivity	А	0.6083
elasticity of matching w.r.t. vacancies	0	0.5000
bargaining power	f»¦g <sub>i;j2fh;lg</sub>	0.5000

Table 3: Behavioural parameters

Labour market variables and probabilities

hirings of high-sk. workers for high-skilled jobs	H <sup>h</sup>	0.0314
hirings of high-sk. workers for low-skilled jobs	Η <sup>h</sup>	0.0547
hirings of low-sk. workers for low-skilled jobs	ΗÌ	0.0282
e¢. factor of high-sk. workers for high-sk. jobs	$\overline{H}_{h}^{h}$	0.2014
e¢. factor sk. workers for low-skilled. jobs	$\overline{H}_{l}^{h}$	0.7234
e¢. factor sk. workers for low-skilled. jobs	$\overline{H}'$	0.3275
wages of high-sk. workers for high-skilled jobs	Wh	0.9027
wages of high-sk.workers for low-sk. jobs	W <sup>h</sup>	0.5632
wages of low-skilled workers	w	0.5801
high-skilled separation rate	S	0.0780
low-skilled separation rate	1	0.0355
probrmIls h-skilled vac. with h-skilled workers	qh	0.7715
probrmIls I-skilled vac. with h-skilled workers	qh	0.7620
probrmIls I-skilled vac. with low-skilled workers	qi	0.7751
high-skilled vacancies	V <sup>h</sup>	0.0407
low-skilled vacancies	۷I	0.1532

Table 4: Calibrated parameters (Labour Market)

Level of Studies for Labor Force (%)

		· ·	/				
	1985	1987	1989	1991	1993	1994	1995
Illiter. and uned.	11,45	11,11	10,90	9,49	8,19	7,41	6,80
Primary School	44,92	40,84	36,24	34,35	30,67	29,29	27,45
Low-Sec. School	34,60	38,30	42,12	44,67	49,06	50,70	52,06
Upper-Sec. School	4.66	5,19	5,52	5,64	5,97	6,04	6,54
Superior	4.37	4,56	5,22	5,85	6,11	6,57	7,15

source: EPA

Table 5: Level of Studies for Labor Force (%)

Level of Studies for Unemployment (%)

		<u> </u>					
	1985	1987	1989	1991	1993	1994	1995
Illite. and uned.	18,63	19,80	16,42	17,87	24,38	25,25	24,84
Primary School.	16,72	15,32	13,76	14,23	21,10	21,09	27,00
Low-Sec. School.	29,19	25,81	20,09	19,49	26,41	26,25	24,80
Upper-Sec. School.	16,30	16,15	12,28	11,12	15,40	16,98	17,67
Superior School.	17,49	15,24	13,77	12,37	17,18	16,92	16,38

source: EPA

Table 6: Level of Studies for Unemployment (%)

### Figures



Figure 1: Flows in the labour market between employment, unemployment and jobs.



Figure 2: Steady state implication of training cost shock



Figure 3: Steady state implication of training cost shock



Figure 4: The ladder exect when { changes



Figure 5: Steady state implication of relative labour force shock



Figure 6: Steady state implication of relative labour force shock



Figure 7: The ladder  $e^{x}$  ect when  $L^{h}=L^{I}$  changes



Figure 8: Transitional dynamics in unemployment and vacancies



Figure 9: Transitional dynamics in employment



Figure 10: Vacancy ratio V<sup>h</sup>=V<sup>I</sup>



Figure 11: High-skilled vacancies



Figure 12: Low-skilled vacancies

#### Appendix A

Appendix A:1 : Decisions Rules of Firm j:

Recall that production function of the ...rm j is given by:

$$Y_{j;t} = AK_{j;t}^{*} (L^{h}N_{h;j;t}^{h})^{\mu} L^{h}N_{l;j;t}^{h} + L^{l}N_{j;t}^{l}^{\Psi_{1}}$$
(A.1.1)

Accumulation of capital is,

$$K_{j;t+1} = I_{j;t} + (1_{j} \pm)K_{j;t}$$
(A.1.2)

The law of motion of each type of employment are given by:

$$N_{h;j;t+1}^{h} = q_{h;t}^{h}V_{j;t}^{h} + (1 i s)N_{h;j;t}^{h}$$
(A.1.3)

$$N_{l;j;t+1}^{h} = q_{l;t}^{h} V_{j;t}^{l} \frac{L^{l}}{L^{h}}$$
(A.1.4)

$$N_{j;t+1}^{I} = q_{t}^{I} 1_{i} q_{l;t}^{h} V_{j;t}^{I} + (1_{i} 1) N_{j;t}^{I}$$
(A.1.5)

At period t instantaneous pro...t can be expressed as

$$|_{j;t} = Y_{j;t} |_{i} W_{t}^{h} N_{h;j;t}^{h} L^{h} |_{i} W_{t}^{l} N_{l;j;t}^{h} L^{h} |_{i} W_{t}^{l} N_{j;t}^{l} L^{l} |_{i} I_{j;t} |_{i} V_{j;t}^{h} L^{h}$$
(A.1.6)  
$$|_{i} |_{i} V_{j;t}^{l} L^{l} |_{i} \{ H_{t}^{l} L^{l} :$$

Then the ...rm solves the recursive problem

$$\max^{\cdots} (S_{j;t}^{F}) = \frac{1}{j;t} + \frac{1}{1 + r_{t+1}} (S_{j;t+1}^{F})$$

subject to (A.1.1)–(A.1.6). We form the Lagrangian for this problem. Let us denote the Lagrange multipliers associated to  $K_{j;t}$ ;  $N_{h;j;t'}^h$ ,  $N_{l;j;t}^h$  and  $N_{j;t}^l$  respectively by  $X_{j;t'}^k$ ,  $X_{h;j;t'}^h$ ,  $X_{l;j;t}^h$  and  $X_{j;t}^l$ .

 $X_{1;j;t}^{h}$  and  $X_{j;t}^{l}$ . The ...rst order conditions associated to the control variables investment,  $I_{j;t}$  and vacancies,  $V_{j;t}^{h}$  and  $V_{j;t}^{l}$ ; are given by

$$X_{j;t}^{k} = 1$$
 (A.1.7)

$$X_{h;j;t}^{h} = \frac{! h}{q_{h;t}^{h}}$$
 (A.1.8)

$$q_{l;t}^{h} X_{l;j;t}^{h} + q_{t}^{l} (1_{i} q_{l;t}^{h}) X_{j;t}^{l} = !_{l} + \{ q_{t}^{l} (1_{i} q_{l;t}^{h}):$$
(A.1.9)

Equation (A.1.7) represents optimal level of investment, the marginal cost of capital goods for the ...rm that is one. Equation (A.1.8) represents the optimal level of highly–skilled vacancies posted by a ...rm. The ...rst order condition (A.1.9) is the marginal value for the ...rm to ...II a low-skilled job complemented by an additional cost of training to hire a low-skilled worker.

The marginal values of the capital and the di¤erent types of employment for the ...rm are given by the envelope theorem as:

$$\begin{split} - {}_{jt}^{k} &= \frac{{}_{j:t}^{m}(S_{j;t}^{F})}{{}_{@}K_{j;t}} = {}_{N}^{m}\frac{Y_{j;t+1}}{K_{j;t+1}} + 1_{j} \pm \\ - {}_{h;jt}^{h} &= \frac{{}_{i:jt}^{m}(S_{j;t}^{F})}{{}_{@}N_{h;j;t}^{h}} = \mu \frac{Y_{j;t}}{L^{h}N_{h;j;t}^{h}}_{i;j;t} + (1_{j} - s)X_{h;j;t}^{h} \\ - {}_{l;jt}^{h} &= \frac{{}_{i:jt}^{m}(S_{j;t}^{F})}{{}_{@}N_{l;j;t}^{h}} = (1_{j} - {}_{N}^{m} + \mu)\frac{Y_{j;t}}{(L^{h}N_{l;j;t}^{h} + L^{l}N_{j;t}^{l})}_{(L^{h}N_{l;j;t}^{h} + L^{l}N_{j;t}^{l})}_{i} + (1_{j} - {}_{1}^{l})X_{j;t}^{h} \\ - {}_{jt}^{h} &= \frac{{}_{i:jt}^{m}(S_{j;t}^{F})}{{}_{@}N_{j;t}^{l}} = (1_{j} - {}_{N}^{m} + \mu)\frac{Y_{j;t}}{(L^{h}N_{l;j;t}^{h} + L^{l}N_{j;t}^{l})}_{i} + w_{j;t}^{l} + (1_{j} - {}_{1}^{l})X_{j;t}^{l}. \end{split}$$

Combining both envelope conditions and ...rst order conditions, we get the Euler equations related to  $K_{j;t'}$ ,  $N_{h;j;t'}^h$ ,  $N_{l;j;t}^h$  and  $N_{j;t'}^l$ 

$$X_{h;j;t}^{h} = \frac{1}{1 + r_{t+1}} \prod_{\mathbf{\tilde{A}}} \frac{\mu \frac{Y_{j;t+1}}{L^{h} N_{h;j;t+1}^{h}} i W_{j;t+1}^{h} + (1 i s) X_{h;j;t+1}^{h}}{\mathbf{\tilde{A}}}$$
(A.1.11)

$$X_{l;j;t}^{h} = \frac{1}{1 + r_{t+1}} \begin{pmatrix} (1 \ i \ ^{\mathbb{B}} \ i \ \mu) \frac{Y_{j;t+1}}{(L^{h}N_{l;j;t+1}^{h} + L^{1}N_{j;t+1}^{l})} i \\ i \ W_{l;j;t+1}^{h} \end{pmatrix}$$
(A.1.12)

$$X_{j;t}^{I} = \frac{1}{1 + r_{t+1}} \begin{pmatrix} A & & Y_{j;t+1} & I \\ (1_{i} \ {}^{\mathbb{B}} i \ \mu) \frac{Y_{j;t+1}}{(L^{h}N_{1;j;t+1}^{h} + L^{l}N_{j;t+1}^{l})} i \\ i \ W_{j;t+1}^{I} + (1_{i} \ {}^{1})X_{j;t+1}^{I} \end{pmatrix} i \qquad (A.1.13)$$

Furthermore, parameters will be chosen in such a way that the ...rm will prefer to hire a highly skilled worker instead of a low skilled worker for an unskilled vacancy,

$$X_{l;j;t}^{h} > X_{j;t}^{l}$$

This condition will be satis...ed at the steady state.

#### Appendix A:2 : The households

We follow Andolfatto (1996), in order to solve the problem of households. Workers tows are determined according to the matching process we described in section 2.3. Therefore, workers are randomly selected playing a game of "musical chairs". At the beginning of each period, the whole labour force is randomly shu- ed across a given set of jobs.

#### A.2.1 The low-skilled consumer problem

This section presents the derivation of the optimal behaviour of the low-skilled consumer, insisting on insurance issues. At the beginning of each period low-skilled households face di¤erent probabilities of being employed or unemployed as this is contingent on its status in the labour market in the previous period. This therefore implies that 2<sup>N</sup> di¤erent possible stories in the labour market are to be considered after N periods, each of which corresponds to a particular worker employment path and therefore a di¤erent story of accumulation. This leads to heterogeneity, which makes the resolution of the model extremely complicated. For the sake of simplicity, we follow Hansen (1985), and assume that there exists a perfect insurance system which may eliminate ex-post heterogeneity.

The instantaneous utility functions are given by

$$u_{i;t}^{l} = \log(C_{i;t}^{l} i i^{l}) \text{ (employed)}$$
(A.2.1)

$$u_{i;t}^{l^{\mu}} = \log(C_{i;t}^{l^{\mu}} i i^{l^{\mu}})$$
 (unemployed) (A.2.2)

where  $u_{i;t}^{l}$  and  $u_{i;t}^{l^{\alpha}}$  are the respective instantaneous utility functions for employed and unemployed households. As in the main body of the text, we let  $C_{it}^{l}$  and  $C_{it}^{l^{\alpha}}$  denote the respective low-skilled household's consumption when employed and unemployed.  $i^{l}$  and  $i^{l^{\alpha}}$  can be interpreted as a utility cost, expressed in terms of goods, associated with the situation of the household in the labour market. We assume  $i^{l} > i^{l^{\alpha}}$ , which ensures that the consumption of an employee is greater than that of an unemployed household.

At the very beginning of each period — before the matching process has taken place — the low-skilled household does not know what the situation, either employed or unemployed, will be in that period. As a consequence, the household seeks to maximise her expected value. Since there  $N_t^I$  denotes the percentage of low–skilled households that are employed. And <sup>-</sup> 2 (0; 1) is the discount factor of the household. The low-skilled household maximises the problem

$$V^{L}(S^{I}_{i;t}) = N^{I}_{i;t}u^{I}_{i;t} + (1_{i} N^{I}_{i;t})u^{I^{\alpha}}_{i;t} + {}^{-}V^{L}(S^{I}_{i;t+1});$$
(A.2.3)

where state variables are  $S_{i;t}^{I} = S_{i;t}^{I} f B_{j;t}^{I}; B_{i;t}^{I^{\mu}}g$  depending whether the household is employed or unemployed, are to be considered. We made use of the fact that by de...nition of  $p_{t}^{I}$  and the law of motion of  $N_{t+1}^{I}$ 

$$N_{t+1}^{l} = p_{t}^{l} (1_{j} N_{t}^{l}) + (1_{j} {}^{1})N_{t}^{l}$$
(A.2.4)

The household faces the two budget constraints

$$C_{it}^{I} + \dot{z}_{t}^{I} \%_{it}^{I} + B_{it+1}^{I}$$
 (1 + r<sub>t</sub>) $B_{it}^{I} + w_{it}^{I}$ , if employed (A.2.5)

$$C_{it}^{I?} + \dot{c}_{t}^{I} \%_{it}^{I} + B_{it+1}^{I?}$$
 (1 + r<sub>t</sub>) $B_{it}^{I?} + \%_{it}^{I}$ , if unemployed (A.2.6)

where  $B_{i;t}^{!}$  and  $B_{i;t}^{!^{*}}$  denote bond holdings carried over from the previous period. At the beginning of each period. The household receives the real wage,  $w_{it}^{!}$  when employed and the insurance payment,  $\mathscr{W}_{it}^{!}$ , when unemployed. Its expenditures, either employed or unemployed, are consumption, insurance contracts purchased at price  $z_{t}^{!}$  and bonds.

The problem of the household is therefore to solve the Bellman equation — and therefore maximise the intertemporal utility function — subject to (A.2.5) and (A.2.6).  $x_{i;t}^{l}$  and  $x_{i;t}^{l^{\alpha}}$ ; denote the Lagrange multipliers associated with budget constraint of the representative low-skilled household when employed and unemployed respectively. The problem may be stated as a Lagrangian in the following way:<sup>20</sup>

$$L = N_{t}^{I} u_{i;t}^{I} + (1_{i} N_{t}^{I}) u_{i;t}^{I^{\alpha}} + {}^{-} V^{L} (S_{i;t+1}^{I}) + N_{t}^{I} u_{i;t}^{I} \stackrel{i}{(1 + r_{t})} B_{it}^{I} + w_{it}^{I} \stackrel{i}{_{i}} C_{it}^{I} \stackrel{i}{_{i}} \overset{i}{_{i}} \frac{B_{it+1}^{I}}{B_{it+1}^{I}} + (1_{i} N_{t}^{I}) u_{i;t}^{I^{\alpha}} \stackrel{i}{_{i}} (1 + r_{t}) B_{it}^{I^{2}} + w_{it}^{I} \stackrel{i}{_{i}} C_{it}^{I^{2}} \stackrel{i}{_{i}} \overset{i}{_{i}} \frac{b_{it+1}^{I}}{B_{it+1}^{I^{2}}}$$

The ...rst order necessary conditions associated to the problem are therefore First order conditions with respect to  $C_{it}^{l}$ ,  $C_{it}^{l^{2}}$ ;  $\%_{it}^{l}$  are:

$$(C_{it}^{I}_{i}_{i}_{j}^{I})^{i}^{1} = \alpha_{it}^{I}$$
(A.2.7)

$$(C_{it}^{I^{?}} i i^{I^{?}})^{i} = \alpha_{it}^{I^{?}}$$
(A.2.8)

$$i_{i;t} \overset{\mathbb{R}^{1}}{\underset{t}{i}} \overset{\mathbb{R}^{1}}{\underset{t}{i}} + (1_{i} \overset{\mathbb{R}^{1}}{\underset{i;t}{i}})(1_{i} \overset{\mathbb{L}^{1}}{\underset{t}{i}}) \overset{\mathbb{R}^{1}}{\underset{t}{i}} = 0$$
(A.2.9)

Equations (A.2.7) and (A.2.8) state that the Lagrange multipliers  $x_{it}^{l}$  and  $x_{it}^{l^{2}}$  are equal to the marginal value of consumption, considering the alternative states of being employed or unemployed respectively. The ...rst order condition (A.2.9) represents the marginal value to be fully insured against unemployment.

The ...rst order conditions related to  $B_{it+1}^{I}$  and  $B_{it+1}^{I^{?}}$  are:

$$\alpha_{it}^{I} = -\frac{@V^{L}(S_{i;t+1}^{I})}{@B_{it+1}^{I}}$$
(A.2.10)

$$\alpha_{it}^{I^{\alpha}} = -\frac{@V^{L}(S_{i;t+1}^{I})}{@B_{it+1}^{I^{\alpha}}}$$
(A.2.11)

Finally the envelope therefore implies

$$\frac{{}^{@}V^{L}(S_{i;t}^{I})}{{}^{@}B_{it}^{I}} = (1 + r_{t}) {}^{\alpha}{}^{I}_{it}$$
(A.2.12)

$$\frac{{}^{@}V^{L}(S_{i;t}^{I})}{{}^{@}B_{it}^{I*}} = (1 + r_{t}) {}^{\mathbb{R}}{}_{it}^{I*}$$
(A.2.13)

Envelope conditions together with ...rst order conditions related to optimal portfolio composition yields the following Euler conditions:

$$\mathbf{x}_{it}^{I} = (1 + r_{t+1})\mathbf{x}_{it+1}^{I}$$
(A.2.14)

$$\mathbf{x}_{it}^{l'} = (1 + r_{t+1})\mathbf{x}_{it+1}^{l'}$$
(A.2.15)

Insurance Company for low-skilled households: The expected pro...t of the insurance company is given by the di¤erence from the gain of the prime of insurance times the

<sup>&</sup>lt;sup>20</sup>Note that the Lagrange multipliers are just normalized by  $N_t^1$  and  $1_i N_t^1$  for convenience.

insurances and the payment of insurance to people who were unemployed. We assume free entry on the insurance market, pro...ts are driven to zero, such that

$$t_{t} = \dot{z}_{t}^{I} \%_{it}^{I} i (1 i N_{t}^{I}) \%_{it}^{I} = 0$$

We assume the company insures the current unemployment risk. The company can not di¤erentiate from the state of low-skilled households. People insurance against the probability of being unemployed and the optimal prime for the company is

$$\dot{z}_{t}^{I} = 1_{i} N_{t}^{I}$$
: (A.2.16)

Using these results in the optimal choice for insurance (A.2.16) in equation (A.2.9), we end up with

$$\alpha_{i:t}^{I} = \alpha_{i:t}^{I^{n}}$$

for low-skilled households, whether they were employed or unemployed in the previous period. This condition actually states that the marginal utility of wealth is independent from the state of the household in the labour market, which therefore eliminates heterogeneity in saving behaviour. Using this condition together with equations (A.2.7) and (A.2.8), we obtain  $C_{it}^{I} = C_{it}^{I^{\alpha}} + i \, {}^{I} \, {}^{I} \, {}^{I^{\alpha}}$ : Low-skilled households have dimerent consumption levels when employed or unemployed,  $C_{i;t}^{I}$  and  $C_{i;t}^{I^{\alpha}}$ ; but they prefer to be completely insured in terms of utility.

It also implies that equations (A.2.14) and (A.2.15) are equivalent and households accumulate the same quantity of bonds whether they are employed or unemployed,  $B_{it+1}^{I} = B_{it+1}^{I^{*}} = B_{it+1}$ : As a matter of fact, households choose to be completely insured, and have the same wealth in any state. This implies that saving decisions are independent of the employment history of the household. The optimal insurance is found using constraints (A.2.5) and (A.2.6):

$$%_{it}^{I} = W_{ij;t}^{I} + i_{i}^{I} i_{i}^{I^{*}}$$

As soon as households choose to be fully insured, and provided the employed labour force is selected randomly across jobs at the beginning of each period, we are back with the standard representative consumer, and the notation used to distinguish employment status may be eliminated. The representative low-skilled household i maximises the expectation of the discounted sum of its instantaneous utility with respect to the consumption and the assets she holds:

$$\mathbf{\dot{X}}_{t=0} \overset{\circ}{\overset{-t}{\overset{\circ}{\overset{\circ}{\overset{}}}}} N_t^{\mathsf{I}} u_{i;t}^{\mathsf{I}} + (1_{\mathsf{I}} N_t^{\mathsf{I}}) u_{i;t}^{\mathsf{I}^2}$$

subject to (A.2.17). Equation (A.2.17) is the consolidated budget constraint after we introduce perfect assurance:

$$N_{t}^{I}C_{i;t}^{I} + (1_{i} N_{t}^{I})C_{i;t}^{I^{2}} + B_{i;t+1}^{I} N_{t}^{I}w_{ijt}^{I} + (1 + r_{t})B_{i;t}^{I}:$$
(A.2.17)

#### A.2.2. The highly-skilled consumer problem

Like the problem of the low-skilled worker, the highly-skilled worker faces di¤erent states on the labour market.

 $C_{h;it}^{h}$ ,  $C_{l;it}^{h}$ , and  $C_{it}^{h^{?}}$  denote the respective highly-skilled household's consumption and  $i^{h}$ ,  $i^{l}$  and  $i^{h^{n}}$  can be interpreted as a utility cost, expressed in terms of goods, associated with the situation of the household in the labour market. We suppose that  $i^{h}_{l} = i^{l}$ , given that they work in a low-skilled job. We further assume  $i^{h} > i^{l} > i^{h^{n}}$ , which ensures that consumption of an employee in a highly-skilled vacancy is greater than that in a low-skilled vacancy and both greater than that of an unemployed household.

 $B_{h;it}^{h}$ ,  $B_{l;it}^{h}$  and  $B_{it}^{h^{2}}$  denote contingent claims purchased by the highly-skilled household in the previous period. At the beginning of each period, the household receives the value of bonds purchased in the previous period, either highly-employed, low-employed or unemployed. She also receives the wage when employed as a highly-skilled, the wage when employed as a low-skilled plus the insurance payment for this unsatisfactory situation and the insurance payment when unemployed. Its expenditures, either employed or unemployed, are consumption, insurance and purchase of bonds.

The instantaneous utility contingent to be employed in a highly-skilled or low-skilled job and unemployed are,

$$u_{h;it}^{h} = \log(C_{h;it}^{h} i i^{h})$$
 (employed in a highly-skilled job) (A.2.18)

$$u_{l;it}^n = \log(C_{l;it}^n i i')$$
 (employed in a low-skilled job) (A.2.19)

$$u_{it}^{h^{\alpha}} = \log(C_{i,t}^{I^{\alpha}} i_{j}^{I^{\alpha}}) \text{ (unemployed)}$$
(A.2.20)

As in the problem of a low-skilled worker, the highly-skilled household maximises the recursively problem

$$V^{H}(S^{h}_{it}) = N^{h}_{h;t}u^{h}_{h;i;t} + N^{h}_{l;t}u^{h}_{l;i;t} + (1_{i} N^{h}_{h;t} N^{h}_{l;t})u^{h^{\alpha}}_{l;t} + {}^{-}V^{H}(S^{h}_{i;t+1});$$
(A.2.21)

where state variables have three alternative states depending whether the household is employed as highly-skilled or low-skilled or unemployed.  $^- 2$  (0; 1) is the discount factor. We made use of the fact that by de...nition of  $p_{h;t'}^h$  and the law of motion of  $N_{h;t+1}^h$  and  $N_{l;t+1}^h$ .

$$N_{h;t+1}^{h} = p_{h;t}^{h}(1_{i} N_{h;t}^{h}) + (1_{i} s)N_{h;t}^{h}$$
 (A.2.22)

$$N_{h;t+1}^{h} = p_{h;t}^{h}(1 | N_{h;t}^{h}) + (1 | 3)N_{h;t}^{h}$$

$$(A.2.23)$$

Highly skilled worker maximises her household's problem taking into account her discounted expected value — solve the Bellman equation subject to the budget constraints

$$C_{h;it}^{h} + \dot{z}_{t}^{h} W_{it}^{h} + \dot{z}_{t}^{h^{n}} W_{it}^{h^{n}} + B_{h;it+1}^{h} \qquad (1 + r_{t}) B_{h;it}^{h} + W_{h;it}^{h} \qquad (A.2.24)$$

$$C_{l;it}^{h} + \dot{z}_{t}^{h} \ddot{w}_{it}^{h} + \dot{z}_{t}^{h^{a}} \ddot{w}_{it}^{h^{a}} + B_{l;it+1}^{h} \qquad (1 + r_{t})B_{l;it}^{h} + w_{l;it}^{h} + w_{it}^{h} \qquad (A.2.25)$$

$$C_{it}^{h^{?}} + \dot{z}_{t}^{h} {}^{h}_{it} + \dot{z}_{t}^{h^{u}} {}^{h^{u}}_{it} + B_{it+1}^{h^{?}} \qquad (1 + r_{t}) B_{it}^{h^{?}} + {}^{h^{u}}_{it} \qquad (A.2.26)$$

where  $\$_{it}^{h^{a}}$  and  $\$_{it}^{h}$ ; are the insurance contracts with respective prices  $\natural_{t}^{h^{a}}$  and  $\natural_{t}^{h}$ . The household chooses among both type of contracts to be insured against the probability of being unemployed or employed in a low-skilled job. When she works as a low-skilled employee, she receives  $\$_{it}^{h}$  to compensate the wage dimerential with respect to a highly-skilled job. When unemployed, she receives  $\$_{it}^{h^{a}}$  as unemployment bene...t.  $m_{h;tt}^{h}$ ,  $m_{l;tt}^{h}$  and  $m_{it}^{h^{2}}$  denote the Lagrange multipliers associated to budget constraint of the representative highly-skilled household when highly-employed, low-employed and unemployed, respectively.

As in the problem of the low-skilled worker, we ... rst form the Lagrangian

$$L = N_{h;t}^{h} u_{h;i;t}^{h} + N_{l;t}^{h} u_{l;i;t}^{h} + (1 i N_{h;t}^{h} i N_{l;t}^{h}) u_{i;t}^{h*} + {}^{-}V^{H}(S_{it+1}^{h})$$

$$+ N_{h;t}^{h} u_{h;i;t}^{h} i (1 + r_{t}) B_{h;it}^{h} + w_{h;it}^{h} i C_{h;it}^{h} i$$

$$+ N_{h;t}^{h} u_{h;i;t}^{h} i i i i i t_{t}^{h*} u_{i;t}^{h*} i B_{h;it+1}^{h} \eta$$

$$+ N_{l;t}^{h} u_{l;i;t}^{h} i i i i t_{t}^{h*} u_{i;t}^{h*} + u_{l;it}^{h} i B_{h;it+1}^{h} \eta$$

$$+ N_{l;t}^{h} u_{l;i;t}^{h} i i i i i t_{t}^{h*} u_{i;t}^{h*} i B_{l;it+1}^{h*} \eta$$

$$+ N_{l;t}^{h} u_{l;i;t}^{h} i i i t_{t}^{h*} u_{i;t}^{h*} i B_{l;it+1}^{h*} \eta$$

$$+ N_{l;t}^{h} u_{l;i;t}^{h} i i t_{t}^{h*} u_{i;t}^{h*} i B_{l;it+1}^{h*} \eta$$

$$+ N_{l;t}^{h} u_{l;i;t}^{h*} i i t_{t}^{h*} u_{i;t}^{h*} i B_{l;t+1}^{h*} \eta$$

$$+ N_{h;t}^{h} u_{l;t}^{h} i N_{l;t}^{h}) u_{i;t}^{h*} u_{i;t}^{h*} i I B_{h;t+1}^{h*} \eta$$

The ...rst order conditions with respect to  $C_{h;it}^{h}$ ,  $C_{l;it}^{h}$ ,  $C_{it}^{h^{2}}$ ;  $%_{it}^{h}$  and  $%_{it}^{h^{u}}$  are:

$$(C_{h;it}^{h}_{i}_{i}_{j}^{i}_{j}^{h})^{i}_{i}^{1} = x_{h;it}^{h}$$
(A.2.27)

$$(C_{l;it}^{h} i i^{l})^{i} = x_{l;it}^{h}$$
(A.2.28)

$$(C_{it}^{h^{2}} i i^{h^{\alpha}})^{i} = \alpha_{it}^{h^{2}}$$
(A.2.29)

$$i N_{h;t}^{h} \dot{z}_{t}^{h} \mathbf{x}_{h;it}^{h} + N_{l;t}^{h} (1_{i} \dot{z}_{t}^{h}) \mathbf{x}_{l;it}^{h} i (1_{i} N_{h;t}^{h} i N_{l;t}^{h}) \dot{z}_{t}^{h} \mathbf{x}_{it}^{h'} = 0$$
(A.2.30)

$$i N_{h;t\dot{c}t}^{h^{a}} \alpha_{h;tt\,i}^{h} N_{l;t\dot{c}t}^{h,i} \alpha_{l;t\dot{c}t}^{h} \alpha_{l;t\dot{c}t}^{h} + (1 i N_{h;t\,i}^{h} N_{l;t}^{h})(1 i \dot{c}_{t}^{h^{a}}) \alpha_{it}^{h^{2}} = 0: \quad (A.2.31)$$

Equations (A.2.27), (A.2.28) and (A.2.29) state that the Lagrange multipliers  $a_{h;it}^{h}, a_{l;it}^{h}$  and  $a_{it}^{h^{2}}$  are equal to the marginal value of consumption in the corresponding state on the labour market. The ...rst order conditions (A.2.30) and (A.2.31) represent the marginal values to be fully assured taking into account the probability of being employed in a low-skilled job or unemployed respectively.

The ...rst order conditions related to  $B_{h;it+1}^{h}$ ,  $B_{l;it+1}^{h}$  and  $B_{it+1}^{h^{?}}$  are:

$$x_{l;it}^{h} = -\frac{@V^{H}(S_{it+1}^{h})}{@B_{l;it+1}^{h}}$$
(A.2.33)

Envelope theorem related to  $B_{h;t}^h$ ,  $B_{l;t}^h$  and  $B_{it}^{h^2}$  yields

$$\frac{{}^{@}V^{H}(S_{it}^{h})}{{}^{@}B_{h;it}^{h}} = (1 + r_{t}) {}^{a}{}^{h}_{h;it}$$
(A.2.35)

$$\frac{@V^{H}(S_{it}^{h})}{@B_{l;it}^{h}} = (1 + r_{t}) \alpha_{l;it}^{h}$$
(A.2.36)

$$\frac{{}^{@}V^{H}(S_{it}^{h})}{{}^{@}B_{it}^{h^{n}}} = (1 + r_{t}) {}^{n}{}^{h^{n}}: \qquad (A.2.37)$$

Envelope conditions together with ...rst order conditions related to optimal portfolio composition yields the following Euler conditions:

$$\mathbf{x}_{h;it}^{h} = (1 + r_{t+1})\mathbf{x}_{h;it+1}^{h}$$
(A.2.38)

$$\mathbf{x}_{l;it}^{h} = (1 + r_{t+1})\mathbf{x}_{l;it+1}^{h}$$
(A.2.39)

$$\mathbf{x}_{it}^{h^{2}} = (1 + r_{t+1})\mathbf{x}_{it+1}^{h^{2}}$$
(A.2.40)

Similarly to the low-skilled problem, we can express the problem of an insurance company for highly-skilled households as

$$I_{i} = i_{t}^{h} N_{it}^{h} + i_{t}^{h^{a}} N_{it}^{h^{a}} i N_{l;t}^{h} N_{it}^{h} i (1 i N_{h;t}^{h} i N_{l;t}^{h}) N_{it}^{h^{a}} = 0;$$

from which we get  $\dot{z}_t^h = N_{I;t}^h$  and  $\dot{z}_t^{h^{\alpha}} = (1 \ _i \ N_{h;t}^h \ _i \ N_{I;t}^h)$ .

These results together with household's ... rst order conditions implies:

which, together with the Frischian demand for consumption, yields

$$C_{h;it}^{h} i i^{h} = C_{l;it}^{h} i^{l} = C_{it}^{h?} i^{r} i^{?}$$
:

Therefore, the levels of insurance are given by  $\%_{it}^h = w_{h;it}^h + i_i i_j h_i w_{l;it}^h$  and  $\%_{it}^{h^a} = w_{h;it}^h + i_j h^a i_j h^a$ .

To summarise, as in the low-skilled worker problem, as soon as households choose to be fully insured, and provided the employed labour force is selected randomly across jobs at the beginning of each period, we are back with the standard representative consumer, and the notation used to distinguish employment status may be eliminated. A representative highly-skilled household i therefore maximises the expectation of the discounted sum of her instantaneous utility with respect to the consumption and the assets she holds:

$$\overset{\mathbf{A}}{\underset{t=0}{\overset{-t}{\overset{\otimes}}}} N^{h}_{h;t} u^{h}_{h;i;t} + N^{h}_{l;t} u^{h}_{l;i;t} + (1 \ i \ N^{h}_{h;t} \ i \ N^{h}_{l;t}) u^{h^{2}}_{i;t} }$$

subject to (A.2.41). Equation (A.2.42) is the consolidated budget constraint when we introduce perfect assurance:

$$N_{h;t}^{h}C_{h;i;t}^{h} + N_{l;t}^{h}C_{l;i;t}^{h} + U_{t}^{h}C_{i;t}^{h^{2}} + B_{i;t+1}^{h} - N_{h;t}^{h}w_{h;ijt}^{h} + N_{l;t}^{h}w_{l;ijt}^{h} + (1 + r_{t})B_{i;t}^{h}: (A.2.41)$$

#### Appendix A:3 : Wage determination

This section is devoted to the exposition of the wage bargaining process, which is determined by a Nash bargaining criterion, therefore yielding a surplus sharing rule. At the beginning of every period a re-negotiation simultaneously occurs between the ...rm and workers of each group. Otherwise there will be many wages as workers and the macroeconomic dynamic would be done with respect to a wages distribution. Therefore, workers negotiate their wages with the ...rm and they account for separations and hirings probabilities.

The wage setting behaviour is obtained maximising the following Nash criterion with respect to the wages which maximise the weighted product of the workers' and the ...rm's net return from the di¤erent job match.

The gains of the ...rm corresponds to marginal values. Thus, let  $-\dot{\epsilon}_{;jt}$  the surplus of the ...rm associated to each group of employment. The gains for workers' correspond to the sum of the utilities when they are employed minus the sum of utilities when the negotiation fails and they become unemployed. Let  $\frac{a_{it}}{a_{it}}$  the dimerent surplus of workers associated to each group of workers in terms of goods. Where (i; i) = f(h; h); (h; l); (l; i)g and  $w^{i}$  are the exogenous parameters of bargaining powers of each group of workers. Thus, the Nash bargaining criterion problem to solve is

$$\max_{\substack{w_{i_{1}t} \\ w_{i_{1}t}}} i_{-\frac{i_{1}}{2}t} + \frac{\mu_{a_{i_{1}}}}{\frac{i_{1}}{2}} + \frac{\mu_{a_{i_{1}}}}{\frac{i_{1}}{2}} + \frac{1}{2}$$

We de...ne the surplus of the ...rm and the workers as following:

#### A.3.1. The ...rm j surplus

Let us ...rst characterise the surplus which accrues to a ...rm j when it employs a highlyskilled worker on a highly-skilled position. This is essentially given by the marginal value of a highly-skilled employment,  $-\frac{h}{h;jt}$ . From the optimal condition associated with a highly-skilled employment, we get

$$-{}^{h}_{h;jt} = \mu \frac{Y_{jt}}{L^{h}N^{h}_{h;jt}} i w^{h}_{h;ijt} + (1 i s)X^{h}_{h;jt}$$
(A.3.1)

Likewise in the case of a highly-skilled worker employed on a low-skilled position, the marginal value of employment is given by

$$-_{l;jt}^{h} = (1 i \ ^{\text{e}} i \ \mu) \frac{Y_{jt}}{L^{h} N_{l;jt}^{h} + L^{l} N_{jt}^{l}} i \ w_{l;ijt}^{h}:$$
(A.3.2)

Finally, when it hires an additional low-skilled worker, this gain is given by

$$-_{jt}^{I} = (1 i \ ^{\text{e}} i \ \mu) \frac{Y_{jt}}{L^{h} N_{l;jt}^{h} + L^{l} N_{jt}^{l}} i \ w_{ijt}^{l} + (1 i^{-1}) X_{jt}^{l}$$
(A.3.3)

where  $X_{h;j\,t}^{\,h}$  and  $X_{j\,t}^{\,l}$  are the Lagrange multipliers associated to employment laws of motion.

#### A.3.2. The household i surplus

Let us now present the determination of the surplus which accrues to each type of worker.

The low-skilled worker:

A low-skilled worker, when employed in period t, instantaneously derives utility associated to her extra gains in the labour market ( $x_{it}$  times the wage revenues net of disutility of labour in terms of goods). Furthermore, in the next period, she may still be employed with probability (1 i <sup>1</sup>) or laid o<sup>x</sup> with probability <sup>1</sup>. Therefore, the utility gain of an employed low-skilled worker in the labour market,  $\frac{e}{1}$  is given by

$${}^{\circ}{}^{e}{}_{i;it} = {}^{\alpha}{}_{it}(w_{ijt}^{i}{}_{i}{}_{i}{}_{i}{}^{i}) + {}^{-i}(1{}_{i}{}^{-1}){}^{\circ}{}^{e}{}_{i;it+1} + {}^{1}{}^{\circ}{}^{u}{}_{i;it+1}$$

where  $a_{it}^{21}$  is the marginal utility of consumption. Likewise, when unemployed, the low-skilled household gets,  $u_{it}^{u}$ 

$$\overset{u}{}_{l;it} = i \, \alpha_{it\,i} \, I^{\alpha} + {}^{-i} p_{l;t} \overset{e}{}_{l;it+1} + (1 \, i \, p_{l;t})^{\cdots} {}^{c}_{l;it+1}$$

Then, the net surplus of a low-skilled worker in the labour market is given by

$${}^{a}{}^{l}_{it} {}^{\cdots}{}^{e}_{l;it} {}^{i}_{l} {}^{u}_{l;it} = {}^{a}{}^{i}_{it} ({w}^{l}_{ijt} + {}^{l?}{}^{i}_{l} {}^{i}_{l}) + {}^{-} (1 {}^{i}_{l} {}^{1}_{l} {}^{p}_{l;t})^{a}{}^{l}_{it+1}:$$
(A.3.4)

The highly-skilled worker:

A highly-skilled worker may be employed either in a highly-skilled or a low-skilled vacancy, or may be unemployed. When employed as a highly-skilled worker, she instantaneously derives utility  $a_{it}(W_{t\ i\ i\ }^{h})$ , and in the following period she may be unemployed or employed on either a highly or a low-skilled position. Therefore, the utility gain in the labour market for a highly-skilled worker employed on a highly-skilled position is given by

$$\overset{"}{}_{h;it}^{h} = \varkappa_{it}(w_{h;ijt}^{h}, i, i^{h}) + \overset{-}{\overset{\mu}{}} (1 ; s) \overset{"}{}_{h;it+1}^{h} + sp_{l;t}^{h}, \overset{"}{}_{l;it+1}^{h} + \\ + s(1 ; p_{l;t}^{h}) \overset{"}{}_{u;it+1}^{h}$$

where  $h_{i,i}$ ,  $h_{i,i}$  and  $h_{i,i}$  respectively denote the utility gain when a highly-skilled worker employed in a highly-skilled job, in a low-skilled job or unemployed worker.

Following the same procedure, the value of a highly-skilled worker employed in a low-skilled position is

$$\overset{n}{}_{l;it} = \mathtt{m}_{it}(\mathsf{W}_{l;ijt}^{h} \ i \ i') + - \underbrace{\overset{\mu}{}_{p_{h;t}^{h}} \overset{n}{}_{h;it+1}^{h} + (1 \ i \ p_{h;t}^{h}) p_{l;t}^{h} \overset{n}{}_{l;it+1}^{h} + (1 \ i \ p_{h;t}^{h}) p_{l;t}^{h} \overset{n}{}_{u;it+1}^{h} + (1 \ i \ p_{h;t}^{h}) (1 \ i \ p_{h;t}^{h}) \overset{n}{}_{u;it+1}^{h}$$

<sup>21</sup>From Appendix A.1, we can see that  $\mathbf{x}_{i;t}^{I} = \mathbf{x}_{i;t}^{I} = \mathbf{x}_{i;t}$ 

and similarly the value when unemployed is

The surplus of a highly-skilled worker, when employed on a highly-skilled position, is actually given by the gain from being employed as a highly-skilled worker minus the gain from being employed as a low-skilled worker times the probability that this event occurs, minus the gain from being unemployed, times the probability of being unemployed. Such that the overall surplus is given by (using the law of large number)

$$\begin{array}{rcl} {}^{a}{}^{h}{}_{h;it} & & \stackrel{h}{\underset{h;it}{i}} & i & p_{l;t_{1}}^{h}{}_{1} & \stackrel{h}{\underset{l;it}{i}} & i & (1_{i} & p_{l;t_{1}}^{h}{}_{1})^{\cdots}{}_{u;it}^{h} = & \mathbb{x}_{it} (w_{l;ijt}^{h} + i & i^{h'} i & i^{l}) \\ & & \mu \\ {}^{a}{}^{h}{}_{h;it} & = & \mathbb{x}_{it} & w_{h;ijt}^{h}{}_{ijt} & i^{h} i & \frac{N_{l;t}^{h}}{1 & N_{h;t}^{h}} (w_{l;ijt}^{h} i & i^{l}) \\ & & \tilde{A} & & I \\ & & + & 1_{i} & \frac{N_{l;t}^{h}}{1_{i} & N_{h;t}^{h}} & i^{h''} + (1_{i} & s_{i} & p_{h;t}^{h})^{a}{}_{h;it+1}^{h'}: \end{array}$$
(A.3.5)

When bargaining on a low-skilled job, the only opportunity left is unemployment such that the surplus is now given by the di¤erence between the utility gains from being employed on a low-skilled position minus the utility gains from being unemployed

$$a_{l;it}^{h} \cdots_{l;it}^{h} i_{u;it}^{l} = a_{it}(W_{l;ijt}^{h} + i_{i}^{h^{2}} i_{i}^{l}):$$
(A.3.6)

We now examine the Nash bargaining process that determines the real wage in each case.

The Nash–Bargaining process

The wage setting behaviour is obtained maximizing the following Nash criterion

$$\max_{W_{i_{j_{t}}}} \mathbf{i}_{-\frac{\dot{c}_{j_{t}}}{y_{t}}} \mathbf{\mu}_{\frac{\mathbf{a}}{\dot{c}_{i_{t}}}} \mathbf{\mu}_{\frac{\mathbf{a}}{\dot{c}_{i_{t}}}} \mathbf{u}_{\frac{\mathbf{a}}{i_{t}}} \mathbf{u}_{\frac{\mathbf{a}}{i_{t}}} \mathbf{u}_{\frac{\mathbf{a}}{i_{t}}}$$

where (i; i) = f(h; h); (h; l); (l; i)g. The ...rst order condition associated to this program – making use of the de...nitions of the corresponding surpluses — yields

$$\frac{\overset{i}{\mathbf{x}^{i}}}{\mathbf{1}_{\mathbf{j}}} \overset{i}{\mathbf{x}^{i}} \overset{i}{\mathbf{x}_{\mathbf{j}}} = \frac{\overset{a}{\mathbf{z}_{\mathbf{j}}}}{\overset{i}{\mathbf{x}_{\mathbf{i}}}}:$$

The marginal values of employment of the household and the ...rm together with the above equation implies  $\tilde{a}$ 

$$w_{h;t}^{h} = w_{h}^{h} \mu \frac{Y_{t}}{L^{h}N_{h;t}^{h}} + p_{h;t}^{h}X_{h;t}^{h} + (1_{i} w_{h}^{h}) + \frac{N_{l;t}^{h}}{1_{i} N_{h;t}^{h}}(w_{l;t i}^{h} i^{l})$$
(A.3.7)  
$$\tilde{A}$$

$$w_{l;t}^{h} = w_{l}^{h} (1_{i} \otimes \mu) \frac{Y_{jt}}{L^{h}N_{l;jt}^{h} + L^{l}N_{jt}^{l}} + (1_{i} \otimes_{h}^{h})_{i}^{l}$$
(A.3.8)  
$$\tilde{\mathbf{A}}$$

$$w_{t}^{I} = *^{I} (1_{i} \otimes i_{j} \mu) \frac{Y_{jt}}{L^{h}N_{1;jt}^{h} + L^{I}N_{jt}^{I}} + p_{t}X_{t}^{I} + (1_{i} *^{I})_{i}^{I}$$
(A.3.9)

assuming a symmetric equilibrium, and using the fact that, in our calibration, We have assumed  $i^{l^{\pi}} = i^{h^{\pi}} = 0$ .

#### Appendix B: Stylised facts and data de...nitions

Spanish education levels have steadily grown from the beginning of the 80's and especially from the middle of the 80's. We can see in Table 5 the evolution of education level for the Spanish labour force. The Illiterate and Primary Schooling have steadily reduced the weight in the Spanish labour force. The increase in education has greatly enlarged the secondary group and has made the top educated people (Upper-secondary +Superior schooling) increase its weight by 100% in the labour force.

But the most problematic issue is the high persistence of the unemployment rates in those low-skilled groups. Illiterate and Primary Schooling have worsened their unemployment rates during the last decade while the top educated largely reduce unemployment rates in the booms and do not increase so much in the crisis (see Table 6).

#### Employment data

Linked labour Population Survey (Encuesta de Población Activa enlazada, EPA hereafter). This quarterly survey collects data of same individuals during six consecutive quarters periods. It covers a large number of individuals and characteristics, such as formal education attainment, occupation, employment status, age and gender. For our estimations and stylised facts we have used the de...nitions of education and occupation in the linked EPA survey.<sup>22</sup>. We have estimated our data in the introduction and in the section data and calibration at following. We have divided as below between high and low skilled categories the education and the occupation. Then, we have observed the state of worker in the period and her situation by education and by occupation to estimate the percentages of labour market (i.e.  $N_h^h = a$  worker who is highly-educated and is employed in a highly-skilled category). To estimate probabilities we have observed an unemployed worker with her education in a period and we estimated what the probability would be that the same worker take a high or low skilled job in the next period. Data are estimated by quarters from 1988 to 1996.

De...nitions:

- <sup>2</sup> We use 5 groups of education:
- 1. Illiterate and uneducated persons.
- 2. Primary education.
- 3. Low-secondary education.
- 4. Upper-secondary education.
- 5. Superior.

<sup>&</sup>lt;sup>22</sup>For estimations of proportions and probabilities we have used the SAS (Statitical Analysis System) program.

Highly-skilled individuals are de...ned, for our purpose, as those with a level of education greater of equal to upper-secondary education. Therefore, low-skilled individuals essentially consist of illiterate and uneducated persons, primary or low-secondary educated people.

- <sup>2</sup> We use 10 groups of occupation (range 0–9 in EPA and INEM classi...cation):
  1.Business and civil service management.
- 1. 2. Technicians and scienti...cs and intellectual professionals.
  - 3. Technicians and support professionals.
  - 4. Clerical employees.
  - 5. Workers in restaurant, catering, personal and security.
  - 6.Skilled agricultural and ...shing workers.
  - 7. Artesans and skilled manufacturing, construction and mining (Craft-Trade workers).
  - 8. Facilities and machine operators, ...tters (Plant-Machine Operators).
  - 9. Unskilled workers.
  - 10.Armed Forces.

Highly-skilled occupations are taken to be managers, professionals, technicians and support professionals (range 1–3 in EPA classi...cation), the rest (range 0,4–9 in EPA) are armed forces, clerks, service, skilled agriculturaln...shing, Craft-Transport, Plant-Manufacture and unskilled group) is taken to de...ne low-skilled occupations.

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