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SEPARABILITY AND PUBLIC FINANCE

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Executive Summary

In a first best setting, all information is publicly available and the government redistributes income at will. Efficiency can be achieved independently of equity concerns through lump sum transfers. In the absence of market failures, indirect taxation is superfluous and intermediary products should not be taxed. Externalities, if any, can be corrected through Pigovian taxes, and the Samuelson rule applies to the provision of public goods. In a second best environment, where some individual characteristics are privately known and not publicly observable, the associated incentive constraints restrict the scope of redistribution. The optimal rules of taxation then typically differ from the first best ones, and efficiency can no longer be disconnected from equity considerations.

Still the public finance literature exhibits cases in which the optimal second best rules have a first best flavor. In all these cases the economy displays some form of separability, either in the individual preferences or in the production side, coupled with a specific information structure. The purpose of this note is to put forward their common underlying structure. It appears that one can isolate in the second best program a part which has the shape of the first best, in the sense that the incentive constraints drop out. Thus the properties of first best allocations apply to the variables that are determined in this piece of the program. This observation allows both to simplify the arguments and to generalize the results.

Abstract

In a second best environment, the optimal policy choice sometimes follows the first best rules. This note lays down the information structure and separability assumptions under which this property holds in a variety of setups.

1 Introduction

In a first best setting, all information is publicly available and the government redistributes income at will. Efficiency can be achieved independently of equity concerns through lump sum transfers. In the absence of market failures, indirect taxation is superfluous and intermediary products should not be taxed. Externalities, if any, can be corrected through Pigovian taxes, and the Samuelson rule applies to the provision of public goods. In a second best environment, where some individual characteristics are privately known and not publicly observable, the associated incentive constraints restrict the scope of redistribution. The optimal rules of taxation then typically differ from the first best ones, and efficiency can no longer be disconnected from equity considerations.

Still the public finance literature exhibits cases in which the optimal second best rules have a first best flavor. The best known instance is the Atkinson and Stiglitz (1976) theorem, according to which indirect taxation is useless though individual productivity is private information (see also Kaplow (2006) and Laroque (2005)). In a different vein, Bovenberg and Jacobs (2005) present a set of assumptions under which education, as an intermediary input, should be tax-free. In Cremer, Gahvari, and Ladoux (1998) Pigovian taxes are the appropriate way to handle externalities, while a version of the Samuelson rule applies in Boadway and Keen (1993), Guesnerie (1995), Nava, Schroyen, and Marchand (1996) and Hellwig (2008b).

All these examples display some form of separability, either in the individual preferences or in the production side, coupled with a specific information structure. The purpose of this note is to put forward their common underlying structure. It appears that one can isolate in the second best program a part which has the shape of the first best, in the sense that the incentive constraints drop out. Thus the properties of first best allocations apply to the variables that are determined in this piece of the program. This observation allows both to simplify the arguments and to generalize the results.

The approach is simpler since it avoids solving for the second best allocation, thus bypassing the difficulties associated with first order conditions and bunching. The technique also allows to study unexplored setups. For instance, indirect taxation appears to be superfluous in situations where agents differ along other dimensions than in Atkinson and Stiglitz (1976). This is the case whenever this additional heterogeneity is publicly observable. If the differences are privately known, the property holds when they do not affect the tastes for consumption goods. Otherwise, indirect taxation typically is a useful instrument on top of direct taxation.

This note is organized as follows. The next section presents a regularity assumption that must be satisfied by the second best allocations under study. Three different models are then examined, dealing with indirect taxation, the financing of education and the provision of public goods.

2   A non-satiation property

Consider an economy with consumers indexed by a possibly multidimensional characteristic n whose cumulative distribution function in the population is $F(\cdot)$. Let the preferences of consumer n be represented by a utility function $U(c_n, g, n)$, where c_n is a finite vector of private goods and g is a public good. In this economy the feasibility constraints write

$$\int_n c_n dF(n) + \gamma(g) \leq \int_j y_j.$$

The cost in private goods of producing g is $\gamma(g)$, and the output y_j belongs to some production set Y_j satisfying Arrow Debreu assumptions.

In a first best setup, the government can impose the allocation it prefers on the private sector. When goods are desirable, a benevolent government chooses an allocation that exhausts resources, so that the feasibility constraints are strictly binding. A (small) increase of resources in private goods then leads to a Pareto improvement. We designate an allocation satisfying this property as a (locally) *non-satiated* optimum.

In a second best situation, the government has more limited power, which translates into additional constraints. We shall consider second best optima which are locally non-satiated, in spite of these constraints. The production efficiency lemma of Diamond and Mirrlees (1971) states conditions under which the non-satiation property is satisfied in an economy with linear taxation. This property is also valid with non-linear taxation in the Mirrlees (1971) setup, under the single-crossing condition (Hellwig (2007)). It seems plausible that second best optima are locally non-satiated in fairly general circumstances, for instance in the absence of single-crossing under multidimensional heterogeneity.

3   Indirect taxes

3.1   Ability is private information

There are k consumption goods indexed by i ($i = 1, \ldots, k$) and labor. The goods are produced from labor according to a linear technology such that one unit of good i is obtained with p^i units of effective labor. The typical agent has a before tax income $y = n\ell$, the product of her productivity n by her labor supply ℓ. Her preferences are represented by a utility function $U(V(c), \ell)$, where c is her consumption vector. This function is separable between consumption goods on the one hand, and labor supply on the other hand. The argument extends to situations where preferences differ (see Section 3.2).

The government observes individual income y but not separately individual productivity nor labor supply. It announces a non linear income tax schedule $R(\cdot)$

which relates before tax income y to after tax income $R(y)$. It may also impose linear taxes $q - p$ on consumption goods, where q is the vector of consumer prices. Given q and $R(\cdot)$, an agent whose before tax income is y chooses a bundle of consumption goods which maximizes $V(c)$ subject to the budget constraint $qc = R(y)$. The solution to this problem is the conditional demand function $c(q, R(y))$.

Lemma 1. *Consider a* non-satiated *second best allocation in which agent n has before tax income y_n^* and consumes c_n^*. Given (y_n^*), (c_n^*) is a first best allocation of the economy where all the agents have the same quasi-concave and increasing utility function $V(\cdot)$ and the aggregate production set is*

$$\int_n pc_n dF(n) \leq \int_n y_n^* dF(n). \tag{1}$$

Indirect taxation is therefore superfluous.

Proof. The government chooses a vector of consumer prices q, an income profile (y_n) and the corresponding after tax income profile $R(y_n)$, which maximize a weighted sum of utilities subject to the incentive compatibility constraints

$$U(V(c(q, R(y_n))), y_n/n) \geq U(V(c(q, R(y_m))), y_m/n)$$

for every (m, n), and the resource constraint

$$\int_n pc(q, R(y_n)) dF(n) = \int_n y_n dF(n).$$

Let $(q^*, (y_n^*, R^*(y_n^*)))$ be the second best optimum. Let V_n^* be the sub-utility derived from consumption at this optimum, i.e., $V_n^* = V(c(q^*, R^*(y_n^*)))$.

For a given couple $(q, R(\cdot))$, let $W_{q,R}(y) = V(c(q, R(y)))$, a definition that implicitly uses the assumption that the function $V(\cdot)$ is identical across agents. Then an income profile (y_n) satisfies the incentive compatibility constraints if

$$U(W_{q,R}(y_n), y_n/n) \geq U(W_{q,R}(y_m), y_m/n).$$

Therefore, given the optimal distribution (y_n^*) of before tax income, any $(q, (R(y_n^*)))$ that leaves the utility profile (V_n^*) unchanged is incentive compatible. Under the second welfare theorem (Mas-Colell, Whinston, and Green (1995), Proposition 6.D.1), any first best optimum of the economy where agents have preferences $V(\cdot)$ and where the feasible set is defined by

$$\int_n pc_n dF(n) \leq \int_n y_n^* dF(n),$$

can be decentralized as a quasi-equilibrium with an appropriate choice of $(q, R(\cdot))$. Note that a non linear income tax here is needed for the government to be able to

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select freely the after tax income distribution. Now, we proceed by contradiction to prove Lemma 1. Suppose that $(q^*, (R^*(y_n^*)))$ is such that the second best allocation (c_n^*) is not a first best optimum of the economy. Then one can achieve the same utility profile with aggregate resources less than $\int_n y_n^* dF(n)$. The non-satiation property gives the desired contradiction. $\square$

Remark 1. The property holds for an economy with a general production set Y, where the input is the aggregate supply of labor, measured in efficiency units. One just needs to replace (1) with

$$\int_n (-y_n, c_n) \in Y.$$

The production set Y can be derived from a more detailed structure, involving intermediary inputs that are produced and used in the production process. Then Lemma 1 implies no price distortions in the allocation of these inputs.

Remark 2. The previous argument extends to any situation where labor supply is a function of the observable y and productivity n. However with a general production set, as in the previous remark, it is natural to have wage, or labor productivity, function of aggregate variables, e.g., aggregate capital or production. As discussed in Stiglitz (1985), Lemma 1 then does not carry through. For instance, if earnings depend on aggregate consumption, consumption enters the incentive compatibility constraints outside the sub-utility $V(\cdot)$ once income is substituted for labor supply, and separability fails.

3.2 More general heterogeneity

This section studies whether and how additional heterogeneity may affect the result. The answer depends on whether this heterogeneity is observable.

Assume first that, on top of ability n, the agents differ along other characteristics z, such as family size, that are both *non-observable* and exogenous. On the one hand, if the utility functions are of the form $U(V(c), \ell, z)$, the argument trivially extends and Lemma 1 applies. On the other hand, if the tastes for consumption goods vary with the size of the family, i.e., z is an argument of the function V, the utility derived from consumption goods when agent (n, z) imitates agent (n', z') depends on his type z and Lemma 1 in general no longer holds.

When z is non-observable but chosen endogenously, the situation depends on how the choice of z varies with the agent's type n, given the government instruments. If z is not an argument of V, the Lemma holds provided that z only depends on variables not in V. Otherwise separability breaks down.

Lemma 1 remains valid when the exogenous characteristics z are *observable*. Then an agent can only imitate someone with the same z. If the utility functions are of the form $U(V(c,z),\ell,z)$, the allocation of consumption goods is first best for the economy where agents have utility functions $(V(c,z))$.

Consider finally the case where the *observable z* are endogenously chosen by the agents. Everyone has a common utility function $U(V(c,z),\ell,z)$ and picks a z from a common set Z, again think of family size. The policy instruments are the vector of consumer prices q, and the after tax income $R(y_n,z_n)$ associated with the profile (y_n,z_n). Then indirect taxation still is superfluous. A sketch of the proof is as follows. Let $W_{q,R}(y,z) = V(c(q,R(y,z)),z)$. The profile (y_n,z_n) satisfies the incentive compatibility constraints if

$$U(W_{q,R}(y_n,z_n),y_n/n,z_n) \geq U(W_{q,R}(y_m,z_m),y_m/n,z_m).$$

Let $V_n^* = V(c(q^*,R(y_n^*,z_n^*)),z_n^*)$ be the sub-utility obtained by agent n at the second best optimum. Given (y_n^*,z_n^*), any $(q,(R(y_n^*,z_n^*)))$ that leaves (V_n^*) unchanged is incentive compatible.[1]

To summarize, indirect taxation is superfluous in situations where there is more heterogeneity than in Atkinson and Stiglitz (1976). If this heterogeneity is privately known, the result obtains when the tastes for consumption goods are independent of this extra heterogeneity, something that is not required when the relevant information is observed by the government. However, when hidden private heterogeneity enters the sub-utility V, Lemma 1 fails.

4   Individual production sets

There is a single physical good, which is consumed or serves as an input in the production process. All agents have identical utility functions $U(c,\ell)$, with c final consumption. Net production of agent n is

$$y = \Phi(e,\ell,n) - e,$$

where e represents an individual investment in education. The government is assumed to observe (c,e,y), but neither n nor ℓ. It announces a schedule $((c_n,e_n,y_n))$ among which agents make their choice. If agent n chooses (c_n,e_n,y_n), she must accordingly supply a quantity of labor ℓ_n such that $y_n = \Phi(e_n,\ell_n,n) - e_n$. Following Bovenberg and Jacobs (2008), we assume the following property of the production function

$$\Phi(e,\ell,n) = \phi[\psi(n,\ell),e],$$

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[1] The same argument formally applies when the sub-utilities depend on the variable y that is observed by the government, as pointed out by Hellwig (2008a).

with ϕ monotone in its first argument. This implies that there is a function A such that the individual net production $y = \phi[\psi(n, \ell), e] - e$ can be rewritten as

$$A(y, e) = \psi(\ell, n).$$

In economic terms, this formulation seems demanding. The unknown labor supply and characteristics of the typical agent are combined into an *ability adjusted labor supply* $\psi(\ell, n)$ of the kind discussed in Remark 2. Since the individual production is a known function of this quantity and of other observables, and since this function is identical across agents, the ability adjusted labor supply can actually be inferred by the government from its observations.

Lemma 2. *Consider a* non-satiated *second best allocation in which agent n chooses (c_n^*, e_n^*, y_n^*). Given (c_n^*) and $A_n^* = A(y_n^*, e_n^*)$, at the second best the individual investment levels e_n^* and productions y_n^* are first best efficient. They maximize $y = \phi(A_n^*, e) - e$.*

Given the level of ability adjusted labor supply, investment is not distorted at a non-satiated second best allocation.

Proof. The second best program maximizes a weighted sum of utilities subject to the feasibility constraints

$$\int_n (y_n - c_n) dF(n) \geq 0, \quad y_n = \phi[\psi(\ell_n, n), e_n] - e_n$$

for every n, and subject to the incentive compatibility constraints

$$U(c_n, \ell_n) \geq U(c_m, \ell_{nm}),$$

for every (m, n) such that there is a ℓ_{nm} verifying $y_m = \phi[\psi(\ell_{nm}, n), e_m] - e_m$, or equivalently $\psi(\ell_{nm}, n) = A(y_m, e_m) \equiv A_m$. From the point of view of incentive compatibility, announcing a schedule $((c_n, e_n, y_n))$ is equivalent to announcing a schedule (c_n, A_n). Let $((c_n^*, e_n^*, y_n^*))$ be the second best optimum, and $A_n^* = A(y_n^*, e_n^*)$ the ability adjusted labor supply of agent n at this optimum. Given (c_n^*), any $((y_n, e_n))$ which leaves (A_n^*) unchanged, i.e., such that $A(y_n, e_n) = A_n^*$ for each n, satisfies the incentive compatibility constraints. One can now prove Lemma 2 by contradiction. If (e_n^*) is not first best efficient, there are resources which the second best government could use, while keeping utilities at their second best levels $(U(c_n^*, \ell_n^*))$, with ℓ_n^* satisfying $A_n^* = \psi(\ell_n^*, n)$. Under the non-satiation hypothesis, this is a contradiction. $\qquad\square$

5   Public good provision

There is a single good which can be used for private or public consumption g and labor. Labor can be transformed into goods at constant returns to scale, one unit

of labor giving one unit of good. Everyone in the economy has the same utility function $U(V(c, g), \ell)$, but productivities n differ. As in Section 3, agent n must work $\ell = y/n$ hours to get before tax income y.

The government observes individual incomes y, but neither individual productivity n nor labor ℓ separately. It chooses a quantity of public good g and a tax schedule which associates with any before tax income y an after tax income $R(y)$.

Given g, person n chooses the income y maximizing $U(V(R(y), g), y/n)$.

Lemma 3. *Consider a* non-satiated *second best allocation* $((y_n^*, c_n^*), g^*)$. *Then, given* $((y_n^*))$, $((c_n^*), g^*)$ *is a first best allocation of the economy with utility functions* $V(c_n, g)$ *and production set*

$$\int_n c_n dF(n) + g = \int_n y_n^* dF(n).$$

As a consequence, the provision of public good satisfies the Samuelson rule

$$\int_n \frac{\partial V/\partial g}{\partial V/\partial c} dF(n) = 1.$$

The sum of the willingness to pay for public good of all consumers is equal to the marginal cost of production.

Proof. It is similar to the proof of Lemma 1. Let $V_n^* = V(R^*(y_n^*), g)$ be the sub-utility profile associated with the second best allocation. Given $((y_n^*))$, any consumption schedule $((c_n), g)$ which keeps $((V_n^*))$ unchanged satisfies the incentive compatibility constraints. It is therefore attainable by the second best government provided it satisfies the feasibility constraint

$$\int_n c_n dF(n) + g = \int_n y_n^* dF(n).$$

This implies that the allocation (c_n^*, g^*) is first best for the economy with utility function V and aggregate resources $\int_n y_n^* dF(n)$. Otherwise, the same utility profile could be achieved with extra resources, a contradiction with the non-satiated hypothesis. $\square$

Remark 3. It is possible to have more heterogeneity in tastes for public good, as in Hellwig (2008b). Indeed, when preferences take the form $u(\phi(c), \ell) + \psi(g, \zeta)$, the incentive constraints are independent of the heterogeneity ζ. The problem separates into pieces: given the income tax schedule, one can look for the optimal public goods provision that allows to reach the reference utility levels, in a program which has the first best structure. When the government knows the distribution of ζ, the optimal provision of public goods is thus governed by the Samuelson rule.

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Remark 4. A similar argument applies in the presence of externalities as in Cremer, Gahvari, and Ladoux (1998). Suppose the agents' utility functions are of the type

$$U(V(c, \mathbf{c}), \ell)$$

where c is the individual consumption vector of private goods $i = 1, \ldots, k$, and $\mathbf{c}$ denotes the collection of consumptions in the economy, which may influence individual welfare but are treated as an externality. As in Section 3, one unit of good i is produced from p^i units of labor according to a linear technology. The government can non linearly tax income through an after tax income schedule $R(\cdot)$, and linearly tax goods by quoting a vector of consumer prices q different from p. Then at a non-satiated second best allocation, there is a first best Pigovian tax designed for the economy with utility functions V and feasibility constraint $\int_n p c_n dF(n) \leq \int_n y_n dF(n)$.

References

ATKINSON, A., AND J. STIGLITZ (1976): "The design of tax structure: direct versus indirect taxation," *Journal of Public Economics*, 6, 55–75.

BOADWAY, R., AND M. KEEN (1993): "Public goods, self selection and optimal income taxation," *International Economic Review*, 34, 463–478.

BOVENBERG, A. L., AND B. JACOBS (2005): "Redistribution and education subsidies are Siamese twins," *Journal of Public Economics*, 89, 2005–35.

——— (2008): "Optimal taxation of human capital and the earnings function," Discussion Paper 2250, CESIFO.

CREMER, H., F. GAHVARI, AND N. LADOUX (1998): "Externalities and optimal taxation," *Journal of Public Economics*, 70, 343–364.

DIAMOND, P., AND J. MIRRLEES (1971): "Optimal Taxation and Public Production, Part I: Production Efficiency," *American Economic Review*, 61, 8–27.

GUESNERIE, R. (1995): *A contribution to the pure theory of taxation.* Cambridge University Press.

HELLWIG, M. (2007): "A contribution to the theory of optimal utilitarian income taxation," *Journal of Public Economics*, 91, 1449–1471.

——— (2008a): "A generalization of the Atkinson-Stiglitz (1976) theorem on the undesirability of nonuniform excise taxation," Discussion paper, Max Planck Institute for Resaerch on Collective Goods.

———— (2008b): "Optimal income taxation, public goods provision and public sector pricing: a contribution to the foundations of public economics," Discussion paper, Max Planck Institute for Resaerch on Collective Goods.

KAPLOW, L. (2006): "On the Undesirability of Commodity Taxation Even When Income Taxation Is Not Optimal," *Journal of Public Economics*, 90(6-7), 1235–1250.

LAROQUE, G. (2005): "Indirect taxation is superfluous under separability and taste homogeneity: A simple proof," *Economics Letters*, 87, 141–144.

MAS-COLELL, A., M. D. WHINSTON, AND J. R. GREEN (1995): *Microeconomic Theory*. Oxford University Press.

MIRRLEES, J. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175–208.

NAVA, M., F. SCHROYEN, AND M. MARCHAND (1996): "Optimal fiscal and public expenditure policy in a two-class economy," *Journal of Public Economics*, 61, 119–137.

STIGLITZ, J. (1985): "Inequality and Capital Taxation," Discussion Paper 457, IMSSS.