R\&D, PRODUCTIVITY AND Market Value

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#### Abstract

Measuring the private returns to $\mathrm{R} \& \mathrm{D}$ requires knowledge of its private depreciation or obsolescence rate, which is inherently variable and responds to competitive pressure. Nevertheless, most of the previous literature has used a constant depreciation rate to construct R\&D capital stocks and measure the returns to R\&D, a rate usually equal to 15 per cent. In this paper I review the implications of this assumption for the measurement of returns using two different methodologies: one based on the production function and another that uses firm market value to infer returns. Under the assumption that firms choose their R\&D investment optimally, that is, marginal expected benefit equals marginal cost, I show that both estimates of returns can be inverted to derive an implied depreciation rate for R\&D capital. I then test these ideas on a large unbalanced panel of U.S. manufacturing firms for the years 1974 to 2003. The two methods do not agree, in that the production function approach suggests depreciation rates near zero (or even appreciation) whereas the market value approach implies depreciation rates ranging from 20 to 50 per cent, depending on the period. The concluding section discusses the possible reasons why this is true.


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# R\&D, Productivity, and Market Value 

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## 1. Introduction

Throughout his long career, the measurement of the returns to $R \& D$ investments was a central concern of Zvi Griliches’ research endeavor. His first paper on this topic was published in the Journal of Political Economy in 1958 and his last thoughts on it are contained in the small volume based on his Kuznets lectures and entitled $R \& D$, Education, and Productivity that was published posthumously in 2000 by Harvard University Press. Between these two poles lie a number of works that pushed the analysis forward but at the same time identified a series of issues that had to be dealt with in order to be confident in the measures being estimated. This article reviews and summarizes these issues and highlights the problem of reconciling measures obtained using differing methodologies that was raised by Griliches in his earlier 1996 article, "R\&D and Productivity: The Unfinished Business". ${ }^{2}$ It then goes on to present updated estimates of the private returns to R\&D in U.S. manufacturing firms during the past 30 years, and to attempt such a reconciliation.

The first Griliches paper (JPE 1958) on measuring the social returns to the research that lay behind the development of hybrid corn was drawn from his thesis research. Looking back on this paper today, two things strike one: first, the effort he devoted to ensuring that the rate measured was the social rate (incorporated all of the supply and demand response) and second, the clear understanding and discussion of the problem of ex ante and ex post returns and the importance of timing in measuring the rate of return. This paper was followed in 1967 by the well-known work with Jorgenson that sought explanations for the productivity residual via improvements in measurement of the inputs. In Griliches case, this particular exploration, which relied heavily on the quality improvement of the inputs to production, lead to interest in measuring the returns to human capital formation in the early 1970s (Gronau, this issue), and renewed attention to the problem of measuring the returns to R\&D, this time outside the agricultural sector, in the late 1970s. The latter interest was partly inspired by the productivity slowdown during the 1970s following the first oil price shock in 1973-1974. In 1978, Griliches undertook a large project at the NBER jointly with Nadiri and various junior researchers funded by the National Science Foundation that aimed to study a number of questions related to the productivity of R\&D. This project created and used a new dataset of publicly traded U.S. manufacturing firms that had been matched to their patent portfolios. The data used in the

[^1]present paper is an extension and outgrowth of that project. ${ }^{3}$ The results of much of the research done by participants in the project are contained in the 1984 conference volume edited by Griliches.

The 1979 article in the Bell Journal laid the groundwork for this exercise and raised a number of issues that have informed much of subsequent work by Griliches himself, his students, and other researchers. He returned to this subject towards the end of his life in three articles: the AEA Presidential address of 1994, the 1996 article already mentioned, and the Kuznets lectures (2000). In the lecture on R\&D and productivity he raised the following questions: First, has the productivity of R\&D declined, as seemed to be indicated by the decline in the market value of US manufacturing firms (Hall 1993, Hall and Kim 1999)? Second, how do we reconcile the apparently very different estimates of R\&D returns that are obtained from difference specifications of the productivity-R\&D relationship, from different levels of aggregation, and from the market value equation vis-à-vis the production function? Third, how should we model knowledge accumulation in a better way than the simple declining balance method traditionally used? Along with Klette (1996) and Hall and Hayashi (1989), he argued that we need a formulation that explicitly recognizes the intertemporal nature of knowledge production, and that we need to distinguish more carefully between private obsolescence of knowledge and social depreciation (which is presumably much slower).

On the econometric side, he was concerned about the simultaneity between $R \& D$ and output, which is accompanied by the paucity of firm-level price or technological opportunity instruments. A second concern was heterogeneity across firms and how to model it. It is very clear that both product prices and the productivity of R\&D vary substantially across firms in ways that are not observed except in the form of total firm profits or revenue, which are already rather aggregate measures. It is possible that such heterogeneity can help to explain why results vary so much across different samples or specification and he viewed this as one of the areas requiring further work.

In this paper I make some modest progress in answering a number of the questions raised by Griliches: first, how do we reconcile the R\&D returns estimates from the productivity and market value equations. Second, how do we explain the changes in the marginal productivity of R\&D over time? Third, can we use the fact that the returns or productivity results vary across broad industrial sectors to inform us on the first two questions? In attempting to answer these questions, my focus is on the role of R\&D depreciation or obsolescence at the firm level and its effects on the measurement of the returns to R\&D. Thus the interpretation offered here is a Schumpeterian one, centering on the

[^2]implications of dynamic technological competition for the ex ante and ex post measurement of R\&D returns.

An important problem in estimating the private returns to $\mathrm{R} \& \mathrm{D}$ using either a revenue production function (flow method) or market value equation (present discounted value of future flows method) is the same: computing the net rate of return or interpreting the shadow value of the $\mathrm{R} \& \mathrm{D}$ stock requires an assumption about the private depreciation or obsolescence of the asset generated by the R\&D investments. But determining the appropriate depreciation rate is difficult if not impossible, for at least two reasons. First, from the firm's perspective, the appropriate depreciation rate is endogenous to its behavior and that of its competitors, in addition to depending to some extent on the progress of public research and science. Therefore there is no reason to assume that it is constant over time or over firm, although it will usually (but not always) change slowly in the time dimension.

Second, identifying the depreciation rate independently from the return to $\mathrm{R} \& \mathrm{D}$ requires determination of the lag structure of R\&D in generating returns. But years of experience with the specification of production functions, market value equations, or even patent production functions (Hall, Griliches, and Hausman 1989) has shown convincingly that this is extremely difficult, because of the lack of appropriate natural experiments. That is, $\mathrm{R} \& \mathrm{D}$ does not vary much over time within firm, so that trying to identify more than one coefficient of $R \& D$ is problematic and leads to very unstable results. Even in the data used in this paper, which is a fairly heterogeneous time series-cross section of firms, the variance of $R \& D$ growth rates is only about 4 per cent of the variance of the levels. As has been observed before (Hall and Mairesse 2005), the $\log$ R\&D series exhibits close to random walk behavior [the correlogram for the first three lags of the data used here is $(0.99,0.97$. $0.96)$ and the partial correlogram is $(0.99,0.00,0.00)]$. The implication of these properties is that including more than one linear function of the $(\log ) R \& D$ series in an equation will be a futile exercise.

Given the problem of estimating R\&D depreciation from a distributed lag, in this paper I turn the problem on its head and try to infer the private $\mathrm{R} \& \mathrm{D}$ depreciation rate from the measured returns to R\&D by assuming that the net return is determined by the financial markets and therefore roughly equal across comparable firms. Thus the paper offers an interpretation of the widely varying market value and productivity of $R \& D$ at the firm level over time, one that is based on the idea of continuous creative destruction of the rents created by R\&D investments. I find that the two different approaches do not agree, in that the production function approach suggests depreciation rates near zero (or even appreciation) whereas the market value approach implies depreciation rates ranging from 20 to 50 per cent, depending on the period. The concluding section of the paper discusses the possible reasons why this is true.

The paper begins with a brief discussion of the data and the basic facts on $R \& D$, productivity, and market value that it contains. The dataset is a comprehensive unbalanced panel of publicly traded U.S. manufacturing firms observed from 1974 to 2003, a thirty year period. The next section of the paper presents the familiar framework for the estimation of the returns to $\mathrm{R} \& \mathrm{D}$ using the production function, discusses the interpretation of the results and presents some estimates, for the manufacturing sector as a whole and for six subsectors, grouped by technology. This is followed by a presentation of the alternative methodology for measuring R\&D returns, the market value approach pioneered by Griliches. A version of a theoretical model of firm-level Tobin's Q with more than one asset from Hayashi and Inoue (1991) is presented in an appendix and then applied in this section of the paper to help interpretation of the results of market value estimation. Empirical results from estimating the market value equation with $R \& D$ capital are then presented and the implied depreciation of $R \& D$ is computed. Conclusions follow.

## 2. Data

The starting point for the panel used in this paper was the population of manufacturing firms on the Compustat files between 1970 and 2004, which consists of more than 100,000 observations on 9,000 firms. Non-U.S. firms, firms with fewer than three years of good data, and observations without good sales, employment, or Tobin's q were removed. ${ }^{4}$ The result of this selection was a dataset of approximately 60,000 observations ( 6,000 firms) in an unbalanced panel for the years 1973 to 2003, of which about two thirds reported $\mathrm{R} \& \mathrm{D}$ expenditures.

As is well known to anyone working with data from this source, in many cases there are large changes in one or more variables from year to year, in some cases due to mergers and divestitures and sometimes simply due to incorrectly entered data. Because of the size of the file it is not feasible to clean the data manually, so another approach was necessary. First, ratios and growth rates of key variables were checked for validity using rules given in the appendix to this paper. Second, where there were substantial breaks in a firm's series, a new firm was created after the break. This procedure is justified by the fact that where large changes occur it is not reasonable to think of the firm as being the same entity over time. Third, most of the estimates have been computed using a least absolute deviations estimator rather than ordinary least squares, to reduce the sensitivity to outliers. ${ }^{5}$

[^3]Cleaning the data in this way and keeping only those firms that engaged in $R \& D$ at least some of the time resulted in a panel with 28,937 observations for the 1974-2003 period. Considerable experimentation with year-to-year estimates from this unbalanced panel revealed some instability in the coefficients due to fluctuations in variables and samples (see Figures 1 and 2 for some examples). Accordingly, it seemed preferable to focus the estimates in the tables on a series of panels balanced over 5-year periods rather than presenting possibly unstable yearly estimates. ${ }^{6}$ However, this decision comes at the cost of reducing the sample even further. The periods chosen were 1974-1978, 19791983, 1984-1988, 1989-1993, 1994-1998, and 1999-2003; an additional year of data prior to the period was also required in order to ensure that beginning of year capitals could be used where necessary. Table A1 in the appendix gives some information on the number of observations in the population and in the sample. The appendix also shows that although the balanced panel observations comprise only about a quarter of the original population, they cover more than three quarters of the R\&D in the publicly traded manufacturing sector. The unbalanced panel includes about 90 per cent of the $R \& D$.

Table 1 gives basic statistics on the two samples used for estimation, unbalanced and partially balanced. The median firm in both panels is large, with that from the balanced panel somewhat larger than from the unbalanced panel: 178 versus 300 million (1996) dollars of revenue, 1500 (2500) employees, ordinary capital stock of 77 (133) million dollars, and R\&D capital of 24 (37) million dollars. ${ }^{7}$ The median R\&D-sales ratio is 2.8 per cent, with a range from 0.1 per cent to 100 per cent (due to the cutoffs used in trimming the data). Tobin's $q$ is highly variable and averages substantially higher than unity, even when it is corrected for the fact that R\&D capital has been left out of the denominator.

## [Table 1 about here]

twofold: first, in the case of the market value equations, it is nearly impossible to find appropriate instruments after differencing to remove firm effects, and it is not even clear that it is desirable to remove these effects when estimating the market valuation of R\&D strategies. Second, in the case of production functions, the best estimator (GMM-system) tends to yield results that are similar to the estimates in levels (Blundell and Bond 1998). Finally, given the focus of the current paper on the macro-economic properties of R\&D investment over time rather than causal models, it is inappropriate to condition on the individual firm effects as that omits the contribution from changes in their distribution.
${ }^{6}$ A potential additional benefit of this decision is that it is far easier to estimate using GMM methods on a balanced panel.
${ }^{7}$ R\&D capital is measured as described in equation (3) of the next section of the paper, using a depreciation rate of 15 per cent.

Figures 1 and 2 report the time series trends in a number of key variables and estimates, computed using models described in more detail later in the paper. Figure 1 shows evolution of four series over the thirty-year period from 1974 to 2003: the R\&D capital stock-output ratio of the median firm, the marginal shadow value of R\&D capital estimated using a hedonic market value equation, the productivity of R\&D capital estimated using a Cobb-Douglas production function, and the same productivity adjusted for double-counting of R\&D spending. ${ }^{8}$ The R\&D capital intensity and the two $\mathrm{R} \& \mathrm{D}$ productivity measures rise in parallel during the entire period, from $0.1,0.0$ and 0.05 respectively to $0.25,0.15$, and 0.2 . This is what one might have expected if the cost of R\&D capital were roughly constant, R\&D capital was measured correctly, and R\&D investment was in equilibrium. It implies a trend during the period of continuously increasing productivity of R\&D investment. On the other hand, the market value of R\&D capital shows much larger swings, from about 0.5 in the mid-1970s to 1.2 , then falling again during the 1980 s , and finally rising to well above one during the late 1990s.

## [Figures 1 and 2 about here]

Figure 2 shows the same four series, but this time relative to ordinary capital. The advantage of such a formulation is that it abstracts from variations in the production or value function due to 1 ) differences in the share of purchased inputs that are not controlled for by the sector dummies, 2) scale economies or diseconomies, 3 ) imperfect competition effects that are consistent with a constant elasticity of demand function, and 4) macroeconomic effects in capital valuation. In this figure, the R\&D capitalcapital ratio rises smoothly from 0.2 to 0.4 and then falls again to 0.3 , whereas the relative productivity rises somewhat more steeply from around zero ( 0.05 if adjusted for double counting to 0.4 (0.5), and not in parallel with the capital stock ratio, which implies that the cost of R\&D capital has risen relative to the cost of ordinary capital. The relative market value of $\mathrm{R} \& \mathrm{D}$ capital exhibits a similar pattern as the absolute market value, except that the earlier period is lifted somewhat higher relative to the later, a result that is consistent with the fact that $R \& D$ investment in the 1974-1990 period increased in spite of its relatively low productivity. That is, the financial markets signaled that higher future returns were to be expected. Note, however, that the relative shadow value of R\&D capital is almost everywhere less than unity, which suggests either that it is mismeasured or that adjustment costs are higher for R\&D.

Broadly speaking, these data reveal that the conundrum identified by Griliches is still there, but in a somewhat altered form. The productivity of $\mathrm{R} \& \mathrm{D}$ has risen to the point where it is approximately equal to its share in production (under the assumption that the costs of capital do not differ too much),

[^4]but the market value of R\&D is just as high as it was in the 1977-1983 period, when its productivity was much lower. There are a number of possible explanations for these findings and in the following two sections of the paper, I explore whether changing depreciation (increased obsolescence) of R\&D may help to explain why higher productivity has not lead to a much higher market value. In support of this investigation, I also present results for six major subsectors (chemicals, rubber, and plastics; pharmaceuticals and medical instruments; electrical machinery; computers and scientific instruments; metals, machinery, and transport equipment; and miscellaneous manufacturing), where we might suspect differing R\&D obsolescence to be at work.

## 3. Productivity

The framework used by Griliches (and a host of subsequent researchers) to estimate the productivity of $R \& D$ or the private returns to $R \& D$ relies on the usual Cobb-Douglas production function augmented to include an additional input that he called "knowledge" capital. A recent survey of results obtained using this model is given by Mairesse and Mohnen (1990) and the econometric issues that arise in estimating production functions more generally have been reviewed well in Griliches and Mairesse (1997). This section of the paper presents the model and discusses the issues that arise when using it to measure the rate of returns to R\&D.

The model commonly used to estimate the productivity of $\mathrm{R} \& \mathrm{D}$ is a Cobb-Douglas production function augmented with a knowledge capital term:

$$
\begin{equation*}
Y=A L^{\alpha} C^{\beta} K^{\delta} e^{u} \tag{1}
\end{equation*}
$$

where $L$ is a measure of labor input, $C$ is ordinary (tangible) capital, $K$ is knowledge (intangible) capital, and $u$ is a disturbance. To implement this equation for estimation using a panel of firms followed over time, take logarithms and write it using $i$ to denote firms and $t$ to denote time:

$$
\begin{equation*}
y_{i t}=\eta_{i}+\lambda_{t}+\alpha l_{i t}+\beta c_{i t}+\gamma k_{i t}+u_{i t} \tag{2}
\end{equation*}
$$

$\eta_{i}$ denotes a firm specific effect constant over time and $\lambda_{\mathrm{t}}$ denotes a time-specific effect that is constant across firms. The shock $u_{i t}$ and the firm effect $\eta_{i}$ may possibly be correlated with the current (and future) input levels. In principle, both of these econometric problems can be solved by estimating with GMM on first differences of the equation, provided appropriate instruments are available (Blundell and Bond 1998).

A number of measurement issues arise in implementing this equation for estimation and in interpreting these results. At the macro-economic level they include the definition and scope of output, whether to use GDP or a broader welfare measure that includes nonpecuniary benefits such as
defense or health improvements, as well as how to incorporate new goods into the measure (Griliches 1979). In the present paper I focus on private returns to R\&D, which makes the problem a bit simpler. Although equations (1) and (2) are usually labeled production functions, in the case of individual firms they are more properly called revenue production functions. That is, in the absence of firmspecific price deflators, the measure of output $Y$ is the firm-level price multiplied by the quantity sold (or a sum of such terms in the usual case where there is more than one product). This fact implies that $\gamma$ measures the joint contribution of R\&D to productivity and to the prices charged by the firm (which could be declining if R\&D makes the firm more efficient in a competitive market or increasing if R\&D is primarily used to improve quality). But from the perspective of measuring private returns, this is not a problem, because either (cost reduction or product improvement) are outcomes that end up in the bottom line. Nevertheless, it is important that we not confuse these measures with true productivity measures, which remove the effects of $R \& D$ on price. The latter are the relevant concept for social welfare.

A key problem in implementing estimation using equation (2) is the construction of the knowledge capital $K$. Beginning with the work conducted during the large NBER project on R\&D, patents, and productivity, Griliches and his co-authors used a conventional declining balance formula for the construction of (real) $K$, by analogy with ordinary investment and capital:

$$
\begin{equation*}
K_{i t}=(1-\delta) K_{i, t-1}+R_{i t} \tag{3}
\end{equation*}
$$

Although a variety of choices for the depreciation rate have been explored in the past, the choice makes little difference for estimation using the first differenced equation, and most researchers use the 15 per cent that Griliches had settled on in his work (see Hall and Mairesse 1995 for some experiments with different rates). It is easy to see why this might be the case: assume that $\mathrm{R} \& \mathrm{D}$ grows over a sufficiently long period at a constant (firm-specific) rate $g_{i}$ and that the knowledge capital K depreciates at a firm-specific rate $\delta_{i}$. Then one can show that

$$
\begin{equation*}
K_{i t} \cong \frac{R_{i t}}{\delta_{i}+g_{i}} \text { or } \log K_{i t}=\log R_{i t}-\log \left(\delta_{i}+g_{i}\right) \tag{4}
\end{equation*}
$$

where $R$ denotes real R\&D investment at time $t$ and $\delta$ is a suitably chosen (private) depreciation rate. For future reference, I note that under the assumption of constant depreciation at the firm level, equation (4) implies that the "true" $\mathrm{R} \& \mathrm{D}$ capital $K^{*}$ ( $K$ computed using correct economic depreciation) is given by the following equation:

$$
\begin{equation*}
K_{i t}^{*}=K_{i t} \frac{\delta_{i}+g_{i}}{\delta_{i}^{0}+g_{i}} \tag{5}
\end{equation*}
$$

where $\delta_{i}^{0}$ is the depreciation rate used to construct the measured $K$ (usually 15 per cent).

Equation (4) also implies that as long as the growth rate and depreciation do not change very much within firm over time, they will be incorporated into the firm effect in equation (2), and the estimated elasticity of output with respect to either $K$ or $R$ will be the same, and that for $K$ will not depend on the choice of depreciation rate. However, although the elasticity of output with respect to R\&D is not affected by the choice of the depreciation rate, I show later in this section of the paper that the estimated rate of return derived from the elasticity is.

Earlier researchers (Mansfield 1965; Terleckyj 1974; Griliches 1980) had used an alternative formulation of this equation that was derived from the first differenced version of equation (2), under the assumption that the depreciation of $R \& D$ is approximately zero:

$$
\begin{gather*}
\Delta y_{i t}=\Delta \lambda_{t}+\alpha \Delta l_{i t}+\beta \Delta c_{i t}+\gamma \Delta k_{i t}+\Delta u_{i t}  \tag{6}\\
\Delta k_{i t}=\frac{R_{i t}-\delta K_{i, t-1}}{K_{i, t-1}} \\
\Rightarrow \gamma \Delta k_{i t}=\left(\gamma \frac{Y_{i t}}{K_{i, t-1}}\right) \frac{R_{i t}}{Y_{i t}}=\rho \frac{R_{i t}}{Y_{i t}}
\end{gather*}
$$

where $\rho$ is the gross rate of return to R\&D capital, and the appropriate independent variable is the firm's R\&D intensity (R\&D to sales ratio). ${ }^{9}$ This version of the model had the advantage that the gross rate of return, rather than the production function coefficient, was assumed to be equalized across firms. Of course, it would be preferable for the net rate of return to be equalized, but that takes us back to the problem of measuring the depreciation of R\&D.

Note that in order to obtain equation (6), R\&D depreciation was assumed to be zero, which is clearly counterfactual. ${ }^{10}$ The true net investment rate may be substantially lower than that measured by the gross $R \& D$ spending-sales ratio. To see the consequences of this assumption, use equation (4) to construct an approximation to net $R \& D$ investment:

[^5]\[

$$
\begin{aligned}
& \Delta k_{i t}=\frac{R_{i t}-\delta K_{i, t-1}}{K_{i, t-1}} \cong \frac{R_{i t}-\delta R_{i, t-1} /\left(\delta_{i}+g_{i}\right)}{K_{i, t-1}}=\frac{R_{i t}}{K_{i, t-1}}\left[1-\frac{\delta_{i}}{\left(1+g_{i}\right)\left(\delta_{i}+g_{i}\right)}\right] \\
& \Rightarrow \hat{\rho} \cong\left[1-\frac{\delta_{i}}{\left(1+g_{i}\right)\left(\delta_{i}+g_{i}\right)}\right] \rho \simeq\left[\frac{g_{i}}{\delta_{i}+g_{i}}\right] \rho
\end{aligned}
$$
\]

This equation shows that the estimated gross rate of return in the $R \& D$ intensity formulation underestimates the true rate of return by the ratio of $R \& D$ growth to the sum of $R \& D$ growth plus depreciation. In the sample used in this paper, the median past growth of real $R \& D$ ranges from about 4 per cent per annum in the earlier period to 10 per cent in the latter. If the depreciation rate is 15 per cent, this implies that the true gross rate of return will be 2.5 to 5 times the estimated value, a point which seems to have gone unnoticed in the literature.

The conclusion is that using either equation (2) or the version of equation (6) that includes $R \& D$ intensity to estimate the rate of return to $\mathrm{R} \& D$ from production function estimates requires an assumption about the rate at which the privately productive knowledge asset depreciates. In the first case the gross and net rates of return to K are:

$$
\begin{equation*}
\rho^{G} \equiv \frac{\partial Y}{\partial K}=\gamma \frac{Y}{K^{*}} \text { and } \rho=\gamma \frac{Y}{K^{*}}-\delta \tag{7}
\end{equation*}
$$

Therefore the production function approach to measuring returns requires knowledge of $\delta$ both to compute the correct level of K and to convert gross to net. In the second case ( $\mathrm{R} \& \mathrm{D}$ intensity), $\rho^{G}$ is measured directly, but one needs knowledge of $\delta$ to convert it to a net rate, and also to correct for the use of gross R\&D investment rather than net R\&D investment in the equation. In the next section, we discuss how to invert the relationship in equation (7) in order to estimate a depreciation rate given an elasticity estimate and a required net rate of return.

### 3.1 Deriving depreciation estimates from the production function

Two possibilities suggest themselves as ways to derive estimates of R\&D depreciation from an estimate of the productivity of R\&D capital. ${ }^{11}$ The first is to make the strong assumptions of perfect competition and price-taking and set the share of $\mathrm{R} \& \mathrm{D}$ capital $\left(c_{K} K / p_{Y} Y\right)$ equal to its productivity. This has a number of disadvantages: these firms are far from perfectly competitive, much of the R\&D is devoted to creating demand-enhancing product improvement, and probably most important, the

[^6]productivity coefficient is seriously biased upward due to the omission of purchased inputs and materials from the production function.

The alternative is simply to assume that the firm is a cost minimizer facing a Cobb-Douglas production function with neutral scale and demand effects. In this case the productivities of the two types of capital (R\&D and other) will be proportional to their shares, even though each share will not be equal to its own productivity coefficient. That is, the biases due to scale and markup cancel, which implies the following relationship:

$$
\begin{equation*}
\frac{\gamma}{\beta}=\frac{c_{K}^{*} K^{*}}{c_{A} A} \tag{8}
\end{equation*}
$$

$\gamma$ is the output elasticity of true knowledge capital $K^{*}$ and $\beta$ is the output elasticity of tangible capital $A$. The neoclassical cost of capital $c_{J},(J=A, K)$ is given by the following equation:

$$
\begin{equation*}
c_{J}(t)=p_{J}(t)\left[1-\frac{\left(1-\delta_{J}\right)\left[p_{J}(t+1) / p_{J}(t)\right]}{(1+\rho)}\right] \tag{9}
\end{equation*}
$$

where $\delta_{J}$ is the appropriate depreciation rate and $\rho$ is the required rate of return. Given estimates of $\gamma$ and $\beta$, and knowing $c_{A} A$, this relation allows us to compute $c_{K} K$ and compare it with the observed measure of $K$ in order to derive an estimate of the cost of $\mathrm{R} \& \mathrm{D}$ capital $c_{K}$ and therefore of its depreciation rate.

However, note that both $c_{K}$ and $K$ contain depreciation, the former because the cost of capital includes the cost of its replacement and the latter because the current measured stock depends on past depreciation. Higher depreciation implies a higher true cost of capital but it also implies that we have overestimated $K$, so the true $K\left(K^{*}\right)$ will be lower. The existence of this tradeoff makes identification of the actual depreciation rate from the production function difficult, even if we are willing to make the assumptions necessary for equation (8) to hold. Assuming that the depreciation rate used to construct $K$ was 15 per cent, equation (5) implies the following relationship between $K^{*}$ and $K$ :

$$
\begin{align*}
& c_{K}^{*} K^{*}=\tilde{p}_{K}\left[1-\frac{\left(1-\delta_{K}\right)\left(1-\Delta \tilde{p}_{K} / \tilde{p}_{K}\right)}{(1+\rho)}\right] \frac{g_{R}+0.15}{g_{R}+\delta_{K}} K \\
& \cong \tilde{p}_{K} K \frac{\left(\rho+\delta_{K}\right)}{\left(g_{R}+\delta_{K}\right)} \frac{\left(g_{R}+0.15\right)}{(1+\rho)} \tag{10}
\end{align*}
$$

The approximation holds because the year-to-year rate of change in the relative price of $R \& D$ is very small. ${ }^{12}$ Clearly, even under the heroic assumption that the output elasticities of R\&D and ordinary capital are proportional to their shares, R\&D depreciation will be difficult to identify if the growth of $R \& D$ is approximately equal to the interest rate, as it is for much of my data.

There is one final measurement issue that needs to be considered if we wish to derive firm-level R\&D rate of return estimates from the existing data on $R \& D$ expenditure and output: the double counting of R\&D inputs. Schankerman (1981) pointed out long ago that because R\&D spending is composed of spending on labor, capital, and materials, these expenditures have already been included in the ordinary input measures of the production function, and the coefficient of R\&D capital therefore has a substantial downward bias. Using a cross section of US manufacturing firms in 1963, he estimated that the magnitude of the downward bias ranged from 0.02 to 0.09 , and was higher in industries with higher R\&D intensities, as theory would suggest. Hall and Mairesse (1995) estimated the downward bias for French manufacturing firms using measured R\&D labor and capital input and found that it was approximately equal to 0.07 . In this paper I suggest a method for approximate bias correction when these measures are not available, a method which uses the fact that the bias is correlated with R\&D intensity.

Using an omitted variable approach to the problem, Schankerman showed that the bias in the R\&D capital coefficient could be written as follows: ${ }^{13}$

$$
\begin{equation*}
\operatorname{plim}(\hat{\gamma}-\gamma)=-\alpha b_{s_{L} k}-\beta b_{s_{K} k} \tag{11}
\end{equation*}
$$

where the coefficients $b_{s L k}$ and $b_{s K k}$ are the coefficients of $k$ from auxiliary regressions of $s_{L}$ and $s_{K}$ on $k, l$, and $c$ (and the industry and year dummies). $s_{L}$ and $s_{K}$ are the ratios of expenditure on R\&D labor and capital to other expenditures on labor and capital respectively. Unfortunately we do not observe $s_{L}$ and $s_{K}$ in our data. The National Science Foundation (2006) reports the following breakdown of R\&D costs by type of expense for manufacturing firms in 2002: 55\% labor, $14 \%$ materials, $4 \%$ capital depreciation, and $30 \%$ other costs, which include overhead, property taxes, and repairs. These figures suggest that the bias due to double counting of capital is negligible, and that except for labor, the other

[^7]costs will be treated as materials input and subsumed in the sector dummies. Therefore if there is bias, it will come from the share of employment that is R\&D employment. I assume that this share is proportional to the $\mathrm{R} \& \mathrm{D}$ intensity of the firm, and include the interaction of $R \& D$ intensity with the $\log$ of $R \& D$ capital to control for the bias. To mitigate problems with simultaneity, $R \& D$ intensity is lagged one period.

I present the results of computations based on equations (8) and (10) in Figure 3 and Table 2. Figure 3 displays four series: the first two are the costs of capital $c_{A}$ and $c_{K}$ computed using the assumed depreciation rates of 10 and 15 per cent respectively, and required net rate of return computed as the US Treasury Bill rate plus a 5 per cent equity premium (also shown on the graph). The divergence in these series that appears in the mid-1980s is due to differential trends in their relative prices. The investment deflator is from NIPA and incorporates the hedonic estimates for the prices of ICT goods, whereas the R\&D deflator is based on the Griliches-Jaffe methodology which uses labor cost and the GDP deflator as components (Hall 1990). The final series shown is the cost of R\&D capital for the median firm derived from equation (8), using the estimated relative productivities of capital and R\&D capital (adjusted for double counting) and measured R\&D capital $K$ rather than (unobserved) true capital $K^{*}$. The derived series is therefore equal to the last expression in equation (10) without the $K$.

## [Figure 3 about here]

Several things can be noted about this derived cost of R\&D capital: first, recall that it is based on the following assumptions and measurements: 1) static cost minimization with optimal capital stocks; 2) estimated revenue productivities of R\&D and ordinary capitals; and 3) a cost of ordinary capital based on interest rates plus an equity premium of 5 per cent and depreciation of 10 per cent per annum. This derived cost of capital is not necessarily the cost the firm actually faces, but the cost implied by the firm's investment behavior if it is a cost minimizing firm. The two may diverge if static expectations do not hold. Second, until the mid-1990s it is lower than a cost of R\&D capital computed using the usual neo-classical formula, which might imply that true depreciation is actually lower than the assumed 15 per cent, except for the fact that lower depreciation also implies that the true cost of capital should be higher [equation (10)]. Third, it increases from about $12 \%$ in the late 1970s to almost 25 per cent in the 1990s (with a spike in the years 1999 and 2000 that is due to the productivity of R\&D investment in those years). During the past ten or fifteen years, it is essentially equal to the cost of R\&D capital computed in the conventional way.

The conclusion is that if R\&D investment depreciates throughout the period at the assumed rate of 15 percent, then up until the mid-1980s, firms were investing as though the cost of R\&D capital was much lower than the cost of ordinary capital. The gap between the two costs of capital did not close until the early 1990s. This could reflect expectations about the future productivity of such R\&D.

However, an alternative interpretation is that the depreciation rate for R\&D investment increased somewhat during the thirty years, raising the effective cost of R\&D capital.

Table 2 shows the results of estimating the model in equation (2) for the partially balanced panel, estimated for the six 5-year subperiods of the 1974-2003 period. All the regressions included twodigit industry dummies and year dummies. The top panel shows some statistics for the sample: its size, the medians of the real R\&D capital-sales and real R\&D capital-ordinary capital ratios, and the median of the annual real $R \& D$ growth rate during the period. Note that the $R \& D$ capital-capital ratio rises to a maximum of 35 per cent in the early 1990s and then declines again, probably because of exit and entry into and out of the sample. The second panel shows the estimated production function coefficients and their ratio, along with the standard errors of these estimates, and the cost of capital and depreciation rate that they imply [equations (8) and (10)]. The third panel shows the same quantities computed using a regression with the correction for double counting bias.

## [Table 2 about here]

The depreciation rate for each firm is computed from the implied cost of R\&D capital by inverting equation (10) with $\rho$ equal to the Treasury Bill rate plus a $5 \%$ risk or equity premium and $g_{R}$ equal to the firm's R\&D growth rate. The "delta" method is used to compute the standard error of each depreciation rate as a function of the variance of the production function coefficient estimates. Medians across the firms for both the depreciation rate and its standard error are shown in the table.

It is notable that the productivity of R\&D increases much more rapidly than the ratio of the stocks, and does not show a similar decline in the last two periods. As was suggested by Figure 2, the cost of R\&D capital rises from a low of less than 5 per cent to a high of over 20 per cent over the period ( 10 per cent to 37 per cent after correcting for double counting), leading to a corresponding increase in the depreciation rate for $\mathrm{R} \& \mathrm{D}$ of about 8 percentage points. However, although increasing, this depreciation rate is everywhere negative, suggesting appreciation rather than depreciation of R\&D capital over the period (albeit at a declining rate towards the end of the period).

A number of explanations of the implied negative private obsolescence (that is, appreciation) of R\&D capital are possible. The most obvious is that R\&D capital was indeed expected to increase in productivity during the 1970s and early 1980s, so that firms invested heavily in R\&D even though its current revenue productivity was low. Another possibility is that ordinary capital actually had higher private obsolescence rates than 10 per cent, implying a higher cost of capital, which would also raise the derived cost of R\&D capital. However, this would not remove the gap between the two costs of capital. Other explanations include mismeasurement of the revenue productivities or the deflators that are used in the cost of capital computations.

This result that firms behaved as though R\&D productivity was expected to increase during the twenty or so years following 1974 implies that using the production function methodology to estimate the returns to $R \& D$ can be problematic if $R \& D$ investment is not in equilibrium. To put it another way, the ex post production function approach actually produces implications about ex ante returns if we combine it with the observed cost of capital. But another way to look at the ex ante returns is to use an explicitly expectations-based methodology as described in the next main section of the paper.

### 3.2 Productivity estimates by sector

Obviously, the assumption of a common production function for all of manufacturing is a vast oversimplification, and in this section of the paper I relax this assumption by dividing the firms into six broad technology classes and repeating the estimations in Table 2 sector by sector. The sectors are Chemicals and chemical-based industries, Pharmaceuticals and medical instruments, Electrical equipment and engines, Computers, communication equipment, and scientific instruments, Metals and machinery, and Miscellaneous. The precise definitions are given in Table A3 in the appendix.

Table 3 shows the results of estimating separate production functions for each sector and time period, in five panels: the sample size, the median of real R\&D capital intensity, the R\&D capital coefficient relative to the ordinary capital coefficient and the standard error of this ratio, the cost of R\&D capital that is implied by that ratio, and the median implied depreciation rate for $R \& D$, together with its median standard error. As expected, the R\&D capital intensity varies greatly across sector, from around 0.1 in the miscellaneous (low tech) sector to almost 0.6 in the computing sector. Note that it tends to decline somewhat over the last two periods, which reflects an increase in both exit from and entry to the sample. The relative productivity of $\mathrm{R} \& \mathrm{D}$ is also highly variable, highest in the later years in the electrical and computing sectors, and lowest in the early years.

## [Table 3 about here]

These estimates also imply highly varying costs of R\&D capital. In the miscellaneous sector, the implied costs are extremely high, reflecting the high relative productivity of R\&D in this sector. Some of this result is due to the fact that there are a few "high technology" industries, such as video games, mixed into this sector, especially in the latter half of the period. It is noteworthy that the lowest implied cost of capital is in the pharmaceutical sector, where investment has been very heavy throughout the period. This sector also has the lowest depreciation rate (highest appreciation rate); for the other sectors depreciation is essentially zero. Thus the sector estimates suggest that depreciation is lower for pharmaceuticals in almost all periods and for electrical and computing in the earlier years, but they also confirm the aggregate finding that depreciation rates estimated from the revenue productivity of R\&D tend to either have the wrong sign or be very close to zero.

## 4. Market value

The main alternative approach to valuing the output of $\mathrm{R} \& \mathrm{D}$ investment is more forward-looking, in that it relies on the financial market's assessment of the value of the firm that has undertaken the investment. The focus is therefore on expected returns, rather than realized profits. A number of indicators have been used as measures of $R \& D$ output in the past, such as capitalized $R \& D$ inputs, patent counts or citation-weighted patent counts, profits or their present discounted value, the value of technology licenses, and so forth. Since the seminal work of Griliches (1981), a frequently used proxy for the private value of $R \& D$ output is the market value of the firm.

The justification for using firm market value as a proxy for $\mathrm{R} \& \mathrm{D}$ output value is the idea that although on average we might expect that the value of spending another dollar on R\&D would be equal to that dollar, therefore allowing the use of $R \& D$ input as a measure of $R \& D$ value, there are a number of factors that intervene to make this a rather poor measure: first and foremost is the risk of any R\&D program, which leads to considerable dispersion in the value of its output ex post even if all $\mathrm{R} \& \mathrm{D}$ input is valued at cost ex ante. In addition, the behavior of competitors in the product market or changes in the prices of inputs or the macro-economy will influence the realized value of an R\&D project. At the least, looking at firm value to obtain a shadow price for $\mathrm{R} \& \mathrm{D}$ output incorporates all the information currently available about the likely success or failure of the sunk R\&D investments in generating future profits for the firm.

Griliches’ 1981 approach to the problem of valuing R\&D output was grounded in the theory of hedonic prices: the central idea was that a firm could be considered as a bundle of assets, both tangible (physical capital and inventories), and intangible (R\&D assets or knowledge stock, reputation, and so forth). To compute the shadow price of an asset at any point in time, one could therefore regress the prices or values of a set of firms on their portfolio of assets of different types, and interpret the slope coefficient of any particular type of asset as its marginal shadow value. When implementing this idea, most researchers have followed Griliches original article and used a first order approximation to the value of the assets, sometimes in logarithmic (Cobb-Douglas) form, but usually in a simple additive specification. The equation estimated looks like this:

$$
\begin{align*}
\log V_{i t} & =\log \left[\alpha_{t} p_{i t}^{I} A_{i t}+\gamma_{t} p_{i t}^{R} K_{i t}\right]+\varepsilon_{i t} \\
\text { or } \log Q_{i t} & \equiv \log \left(V_{i t} / p_{i t}^{I} A_{i t}\right)=\log \alpha_{t}+\log \left[1+\frac{\gamma_{t}}{\alpha_{t}}\left(\frac{p_{i t}^{R} K_{i t}}{p_{i t}^{I} A_{i t}}\right)\right]+\varepsilon_{i t} \tag{12}
\end{align*}
$$

where $V_{i t}$ denotes the market value of firm $i$ at time $t, p^{I}{ }_{i t} A_{i t}$ denotes its (nominal) tangible assets, and $p^{R}{ }_{i t} K_{i t}$ denotes the (nominal) R\&D or knowledge asset. ${ }^{14}$ The second form of the equation is often simplified using the $\log (l+\varepsilon) \approx \varepsilon$ approximation to linearize the model.

Thus, the methodology allows the measurement of the shadow value of $R \& D$, provided one can construct a measure of the R\&D asset. However, there is an obvious problem in constructing the asset associated with R\&D, namely, how should one add up past R\&D investments in order to construct this measure? A second implication of the foregoing discussion is that nothing in the theory of hedonics suggests that this measured shadow value should be constant across time, or even across industries. In fact, our interest in the exercise is driven by the fact that it will not be constant. ${ }^{15}$

In practice, both coefficients ( $\alpha_{t}$ and $\gamma_{t}$ ) tend to fluctuate a great deal over time and it is not clear how to interpret the fluctuations. $\alpha_{t}$ represents the overall premium or discount in the market for ordinary capital assets and $\gamma_{t}$ the relative premium or discount for knowledge (R\&D) assets.

Second, and more importantly, construction of the R\&D asset requires adding up past $R \& D$ in some way, and assuming some form of depreciation. Usually the declining balance form of equation (3) is used, with $\delta$ set equal to 15 per cent,. Although this value may be appropriate on average, in many individual firms and industries, depreciation of past $R \& D$ stock in any given year can deviate considerably from this value and this deviation will show up in the estimated value of $\gamma$. In effect, the knowledge stock that matters for the firm is that which generates privately appropriable returns. From the perspective of $Q$ theory, when the depreciation of knowledge assets is high and therefore $Q$ is low, either the book value of the knowledge asset should be lower (implying that $K$ should have been constructed using a higher $\delta$ ) or its market value should be lower (implying that the estimated $\gamma_{t}$ will be lower).

Although time-varying (and endogenous) depreciation rates can also be a property of ordinary tangible assets, in principle they are much less common because many of these assets trade on a

[^8]second hand market, which implies that their "market" value is much less volatile and also that it may not vary as much across industries. The problem with the $\mathrm{R} \& \mathrm{D}$ asset variable is that it is often quite specialized and that once past investment has been written off by the emergence of a better competing product, it has little residual private value. Therefore, we would like our model both to allow for this feature of the data and to provide a way to interpret the results.

Another way to look at the problem of constructing the R\&D stock or knowledge asset is to use the Tobin's $q$ interpretation of the market value equation, in which the market to book ratio for a firm is a measure of the average premium or discount at which a particular asset will sell. For this interpretation, the correct measure of the $\mathrm{R} \& \mathrm{D}$ asset in the denominator (the book value) is its replacement cost. But what does this mean? In the case of a machine or tangible asset, the meaning is obvious - it is the cost of acquiring one just like it or of manufacturing a new one. But replacing the knowledge stock created by $\mathrm{R} \& \mathrm{D}$ is quite a different matter: it may be essentially free, if the stock is simply to be duplicated within the firm, that is, spread over more output. It may be low cost, if it is simply to be imitated or rediscovered. It may be high cost, if the corresponding invention is patented and another firm wishes to enter and reproduce it, either by taking out a license, or by inventing around the patent.

This feature of $R \& D$ implies that valuation of the asset it creates can be highly variable across firms and time. More importantly, it implies that it is not plausible to try to incorporate "true" economic depreciation into the book estimate of R\&D capital. It would be far better to try to estimate depreciation from the valuation itself. However, there is a difficulty, because we will need some kind of depreciation to construct a stock from past R\&D flows. Simply adding the flows up is not attractive because it places too much weight on R\&D done long ago and because the stock then becomes quite sensitive to the number of time periods over which the firm has been observed. So the best we can do is what was originally done by Griliches (1981) and followed by his successors: use a plausible depreciation rate such as 15 per cent and then try to infer the true value from the estimated shadow value of the $R \& D$. In the next section of the paper this is what is done.

In appendix B, I present a sketch of a theoretical model due to Hayashi and Inoue (1991), which provides a framework for interpretation of the hedonic market value regression results. Under the usual $Q$ model assumptions and an additional assumption that the market value equation is weakly separable so that there is a single capital aggregator, Appendix B shows that the estimating equation for a firm with two capitals, tangible and knowledge, is the following:

$$
\begin{equation*}
\log Q_{i t}=\log \alpha_{t}+\log \left[1+\frac{\gamma_{t}}{\alpha_{t}} \frac{\left(1-\delta_{R}\right) p_{t}^{R} K_{i t}}{\left(1-\delta_{I}\right) p_{i t}^{I} A_{i t}}+\frac{\lambda_{t}}{\alpha_{t}} \varphi^{*}\left(K_{i t} / A_{i t}\right)\right]+\varepsilon_{i t} \tag{13}
\end{equation*}
$$

In this equation $p^{R} K$ and $p^{I} A$ are the nominal values of beginning of period capital stocks, K and A are real values, $\alpha_{t}$ corresponds to overall market movements in Tobin's $Q, \gamma_{t} / \alpha_{t}$ is the relative shadow value of R\&D capital, and $\varphi^{*}(K / A)$ is a capital aggregator, constructed as a Divisia index of $K$ and $A$, using their respective costs of capital (see the appendix for details), with $\lambda_{t} / \alpha_{t}$ its coefficient. There are two K terms: if the depreciation rates are accurate, the first term simply corrects the assets used to construct $Q$ for the omission of R\&D capital, which implies that $\gamma_{t} / \alpha_{t}$ should be unity. The second term represents the contribution of variations in $R \& D$ capital intensity to deviations of the corrected Q ratio from unity. Thus $\lambda_{t} / \alpha_{t}$ should also be zero in equilibrium if everything is correctly measured and there are no unanticipated shocks that impact R\&D-intensive firms differentially.

Comparing equations (12) and (13), we see that there are two differences: first, because the model and data are discrete time, the assets need to be corrected by one minus their depreciation rates, to capture the fact that the appropriate dating is the same as that of market value, end of period. Second, the theoretical model includes the capital aggregator term, $\varphi^{*}(K / A)$, whose coefficient is a function of the random (exogenous to the firm at time $t$ ) shocks driving the valuation process. Unfortunately the model provides no further information about or predictions of the magnitude of this coefficient. In the next section of the paper, I explore the use of equation (13) to estimate the market value equation and compare the results to the simple specification in equation (12).

### 4.1 Market value estimates

Table 4 shows the result of estimating several versions of equation (13) for each of the six 5-year panels, with industry and time dummies included. The first column includes only a term for the nominal $K / A$ ratio $\left.\left[p^{R} K / p^{I} A\right)\right]$ along with the dummies. As is familiar from previous work, the coefficients of this ratio are less than unity in all periods, suggesting that the measured ratio is biased upwards, that is, that $K$ actually depreciates more quickly than implied by the 15 per cent rate that was used to construct it.

## [Table 4 about here]

The second column adds the capital aggregator $\varphi^{*}(K / A)$ to the equation, with an interesting effect: the coefficient of K/A is now closer to the theoretical value of unity in all periods, whereas the coefficient of $\varphi^{*}$ is substantially negative everywhere. The third column imposes the restriction that the coefficient is unity, with little change in the estimates. The interpretation of the negative coefficient on $\varphi^{*}$ is that if the $\mathrm{R} \& \mathrm{D}$ stock is computed correctly (so that its coefficient should be one in equilibrium), then the more $R \& D$ intensive the capital aggregate is, the lower is the market value of the firm, controlling for the year, industry, and the level of tangible and intangible assets. In short, the expected supranormal rents for a firm that is more $R \& D$-intensive than the typical firm in its industry
are negative, with the possible exception of the last period, where the coefficient of is near zero. However, this conclusion relies heavily on having used the "correct" depreciation rate for R\&D when constructing the R\&D capital stock. If the stock is overestimated, the negative coefficient on $\varphi^{*}$ can be due to mismeasurement in two highly correlated variables, which often leads to coefficients of opposite sign (Hall 2004).

The fourth and final column includes two measures of $K / A$ separately: the first is the news in R\&D, computed as the difference between the actual $R / A$ ratio and that predicted from past $K$, time, and year dummies. The second is simply the lagged value of $K / A .{ }^{16}$ Except in the final period, lagged $K / A$ enters with a coefficient that is somewhat higher than that of contemporaneous $K / \mathrm{A}$ in the previous estimates. In spite of this, the "news in R\&D" term is highly significant and the coefficient is about 8 times that for the stock of R\&D. That is, an additional dollar of R\&D that is not simply a continuation of the past spending history is valued 8 times as much as a dollar that has already been sunk. So although high R\&D intensity relative to the industry intensity may be negative for value, increasing $R \& D$ is clearly a positive signal. ${ }^{17}$

Given the uncertainty that surrounds the estimates based on the theoretical model in Table 4, Tables 5 and 6 take a step backward: in these tables, the usual hedonic regression [equation (12)] is estimated and the implied depreciation rate for $\mathrm{R} \& \mathrm{D}$ is backed out of the estimated coefficient of $K / A$, using the relationship in equation (5):

$$
\begin{equation*}
\hat{\delta}_{i t}=\frac{.15+g_{i t}}{\hat{\gamma}_{t}}-g_{i t} \tag{14}
\end{equation*}
$$

Table 5 shows estimates for all of manufacturing and Table 6 for each of the 6 subsectors. In Table 5, three different estimation methods are used for comparison: ordinary least squares (OLS) and least absolute deviations (LAD) on the linear version of equation (12) and nonlinear least squares (NLS) on the version that does not use the $\log (1+\varepsilon)$ approximation. The top panel of the table shows a few characteristics of the samples, including the median of the $R \& D$ growth rates that will be used to construct the estimates of the depreciation rate. K/A and the R\&D growth rate increase until the mid1990s and then fall off a bit in the last period. By the end of the period, the median R\&D capital stock is over one-third of the total tangible assets, having grown from about one-fifth in the mid-1970s.

[^9]
## [Table 5 about here]

The bottom three panels of Table 5 show the estimated K/A coefficient from a regression that also includes 14 industry dummies and dummies for each year. The LAD estimates are higher than the OLS estimates in general, and the NLS estimates are higher still. ${ }^{18}$ The standard errors are roughly comparable across all the estimates and fairly small; the fits are similar to those in Table 4. Given the large size of the K/A ratios, the preferred specification is that estimated by NLS which does not rely on approximating the original equation. These estimates are reasonably consistent with earlier work using the same data, showing an increased valuation of R\&D in the late 1970s and early 1980s followed by a precipitous decline in the 1980s. However, they also show a substantial recovery during the mid-1990s, one that does not reflect declining R\&D (see the first panel) but is more likely to be due to an increase in technological opportunity.

The last two columns of these panels show the median of the depreciation rate for $\mathrm{R} \& \mathrm{D}$ that is implied by these estimates under the assumption that $\mathrm{R} \& \mathrm{D}$ capital is valued equally with tangible capital (that is, the coefficient of $K / A$ is unity), along with the median of the standard error on the depreciation rate computed from the standard error on the coefficient using the "delta" method and conditioning on the observed firm-level growth rate of R\&D. These depreciation rates are large, but not implausible: they follow the same pattern as the coefficient (as they should), and they indicate that up to $50 \%$ of R\&D might have been written off in any one year during the 1980s; the average rate for the whole thirty years is 27 per cent, somewhat higher than the 15 per cent usually used.

Table 6 gives a hint of the source of these relatively high depreciation rates, although the estimates are very imprecise in places, because they are a function of the inverse of the $K / A$ coefficient and it is sometimes close to zero. The most consistent sectors are the drugs and computers sectors, which are of course the ones with the highest R\&D intensity. These two sectors exhibit very different depreciation patterns. For drugs and medical instruments, R\&D depreciation is roughly $15 \%$, somewhat lower in the mid-1970s and mid-1980s and higher thereafter. On the other hand, the computers and instruments sector has write-offs of $50 \%$ or higher during the 1979-1998 period, with a high of $91 \%$ in the mid-1980s. It is not hard to see this as the reaction of the sector to the personal computer and internet revolution, which was accompanied by a number of exits of older computer firms and restructuring of those that remained. The dotcom boom seems to have restored the

[^10]depreciation rate to something more normal, although still slightly higher than that for the pharmaceutical sector. Thus there is some indication that the private returns depreciation rates derived from market value equations for these two sectors capture something of the different nature of competition in the sectors, with pharmaceuticals simultaneously having a longer product cycle, and a greater ability to exclude competitors over a longer period. Both of these features suggest that the private $R \& D$ depreciation rate should be lower for this sector than for computers and instruments.

## [Table 6 about here]

Turning to the other sectors, Chemicals looks a lot like Pharmaceuticals, but with the exception of high deprecation in the late 1980s-early 1990s, a write-off that also occurred in the metals, machinery, and transportation equipment sector somewhat earlier. This is the period of leveraged buyouts and shrinkage of the traditional rustbelt industries in the United States, clearly reflected in their R\&D valuations (also notice the decline in number of observations as firms exit from these industries, mostly via merger). The electrical industry did not suffer as much in the early part of this period, but its valuations fell later, which suggests a speed-up of competition or a change in industry composition, possibly related to the growth of the computing sector.

From these results I conclude that the market value equation is capable of giving an approximate indication of relative depreciation, but that the estimates are probably not precise enough for measuring a rate of return. They may also be systematically high, which is what one might expect of the coefficient estimates are biased downward, either because of measurement error, or because they have not been adjusted for the fact that R\&D investment may be riskier than ordinary investment.

## 5. Conclusions

This paper has presented a detailed exploration of the consequences of variable and unknown private depreciation rates of $R \& D$ capital for the measurement of the rate of return to $R \& D$, going further down one of the many paths left open for exploration by Zvi Griliches in his final monograph published in 2000. Although the paper has focused specifically on $\mathrm{R} \& D$, the results are applicable to returns measurement for any investment where the capital created is not traded on a secondhand market (so there is no independent measure of depreciation) and where the capital does indeed depreciate, at least from the perspective of its "owner." In particular, the discussion applies to innovation investments more broadly.

The investigation has revealed several things: first, it has shown that one of the methodologies used in earlier literature, measuring returns by regressing productivity on R\&D intensity, is seriously downward biased if depreciation is of any concern. Second, estimates of private R\&D depreciation
have been obtained in two different ways: first, by assuming the firms invest in R\&D until its relative marginal product equals the relative cost of capital and deriving the implied depreciation rate, and second by assuming that the market values properly depreciated R\&D capital one for one with ordinary capital and deriving the implied depreciation rate from that fact. For the manufacturing sector during the entire 1974-2003 period, these two methods yielded quite different estimates: -6 per cent from R\&D productivity and 27 per cent from market value. At the broad industry level, there is little systematic relationship between the results either, with the exception that rates are relatively lower in pharmaceuticals and somewhat higher in electrical/computing.

Thus we are no further in resolving the conundrum identified by Griliches, although we have perhaps a better understanding of its origins. The assumptions required to perform the computations in this paper are heroic and the heterogeneity across the firms substantial, which helps to explain the variability of the results to some extent. Nevertheless, the persistent gap between productivity and market value results requires some explanation. In order to derive depreciation from the productivity results I had to assume that the firms' current level of R\&D stock was chosen so that its marginal product covered the required rate of return plus depreciation. On the other hand, to derive depreciation from market value I had to assume that financial markets were efficient and that the firm had optimized its choice of R\&D capital. Both sets of assumption ignore adjustment costs in R\&D capital, which may be substantial. However, incorporating these costs into these frameworks is difficult and beyond the scope of the present paper. Both sets of estimates also are highly volatile because of the huge gap between expected and realized returns: for market value this is obvious, but it is no less true for the production function if adjustment costs are large.

With respect to the "bottom line," what do these estimates tell us about the rate of return to R\&D? The simplest answer is "nothing," because we assumed a normal rate of return to compute them. But two further conclusions can be drawn: first, private depreciation is likely to be more variable and higher than the 15 per cent normally assumed. Second, researchers and policy analysts should be strongly cautioned against treating the rate of return to $\mathrm{R} \& D$ as a parameter: clearly it is the complex outcome of a moving equilibrium.

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Figure 1
Productivity and market value of R\&D in US manufacturing firms


Figure 2
Productivity and market value of R\&D in US manufacturing firms (relative to capital)


Figure 3


Table 1
Sample statistics 1974-2003

|  | Mean | Std. dev. | Median | IQ range | Minimum | Maximum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Unbalanced panel: 28937 observations |  |  |  |  |  |  |
| Sales* | 1778 | 7653 | 178 | 794 | 2.0 | 174710 |
| Employment (1000s) | 10.1 | 33.6 | 1.5 | 5.9 | 0.1 | 876.8 |
| Capital stock* | 1070 | 4680 | 77 | 395 | 0.7 | 141190 |
| K stock* | 276 | 1436 | 24 | 86 | 0.0 | 43011 |
| Capital-sales ratio | $54.8 \%$ | $31.3 \%$ | $47.5 \%$ | $29.0 \%$ | $3.3 \%$ | $498.9 \%$ |
| K-sales ratio | $24.1 \%$ | $32.2 \%$ | $14.3 \%$ | $24.1 \%$ | $1.0 \%$ | $486.5 \%$ |
| R\&D-sales ratio | $5.3 \%$ | $7.7 \%$ | $2.8 \%$ | $5.5 \%$ | $0.1 \%$ | $99.1 \%$ |
| Tobin's q = V/A | 2.84 | 3.58 | 1.83 | 1.74 | 0.12 | 70.59 |
| Tobin's q = V/(A+K) | 1.98 | 1.81 | 1.45 | 1.21 | 0.10 | 19.93 |


| Balanced panel: 16750 observations |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Sales* | 2518 | 9538 | 300 | 1350 | 5.1 | 164843 |
| Employment (1000s) | 14.2 | 41.6 | 2.5 | 10.4 | 0.1 | 876.8 |
| Capital stock* | 1487 | 5685 | 133 | 696 | 1.9 | 141190 |
| K stock* | 402 | 1823 | 37 | 153 | 0.2 | 43011 |
| Capital-sales ratio | $54.6 \%$ | $27.8 \%$ | $48.1 \%$ | $26.9 \%$ | $5.6 \%$ | $460.8 \%$ |
| K-sales ratio | $20.9 \%$ | $22.0 \%$ | $13.9 \%$ | $21.3 \%$ | $1.0 \%$ | $332.4 \%$ |
| R\&D-sales ratio | $4.4 \%$ | $5.0 \%$ | $2.7 \%$ | $4.6 \%$ | $0.1 \%$ | $83.8 \%$ |
| Tobin's q = V/A | 2.48 | 2.69 | 1.75 | 1.51 | 0.12 | 50.67 |
| Tobin's q = V/(A+K) | 1.85 | 1.56 | 1.41 | 1.11 | 0.10 | 19.43 |

[^11]Table 2
Production function estimates

|  | Observ- <br> ations <br> (firms) | Median of <br> Real <br> K/Output | Median of <br> Real <br> K/Capital | Median <br> growth of <br> R\&D* |
| :--- | :---: | :---: | :---: | :---: |
| Period | R | (609) | 0.112 | 0.212 |
| $1974-1978$ | $3045-1983$ | $2990(598)$ | 0.124 | 0.238 |
| $1984-1988$ | $2535(507)$ | 0.171 | 0.310 | $6.1 \%$ |
| $1989-1993$ | $2915(583)$ | 0.195 | 0.350 | $6.8 \%$ |
| $1994-1998$ | $2735(547)$ | 0.195 | 0.327 | $8.1 \%$ |
| $1999-2003$ | $2530(506)$ | 0.194 | 0.250 | $5.6 \%$ |
| All years | $16750(1521)$ | 0.139 | 0.272 | $6.0 \%$ |


|  | LAD coeff. <br> for log <br> Capital | Standard <br> error | LAD <br> coefficient <br> for log K | Standard <br> error | Ratio of <br> LAD <br> coefficients | Standard <br> error | Implied cost <br> of R\&D <br> capital** | Implied <br> deprec. <br> rate*** | Median <br> standard <br> error*** |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Period | 0.466 | 0.016 | 0.011 | 0.007 | 0.023 | 0.016 | $2.5 \%$ | $-12.0 \%$ | $1.2 \%$ |
| $1974-1978$ | 0.1983 | 0.444 | 0.017 | 0.042 | 0.010 | 0.095 | 0.024 | $10.9 \%$ | $-17.8 \%$ |
| $4.5 \%$ |  |  |  |  |  |  |  |  |  |
| $1984-1988$ | 0.398 | 0.017 | 0.043 | 0.008 | 0.109 | 0.023 | $7.6 \%$ | $-12.7 \%$ | $1.1 \%$ |
| $1989-1993$ | 0.369 | 0.013 | 0.086 | 0.009 | 0.232 | 0.028 | $13.8 \%$ | $-8.5 \%$ | $0.6 \%$ |
| $1994-1998$ | 0.376 | 0.013 | 0.123 | 0.009 | 0.328 | 0.030 | $18.5 \%$ | $-8.3 \%$ | $0.4 \%$ |
| $1999-2003$ | 0.328 | 0.016 | 0.133 | 0.011 | 0.406 | 0.048 | $26.6 \%$ | $-4.7 \%$ | $0.4 \%$ |
| All years | 0.395 | 0.001 | 0.075 | 0.003 | 0.191 | 0.011 | $12.5 \%$ | $-6.6 \%$ | $0.2 \%$ |
| Period |  |  | With correction for double counting of R\&D |  |  |  |  |  |  |

LAD estimates of $\log$ deflated sales on $\log$ employees, $\log$ capital stock, $\log R \& D$ stock $K, 14$ industry dummies, and year dummies.
The estimated coefficients and bootstrap standard errors of $\log$ R\&D stock, log capital and their ratio are shown.
*This is the average growth of R\&D between 1970 (or the year of entry) and the beginning of the year in question, for the median firm.
**The median of the ratio of the coefficients to the R\&D-capital ratio times the cost of ordinary capital.
***The median and median standard error of the estimated depreciation rate for $R \& D$ capital, computed using equation (..) of the paper.

Table 3
Production function estimates by sector

| Period |  <br> chem-based | Drugs \& med <br> inst | Electrical |  <br> inst |  <br> machinery | Misce-Ilaneous |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |


| Ratio of Log K to Log C coefficient (standard error)* |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1974-1978$ | $0.28(0.07)$ | $0.28(0.09)$ | $0.09(0.05)$ | $0.18(0.05)$ | $0.25(0.12)$ | $0.12(0.08)$ |
| $1979-1983$ | $0.35(0.10)$ | $0.27(0.05)$ | $-0.01(0.09)$ | $0.41(0.12)$ | $0.29(0.07)$ | $0.37(0.12)$ |
| $1984-1988$ | $0.20(0.06)$ | $0.17(0.10)$ | $0.57(0.27)$ | $0.70(0.13)$ | $0.19(0.10)$ | $0.35(0.18)$ |
| $1989-1993$ | $0.26(0.11)$ | $0.25(0.11)$ | $1.41(0.63)$ | $0.76(0.20)$ | $0.63(0.22)$ | $0.69(0.22)$ |
| $1994-1998$ | $0.79(0.26)$ | $0.49(0.12)$ | $0.34(0.07)$ | $0.81(0.16)$ | $0.36(0.08)$ | $0.77(0.15)$ |
| $1999-2003$ | $0.55(0.18)$ | $0.33(0.08)$ | $1.34(0.58)$ | $1.84(0.29)$ | $0.46(0.07)$ | $0.52(0.09)$ |
| All years | $0.29(0.04)$ | $0.13(0.03)$ | $0.28(0.04)$ | $0.67(0.07)$ | $0.23(0.03)$ | $0.29(0.05)$ |
|  |  |  |  |  |  |  |
| $1974-1978$ | $31.6 \%$ | $18.1 \%$ | Implied cost of R\&D capital |  |  |  |
| $1979-1983$ | $49.5 \%$ | $20.4 \%$ | $8.4 \%$ | $9.1 \%$ | $37.6 \%$ | $26.2 \%$ |
| $1984-1988$ | $17.0 \%$ | $8.1 \%$ | $-0.7 \%$ | $22.9 \%$ | $48.9 \%$ | $94.0 \%$ |
| $1989-1993$ | $22.7 \%$ | $10.5 \%$ | $78 \%$ | $26.7 \%$ | $22.8 \%$ | $62.7 \%$ |
| $1994-1998$ | $79.2 \%$ | $20.1 \%$ | $24.2 \%$ | $23.0 \%$ | $68.1 \%$ | $123.5 \%$ |
| $1999-2003$ | $55.1 \%$ | $13.9 \%$ | $98.0 \%$ | $21.7 \%$ | $34.9 \%$ | $108.7 \%$ |
| All years | $31.1 \%$ | $6.2 \%$ | $22.5 \%$ | $53.1 \%$ | $50.9 \%$ | $63.2 \%$ |


| Estimated R\&D depreciation rate in per cent (median standard error)** |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $1974-1978$ | $1.9(2.8)$ | $-9.0(6.2)$ | $-12.5(6.8)$ | $-12.3(2.8)$ | $2.7(1.7)$ | $3.7(8.3)$ |
| $1979-1983$ | $1.0(2.1)$ | $-3.6(6.9)$ | $-14.9(6.4)$ | $-2.0(7.5)$ | $0.6(1.8)$ | $-1.3(0.9)$ |
| $1984-1988$ | $-11.3(4.3)$ | $-13.7(3.3)$ | $-3.5(3.7)$ | $-6.4(3.0)$ | $-4.2(7.1)$ | $-4.2(1.5)$ |
| $1989-1993$ | $-6.1(3.5)$ | $-8.9(2.8)$ | $-4.2(0.8)$ | $-6.0(2.6)$ | $-3.2(0.7)$ | $-5.2(0.3)$ |
| $1994-1998$ | $-4.7(0.3)$ | $-8.4(1.7)$ | $-9.4(2.1)$ | $-7.6(1.5)$ | $-5.6(1.1)$ | $-7.2(0.1)$ |
| $1999-2003$ | $-1.2(0.9)$ | $-6.8(1.0)$ | $-4.3(0.4)$ | $-5.3(0.6)$ | $-3.7(0.3)$ | $-2.9(0.3)$ |
| $1999-2003$ | $-2.3(1.6)$ | $-10.9(0.7)$ | $-3.0(2.1)$ | $-5.0(1.0)$ | $-1.8(1.2)$ | $-2.3(0.6)$ |

LAD estimates of $\log$ deflated sales on $\log$ employees, $\log$ capital stock, $\log$ R\&D stock $K$, and industry and year dummies.
*The ratio of the estimated coefficients of $\log$ R\&D stock and log capital and its standard error are shown.
**The median and median standard error of the estimated depreciation rate for $R \& D$ capital, computed using equation (9).

Table 4
The Market Value-R\&D Relation: Exploring the Functional Form
US Manufacturing Firms

|  | (0) |  | (1) |  | (2) |  | (3) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1974-1978 3045 observations |  |  |  |  |  |  |  |
| PHI(K/A) |  |  | -0.41 | (0.03) | -0.38 | (0.03) |  |  |
| R/A surprise* |  |  |  |  |  |  | 4.71 | (0.03) |
| K/A** | 0.53 | (0.03) | 0.72 | (0.03) | 1.00 |  | 0.61 | (0.03) |
| R-squared |  |  |  |  | 0.188 |  |  |  |
| Standard error |  |  |  |  | 0.442 |  |  |  |
|  | 1979-1983 2990 observations |  |  |  |  |  |  |  |
| PHI(K/A) |  |  | -0.43 | (0.02) | -0.45 | (0.02) |  |  |
| R/A surprise* |  |  |  |  |  |  | 6.76 | (0.02) |
| K/A** | 0.60 | (0.02) | 0.79 | (0.02) | 1.00 |  | 0.80 | (0.02) |
| R-squared |  |  |  |  | 0.289 |  |  |  |
| Standard error |  |  |  |  | 0.439 |  |  |  |
|  | 1984-1988 2535 observations |  |  |  |  |  |  |  |
| PHI(K/A) |  |  | -0.34 | (0.03) | -0.30 | (0.03) |  |  |
| R/A surprise* |  |  |  |  |  |  | 3.24 | (0.03) |
| K/A** | 0.39 | (0.03) | 0.62 | (0.03) | 1.00 |  | 0.47 | (0.03) |
| R-squared |  |  |  |  | 0.209 |  |  |  |
| Standard error |  |  |  |  | 0.453 |  |  |  |
|  | 1989-1993 2915 observations |  |  |  |  |  |  |  |
| PHI(K/A) |  |  | -0.40 | (0.03) | -0.39 | (0.03) |  |  |
| R/A surprise* |  |  |  |  |  |  | 3.43 | (0.03) |
| K/A** | 0.38 | (0.03) | 0.64 | (0.03) | 1.00 |  | 0.50 | (0.03) |
| R-squared |  |  |  |  | 0.222 |  |  |  |
| Standard error |  |  |  |  | 0.546 |  |  |  |
|  | 1994-1998 2735 observations |  |  |  |  |  |  |  |
| PHI(K/A) |  |  | -0.38 | (0.03) | -0.38 | (0.03) |  |  |
| R/A surprise* |  |  |  |  |  |  | 5.15 | (0.03) |
| K/A** | 0.55 | (0.03) | 0.71 | (0.03) | 1.00 |  | 0.62 | (0.03) |
| R-squared |  |  |  |  | 0.181 |  |  |  |
| Standard error |  |  |  |  | 0.586 |  |  |  |
|  | 1999-2003 2530 observations |  |  |  |  |  |  |  |
| PHI(K/A) |  |  | -0.16 | (0.04) | -0.10 | (0.04) |  |  |
| R/A surprise* |  |  |  |  |  |  | 6.93 | (0.04) |
| K/A** | 0.79 | (0.04) | 0.80 | (0.04) | 1.00 |  | 0.94 | (0.04) |
| R-squared |  |  |  |  | 0.250 |  |  |  |
| Standard error |  |  |  |  | 0.639 |  |  |  |

Nonlinear least squares estimates with dependent variable $\log \mathrm{Q}$, robust standard errors.
14 sector dummies and 4 year dummies are included in the equation.
*This variable is the actual R/A less the fitted value from a regression of $\log \mathrm{R} / \mathrm{A}$ on $\log \mathrm{K} / \mathrm{A}$ lagged plus time and industry dummies.
**In the last column this variable is lagged.

Table 5
Market value equation estimates

| Period | Observations | Median K/A (nominal) | Interquartile range for K/A | Median real growth of R\&D |
| :---: | :---: | :---: | :---: | :---: |
| 1974-1978 | 3045 | 0.176 | 0.233 | 3.0\% |
| 1979-1983 | 2990 | 0.211 | 0.290 | 5.3\% |
| 1984-1988 | 2535 | 0.285 | 0.404 | 6.9\% |
| 1989-1993 | 2915 | 0.349 | 0.561 | 6.7\% |
| 1994-1998 | 2735 | 0.335 | 0.606 | 8.1\% |
| 1999-2003 | 2530 | 0.308 | 0.571 | 4.7\% |
| All years | 16750 | 0.261 | 0.409 | 5.9\% |
| Period | K Coefficient | Implied depreciation rate ${ }^{* * *}$ |  |  |
|  |  | Std. err. | Median | Median s.e. |
|  |  | Ordinary least squares* |  |  |
| 1974-1978 | 0.352 | 0.053 | 48.2\% | 7.7\% |
| 1979-1983 | 0.416 | 0.046 | 43.6\% | 5.4\% |
| 1984-1988 | 0.287 | 0.032 | 69.2\% | 8.5\% |
| 1989-1993 | 0.239 | 0.024 | 84.1\% | 9.0\% |
| 1994-1998 | 0.302 | 0.028 | 68.3\% | 7.2\% |
| 1999-2003 | 0.366 | 0.026 | 49.0\% | 3.8\% |
| All years | 0.297 | 0.013 | 40.6\% | 1.2\% |
|  | Least absolute deviations** |  |  |  |
| 1974-1978 | 0.390 | 0.049 | 43.1\% | 5.8\% |
| 1979-1983 | 0.466 | 0.052 | 38.3\% | 4.9\% |
| 1984-1988 | 0.271 | 0.034 | 73.8\% | 10.1\% |
| 1989-1993 | 0.274 | 0.033 | 72.3\% | 9.5\% |
| 1994-1998 | 0.316 | 0.043 | 65.0\% | 9.9\% |
| 1999-2003 | 0.404 | 0.033 | 44.0\% | 4.0\% |
| All years | 0.297 | 0.017 | 40.6\% | 1.5\% |
|  | Nonlinear estimation* |  |  |  |
| 1974-1978 | 0.526 | 0.025 | 31.2\% | 5.1\% |
| 1979-1983 | 0.595 | 0.025 | 28.8\% | 4.1\% |
| 1984-1988 | 0.385 | 0.028 | 49.9\% | 7.3\% |
| 1989-1993 | 0.382 | 0.031 | 50.0\% | 6.2\% |
| 1994-1998 | 0.551 | 0.037 | 33.8\% | 4.5\% |
| 1999-2003 | 0.794 | 0.040 | 20.1\% | 2.4\% |
| All years | 0.503 | 0.032 | 27.5\% | 1.2\% |

Year dummies and 14 industry dummies are included in the equations.
*Robust standard errors are shown for these two estimators.
**Standard errors computed via the bootstrap with 200 replications.
***The R\&D depreciation rate computed using the estimated coefficient and each firm's R\&D growth rate under the assumption that the true K/A coefficient is one. The median and the median standard error are shown.

Table 6
Market value equation estimates by sector

| Period | Chemicals | Drugs \& med <br> inst | Electrical |  <br> inst |  <br> machinery | Misce-llaneous |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $1974-1978$ | 400 | 310 | Number of observations |  |  |  |
| $1979-1983$ | 350 | 325 | 290 | 695 |  |  |
| $1984-1988$ | 290 | 305 | 290 | 820 | 760 | 490 |
| $1989-1993$ | 310 | 415 | 230 | 760 | 605 | 455 |
| $1994-1998$ | 250 | 455 | 185 | 1045 | 610 | 345 |
| $1999-2003$ | 235 | 470 | 205 | 920 | 615 | 350 |
| All years | 1835 | 2280 | 200 | 750 | 570 | 290 |


|  | Median of Nominal K/tangible assets ratio |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1974-1978$ | 0.167 | 0.281 | 0.208 | 0.362 | 0.122 | 0.082 |
| $1979-1983$ | 0.167 | 0.315 | 0.229 | 0.428 | 0.140 | 0.094 |
| $1984-1988$ | 0.231 | 0.429 | 0.294 | 0.532 | 0.168 | 0.109 |
| $1989-1993$ | 0.233 | 0.486 | 0.354 | 0.706 | 0.188 | 0.115 |
| $1994-1998$ | 0.193 | 0.472 | 0.274 | 0.706 | 0.195 | 0.137 |
| $1999-2003$ | 0.198 | 0.473 | 0.274 | 0.706 | 0.181 | 0.163 |
| All years | 0.192 | 0.394 | 0.264 | 0.547 | 0.155 | 0.109 |


| Median of real R\&D growth rate |  |  |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| $1974-1978$ | $2.7 \%$ | $4.4 \%$ | $2.2 \%$ | $4.0 \%$ | $2.6 \%$ | $2.5 \%$ |
| $1979-1983$ | $4.7 \%$ | $6.1 \%$ | $5.2 \%$ | $7.4 \%$ | $4.6 \%$ | $4.6 \%$ |
| $1984-1988$ | $5.9 \%$ | $7.7 \%$ | $6.8 \%$ | $9.1 \%$ | $5.8 \%$ | $6.4 \%$ |
| $1989-1993$ | $5.6 \%$ | $8.4 \%$ | $6.0 \%$ | $8.7 \%$ | $5.0 \%$ | $5.9 \%$ |
| $1994-1998$ | $6.2 \%$ | $9.5 \%$ | $8.1 \%$ | $9.4 \%$ | $6.8 \%$ | $7.6 \%$ |
| $1999-2003$ | $2.9 \%$ | $5.5 \%$ | $3.9 \%$ | $6.3 \%$ | $4.2 \%$ | $3.7 \%$ |
| All years | $4.9 \%$ | $7.1 \%$ | $5.2 \%$ | $7.9 \%$ | $4.9 \%$ | $5.0 \%$ |
|  |  | K/tangible assets coefficient (s.e.)* |  |  |  |  |
| $1974-1978$ | $0.63(0.08)$ | $1.69(0.10)$ | $0.35(0.07)$ | $0.60(0.06)$ | $-0.11(0.04)$ | $0.46(0.06)$ |
| $1979-1983$ | $1.21(0.08)$ | $0.92(0.09)$ | $0.81(0.07)$ | $0.34(0.06)$ | $0.78(0.05)$ | $0.44(0.06)$ |
| $1984-1988$ | $1.23(0.07)$ | $1.58(0.10)$ | $0.69(0.06)$ | $0.24(0.05)$ | $0.15(0.04)$ | $0.02(0.07)$ |
| $1989-1993$ | $0.45(0.08)$ | $0.76(0.10)$ | $0.03(0.09)$ | $0.39(0.06)$ | $0.11(0.06)$ | $0.32(0.09)$ |
| $1994-1998$ | $0.70(0.09)$ | $0.83(0.11)$ | $0.33(0.09)$ | $0.45(0.08)$ | $0.47(0.06)$ | $1.84(0.08)$ |
| $1999-2003$ | $0.45(0.07)$ | $0.87(0.11)$ | $0.33(0.09)$ | $0.71(0.09)$ | $0.97(0.07)$ | $2.65(0.09)$ |
| All years | $0.73(0.08)$ | $0.95(0.11)$ | $0.35(0.09)$ | $0.46(0.07)$ | $0.42(0.06)$ | $0.69(0.08)$ |


| Median estimated R\&D depreciation rate in per cent (median standard error)** |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $1974-1978$ | $25.2(11.4)$ | $7.0(2.7)$ | $47.1(32.5)$ | $27.8(6.4)$ | $<-100$ | $35.7(26.1)$ |
| $1979-1983$ | $11.6(3.0)$ | $16.9(5.7)$ | $19.7(7.6)$ | $58.6(17.5)$ | $20.4(5.1)$ | $40.1(25.9)$ |
| $1984-1988$ | $11.1(2.7)$ | $6.6(3.3)$ | $24.8(6.5)$ | $91.4(27.5)$ | $>100$ | $>100$ |
| $1989-1993$ | $39.8(16.7)$ | $22.3(7.0)$ | $>100$ | $52.8(8.4)$ | $>100$ | $60.3(34.4)$ |
| $1994-1998$ | $24.1(10.5)$ | $20.1(5.8)$ | $62.4(26.3)$ | $44.2(9.4)$ | $39.5(12.3)$ | $4.7(2.4)$ |
| $1999-2003$ | $36.8(15.3)$ | $18.0(4.9)$ | $55.1(27.3)$ | $23.9(4.8)$ | $15.5(3.8)$ | $3.4(1.2)$ |
| All years | $22.2(3.2)$ | $16.1(2.2)$ | $52.1(10.8)$ | $42.0(3.9)$ | $43.0(6.2)$ | $24.1(5.5)$ |

[^12]
## Appendix A: Choosing the Sample of Firms

This appendix describes the creation of the sample. The starting point was 104,000 Compustat observations on firms between 1970 and 2004 in SIC 2000-2999 inclusive. Observations with sales, employment, market value, or capital stock missing or R\&D missing or zero were removed, as were firms (or parts of firms) that had 3 or fewer observations in successive years. Foreign firms (either recorded as located outside the US by Compustat or containing -ADR, NV, PLC, or LTD in the name) were also removed. Following this, observations that did not satisfy the restrictions given in Table A1 below were removed from the sample.

## Table A1

## Selecting the sample

| Variable | Restriction for sample |
| :--- | :--- |
| Employment | Greater than 100 employees |
| Modified Tobin's q | Between 0.1 and 20 |
| Modified Tobin's q | Absolute value of growth < 500\% |
| Intangibles-capital stock ratio | Current and lagged $<100 \%$ |
| K stock-capital stock ratio | Between 1\% and 500\% |
| R\&D-capital stock ratio | Between 0.1\% and 200\% |
| R\&D-sales ratio | Between 0.1\% and 100\% |
| K-sales ratio | Between $1 \%$ and 500\% |
| Capital-sales ratio | Between $1 \%$ and 500\% |
| Capital-employment ratio | Between $\$ 1000$ and \$2M |
| Growth in sales | Absolute value $<150 \%$ |
| Growth in employment | Absolute value $<100 \%$ |
| Growth in capital stock | Absolute value $<100 \%$ |
| Growth in K stock | Absolute value $<100 \%$ |
| Growth in R\&D | Absolute value $<150 \%$ |
| Sales-employment ratio | Between \$5000 and \$2M |
| K stock-employment ratio | Between \$500 and \$2M |

The above procedure left about 29,000 observations in an unbalanced panel from 1973 to 2003. From this, 6 balanced panels with 5 years of data each ( 6 years including lagged capital) were chosen for a total of 16,750 observations. Some information on the selection is given in Table A2. Although the selection is fairly severe, Figure A1 shows that there is still significant coverage of the total R\&D in the economy. The unbalanced sample includes 98 per cent of the $\mathrm{R} \& \mathrm{D}$ in the manufacturing sector master file until 1993; coverage then drops to 90 per cent in the final ten years. The partially balanced
panels display a similar but somewhat lower pattern. The decline in the later years appears to be caused by a number of new entrants whose data series are not long enough to be included in the sample.

Table A3 gives the distribution of firms and observations by two-digit industry (and a few three-digit industries) and the allocation of the industries to the six main technology sectors that are used in Tables 3 and 6.

Figure A1
R\&D coverage of sample


Table A2
Choosing the sample of firms from Compustat*

| Period | Raw data (manufacturing) Observations | Domestic Manufacturing Firms with sales, employment and q |  | Unbalanced R\&D sample Observations | 5-year balanced samples with R\&D |  | Sample share |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Observations | Firms |  | Observations | Firms | Observations | Firms |
| 1974-1978 | 14971 | 9001 | 2188 | 4822 | 3045 | 609 | $34 \%$ | 28\% |
| 1979-1983 | 14540 | 9414 | 2425 | 4662 | 2990 | 598 | 32\% | 25\% |
| 1984-1988 | 16285 | 9792 | 2660 | 4776 | 2535 | 507 | 26\% | 19\% |
| 1989-1993 | 16574 | 9873 | 2654 | 4696 | 2915 | 583 | 30\% | 22\% |
| 1994-1998 | 21272 | 12142 | 3113 | 5383 | 2735 | 547 | 23\% | 18\% |
| 1999-2003 | 21024 | 10418 | 2644 | 4598 | 2530 | 506 | 24\% | 19\% |
| Total | 104666 | 60640 | 6189 | 28937 | 16750 | 1350 | 28\% | 22\% |

*The source is the Compustat annual industrial file as of 2005, all years, SIC $=2000$ to 3999

Table A3
Distribution of the Sample by Industry

| Sector | Two-digit industry | Firms | Observations |
| :--- | :--- | :---: | ---: |
| Chemicals \& chemical-based | 28 Chemicals ex pharmaceuticals | 82 | 1,095 |
|  | 29 Oil | 19 | 255 |
|  | 30 Rubber \& plastics | 47 | 485 |
| Pharmaceuticals \& medical |  |  |  |
| instruments | 283 Pharmaceuticals \& biotechnology | 106 | 1,395 |
|  | 384 Medical \& dental instruments | 100 | 885 |
| Electrical equipment | 36 Electrical equipment | 88 | 940 |
|  | 37 Aircraft | 34 | 460 |
| Computers, communnication |  |  |  |
| eq., \& scientific instruments | 357 Office machinery | 110 | 1,035 |
|  | 367 Commmunication equipment | 218 | 2,245 |
|  | 38 Scientific instruments | 148 | 1,710 |
| Metals \& machinery | 33 Primary metals | 42 | 465 |
|  | 34 Fabricated metals | 770 |  |
|  | 35 Machinery | 79 | 1,960 |
|  | 37 Autos \& other transport | 185 | 715 |
| 20 Food \& kindred products | 37 | 355 |  |
|  | 21 Tobacco | 32 | 25 |
|  | 22 Textile mill products | 3 | 215 |
|  | 23 Apparel \& other textile products | 27 | 3 |
|  | 24 Lumber \& wood products | 7 | 35 |
|  | 25 Furniture \& fixtures | 65 |  |
|  | 26 Paper \& allied products | 25 | 360 |
|  | 27 Printing \& publishing | 35 | 400 |
|  | 31 Leather \& allied products | 10 | 105 |
|  | 32 Stone, clay, \& glass | 6 | 55 |
|  | 39 Miscellaneous NEC | 23 | 270 |
|  | All manufacturing | 32 | 350 |
| Totals | $\mathbf{1 5 2 1}$ | $\mathbf{1 6 , 7 5 0}$ |  |

## Appendix B: Hayashi-Inoue Model Applied to R\&D Assets

This appendix presents a theoretical $Q$ model of investment with two or more capital assets that is due to Hayashi and Inoue (1991), applied to the case of R\&D assets. The $Q$ model, originally due to Tobin, defines $Q$ as the ratio of the market value $V$ of a (unique) asset to its replacement $\operatorname{cost} A$. The theory had the following implications:

- $\mathrm{Q}>1=>$ the firm or other similar firms should invest to create more of the asset and drive its value down to replacement cost.
- $\mathrm{Q}<1 \Rightarrow>$ the firm should shrink or disinvest to reduce the amount of the asset until its value rises to its replacement cost.
- $\mathrm{Q}=1$ in equilibrium.

If the market value of a firm is equal to the value of its dynamic program of investment, the ideal theoretical specification would be derived by modeling investment in tangible and intangible (knowledge) assets under uncertainty using a dynamic program for the firm. From this we could obtain a value function of the assets (state variables) of the firm and use it as a basis for estimation.

Under the assumption that the asset being modeled is a firm and that its Q is derived from the firm's dynamic program, Hayashi (1982) gave conditions under which marginal $\mathrm{Q}(d V / d A)$ is equal to average (V/A). Hayashi and Inoue (1991) and Wildasin (1984) developed the theory with more than one asset (capital). In this appendix I use Hayashi and Inoue's model of a firm with multiple capitals to derive the value function for a firm with two capitals, tangible and intangible, and discuss the interpretation of the estimating model in the light of the theory.

The Hayashi and Inoue (HI) model is based on the idea that the firm's problem is weakly separable into two stages: the top stage allocates a capital aggregate across time, and the bottom stage chooses levels of the individual capitals that make up the aggregate under cost minimization. Aggregating capital in this way allows for a familiar Tobin's $q$ interpretation where the capital aggregate plays the role that the single capital asset plays in the traditional model. In particular, the result that marginal $q$ is equal to average $q$ can be obtained under the assumption of constant returns to scale in the profit, aggregate, and adjustment cost functions.

In my version of the model the two capitals will be denoted by $A_{t}$ (tangible) and $K_{t}$ (intangible), the corresponding investments by $I_{t}$ and $R_{t}$, their prices by $p_{t}^{I}$ and $p_{t}^{R}$ respectively, and their depreciation rates by $\delta_{I}$ and $\delta_{R}$. The capital aggregate is denoted $\varphi\left(A_{t}, K_{t}\right)$ and the (stochastic) output price by $p_{t}$. In contrast to HI , we assume a single constant discount rate $\beta$; this can easily be generalized. We also
ignore tax considerations for the moment. Given this notation, the firm's profit maximizing problem is given by the following expression: ${ }^{19}$

$$
\begin{align*}
\max V_{0} & =E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\Pi\left(A_{t}, K_{t}, A_{t+1}, K_{t+1} ; p_{t}\right)-p_{t}^{I} I_{t}-p_{t}^{R} R_{t}\right] \\
\text { s.t. } A_{t+1} & =\left(1-\delta_{I}\right) A_{t}+I_{t}  \tag{15}\\
K_{t+1} & =\left(1-\delta_{R}\right) K_{t}+R_{t}
\end{align*}
$$

where the profit function $\pi($.$) incorporates adjustment cost functions for the two capitals and all the$ variable factors are optimized out period by period. The state variables for this problem are $A_{t}$ and $K_{t}$, along with the exogenous processes for the three prices, which vary only over time and possibly industry, and the control variables are the two investment choices. The prices are assumed to be driven by a single first order Markov process $\left(s_{t}, t=0,1, \ldots\right)$.

To obtain the value function used for estimation, it is convenient to substitute in the accumulation constraints, eliminating $I_{t}$ and $R_{t}$ from the problem. The new control variables will be one plus net capital growth (that is, $A_{t+1} / A_{t}$ and $\mathrm{K}_{t+1} / K_{t}$ ), rather than gross investments. After some manipulation, we obtain

$$
\begin{align*}
V_{0} & =p_{0}^{I}\left(1-\delta_{I}\right) A_{0}+p_{0}^{R}\left(1-\delta_{R}\right) K_{0}+W_{0} \\
\text { with } W_{0} & =E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\Pi\left(A_{t}, K_{t}, A_{t+1}, K_{t+1} ; p_{t}\right)-c_{t}^{A} A_{t+1}-c_{t}^{K} K_{t+1}\right] \tag{16}
\end{align*}
$$

where the costs of capital $c_{t}$ are given by equation (9), suitably modified to incorporate expectations:

$$
\begin{align*}
& c_{t}^{A}=p_{t}^{I}-E_{t}\left[\beta\left(1-\delta_{I}\right) p_{t+1}^{I}\right]  \tag{17}\\
& c_{t}^{K}=p_{t}^{R}-E_{t}\left[\beta\left(1-\delta_{R}\right) p_{t+1}^{R}\right]
\end{align*}
$$

The first two terms in the value function describe the current state of the firm, and are not under its control. The last term $\left(W_{0}\right)$ represents the rents that are expected to be earned by the firm because of adjustment costs associated with the two capitals. Each term in $W_{0}$ is the profits earned with the current capital levels minus the adjustment costs incurred from moving to next period's capital and the rental cost of that capital. In equilibrium, when capitals are at their optimal level, so that there are

[^13]no adjustment costs and they simply earn their rental cost, $W_{0}$ will be zero. In that case, only the first two terms of the value function will enter, so that the market value of the two assets will simply be equal to the cost of replacing them at current prices, and aggregate Tobin's $q$ will be unity.

To obtain a tractable form for the rents $W_{0}, \mathrm{HI}$ assume that profits depend only on a capital aggregate, and that both the aggregate and the profit function are linear homogeneous:

$$
\begin{align*}
\Pi\left(A_{t}, K_{t}, A_{t+1}, K_{t+1} ; p_{t}\right) & =\Pi^{*}\left(\varphi\left(A_{t}, K_{t}\right), \varphi\left(A_{t+1}, K_{t+1}\right) ; p_{t}\right) \\
& =\pi\left(\frac{\varphi\left(A_{t}, K_{t}\right)}{\varphi\left(A_{t+1}, K_{t+1}\right)} ; p_{t}\right) \varphi\left(A_{t+1}, K_{t+1}\right) \tag{18}
\end{align*}
$$

Using this form of the profit function, they prove that the value of the expected rents $V_{0}$ is a linear function of the capital aggregate:

$$
\begin{equation*}
W_{0}=Q\left(s_{0}\right) \varphi^{*}\left(K_{0} / A_{0}\right) A_{0} \tag{19}
\end{equation*}
$$

where $Q(s)$ is a function only of the exogenous shocks $s$ and $\varphi^{*}\left(K_{0} / A_{0}\right)$ is the capital aggregator index after constant returns has been imposed. Given this result, the value function for the firm becomes the following:

$$
\begin{equation*}
W_{0}=\left[1+\frac{Q\left(s_{0}\right) \varphi^{*}\left(K_{0} / A_{0}\right)}{p_{0}^{I}\left(1-\delta_{I}\right)}\right] p_{0}^{I}\left(1-\delta_{I}\right) A_{0}+p_{0}^{R}\left(1-\delta_{R}\right) K_{0} \tag{20}
\end{equation*}
$$

which implies that conventionally measured Tobin's q is related to tangible and intangible assets in the following way:

$$
\begin{align*}
& q_{0}=\frac{V_{0}}{p_{0}^{I}\left(1-\delta_{I}\right) A_{0}}=1+\frac{Q\left(s_{0}\right)}{p_{0}^{I}\left(1-\delta_{I}\right)} \varphi^{*}\left(K_{0} / A_{0}\right)+\frac{p_{0}^{R}\left(1-\delta_{R}\right) K_{0}}{p_{0}^{I}\left(1-\delta_{I}\right) A_{0}} \\
& \log q_{0}=\log \left[1+\frac{Q\left(s_{0}\right)}{p_{0}^{I}\left(1-\delta_{I}\right)} \varphi^{*}\left(K_{0} / A_{0}\right)+\frac{p_{0}^{R}\left(1-\delta_{R}\right) K_{0}}{p_{0}^{I}\left(1-\delta_{I}\right) A_{0}}\right] \tag{21}
\end{align*}
$$

This equation states that $q$ is equal to 1 plus a term involving the capital aggregator that captures the possible supranormal rents available to the firm in the future (note that this term contains the index of the exogenous shocks at time 0 ). The final term simply corrects $q$ for the fact that the denominator does not contain the firm's stock of R\&D, using the current price of R\&D and the assumed depreciation rate to measure its book value. The contribution of the theory is to demonstrate that under the assumption that the firm can optimize its capital mix in any period in response to the cost of capital, a single index of the shocks will price that mix. Unfortunately, the theory does not tell us how to construct that index.

The equation above suggests the following regression specifications:

$$
\begin{equation*}
\log q_{i t}=\log \alpha_{t}+\log \left[1+\frac{\lambda_{t}}{\alpha_{t}} \varphi^{*}\left(K_{i t} / A_{i t}\right)+\frac{\gamma_{t}}{\alpha_{t}} \frac{p_{i t}^{R}\left(1-\delta_{R}\right) K_{i t}}{p_{i t}^{I}\left(1-\delta_{I}\right) A_{i t}}\right]+\varepsilon_{i t} \tag{22}
\end{equation*}
$$

where $\gamma$ is included to allow for depreciation rates that change over time or are mismeasured in some way. If a linear specification is desired, we can use:

$$
\begin{equation*}
\log q_{i t}=\log \alpha_{t}+\frac{\lambda_{t}}{\alpha_{t}} \varphi^{*}\left(K_{i t} / A_{i t}\right)+\frac{\gamma_{t}}{\alpha_{t}} \frac{p_{i t}^{R}\left(1-\delta_{R}\right) K_{i t}}{p_{i t}^{I}\left(1-\delta_{I}\right) A_{i t}}+\varepsilon_{i t} \tag{23}
\end{equation*}
$$

Equation (23) allows depreciation rates and the shocks to vary over time, but not across firms. It may be useful to allow them to vary across industry. Note that I have included a parameter $\alpha_{t}$ to accommodate time-varying depreciation of tangible capital and $\gamma_{t}$ for intangible capital. As further discussed below, the difficulty with these equations empirically is that the capital aggregator and the last term $(K / A)$ will be highly correlated across firms and therefore their coefficients will be unstable.

## Computing the capital aggregator

$\varphi^{*}\left(K_{0} / A_{0}\right)$ is constructed from the capital stocks using standard user cost formulas and a Divisia index, with the costs of capital as the price series, and real capital as the quantity series. In the computations here, I used the costs of capital series already shown in Figure 3. The rate of change of the capital aggregator $\varphi$ is defined by the following equation:

$$
\begin{align*}
\frac{\Delta \varphi\left(A_{t+1}, K_{t+1}\right)}{\varphi\left(A_{t+1}, K_{t+1}\right)} & =\frac{c_{t}^{A} A_{t+1}}{c_{t}^{A} A_{t+1}+c_{t}^{K} K_{t+1}} \frac{\Delta A_{t+1}}{A_{t+1}}+\frac{c_{t}^{K} K_{t+1}}{c_{t}^{A} A_{t+1}+c_{t}^{K} K_{t+1}} \frac{\Delta K_{t+1}}{K_{t+1}}  \tag{24}\\
& =\frac{c_{t}^{A} \Delta A_{t+1}+c_{t}^{K} \Delta K_{t+1}}{c_{t}^{A} A_{t+1}+c_{t}^{K} K_{t+1}}
\end{align*}
$$

The initial level is computed from the sum of the capitals in the first year (that is, the price is normalized to one).

Once $\varphi\left(A_{t}, K_{\mathrm{t}}\right)$ has been computed, $\varphi^{*}\left(K_{t} / A_{t}\right)=\varphi\left(A_{t}, K_{\mathrm{t}}\right) / A_{t}$. Note that if the two costs of capital are equal $\left(c^{A}=c^{K}\right)$ during all time periods, $\varphi^{*}\left(K_{t} / A_{t}\right)$ simply becomes $l+K_{t} / A_{t}$. Even if the costs of capital are not identical, if they change similarly over time, $\varphi^{*}\left(K_{t} / A_{t}\right)$ will be highly collinear with $K / A$ itself. Thus if the relative user costs do not vary much, separate identification of the coefficients of $K / A$ (the relative value of current $\mathrm{R} \& \mathrm{D}$ assets) and $\varphi^{*}\left(K_{t} / A_{t}\right)$ (the relative value of the expected rents from these assets) may be impossible.

Because the panel is very unbalanced, there is an extra complication in the capital aggregate computation due to the need to normalize the price series in a base year. My solution to this problem is to first construct aggregate price and quantity series for capital using all the firms in the sample and normalizing price to unity in the first period. I then use equation (24) for each firm, with the initial year value of the price index normalized to the aggregate value. Figure 5 shows the resulting indices (aggregate and averaged over firms) normalized to one for ordinary capital in the first year, for the sample of large US manufacturing firms used later in the paper. The index of aggregate capital (adjusted for the relative costs of capital) is roughly constant at around 1.4 (implying that R\&D capital is 40 per cent of the level of ordinary capital), whereas the index of capital for the average firm falls slightly during the 1990s as smaller firms enter the sample. In addition, the dispersion rises, from an interquartile range of about 0.2 to 0.8 , as more R\&D-intensive firms enter the sample.

As was noted earlier, the coefficients of $\varphi$ and $K / A$ in the regression equations (22) or (23) will only be separately identified when $\varphi$ is sufficiently nonlinear in $K / A$, that is, when the costs of the two capitals follow different paths over time (or when they vary across firms, which is somewhat harder to justify, although variations in the supply of trained personnel in various scientific fields might imply some variation in the cost of R\&D). To assess this possibility, $\varphi$ was regressed on $K / A$, allowing both the slope and the intercept to vary by year of the data. Although $K / A$ explains most of the variance in $\varphi$, about 7 per cent remains to be explained, which is enough to identify the two coefficients separately, but not enough to prevent very unstable behavior due to the imprecision of the estimates. ${ }^{20}$

Two possible solutions exist to this problem. The first is to impose unity on the coefficient of $K / A$ as implied by the HI model, and then simply estimate that coefficient of the term corresponding to the supranormal rents from the capitals. The second takes a slightly different approach and is in the spirit of Griliches original article: because the capital aggregator term is supposed to capture the supranormal rents to be earned in the future from the firm's current portfolio of assets, it might be possible to proxy for that term using the deviation of the firm's R\&D spending in the current year from what was forecast by past R\&D. That is, if the firm is earning an above normal return from its current stock of R\&D, it might be expected to increase its R\&D investment in order to drive the return down. The paper explores these possibilities in Table 4.

[^14]Figure B1


Figure B2



[^0]:    ${ }^{1}$ University of California at Berkeley, University of Maastricht, NBER, and IFS London. email: bhhall@econ.berkeley.edu. This is a revision of a paper presented at the International Conference in memory of Zvi Griliches, Paris, August 2003 and the Conference in memory of Tor Jakob Klette, Oslo, Norway, in August 2004. The paper has benefited from comments by participants in those conferences and especially from the editors of this special issue, Jacques Mairesse and Manuel Trajtenberg.

[^1]:    ${ }^{2}$ A revised version of this article also appeared as Chapter 4 of Griliches (2000).

[^2]:    ${ }^{3}$ Although the panel data set used here is similar to that in Hall, Jaffe, and Trajtenberg (2005), it is in fact a more recent sample drawn from Standard and Poors' Annual Industrial Compustat files, and it therefore does not contain the patent data (yet).

[^3]:    ${ }^{4}$ This last requirement removed about one third of the observations, because Compustat includes firms that file with the U.S. Securities and Exchange Commission for a number of reasons; many of these firms are not publicly traded but have some outstanding debt whose ownership is dispersed.
    ${ }^{5}$ Using LAD comes at the cost of neglecting biases that may be introduced by permanent unobserved differences across the firms that are related to their choice of inputs. The justification for this procedure is

[^4]:    ${ }^{8}$ This correction is discussed in section 3 of the paper.

[^5]:    ${ }^{9}$ Note that equation (5) is written using real rather than nominal variables whereas R\&D intensity is usually measured using nominal variables. This will make little difference in the end to the measured rate of return as long as time dummies are included and prices do not change too quickly over time.
    ${ }^{10}$ This formulation was initially applied to more aggregate date, where the assumption of zero depreciation rate is more appropriate.

[^6]:    ${ }^{11}$ It is helpful to recall that the estimated productivity itself is not greatly affected by the choice of depreciation rate, once industry or firm effects have been controlled for.

[^7]:    ${ }^{12}$ In the implementation reported here, I do take account of this capital gains term, however.
    ${ }^{13}$ Schankerman also included a bias term for the fact that R\&D is expensed rather than capitalized in the firm's accounts, and therefore value added is underestimated. In the data analyzed here we are using sales (revenue) as the dependent variable, so this should not be a problem (recall that we treat R\&D as a capital on the right hand side). Of course using sales introduces other problems because we have no intermediate input measures; the inclusion of sector dummies is a first order correction for this problem if the share of intermediate inputs is roughly constant within sector.

[^8]:    ${ }^{14}$ Most empirical researchers, myself included, write this equation without the price terms, implicitly assuming that $A$ and $K$ are measured in nominal returns. I depart from this practice here to make the notation consistent with that of the theoretical model of Hayashi and Inoue discussed later in the section.
    ${ }^{15}$ Hedonic regressions often give rise to a well-known identification problem because they are based on market equilibrium data (S. Rosen 1974, Brown and H. Rosen (1982)). Ekeland, Heckman, and Nesheim (2003) discuss this issue and show that in the linear-quadratic model of consumer preferences and production with worker and firm heterogeneity, the linear hedonic model identifies consumer preferences (demand for labor) when workers are homogeneous and worker preferences (supply of labor) when consumers are homogeneous. In our setting, that would correspond to assuming that investors are homogeneous in their taste for firms so that what we are estimating is the supply price of R\&D assets, which varies across firms because of heterogeneous skills and adjustment costs. The use of this assumption seems somewhat more reasonable in the setting here than in many other hedonic applications.

[^9]:    ${ }^{16}$ Note that the variables are nearly but not exactly orthogonal. Their correlation is approximately 0.15 .
    ${ }^{17}$ Recall that the market value is measured at the end of the year and R\&D is flow during the year, so that both may be driven by changes in the firm's profit expectations.

[^10]:    ${ }^{18}$ This is consistent with kernel regression estimates (not shown) which indicate an approximately linear relationship for values of K/A between 0.01 and 1 , but considerable flatter below and above that interval. Below $1 \% \mathrm{~K}$ is essentially irrelevant for value (these observations are treated as zero) and above $100 \%$ the relationship is flatter. The log transformation approximates this kind of behavior well. The average measured elasticity (as opposed to the coefficient) from the NLS estimates is approximately the same as the OLS coefficient.

[^11]:    *Millions of \$1996

[^12]:    *Nonlinear least squares estimates with dependent variable $\log \mathrm{Q}$, robust standard errors. Industry and year dummies included.
    **The depreciation rate computed under the assumption that the true K/A coefficient is equal to one.

[^13]:    ${ }^{19}$ Note that the notation is slightly different from that used by Hayashi and Inoue (1991), in that I have used V for the total value of the firm and $W$ for the value of its supranormal rents, whereas HI used $W$ for the value and $V$ for the rents. The notation used here is more consistent with the previous literature on estimating the market value equation.

[^14]:    ${ }^{20}$ Adding to the difficulty is the fact that both are presumably measured with considerable error, and error that is correlated. Hall (2004) shows that using two proxy variables for the same underlying concept in a regression, where one is better measured than the other but their measurement errors are correlated will lead to biased estimates where the two variables actually have opposite signs in the regression.

