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(preliminary, not for quotation without permission)

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Abstract: This paper examines the change in welfare in Ireland over the 1987-1994 period by investigating whether Lorenz and Generalised Lorenz dominance can be observed for household expenditure data. It also calculates bootstrapped standard error measures for Lorenz and Generalised Lorenz curves and finds that the Generalised Lorenz curve for 1994 lies everywhere above that for 1987 thus indicating dominance. It also investigates whether welfare rose using more specific social welfare measures based on average expenditure and the Gini coefficient and finds a statistically significant rise in social welfare.

Was Ireland Better Off in 1994 than in 1987?

1. Introduction

The answer to the question posed in the title can be answered in a number of ways. Perhaps the most standard way is to compare some measure of income per head (such as GNP or GDP) in 1994 to a similar measure evaluated in 1987, adjusted for prices, and then compare the two. However, evaluating whether a country is "better off" purely by looking at income per head is not without problems as any introductory economics textbook will outline. A measure such as GNP/GDP per head only includes marketed output. Thus no value is placed upon voluntary activities. There is no account taken of leisure nor of the environment. GNP/GDP per head also only takes account of *average* income per head and has nothing to say about the *distribution* of income. For example, society may prefer a more equal distribution of income to a less equal one, and may be prepared to sacrifice some income to achieve this.[1] It is this latter issue that is addressed in this paper. We try to examine whether Ireland was better off in 1994 compared to 1987 using measures which take account of income per head and its distribution.

When making comparisons on the basis of average income and its distribution the principal issue to be resolved is the trade-off between the level of income and how equally it is distributed. The adoption of a specific *social welfare function* resolves this issue by incorporating an explicit trade-off between average income and its distribution.[2] The problem of course is that it may be quite difficult to find agreement on exactly which social welfare function to adopt. However, it may be possible to find agreement on certain broad properties that a social welfare function should have. For example, there may be agreement that an increase in average incomes, *ceteris paribus*, should lead to a rise in social welfare. It may also be possible for society to agree that a transfer of income from a richer to a poorer person (thus keeping average income unchanged but making the distribution of income more equal) should lead to a rise (or at least not a fall) in social welfare. Atkinson (1970) and Shorrocks (1983) have shown that for certain broad properties it may be possible to find

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[1] We assume that when making static comparisons between two distributions of income more equality is always preferable to less equality. The possibility that the level of inequality may affect the *growth* of income is not addressed here (see Alesina and Rodrik, 1994, Personn and Tabellini, 1994, and Welch, 1999, for a discussion of these issues).

[2] Thus the only information entering the social welfare function is information regarding income. Sen (1977) discusses the informational content of social welfare functions.

dominance relationships. In other words, as long as we can find agreement on certain broad properties which a social welfare function should possess, then it is possible to make unambiguous statements regarding changes in social welfare i.e. all welfare functions possessing these properties will show a rise/fall in social welfare. This gets around the problem of our results regarding changes in social welfare being sensitive to the specific welfare function chosen. If such dominance relationships can not be found then it is always possible to find social welfare functions which will rank situations differently.

This paper examines whether such dominance relationships exist for Ireland when comparing social welfare in the two years 1987 and 1994. If dominance can be found then it is possible to state that for a broad class of social welfare functions, Ireland was "better off" in 1994 compared to 1987. If dominance can not be found then specific social welfare functions can be evaluated for 1987 and 1994, bearing in mind that it will always be possible to find another social welfare function which will rank the two years differently.

The layout of this paper is as follows: in section 2 we give a more detailed (but still brief!) account of social welfare functions and dominance relationships. In section 3 we describe our data set and discuss precisely which measure of "income" should be used. Section 4 attempts to answer the question posed in the title by investigating whether dominance results hold and also looks at some specific social welfare functions. Section 5 presents concluding comments.

2. Social Welfare Functions and Dominance

For our purposes here, the two seminal papers in the area are those by Atkinson (1970) and Shorrocks (1983). Atkinson (1970) demonstrated the link between Lorenz dominance and social welfare when average incomes in the two situations under comparison are equal. Shorrocks (1983) introduced the Generalised Lorenz curve to take account of the case where average incomes differ. Before outlining these results it is necessary to introduce some notation.

Suppose we have a distribution of income across N recipients which we represent in discrete form by $y_1 \leq y_2 \leq \ldots y_N$. The Lorenz curve can then be defined by

$$L\left(\frac{j}{n}\right) = \sum_{i=1}^{j} \frac{y_i}{Y}, 1 \leq j \leq N$$

where $Y = \sum_{i=1}^{N} y_i$ thus giving the income shares of fractions 1/N, 2/N, 3/N…of the population cumulated upwards from the lowest income y_1.

If we assume that income is continuously distributed then $y_1 \le y_2 \le \ldots y_N$ can be represented by the frequency density function $f(y)$ and cumulative density function $F(y)$. Then for each $p \in (0,1)$ there is just one income level y^* with rank p, which satisfies $p = F(y^*)$ and the income of the first $100p$ per cent of the population is $N\int_0^{y^*} yf(y)dy$ with total income $N\int_0^{\infty} yf(y)dy = Nm$ where m is mean income. A continuous Lorenz curve $L(p)$ can then be defined as

$$p = F(y^*) \Rightarrow L(p) = \int_0^{y^*} \frac{yf(y)dy}{m}, 0 < p < 1.$$

More intuitively, the Lorenz curve can be obtained by ordering all income units, starting with the lowest, and plotting, against the cumulative proportion of the population so ordered (running from zero to one along the horizontal axis) the cumulative proportion of income received by these units. This gives us a curve like Figure 1.

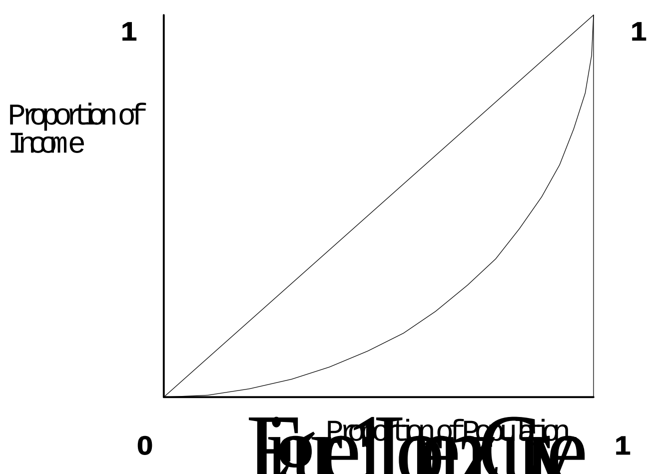

Figure 1: Lorenz Curve

If everybody received the same income then the “curve” would be a straight line from (0,0) to (1,1) i.e. a perfect diagonal. If there is any inequality in the distribution of income than the Lorenz curve will lie below the perfect diagonal. Intuitively the further below the diagonal the curve is, the more unequal is the income distribution. Thus if we wished to represent two income distributions by Lorenz curves, and one Lorenz curve lay everywhere below the other, then that income distribution is more unequal. This gives rise to the idea of "Lorenz Dominance". More formally, suppose we have two distributions, $F(y)$ and $G(y)$, with associated Lorenz curves $L_F(p)$ and $L_G(p)$ then we say that distribution F *Lorenz Dominates* distribution G if $L_F(p) \geq L_G(p) \forall p \in [0,1]$ and $L_F \neq L_G$. Note that Lorenz curves are independent of scale, so that if distribution F was simply a scaled up version of distribution G then their Lorenz curves would be equal.

However, what if there is no Lorenz dominance i.e. if the Lorenz curves cross? Then it is not possible to unambiguously rank one distribution as more unequal that the other. It will always be possible to find two inequality measures giving a different ranking.

What is the relationship between Lorenz curves and social welfare? First we have to define social welfare. Suppose that to each level of income, y, we assign a level of utility, $U(y)$, then we can regard the average utility in society as $W = \int U(y) f(y) dy$. Then we can compare the level of social welfare associated with different distributions. Of course, this is a very specific definition of utility and also of social welfare. It assumes that individual utility is a function of own-income only and that social welfare can be regarded as simply the aggregate of individual utilities.

We can now state the first fundamental result, due to Atkinson (1970), relating Lorenz curves to social welfare. Suppose $F(y)$ and $G(y)$ are two income distributions with equal means $m_F = m_G$, then $L_F(p) \geq L_G(p)$ for all $p \in [0,1] \Leftrightarrow \int U(y) f(y) dy \geq \int U(y) g(y) dy$ for every function $U(y)$ such that $U'(y) > 0$ and $U''(y) < 0$. Thus providing individual utility functions are increasing and strictly concave in incomes, if a distribution F Lorenz dominates another distribution G, then social welfare under F will be higher than under G, provided average incomes are the same. Thus in this case Lorenz dominance is equivalent to social welfare dominance. Note how strong a result this is. Providing we are willing to agree that utility functions should be increasing and concave in income then provided Lorenz dominance is observed we can make an unambiguous welfare statement. If Lorenz dominance is not observed, then no unambiguous social welfare ranking can be obtained. If Lorenz curves cross, then it is always possible to find two increasing and concave social welfare functions which will rank the two income distributions differently. To obtain a ranking it will be necessary to put more restrictions on the form of the social welfare function, with the increasing risk that it will not be possible to find agreement on what those restrictions should be.

What about the assumption of concavity of the utility function $U(y)$? This assumption can be justified on two grounds. First, if $U(y)$ is a true representation of individual preferences then it can be justified on the grounds of diminishing marginal utility of income. Alternatively it can be regarded as the preferences of a *social planner* who is inequality averse and places a higher weight on the utility of the less well-off. Under either interpretation it implies that a transfer of income from a less well-off person to a more well-off person will lead to a fall in average utility or social welfare.

While Atkinson's result is a strong one, in practice it is rarely the case that the distributions being ranked have the same mean. To overcome this, Shorrocks (1983) introduced the *Generalised Lorenz Curve*. This is analogous to the ordinary Lorenz curve but instead of the cumulated proportion of income on the vertical axis, we have the cumulated income per head. Thus the Lorenz curve for distribution F, instead of going from (0,0) to (1,1) will go from (0,0) to $(1, m_F)$. Formally the definition of the Generalised Lorenz curve is $p = F(y^*) \Rightarrow GL_F(p) = \int_0^{y^*} yf(y)dy = m_F L_F(p)$. Figure 2 illustrates generalised Lorenz curves for two distributions, F and G, where $m_F > m_G$.

In the case of Generalised Lorenz curves the relationship between dominance and social welfare is then given by the following result: if $F(y)$ and $G(y)$ are two income distributions then $\int U(y)f(y)dy \geq \int U(y)g(y)dy$ for all increasing strictly concave $U(y) \Leftrightarrow GL_F(p) \geq GL_G(p)$ for all $p \in [0,1]$. So once again we get the extremely powerful result that if Generalised Lorenz dominance holds welfare dominance can be inferred for all increasing strictly concave social welfare functions. If Generalised Lorenz dominance does not hold and the GL curves cross then it is always possible to find two increasing and concave social welfare functions which will rank the two income distributions differently.[3]

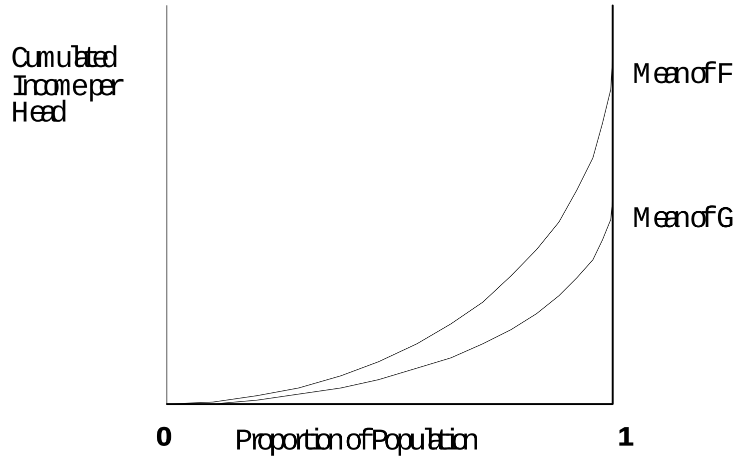

Figure 2 Generalised Lorenz Curves

Thus returning to the question posed in the title, if we observe Generalised Lorenz dominance for Ireland for 1994 over 1987 we will be able to assert that Ireland was indeed better off in the latter year, presuming we equate “better off” with having higher social welfare and there is general agreement on the use of an increasing concave social welfare function. We now investigate whether this is in fact the case.

3. Generalised Lorenz Dominance in Ireland 1987-94.

In this section we investigate whether Generalised Lorenz dominance held in Ireland when comparing 1994 with 1987. If the answer is “yes” then it seems reasonable to suggest that Ireland was better off in 1994. If the answer is “no” then the situation is less clearcut and we will try to answer the question using more specific and restrictive social welfare functions.

In this section we apply the ideas from section 2 to data from the Irish Household Budget Surveys (HBS) of 1987 and 1994. These are nationally representative surveys carried out every seven years and collect a variety of information concerning in excess of 7000 households. Households answer questions over a two-week period about consumption patterns, sources of income plus other information regarding demographic and housing circumstances etc.. Before carrying out the empirical work we must decide on exactly what

[3] Kakwani (1984) examines welfare distributions and finds that for 40 out of 248 pairwise comparisons it is not possible to rank distributions via either Atkinson’s or Shorrocks’ theorem.

measure of income should be used. In the discussion so far we have been assuming that a satisfactory and non-controversial measure of income is available. In fact this is not the case. Given our data, we have a choice between "income" or "expenditure". Broadly the issues are as follows[4]: certain components of income are difficult to measure e.g. income from self-employment. Perhaps more importantly cross-section studies typically provide income measures which are snapshots in time and thus take no account of the difference between transitory and permanent income. Since consumption/expenditure decisions are usually made with reference to permanent income then expenditure measures may be preferable. However, such measures also have drawbacks. Expenditure on items such as alcohol and tobacco are typically under-reported. Also, as mentioned above, expenditure over a two-week period may not be a reliable measure of consumption, particularly for mature households who may have a large stock of durables from which they derive services.

However a further problem specific to the HBS is that income observations are "top-coded" i.e. values of income in excess of £800 per week are simply entered as £800 per week. Thus the distribution of income is censored on the right hand side at a value of £800. This will obviously influence the calculation of both ordinary and Generalised Lorenz curves. Given these problems its seems best to use total expenditure as our measure of "income".

A further issue concerns adjustments which must be made for family size. Since we are examining expenditure for families of differing sizes and composition it is necessary to adjust our measures of expenditure by the appropriate equivalence scale. There is an extensive literature on the appropriate choice of equivalence scale.[5] Here we use a scale which has been widely used in poverty studies in the EU. It is the same as scale "C" used by Callan et al (1996) and is also used by O'Neill and Sweetman (1998). The weights are 1 for the first adult in the household, 0.7 for additional people aged over 14 and 0.5 for people aged less than 14.

Below we present some summary statistics regarding the change in average equivalised expenditure for 1987 and 1994 (in 1987 prices). It is worth noting that total GNP (in constant prices) rose by over 33% and GNP per head by 32% over the same period. This

[4] For a detailed discussion see Blundell and Preston (1998).
[5] See Deaton and Muellbauer (1980) for a discussion.

discrepancy between the change in income and the change in expenditure is a feature of the data over the period and is a topic we hope to return to in future work.

Table 1: Summary of Weekly Equivalised Expenditure 1987 and 1994 (1987 prices)

	1987 (N=7705)	***1994 (N=7877)***	**% change**
Average Equivalised Exp	94.32717	100.2334	+6.261
Standard Deviation	63.69861	64.98494	

First of all we will compare inequality between 1987 and 1994, ignoring for the moment the change in mean weekly equivalised expenditure i.e. we examine Lorenz curves only. In table 2 we present Lorenz ordinates for 1987 and 1994.

Table 2: Lorenz Ordinates by Decile, 1987-1994 (s.e. in brackets)

Decile	1987	1994	% change	Test Statistic
1st	.0316966 (.0014291)	.0309983 (.0014526)	-2.17	30.24845**
2nd	.0769428 (.002484)	.0755242 (.0020679)	-1.86	38.70002**
3rd	.1323981 (.0032062)	.1302807 (.0029946)	-1.62	42.58182**
4th	.198122 (.0041809)	.1953108 (.0038831)	-1.41	43.4664**
5th	.274619 (.0050126)	.2716832 (.0047969)	-1.07	37.33829**
6th	.3632026 (.0049823)	.3616576 (.0054987)	-0.44	18.38739**
7th	.4670441 (.0054774)	.467942 (.005192)	0.20	-10.4976**
8th	.5913745 (.0054866)	.5954423 (.0052917)	0.69	-47.0909**
9th	.7482283 (.0056974)	.7548628 (.0057892)	0.88	-72.0985**

The results here appear to suggest that we do not observe Lorenz dominance. For deciles one to six the Lorenz curve for 1987 lies above that of 1994, but for deciles seven to nine it lies below. Thus the Lorenz curves cross and no unambiguous statement can be made regarding the change in inequality.

However, we must bear in mind that our Lorenz curves are derived from sample data and are thus subject to sampling variability. We also include measures of the standard errors of the ordinates plus the test statistics for the null hypothesis that the ordinates differ.[6] Suppose that L_i is the i^th^ Lorenz ordinate $((i = 1,2,..k)$, where the k^th^ ordinate is equal to one. Then, given estimated Lorenz ordinates from two samples a and b with sample sizes N_a and N_b respectively, we have k-1 pairwise tests of sample Lorenz ordinates:

$$T_i = \frac{\hat{L}_i^a - \hat{L}_i^b}{\sqrt{\frac{\hat{V}_i^a}{N_a} + \frac{\hat{V}_i^b}{N_b}}}, \; i = 1,2,...k-1$$

In large samples, T_i is asymptotically normally distributed. Bishop, Formby and Smith (1991) suggest the following criteria when testing for Lorenz dominance: if there is at least one positive significant difference and no negative significant differences between Lorenz ordinates then dominance holds. Two distributions are ranked as equivalent if there are no significant differences, while the curves cross if the difference in at least one set of ordinates is positive and significant while at least one other set is negative and significant.

As we can see from table 2, under the Bishop, Formby and Smith criteria we cannot reject the hypothesis that the Lorenz curves cross.

What about Generalised Lorenz dominance? In table 3 we present ordinates for the Generalised Lorenz curves.

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[6] Standard errors for the ordinates are calculated via the bootstrap method. See Mills and Zandvakili (1997) for a comparison of bootstrapped standard errors compared to asymptotic standard errors.

Table3: Generalised Lorenz Ordinates by Decile, 1987-1994 (s.e. in brackets)

Decile	1987	1994	% change	Test Statistic
1st	2.989852 (0.1372039)	3.107066 (0.135829)	3.95	-53.5821**
2nd	7.257793 (0.2013511)	7.570046 (0.2556902)	4.29	-84.7913**
3rd	12.48874 (0.3018071)	13.05848 (0.3236103)	4.54	-113.683**
4th	18.68829 (0.3634071)	19.57667 (0.4001215)	4.77	-145.14**
5th	25.90403 (0.4573857)	27.23174 (0.5045446)	5.13	-172.17**
6th	34.25988 (0.4727833)	36.25018 (0.5982737)	5.80	-230.667**
7th	44.05495 (0.5367815)	46.90343 (0.6099869)	6.48	-309.631**
8th	55.78269 (0.6475027)	59.68322 (0.6441411)	7.00	-376.922**
9th	70.57825 (0.6703694)	75.66248 (0.6269808)	7.20	-488.709**
Mean	94.32717 (.13328139)	100.2334 (.0709842)	6.26	

The results in table 3 suggest that we can make an unambiguous statement regarding the change in welfare between 1987 and 1994. Given that the Generalised Lorenz curve for 1994 is everywhere above that for 1987, then all social welfare functions which are increasing and convex in expenditure show an increase in social welfare over the period. We also see from the test statistics that this dominance result is statistically significant.

In some respects this is a very strong statement to make. Yet it only has validity if we accept the form of both the individual utility functions $U(y)$ and their average as a measure of social welfare. As pointed out above, our version of $U(y)$ implies that own-utility is a function of own-income only. Might our results change if we modify this assumption? This is discussed in the next section.

4. Non-Individualistic Social Welfare Functions

The assumption that individual utility is a function of own-income only can be challenged on a number of fronts. First, if utility is dependent upon income only then a whole host of potentially relevant information is being ignored. To give just a simple example, this form of the utility function takes no account of the utility arising from leisure. Nor does it include less tangible, but still important factors such as the value of the environment or the "quality of life". The kind of data provided in surveys such as the HBS typically does not include such information, so while acknowledging its importance we will not be taking account of it.

Even if we could incorporate such features into the utility function, it is arguable that non-utility factors should also be included in social welfare. Sen's example of the sadist gaining more utility from torture than his victim loses may appear somewhat fanciful, but it does bring home the point that rights as well as utilities should ideally feature in measures of social welfare. Once again however, given the data at our disposal, we cannot address this problem.

Finally, it could be argued that utility should not be *individualistic* i.e. dependent just upon own-income or own-leisure. Survey evidence suggests that it is not just own-income but also the incomes of others, particularly those in peer groups, that affects utility. There are a variety of mechanisms whereby this can come about. For example, Runciman (1966) introduced the notion of *relative deprivation*. In this case an individual's utility is a function not just of the commodities (income) he has but also the foregone utility through not having commodities (income) which other persons have. The deprivation approach assumes that the value (marginal utility) of a commodity to the individual, other things being equal, is an increasing function of its scarcity value to the individual. The degree of deprivation inherent in not having something (say the jth unit of income) is an increasing function of the number of those who have it, or a decreasing function of the number who do not. Thus externalities are introduced, since the marginal utility of income is a function of the income distribution as a whole.

Thus the scarcity of a unit of income, y^* is $F(y^*)$, the cumulative income distribution and $1-F(y^*)$ is the frequency of individuals with income above y^*. Let $h[1-F(y^*)], h' > 0$, be the marginal welfare of income. The deprivation of the ith individual is then given by $d(y_i) = \int_{y_i}^{y_{\max}} h[1-F(y)]dy$ where $y_{\max}$ is the maximum income in society so that the integration is over the range of incomes of which the ith individual is deprived. The welfare of the ith individual is given by $U(y_i) = \int_0^{y_i} h[1-F(y)]dy$.

If aggregate welfare and deprivation are given respectively as $W = \int_0^{y_{\max}} U(y)f(y)dy$ and $D = \int_0^{y_{\max}} d(y)f(y)dy$, then Yitzhaki (1979) shows that if $h[1-F(y_i)] = 1-F(y_i)$, then $W = m(1-G)$ and $D = mG$ where m is average income and G is the Gini coefficient. Thus this gives us an (admittedly restrictive) measure of welfare based upon a non-individualistic individual utility function.

One attractive generalisation of the above approach is to assume that $h[1-F(y_i)] = [1-F(y_i)]^v, v > 0$. We then obtain the result that $W = m[1-G(v)]$ where $G(v)$ is Yitzhaki's extended Gini and v is a parameter influencing the weight attached to the lower end of the distribution. If $v = 0$ then $W = m$ while $v \to \infty$ leads to $W = \min_i\ y_i$ i.e. the Rawlsian criterion.

In table 4 below we present estimated Gini coefficients (with bootstrapped standard errors) and the associated welfare and deprivation measures for 1987 and 1994.

Table 4: Gini Coefficients and Welfare and Deprivation Measures, 1987-1994

	1987	1994	% change	Test statistic
Gini Coefficient	.32969223 (0.0032222)	.32933998 (0.002722)	-0.107	7.363941
"Welfare"	63.22823497 (0.56334349)	67.22253405 (0.44445603)	+6.317	490.674
"Deprivation"	31.09893503 (0.56334349)	33.01086595 (0.44445603)	+6.148	234.869

We see that inequality as measured by the Gini coefficient fell slightly between 1987 and 1994 and this fall is also statistically significant. Welfare rose by over 6% and this too is statistically significant. Paradoxically measured deprivation also rose by over 6%. Since the deprivation measure is concerned with "not-having", as average income rises then the amount which people "do not have" also rises. Then since the change in the Gini is only marginal, while average expenditure rises by just over 6%, measured deprivation rises by approximately the same amount as does average expenditure.

What about the extended Gini where the *v* parameter can be altered to reflect increasing concern for the welfare of those at the lower end of the expenditure distribution? Table 5 essentially reproduces table 4 except that we include three different values of *v*, viz. 3, 5 and 8, with the associated test statistics.

Table 5: % Change in Extended Gini and Welfare and Deprivation Measures, 1987-1994 (test statistic in brackets)

v	3	5	8
Extended Gini Coefficient	0.731336 (66.6754)	1.145003 (118.261)	1.205242 (132.404)
"Welfare"	5.641372 (247.623)	4.809737 (173.07)	4.259722 (131.161)
"Deprivation"	7.038558 (246.507)	7.478127 (321.06)	7.542137 (362.968)

Once again we observe the paradoxical situation that both measured welfare and measured deprivation have increased. It is also noticeable that the rise in measured deprivation is now greater than the rise in welfare. This is because higher values of the *v* parameter imply a higher weight is being put on the lower part of the expenditure distribution. As table 2 shows, the Lorenz curve for 1994 is further from the diagonal than that for 1987 at the lower end of the expenditure distribution. Since this part of the distribution now receives a higher weight, we see a larger rise in deprivation.

5. Conclusion

This paper has attempted to answer the question whether welfare in Ireland rose over the period 1987-94, where welfare is interpreted as depending upon average expenditure and its distribution. Using dominance results of Atkinson and Shorrocks we show that all social welfare functions based upon increasing, individualistic and concave utility functions would show a statistically significant rise in social welfare over the period. We also show that when the assumption of an individualistic utility function is dropped and a more specific social welfare function is adopted, measured welfare still rises and this rise is statistically significant. Moreover, this results holds for a variety of assumptions regarding the weight to be put upon the welfare of those at the lower end of the expenditure distribution.

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How do we answer the question in the title?

Examine GNP/GDP per head?

But what about leisure?

And externalities?

And the "quality of life"?

And distribution?

How about a *Social Welfare Function?*

Great!! But what one?

What general properties should a SWF possess?

(1) More is better!

(2) More equal distribution is better than a less equal one

If situation A ranked better than situation B for all SWFs which obey (1) and (2) we have *dominance*

How do we know if one distribution is more equal than another?

Examine *Lorenz Curve*

Lorenz Dominance:

Distribution F *Lorenz Dominates* distribution G if

$$L_F(p) \geq L_G(p) \forall p \in [0,1]$$

and $L_F \neq L_G$**.**

But what is relation to social welfare?

Suppose that to each level of income, y, we assign a level of utility, $U(y)$, then we can regard the average utility in society as $W = \int U(y) f(y) dy$.

Then Atkinson's Theorem says:

Suppose $F(y)$ and $G(y)$ are two income distributions with equal means $m_F = m_G$, then $L_F(p) \geq L_G(p)$ for all $p \in [0,1] \Leftrightarrow \int U(y) f(y) dy \geq \int U(y) g(y) dy$ for every function $U(y)$ such that $U'(y) > 0$ and $U''(y) < 0$.

i.e. link between Lorenz dominance and welfare dominance, provided we are happy about form of $U(y)$ and $W = \int U(y) f(y) dy$

But what if $m_F \neq m_G$? Would we prefer a larger cake less equally distributed?

Shorrocks (1983) introduced *Generalised Lorenz Curve*.

Then we have Shorrocks' Thoerem:

if $F(y)$ and $G(y)$ are two income distributions then $\int U(y)f(y)dy \geq \int U(y)g(y)dy$ for all increasing strictly concave $U(y) \Leftrightarrow GL_F(p) \geq GL_G(p)$ for all $p \in [0,1]$.

i.e. generalisation of Atkinson for case of $m_F \neq m_G$

We analyse Irish data for 1987 and 1994 and examine for Lorenz and Generalised Lorenz Dominance

Table 2: Lorenz Ordinates by Decile, 1987-1994 (s.e. in brackets)

Decile	1987	1994	% change	Test Statistic
1st	.0316966 (.0014291)	.0309983 (.0014526)	-2.17	30.24845**
2nd	.0769428 (.002484)	.0755242 (.0020679)	-1.86	38.70002**
3rd	.1323981 (.0032062)	.1302807 (.0029946)	-1.62	42.58182**
4th	.198122 (.0041809)	.1953108 (.0038831)	-1.41	43.4664**
5th	.274619 (.0050126)	.2716832 (.0047969)	-1.07	37.33829**
6th	.3632026 (.0049823)	.3616576 (.0054987)	-0.44	18.38739**
7th	.4670441 (.0054774)	.467942 (.005192)	0.20	-10.4976**
8th	.5913745 (.0054866)	.5954423 (.0052917)	0.69	-47.0909**
9th	.7482283 (.0056974)	.7548628 (.0057892)	0.88	-72.0985**

What about Generalised Lorenz Dominance?

Table3: Generalised Lorenz Ordinates by Decile, 1987-1994 (s.e. in brackets)

Decile	1987	1994	% change	Test Statistic
1st	2.989852 (0.1372039)	3.107066 (0.135829)	3.95	-53.5821**
2nd	7.257793 (0.2013511)	7.570046 (0.2556902)	4.29	-84.7913**
3rd	12.48874 (0.3018071)	13.05848 (0.3236103)	4.54	-113.683**
4th	18.68829 (0.3634071)	19.57667 (0.4001215)	4.77	-145.14**
5th	25.90403 (0.4573857)	27.23174 (0.5045446)	5.13	-172.17**
6th	34.25988 (0.4727833)	36.25018 (0.5982737)	5.80	-230.667**
7th	44.05495 (0.5367815)	46.90343 (0.6099869)	6.48	-309.631**
8th	55.78269 (0.6475027)	59.68322 (0.6441411)	7.00	-376.922**
9th	70.57825 (0.6703694)	75.66248 (0.6269808)	7.20	-488.709**
Mean	94.32717 (.13328139)	100.2334 (.0709842)	6.26	

So has question been answered unambiguously?

What about form of $U(y)$?

Takes no account of leisure, externalities, quality of life, non-utility information….

Also dependent upon *own-income* only

***Relative Deprivation*: utility also a function of what you *don't* have**

Scarcity of a unit of income, y^* is $F(y^*)$, the cumulative income distribution and $1-F(y^*)$ is the frequency of individuals with income above y^*.

Let $h[1-F(y^*)], h' > 0$, be the marginal welfare of income. The deprivation of the ith individual is then given by $d(y_i) = \int_{y_i}^{y_{max}} h[1-F(y)]dy$ where y_{max} is the maximum income in society so that the integration is over the range of incomes of which the ith individual is deprived.

The welfare of the ith individual is given by $U(y_i) = \int_0^{y_i} h[1-F(y)]dy$.

If aggregate welfare and deprivation are given respectively as $W = \int_0^{y_{\max}} U(y) f(y) dy$ **and** $D = \int_0^{y_{\max}} d(y) f(y) dy$**, then Yitzhaki (1979) shows that if** $h[1 - F(y_i)] = 1 - F(y_i)$**, then** $W = m(1 - G)$ **and** $D = mG$ **where** m **is average income and *G* is the Gini coefficient.**

If $h[1 - F(y_i)] = [1 - F(y_i)]^v, v > 0$**, then** $W = m[1 - G(v)]$ **where *G*(*v*) is Yitzhaki's extended Gini and *v* is a parameter influencing the weight attached to the lower end of the distribution.**

Table 4: Gini Coefficients and Welfare and Deprivation Measures, 1987-1994

	1987	1994	% change	Test statistic
Gini Coefficient	.32969223 (0.0032222)	.32933998 (0.002722)	-0.107	7.363941
"Welfare"	63.22823497 (0.56334349)	67.22253405 (0.44445603)	+6.317	490.674
"Deprivation"	31.09893503 (0.56334349)	33.01086595 (0.44445603)	+6.148	234.869

Table 5: % Change in Extended Gini and Welfare and Deprivation Measures, 1987-1994 (test statistic in brackets)

v	3	5	8
Extended Gini Coefficient	0.731336 (66.6754)	1.145003 (118.261)	1.205242 (132.404)
"Welfare"	5.641372 (247.623)	4.809737 (173.07)	4.259722 (131.161)
"Deprivation"	7.038558 (246.507)	7.478127 (321.06)	7.542137 (362.968)

“…rural Ireland’s boom of decades means that one can no longer even glimpse the magnificent sea on the road from Galway to An Ceathrú Rua. Urban money means that the great Victorian red-brick Tara Street of my youth is no longer a street at all, but a mere gaggle of buildings. Our economic buying power has developed more rapidly than our sense of value. We will grow to regret these things.”

Frank Barry in *Understanding Ireland’s Economic Growth* (ed. Barry).