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The Ambiguous Effect of Minimum Wages on Workers and Total Hours¹

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Abstract

We model a standard competitive labour market where firms choose combinations of workers and hours per worker to produce output. If one assumes that the scale of production has no impact on hours per worker, then the change in the number of workers and hours per worker resulting from a minimum wage are inversely related. We also demonstrate that total hours worked at the firm may rise if there are small fixed costs to hiring workers.

Keywords: Minimum wages, hours, employment.

JEL Classification J22, J38

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Section I: Introduction

Evidence from minimum wage studies have frequently been used to assess whether labour markets are well approximated by the competitive model, which would predict that a minimum wage would have a negative impact on employment, or by alternative models such as the monopsony model, which posits a positive relationship². For example, Neumark and Wascher (2007) note in their review of the empirical literature: “ .. we hope our review will help readers assess alternative models of the labour market” (p.5).³ However, even Neumark and Wascher are careful to stress the limitations of the theoretical predictions noting that “...even in the neoclassical model, the effect of the minimum wage on any given set of workers will depend on, among other things, the elasticities of substitution across different types of workers and cross elasticities of demand across different types of goods.”

In this paper we argue that in fact the results of empirical research on the employment effects of minimum wages have little to say in terms of providing evidence for competing models of the labour market. To demonstrate this we examine the predictions of the standard theoretical model where firms choose the number of workers and hours per worker⁴. Our results show that the impact of minimum wages on hours per worker, the number of workers, and total hours worked are indeed ambiguous. Our results are based on a partial equilibrium models where firms choose one type of worker and are not driven by general equilibrium effects or uncertainty about how the firm will substitute across different types of workers.

² Strobl and Walsh (2007) look at the impact of minimum wage on employment and hours in a monopsony model.

³ Neumark and Wascher in their introduction and throughout chapter 3.2 document several studies that suggest the sign of the employment effect from a minimum wage is evidence for these competing models.

⁴ While there exists an abundance of studies empirically estimating the employment effects of the minimum wage [see Neumark and Wascher (2007) for a survey], this literature often estimates changes in the number of employees or sometimes the number of full-time equivalents. Moreover, the empirical evidence with regard to the effect of a minimum wage on hours per worker is rather mixed; see, for instance, Katz and Kruger (1992), Brown (1999), Zavodney (2000), and Gregory (2002).

Section II: The Model

The theoretical treatment of minimum wages in the literature when firms choose a combination of hours per worker and workers is rather limited.⁵ In this paper we apply a minimum wage to Kinoshita's (1987) model.⁶

Firms are assumed to be able to hire as many workers as they wish at any hour (h), so that hourly wage (w) combination is on the equilibrium locus $w(h)$. The firm's profit function is:

$$\Pi(n, h) = pf[n, h(w)] - wh(w)n - kn \quad (1.1)$$

The output price p is given to the firm and the production function $q = f(h, n)$ satisfies $f_n > 0$, $f_h > 0$, $f_{nn} < 0$, $f_{hh} < 0$, where h is hours per worker, n is the number of workers and q is output. There are fixed costs k per worker. The firm's choice of w and n at an interior solution satisfy the following first order conditions:

$$\Pi_w(w, n) = pf_h[n, h(w)]h_w(w) - wh_w(w)n - h(w)n = 0 \quad (1.2)$$

$$\Pi_n(w, n) = pf_n[n, h(w)] - wh(w) - k = 0$$

One can assess the impact of a minimum wage on the number of workers by totally differentiating the first order condition on n . Evaluating this differential at the initial equilibrium we get:

$$\frac{dn}{dw} = \frac{-\Pi_{nw}}{\Pi_{nn}} = \frac{[\frac{f_h}{n} - f_{nh}]}{f_{nn}} h_w \quad (1.3)$$

⁵ Hamermesh (1993) develops a framework that deals with the firm's choice of workers and hours in a cost minimisation framework and includes a brief discussion of minimum wages, while Michl (2000) outlines a model where firms choose workers and hours and the wage does not increase with hours. Other studies, such as Stewart and Swaffield (2006), Zavodney (2000), Neumark and Schweitzer (2000), and Connolly and Gregory (2002) only contain general discussions on how minimum wages are related to hours.

⁶ Kinoshita derives the equilibrium properties of the compensating differentials model applied to hours work. The models of Lewis (1969) and Rosen (1986) are the precursors to this model.

Next we assume that the scale of production does not affect the optimal choice of hours per worker, other things equal. Hamermesh (1993) p50 notes, for example, that “...*there is no evidence that weekly hours of full-time workers at General Motors differ substantially from hours of workers at the local steel fabricator*”(p.50). If this assumption holds, we show in the Appendix using the firms cost minimisation problem⁷:

$$\frac{dn}{dw} = \frac{[\frac{f_h}{n} - f_{nh}]}{f_m} h_w = -\frac{f_h}{f_n} h_w \quad (1.4)$$

If the hourly wage - hours per worker locus has a positive (negative) slope we expect firms to use the minimum wage to increase (decrease) hours per worker and decrease (increase) the number of workers at a given level of output. A puzzling implication of (1.4) is that since much of the existing empirical evidence suggests a decline in hours from a minimum wage⁸, then (1.4) suggests that affected firms decrease hours and increase the number of workers. This implies that workers are on a negatively sloped hourly wage, hours locus. There is no reason that this should not be so in the theory, but one may suspect that many economists would expect the contrary. For example Hamermesh (1993) assumes the equilibrium locus has a positive slope in his treatment of the theory of hours per workers, while Michl (2000) assumes the locus is flat. In any case equation (1.4) should make us reluctant to conclude that one can infer whether the labour market is competitive or not by looking at the results of studies that focus on the number of workers, as much of the literature does since theory has

⁷ In fact the assumption need only be true for a small deviation from equilibrium output for the analysis to go through. Many of the most commonly used functional forms used for the labour aggregator such as the class of functions $f(n, h) = An^a x(h)$ satisfy this assumption. A and a are positive constants and x(h) is a positive function

⁸ See the review by Neumark and Wascher (2007) or Brown (1999).

no clear prediction on the change in hours per worker and predicts an offsetting change in the number of workers.⁹

While it has been pointed out in the literature that when one accounts for the possibility that hours per worker may fall in response to a minimum wage, this may be associated with an increase in the number of workers, even in the competitive model, the general belief is that total hours cannot increase. As Neumark and Wascher (2007) p166 note “..although much of the literature has focused on the employment effects of the minimum wage, the predictions of theory tend to be about overall labour input rather than employment specifically.. (p.166)”. The empirical studies that do try and estimate the impact on the overall labour input generally focus on total hours¹⁰.

We define the elasticity of output with respect to workers (n) and hours per worker (h), respectively, as $f_h \frac{h}{f} = e_{qh}$ and $f_n \frac{n}{f} = e_{qn}$. It follows from the first order conditions (1.2) that:

$$h_w = \frac{h}{w} \left[\frac{1}{\left(1 + \frac{k}{wh}\right) \frac{e_{qh}}{e_{qn}} - 1} \right] \quad (1.5)$$

If one thinks of employment as total hours (nh), then using (1.4) and (1.5) the employment effect would be:

$$\frac{d(nh)}{dw} = n \frac{dh}{dw} + h \frac{dn}{dw} = n \left(1 - \frac{e_{qh}}{e_{qn}}\right) h_w = -\frac{nh}{w} \left[\frac{\frac{e_{qh}}{e_{qn}} - 1}{\left(1 + \frac{k}{wh}\right) \frac{e_{qh}}{e_{qn}} - 1} \right] \quad (1.6)$$

The elasticity of total hours (nh) with respect to the wage is:

⁹ See Neumark and Wascher (2007) for examples. Some studies do account for hours, Michl (2000) provides evidence and some theory to suggest that decreases in hours could explain the positive employment effects found in Card and Kruegers well known (1995) study of the New Jersey minimum wage increase.

¹⁰ This may be approximated by measuring employment as the number of full-time equivalent workers.

$$\mathbf{e}_{nh,w} = -\left[\frac{\frac{\mathbf{e}_{qh} - 1}{\mathbf{e}_{qn}}}{\left(1 + \frac{k}{wh}\right) \frac{\mathbf{e}_{qh}}{\mathbf{e}_{qn}} - 1} \right] \quad (1.7)$$

Total hours will increase from a minimum wage if:

$$\frac{k}{wh} > \frac{\mathbf{e}_{qn} - \mathbf{e}_{qh}}{\mathbf{e}_{qh}} > 0 \quad (1.8)$$

One should note from (1.7) that when fixed costs (k) are equal to zero then $\mathbf{e}_{nh,w} = -1$ and a minimum wage reduces total hours proportionately. However (1.8) also indicates that when $\mathbf{e}_{qn} > \mathbf{e}_{qh}$, if fixed costs as a fraction of the wage bill lie above a certain threshold then total hours will increase in response to a minimum wage¹¹.

Also, one does not need extreme values for the parameters for total hours to increase. For example, when $\mathbf{e}_{qn} > \mathbf{e}_{qh}$ but the elasticity of output with respect to workers and hours per worker are similar, the presence of small fixed costs will ensure a positive effect.

We can establish the following proposition:¹²

If $\mathbf{e}_{qn} > \mathbf{e}_{qh}$ a minimum wage will increase total hours worked if the hours per worker, hourly wage locus has a positive slope.

Proof: From (1.5) if the hourly wage locus has a positive slope the denominator of (1.7) is positive. This implies that (1.7) is positive since $\mathbf{e}_{qn} > \mathbf{e}_{qh}$.

¹¹ Feldstein (1967) and Michl (2000) amongst others explicitly assume $\mathbf{e}_{qn} > \mathbf{e}_{qh}$.

¹² A maple file solving the model explicitly and showing the model is well behaved over parameter ranges where total hours increase, where there are representative workers and firms with Cobb-Douglas utility and production functions is excluded because of space limitations but is available from the authors.

Section III: Conclusion

The idea that minimum wages may lead to offsetting effects on hours per worker is generally recognised in the literature. However, given the prevalence of studies that focus solely on the number of workers and the willingness to make inferences on the underlying labour market from the results, we suspect that the fact that changes in hours per workers and the number of workers from a minimum wage can be either positive or negative and will typically be inversely related in a simple partial equilibrium competitive labour market is not yet well understood. In addition, the result that total hours may increase in response to a minimum wage when firms have even small fixed costs, should make researchers wary about using empirical studies of minimum wages on employment to make inferences on the nature of the underlying labour market.

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Appendix:

Condition for scale effect on hours to equal zero

We minimise cost subject to the constraint that a given level of output q is produced using the lagrangean \mathcal{L} . The labour aggregator $f(h,n)$ turns combinations of workers (n) and hours per worker (h) into an amount (q) of output. The equilibrium hourly wage hours worked locus is: $w(h)$. A fixed cost k must be Paid per worker hired¹³.

$$\mathcal{L} = wh(w)n + nk + \mathcal{L}\{q - f[n, w(h)]\} \quad (1.9)$$

The first order conditions on h , n and \mathcal{L} respectively are:

$$\mathcal{L}_w(n, h, \mathcal{L}) = wh_w(w)n + h(w)n - \mathcal{L} f_h[n, h(w)]h_w(w) = 0$$

$$\mathcal{L}_n(n, h, \mathcal{L}) = wh(w) + k - \mathcal{L} f_n[n, h(w)] = 0 \quad (1.10)$$

$$\mathcal{L}_{\mathcal{L}}(n, h, \mathcal{L}) = q - f[n, h(w)] = 0$$

¹³ We let the firm choose the level of hours here, which depends on the wage, rather than choose the wage which depends on hours as in the text. This makes no difference to the analysis but for exposition purposes it may be a little clearer to illustrate the scale affect when the firm chooses hours and to illustrate the minimum wage affect when the firm chooses the wage.

Totally differentiating the first order conditions with respect to $h, n, ?$ and q we get that

if $\frac{\partial h}{\partial y} = 0$ then:

$$\mathbf{i}_{wn} \mathbf{i}_{nl} - \mathbf{i}_w \mathbf{i}_{nn} = -\mathbf{l} f_{nn} f_h h_w - f_n (w h_w + h - \mathbf{l} f_{hn} h_w) = 0 \quad (1.11)$$

Note from this:

$$\frac{f_h}{f_n} = - \frac{(\frac{f_h}{n} - f_{ln})}{f_{nn}} \quad (1.12)$$