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On the impact of labor market matching on regional disparities[*]

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Abstract

We propose a model where imperfect matching between firms and workers on local labor markets leads to spatial agglomeration. We show that the occurrence of spatial agglomeration depends on initial size differences in terms of both number of workers and firms. We analyse the effect of different public policies. In our setting, the effect of a higher level of human capital on regional disparities depends on whether it makes workers more mobile on the labour market or more specialised. Policies that increase workers' interregional mobility, increase the likelihood that regions diverge. Finally, competition policy is shown to reduce regional disparities.

Keywords: Economic geography, local labor market, regional disparities, human capital

JEL codes: J61, J42, R12

1   Introduction

In economic geography, there are several reasons why workers and firms agglomerate in a few areas. Most models have explored the agglomeration process based on the home-market effect (Krugman, 1991b), specific non-tradable inputs (Venables, 1996) or technological spillovers (Belleflamme *et al.*, 2000). According to Marshall (1920), the way local labor markets operate is yet another reason why spatial polarization of both workers and firms of an industry will persist. He states:

> "A localized industry gains a great advantage from the fact that it offers a constant market for skill. Employers are apt to resort to any place where they are likely to find a good choice of workers with the special skill which they require; while men seeking employment naturally go to places where there are many employers who need such skills as theirs and where therefore it is likely to find a good market".

Such an explanation of industrial concentration is supported by empirical evidence. Dumais *et al.* (1997) shows that industrial spatial polarization persistence is driven mostly by labor pooling. Three theoretical arguments can be provided in order to explain Marshall's intuition: risk, "hold-up", and skill mismatch. As explained in Krugman (1991a), workers locate where there is the largest number of firms in order to minimize the risk of being unemployed and firms choose to locate where there is a large pool of workers. This argument has been developed in Picard and Toulemonde (2001). In Rotemberg and Saloner (2000), firms are induced to agglomerate in a region, and thereby face tough competition on the labor market, so as to commit not to hold-up workers' specific human capital. Therefore, workers have incentives to invest in human capital where firms are agglomerated. In this paper, we explore a third argument, namely skill mismatch.

The aim of this paper is twofold. First, we provide a very simple setting in which imperfect matching between firms and workers explains spatial polarization of both workers and firms, even in the presence of non-strategic behaviour of firms on the final good market. Our modelling of the labor market is related to Becker's argument according to which a worker is characterized both by his specific human capital (the task he perfectly matches) and his general human capital which allows mobility between tasks. We argue that imperfect matching, even slight, between

firms' requirements and workers' specializations is sufficient to explain spatial polarization. Indeed, imperfect matching gives rise to imperfect competition on the labor market and thereby confers oligopsonic power to firms.[1] Therefore, workers have incentives to locate close to a large number of firms since they do not want to be captured by a firm. But because of urban costs workers prefer not too densely populated areas. Firms, on one hand, prefer to escape from tough competition on the labor market but, on the other hand, want to be close to a large number of workers. In that way, we identify, as in a typical economic geography model, centripetal as well as centrifugal forces. We show that, if a region's size in terms of number of workers and firms is initially sufficiently large, agglomeration forces lead the economy toward complete agglomeration. In terms of global welfare, we show that there is too much agglomeration. The difference between equilibrium outcome and optimum is due to the imperfection on the labor market which confers too much incentives to workers to agglomerate.

Second, we apply our framework to re-examine how public policies such as an improvement of interregional mobility of workers, an increase in the level of human capital or a competition policy, influence the degree of regional disparity. Indeed, workers' low spatial mobility is considered as a critical feature of the European Union. Furthermore, European regional policy puts a heavy emphasis on training in order to reduce spatial disparities (see de la Fuente and Vives, 1995). Moreover, competition policy is one of the leading economic policies at the European level and its impact on regional inequalities is poorly examined. Our main conclusions are the following. First, as in other economic geography models, the presence of immobile workers in each region acts as a dispersion force: we show that the presence of immobile workers can radically change the spatial outcome. Second, whereas the "brain drain" literature stresses that newly skilled workers could be induced to leave depressed areas and always locate in the most developed regions, we show that if an improvement in the level of human capital leads to greater mobility of workers between tasks, an increase in human capital reduces regional disparities. This result is in sharp contrast with existing models (Miyagiwa, 1991) in which an increase in human capital is likely to intensify the "brain drain". If, on the contrary, an increase in human capital leads to a greater specialisation of workers, we obtain the same results as in the brain drain literature. Our result follows from the agglomeration force at work in our setting. Indeed, Becker stresses that an increase in general human capital

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[1]Such oligopsonic power is supported by empirical evidence, see i. e. Staiger, Spetz and Phibbs (1999).

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improves the mobility of workers between qualifications. Therefore, monopsony power decreases and thereby hurts less workers who eventually have less incentives to migrate toward the larger region. In other words, these first two results state that whereas more spatial mobility is likely to increase regional disparities, increasing skill mobility is likely to reduce such disparities. Third, we show that a competition policy aimed at fostering entry of new firms on the market reduces regional disparities.

There are a few papers related to the issue under consideration in this paper. In the urban economics literature, there are two papers which use the same framework to look at specific spatial equilibria. However, these papers do not look at the spatial dynamics and the possibility of regional divergence as in the new economic geography literature. Hesley and Strange (1990) show how the working of the labor market leads the economy to agglomerate within symmetrical cities. Their modelling of the labor market differs from ours in the sense that the wage setting process involves Nash bargaining between firms and workers and does not take into account the existence of rival firms. Another related paper is that of Abdel-Rahman and Wang (1995). In a framework close to that of Hesley and Strange (1990), they look at one spatial equilibrium which consists of a single metropolis within a system of cities and give conditions on the parameters under which it exists. Rioux and Verdier (2000) look at the incentives for local governments to finance general human capital. Here the main difference is that workers are assumed to be immobile between regions while firms are mobile. Finally, our analysis is reminiscent of the model by Papageorgiou and Thisse (1985). In their case, agglomeration takes place because of competition on the product market.

Our framework shows also a strong similarity with the "brain drain" literature that explains why skilled workers are induced to locate close to each other (Miyagiwa, 1991; Reichlin and Rustichini, 1998 or Bénabou, 1996 in an urban context). Nevertheless those papers assume the existence of an exogenous positive local human capital externality whereas in this paper we attempt to bring microeconomic foundations to such externalities. Yet the outcome is the same: ceteris paribus, agglomeration of skilled workers increases their productivity. Such a result receives large empirical support (Peri, 1998, for instance).

The remainder of the paper is organized as follows. Section 2 describes the model. We look at the equilibrium in the labor market and derive the regional wages and profits. Section 3 analyses location decisions of the agents. In section 4, we confront the migration decisions of

the two types of agents and describe the different spatial outcomes. In Section 5, we look at how different public policies can influence the spatial outcome. And finally, Section 6 concludes.

2 The Model

2.1 Workers and firms

There are two regions, region A and region B. In each region, there are both workers and firms. We denote by α_A (respectively $\alpha_B = 1 - \alpha_A$) the share of the population of workers located in region A (respectively in region B). The population of each region consists of mobile and immobile workers. We assume that the total population is equal to 1. Both mobile and immobile workers are endowed with the same level of human capital h. This human capital determines a worker's productivity.

There are N firms. The total number of firms is exogenous. Nevertheless, in Section 5, we discuss the impact of a variation of N. We denote by β_A the share of firms located in region A. We consider N sufficiently high so as to be able to ignore the integer problem. Each firm incurs a fixed cost (which is not specified here since we assume that the number of firms is exogenous) and produces a homogenous good with labor according to the following technology:

$$Y = F(hl) \tag{1}$$

with $F' > 0$ and $F'' \leq 0$. We denote by l the number of workers. Such a technology exhibits non-increasing returns to scale with respect to labor. Since our objective is to focus on the impact of matching in the labor market, we assume that all firms produce the same homogeneous good and are price takers on the good market. Furthermore, trade between the two regional goods markets is assumed to be costless which means that the price of the homogenous good is the same in both regions. We normalize the product price to one. This assumption implies that all interaction effects on the goods market are ruled out and that, unlike in Krugman (1991b) and Ottaviano *et al.* (2002), they do not influence the location decisions of agents. In equilibrium, firms earn positive profits. We assume that all workers have an equal share of total profits.

Each region is a linear segment along which the population of workers is uniformly distributed. There is a central business district (CBD) where firms are located and where pro-

duction takes place.[2] Workers reside outside the city and commute to the CBD to work. Each worker consumes one unit of land and his commuting cost is linear in the distance travelled. The urban costs incurred by the workers consist of both land rent as well as commuting cost. We assume that the land rents collected are equally distributed among the workers. Under these assumptions, the urban cost of living in region j $C(\alpha_j)$ is linear in the size of the population and given by

$$C(\alpha_j) = \frac{t}{4}\alpha_j$$

where $t > 0$ is the unit commuting cost paid in the numéraire good and which is identical to both regions.[3]

Both firms and (mobile) workers choose their spatial location. Firms choose their location in function of the profit they earn and workers base their decision on the income which is the sum of their wage and the share of profit they obtain. Contrary to other economic geography models, in our framework, we do not assume that firms adjust instantaneously and thereby allow for a richer set of dynamics.

2.2 The labor market

Firms and workers are located in one region only and the labor market is local. Workers have heterogenous skills while firms are characterized by a particular technology.[4] Since there is only a finite number of firms present in the labor market, there is a certain mismatch between the skills of the workers and the requirements of the firms. To model this we use the framework of Hamilton *et al.* (2000). The labor market is described by a circle of circumference one, which stands for the skill space. Each position on the circle indicates a specialization. Each firm and each worker has a specific position on the circle. For the worker, it represents the skills he possesses and for the firm it represents the skill requirement of its technology. A worker can produce output only when his skills perfectly match the firm's technology. If his skills do not match any existing technology, then the worker has to undergo training. The training

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[2]See Fujita and Thisse (1996) for reasons why firms want to locate in the CBD.

[3]See Appendix.

[4]As shown by Stevens (1994) firms have an incentive to differentiate their skill requirements in order to obtain market power in the labour market characterised by the heterogeneity of the workers' skills.

cost depends on the distance between the worker's skill and the firm's skill requirement. More specifically, if the position of worker is given by x and that of the firm by x_i, then the training cost function is given by $s(h) |x - x_i|$. The unit cost of this training is given by $s(h)$. The unit cost of training can depend on h in two different ways. A negative relationship between the two means that, as explained in Becker (1964), (general) human capital improves the ability of workers to match a given technology.[5] If, on the contrary, the unit cost of training is positively related to h, an increase in human capital represents an increase in the worker' specialisation.[6]

We assume that there is asymmetric information between workers and firms. First, firms are not able to observe workers' positions on the circle. The only information they have is about the distribution of workers in the skill space. We assume that workers are uniformly distributed on the circle. Consequently, the firms set wages that do not depend on the workers' characteristics. Similarly, workers do not observe the firms' positions before choosing the region where they will be working. The workers only know the number of firms in a particular region and form an expectation concerning the distance to the nearest firm. This amounts to assume that workers migrate before they find a job. Because of the assumption on asymmetric information between firms and workers, we consider here that workers have to bear all the training costs.

Hence the timing of the game is the following. In a first stage, mobile workers and firms choose simultaneously their region. In a second stage, firms compete on the local labor markets. Solving the game by backward induction, we first focus on the wage setting process and then study the migration decisions and the spatial equilibria.

2.3   Regional wages and profits

Since interaction on the labor market is local, regional wages and profits depend on the characteristics of each region. Each region is characterized by the size of its labor market, reflected by its population density and the number of firms. Wages are set by the firms in order to maximise their profits. Workers base their location decisions on the expected wage. This expected wage depends on the gross wage offered by the firms and the expected training cost. Finally, firms infer the profit they will obtain in each region based on the population and the number

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[5]This is the assumption made in Hamilton *et al.* (2000).

[6]While throughout the paper we talk about "training costs", it can represent any type of cost as, for instance, a loss of utility associated to the mismatch.

of firms in that region.

Consider a firm i in region j $(j = A, B)$ located in the skill space at x_i. The labor supply to firm i depends on the wage it proposes as well as on the wages w_{i-1} and w_{i+1} set by the adjacent firms. Denote by $\widehat{x}$ and $\widehat{y}$ respectively the workers indifferent to working in firm i or firm $i-1$ and firm i and $i+1$. This implies that the labor supply to firm i is given by:[7]

$$
\begin{aligned}
l_i\left(w_i, w_{i-1}, w_{i+1}\right) &= \alpha_j\left(\widehat{y} - \widehat{x}\right) & (2) \\
&= \alpha_j\left(\frac{x_{i+1} - x_{i-1}}{2} + \frac{w_i}{s} - \frac{w_{i-1} + w_{i+1}}{2s}\right) & (3)
\end{aligned}
$$

Hence the profit is given by (for ease of notation we use l_i for $l_i\left(w_i, w_{i-1}, w_{i+1}\right)$)

$$
\pi_i = F\left(hl_i\right) - w_i l_i \tag{4}
$$

where firm i maximizes its profit by choosing w_i. The FOC[8] is (dropping the index i for ease of notation)

$$
\frac{\partial l}{\partial w} hF'(hl) - l - w\frac{\partial l}{\partial w} = 0
$$

with

$$
\frac{\partial l}{\partial w} = \frac{\alpha_j}{s}
$$

This expression gives the effect of a change in the firm's own wage on its labor supply. The higher the population density, the smaller the wage increase needed to increase the labor supply to the firm. We focus here on the symmetric Nash equilibrium, that is an equilibrium where firms are equidistant[9] and where $w_i = w$. Given that the firms are symmetrically located on the circle, we have in region j : $l_j = \alpha_j/\beta_j N$. This means that the FOC for region A can be written as:

$$
\frac{\alpha_A}{s}\left(hF'(h\frac{\alpha_A}{\beta_A N}) - w_A\right) = \frac{\alpha_A}{\beta_A N}
$$

---

[7]For a proof see appendix. Note that we assume that no worker stays unemployed. This is the case when the wage is positive for any worker. A sufficient condition for this to be true is that the marginal productivity is high enough. This assumption allows us to focus on skill mismatch as a determinant of spatial agglomeration.

[8]It is readily verified that the SOC is always verified.

[9]Following Economides (1989) and Kats (1995), the equidistant configuration of locations on the circle is likely to be an equilibrium outcome of a game in which firms choose their technologies prior to setting their wages. In our case, this has to be considered as an approximation. Allowing for asymmetric firm locations would only complicate computations without modifying the results.

We can derive the unique symmetric wage equilibrium:

$$w_A\left(\alpha_A, \beta_A\right) = hF'\left(h\frac{\alpha_A}{\beta_A N}\right) - \frac{s}{\beta_A N} \tag{5}$$

We deduce that

$$\frac{\partial w_A}{\partial \beta_A} = \frac{N}{(\beta_A N)^2}\left(-h^2\alpha_A F''\left(h\frac{\alpha_A}{\beta_A N}\right) + s\right) > 0 \tag{6}$$

$$\text{and} \quad \frac{\partial w_A}{\partial \alpha_A} = \frac{h^2}{\beta_A N}F''\left(h\frac{\alpha_A}{\beta_A N}\right) < 0 \tag{7}$$

The expression for the wage equals the marginal productivity of labor (the wage that would prevail with perfect competition on the labor market) minus a term $s/\beta_A N$ which can be interpreted as the impact of imperfect competition on the labor market. In this model, imperfect competition results from imperfect matching between firms and workers. Firms benefit from such an imperfection: they can set a wage lower than the marginal productivity by using the fact that a worker cannot move to another firm at zero cost. Therefore, the higher the unit training cost s, the greater the monopsony power of the firm and the lower its wage. However, this monopsony power is reduced by the number of firms in the region. This effect is captured by the fact that the wage is positively related to the number of firms in the region. In our case, this effect arises because a change in the number of firms in a region changes the intensity of competition between firms, which in turn influences the equilibrium wage. Note that the wage is also positively related to the number of firms through the impact of the number of firms on the marginal productivity. Finally, expression (5) shows that an increase in the density of workers pushes wages down. As the density of workers increases, the number of workers a firm employs increases and the marginal productivity decreases. Such an impact is due to diminishing returns.

Moreover, consider a worker not yet located in a region. This worker only knows the number of firms located in that region but, beforehand, he does not know exactly the distance to the firm where he will be employed. If he expects firms to be symmetrically located on the circle,

his expected wage is the gross wage net of expected training cost $E(TC)$ and is given by:

$$
\begin{aligned}
\overline{w}_A\left(\alpha_A, \beta_A\right) &= w_A\left(\alpha_A, \beta_A\right) - E(TC) \\
&= w_A\left(\alpha_A, \beta_A\right) - s \int_0^{1/2\beta_A N} u\left(2\beta_A N\right) du \\
&= hF'\left(h\frac{\alpha_A}{\beta_A N}\right) - \frac{5s}{4\beta_A N}
\end{aligned}
\tag{8}
$$

Here again, the expected wage increases as the number of firms within the region increases. However, here the effect of β_A is stronger than it is for the worker who perfectly matches the skill needs of a firm since it also takes into account the effect on the training costs that the workers incur.

Using equations (4) and (5), the equilibrium profit is given by

$$
\pi_A(\alpha_A, \beta_A) = F\left(hl_A\right) - hl_A F'\left(hl_A\right) + s\frac{l_A}{\beta_A N}
\tag{9}
$$

where $l_A = \alpha_A/N\beta_A$. In addition, we have that (using π_A for $\pi_A(\alpha_A, \beta_A)$)

$$
\frac{\partial \pi_A}{\partial \beta_A} = \frac{hl_A}{\beta_A}F''(hl_A) - 2s\frac{Nl_A}{(\beta_A N)^2} < 0
\tag{10}
$$

$$
\text{and} \quad \frac{\partial \pi_A}{\partial \alpha_A} = -\frac{h}{\beta_A N}F''(hl_A) + \frac{l_A}{\beta_A N} > 0
\tag{11}
$$

Profits depend negatively on the number of firms located in the region since competition on the labor market gets fiercer. The density of the labor market has a positive impact on profit. The explanation is twofold. First, the wage decreases as α_A increases because of the decreasing returns to scale. Second, ceteris paribus, the number of workers employed becomes higher.

3 Migration decisions

The previous section describes wages and profits for given agents' locations. However, since firms and workers can move from one region to another (for now, we assume that all workers are mobile), these locations are endogenous. We start looking at the location decisions and the location dynamics for each agent separately and then confront the behaviour of the two types of agents to obtain the regional location dynamics.

3.1 Firms

The comparison between the profit obtained in region A and the profit obtained in region B leads to the following expression:

$$\Delta\pi\left(\alpha_A, \beta_A\right) = \pi_A\left(\alpha_A, \beta_A\right) - \pi_B\left(\alpha_B, \beta_B\right)$$

Furthermore, since we assume[10]

$$\alpha_B = 1 - \alpha_A \quad \text{and} \quad \beta_B = 1 - \beta_A$$

we have

$$\Delta\pi\left(\alpha_A, \beta_A\right) = \pi_A\left(\alpha_A, \beta_A\right) - \pi_B\left(1 - \alpha_A, 1 - \beta_A\right) \tag{12}$$

It follows from (10) and (11) that:

$$\frac{\partial\Delta\pi}{\partial\beta_A} < 0 \quad \text{and} \quad \frac{\partial\Delta\pi}{\partial\alpha_A} > 0$$

More agglomeration of firms within region A reduces the difference of profit while agglomeration of workers in region A magnifies this difference. In other words, competition on the labor market is a centrifugal force whereas the size of the labor force is a centripetal force.

In equilibrium, we know that (12) is zero, which holds if and only if

$$\beta_A = M(\alpha_A)$$

Function M gives for any density of workers α_A, the share of firms in region A (β_A) such that profits in both regions are the same: the agglomeration force and the dispersion force offset each other. This means that we have a one-to-one relationship between the number of firms in a region and the population of workers in that region. Moreover, unlike existing economic geography models (e.g. Krugman, 1991b; Ottaviano *et al.*, 2002) where this link is exogenously given by the technology, it is endogenous in our model. We deduce that

$$M'(\alpha_A) = -\frac{\partial\Delta\pi/\partial\alpha_A}{\partial\Delta\pi/\partial\beta_A} = -\frac{>0}{<0} > 0$$

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[10]While the assumption that the total population is constant seems natural, the assumption that the total number of firms is fixed is likely to have important implications. Section 5.3 looks at the effect of a change in N.

Moreover we prove in the appendix that $M''(\alpha_A) > 0$.
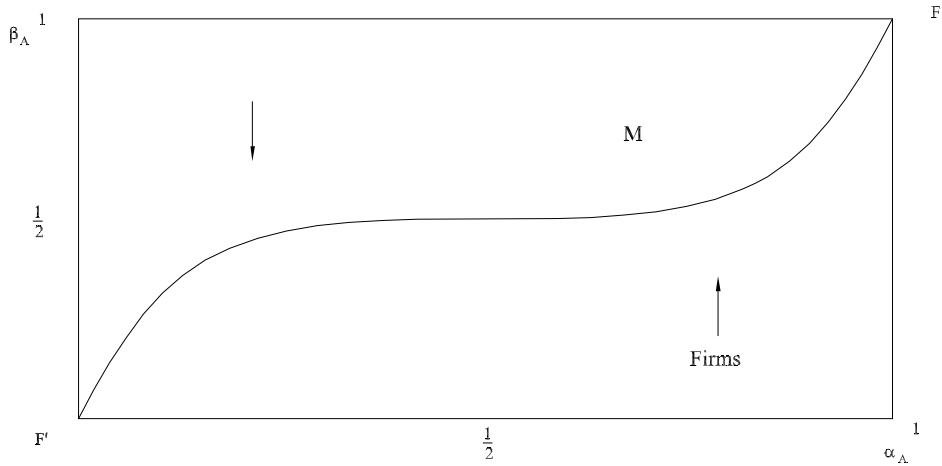
Graphically, this gives us the following result

Figure 1

The figure represents the function M: on the X-axis, the labor force α_A of region A is represented and on the Y-axis the share of firms in region A is represented. The curve in Figure 1 is symmetric with respect to point $(1/2, 1/2)$. In addition, the function M is increasing in α_A. The reason is obvious: as the number of workers in region A increases, the profit difference increases. Therefore, the number of firms in region A must rise to offset such an increase. Moreover, note that below the curve, that is if the number of firms located in region A is relatively low compared with the density of workers, the profit in region A is higher than in region B. Indeed, competition on the labor market is relatively low compared with the density of the labor market: the dispersion force offsets the agglomeration force. Therefore, firms have an incentive to move from region B to region A.

3.2 Workers

In deciding where he wants to locate, a worker compares the net expected income in each region as well as the difference in cost of locating in each region.[11] Since the share of profit a worker obtains is independent of his location, he will base his location decision on the wage difference. More precisely, the worker looks at the difference of expected wages. The reason for considering the expected wage is that before making his decision the worker only knows the number of firms in a particular region. He forms an expectation about the distance to the nearest firm.

The difference between the net expected wage obtained in region A and the net expected salary obtained in region B and the cost difference gives us the following expression:

$$\Delta w\left(\alpha_A, \beta_A\right) = \bar{w}_A\left(\alpha_A, \beta_A\right) - \bar{w}_B\left(1 - \alpha_A, 1 - \beta_A\right) - \frac{t}{2}\left(\alpha_A - \frac{1}{2}\right) \tag{13}$$

From this and from (7) and (6), we have that

$$\frac{\partial \Delta w}{\partial \beta_A} > 0 \quad \text{and} \quad \frac{\partial \Delta w}{\partial \alpha_A} < 0$$

As firms' agglomeration in region A increases, the wage difference increases as well: *tough competition on the labor market is a centripetal force for workers*. Stated differently, imperfect matching on the labor market confers bargaining power on firms and leads workers to move to the region where a larger number of firms are located. This is in contrast with "brain drain" models (Miyagiwa, 1991) where the number of skilled workers is an agglomeration force because of exogenous human capital externalities. Moreover, the wage difference decreases as the mass of workers in region A rises: the mass of workers within one region is a dispersion force. This force consists of two effects. The first effect operates through the decreasing returns in the production function and the second effect is the urban cost effect.

We have that the difference in net expected wages is zero if

$$N_A = G(\alpha_A)$$

---

[11]There is empirical evidence that besides differences in wages, differences in housing costs play an important role in migration decisions, both in the US (Haurin and Haurin 1991) and in several European countries (Tassinopoulos 1998). See also Hall (1990) on the attractiveness of regions and Dieleman and Jobse (1997) on the importance of congestion in location decisions.

where

$$G'(\alpha_A) = -\frac{\partial \Delta w / \partial \alpha_A}{\partial \Delta w / \partial \beta_A} = -\frac{< 0}{> 0} > 0$$

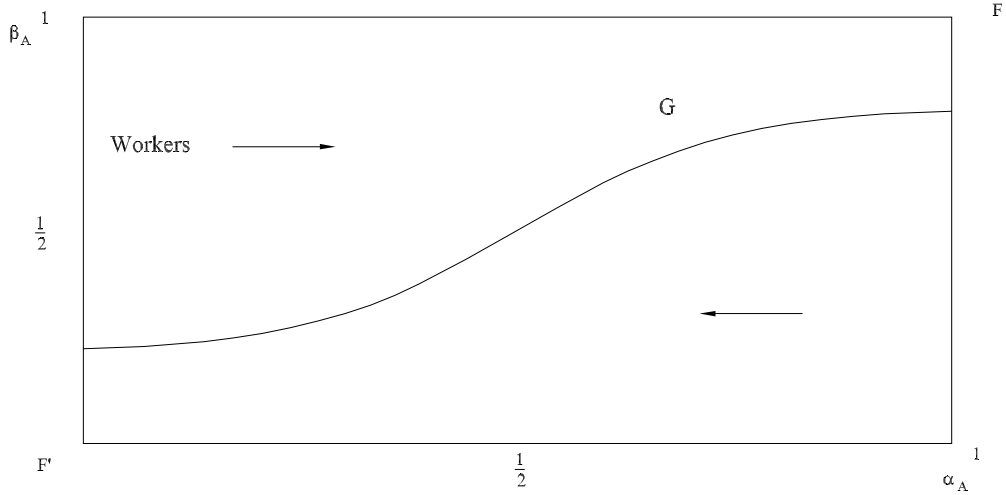Graphically, this gives us the following result.

Figure 2

Function G gives for a mass of workers in region A (α_A) the number of firms that equalize net expected wages in both regions. Above line G, workers enjoy a higher wage in region A than in region B: the large number of firms compared to the number of workers implies a strong competition on the labor market. Consider a young worker that does not work yet and has to choose his spatial location. He observes only the number of firms located in region A or in region B. Thus, this worker compares net expected wages in both regions. Therefore, whenever the economy is above the curve G, he has an incentive to locate in region A. Note that when t increases, the curve G rotates counterclockwise: a higher t for the same number of workers needs to be compensated by a higher wage. This will be the case if the matching is improved through a larger number of firms.

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4 Spatial equilibria

The migration decisions of workers and firms are a priori independent of one another. Thus, in order to get the spatial distribution of economic activity, we must deal with both migration decisions. Confronting the location decisions of the two agents leads us to the following result (the different figures are drawn for a quadratic production function and for $t > \underline{t}$, see appendix 7.5).
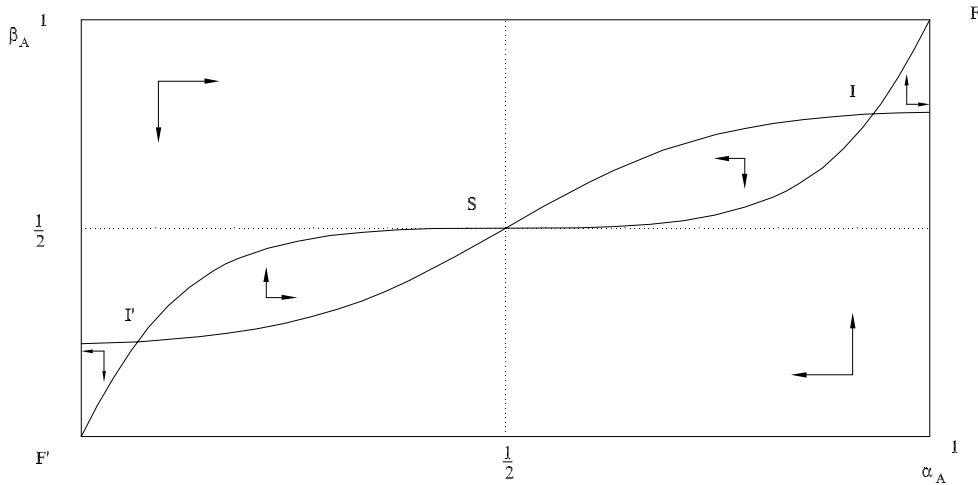
Figure 3

There exist at most three types of equilibria (see appendix 7.5 for the existence):

- Spatial equilibria characterized by an "asymmetric-interior" distribution of both firms and workers at point I and I'. Firms and workers are agglomerated within one region but, such agglomeration is not complete: workers and firms remain located in the other region. We show in appendix (7.5) that points I and I' exist. We should notice that there could exist more than two "asymmetric interior" equilibria.

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- Two equilibria characterized by full agglomeration of mobile workers in either region A or region B (point F or F').

- A symmetric equilibrium where half of workers as well as half of firms locate in both regions (point S).

The spatial outcome will depend both on the initial conditions of the region as well as the speed of adjustment of both types of agents. We analyse three cases. In the first case, we assume that firms adjust themselves instantaneously while workers' adjustment is imperfect. The second case looks at the mirror situation where workers adjust themselves immediately while the mobility of firms is lower. Finally, we look at the case where the adjustment speeds for both agents are finite.

1. The speed of adjustment of firms is infinite. The whole dynamic is described along the M curve. The dynamics of the workers is given by:

$$\dot{\alpha}_A = \begin{cases} \varphi Max\left(\Delta w\left(\alpha_A, \beta_A\right), 0\right) \text{ if } \alpha_A = 0 \\ \varphi \Delta w\left(\alpha_A, \beta_A\right) \text{ if } 0 < \alpha_A < 1 \\ \varphi Min\left(\Delta w\left(\alpha_A, \beta_A\right), 0\right) \text{ if } \alpha_A = 1 \end{cases} \tag{14}$$

with $\varphi > 0$.

Note that as in Ottaviano *et al.* (2002), we consider only myopic agents since the migration decision is driven by the current wage only.

According to equation (14), new workers have an incentive to locate in region A whenever the net expected wage $\overline{w}_A$ is higher than the net expected wage in region B. In the same

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way, the firm locate in region A whenever the profit is higher in region A.
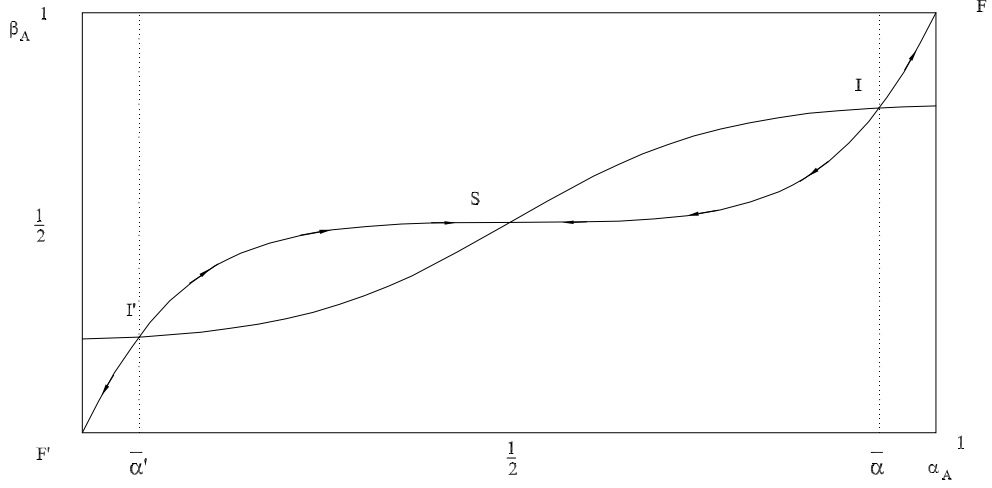

Figure 4

As a result, whenever the density of workers is less than $\bar{\alpha}$, the economy converges toward the symmetric equilibrium while whenever this density of workers in region A is higher than $\bar{\alpha}$, the economy converges toward complete agglomeration. In other words, $\bar{\alpha}$ is a threshold value for the size of region A above which this region is able to attract new workers and thereby new firms. The population level $\bar{\alpha}$ is the critical size above which a cumulative process leads the economy toward complete agglomeration of firms and workers.

2. The speed of the workers is infinite. The dynamics for the firms are given by

$$\dot{\beta}_A = \begin{cases} \gamma Max\left(\Delta\pi\left(\alpha_A,\beta_A\right),0\right) \text{ if } \beta_A = 0 \\ \gamma\Delta\pi\left(\alpha_A,\beta_A\right) \text{ if } 0 < \beta_A < 1 \\ \gamma Min\left(\Delta\pi\left(\alpha_A,\beta_A\right),0\right) \text{ if } \beta_A = 1 \end{cases} \qquad (15)$$
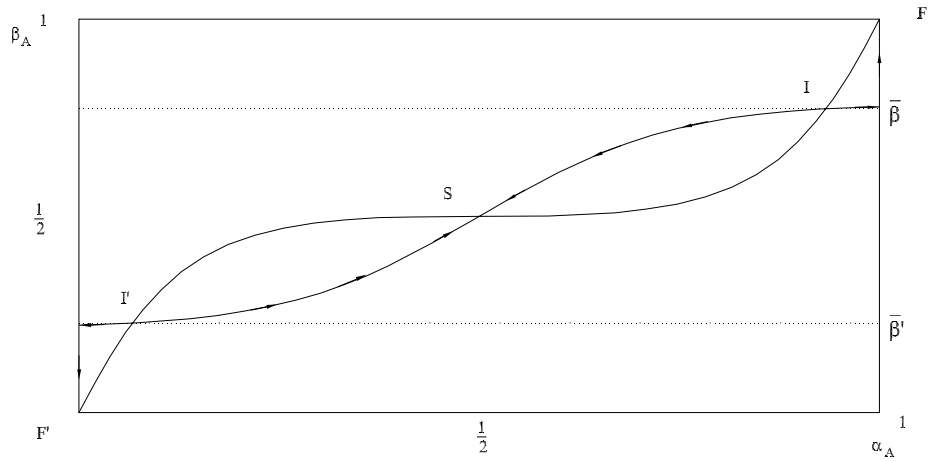
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where $\gamma > 0$.

Figure 5

As it is clear from Figure 5, the stable equilibria remain the same. Here again we find a size effect. But in this case, the size threshold is expressed in terms of a critical number of firms $\bar{\beta}$. If a country is sufficiently large in terms of number of firms, it attracts workers which then attract more firms and so on.

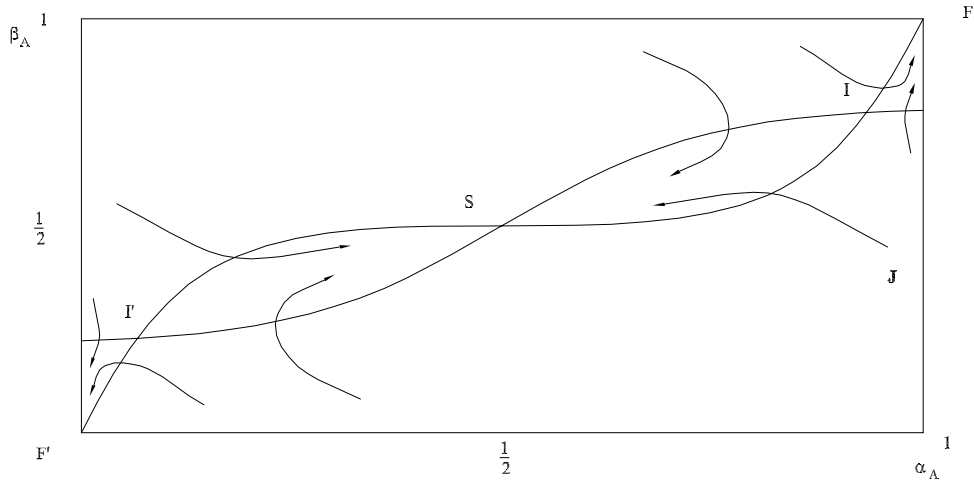3. The adjustment speeds of both agents are finite.

Figure 6

The dynamics of workers and firms are given by equations (14) and (15). Point I and I' are a saddle point (see appendix 7.6).

In this case, the initial conditions, both in terms of population and number of firms, as well as the difference of the speed of adjustment of both types of agents, determine the spatial outcome. Consider point J in Figure 6. Since this point is at the right of $\bar{\alpha}$ the economy would be completely agglomerated if the speed of adjustment of the firms were infinite. But since, in this case, there is some inertia in their location decisions, the combined dynamics lead the economy to a completely different outcome, namely the symmetrical equilibrium. The explanation is the following. At point J, the workers of region A have an incentive to move to region B because the number of firms is not sufficiently large. At the same time, there is an entry of firms in region A because the number of workers there is large. Depending on whether or not the speed of adjustment of the firms is sufficiently important, a threshold is going to be reached where the mass of firms in region A is important enough to attract again workers to region A. If not,

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then, on the contrary, another threshold is going to be reached in terms of population such that firms no longer want to move into region A.

Starting from J, a country like the US might end up in point S while in the EU might reach point F. This can be explained in the following way. The critical size threshold above which the region converges to full agglomeration depends on the population and the number of firms. However, the relative importance of these two elements in determining the threshold is a function of the speeds of adjustment: at given adjustment speed of the workers, the greater the speed of adjustment of the firms, the less important is the number of firms. Point J for a country like the US where the mobility of workers is high might not be in the bassin of attraction of point F and thus end up in point S while for the EU, where typically the workers are less mobile (Bentivogli and Pagano, 1999), this point might, on the contrary, may be in the bassin of attraction of point F.

The spatial equilibria are the result of the existence of a threshold effect: above a certain initial size asymmetry between regions ($\overline{\alpha}$ or $\overline{\beta}$ in the polar cases), a self-reinforcing process leads firms and workers to agglomerate within one region. Therefore, the impact of any public policy depends on its effect on the threshold. In other words, a public intervention that increases the threshold value is likely to push the economy toward less regional agglomeration since such a policy increases the size above which the economy converges toward complete agglomeration.

5   Public policies and regional outcome

We focus on three different public policies: policies aimed at increasing workers' spatial mobility, education policy and competition policy. We focus on these three public policies for two main reasons. First, they make up the core of the European Commission public intervention. Indeed, for instance, a recent report stresses that the Commission considers spatial mobility as a crucial element so as to foster regional cohesion (*Le Monde*, 26th February 2002 on the recent "Action plan for mobility" set up by the European Commission in March 2002). Moreover, similar proportions of Structural Funds, the main instrument of EU regional policy, are devoted to education, infrastructure investment and subsidies to entreprises. Second, from a theoretical point of view, we have shown in our model that worker mobility determines the

spatial outcome. There are different ways in which worker mobility can be defined: workers can be mobile between or within regions or they can be mobile in the labor market. And these three public policies, directly or indirectly, have an impact on worker mobility.

So as to assess the impact of public policy in terms of global efficiency, we first compare both stable equilibria. For technical convenience, we consider hereafter a linear production function: $F(hl) = hl$. In other words, we abstract from any scale effect in production which means that it is not a source of dispersion and thereby allows us to focus on the urban cost and skill mismatch effect. Note that our general results do not depend on whether we assume constant or decreasing returns to scale.

5.1 Spatial outcome and efficiency

Total welfare is measured by taking the region's total production and deducting the total training cost and the urban cost. For a region j, this is given by

$$W_j(\alpha_j) = N\beta_j(\alpha_j) F\left(\frac{\alpha_j}{N\beta_j(\alpha_j)}\right) - 2N\beta_j(\alpha_j) \int_0^{1/2N\beta_j} su\alpha_j du - t\alpha_j^2$$

Total welfare of both regions is given by

$$W(\alpha_A) = W_A(\alpha_A) + W_B(1 - \alpha_A)$$

$$
\begin{aligned}
W(\alpha_A) \quad = \quad & \underbrace{N\beta_A(\alpha_A) F\left(\frac{\alpha_A}{N\beta_A(\alpha_A)}\right) + (N - N\beta_A(\alpha_A)) F\left(\frac{1 - \alpha_A}{N - N\beta_A(\alpha_A)}\right)}_{\text{Total production}} \\
& \underbrace{-2N\beta_A(\alpha_A) \int_0^{1/2N\beta_A} su\alpha_A du - 2(N - N\beta_A(\alpha_A)) \int_0^{1/2(N - N\beta_A)} su(1 - \alpha_A)\, du}_{\text{Total training cost}} \\
& \underbrace{-t\left(\alpha_A^2 + (1 - \alpha_A)^2\right)}_{\text{Total urban cost}}
\end{aligned}
$$

Recall that the training cost is not a transfer but a pure loss. Whether the economy is symmetric or agglomerated, the total production (TP) remains the same while the training costs will be lower in the agglomerated case because of a better labor market matching. In the case of the

symmetrical equilibrium, we have $\alpha_A = 1/2$ and we have

$$W\left(\alpha_A = 1/2\right) = TP - \frac{s}{2N} - \frac{t}{2}$$

For the agglomerated outcome, we have

$$W\left(\alpha_A = 1\right) = TP - \frac{s}{4N} - t$$

This means that *socially the agglomerated equilibrium dominates the symmetrical equilibrium* if and only if

$$t < \frac{s}{2N}$$

We know that $s/2N < \underline{t}$ which means that whenever there is imperfection on the labor market $(s > 0)$, for high values of the urban cost $(t > \underline{t})$ the economy runs the risk of converging to the suboptimal social equilibrium. In other words, when the urban cost is important, there could be too much agglomeration. Such an inefficiency is due to imperfect competition on the labor market. Indeed, the oligopsonic market power of firms on the labor market introduces a difference between the private and social benefit of agglomerating, the former being higher than the latter since workers are induced to avoid market power by migrating toward the region where most of the firms are located. As a corollary, any policy that leads the economy toward dispersion is likely to be beneficial.

5.2 Workers' spatial mobility

Until now the assumption was that all workers are mobile. However, there are a number of regions where immobile workers constitute an important share of the population. This leads us to the following question: what happens when in each region a number of workers are immobile? In terms of public policy implications, the question is whether and when a public intervention aimed at increasing the workers's mobility (i.e. increase the number of mobile workers) will influence the regional distribution of economic activity.

Assume that a share of the total population is immobile and will not migrate even if there is a wage differential. The regional distribution of immobile workers can be symmetric or asymmetric. The skills' space of the immobile workers is the same as the one of mobile workers

and wages are set in exactly the same way. Since all workers have exactly the same level of human capital and since, by assumption, both types of workers are perfect substitutes, within each region both types of workers earn exactly the same wage. In the presence of immobile workers, there is an upper and a lower bound on the population of region A, α_A. The lower bound is given by α_A^i which is the number of immobile workers in region A. The upper bound is given by $1 - \alpha_B^i$ where α_B^i are the immobile workers in region B. The thresholds, depending on their positions can modify the location dynamics and change the spatial outcome. When there are immobile workers in one region, the full agglomeration equilibrium in the other region is obviously ruled out. The symmetrical equilibrium can also disappear if one of the two regions has a share of immobile workers which is more than half of the total population.
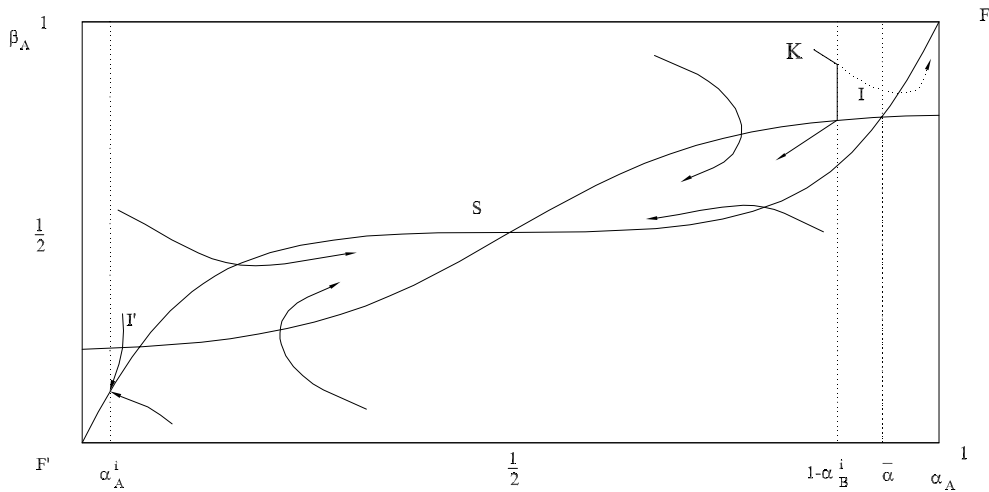

Figure 7

The more important aspect of the presence of immobile workers is the fact that their presence can change the spatial outcome in a fundamental way. Consider point K in Figure 7. When there are no immobile workers in region B, the dynamics would lead the economy to the full agglomeration equilibrium. If there is a sufficiently large number of immobile workers in region B such that the upper boundary is to the left of $\bar{\alpha}$, then the economy no longer fully agglomerates but converges either to the symmetrical equilibrium or the lower boundary.

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The equilibrium on the lower boundary can be shown to be stable.[12] The reason is that the movement of workers towards region A is unable to reach a sufficiently high population level necessary to attract the firms back to region A. This means that although initially region A is large, the presence of immobile workers does not allow the region to reach the critical size in terms of mass of workers to avoid firms leaving the region.

Workers can be mobile between or within regions. Our analysis suggests that increasing the mobility of workers between regions or within regions (by decreasing the urban costs) is likely to result into greater regional disparity. In contrast, increasing a worker's mobility in the labor market and allowing him to move more easily between jobs is shown to be beneficial to reducing spatial disparities.

5.3   Education

The European Commission's Report on Economic and Social Cohesion stresses in its recommendation the benefit of education and training on the development of regions and the reduction of regional disparities: "the Union must support the factors that play a decisive role (...) to reduce the profound imbalances affecting its territory. In short, supporting investment in (...) human capital must remain the key objective" (European Commission, 2001). Nevertheless such a point is controversial since a large literature on brain drain emphasizes that because of local spillovers, well-educated workers have incentives to locate in the same place and thereby leave their native region (Miyagiwa, 1991). Our model departs from this literature in two ways: first, the workers' agglomeration force is not based on a local pure externality and, second, human capital has not only an ad hoc positive impact on firms' profits. Indeed, as stressed in section 2.2, the level of human capital denoted by h has a direct impact on the productivity h as well as an indirect impact on the level of mismatch through the term $s(h)$: unlike specific training, general training[13] improves the workers' mobility on the labor market and reduces the magnitude of mismatch. Therefore the model provides new insight on the impact of human capital formation and training on both workers and firms location choices.

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[12]For a proof, see Appendix 7.6 Point IS.

[13]With specific (general) training, we mean any type of education or training that increases (decreases) a worker's specialisation. An example of specific training are certain types of vocational training schools which focus on a single sector or occupation.

In a first step, we look at the impact of human capital on the size effect when the speed of firms is infinite. In this case, the size effect is only measured through a change in the population. In a second step, we shall look at the more general case where the speed of adjustment is not infinite.

When there is no firm inertia the impact of human capital on spatial disparity is established by looking at the effect of h on $\bar{\alpha}$. The level of human capital has an impact on wages as well as on profits. The effect of h on the wage difference, that is on the location decision of the workers, is given by

$$\frac{d\Delta w}{dh} = \frac{5}{4} s'(h) \left(\frac{1}{N(1 - \beta_A)} - \frac{1}{N\beta_A} \right) \tag{16}$$

If region A is the larger region both in terms of population and in terms of the number of firms, we have that

$$\frac{1}{N(1 - \beta_A)} - \frac{1}{N\beta_A} > 0$$

This implies that

$$\frac{d\Delta w}{dh} = \frac{5}{4} s'(h) \left(\frac{1}{N(1 - \beta_A)} - \frac{1}{N\beta_A} \right) \left\{ \begin{matrix} < \\ > \end{matrix} \right\} 0 \Longleftrightarrow s'(h) \left\{ \begin{matrix} < \\ > \end{matrix} \right\} 0 \tag{17}$$

As the level of human capital increases, workers are more or less likely to move to the region where firms are agglomerated (here region A) depending on whether they are more or less mobile on the labor market following an increase in h. In other terms, the result states that general training reduces the incentives of workers to agglomerate.

Such an impact is the result of a "competition effect": higher human capital results in a change in the mobility of workers in the technology space. *If there is a matching improvement, the increased competition on local labor markets leads to less spatial mobility.* Indeed, the agglomeration of workers arises from imperfect matching with firms. Such imperfect matching gives bargaining power to firms. As a result, workers are likely to locate where competition between firms on the labor market is tough. In other words, the magnitude of the agglomeration force depends on the degree of matching imperfection. An increase in the level of human capital changes the degree in matching, thus changing the oligopsonic power of firms and thereby leading to a change in the bassin of attraction.

Regarding the effect of h on the decision of firms' location of the firms, we have

$$\frac{d\Delta\pi}{dh} = s'(h)\frac{1}{N}\left(\frac{l_A}{\beta_A} - \frac{l_B}{1-\beta_A}\right) = 0 \tag{18}$$

This implies that with this specification, an increase in the level of human capital has no impact on firms' locations. Indeed, because of constant returns to scale, all effects on the difference in profits come from the monopsonic power term $(sl_j/N\beta_j)$. At the equilibrium I, both terms are equal. Moreover, both terms are proportional to $s(h)$.

Thus we have

$$\frac{d\bar{\alpha}}{dh}\left\{\begin{array}{c} > \\ < \end{array}\right\} 0 \Longleftrightarrow s'(h)\left\{\begin{array}{c} < \\ > \end{array}\right\} 0$$

Hence, *if a higher level of human capital increases (decreases) workers' mobility on the labor market, this increase leads to an increase (decrease) of the critical threshold in terms of size above which the economy converges towards complete agglomeration and experiences higher spatial disparity.*

Assume that an increase in the level of human capital leads to greater mobility of the workers on the labor market. Consider an initial level of human capital h_0. Whenever the density of workers in region A is higher than $\bar{\alpha}(h_0)$, the number of firms located in A is high enough to attract new workers and so on. Suppose the level of human capital increases to a level $h_1 (> h_0)$. If the density remains in the neighborhood of $\bar{\alpha}(h_0)$, for that new level of human capital, workers are no longer induced to locate in region A despite the fact that a lot of firms are induced to do so. In other words, the economy no longer converges towards the agglomerated equilibrium, but instead moves towards the symmetrical outcome. As shown by the welfare analysis of section 5.1, this means that such a policy is able to promote regional equity and global efficiency. Our framework also allows us to say something about the desirability of general training versus specific training. From a perspective of reducing regional disparities and improve efficiency, our model shows that general training is to be preferred to specific training.

The second case to be considered is the case where firms do not relocate instantaneously. Extrapolating the linearization around point I, we know that there is an unstable arm which goes through point I.[14] Above the unstable arm, the economy agglomerates completely. Under

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[14]Here again the analysis is the same for point I'.

the unstable arm, the economy moves towards the symmetrical equilibrium. Since we have shown that an increase in h moves point I towards the north-east (respectively south-west) in the case of general training (respectively specific training), we can infer that the probability of ending up in a situation of spatial inequality is reduced.[15]

Few economic geography papers introduce human capital. Nevertheless, our result developed in this section is in line with Martin (1999) in a different context. He stresses that an increase in the researchers' productivity reduces regional inequality by inducing more entry on the good market. Such results are in sharp contrast with standard models of the "brain drain" in which human capital improvement is likely to support emigration of skilled workers to the most developed region. Indeed, in a model à la Miyagiwa (1991), because of positive human externality, the most educated workers are those who have the highest incentives to move toward the most developed region. Therefore, an increase in the level of human capital can magnify the agglomeration force. The main difference with this literature lies in the agglomeration force that leads the economy toward spatial polarization. In our framework, the positive impact of workers located in a region arises from an indirect mechanism: workers attract firms which in turn attract workers; whereas in the "brain drain" literature, the local positive externality needed for the agglomeration effect is exogenous and linked to a peer effect. Therefore, *an increase in human capital magnifies this peer effect while, in our framework, such an increase can make workers less sensitive to labor market competition.* Note that whenever an increase in human capital increases the specialization of workers, the model leads to the same result as the "brain drain" literature since workers are then more induced to move toward the region where firms are agglomerated.

We can summarize the impact of these two mobility policies in the following way: any improvement of spatial mobility is likely to increase regional disparities whereas improved mobility between skill requirements is likely to reduce such disparities.

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[15]This point can be illustrated with experiences of countries like the Czech Republic and Poland. The existence of education structures leading to a workforce which was too specialised was seen as a major barrier to their development. Recently, these countries have tried to make vocational programs more general (Gill *et al.* 2000). Similarly, Campbell *et al.* (1989) attribute the economic problems of the UK in the 80s to the same reasons.

5.4 Competition policy

Competition policy can either accommodate or deter entry. Since profits and wages depend on the number of firms, such a public intervention has an impact on spatial outcome. Not surprisingly, an increase in the number of firms has the same qualitative effect on the spatial equilibrium as an education policy which makes the workers more mobile on the labor market. Indeed, eventually, both policies tend to increase competition on the labor market.

As before, since profits are proportional to $1/N$, an increase in N has the same effect in both regions:

$$\frac{d\Delta\pi}{dN} = -s(h)\frac{1}{N^2}\left(\frac{l_A}{\beta_A} - \frac{l_B}{1-\beta_A}\right) = 0 \tag{19}$$

Since competition increases in the small region as well, workers have less incentives to move toward the large region where they suffer from high urban costs. More formally, an increase in the number of firms reduces the wage differential:

$$\frac{d\Delta w}{dN} = \frac{5}{4}s(h)\frac{1}{N^2}\left(\frac{1}{\beta_A} - \frac{1}{1-\beta_A}\right) < 0 \tag{20}$$

As a result:

$$\frac{d\bar{\alpha}}{dN} > 0$$

Hence, *a higher level of entry of firms increases the critical size threshold above which the economy converges towards complete agglomeration and experiences higher spatial disparity.* Therefore, an increase in the number of firms pushes the economy toward spatial dispersion. In other words, we show that entry plays as a dispersion force. In this model, we have a fixed number of firms. Nevertheless, the former result suggests that free entry would push the economy toward the symmetric equilibrium.

As the previous policy, competition policy influences the degree of regional disparity by modifying competition on the labor market. Since oligopsony power on the labor market affects wages and thereby leads workers to migrate where firms are agglomerated, any pro-competitive policy, whether it promotes firm entry or improves workers' mobility between task through education or training, makes complete agglomeration less likely.

6 Conclusion

We show in this paper that labor market imperfection due to specific human capital can give rise to increasing regional disparities. Indeed, workers, in order to avoid firms' monopsony power, have incentives to move to the region where firms are located and firms benefit from locating in regions with large labor markets.

In this framework, we show how certain public policies can influence the spatial distribution of economic activity. Any policy that increases the mobility of workers between tasks, increases competition on the labor market. As a result, workers are less likely to move from the less populated region to the most populated region. Stated differently, such an improvement undermines size effects and can therefore bring away the economy from regional disparities. Our analysis suggests that not all types of education policies favor regional equality: only the education policies which increases the workers' mobility on the labor market will decrease the probability of having regional disparity.

Finally, the presence of immobile workers can modify the regional outcome. First, obviously, immobile workers prohibit full agglomeration of either region. More importantly, when the share of immobile workers is sufficiently important, an economy which, without immobile workers would be spatially polarized, could with their presence, on the contrary, converge towards the symmetrical equilibrium. The reason is that the presence of immobile workers prevents the economy to reach a critical size threshold that would allow the polarization of the regions.

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7   Appendix

7.1   Labour supply

Formally we have for the workers who are indifferent between working in firm $i-1$ and i $(\hat{x})$ and between firm i and $i+1$ $(\hat{y})$

$$
\begin{aligned}
\hat{x} &= \frac{w_{i-1} - w_i}{2s} + \frac{x_{i-1} + x_i}{2} \\
\hat{y} &= \frac{w_i - w_{i+1}}{2s} + \frac{x_{i+1} + x_i}{2}
\end{aligned}
$$

This means that the supply of labor to firm i in region j consists of all workers between $\hat{x}$ and $\hat{y}$ and is given by $\alpha_j(\hat{y} - \hat{x})$.

7.2   Urban cost

Without loss of generality, the opportunity cost of land is normalized to zero. The workers of region j are equally distributed around the CBD. In equilibrium, since all workers choosing to reside in region j expect to earn the same wage, they have the same expected utility level. Furthermore, since they all consume one unit of land, the equilibrium land rent at distance $x < \alpha_j/2$ from the CBD in j is given by

$$
R^*(x) = t(\alpha_j/2 - x)
$$

This means that a worker located at the average distance $\alpha_j/4$ from the CBD bears a commuting cost equal to $t\alpha_j/4$ and pays the average land rent $t\alpha_j/4$. We assume that all the land rents are collected and equally redistributed among the workers of the region. Consequently, the individual urban costs after redistribution of the land rents are equal to $t\alpha_j/4$.

7.3   Convexity of function $M(\alpha_A)$

Function M is convex if and only if an increase in α_A leads to an increase in $(1-\beta_A)/(1-\alpha_A)$.

In other words

$$
\frac{\partial \left(\frac{1-M(\alpha_A)}{1-\alpha_A} \right)}{\partial \alpha_A} > 0 \Leftrightarrow -M'(\alpha_A)(1-\alpha_A) - (1-M(\alpha_A))(-1) > 0 \Leftrightarrow
$$

$$\frac{1 - \beta_A(\alpha_A)}{1 - \alpha_A} > M'(\alpha_A)$$

$$M'(\alpha_A) = -\frac{\partial \Delta\pi/\partial\alpha_A}{\partial \Delta\pi/\partial\beta_A}$$

Denote by $F_j'' = F''\left(\alpha_j/N\beta_j\right)$

$$
\begin{aligned}
\frac{\partial \Delta\pi}{\partial \alpha_A} &= -\frac{h^2}{N^2}\left(\frac{\alpha_A}{\beta_A^2}F_a'' + \frac{(1-\alpha_A)}{(N-\beta_A)^2}F_b''\right) + s\frac{1}{N^2}\left(\frac{1}{\beta_A^2} + \frac{1}{(N-\beta_A)^2}\right) \\
&= -\frac{h^2}{N^2}\left(\frac{\alpha_A}{\beta_A^2}F_a''\right) + s\frac{1}{\beta_A^2} - \frac{h^2}{N^2}\left(\frac{(1-\alpha_A)}{(N-\beta_A)^2}\right)F_b'' + s\frac{1}{N^2}\frac{1}{(1-\beta_A)^2} \\
&= \Phi + \Psi
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \Delta\pi}{\partial \beta_A} &= \frac{h^2}{N^2}\left(l_a\frac{\alpha_A}{\beta_A^2}F_a'' + l_b\frac{(1-\alpha_A)}{(1-\beta_A)^2}F_b''\right) - 2s\frac{1}{N^2}\left(\frac{l_a}{\beta_A^2} + \frac{l_b}{(1-\beta_A)^2}\right) \\
&= \frac{h^2}{N^2}\left(l_a\frac{\alpha_A}{\beta_A^2}F_a''\right) - 2s\frac{1}{N^2}\frac{l_a}{\beta_A^2} + \frac{h^2}{N^2}l_b\left(\frac{(1-\alpha_A)}{(1-\beta_A)^2}\right)F_b'' - 2s\frac{1}{N^2}\frac{l_b}{(1-\beta_A)^2} \\
&= -l_a'\Phi - l_b'\Psi
\end{aligned}
$$

with

$$\Phi' > \Phi \text{ and } \Psi' > \Psi$$

This means that

$$N_A'(\alpha_A) = \frac{\Phi + \Psi}{l_a\Phi' + l_b'\Psi'}$$

We also have

$$\frac{\Phi + \Psi}{l_a\Phi' + l_b'\Psi'} < \frac{\Phi + \Psi}{l_a\Phi + l_b\Psi}$$

we know that M is convex if

$$\frac{1}{l_b} > \frac{\Phi + \Psi}{a\Phi + l_b\Psi}$$

or, equivalently,

$$l_a\Phi + l_b\Psi > l_b\Phi + l_b\Psi$$

which is true since

$$l_a > l_b$$

7.4 Concavity of function $G(\alpha_A)$

Showing the concavity of $G(\alpha_A)$ for a general production function proves to be rather complicated. It is possible to find numerical examples to show this property. In addition, we are able to show the concavity property analytically in the special case when the production function is quadratic:

$$F(hl) = hl \left(\Gamma - \frac{\theta}{2} hl \right) \tag{21}$$

At point I, we know that

$$\Delta w = 0 \Leftrightarrow$$

$$\theta h^2 \left(N\beta_A - \alpha_A N \right) + \frac{5}{4} s \left(2N\beta_A - N \right) - \left(N - N\beta_A \right) N\beta_A \left(2\alpha_A - 1 \right) t = 0$$

Moreover, using the Implicit Function Theorem, we obtain the following result

$$G'(\alpha_A)$$
$$= \frac{\theta h^2 N + 2 \left(N - N\beta_A \right) N\beta_A t}{\theta h^2 + \frac{5}{2} s - \left(N - 2N\beta_A \right) \left(2\alpha_A - 1 \right) t} = \frac{K}{D}$$

Furthermore, we have

$$G''(\alpha_A) = \frac{\left[2NG' \left(N - 2N\beta_A \right) t \right] D - K \left[2NG' \left(2\alpha_A - 1 \right) t + 2t \left(2N\beta_A - N \right) \right]}{D^2} < 0$$

7.5 Existence of equilibria (point I)

The behavior between the two curves can be analyzed by looking at the difference of the two curves:

$$P(\alpha_A) = G(\alpha_A) - M(\alpha_A)$$

This expression is symmetrical with respect to $\alpha_A = \frac{1}{2}$:

$$P(\alpha_A) = -P(1 - \alpha_A)$$

In addition, we know that

$$P(\alpha_A = \frac{1}{2}) = 0$$

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In the quadratic case, P is concave. Thus if I exists and it is unique.

Let us consider the general case.

For $\alpha_A = 1$,

$$\Delta w = 0$$

$$\Leftrightarrow hF'(\frac{h}{N\beta_A}) - \frac{5s}{4\beta_A} - \frac{t}{4} - hF'(0) + \frac{5}{4}\frac{s}{N - N_A} = 0$$

$$\Leftrightarrow \frac{5}{4}s\frac{(2\beta_A - 1)}{N\beta_A(1 - \beta_A)} = h\left(F'(0) - F'(\frac{h}{N\beta_A})\right) + \frac{t}{4}$$

As far as $F'(0)$ is finite, we have always $G(1) < N$ and thus $P(\alpha_A = 1) < 0$. In other words, at $\alpha_A = 1/2$, P takes the value zero. At $\alpha_A = 1$, P has a negative value. If $F'(0)$ is infinite, $P(\alpha_A = 1) > 0$.

The question is whether at $\alpha_A = 1/2$, P increases before decreasing. If it does, it intersects the X-axis To check this, we have

$$P'(\alpha_A = 1/2) = G'(\alpha_A = 1/2) - M'(\alpha_A = 1/2)$$

Applying the Implicit Function Theorem to Δw for function G, we arrive at the following result:

$$G'(\alpha_A = 1/2) = \frac{N\left(-h^2F'' + \frac{Nt}{2}\right)}{-h^2F'' + \frac{5s}{2}}$$

For function M, the following result is easily computed:

$$M'(\alpha_A = 1/2) = \frac{N\left(-h^2F'' + 2s\right)}{\left(-h^2F'' + 4s\right)}$$

This implies that

$$P'(\alpha_A = 1/2) = \frac{N\left(-h^2F'' + \frac{Nt}{2}\right)}{-h^2F'' + \frac{5s}{2}} - \frac{N\left(-h^2F'' + 2s\right)}{\left(-h^2F'' + 4s\right)} > 0$$

$$\Leftrightarrow t > \frac{s\left(-h^2F''(\frac{1}{N}) + 10s\right)}{N\left(-h^2F''(\frac{1}{N}) + 4s\right)} = \underline{t}$$

In other words, (i) when $F'(0)$ is finite at least a point I exists if $\underline{t} < t$ and (ii) when $F'(0)$ is infinite, at least a point I exists if $t > \underline{t}$.

Assume that there is only one point I corrsesponding to the case previous case (i). We denote $\bar{\alpha}$ and $\overline{\beta}$ the coordinates of the intersection point I. Since

$$P'(\alpha_A = \frac{1}{2}) > 0 \quad \text{and} \quad P(\alpha_A = 1) < 0$$

we have

$$P'(\bar{\alpha}) < 0$$

which implies that

$$G'(\bar{\alpha}) < M'(\bar{\alpha})$$

7.6   Equilibria stability

We assume here that there is only one point I.

We introduce equations of motion for both variables:

$$\dot{\alpha}_A = \begin{cases} \varphi Max \left(\Delta w \left(\alpha_A, \beta_A \right), 0 \right) \text{ if } \alpha_A = \alpha_A^L \\ \varphi \Delta w \left(\alpha_A, \beta_A \right) \text{ if } \alpha_A^L < \alpha_A < \alpha_A^H \\ \varphi Min \left(\Delta w \left(\alpha_A, \beta_A \right), 0 \right) \text{ if } \alpha_A = \alpha_A^H \end{cases} \quad (22)$$

$$\dot{\beta}_A = \begin{cases} \gamma Max \left(\Delta \pi \left(\alpha_A, \beta_A \right), 0 \right) \text{ if } \beta_A = 0 \\ \gamma \Delta \pi \left(\alpha_A, \beta_A \right) \text{ if } 0 < \beta_A < 1 \\ \gamma Min \left(\Delta \pi \left(\alpha_A, \beta_A \right), 0 \right) \text{ if } \beta_A = 1 \end{cases} \quad (23)$$

with $\varphi, \gamma > 0$. These equations of motion show that when the variables are between their respective boundaries then the movement has the same sign as the difference in either wages or profits. These equations also show us that when a variable reaches one of the threshold, the movement stops. Note that expression (22) is more general than (14). In (14), we have $\alpha_A^L = 0$ and $\alpha_A^H = 1$.

To check for the stability of the different equilibria, we look at the Jacobian matrix of the system of equations given by (22) and (23) as far as both functions are continuous i. e. everywhere except at the different treshold where we use a graphical argument. For a stable equilibrium, it is sufficient to show that the determinant is positive while the trace is negative. For a saddle point, it is sufficient to show that the determinant is negative.

The trace of the Jacobian matrix is negative since

$$tr\left(J\right)=\frac{\partial\dot{\alpha}_A}{\partial\alpha_A}+\frac{\partial\dot{\beta}_A}{\partial\beta_A}$$

and both elements are negative or zero.

Proof:

$$\frac{\partial\dot{\alpha}_A}{\partial\alpha_A}=\varphi\frac{\partial\Delta w}{\partial\alpha_A}<0$$

and:

$$\frac{\partial\dot{\beta}_A}{\partial\beta_A}=\gamma\frac{\partial\Delta\pi}{\partial\alpha_A}<0$$

The determinant of the Jacobian matrix can be written as

$$\frac{\partial\dot{\alpha}_A}{\partial\alpha_A}\frac{\partial\dot{\beta}_A}{\partial\beta_A}-\frac{\partial\dot{\alpha}_A}{\partial\beta_A}\frac{\partial\dot{\beta}_A}{\partial\alpha_A}$$

The off-diagonal line elements can be written as

$$\frac{\partial\dot{\alpha}_A}{\partial\beta_A}=\varphi\frac{\partial\Delta w}{\partial\beta_A}$$

and

$$\frac{\partial\dot{\beta}_A}{\partial\alpha_A}=\gamma\frac{\partial\Delta\pi}{\partial\alpha_A}$$

Two possible equilibria have to be looked at: S (symmetrical), I (intermediate). We have the additional information that $\frac{\partial\Delta w}{\partial\alpha_A}<0$ and $\frac{\partial\Delta\pi}{\partial\alpha_A}<0$.

Point S In the case of point S, the determinant can be written as

$$\varphi\frac{\partial\Delta w}{\partial\alpha_A}\gamma\varphi\frac{\partial\Delta\pi}{\partial\alpha_A}-\varphi\frac{\partial\Delta w}{\partial\beta_A}\gamma\frac{\partial\Delta\pi}{\partial\alpha_A}$$

Rearranging the terms, we get

$$\left(\frac{\frac{\partial\Delta w}{\partial\alpha_A}\frac{\partial\Delta\pi}{\partial\alpha_A}}{\frac{\partial\Delta w}{\partial\beta_A}\frac{\partial\Delta\pi}{\partial\alpha_A}}-1\right)\varphi\frac{\partial\Delta w}{\partial\beta_A}\gamma\frac{\partial\Delta\pi}{\partial\alpha_A}$$

We know that the expression outside the brackets is always positive and the terms within the brackets can be rewritten.

For any point located on the demarcation lines, we know that

$$g(\alpha_A, \beta_A) \;=\; 0$$
$$m(\alpha_A, \beta_A) \;=\; 0$$

Taking the total derivatives of both equations and rearranging the terms we get

$$\frac{\frac{\partial \Delta w}{\partial \alpha_A} \; \frac{\partial \Delta \pi}{\partial \beta_A}}{\frac{\partial \Delta w}{\partial \beta_A} \; \frac{\partial \Delta \pi}{\partial \alpha_A}} = G'(\alpha_A)/M'(\alpha_A)$$

This leads us to

$$(G'/M' - 1)$$

We know that at point S we have $G' > M'$ which means that the determinant is positive.

Point I   In the case of I, we can apply the same logic as for point S, which means that the sign of the determinant is given by the sign of

$$(G'/M' - 1)$$

At point I, we know that $G' < M'$ which means that the determinant is negative and that we have a saddle point.

Points F, IS and IF   Points are stable according to a graphical argument based on the phase diagram of figures 6 and 7. Note that an analytical result is hard to derive because of the non continuity of the dynamic around these points.