CENTRE FOR ECONOMIC RESEARCH

WORKING PAPER SERIES

2001

The Effect of Payroll Taxes in the Monopoly Union Model: Four Lemmas and a Corollary

Kevin J Denny, University College Dublin

WP01/12

May 2001

DEPARTMENT OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4

The effect of payroll taxes in the monopoly union model: four lemmas and a corollary

Kevin J. Denny*

Department of Economics
University College Dublin

24 May 2001

version 1.02

Abstract:

This paper models the effect of a simple linear payroll tax in a monopoly union model. Previously derived ambiguous results are given a more intuitive interpretation and conditions under which the effect on wages is unambiguously positive are given. It is shown that after tax wages are invariant to changes in the tax rate.

JEL Classification: J51,H24

Keywords: Trade union, wages, payroll tax

* Address for correspondence: Department of Economics, University College Dublin, Belfield, Dublin 4, Ireland. (tel: 353-1-716 8399, fax: 353-1-283 0068). Email: kevin.denny@ucd.ie. My thanks to Andrew Oswald for comments. Opinions and errors, if any, are all mine.

In a well known paper published in 1985 Andrew Oswald summarized the main comparative static results for the best known models of union behaviour, the monopoly union model and the efficient bargain model. The main purpose of this note is to demonstrate that one of the results, the impact of a payroll tax on wages, given there as ambiguous, can in fact be signed under certain conditions. Some other interesting properties are also derived of the model are explored.

Recall that the monopoly union model is given by:

$$Max_w \quad R = u(\omega).n(w) + (m - n).u(b) \tag{1}$$
$$\omega \equiv w(1 - t) + s \qquad m \geq n$$

n is employment and is set by the firm as a function of the wage, m is membership, assumed exogenous, t is a proportional tax rate and s is a lump sum transfer. The first order and second order conditions for a maximum are respectively:

$$R_w = u'(\omega).(1 - t)n + [u(\omega) - u(b)]n_w = 0 \tag{2}$$
$$R_{ww} = u''(\omega)(1 - t)^2 n + 2u'(\omega)(1 - t)n_w + [u(\omega) - u(b)]n_{ww} < 0 \tag{3}$$
$$= u''(\omega)(1 - t)^2 n + 2u'(\omega)(1 - t)n_w - \left[\frac{u'(\omega)(1 - t)n}{n_w}\right]n_{ww} < 0 \qquad \text{using (2)}$$

Using the implicit function rule the sign of the effect of t on w is given by the partial derivative of the first order condition w.r.t t : $Sign \frac{\partial w}{\partial t} = Sign\, R_{wt}$

$$R_{wt} = -u'(\omega)n - u''(\omega)w.n(1 - t) - u'(\omega)n_w w \tag{4}$$

which is ambiguous. It would appear, as Oswald notes, that one cannot determine the qualitative effect of taxes on wages. Note however that (4) can be rewritten as:

$$R_{wt} = -u''(\omega)wn(1 - t) - u'(\omega)n\left[1 + n_w \frac{w}{n}\right]$$
$$= n\left(- u''(\omega)w(1 - t) - u'(\omega)[1 - \varepsilon]\right) \tag{4'}$$
$$\varepsilon \equiv -n_w.\frac{w}{n} > 0$$

So $1<\varepsilon$ is sufficient but not necessary, given the absence of risk loving (i.e. given $u''(\omega)\leq 0$), for the effect of taxes on wages to be positive. If one assumes the lump sum transfer is zero, s=0 then this simplifies further:

$$R_{wt} = u'(\omega)n\left(-\frac{u''(\omega).\omega}{u'(\omega)} + \varepsilon - 1\right) = u'(\omega)n(c + \varepsilon - 1) \qquad (4'')$$

$$c \equiv -\frac{u''(\omega).\omega}{u'(\omega)}$$

The parameter c is the coefficient of Relative Risk Aversion. This gives a simple necessary and sufficient condition for the effect of the tax rate on wages to be positive (or negative):

Lemma 1:

An increase in the payroll tax rate increases (decreases) the monopoly union's wage iff the sum of the elasticity of labour demand and the coefficient of relative risk aversion is greater (less) than one.

So the worker's marginal utility schedule and the firm's labour demand curve must be jointly sufficiently curved. To get the magnitude of the effect depends on the second order condition so it is worth asking whether it is possible to say any more then the above result. The answer is yes if we assume a constant elasticity of labour demand. Let the firm be characterised by:

$$Max_n \pi = pn^{\theta} - wn \qquad 0 < \theta < 1$$

$$\Rightarrow \frac{d\pi}{dn} = \theta pn^{\theta - 1} - w = 0 \qquad (5)$$

The elasticity of labour demand is given by $\varepsilon \equiv \frac{1}{1-\theta} > 1$. Noting that:

$$n_{ww} = \left(\frac{1}{\theta - 1}\right)\left(\frac{1}{\theta - 1} - 1\right)n.\frac{1}{w^2} \qquad (6)$$

The second order condition, after some manipulation, can be rewritten as follows:

$$R_{ww} = \frac{u'(\omega)n(1-t)}{w}\left(-c + \frac{\theta}{\theta - 1}\right)$$

$$= -\frac{u'(\omega)n(1-t)}{w}(c + \varepsilon - 1) \qquad (3')$$

One can see at this point that the union's second order condition allows one to sign (4") since the term in parentheses there is the same as in (3'). That is (3') implies $c + \varepsilon - 1 > 0$. Assume henceforth that the elasticity of labour demand is, indeed, constant[1]. It follows that:

Lemma 2:

An increase in the payroll tax increases the monopoly union wage.

Proof:

$$\frac{\partial \boldsymbol{w}}{\partial \boldsymbol{t}} = -\frac{\boldsymbol{R_{wt}}}{\boldsymbol{R_{ww}}} = \frac{\boldsymbol{w}}{1-\boldsymbol{t}} > 0 \qquad \boldsymbol{or}$$

$$\frac{\partial \boldsymbol{w}}{\partial \boldsymbol{t}} \cdot \frac{1}{\boldsymbol{w}} = \frac{1}{1-\boldsymbol{t}} \qquad (\approx 1 + \boldsymbol{t} \text{ if } \boldsymbol{t} \text{ "}\boldsymbol{small}\text{"}) \tag{7}$$

Given constant elasticity of demand, the effect of taxes is, in fact, unambiguously positive. Moreover the higher the tax rate is to begin with the more a given tax increase is passed on in higher wages. The (marginal) introduction of a tax has, to first order, a unit elastic effect.

The effect of a tax change on after-tax wages may be of more interest, to the workers at least:

Corollary:

The after-tax wage is invariant to changes in the tax rate.

Proof:

Since $\frac{\partial[\boldsymbol{w}(1-\boldsymbol{t})]}{\partial \boldsymbol{t}} = (1-\boldsymbol{t})\frac{\partial \boldsymbol{w}}{\partial \boldsymbol{t}} - \boldsymbol{w}$ it follows from (7) that:

$$\frac{\partial[\boldsymbol{w}(1-\boldsymbol{t})]}{\partial \boldsymbol{t}} = 0 \tag{8}$$

This result could also be obtained by noting that (2) can be written in the form:

$\boldsymbol{u}'(\omega).\omega + (\boldsymbol{u}(\omega) - \boldsymbol{u}(\boldsymbol{b})).\varepsilon = 0$ from which the constancy of ω follows immediately from the constancy of ε. So an increase in the tax rate causes the union to pick a new wage sufficiently higher to leave net wages unchanged. This is quite a strong- and testable- result. It is not entirely surprising, however, since it is known that under the key assumption made here (constant elasticity of labour demand) iso-elastic shifts in the labour demand curve or

[1] Santoni(1995) shows that another one of Oswald's results, the effect of an employment subsidy, relies on the implicit assumption that the labour demand curve is linear.

changes in the firm's price will leave the wage unchanged. Employment will be lower and the union worse off nonetheless.

The revenue from the payroll tax levied on the workers in this firm is simply the tax rate times the wage bill R=t.w(t).n(w(t)). So one can derive two further propositions of interest:

Lemma 3:

An increase in the tax rate reduces the firm's wage bill.

Proof:

$$\frac{\partial(w(t)n(w(t)))}{\partial t}\frac{1}{w(t)n(w(t))} = \frac{1-\varepsilon}{1-t} < 0 \qquad (9)$$

This does not mean, of course, that the firm is better off since combining (7) with the Envelope Theorem shows that profits are decreasing in t.

Lemma 4:

The Laffer Curve slopes down iff $t.\varepsilon>1$

Proof:

$$\frac{\partial R}{\partial t} = w.n + t.\frac{\partial(w(t)n(w(t)))}{\partial t} = \frac{w.n}{1-t}(1 - t.\varepsilon) \qquad (10)$$

Following the early contributions of Oswald(1985) and Hersoug(1984) there is now a substantial theoretical literature on payroll taxes in unionized labour markets. These papers have typically had focused on issues of tax structure such as changes in progressivity (e.g. Koskela and Vilmounen(1996), Lockwood and Manning (1993), optimal taxation (e.g. Palokongas 1987, Boeters and Schneider(1999)) and typically have more complex forms of union/firm interaction such as bargaining models . In general these settings do not yield the simple results derived here and it would appear that they have been overlooked by recent developments.

References

Stefan Boeters and Kerstin Schnieder (1999) The structure of optimal taxation in a unionized labor market *FinanzArchiv* 56(2) 174-187

Tor Hersoug (1984) Union wage responses to tax changes *Oxford Economic Papers* 36(21) 37-51

Erkki Koskela and Jouko Vilmunen (1996) Tax progression is good for employment in popular models of trade union behaviour *Labour Economics* 3(1) 1996 65-80

Ben Lockwood and Alan Manning (1993) Wage setting and the tax system: theory and evidence for the UK *Journal of Public Economics* , 52 , 1-29

John Creedy (1990) Flattening the tax structure, changing the tax mix and union's wage demands *Journal of Economic Studies* 17(1), 5-15

Andrew J. Oswald (1985) The Economic Theory of Trade Unions: an Introductory Survey *Scandinavian Journal of Economics* , 87(2), 160-193

Tapio Palokongas (1987) Optimal Taxation and Employment Policy with a centralized wage setting Oxford Bulletin of Economics and Statistics, 39, 799-812

Santoni Michele (1995) A Note on Oswald's "The Economic Theory of Trade Unions" *Scandinavian Journal of Economics* ,97(1), 169-171