UCD CENTRE FOR ECONOMIC RESEARCH

WORKING PAPER SERIES

2007

Modelling Ireland's Exchange Rates: From EMS to EMU

Derek Bond, University of Ulster,
Michael J Harrison, University College Dublin and
Edward J O'Brien, European Central Bank

WP07/18

November 2007

**UCD SCHOOL OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4**

Modelling Ireland's Exchange Rates: From EMS to EMU[*]

Derek Bond[a]   Michael J. Harrison[b]   Edward J. O'Brien[c,†]

November 17, 2007

a University of Ulster, Coleraine, Co. Londonderry, BT52 1SA, United Kingdom

b School of Economics, University College Dublin, Belfield, Dublin 4, Ireland

c European Central Bank, Kaiserstraße 29, D-60311 Frankfurt am Main, Germany

Abstract

This paper attempts to model the nominal and real exchange rate for Ireland, relative to Germany and the UK from 1975 to 2003. It offers an overview of the theory of purchasing power parity (Ppp), focusing particularly on likely sources of nonlinearity. Potential difficulties in placing the analysis in the standard $I(1)/I(0)$ framework are highlighted and comparisons with previous Irish studies are made.

Tests for fractional integration and nonlinearity, including random field regressions, are discussed and applied. The results obtained highlight the likely inadequacies of the standard cointegration and Star approaches to modelling, and point instead to multiple structural changes models. Using this approach, both bilateral nominal exchange rates are effectively modelled, and in the case of Ireland and Germany, Ppp is found to be valid not only in the long run, but also in the medium term.

J.E.L. Classification: C22, C51, F31, F41

Keywords: Purchasing power parity; fractional Dickey-Fuller tests; smooth transition autoregression; random field regression; multiple structural changes models.

1 Introduction

Purchasing power parity (PPP) continues to be a major subject of applied economic research. The extensive study of PPP is unsurprising, given its crucial role in international finance and in the theory and policy of exchange rate determination and the conduct of monetary regimes. Results of empirical studies of PPP have been very heterogeneous (see, for example, Taylor and Taylor, 2004). From general acceptance in the 1970s to firm rejection in the 1980s, PPP has generally been accepted, albeit cautiously, in more recent decades (Taylor, 2006). These developments are, in part, due to contemporaneous developments in econometric theory. Another important factor throughout this period has been the changing monetary landscape. The 1970s saw the end of the Bretton-Woods era and the inception of the European Monetary System (EMS); more recently, European Monetary Union (EMU) occurred.

Early empirical investigations of PPP generally took one of two approaches, examining either the co-movement of price indices or the behaviour of the real exchange rate, with a particular emphasis on the long run (see, for example, Sarno and Taylor, 2002). The perceived difficulties with these approaches, which frequently employed cointegration techniques, were generally attributed to the low power of unit root test procedures. Efforts to overcome these difficulties focused on obtaining long-span data series, using alternative testing procedures and panel data approaches (see, for example, Papell, 2006).

More recently, however, two new approaches have grown in importance, focusing on the persistence in deviation of the real exchange rate and nonlinearity. Persistence may be due to aggregation bias in the data and nonlinearity may arise from asymmetric adjustment to PPP (Rogoff, 1996). Several studies have placed PPP in the fractional (co)integration framework in an attempt to capture persistence, but these have not addressed the power issues relating to unit root tests and the estimation of long memory models (see, for example, Villeneuve and Handa, 2006). The most commonly used nonlinear technique has been smooth transition autoregression (Sarno, 2005). Although this approach may be appealing theoretically, it tests the null of linearity against just one nonlinear specification, thereby disregarding any other form of nonlinearity; a more general approach may be more appropriate. Also, these approaches have usually been considered in isolation, although it is clear from the econometrics literature that nonstationarity, be it fractional or otherwise, and nonlinearity are closely related.

This paper aims to model the nominal and real exchange rate for Ireland, relative to Germany and the United Kingdom (UK), from 1975 to 2003, with a particular emphasis on persistence and nonlinearity. Adopting an approach similar to Johansen and Juselius (1992), the paper initially explores PPP in a cointegration framework. The possibilities of both persistent deviation from PPP and nonlinearity are then considered. Two approaches, which have yet to be employed in the study of PPP and which have the potential to overcome the difficulties encountered in previous studies, are introduced. The first, the fractional augmented Dickey-Fuller test, examines the hypothesis of fractional against integer integration, and may help distinguish between stationary, nonstationary and long memory processes. The second, random field regression, offers a new approach to testing for and specifying nonlinear models. Crucially, this technique assumes no prior knowledge

of the likely form of nonlinearity.

The structure of the paper is as follows. Section 2 provides relevant background material, describing the theory of PPP, the results of previous studies using Irish data and a brief history of important monetary developments. Section 3 explains the concept of fractional integration and some approaches to modelling nonlinearity, in particular, random field regression. Section 4 describes the data, the precise methodology used in the paper and presents and discusses the results. Finally, Section 5 concludes by considering how the methodology might assist in the development of the general discussion of PPP.

2 Purchasing Power Parity

A simple statement of the purchasing power parity hypothesis is that national price levels should be equal when expressed in a common currency. More formally, if s_t is the logarithm of the nominal exchange rate (expressed as units of foreign currency per unit of domestic currency), p_t and p_t^* are the logarithms of the domestic and foreign price levels, respectively, and q_t is the logarithm of the real exchange rate in period $t = 1, 2, ..., T$, then for all t,

$$q_t = s_t + p_t - p_t^*. \tag{1}$$

It follows that q_t must be stationary for long-run PPP to hold. If the mean of q_t, $E(q_t)$, is zero, PPP is absolute, whereas if $E(q_t) \neq 0$, PPP is relative. Most of the empirical studies of PPP have either been concerned with testing whether q_t has a mean reversion tendency over time or whether s_t, p_t and p_t^* move together over time.

This latter work has generally been concerned with models whose simplest form is

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \epsilon_t, \tag{2}$$

where ϵ_t is white noise. Early studies were concerned with whether the estimated values of the parameters of various versions of Equation (2) were as predicted (see, for example, MacDonald and Taylor, 1992). As awareness of time series dynamics increased, the issue changed to one of whether Equation (2) is a cointegrating regression. Papers such as those by Wright (1994) and Kenny and McGettigan (1999) take such an approach with Irish data, using the now well-known Engle-Granger (1987) two-step method or Johansen (1988) approach to cointegration.

In recent years, the emphasis has generally shifted from considering models like Equation (2), to considering directly the behaviour of $\{q_t\}_{t=1}^T$, the sequence of real exchange rate values. Within the $I(1)/I(0)$ framework, most initial studies failed to reject the hypothesis that real exchange rates were $I(1)$ for recent periods of flexible exchange rates. This failure to reject the possibility of unit roots in real exchange rate series implies a lack of mean reversion, which undermines the PPP hypothesis. The explanation often given for this non-rejection is the recognised low power of traditional unit root tests, such as the standard Dickey-Fuller (1981) test. To overcome this problem, two general approaches have been adopted. The first has been the construction and use of long series of exchange rate data and more powerful asymptotic tests (see, for example, Taylor, 2002). The second, using panel data, attempts to estimate the half life of the mean reversion of the

real exchange rate (Cashin and McDermott, 2004). There is, though, another possibility that is receiving increasing attention, and this is described in some detail in the following subsection.

2.1 Nonlinearity and purchasing power parity

Among the various alternative approaches to modelling the Ppp relationship that have been put forward, much recent interest has focused on nonlinearity. Taylor (2006) details three of the most commonly cited sources of potential nonlinearity in Ppp. The first relates to the assumption underlying Ppp that transport costs, tariffs and other barriers to trade are negligible or non-existent. If this assumption is false, these costs may cause frictions in the markets for goods and services. Such frictions can lead to so-called 'bands of inaction', within which it is unprofitable to arbitrage the deviations from the law of one price. These bands may cause discontinuities in the relationship. Bands of inaction may also arise from sunk costs (Schnatz, 2006). Taylor (2001) modelled such bands of inaction as two-regime threshold autoregressions. Similarly, Taylor, et al. (2001) used a smooth transition autoregressive model where the speed of adjustment to Ppp was proportional to the transaction costs and resulted in smooth rather than discreet adjustments.

A second source of nonlinearity in Ppp has been proposed by Kilian and Taylor (2003). They suggest that the interaction of heterogeneous agents in the foreign exchange market may result in nonlinearity. When the exchange rate is close to its Ppp equilibrium level, agents would hold a diverse range of views regarding its (mis)alignment. But as the exchange rate deviates further from its equilibrium level, the range of views regarding future movements converge.

The third possible source of nonlinearity, proposed by Sarno and Taylor (2001) relates to official intervention in the foreign exchange market. If misalignments in the equilibrium level of exchange rates are viewed as co-ordination problems between traders and monetary authorities, official intervention may be required to correct the misalignment. This view is supported empirically by Taylor (2005).

The persistence of deviations from Ppp has been a source of much study. While these deviations may result from nonlinearities such as those described in previous paragraphs, there is a further possibility. Persistent deviations from Ppp may be due to long memory processes in the data and these in turn may arise from data aggregation (Granger, 1980). Taylor (2006) discusses the role of aggregation bias in the Ppp 'puzzle', but fails to make the link between the aggregation of data and fractional integration. Data aggregation in this context may be temporal or cross-sectional (see Imbs, et al., 2005). Interestingly, they find that this bias may be more significant for data which excludes the non-traded sector, but that the bias may be overcome by using nonlinear models.

Taylor and Peel (2000), and Kilian and Taylor (2003) find that both nominal and real exchange rates are well characterised by nonlinear processes, specifically smooth transition models. Several studies have also placed Ppp in a fractional integration framework, with varying degrees of success (see Villeneuve and Handa, 2006). It is also of interest to note, however, that Sarno and Taylor (2001) found that it would require very long time series to correctly reject the unit root in real exchange rates, using standard tests, if the true data

generating process was indeed stationary with slow mean reversion. This suggests that a potentially more powerful approach, such as the fractional augmented Dickey-Fuller test (Dolado, et al., 2002), may be useful, particularly when the long time series are likely to contain numerous structural breaks resulting from fluctuations in exchange rates, international trade and the underlying policy environment (Schnatz, 2006).

While persistent deviations from Ppp may result from nonlinearity in the data generating process, what appear to be long memory processes may result from an inability to distinguish between nonstationarity and nonlinearity. From the econometrics literature, it is clear that nonstationarity and nonlinearity are closely related. It has been well known for many years that it is difficult to distinguish statistically between difference stationary series and nonlinear but stationary series (see Perron, 1989). Recent works in this area include Lee, et al. (2005) and Hong and Phillips (2005). Increasingly, the analysis uses the fractional integration framework rather than the 'knife-edge' $I(1)/I(0)$ approach to consider the interaction between nonlinearity and nonstationarity. Other recent work by Dolado, et al. (2005), Gil-Alana (2004) and Mayoral (2005) has devised new test procedures for fractionality and/or nonlinearity. However, in most cases the form of the nonlinearity needs to be known.

2.2 The Irish experience

Empirically testing Ppp for Ireland has produced varying results. In some cases, Ppp could not be accepted, whereas in others it could not be rejected. Bradley (1977) found evidence in favour of short-run and long-run Ppp, using pre-Ems data for Ireland and the UK. Thom (1989) failed to reject the hypothesis of stationarity in the real exchange rate for Ireland, Germany and the United States, while Callan and Fitzgerald (1989) rejected Ppp for Irish, German and UK data.

While rejection was common, particularly when data from the Ems period was used, non-rejection seemed most common when either alternative price indices were used or other variables were included in the model. For instance, Kenny and McGettigan (1999) distinguished between prices in the traded and non-traded sectors, and Wright (1994) considered interest rate differentials, along with the variables in Equation (2). Finally, Lane and Milesi-Ferretti (2002) found evidence of a time-varying real exchange rate.

2.3 Ireland and the European Monetary System

In an effort to explore the implied long-run Ppp relationship, this study uses data from 1975 to 2003. This period, however, saw the inception of Ems and Emu. It is important, therefore, to understand the events relating to monetary integration in this period.

Ireland joined Ems at its outset in 1979, as did Germany; the UK did not. This brought to an end the period where the Irish pound was pegged to Sterling. During the early years of Ems, the Irish currency depreciated against the basket of European currencies of Ems participants, known as the European Currency Unit (Ecu), as the Deutsche-Mark was re-valued in 1979, 1981 and 1982. The Irish pound continued to depreciate against the Deutsche-Mark until 1985, but remained stable within Ems, until its re-alignment in August 1986, when it devalued by 8 per cent relative to the Ecu. This devaluation was

brought about by a loss of competitiveness vis-à-vis the UK, due to movements in the Deutsche-Mark/Sterling exchange rate.

From 1987 to 1992, the Irish pound was stable against the Deutsche-Mark. This period was notable, as the UK joined EMS in 1989 and Germany re-unified in 1990. These events were followed by a period of sustained pressure on the Irish pound within EMS, culminating in another devaluation in January 1993. This followed Sterling's devaluation in September 1992 and ultimate exit from the system shortly after. This was a period of crises for EMS and resulted in a widening of the currency fluctuation bands. These so-called wide bands applied until 1999, when EMS was overtaken by EMU. The penultimate step towards monetary union was taken in 1996-97, in the form of the new exchange rate mechanism.

According to Bini-Smaghi and Ferri (2006), the Irish pound was one of the most frequently attacked currencies during the EMS period, and was also one of the most susceptible to resultant re-alignments. Both Thom (1989) and Honohan and Leddin (2006), however, have argued that these re-alignments should not necessarily be viewed as shocks, but rather as corrective adjustments, which are not necessarily inconsistent with PPP. This view coincides with that of Taylor (2004, 2005) regarding official intervention in the foreign exchange market, and suggests that this may be a likely cause of nonlinearity in the PPP relationship.

3 Nonstationarity and Nonlinearity

3.1 Fractional integration and long memory models

The concept of long memory can be related to the issues of nonstationarity and nonlinearity. However, long memory has not played a central role in the discussion of PPP, despite being used extensively in other areas of exchange rate analysis, such as the forward rate anomaly (see Bond, et al., 2006), and being used in the early and heavily cited works by Diebold, et al. (1991) and Cheung and Lai (1993). The papers by Robinson and Iacone (2005), and Villeneuve and Handa (2006) are two of the few recently published works that apply the concept to PPP.

A series $\{y_t\}_{t=0}^{\infty}$ is said to be integrated to order d, denoted by $I(d)$, if the series has to be differenced d times before it is (asymptotically) stationary, $I(0)$. In the classical analysis, d is an integer and the majority of investigation has involved the $I(1)/I(0)$ framework. That is, either $\Delta y_t = y_t - y_{t-1}$ or y_t is $I(0)$. In fractional integration analysis, the restriction that d is an integer is relaxed. This leads to a more general formula for an integrated series of order d given by

$$\Delta^d y_t = y_t - d y_{t-1} + \frac{1}{2!} d(d-1) y_{t-2} - \ldots + \frac{(-1)^j}{j!} d(d-1)\ldots(d-j+1) y_{t-j} + \ldots, \quad (3)$$

which is $I(0)$. In the case where $0 < d < 1$, it follows that not only the immediate past values of y but values from previous time periods influence the current value. If $0 < d < 0.5$, then the series $\{y_t\}_t^{\infty}$ is stationary; and if $0.5 \leq d < 1.0$, then $\{y_t\}_t^{\infty}$ is nonstationary. Both estimation and inference in the case where d is not an integer is more complex than in the standard integer d case (see Bond, et al., 2007a) and this could be

an explanation for the lack of uptake of the concept in the analysis of PPP.

3.2   The fractional augmented Dickey-Fuller test

The Dolado, et al. (2002) approach to testing for fractionality is based on the distribution of the t-statistic on ϕ from the generalised ADF regression

$$\Delta^{d_0} y_t = \phi \Delta^{d_1} y_{t-1} + \sum_{i=1}^{p} \zeta_i y_{t-i} + v_t, \tag{4}$$

where v_t is a hypothesised white noise error. For testing purposes, Dolado, et al. (2002) set d_0 equal to 1. The test of the null hypothesis $H_0 : \phi = 0$ is then a test that the series $\{y_t\}_{t=0}^{\infty}$ is $I(1)$ against the alternative hypothesis that the series is $I(d_1)$. They showed that if $0.5 \leq d_1 < 1.0$, the t-statistic for ϕ under H_0 follows an asymptotic normal distribution, while if $0 < d_1 < 0.5$, the t-statistic follows a non-standard distribution of fractional Brownian motion. However, they also showed that in the practically realistic case in which d_1 is unknown, the t-statistic has an asymptotic normal distribution for $0 \leq d_1 < 1.0$, provided that a $T^{-\frac{1}{2}}$-consistent estimator of d_1 is used.

3.3   Smooth transition autoregressive models

The standard way to model the nonlinearities in the PPP context has been to use STAR models (see Teräsvirta, 1994). Assuming that the real exchange rate is a stationary process, the STAR representation can be written as

$$q_t = \varphi' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma, c, \tau_t) + \epsilon_t, \tag{5}$$

where ϵ_t is white noise, $\mathbf{z}_t = [1 \, q_{t-1} \, \ldots \, q_{t-p}]'$, and φ and θ are $(p+1)$-vectors of parameters. The transition function $G(\cdot)$ determines the degree of mean reversion and is a function of γ, the slope coefficient, c the location parameter and τ_t the transition variable. Normally, τ_t is assumed to be an element of $\mathbf{z}_t$.

There has been little discussion about the choice of specification of the transition function, G, for PPP applications. It is generally accepted, following Taylor, et al. (2001), that its form is exponential:

$$G(\gamma, c, \tau_t) = 1 - \exp\left[-\gamma(\tau_t - c)^2\right], \tag{6}$$

and the resultant model is known as the exponential smooth transition autoregressive (ESTAR) model. The reason for this choice is that it is felt that the movement of the real exchange rate is symmetrical. However others, such as Baharumshah and Liew (2006), argue that the asymmetric logistic function (and hence the LSTAR model) should also be considered, i.e.,

$$G(\gamma, c, \tau_t) = [1 + \exp\left[-\gamma(\tau_t - c)\right]]^{-1}, \tag{7}$$

on the grounds that there is little empirical evidence to support the use of ESTAR models.

A more general alternative to the ESTAR model is the LSTAR2 model:

$$G(\gamma, c, \tau_t) = \left[1 + \exp\left[-\gamma \prod_{k=1}^{2}(\tau_t - c_k)\right]\right]^{-1}. \qquad (8)$$

Using the LSTAR2 model overcomes the problem that, as $\gamma \to \infty$, Equation (6) becomes linear.

Tests for nonlinearity can be derived in this context from the model

$$q_t = \beta_0 + \sum_{j=1}^{3} \beta_j \ \widetilde{z}_{tj}\tau_t^j + u_t^*, \quad t = 1, 2, ..., T, \qquad (9)$$

where τ_t is the tth observation on the transition variable, $\widetilde{z}_{tj}, t = 1, 2, 3$, is the tth observation on the jth explanatory variable, which in the simple autoregressive case is just the j-period lagged value of q_t, and u_t^* is a white noise disturbance. The four standard tests have the null hypotheses $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$, $H_{04} : \beta_3 = 0$, $H_{03} : \beta_2 = 0|\beta_3 = 0$ and $H_{02} : \beta_1 = 0|\beta_2 = \beta_3 = 0$. A very different and little-known alternative to modelling nonlinearity, however, is available.

3.4 Random field regression models

This alternative approach to modelling nonlinearity is provided by random field regression. Dahl (2002) showed that the random field approach has relatively better small sample fitting abilities than a wide range of parametric and nonparametric alternatives, including LSTAR and ESTAR models. The idea of using random field models to estimate and test for nonlinear economic relationships was introduced by Hamilton (2001) and is as follows.

If y_t is a stationary process, $\epsilon_t \sim$ n.i.d.$(0, \sigma^2)$, and $\mathbf{x}_t$ is a k-vector, that may include lagged dependent variables, then the basic model is

$$y_t = \mu(\mathbf{x}_t) + \epsilon_t, \qquad (10)$$

where the form of the conditional expectation functional, $\mu(\mathbf{x}_t)$, is unknown and assumed to be determined by the outcome of a random field. Hamilton suggests representing $\mu(\mathbf{x}_t)$ as consisting of two components. The first is the usual linear component, while the second, a nonlinear component, is treated as stochastic and hence unobservable. Both the linear and nonlinear components contain unknown parameters that need to be estimated. Following Hamilton, the conditional mean function is written as

$$\mu(\mathbf{x}_t) = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t + \lambda m(\bar{\mathbf{x}}_t), \qquad (11)$$

where $\bar{\mathbf{x}}_t = \boldsymbol{g} \odot \mathbf{x}_t$, $\boldsymbol{g}$ is a k-vector of parameters and $\odot$ denotes the Hadamard (element-by-element) product of matrices. The function $m(\bar{\mathbf{x}}_t)$ is referred to as the random field. If the random field is Gaussian, it is defined fully by its first two moments. If $\mathbf{H}_k$ is the covariance matrix of the random field, with a typical element $H_k(\mathbf{x}, \mathbf{z}) = E[m(\mathbf{x})\, m(\mathbf{z})]$,

Equation (10) can be rewritten as

$$y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t + u_t, \tag{12}$$

where $u_t = \lambda m(\bar{\mathbf{x}}_t) + \epsilon_t$, or in matrix form

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \tag{13}$$

where $\boldsymbol{\beta} = [\alpha_0 \; \boldsymbol{\alpha}_1']'$. It follows that

$$\mathbf{u} \sim N(0, \lambda^2 \mathbf{H}_k + \sigma^2 \mathbf{I}_T). \tag{14}$$

Treating equations (13) and (14) as a generalised least squares problem, the associated profile maximum likelihood function can be obtained and estimated. The only problem is that the form of the covariance matrix is unknown. Hamilton derives $\mathbf{H}_k$ as a simple moving average representation of the random field based on $\boldsymbol{g}$, using an L_2-norm measure. He shows that even under fairly general misspecification, it is possible to obtain consistent estimators of the conditional mean.

The additive random field function used by Hamilton (2001) suggests that a simple method of testing for nonlinearity is to check if λ, or λ^2, is zero or not. Hamilton showed that if $\lambda^2 = 0$ and the nonlinear model is estimated for a fixed $\boldsymbol{g}$, the maximum likelihood estimator $\widetilde{\lambda}$ is consistent and asymptotically normal. Hamilton showed that provided the covariance function of the random field can be derived, for a fixed $\boldsymbol{g}$ (Hamilton uses the mean of its prior distribution), testing only requires a single linear regression to be estimated. Hamilton derived the appropriate score vectors of first derivatives, for $k = 1, 2, .., 5$, and the associated information matrices, and proposed a form of the LM test for practical application. The test statistic is

$$\lambda_H^E(\boldsymbol{g}) = \frac{[\widehat{\boldsymbol{\epsilon}}'\mathbf{H}\widehat{\boldsymbol{\epsilon}} - \widehat{\sigma}^2 tr(\mathbf{M}_T \mathbf{H})]^2}{\widehat{\sigma}^4 \left[2tr \left([\mathbf{M}_T \mathbf{H} \mathbf{M}_T - (T-k-1)^{-1} \mathbf{M}_T tr(\mathbf{M}_T \mathbf{H})]^2 \right) \right]}, \tag{15}$$

where $\widehat{\boldsymbol{\epsilon}}$ is the vector of residuals from the OLS estimation of the standard linear regression $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, $\widehat{\sigma} = (T-k-1)^{-\frac{1}{2}}\sqrt{\widehat{\boldsymbol{\epsilon}}'\widehat{\boldsymbol{\epsilon}}}$ is the standard error of estimate and $\mathbf{M}_T = \mathbf{I}_T - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is the familiar symmetric idempotent matrix.

As the test statistic, $\lambda_H^E(\boldsymbol{g})$, is distributed as χ_1^2 under the null hypothesis, linearity would be rejected if $\lambda_H^E(\boldsymbol{g})$ exceeded the critical value $\chi_{1,\alpha}^2$ for the chosen level of significance, α.[1]

The usefulness of the Hamilton LM test depends on certain nuisance parameters that are only identified under the alternative hypothesis. As Hansen (1996) shows, dealing with unidentified nuisance parameters by assuming full knowledge of the parameterised stochastic process that determines the random field may have adverse effects on the power of the test. To take account of this, Dahl and González-Rivera (2003) introduce other LM

---

[1]The notation used here for the λ statistic is that of Dahl and González-Rivera (2003). The superscript E shows that full knowledge of the parametric nature of the covariance function is assumed. The alternative is superscript A, which signals that no assumption about the covariance function is assumed. The subscript H shows that the Hessian of the loglikelihood function is used. The alternative is subscript OP, which indicates that the outer product of the score function is used.

tests that extend the Hamilton approach. The first, based on the statistic $\lambda_{OP}^{E}(\boldsymbol{g})$, assumes, like Hamilton's test, knowledge of the covariance matrix, but its behaviour is based on the L_1-norm. The nuisance parameters are still present but now only enter the test in a linear fashion. The second, the λ_{OP}^{A} test, only assumes that the covariance function is smooth enough to be depicted by a Taylor expansion. The final test is a test of the null hypothesis $H_0 : \boldsymbol{g} = 0$; this g_{OP} test makes no assumption about either the covariance function or λ. Dahl and González-Rivera (2003) show that in many circumstances, the λ_{OP}^{A} and g_{OP} tests have better power than other tests of nonlinearity.

The full importance of Hamilton's random field approach is only realised when the parameters λ and $\boldsymbol{g}$ are estimated. In particular, the estimated value of $\boldsymbol{g}$ can be used for inference on the form of the nonlinearity. A highly significant $g_i, i = 1, 2, ..., k$, suggests that the corresponding variable plays an important role in the nonlinearity of the model. Hamilton showed that estimating the unknown parameters $\boldsymbol{\varphi} = \{\alpha_0, \boldsymbol{\alpha}_1, \boldsymbol{g}, \sigma^2, \lambda\}$ can be reduced to maximum likelihood estimation of a reparameterisation of equations (10) and (11):

$$\eta\left(\mathbf{y}, \mathbf{X}; \boldsymbol{g}, \zeta\right) = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln\sigma^2\left(\boldsymbol{g}, \zeta\right) - \frac{1}{2}\ln\left|\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}, \zeta\right)\right| - \frac{T}{2}, \tag{16}$$

and

$$\widetilde{\boldsymbol{\beta}}\left(\boldsymbol{g}, \zeta\right) = \left[\mathbf{X}'\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}, \zeta\right)^{-1}\mathbf{X}\right]^{-1}\left[\mathbf{X}'\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}, \zeta\right)^{-1}\mathbf{y}\right], \tag{17}$$

$$\widetilde{\sigma}^2\left(\boldsymbol{g}, \zeta\right) = \frac{1}{T}\left[\mathbf{y} - \mathbf{X}\widetilde{\boldsymbol{\beta}}\left(\boldsymbol{g}; \zeta\right)\right]'\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}; \zeta\right)^{-1}\left[\mathbf{y} - \mathbf{X}\widetilde{\boldsymbol{\beta}}\left(\boldsymbol{g}; \zeta\right)\right], \tag{18}$$

where $\zeta = \lambda/\sigma$ and $\mathbf{W}\left(\mathbf{X}; \mathbf{g}, \zeta\right) = \zeta^2\mathbf{H}_k + \sigma^2\mathbf{I}_T$. The profile likelihood can be maximised with respect to $(\boldsymbol{g}, \zeta)$ using standard optimisation algorithms, though as Bond, et al. (2005) point out, care needs to be taken because of computational difficulties. Also, as Hamilton (2005) explains, other computational issues make it possible for the nonlinearity tests based on λ to be strongly significant but the results of the nonlinear maximisation of the likelihood function to suggest that ζ is insignificant. Once estimates for $\boldsymbol{g}$ and ζ have been obtained, equations (17) and (18) can be used to obtain estimates of $\boldsymbol{\beta}$ and σ.

The Hamilton (2001) method is concerned with inferring the form of nonlinearity appropriate to a given dataset and can, therefore, aid the specification of a final nonlinear model. While in some cases this may be straightforward, in others, it may lead to the use of further techniques. One such method that may work very well with random field regression is Bai and Perron's (1998, 2003) multiple structural changes approach.

3.5 Multiple structural changes models

The final nonlinear method introduced here is Bai and Perron's (1998, 2003) approach to estimating and testing structural changes models. The usefulness of this approach to exploring Ppp for Ireland, in tandem with random field regression, will become evident in later sections. This approach is based on the multiple linear regression

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{z}_t'\boldsymbol{\delta}_j + u_t, \qquad t = T_{j-1} + 1, \ldots, T_j, \;\; j = 1, \ldots, m+1, \tag{19}$$

where y_t is the observed dependent variable, $\mathbf{x}_t$ is a p-vector of explanatory variables whose corresponding coefficient vector, $\boldsymbol{\beta}$, is not subject to change, $\mathbf{z}_t$ is a q-vector of explanatory variables, whose corresponding coefficient vector $\boldsymbol{\delta}_j$, is subject to change, and u_t is the disturbance term. The model is tested for $T_1 - T_m$ break points. This model can be estimated by least squares, as for each regime the least squares estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\delta}_j$ are found by minimising the sum of squared residuals

$$\min SSR = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} \left[y_t - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{z}_t'\boldsymbol{\delta}_i \right]^2. \tag{20}$$

To specify such a model, Bai and Perron (1998, 2003) propose a range of tests. The $\sup F_T$ test examines the null of no structural breaks ($m = 0$) against $m = k$ breaks. For a partition $(T_1, \ldots, T_k)$, where $T_i = [T\lambda_i]$ and $\lambda_i = T_i/T$, it can be shown that

$$F_T(\lambda_i, \ldots, \lambda_k; q) = \frac{1}{T} \left(\frac{T - (k+1)q - p}{kq} \right) \widehat{\boldsymbol{\delta}}' \mathbf{R}' \left(\mathbf{R}\widehat{\mathbf{V}}(\widehat{\boldsymbol{\delta}})\mathbf{R}' \right)^{-1} \mathbf{R}\widehat{\boldsymbol{\delta}}, \tag{21}$$

where $\mathbf{R}$ is defined such that $(\mathbf{R}\boldsymbol{\delta})' = \left(\widehat{\delta}_1 - \widehat{\delta}_2, \ldots, \widehat{\delta}_k - \widehat{\delta}_{k+1} \right)$ and $\widehat{\mathbf{V}}(\widehat{\boldsymbol{\delta}})$ is an estimate of the variance-covariance of $\widehat{\boldsymbol{\delta}}$. This test can be augmented to provide tests of $l + 1$ breaks against l breaks, as the $\sup F_T(l+1 \mid l)$ test. Two further tests explore the data with a pre-specified number of breaks; the *UD max* and *WD max* procedures test sequentially the hypothesis of m unknown breaks against the null of no break. The *UD max* test is defined as

$$UD\ max\ F_T(M, q) = \max_{1 \geq m \geq M} F_T\left(\widehat{\lambda}_1, \ldots, \widehat{\lambda}_m; q \right). \tag{22}$$

The *WD max* test is similar to the *UD max* test, but applies weights to the individual tests so that the marginal p-values are equal across all values of m.

4 Results and Discussion

Having introduced the main methods to be used in this paper, particularly those likely to be less well known, the paper proceeds to estimate models for the nominal and real exchange rate for Ireland, Germany and the UK. This section introduces the data to be used and reports on some preliminary analysis. The fractional augmented Dickey-Fuller test is then implemented, before a more standard cointegration approach is taken. Nonlinearity tests are then applied to both the nominal and real exchange rates, before random field regressions are estimated. Finally, multiple structural changes models are fitted to the data.

4.1 Data

The explanatory model used throughout this analysis follows Johansen and Juselius (1992), and Wright (1994). The specification is

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \alpha_3 i_t + \alpha_4 i_t^* + \epsilon_t, \tag{23}$$

where, in addition to the variables defined in Section 2, i_t and i_t^* are the domestic and foreign short-term interest rates.[2] The real exchange rate series, $\{q_t\}_{t=1}^T$, is constructed using Equation (1). Wholesale price indices are used in preference to consumer price indices. Wholesale indices offer a better approximation of price developments in the traded sector, and have frequently been employed in PPP studies, as deviations from PPP are less likely in the traded sector.

As previously stated, the data is quarterly for the period 1975 Q1 to 2003 Q3, a total of 115 observations. As the period pre-dates EMS and the break with Sterling, the Sterling/Irish Pound nominal exchange rate is fixed from 1975 until 1978. Likewise, the Deutsche-Mark/Irish Pound rate is fixed from 1999 to the end of the sample, as a result of EMU membership. As discussed earlier, these data clearly span several monetary regimes and crises. Unlike Wright (1994), however, who used data for 1981 to 1992 to avoid regime change and crises, this paper aims to explore the long-run PPP relationship throughout this entire period. Indeed, if nonlinearity in PPP may result from regime change, excluding such data may not prove beneficial.

4.2 Preliminary analysis

To place the long memory and random field analysis into context, the standard $I(1)/I(0)$ analysis using the ADF unit root test was conducted. The strategy of Dolado, et al. (1990), to determine whether the ADF regressions have significant constants or trends, was adopted. The lag length for the ADF test was determined using the modified Akaike information criterion (MAIC), which Ng and Perron (2001) showed to be a generally better decision criteria, as it takes account of the persistence found in many series. The alternative KPSS and NP unit root tests were also applied, the latter being generally more powerful against the alternative of fractional integration than the standard ADF (see Kwiatkowski, et al., 1992 and Perron and Ng, 1996, respectively).

The results of this basic unit root analysis are given in Table 1.[3] In half of the cases, the Dolado, et al. (1990) testing strategy suggests that the existence of a trend in the ADF test regressions, or drift in the series in question, cannot be rejected; the associated probabilities given in Table 1 are therefore from the standard normal distribution. In the other half of the cases, the existence of a constant and trend is rejected so the probabilities given are from MacKinnon (1996).

These results generally seem to suggest that most series are $I(1)$. The performance of the KPSS test, which has a null hypothesis of stationarity, is strange for the Ireland/UK data as the test does not reject this null in three of the six cases. Also, it is interesting that the traditional ADF test rejects the unit root hypothesis for one of the real exchange rates, whereas the 'more powerful' NP test fails to reject for both series.

As the data used here are quarterly, the possibility of seasonal (co)integration arises. Tests for seasonal unit roots, using the Hylleberg, Engle, Granger, and Yoo (1990) test, were computed and are available from the authors on request. In general, results for the series suggest that they are in fact $I(1)$ and that no seasonal integration is present. The

---

[2]The short-term (3-month) interest rates were obtained from EcoWin; the remainder of the series were provided by Jonathan H. Wright. The data are available on request from the authors.

[3]All tables are in the Appendix.

only exception is the Irish price level. As with standard ADF tests, there is some evidence to suggest that this series is $I(0)$.

4.3 Fractional integration analysis

Following this traditional analysis, the issue of fractional integration was investigated. The approach to applying the FADF test suggested by Dolado, et al. (2002), is to obtain a consistent parametric estimate of d and apply the FADF test for this value. The 'over differenced' *ARFIMA* model, which uses the first differences of the observations on a variable rather than the raw levels observations themselves, was estimated to avoid the problems associated with drift, as recommended by Smith, et al. (1997). Two parametric estimates of d were calculated using the Doornik and Ooms (1999) `ARFIMA` package, namely, the exact maximum likelihood (EML) estimate produced by the algorithm suggested by Sowell (1992),[4] and an approximate maximum likelihood estimator based on the conditional sum of squared naïve residuals, developed by Beran (1995) and referred to by Doornik and Ooms (1999) as a nonlinear least squares (NLS) estimator. The nonparametric estimate of d from the logperiodogram method of Geweke and Porter-Hudak (1983) (GPH) and the semiparametric estimate from the Gaussian method (GSP) discussed by Robinson and Henry (1998) are also available in `ARFIMA`; these were also calculated. The estimates of d were then used in the FADF test, with the MAIC being used to set the lag length for the test.

Table 2 gives the results of the simple fractional integration analysis. For each series, four different estimates of d are given, together with their estimated standard errors and associated FADF test statistic values, where computed. The FADF test is only meaningful, and hence reported, if $d \leqslant 1$, when the probabilities to be applied to the test statistics are the standard normal ones. The results are interesting and would seem to imply that the only series that is likely to be unambiguously fractionally integrated is Irish interest rates. While all the estimates of d for the nominal exchange rate between Ireland and the UK are less than one, the FADF test fails to reject the null hypothesis of a unit root. For all other series, the estimates of d gave conflicting values, although the suggestion is of a unit root in the Ireland/UK real exchange rate. The FADF test only gave strong evidence of fractional integration in the case of the Ireland/Germany nominal and real exchange rates when the GPH and GSP estimates of d were used.

4.4 Cointegration analysis

Traditional cointegration analysis was then applied to the simple PPP model of Equation (23). Firstly, the Engle and Granger (1987) two-step procedure was used, with the lagged residuals from the levels regression serving as the error-correction term. Then the Johansen (1988) *VAR* approach was applied to the data. The effect of applying the Johansen (2002) small-sample correction factor was also investigated.

The results of applying the standard Engle-Granger analysis in the context of model (23) are given in tables 3 and 4. Table 3 reports the findings of the levels analysis and

---

[4]The Sowell algorithm requires that $d < 0.5$, which is another reason for using the 'over-differenced' model.

in all cases both the traditional ADF test on residuals (augmented Engle-Granger test) and the NP test fail to reject the null hypothesis that the residuals have a unit root. The KPSS test also rejects the null of stationary residuals in all but one case. Therefore, treating the variables as $I(1)$, it seems that cointegration of the nominal exchange rate, price levels and interest rates is overwhelmingly rejected for both the Ireland/UK and the Ireland/Germany data. These results are confirmed by the findings of CRDW tests.

Table 4 gives the results of trying to estimate parsimonious error-correction models, using the first lag of the residuals from the corresponding levels model as the error-correction term in each of the two cases. While the coefficients of the error-correction terms have the 'right' sign, the t-ratios are small in absolute value, confirming the conclusion about the lack of cointegration. The ECM test also rejects cointegration in all cases. Dropping the insignificant constant terms has a minimal effect on the results.

Table 5 shows evidence of one cointegrating vector in the Ireland/Germany case, when interest rates are excluded from the equation. Importantly, this result is overturned by the trace test when Johansen's small-sample correction to that test is applied. However, when interest rates are included, one cointegrating vector is suggested whether or not the small-sample correction is used. In this case, the trace and maximal eigenvalue tests concur. Table 6 presents the results for the Ireland/UK relationship. As with the previous case, the finding of one cointegrating vector in the specification without interest rates is overturned by the adjusted trace test. In contrast, two vectors are suggested when the interest rates are included, and this result is unaffected by the small-sample correction factor, which strangely is less than 1.

Taken together, the results so far are rather mixed and indicate that there is little evidence of cointegration in a traditional PPP setting, but that the introduction of interest rates appears to be significant. Overall, as in previous studies, this attempt to place the PPP analysis of Irish data in a cointegrating framework is not entirely satisfactory. We therefore turn to the results from the alternative nonlinear methodologies.

4.5   Nonlinearity tests

The analysis next considered the possibility of nonlinearity in the data. For the causal models, the standard RESET test was applied, together with the random field-based tests described above. Also, for an autoregressive model involving q_t, the now standard STAR tests for nonlinearity were applied, as discussed previously.

Tables 7 and 8 give the results of the various nonlinearity tests. In all tests, the null hypothesis is that the model/series is linear. For the RESET test, both the F and LR variants are given. For the STAR nonlinearity test, an F-test version is used, with F being the test statistic for H_0 and $F4$, $F3$ and $F2$ being, respectively, the test statistics for the hypotheses H_{04}, H_{03} and H_{02}, specified previously. The AIC suggested a lag length of three for the STAR test in the case of the Ireland/Germany exchange rate and a lag length of two for the Ireland/UK case. The SIC suggested a lag length of one in both cases.

As can be seen from Table 7, the RESET test and the four random field-based tests emphatically reject linearity at the 5 per cent significance level in the case of the Ireland/Germany model. For the Ireland/UK model, however, there is a marked contrast

between the findings from the two test approaches, with the RESET test failing to reject linearity but all of the random field tests strongly rejecting it.

Table 8 contains similar, though opposite findings. The RESET test, STAR tests and random field-based tests all suggest that the assumption of linearity is adequate for the Ireland/UK real exchange rate taken on its own; but whereas the random field tests overwhelmingly support linearity of the Ireland/Germany real exchange rate, the STAR test based on the use of three lags gives some indications of nonlinearity and the RESET test rejects linearity very strongly. It is difficult to explain these conflicting outcomes in tables 7 and 8, especially in the absence of information on the relative power of the different types of test.[5] Nonetheless, there is limited evidence of nonlinearity in the real exchange rate. This suggests that following a STAR approach may not be optimal. The remainder of the paper, therefore, concentrates on modelling the nominal exchange rate.

4.6 Random field estimation

Given the results of the nonlinearity tests, the parameters of the random field model were estimated for the nominal exchange rate. The GAUSS code provided by Hamilton (2001) was adapted to apply the algorithm switching approach to the numerical optimisation suggested by Bond, et al. (2005).[6] Specifically, algorithm switching between the *Steepest Descent* and *Newton* methods were employed.

Given that the bulk of the results in Table 7 suggest that the linear equation used in the analysis of PPP is not an appropriate specification, interest focuses on the results of the nonlinear estimation of the random field regression. These are given in Table 9. Interestingly, in the case of both country pairings, the standard model and the augmented model exhibit nonlinearity with respect to the two price variables, the price coefficients in the nonlinear component of the models being highly significant. However, in the augmented Ireland/Germany model, the German interest rate is nonlinearly significant, while in the Ireland/UK model it is the Irish interest rate that appears to have a significantly nonlinear influence on the nominal exchange rate.

Most strikingly, perhaps, is the fact that when nonlinearity is modelled by means of a random field, the coefficients on the domestic and foreign prices in the specifications with and without interest rates, are not statistically significantly different from their -1 and 1 values under purchasing power parity theory. This finding contrasts with the findings in the earlier Irish studies by, for example, Thom (1989) and Wright (1994), both of whom report cointegrating vectors, corresponding to the vector of variables s_t, p_t and p_t^*, that are markedly different from (1, -1, 1).

These results have found significant nonlinearity and attributed that nonlinearity to certain variables. The next stage was to infer a suitable nonlinear model. As three variables have been found to be nonlinearly significant in each case, Hamilton's (2001) approach to inference, using the conditional expectation function to infer nonlinearity, is not possible. An alternative approach suggested by Bond, et al. (2007b) is therefore

---

[5]In particular, no results appear to be available on the power of the RESET test relative to random-field based LM tests for nonlinearity. This is a subject of ongoing research and the findings will be presented in a forthcoming paper.

[6]Hamilton's (2001) GAUSS code is available at http://weber.ucsd.edu/~jhamilto/.

used. This approach exploits the fact that the random field estimation consists of two components: a linear and a nonlinear term. In the context of PPP, these two components can be viewed as a linear long-run approximation to PPP over the sample period and a nonlinear dynamic or deviation component.

The procedure outlined above was applied to the Irish/German data.[7] An estimate of the linear term, $\alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$, was plotted as the 'fitted' term along with the actual dependent variable against time. This is shown in Figure 1. As can be seen, the fit here is reasonably good, and underlines how $\alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$ can be viewed as a linear long-run approximation to PPP. Figure 2 plots the 'residual' of this, as the difference between actual and linear fitted observations. Several breaks are clearly evident from this plot, particularly around 1978, 1986, and 1996. To infer the form of nonlinearity which may account for these breaks, the residuals were plotted against the three significantly nonlinear variables, respectively.[8] Clear evidence of regimes was found in these plots. They corresponded approximately to breaks at 1978, 1986, 1990 and 1996.

It should be noted at this point that the break dates suggested by the Hamilton approach are very much in line with monetary developments affecting the Irish nominal exchange rate. The year 1978 saw the end of the peg to Sterling and the commencement of EMS the following year, the Irish currency was devalued in 1986 and in 1989-1990, the UK joined EMS and Germany re-unified. The final break, 1996, may relate to the introduction of the new exchange rate mechanism around that time, in preparation for EMU.

4.7   Multiple structural changes models

Based on these findings, break-date tests and time-varying parameter estimation, following Bai and Perron (1998, 2003), were used.[9] The sample size was truncated for computational reasons, however, to remove the period of fixed exchange rates under EMU. Table 10 shows the results of this approach for Ireland/Germany. Four significant breaks are identified at 1978 Q2, 1986 Q2, 1990 Q3 and 1995 Q3. The $\sup F_T(l)$, $\sup F_T(l+1 \mid l)$, *UD max* and *WD max* tests are all significant at the 5 per cent level for four breaks. Figure 3 shows a plot over time of actual versus fitted s_t. The plot is based on estimates from the time-varying parameter model and is much improved on that seen in Figure 1. Even more noteworthy are the coefficients reported in Table 10. In three out of five regimes, the coefficients for p_t and p_t^* are not statistically significantly different from -1 and 1, the values predicted by theory. For the second regime, coefficients of -0.725 and 0.813, are statistically significantly different from 1, yet remain plausible in magnitude. It is only for the fourth regime that the parameter estimates deviate substantially from theory, at approximately ± 2. This regime is for the period 1990 Q3 to 1995 Q3, and the results remain to be explained. There is some limited evidence of a further break at 1993, but this was not found using the Bai and Perron approach.[10] Recall also that this period can be characterised as one of crisis for EMS, and this may go some way to explaining this

---

[7]For this analysis, the data sample was truncated to exclude the period of fixed exchange rates under EMU. The motivation for this will become clear in the next section.

[8]While not reported here, these plots are available from the authors on request.

[9]The GAUSS code to implement these techniques is available from http://people.bu.edu/perron/code.html.

[10]The Irish currency devalued relative to ECU in 1993.

16

result. Nevertheless, these findings do not detract greatly from the overall results, which suggest that Ppp does in fact hold for Ireland, in both the medium and long run.

A similar approach was undertaken for the UK, the results of which are available from the authors on request. Although the fit achieved and the coefficients obtained were not as noteworthy as in the German case, the results are nevertheless encouraging. It appears that modelling Ppp for Ireland, Germany and the UK is best done with time-varying parameter models. The breaks found using this method in tandem with the random field approach, are as stated previously, very much in line with monetary developments. The failure to 'find' a break a 1993 may result from the fact that this was a period of crisis, making it difficult to separate the effects of the Irish devaluation from volatility in the other series; recall, for example, that around this time the UK devalued its currency and then exited Ems.

These results should not be surprising. As mentioned previously, official intervention by monetary authorities in the foreign exchange market has been proposed as a source of potential nonlinearity in the Ppp relationship, and several authors have suggested that this is in line with Ppp theory. Several authors have also found evidence to support these findings. Using Irish data, Lane and Milesi-Ferretti (2002) found evidence of time-varying parameters, albeit for the real exchange rate. Lahtinen (2006), using a model that allowed for adjustments towards long-run equilibrium, found that adjustment was sudden rather than smooth for the Dollar/Euro exchange rate. Such sudden adjustment may result from market intervention, as appears to have been the case here. Finally, Sager (2006), using three major exchange rates, also found shocks to be important and that there was no benefit in modelling Ppp as a nonlinear process once those shocks were accounted for.

5   Summary and Conclusions

This paper has empirically modelled the nominal exchange rate for Ireland, relative to Germany and the UK, from 1975 to 2003. It has used new approaches, yet to be applied in this area: the fractional augmented Dickey-Fuller test, random field regression and multiple structural changes models. It has shown that Ppp can be effectively modelled for those bilateral exchange rates by using such structural changes models.

The theoretical background to Ppp has been sketched, paying particular attention to recent advances in the literature concerning nonlinearity and its likely causes in Ppp. Importantly, the link between fractional integration and data aggregation has been highlighted, as a source of potential deviation from Ppp that has been previously overlooked. Investigating the occurrence of fractionality in aggregated time series represents an interesting agenda for future research.

A battery of unit root tests was applied and found that most series could be characterised as nonstationary. The fractional augmented Dickey-Fuller test, not before used in this area, was also applied. Little evidence of fractionality was found, however, indicating that there was no persistent deviation in the real exchange rate from its Ppp equilibrium.

Attempts to model the nominal exchange rate used standard cointegration techniques, including both the Crdw and Ecm tests along with the more standard Engle-Granger and Johansen approaches. Using a similar approach to Johansen and Juselius (1992), this

illustrated the potential difficulties inherent in placing the study of PPP in the $I(1)/I(0)$ framework. These difficulties were implicit in the very mixed results of previous Irish studies using this approach, an overview of which was provided. The implementation of Johansen's (2002) correction highlighted the need for caution when using small samples, as the correction factor had a significant impact on inference regarding the number of cointegrating vectors found.

Nonlinearity was then tested using a range of approaches. Although these produced varying results, the random field-based tests strongly indicated nonlinearity, while the STAR-based tests were much more ambiguous, frequently failing to reject linearity. It should be borne in mind that the STAR procedure tests the null of linearity against an alternative of threshold nonlinearity, whereas the random field-based methods test a null of linearity against an alternative of nonspecific nonlinearity. These results suggested that there was little if any nonlinearity in the real exchange rates. This, taken with the evidence of the FADF tests, suggested that modelling the real exchange rate as a long memory or nonlinear process was not warranted in these cases. The remainder of the paper concentrated on the nominal exchange rate, therefore.

Given the findings of nonlinearity in the nominal exchange rate, random field regressions, which had been outlined previously, were estimated. These produced striking results; the estimated coefficients of the linear component of the model were not significantly different from those expected under PPP and both price indices were found to be nonlinearly significant in each case. This further underlines the difficulties likely to be encountered with a STAR approach here, as there are two, if not more transition variables. Specification of STAR models in such cases is not straightforward, although this is the subject of ongoing research.

It was clear from the random field regression that although a series of significant breaks occurred in the data, the long-run approximation to PPP derived from the estimation was reasonable. The breaks were found to coincide accurately with monetary developments in the economies in question, and these results suggested that a multiple structural changes model may be appropriate for both bilateral exchange rates. Using Bai and Perron's (1998, 2003) approach, structural changes models were estimated and break dates tested. Interestingly, this approach found very similar breaks to those found previously, and these were highly statistically significant. The estimated coefficients from these models were also very close to those theoretically predicted by PPP in the case of Ireland/Germany. The good fit achieved by this model was also noteworthy.

These results provide strong evidence for nonlinearity in the PPP relationship for these data, resulting from monetary developments. This supports the theory that shocks relating to official intervention in the foreign exchange market may result in nonlinearity, but that when such shocks are modelled, the PPP relationship is linear. This certainly appears to be the case for the Ireland/Germany data, as PPP holds in some of the short periods between structural changes. It remains to be seen whether similar findings to these apply to other currencies.

References

Baharumshah, A. Z. and Liew, V. K.-S. (2006). "Forecasting Performance of Exponential Smooth Transition Autoregressive Exchange Rate Models", Open Economies Review, Vol. 17, No. 2, pp. 235-251.

Bai, J. and Perron, P. (1998). "Estimating and Testing Linear Models with Multiple Structural Changes", Econometrica, Vol. 66, No. 1, pp. 47-78.

Bai, J. and Perron, P. (2003). "Computation and Analysis of Multiple Structural Changes Models", Journal of Applied Econometrics, Vol. 18, No. 1, pp. 1-22.

Beran, J. (1995). "Maximum Likelihood Estimation of the Differencing Parameter for Invertible Short and Long Memory Autoregressive Integrated Moving Average Models", Journal of the Royal Statistical Society, Series B, Vol. 57, No. 4, pp. 659-672.

Bini-Smaghi, L. and Ferri, G. (2006). "Revisiting the European Monetary System Experience: Were Some Members More Equal Than Others?", Economic Notes, Vol. 35, No. 2, pp. 151-172.

Bond, D., Harrison, M. J., Hession, N. and O'Brien, E. J. (2006). "Some Empirical Observations on the Forward Exchange Rate Anomaly", Trinity Economic Papers, No. 2.

Bond, D., Harrison, M. J. and O'Brien, E. J. (2005). "Investigating Nonlinearity: A Note on the Estimation of Hamilton's Random Field Regression Model", Studies in Nonlinear Dynamics and Econometrics, Vol. 9, No. 3, Article 2.

Bond, D., Harrison, M. J. and O'Brien, E. J. (2007a). "Demand for Money: A Study in Testing Time Series for Long Memory and Nonlinearity", The Economic and Social Review, Vol. 38, No. 1, pp. 1-24.

Bond, D., Harrison, M. J. and O'Brien, E. J. (2007b). "Exploring Nonlinearity With Random Field Regression", Applied Economics Letters, forthcoming.

Bradley, J. (1977). "Lags in Transmission of Inflation", The Economic and Social Review, Vol. 8, No. 2, pp. 149-154.

Callan, T. and Fitzgerald, J. (1989). "Price Determination in Ireland: Effects of Changes in Exchange Rates and Exchange Rate Regimes", The Economic and Social Review, Vol. 20, No. 2, pp. 165-188.

Cashin, P. and McDermott, C. J. (2004). "Parity Reversion in Real Exchange Rates: Fast, Slow or Not at All?", International Monetary Fund Working Papers, No. 128.

Cheung, Y. W. and Lai, K. S. (1993). "A Fractional Cointegration Analysis of Purchasing Power Parity", Journal of Business & Economic Statistics, Vol. 11, No. 1, pp. 103-112.

Dahl, C. M. (2002). "An Investigation of Tests for Linearity and the Accuracy of Likelihood Based Inference Using Random Fields", Econometrics Journal, Vol. 5, No. 2, pp. 263-284.

Dahl, C. M. and González-Rivera, G. (2003). "Testing for Neglected Nonlinearity in Regression Models Based on the Theory of Random Fields", Journal of Econometrics, Vol. 114, No. 1, pp. 141-164.

Dickey, D. A. and Fuller, W. A. (1981). "Likelihood Ratio Tests for Autoregressive Time Series with a Unit Root", Econometrica, Vol. 49, No. 4, pp. 1057-1072.

Diebold, F. X., Husted, S. and Rush, M. (1991). "Real Exchange Rates Under the Gold Standard", Journal of Political Economy, Vol. 99, No. 6, pp. 1252-1271.

Dolado, J. J., Gonzalo, J. and Mayoral, L. (2002). "A Fractional Dickey-Fuller Test for Unit Roots", Econometrica, Vol. 70, No. 5, pp. 1963-2006.

Dolado, J. J., Gonzalo, J. and Mayoral, L. (2005). "What is What?: A Simple Test of Long Memory vs Structural Breaks in the Time Domain", Working Paper, Universitat Pompeu Fabra.

Dolado, J. J., Jenkinson, T. and Sosvilla-Rivero, S. (1990). "Cointegration and Unit Roots", Journal of Economic Surveys, Vol. 4, No. 3, pp. 249-273.

Doornik, J. A. and Ooms, M. (1999). "A Package for Estimating, Forecasting and Simulating $ARFIMA$ Models: $ARFIMA$ Package 1.0 for Ox", Discussion Paper, Nuffield College, University of Oxford.

Engle, R. F. and Granger, C. W. J. (1987). "Cointegration and Error Correction: Representation, Estimation and Testing", Econometrica, Vol. 55, No. 2, pp. 251-276.

Geweke, J. and Porter-Hudak, S. (1983). "The Estimation and Application of Long Memory Time Series Models", Journal of Time Series Analysis, Vol. 4, No. 4, pp. 221-238.

Gil-Alana, L. A. (2004). "A Joint Test of Fractional Integration and Structural Breaks at a Known Period of Time", Journal of Time Series Analysis, Vol. 25, No. 5, pp. 691-700.

Granger, C. W. J. (1980). "Long Memory Relationships and the Aggregation of Dynamic Models", Journal of Econometrics, Vol. 14, No. 2, pp. 227-238.

Hamilton, J. D. (2001). "A Parametric Approach to Flexible Nonlinear Inference", Econometrica, Vol. 69, No. 3, pp. 537-573.

Hamilton, J. D. (2005). "Comment on 'Investigating Nonlinearity'", Studies in Nonlinear Dynamics and Econometrics, Vol. 9, No. 3, Article 3.

Hansen, B. E. (1996). "Inference When a Nuisance Parameter is Not Identified Under the Null Hypothesis", Econometrica, Vol. 64, No. 2, pp. 413-430.

Hong, S. H. and Phillips, P. C. B. (2005). "Testing Linearity in Cointegrating Relations with an Application to Purchasing Power Parity", Cowles Foundation, Yale University, Discussion Papers, No. 1541.

Honohan, P. and Leddin, A. (2005). "Ireland in EMU: More Shocks, Less Insulation", The Economic and Social Review, Vol. 37, No. 2, pp. 263-294.

Hylleberg, S., Engle, R. F., Granger, C. W. J. and Yoo, B. S. (1990). "Seasonal Integration and Cointegration", Journal of Econometrics, Vol. 44, No. 1-2, pp. 215-248.

Imbs, J., Mumtaz, H., Ravn, M. and Rey, H. (2005). "PPP Strikes Back: Aggregation and the Real Exchange Rate", The Quarterly Journal of Economics, Vol. 120, No. 1, pp. 1-43.

Johansen, S. (1988). "Statistical Analysis of Cointegration Vectors", Journal of Economic Dynamics and Control, Vol. 12, No. 2-3, pp. 231-254.

Johansen, S. (2002). "A Small Sample Correction for the Test of Cointegrating Rank in the Vector Autoregressive Model", Econometrica, Vol. 70, No. 5, pp. 1929-1961.

Johansen, S. and Juselius, K. (1992). "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for UK", Journal of Econometrics, Vol. 53, No. 1-3, pp. 211-244.

Kenny, G. and McGettigan, D. (1999). "Modelling Traded, Non-traded and Aggregate Inflation in a Small Open Economy: The Case of Ireland", The Manchester School, Vol. 67, No. 1, pp. 60-88.

Kilian, L. and Taylor, M. P. (2003). "Why is it So Difficult to Beat the Random Walk Forecast of Exchange Rates", Journal of International Economics, Vol. 60, No. 1, pp. 85-107.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root", Journal of Econometrics, Vol. 54, No. 1-3, pp. 159-178.

Lahtinen, M. (2006). "The Purchasing Power Parity Puzzle: A Sudden Nonlinear Perspective", Applied Financial Economics, Vol. 16, No. 1-2, pp. 119-125.

Lane, P. R. and Milesi-Ferretti, G. M. (2002). "Long-run Determinants of the Irish Real Exchange Rate", Applied Economics, Vol. 34, No. 5, pp. 549-553.

Lee, Y. S., Kim, T.-H. and Newbold, P. (2005). "Spurious Nonlinear Regression in Econometrics", Economics Letters, Vol. 87, No. 3, pp. 301-306.

MacDonald, R. and Taylor, M. P. (1992). "Exchange Rate Economics: A Survey", International Monetary Fund Staff Papers, Vol. 39, No. 1, pp. 1-57.

MacKinnon, J. G. (1996). "Numerical Distribution Functions for Unit Root and Cointegration Tests", Journal of Applied Econometrics, Vol. 11, No. 6, pp. 601-618.

Mayoral, L. (2005). "Is the Observed Persistence Spurious or Real? A Test for Fractional Integration vs Short Memory and Structural Breaks", Working Paper, Universitat Pompeu Fabra.

Ng, S. and Perron, P. (2001). "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power", Econometrica, Vol. 69, No. 6, pp. 1519-1554.

Papell, D. H. (2006). "The Panel Purchasing Power Parity Puzzle", Journal of Money, Credit and Banking, Vol. 38, No. 2, pp. 447-467

Perron, P. (1989). "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis", Econometrica, Vol. 57, No. 6, pp. 1361-1401.

Perron, P. and Ng, S. (1996). "Useful Modifications to Some Unit Root Tests with Dependent Errors and their Local Asymptotic Properties", Review of Economic Studies, Vol. 63, No. 3, pp. 435-463.

Robinson, P. M. and Iacone, F. (2005). "Cointegration in Fractional Systems with Deterministic Trends", Journal of Econometrics, Vol. 127, No. 1-2, pp. 263-298.

Robinson, P. M. and Henry, M. (1998). "Long and Short Memory Conditional Heteroscedasticity in Estimating the Memory Parameter of Levels", Econometric Theory, Vol. 15, No. 3, pp. 299-336.

Rogoff, K. (1996). "The Purchasing Power Parity Puzzle", Journal of Economic Literature, Vol. 34, No. 2, pp. 647-668.

Sager, M. (2006). "Explaining the Persistence of Deviations from PPP: A Nonlinear Harrod-Balassa-Samuelson Effect?", Applied Financial Economics, Vol. 16, No. 1-2, pp. 41-61.

Sarno, L. (2005). "Towards a Solution to the Puzzles in Exchange Rate Economics: Where Do We Stand?", Canadian Journal of Economics, Vol. 38, No. 3, pp. 673-708.

Sarno, L. and Taylor, M. P. (2001). "Official Intervention in the Foreign Exchange Market: Is it Effective and, if So, How Does it Work?", Journal of Economic Literature, Vol. 39, No. 3, pp. 839-868.

Sarno, L. and Taylor, M. P. (2002). "Purchasing Power Parity and the Real Exchange Rate", International Monetary Fund Staff Papers, Vol. 49, No. 1, pp. 65-105.

Schnatz, B. (2006). "Is Reversion to PPP in Euro Exchange Rates Non-linear?", European Central Bank Working Paper Series, No. 682

Smith, A. A., Sowell, F. and Zin, S. E. (1997). Fractional Integration with Drift: Estimation in Small Samples, Empirical Economics, Vol. 22, No. 1, pp. 103-116.

Sowell, F. (1992). "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models", Journal of Econometrics, Vol. 53, No. 1-3, pp. 165-188.

Taylor, A. M. (2001). "Potential Pitfalls for the Purchasing Power Parity Puzzle? Sampling and Specification Biases in Mean Reversion Tests of the Law of One Price", Econometrica, Vol. 69, No. 2, pp. 473-498.

Taylor, A. M. (2002). "A Century of Purchasing Power Parity", The Review of Economics and Statistics, Vol. 84, No. 1, pp. 139-150.

Taylor, A. M. and Taylor, M. P. (2004). "The Purchasing Power Parity Debate", Journal of Economic Perspectives, Vol. 18, No. 4, pp. 135-158.

Taylor, M. P. (2004). "Is Official Exchange Rate Intervention Effective?", Economica, Vol. 71, No. 281, pp. 1-12.

Taylor, M. P. (2005). "Official Foreign Exchange Intervention as a Coordinating Signal in the Dollar-Yen Market", Pacific Economic Review, Vol. 10, No. 1, pp. 73-82.

Taylor, M. P. (2006). "Real Exchange Rates and Purchasing Power Parity: Mean-Reversion in Economic Thought", Applied Financial Economics, Vol. 16, No. 1-2, pp. 1-17.

Taylor, M. P. and Peel, D. A. (2000). "Nonlinear Adjustment, Long-run Equilibrium and Exchange Rate Fundamentals", Journal of International Money and Finance, Vol. 19, No. 1, pp. 33-53.

Taylor, M. P., Peel, D. A. Sarno, L. (2001). "Nonlinear Mean-Reversion in Real Exchange Rates: Towards a Solution of the Purchasing Power Parity Puzzle", International Economic Review, Vol. 42, No. 4, pp. 1015-1042.

Teräsvirta, T. (1994). "Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models", Journal of the American Statistical Association, Vol. 89, No. 425, pp. 208-218

Thom, R. (1989). "Real Exchange Rates, Cointegration and Purchasing Power Parity: Irish Experience in the EMS", The Economic and Social Review, Vol. 20, No. 2, pp. 147-163.

Villeneuve, J.F. and Handa, J. (2006). "Purchasing Power Parity as a Long-term Memory Process: Evidence From Canada", Applied Financial Economics, Vol. 16, No. 1-2, pp. 109-117.

Wright, J. (1994). "A Cointegration Based Analysis of Irish Purchasing Power Parity Relationships using the Johansen Procedure", The Economic and Social Review, Vol. 25, No. 3, pp. 261-278.

A   Appendix

A.1   Tables

Table 1: Unit Root Tests.

Variables	Adf	P-value	No. of Lags	Kpss[a]	Np[a]
	IRELAND & GERMANY				
NOMINAL EXCHANGE RATE	-1.119	0.266	7	Yes	No
IRISH PRICE LEVEL	-2.155	0.034	4	Yes	No
GERMAN PRICE LEVEL	-1.933	0.056	2	Yes	No
IRISH INTEREST RATE	-1.085	0.250[b]	2	Yes[c]	No[c]
GERMAN INTEREST RATE	-0.936	0.309[b]	1	Yes	No
REAL EXCHANGE RATE	-3.543	0.000	2	Yes	No
	IRELAND & UNITED KINGDOM				
NOMINAL EXCHANGE RATE	-1.221	0.203[b]	0	No	No
IRISH PRICE LEVEL	-2.155	0.034	4	Yes	No
UK PRICE LEVEL	-1.722	0.088	8	Yes	No
IRISH INTEREST RATE	-1.085	0.250[b]	2	Yes[c]	No[c]
UK INTEREST RATE	-0.645	0.436[b]	10	No	No
REAL EXCHANGE RATE	-1.103	0.240[b]	2	No	No

a Yes - significant at 5 per cent level. No - not significant at 5 per cent level.
b Trend and constant not included. MacKinnon (1996) p-values used.
c Not significant at 1 per cent level.

Variables	Eml	Nls	Gph	Gsp
			Fadf	
Common Series				
Irish Price Level	1.46 (0.04)	1.50 (0.07)	1.01 (0.11)	0.89 (0.07)
	-	-	-	4.5
Irish Interest Rate	0.79 (0.10)	0.78 (0.10)	0.97 (0.10)	0.80 (0.06)
	-3.22	-3.21	-3.35	-3.23
Ireland & Germany				
Nominal Exchange Rate	1.49 (0.14)	1.89 (0.10)	0.94 (0.11)	0.82 (0.07)
	-	-	-5.48	-5.51
German Price Level	1.46 (0.05)	1.57 (0.09)	1.02 (0.11)	0.92 (0.07)
	-	-	-	2.89
German Interest Rate	0.69 (0.24)	0.65^a (0.23)	1.12 (0.11)	1.03 (0.07)
	-1.49	-1.48	-	-
Real Exchange Rate	1.41 (0.08)	1.48 (0.08)	0.98 (0.11)	0.85 (0.07)
	-	-	-5.05	-5.12
Ireland & United Kingdom				
Nominal Exchange Rate	0.95 (0.09)	0.95 (0.09)	0.88 (0.11)	0.91 (0.07)
	-1.60	-1.60	-1.608	-1.60
UK Price Level	1.48 (0.02)	1.55 (0.06)	0.99 (0.11)	0.87 (0.07)
	-	-	5.03	4.69
UK Interest Rate	1.07 (0.09)	1.08 (0.10)	1.00 (0.11)	0.94 (0.07)
	-	-	-	-2.53
Real Exchange Rate	1.07 (0.09)	1.08 (0.09)	1.15 (0.11)	0.97 (0.07)
	-	-	-	-1.09

a Trend and constant not included. McKinnon (1996) p-values used.

- Indicates Fadf test not applicable.

Note: standard errors in parentheses.

Table 3: $I(1)/I(0)$ Levels Regression Analysis.

VARIABLES	IRELAND & GERMANY		IRELAND & UNITED KINGDOM	
CONSTANT	2.854 (0.549)	1.804 (0.575)	0.859 (0.108)	0.833 (0.108)
PRICE LEVELS				
IRISH	−0.568 (0.083)	−0.672 (0.081)	−0.875 (0.111)	−1.029 (0.123)
FOREIGN	0.007 (0.200)	0.329 (0.203)	0.670 (0.095)	0.825 (0.110)
INTEREST RATES				
IRISH		0.005 (0.002)		0.007 (0.003)
FOREIGN		0.002 (0.003)		−0.003 (0.003)
AEG	−2.475 [−3.817]	−2.835 [−4.540]	−2.653 [−3.817]	−2.728 [−4.540]
CRDW test	0.186 [0.48]	0.245 [0.68]	0.239 [0.48]	0.250 [0.68]
NG-PERRON[a]	No	No	No	No
KPSS[a]	No	Yes[b]	Yes[b]	Yes[b]

a Yes - significant at 5 per cent level. No - not significant at 5 per cent level.
b Significant at 5 per cent level but not the 1 per cent level.
Note: standard errors in round brackets; 5 per cent AEG and CRDW
    critical values in square brackets.

Table 4: Error Correction Analysis.

Variables	Ireland & Germany		Ireland & United Kingdom	
Constant	−0.004 (−0.003)	−0.004 (−0.003)	0.004 (0.005)	0.001 (0.004)
Δ Price Levels				
Irish	−0.686 (0.157)	−0.667 (0.164)	−1.105 (0.282)	−1.020 (0.284)
Foreign	1.021 (0.428)	0.927 (0.502)	0.831 (0.361)	0.715 (0.357)
Δ Interest Rates				
Irish		0.0004 (0.001)		0.005 (0.001)
Foreign		0.001 (0.004)		0.00006 (0.003)
ECM	−0.108 (0.039)	−0.107 (0.040)	−0.133 (0.049)	−0.124 (0.052)
ECM test critical values	−3.244 [0.134]	−3.787 [0.326]	−3.244 [0.148]	−3.787 [0.444]

Note: standard errors in round brackets; p-values in square brackets.

Table 5: Johansen Results for Ireland & Germany.

Hypotheses		Test Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value

Johansen Results for Ireland & Germany excluding Interest Rates

Cointegration Rank Test (Trace)[a]

$r = 0$	$r \geq 1$	39.203	34.870	31.930	45.68[c]
$r \leq 1$	$r \geq 2$	13.347	20.180	17.880	-
$r \leq 2$	$r = 3$	5.903	9.160	7.530	-

Cointegration Rank Test (Maximum Eigenvalue)[a]

$r = 0$	$r = 1$	25.856	22.040	19.860	
$r \leq 1$	$r = 2$	7.444	15.870	13.810	
$r \leq 2$	$r = 3$	5.903	9.160	7.530	

Johansen Results for Ireland & Germany including Interest Rates

Cointegration Rank Test (Trace)[b]

$r = 0$	$r \geq 1$	111.587	87.170	82.880	98.328[d]
$r \leq 1$	$r \geq 2$	57.298	63.000	59.160	-
$r \leq 2$	$r \geq 3$	31.448	42.340	39.340	-
$r \leq 3$	$r \geq 4$	15.809	25.770	23.080	-
$r \leq 4$	$r = 5$	6.057	12.390	10.550	-

Cointegration Rank Test (Maximum Eigenvalue)[b]

$r = 0$	$r = 1$	54.290	37.860	35.040	
$r \leq 1$	$r = 2$	25.850	31.790	29.130	
$r \leq 2$	$r = 3$	15.639	25.420	23.100	
$r \leq 3$	$r = 4$	9.751	19.220	17.180	
$r \leq 4$	$r = 5$	6.057	12.390	10.550	

a Cointegration with restricted intercepts and no trends in the *VAR*.
b Cointegration with unrestricted intercepts and restricted trends in the *VAR*.
c The correction factor is 1.310.
d The correction factor is 1.128.

Table 6: Johansen Results for IRELAND & UK.

HYPOTHESES		TEST STATISTIC	0.05 CRITICAL VALUE	0.10 CRITICAL VALUE	MODIFIED 0.05 CRITICAL VALUE

Johansen Results for IRELAND & UK excluding Interest Rates

Cointegration Rank Test (Trace)[a]

$r = 0$	$r \geq 1$	57.532	42.340	39.340	70.030[b]
$r \leq 1$	$r \geq 2$	21.695	25.770	23.080	-
$r \leq 2$	$r = 3$	4.788	12.390	10.550	-

Cointegration Rank Test (Maximum Eigenvalue)[a]

$r = 0$	$r = 1$	35.838	25.420	23.100	
$r \leq 1$	$r = 2$	16.907	19.220	17.180	
$r \leq 2$	$r = 3$	4.788	12.390	10.550	

Johansen Results for IRELAND & UK including Interest Rates

Cointegration Rank Test (Trace)[a]

$r = 0$	$r \geq 1$	127.997	87.170	82.880	85.427[c]
$r \leq 1$	$r \geq 2$	77.194	63.000	59.160	61.740[c]
$r \leq 2$	$r \geq 3$	41.665	42.340	39.340	41.493[c]
$r \leq 3$	$r \geq 4$	21.103	25.770	23.080	-
$r \leq 4$	$r = 5$	4.707	12.390	10.550	-

Cointegration Rank Test (Maximum Eigenvalue)[a]

$r = 0$	$r = 1$	50.803	37.860	35.040	
$r \leq 1$	$r = 2$	35.530	31.790	29.130	
$r \leq 2$	$r = 3$	20.562	25.420	23.100	
$r \leq 3$	$r = 4$	16.395	19.220	17.180	
$r \leq 4$	$r = 5$	4.707	12.390	10.550	

a Cointegration with unrestricted intercepts and restricted trends in the *VAR*.
b The correction factor is 1.654.
c The correction factor is 0.980.

Table 7: Nonlinearity Tests - Causal Models.

Test	Test Statistic	P-value	Bootstrap p-value	Test Statistic	P-value	Bootstrap p-value
	Ireland & Germany			Ireland & United Kingdom		
Reset						
	excluding interest rates					
F	35.04	0.000		0.948	0.431	
Lr	77.646	0.000		3.969	0.414	
	including interest rates					
F	24.474	0.000		0.882	0.477	
Lr	60.085	0.000		3.765	0.439	
Random Field						
	excluding interest rates					
$\lambda_H^E(\boldsymbol{g})$	575.388	0.000	0.001	648.928	0.000	0.001
λ_{OP}^A	324.321	0.000	0.001	151.160	0.000	0.001
$\lambda_{OP}^E(\boldsymbol{g})$	233.907	0.000	0.001	233.152	0.000	0.001
g_{OP}	11.380	0.044	0.001	104.661	0.000	0.001
	including interest rates					
$\lambda_H^E(\boldsymbol{g})$	179.66	0.000	0.001	205.475	0.000	0.001
λ_{OP}^A	224.382	0.000	0.001	545.731	0.000	0.001
$\lambda_{OP}^E(\boldsymbol{g})$	180.758	0.000	0.001	161.323	0.000	0.001
g_{OP}	156.695	0.000	0.001	211.304	0.000	0.001

Table 8: Nonlinearity Tests - Real Exchange Rates.

Test	Test Statistic	P-value	Bootstrap p-value	Test Statistic	P-value	Bootstrap p-value
		Ireland & Germany			Ireland & United Kingdom	
Reset						
F	8.136	0.000		1.043	0.376	
Lr	23.606	0.000		3.969	0.349	
Star				lag length 1		
F		0.236			0.576	
F_4		0.379			0.952	
F_3		0.121			0.169	
F_2		0.303			0.764	
		lag length 3			lag length 2	
F		0.010			0.207	
F_4		0.054			0.108	
F_3		0.010			0.236	
F_2		0.039			0.591	
Random Field						
$\lambda_H^E(\boldsymbol{g})$	2.410	0.121	0.058	0.187	0.665	0.653
λ_{OP}^A	4.481	0.923	0.369	6.721	0.751	0.394
$\lambda_{OP}^E(\boldsymbol{g})$	0.035	0.852	0.922	1.056	0.304	0.562
g_{OP}	4.551	0.871	0.367	2.847	0.970	0.458

Table 9: Random Field Analysis - IRELAND, GERMANY & UK.

	IRELAND & GERMANY		IRELAND & UNITED KINGDOM	
	ESTIMATES			
LINEAR				
c	0.332 (1.488)	0.769 (1.121)	1.176 (0.751)	0.907 (0.213)
p_t	−0.896 (0.191)	−0.836 (0.152)	−1.439 (0.308)	−1.093 (0.239)
p_t^*	0.892 (0.502)	0.724 (0.390)	1.164 (0.320)	0.882 (0.218)
i_t		−0.0004 (0.002)		0.009 (0.004)
i_t^*		0.007 (0.005)		−0.009 (0.004)
NONLINEAR				
σ	0.019 (0.002)	0.010 (0.004)	0.021 (0.003)	0.009 (0.004)
ζ	3.987 (0.817)	5.859 (2.551)	9.572 (2.109)	8.148 (4.368)
p_t	4.265 (0.375)	4.609 (1.103)	0.480 (0.116)	2.777 (1.214)
p_t^*	11.068 (0.733)	16.971 (3.021)	−1.864 (0.044)	10.454 (1.846)
i_t		−0.032 (0.023)		0.118 (0.039)
i_t^*		−0.146 (0.052)		$-2.26E-7$ (0.040)

Note: standard errors in parentheses.

Table 10: Multiple Structural Changes Model Estimation: IRELAND-GERMANY.

		ESTIMATE	STANDARD ERROR	P-VALUE
$\widehat{\delta}_1$	p_t	-1.034	0.059	0.000
	p_t^*	1.077	0.051	0.000
$\widehat{\delta}_2$	p_t	-0.725	0.043	0.000
	p_t^*	0.813	0.042	0.000
$\widehat{\delta}_3$	p_t	-0.787	0.386	0.045
	p_t^*	0.849	0.385	0.030
$\widehat{\delta}_4$	p_t	-1.961	0.311	0.000
	p_t^*	1.999	0.312	0.000
$\widehat{\delta}_5$	p_t	-0.843	0.499	0.094
	p_t^*	0.894	0.499	0.077
	i_t	-0.003	0.002	0.070
	i_t^*	0.008	0.002	0.000
R^2		0.985		
$\bar{R}^2$		0.983		
$F(12, 85)$		468.237		0.000
DW		0.854		

Confidence Intervals[a]

$\widehat{T}_1$	1978 Q2	1978 Q1$-$1981 Q2
$\widehat{T}_2$	1986 Q2	1986 Q1$-$1986 Q3
$\widehat{T}_3$	1990 Q3	1990 Q2$-$1990 Q4
$\widehat{T}_4$	1995 Q3	1994 Q2$-$1996 Q2

Break Tests

$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$
90.144	99.056	160.258	110.216
[11.470]	[9.750]	[8.360]	[7.190]

$\sup F_T(2 \mid 1)$	$\sup F_T(3 \mid 2)$	$\sup F_T(4 \mid 3)$
96.265	12.223	19.191
[11.470]	[12.950]	[14.030]

$UD\ max$	$WD\ max$
160.258	219.875
[11.700]	[12.810]

Note: 5 per cent critical values in parenthesis.
a The 95 per cent confidence interval for break date.
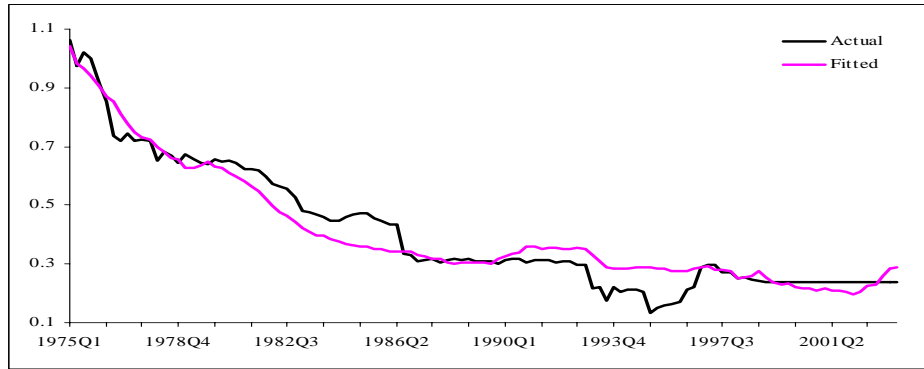
A.2 Figures

Figure 1: IRELAND/GERMANY: actual versus fitted based on random field regression.

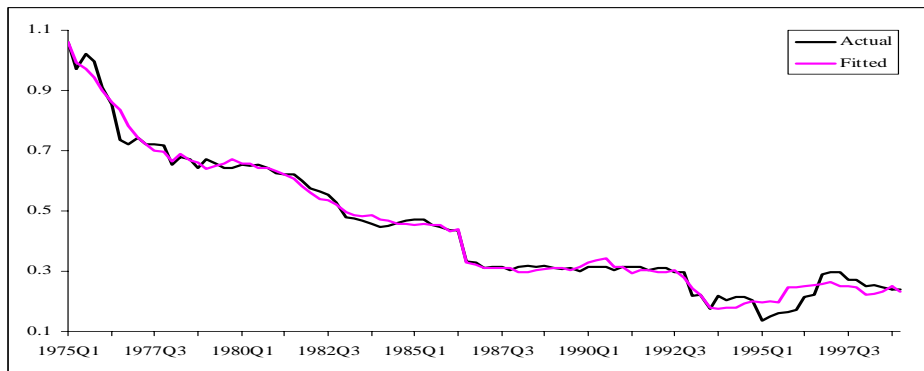Figure 2: IRELAND/GERMANY: actual *minus* fitted.

Figure 3: IRELAND/GERMANY: actual versus fitted based on structural changes model.