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Inequality, Mobility and Income Distribution Comparisons

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Abstract

This paper examines the relationship between the cross-sectional and lifetime income distributions using a simple model of relative income mobility. It asks whether cross-sectional comparisons between countries can provide a good indication of lifetime inequality differences if income mobility is similar, and whether lifetime inequality increases by less than cross-sectional inequality if the latter increases as a result of higher mobility. Analytical and simulation methods are used to show that the answer to both questions is negative. Comparisons must allow for different types of mobility, the nature of the age–income profile and the age distribution in each country.

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I. INTRODUCTION

Income distribution comparisons between countries are typically made using cross-sectional data. In most cases, data limitations rule out comparisons on the basis of a longer-period measure of income, although limited information about relative income mobility is sometimes available. The question therefore arises of whether some general statements regarding alternative distributions can be made on the basis of cross-sectional data combined with summary measures of mobility. This requires an analysis of the relationships among alternative types of

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distribution. In particular, the aim of this paper is to examine the general validity of the following two statements:

> *Statement 1:* If two countries have broadly similar degrees of income mobility, cross-sectional comparisons provide a good indication of differences in lifetime inequality.
> *Statement 2:* If an increase in cross-sectional inequality in a country is associated with an increase in income mobility, lifetime inequality increases by less than cross-sectional inequality.

Both of these statements may at first sight appear to be quite reasonable, and if true they would be very useful. The first extends the range of international comparisons that can be made with limited data, and the second statement suggests, for example, that greater ‘labour market flexibility’ has a smaller impact on inequality when a longer-period measure is considered. Indeed, both statements can be found in a lengthy report by the OECD (1996).[1]

The measurement of both mobility and inequality raises many well-known problems.[2] The present paper does not explore these problems and instead considers the two statements using a very simple model of income mobility; the simplicity serves to highlight the essential elements of the comparisons. It illustrates the difficulty of providing formal results concerning the relationships among the various distributions. The model is presented in Section II. This section uses algebraic methods with a very simple mobility process to demonstrate that the two statements are not necessarily true, when using the variance of logarithms of lifetime and cross-sectional income distributions. The section then introduces an extra dimension to mobility, which is seen to complicate comparisons. In order to reinforce and illustrate the earlier analytical results, some simulations are presented in Section III. In addition, the Gini inequality measure, for which analytical results are much more difficult to obtain, is used (the variance of logarithms is to some extent open to the criticism that in some cases it can give perverse results). This section also presents results for the more complex mobility process.

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[1]Thus ‘Countries with higher cross-sectional earnings inequality do not appear to have correspondingly higher relative earnings mobility, so that international differences in earnings inequality at a single point in time probably provide a good approximation of the differences in life-time earnings inequality’ (OECD, 1996, p. 60; see also pp. 83 and 94); and ‘If the forces causing wider earnings dispersion within a single year also create a more fluid labour market, in which the relative position of workers within the earnings distribution varies more over time, then life-time inequality of earnings will increase by less than what is observed cross-sectionally’ (OECD, 1996, p. 76). However, it was acknowledged that ‘the analysis [of mobility] undertaken in the chapter is exploratory. In particular, attempts to compare earnings mobility across different countries are hindered by fundamental conceptual and empirical difficulties’ (OECD, 1996, p. 59).
[2]Some approaches to the measurement of mobility are based explicitly on the extent to which an inequality measure changes as the time period is increased.

II. A SIMPLE MODEL

1. Cohort Income Profiles

Consider a cohort of individuals, each of whom enters the labour market at the same age. Let y_{it} and m_t denote respectively the income of individual *i* (*i*=1,...,*N*) and the geometric mean income (defined by $\ln m_t = \frac{1}{N}\sum_i \ln y_{it}$) in age-group *t* (*t*=1,...,*T*). If, furthermore, z_{it} is the logarithm of the ratio of person *i*'s income to the geometric mean, so that $z_{it} = \ln\left(\frac{y_{it}}{m_t}\right)$, suppose that

$$(1) \qquad z_{it} = z_{i,t-1} + u_{it}$$

where u_{it} is a random variable that is assumed to be independently normally distributed as N(0, s_u^2). Hence the mobility process is one in which individuals make random relative proportional income changes from year to year, and the variance, s_u^2, is a direct measure of the extent of mobility.[3] Taking variances of equation (1) gives

$$(2) \qquad s_t^2 = s_1^2 + (t-1)s_u^2$$

and the variance of logarithms at age *t*, s_t^2, is a linear function of age.

Suppose also that, in addition to the random proportionate change determined by u_{it}, all incomes are subject to growth at the constant rate a. This means that the arithmetic mean of logarithms of income at age *t*, m_t, is given by

$$(3) \qquad m_t = m_1 + (t-1)a.$$

2. Lifetime Income

The lifetime income of individual *i*, Y_i, is given (ignoring discounting) by

$$(4) \qquad Y_i = \sum_{t=1}^{T} \exp(z_{it} + m_t)$$

so that

[3]This is a simple Markov process which, in the present context, is known as a Gibrat process.

$$(5) \qquad \ln Y_i = (z_{i1} + m_1) + \ln\left[1 + \sum_{t=2}^{T} \exp\left\{\sum_{s=2}^{t} u_{is} + a\,(t-1)\right\}\right]$$

and the variance of logarithms of lifetime income, $s^2_{(T)}$, is equal to

$$(6) \qquad s^2_{(T)} = s^2_1 + V[X]$$

where X is the second term on the right-hand side of equation (5) and depends in a rather awkward way on a, T and, of course, s^2_u. Further progress can be made by using the linear approximation[4] $V[f(u)] = f'[E(u)]^2 V(u)$. Noting that all u_{it} are from the same distribution with mean 0, an approximation to equation (6) is given by

$$(7) \qquad s^2_{(T)} = s^2_1 + s^2_u \left[\frac{\sum_{t=2}^{T} \exp\{a\,(t-1)\}}{1 + \sum_{t=2}^{T} \exp\{a\,(t-1)\}}\right]^2 .$$

If there is no mobility, then of course the variance of logarithms of lifetime income is the same as that in the first year.[5] If there is mobility, then $s^2_{(T)}$ is greater than s^2_1 by an amount increasing in s^2_u. It is also possible for the ranking of countries according to $s^2_{(T)}$ to change as the length of time over which incomes are measured, T, is gradually increased.

3. The Cross-Sectional Distribution

The cross-sectional distribution consists of individuals from each of the T cohorts. In order to avoid problems arising from growth and other factors leading to labour market differences between cohorts, which produce changes in the cross-sectional distribution over time as different cohorts become older, suppose that each cohort has similar values of a, s^2_1 and s^2_u. The cross-sectional distribution is obtained by aggregating over many cohorts, which requires an age distribution to be specified. Suppose that h_t denotes the proportion of the population who are aged t in a given time period, so that $\sum_{t=1}^{T} h_t = 1$. The mean of logarithms in the cross-

[4]See, for example, Greene (1991, p. 61).
[5]For treatment of the coefficient of variation of lifetime income, see Creedy (1985, pp. 101–18).

section, m, is therefore equal to $\sum_{t=1}^{T} m_t h_t$ and the variance of logarithms, s^2, is given, following the standard decomposition into within- and between-age components, by[6]

$$s^2 = \sum_{t=1}^{T} h_t s_t^2 + \sum_{t=1}^{T} h_t \left(m_t - m\right)^2. \tag{8}$$

For $t \geq 2$, the terms in m_t and s_t^2 are given by simple expressions involving a and s_u^2, so it is possible to expand equation (8). It can be shown that

$$s^2 = s_1^2 + s_u^2 \sum_{t=2}^{T} (t-1) h_t + a^2 \sum_{t=1}^{T} h_t \left(t - \sum_{s=1}^{T} s h_s \right)^2. \tag{9}$$

Hence s_1^2 provides a lower bound to the variance of logarithms in both cross-sectional and lifetime contexts. Furthermore, the value of m_1 is irrelevant, while s_u^2 and a affect the variance of logarithms of the cross-sectional distribution through the second and third terms respectively on the right-hand side of equation (9). The distribution of h_t is seen to play a fundamental role in determining s^2.

4. Comparisons between Distributions

A comparison of equations (7) and (8) shows immediately that, in the extreme case where there is no relative income mobility within cohorts, $s_u^2 = 0$ and $s_{(T)}^2 = s_1^2$, while $s^2 > s_1^2$ because cross-sectional inequality depends on the steepness of the age–income profile and the age distribution. Two countries can therefore both have no mobility, but the cross-sectional distributions can give quite misleading indications of lifetime inequality, depending on the values of a and the distributions of h_t. Similarly, for common non-zero values of s_u^2, the cross-section can be equally misleading. This argument therefore shows that Statement 1 above is incorrect. The idea that it would be useful to allow for differences in age distributions when comparing cross-sectional income distributions is, of course, not new, and a variety of earlier studies have proposed suitable adjustment

[6]For an extensive treatment of the problem of aggregation over ages under alternative assumptions about the age–income profile and the age distribution, see Creedy (1985, pp. 84–94).

methods. Nevertheless, the OECD study cited above made no mention of age distributions or income profiles.[7]

Consider next the argument of Statement 2. By differentiating both equation (7) and equation (9) with respect to σ_u^2, it can be seen that $\partial \sigma^2 / \partial \sigma_u^2$ depends only on the form of the age distribution, while the term $\partial \sigma_{(T)}^2 / \partial \sigma_u^2$ depends only on the parameter α (and, of course, T). Hence there is no reason why an increase in mobility, reflected in an increase in σ_u^2, should be expected to increase lifetime inequality by less than cross-sectional inequality; they depend on quite different factors. The analysis has, for convenience, been in terms of the variance of logarithms and has required the use of a linear approximation, but the same basic properties may be expected to hold for other measures of inequality.[8] This is to some extent demonstrated by the simulations reported in Section III, which use a different measure — the popular Gini inequality measure.

5. Regression towards the Mean

The above discussion has been confined to just one type of mobility, measured by the term σ_u^2. However, mobility can take other forms. For example, it is possible that there is a systematic tendency for relatively higher incomes to increase by an amount that is different from that of lower incomes. In the present context, this phenomenon is referred to as regression towards or away from the (geometric) mean, following the famous use of the term by Galton (1889) when examining the heights of fathers and sons. In this type of process, equation (1) can be modified to give, for $t \geq 2$,

$$z_{it} = \beta z_{i,t-1} + u_{it} \tag{10}$$

where $\beta < 1$ indicates regression towards the mean and $\beta > 1$ indicates regression away from the mean.[9] This considerably complicates the type of analysis given above. The next section provides numerical examples of the role of regression, and reinforces the earlier results, using simulation methods.

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[7]It actually went further than Statement 1 by suggesting that mobility is in fact similar in different countries.
[8]This is notwithstanding the criticism of the variance of logarithms, that it can violate the principle of transfers.
[9]For further discussion of this type of process, see Creedy (1985, pp. 35–8).

III. SOME SIMULATIONS

1. The Assumptions

The model can be used to generate a simulated cohort of individuals, on the further assumption that incomes in the first period are log-normally distributed with mean and variance of logarithms of m_1 and s_1^2 respectively. Consider two countries, denoted A and B. To reduce the number of results to be reported, suppose that all individuals work for just five periods. It is useful to take more than two periods because comparisons are shown to depend on the length of time over which incomes are cumulated. Suppose that the values of a and s_1^2 in each country are as given in Table 1. Country A is assumed to have a relatively flat age–income profile, while it has a substantially higher degree of inequality in the first year of the life cycle, compared with country B. There is much movement within the cross-sectional distribution that does not involve mobility within the distribution of members of the same cohort, simply because of the shape of the age–income profile. These numerical values are chosen purely for illustrative purposes.[10]

In order to obtain the cross-sectional age distribution at any date, it is necessary to aggregate over five different cohorts. Information is required about the age distribution — that is, the number of people in each cohort existing at the specified date. The number of individuals in any cross-section is obviously very much larger than that in any single cohort; in the present context, this can be up to five times as large. For present purposes, suppose that countries A and B have age distributions as shown in Table 1. Hence, country A has a relatively young population while country B's population is relatively old.

TABLE 1

Parameter Values

	Country A	*Country B*
Income growth a	0.025	0.15
Initial inequality s_1^2	0.4	0.05
Size of age-group 1	1,000	200
2	800	400
3	600	600
4	400	800
5	200	1,000

[10]It has been shown above that the value of m_1 does not affect the various relative measures of inequality.

2. Numerical Examples

There are, of course, many inequality measures that could be used in comparing the alternative income distributions. In addition, a variety of lifetime income concepts are available, such as annuity measures, present values or annual averages. This section reports simulation results using the present value of income, discounted back to the first period at the rate of 5 per cent. Instead of using the variance of logarithms, for which analytical results have demonstrated the problems with the two statements, the Gini inequality measure is used.

Table 2 presents the inequality measures for the two countries, A and B, under alternative assumptions regarding mobility. The first part of the table gives the Gini measures for each year of the five-period life cycle, based on a simulated cohort size of 1,000 individuals. These cohort profiles are, by assumption, the same for every cohort. The increase in the Gini measure reflects the random proportional changes within the distribution of contemporaries that are influenced by the term s_u^2. The second part of the table shows Gini inequality measures of the present value of income for alternative time periods. It can be seen that lifetime inequality is higher in country A than in country B, partially reflecting the higher value of s_1^2 in that country.

The Gini measure of inequality of the cross-sectional income distribution, based on the age distribution given in Table 1, is given in the last row of the table. Each cross-sectional distribution contains 3,000 individuals. Country A, which has the flatter age–income profile and the younger population, has a lower degree

TABLE 2

Gini Inequality Measures

		Country A	*Country B*	*Country A*	*Country B*	*Country A*	*Country B*
Mobility s_u^2		1	1	1.5	1.5	1.5	1.5
Regression to the mean *b*		1	1	1	1	0.85	0.85
Year	1	0.343	0.123	0.343	0.123	0.343	0.123
	2	0.605	0.537	0.676	0.624	0.661	0.622
	3	0.717	0.682	0.789	0.770	0.744	0.735
	4	0.764	0.750	0.839	0.834	0.777	0.776
	5	0.836	0.822	0.895	0.888	0.823	0.819
Present value over first	2 yrs	0.468	0.362	0.530	0.449	0.511	0.446
	3 yrs	0.557	0.509	0.647	0.621	0.591	0.576
	4 yrs	0.616	0.597	0.714	0.712	0.628	0.633
	5 yrs	0.688	0.685	0.790	0.795	0.669	0.683
Cross-section		0.693	0.780	0.799	0.881	0.717	0.804

of cross-sectional inequality than country B, which has the steeper age–income profile and the older population.[11] The rankings of the countries by cross-sectional and lifetime inequality are different. These results provide a clear example of a situation in which the first statement — that cross-sectional comparisons give an indication of lifetime inequality when relative earnings mobility is the same in each country — is incorrect.

3. Higher Mobility

The second statement suggests that an increase in relative mobility increases cross-sectional inequality more than lifetime inequality. This can be examined by increasing the value of s_u^2 used in the simulations, and the second pair of columns in Table 2 are the results for the case where $s_u^2 = 1.5$. The increase in mobility means that inequality increases more rapidly over the life cycle. Comparison of results for $s_u^2 = 1$ and $s_u^2 = 1.5$ shows that, for country A, the Gini measure of lifetime inequality increases by less than the Gini measure for the cross-sectional distribution, as implied by Statement 2. However, for country B, which has the steeper age–income profile and the lower inequality in the first age-group, the Gini measure of lifetime income increases by more than that of the cross-sectional distribution, contradicting Statement 2. As shown earlier, the effects of an increase in relative mobility on cross-sectional and lifetime inequality depend on the age distribution as well as on the precise nature of age–income profiles.

4. Regression towards the Mean

The effect of an increase in relative mobility is simplified by the convenient assumption that mobility can be described in terms of a single parameter. An increase in s_u^2 inevitably involves an increase in the extent to which the inequality of income increases over the life cycle. Lifetime and cross-sectional inequality must also increase as a result of an increase in this type of mobility. However, different types of relative mobility may take place simultaneously. Suppose that, in addition to the independent random variation governed by s_u^2, there is some 'regression towards the mean', reflected by the extent to which b is less than unity.

Instead of the previous case where only s_u^2 is increased to 1.5, suppose also that b is reduced from its implicit value of unity to 0.85. The inequality measures

[11]Even if the age distributions in each country were completely flat, the ranking would remain the same, with Gini measures for countries A and B respectively of 0.768 and 0.776.

resulting from this process are given in the last pair of columns in Table 2. The Gini measure increases with age, despite the introduction of the egalitarian changes. However, the rate of increase is not so great as when $b = 1$.

Comparisons can now be made between the first and third pairs of columns in Table 2. In both countries, the inequality of the cross-sectional distribution increases whereas the inequality of the distribution of lifetime income falls. In contradiction to Statement 2 , additional mobility can produce changes that, from the cross-sectional perspective, appear to increase inequality while at the same time it can reduce lifetime inequality. The extra mobility has a sufficient egalitarian element which can generate a reduction in lifetime income inequality. Hence, when discussing the effects of changes in mobility, it is very important to distinguish precisely which type of mobility is affected.

The last two columns also show that the present value of income over four and five years is greater in country B than in country A, which is the reverse of the ranking obtained when using the present value of income over the first two and three years. Furthermore, country B has lower inequality within each cohort in each year over the life cycle, but greater lifetime inequality.

IV. CONCLUSIONS

This paper has examined the relationship between the cross-sectional and lifetime income distributions, using a simple analytical model. The analytical results are reinforced by the use of simulation methods. The analysis was motivated by the consideration of two initially appealing statements concerning international comparisons and comparative static changes in mobility. These two statements are the same as those made by the OECD (1996). It was shown that neither of the statements necessarily follows from the assumptions made. It is extremely important to distinguish different types of mobility and to make appropriate allowance for the properties of age–earnings profiles and the age distribution of the population.

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