On the Evolutionary Stability of 'Tough' Bargaining Behavior

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Abstract

This paper investigates whether 'tough' bargaining behavior, which gives rise to inefficiency, can be evolutionary stable. We show that in a two-stage Nash Demand Game tough behavior survives. Indeed, almost all the surplus may be wasted. These results differ drastically from those of Ellingsen's model (Ellingsen (1997)), where bargaining is efficient. We also study the Ultimatum Game. Here evolutionary selection wipes out all tough behavior, as long as the Proposer does not directly observe the Responder's commitment to rejecting low offers.

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1 Introduction

An important insight, due to Thomas Schelling (Schelling (1960)), is the role *commitment* can play in bargaining. The *benefits* from commitment come about when the committed player persuades the opponent to give away a large share of the surplus. The *costs* of commitment is that the opponent may also have committed himself, such that disagreement may result. Furthermore, even when the opponent has not committed himself, he may not, due to insufficient information, realize that the player is committed, in which case disagreement may again occur.

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In this paper we investigate how, in an evolutionary equilibrium, these benefits and costs are related. We will say that a player who is committed to getting more than half of the surplus is a '*tough*' player. Since any two such players will disagree, the existence of tough players implies bargaining inefficiency. We wish to analyze the extent to which such bargaining behavior can be part of an evolutionarily stable outcome. A result that tough bargaining behavior survives would differ substantially from rationality-based models of bargaining with commitment. Here players do not make incompatible commitments and bargaining is efficient, unless there is some uncertainty (e.g. about players' payoffs or the size of the surplus). We refer the reader to Muthoo (1999), Ch. 8, where such bargaining models, and others that are related to the ones studied here, are analyzed and discussed.

We consider two well-known and simple bargaining games: A variant of the Nash Demand Game (Nash (1953)) and an Ultimatum Game. In the evolutionary model, a large population of players is randomly and repeatedly matched over time to play some 'stage game', this being one of the two above-mentioned games. Some players are socially 'programmed', or committed, to a certain fixed bargaining behaviour, while other players, called 'optimizers', respond optimally to opponents' behaviour.[1]

Our results are the following. In the two-stage Nash Demand Game, there exists an evolutionary stable outcome where tough players are present. Hence there is inefficiency. In this population the net benefit from commitment equals the net benefit from not being committed. Furthermore, in the limit, as the grid of feasible demands becomes very fine, there is *extreme inefficiency*: All the surplus is wasted. This, we believe, is a very strong result, which, to our knowledge, has not been seen in other models of bargaining under certainty.

In the Ultimatum Game, being in the Proposer role is itself a commitment device. If Proposers have no specific information about the Responders' smallest acceptable offers, there is only a cost associated with being tough, so in the evolutionarily stable outcome no player is tough. If, on the other hand, Proposers observe the Responder's smallest acceptable offers before the Proposers make offers, evolution favors Responders who are committed to rejecting low offers. This implies that tough players survive, as in the two-stage Nash Demand Game.

Our main contribution is to point out that whether bargaining toughness survives depends on the bargaining protocol. However, another contribution is obtained by relating our results for the two-stage Nash Demand Game to Ellingsen's interesting evolutionary model (Ellingsen (1997)). Here the stage game is the Nash Demand Game. In his model there are no stable outcomes with toughness and bargaining is efficient. This striking difference occurs even though his and our model differ essentially only in the assumption about what happens when the opponent demands *literally all* the surplus. Ellingsen assumes that an optimizing player always lets such an opponent have all the surplus, i.e., the optimizer demands zero. However, note that the optimizer in this situation is completely indifferent between all feasible demands. Moreover, the zero demand is weakly dominated, so it seems just as plausible that the optimizer in this situation would make any (e.g. an arbitrarily small) demand, or simply randomize. We show that under either assumption there exists an evolutionarily stable strategy with tough behavior. Another insight of this paper is thus to show

[1]Such a set-up has been analyzed in other papers, e.g. Banerjee and Weibull (1995).

that in an evolutionary model what goes on at the 'boundary' of the strategy set can be crucial for the efficiency of bargaining.

The rest of the paper is organized as follows: In Section 2 we analyze the two-stage Nash Demand Game and in Section 3 we consider the Ultimatum Game. Section 4 summarizes and Section 5 is the Appendix, which contains all proofs.

2 A two-stage Nash Demand Game

Consider the following two-player two-stage game: Each player makes a demand for a unit-size surplus at either Stage 1 or Stage 2. If the two demands do not exceed the surplus, each player gets his demanded share. If the two demands are not compatible, each player receives zero.

We interpret a demand made at Stage 1 as a *commitment* to getting a certain share of the cake. The potential cost of committing oneself to a demand at Stage 1 is that the opponent does the same and that the two demands turn out to be incompatible. The potential benefit is that the opponent has decided to make a demand at the second stage and will, so to speak, simply 'take the rest'.

There is a finite set of feasible demands that can be made at each stage:

$$\{x_0, x_1, x_2, ..., 1/2, ..., x_{n-1}, x_n\} = \{0, 1/n, 2/n, ..., 1/2, ..., (n-1)/n, 1\} \equiv X.$$

We assume that n is even (ensuring that $1/2 \in X$) and $n > 2$ (such that X contains more elements than just 0, 1/2 and 1), i.e., $n \in \{4, 6, 8, ...\}$.

We may think of bargaining as taking place over a sum of money (normalized to one), with $1/n$ being the smallest monetary unit. The 'fair' demand is indexed by f: $x_f = 1/2$. For simplicity, the demand $x_i \in X$, where $i = 0, ..., n$, will be denoted as $i \in X$. Statements such as $x_i + x_j > 1$ will be written as $i + j > 1$, and so on. The payoffs are those for the Nash Demand Game: If a player demanded x and the opponent demanded x', where $x, x' \in X$, the payoff to the first player equals $x/(x+x')$ if $x + x' \leq 1$ and equals 0 if $x + x' > 1$.[2]

We will consider the following strategies for the evolutionary game: An *i-strategy*, where $i = 0, ..., n$, always demands $i \in X$ at Stage 1. The set of i-strategies can be decomposed as follows: The *fair* strategy: Make demand $f = 1/2$ at Stage 1; a *soft* strategy: Make a demand i with $i = 0, 1, ..., f - 1$ at Stage 1; and a *tough* strategy: Make a demand i with $i = f + 1, ..., n$ at Stage 1. A player using the *optimizing* strategy makes a demand at Stage 2 as follows: If the opponent demanded $x < 1$ at Stage 1, then demand $1 - x$ at Stage 2; if the opponent did not make a demand at Stage 1, then demand one-half at Stage 2. The latter may be justified as follows: When the optimizing player learns that the opponent did not make a demand at Stage 1, she concludes (correctly) that the opponent is an optimizer, too. We then assume that two optimizers play the unique symmetric and Pareto-optimal Nash equilibrium, which is that each player demands one-half.

[2] We assume that when demands are compatible, a player gets his demanded share plus a proportional share of any surplus that is left on the table. Our results are not sensitive to this assumption.

We must also specify what an optimizing player does at Stage 2 when the opponent demanded *literally* all the surplus at Stage 1. However, as mentioned in the Introduction, we note that no matter what the optimizer demands, her payoff will be zero. Furthermore, the zero demand is weakly dominated by all the other demands. We therefore assume that when the optimizing player learns that the opponent demanded *literally* all the surplus at Stage 1, the optimizer makes some fixed and strictly positive demand at Stage 2. Another assumption, which gives the same results, is that the optimizer randomly chooses a demand.

We will denote the optimizing strategy as strategy 'o'. The set of strategies for the evolutionary game is therefore $S = I \cup \{o\}$, where I denotes the set of i-strategies. Let $\pi(t, u)$ denote the payoff to a player using strategy $t \in S$ when the opponent uses $u \in S$. If $i, j \in I$, the payoffs for the evolutionary game are: $\pi(i, j) = i/(i + j)$ if $i + j \leq 1$; $\pi(i, j) = 0$ if $i + j > 1$; $\pi(o, i) = 1 - i/n$ if $i < n$; $\pi(i, o) = i/n$ if $i < n$; $\pi(o, i) = \pi(i, o) = 0$ if $i = n$; and, finally, $\pi(o, o) = 1/2$. The evolutionary selection dynamic is the Replicator Dynamic (Taylor and Jonker (1978))). We will use the well-known Evolutionarily Stable Strategy (ESS) and Neutrally Stable Strategy (NSS) concepts (Maynard-Smith and Price (1973); Maynard-Smith (1982)) to describe the stability properties of population behavior for this dynamic. The reader is referred to e.g. Weibull (1995) for details.

2.1 Tough Behavior

An $(n - 1)$-player always demands $(n - 1)/n > 1/2$. The following result shows that these tough players survive:

Proposition 1 *Consider the population where a share* $(n - 2)/n$ *are tough* $(n - 1)$*-players and the remaining share,* $2/n$*, are optimizers.*

(a). This population is the unique Evolutionarily Stable Strategy (ESS). It is asymptotically stable for the Replicator Dynamic.

(b). The average share of the surplus to a player in the ESS is $c = 2(n - 1)/n^2$*. Thus, in the limit as* $n \to \infty$*, we have* $c \to 0$*: There is only conflict.*

We may interpret this result in terms of the benefits and costs from commitment: The cost of commitment for a tough player is that he gets zero payoff when he meets another tough player. The benefit from commitment is that he manages to get more than half of the surplus when meeting an optimizer. In the evolutionarily stable population the tough players act as if they maximize the benefits from commitment. Moreover, the population shares are such that the net benefit from being committed, i.e., tough, equals the benefits from not being committed, i.e., optimizing. The inefficiency result in part (b) is due to the fact that when n increases the share of tough $(n - 1)$-players also increases, because they get more when meeting the optimizers. Since two tough players waste the surplus, this increases disagreement in the population. In the limit, as $n \to \infty$, the surplus is almost completely wasted. Put in the usual framework of rational players, as $n \to \infty$, both players choose to commit themselves with probability 1 to demanding all the surplus at Stage 1.

2.2 The 'fair' outcome

In addition to the ESS described in the previous section, there are stable populations where players are either fair or optimizing. Then all players divide the surplus equally. Such a population is stable if there are not too many optimizers. Precisely, any population where a share s of players are optimizers and the rest are fair players is a Neutrally Stable Strategy, and hence stable, as long as the share of optimizers satisfies $s < n/[2(n-1)]$.

In Ellingsen's model, a very similar result is found. However, in his model there are *no* stable outcomes with tough behavior. The reason is that there any population composed of tough players and optimizing players can be invaded by even tougher players. In particular, an n-player can invade since the optimizer is assumed to demand zero when meeting such a player. However, the population with only n-players and optimizers is also unstable[3], as is the population composed only of n-players. We have thus obtained a qualitatively different result than in Ellingsen's model, only by a small change in what might seem an innocuous assumption about how optimizing players react when meeting a player who demands *literally* all the surplus.

2.3 Multiplicity and evolutionary drift

We have seen that, in addition to the inefficient tough-optimizer ESS, there is an efficient outcome where fair and optimizing players all make the demand of one-half. Precisely, there is in the state space a component of populations where players are either fair or optimizing and where the share of optimizers satisfies the inequality stated at the end of the previous section. Each such population is stable, but *not* asymptotically stable: Since the fair and the optimizing players perform equally well in the population, the population shares of each strategy may unnoticeably change, due to 'noise' and other stochastic factors. This phenomenon is called 'evolutionary drift'; the reader is referred to Binmore and Samuelson (1999). Following that paper, we may call the component a *hanging valley*, namely a flat floor of equilibria with a 'cliff edge' at one end. At this cliff edge the share of optimizers is such that a tough $(n-1)$-mutant would earn exactly the same as the incumbents; and were the share of optimizers to increase even more due to evolutionary drift, this mutant can invade. Then the population (the 'ball') would be pushed out over the cliff edge and plunge to the basin of attraction of the ESS. Once there, the 'ball' rolls down in a pit (the ESS) and stays there. We expect that in the presence of evolutionary drift, the population will be observed at the inefficient ESS in the 'ultra-long' run. The reader is once more referred to Binmore and Samuelson (1999) for an insightful analysis of different time spans in evolutionary models.

[3] An n-player performs strictly better against an optimizer, and equally well against another n-player, as does an optimizer.

3 An Ultimatum Game

The Ultimatum Game differs from the Nash Demand Game in that being Proposer is itself a commitment device. Intuitively, since Responders can do nothing better than picking up Proposers' offers, evolutionary selection should reward those Responders who accept all offers and so toughness should not be present. Indeed, we will see that it is only if the Responder's commitment to rejecting low offers is somehow *visible* to the Proposer that toughness is viable.

In the Proposer role a strategy is a demand $i \in X$, as before, with the understanding that $1 - x$ is the *offer* to the Responder. In the Responder role, a strategy is a Smallest Acceptable Offer (SAO): If a Responder uses SAO y she accepts any offer at least as large as y and otherwise rejects it. We assume that no Responder accepts an offer of *literally* zero and that no Responder accepts *only* an offer of *literally* all the surplus. The set of feasible SAOs is thus given by $Y = \{1/n, 2/n, ..., (n-1)/n\}$. A strategy is then a pair $(i, j) \in X \times Y$, where $i \in X$ is the demand made when Proposer and $j \in Y$ is the SAO employed when Responder. The payoffs to a player is a weighted average of the payoffs in the Proposer and the Responder role.

We can, as before, decompose the set of (i, j)-strategies in a rather intuitive way: A strategy (i, j) is *balanced* if $i + j = 1$, *soft* if $i + j < 1$ and *tough* if $i + j > 1$. We may interpret these strategies as follows: A balanced (soft) [tough] player is committed to getting half (less than half) [more than half] of the surplus, taken as an average over the two player roles. Two players using the same tough strategy always disagree. An (i, j)-*player* always uses strategy (i, j). An *optimizing* player makes as Proposer an optimal offer and accepts any positive offer as Responder. Denoting the optimizing strategy once more as o and the set of (i, j)-strategies by I, the pure strategy set for the evolutionary game is $I \cup \{o\}$.

We will distinguish between two scenarios: *Scenario 1* is when the Proposer, when making offers, only knows the distribution of SAOs in the population. *Scenario 2* is when a Proposer, *before* making an offer, observes what the Responder's SAO is. Scenario 2 clearly makes a very strong assumption that the Responder's SAO is perfectly observable by the Proposer (in Ellingsen's model it is likewise assumed that a player can observe what demand the opponent will make).

Proposition 2 *(a). Under Scenario 1, there are no tough players in any stable population. Moreover, stability implies efficiency.*

(b). Under Scenario 2, consider the population where a share $(n-2)/n$ *are tough* $(n-1, n-1)$*-players and the remaining share,* $2/n$*, are optimizers. This population is the unique ESS. Furthermore, the average share of the cake and its limiting behavior is exactly as in Proposition 1.*

We may interpret this result as follows: Under Scenario 1, the ever-present cost from being tough is that there is disagreement when such a player meets similar tough players. The benefit, that one may be able to induce the opponent to give more than one-half (as an average over the roles), however, is non-existent, since no Proposers condition offers on the Responder's SAO. The second part of the proposition follows from this: Any population where some Responders turn down offers is unstable, since players who accept the offers (and make the same offers as Proposer) perform strictly better.

When Scenario 2 applies, an optimizing Proposer will condition his offer on the observed SAO. Thus, whenever a Responder faces such a Proposer, the Responder's SAO is effectively a *demand* for a share of the surplus. This means that the benefit from being tough is now positive. Indeed, the most successful tough players will be those who 'demand' the most in each role. We then obtain co-existence between tough players and optimizers, exactly as in our two-stage Nash Demand Game. There are also stable (but not asymptotically stable) populations with balanced and optimizing behavior. Stability requires that there are sufficiently few optimizers in the population; the logic behind this condition is exactly the same as the one for the two-stage Nash Demand Game and the one in Ellingsen (1997). From the argument in Section 2.2, we predict that evolutionary drift will, with sufficient time, bring the population to the inefficient ESS.

4 Summary

When rational players bargain under certainty commitment and conflict is viewed as anomalous, since the bargainers will exploit gains from trade efficiently. However, studying two bargaining games in an evolutionary context revealed that there can exist an evolutionarily stable outcome that is (potentially extremely) inefficient. Our analysis shows that the evolutionary stability of bargaining 'toughness', and of the associated inefficiency, depends on how the costs and benefits from being tough compare. This, in turn, depends on the bargaining protocol. These results may be interpreted as a formalization, in an evolutionary context, of many of the important insights in Thomas Schelling's book (Schelling (1960)).

5 Appendix

Proof of Proposition 1:

Consider a population with support $\{o, n-1\}$ and with s denoting the share of $(n-1)$-players. Expected payoffs to the optimizing players and $(n-1)$-players are $(1-s)(1/2) + s(1/n)$ and $(1-s)[(n-1)/n]$, respectively. These are equal at $s = (n-2)/n \equiv s'$. At this population, which we denote by s^*, average payoff is $(1-s')[(n-2)/n] = (2/n) - 2/(n^2)$. Next, let $\pi(p, s^*)$ denote the expected payoff to a strategy $p \in \{I \cup o\}$ at s^*. We verify that any pure strategy p other than o and $n-1$ satisfies $\pi(p, s^*) < \pi(s^*, s^*)$. In particular, since the optimizer makes a strictly positive demand when meeting an n-player, we have $\pi(n, s^*) = 0$. Thus we have $\pi(s, s^*) \leq \pi(s^*, s^*)$ for all populations s, so s^* is a Nash equilibrium. To show that s^* is an Evolutionarily Stable Strategy (ESS) (Maynard-Smith and Price (1973)), and hence asymptotically stable for the Replicator Dynamic (Taylor and Jonker (1978)), we must verify that we have $\pi(s^*, s) > \pi(s, s)$ for all populations s satisfying $\pi(s, s^*) = \pi(s^*, s^*)$. From above, we need only check this inequality for populations s with support in $\{o, n-1\}$. We have $\pi(s^*, s) - \pi(s, s) = (s' - s)[\pi(n-1, s) - \pi(o, s)]$ or, after some simplifications, $\pi(s^*, s) - \pi(s, s) = 2[2((n-1)/n) - 1 - s]^2$, This is strictly positive for any $s \neq s'$. Hence s^* is an ESS.

We will also consider the possibility that an optimizer, instead of choosing some positive fixed demand, randomly chooses a demand when meeting an n-player. Then the n-player earns $\pi(n, s^*) = [1/(n+1)](1 - s')$ at s^*. Since an $(n-1)$-player earns $[(n-1)/n](1 - s') = \pi(s^*, s^*)$, we have $\pi(n, s^*) < \pi(s^*, s^*)$ whenever $(n-1)/n > 1/(n+1)$ or $n^2 - n - 1 > 0$. This clearly holds for all $n \geq 4$. Thus s^* is again a Nash equilibrium and the proof that s^* is also an ESS is exactly as above. The proof that this is the unique ESS is given below.

Proof of the stability of fair and optimizing behavior:

Here we prove the claim made in Section 2.2 about the stability of optimizing and fair behavior. In any population with only fair and optimizing players, average payoff is one-half. It is not difficult to see that this population is an NSS, and hence stable, if an $(n-1)$-mutant cannot invade. Letting s denote the population share of optimizers, this is the case when $1/2 > s(n-1)/n$. Re-arranging this expression gives the inequality in the main text.

To prove that the ESS just described is unique, we note that all Nash equilibria other than the Nash equilibrium s^* have the following form: A player uses some mixture of the fair strategy and the optimizing strategy. Of those only the ones who satisfy the inequality just stated are NSS. However, none of these NSS is an ESS. To see this, we may note that a mutant using the optimizing strategy is an alternative best reply and, moreover, the mutant performs as well as the incumbents against the same mutant. This violates the ESS conditions (the reader is once more referred to e.g. Weibull (1995) for details). ■

Proof of Proposition 2:

Part (a): We prove that any population with tough strategies is unstable. Consider a population where some players use a tough strategy (i, j). Suppose first the offer, $1 - i$, is strictly positive, i.e., $i < n$. An (i, k)-player, with $k \leq 1 - i$, then earns the same in the Proposer role as the (i, j)-player. However, he earns strictly more in the Responder role, since he, unlike the (i, j)-player, accepts a proposing (i, j)-player's strictly positive offer. Thus the (i, k)-player's expected payoff is strictly larger than the (i, j)-player's expected payoff, so the population is unstable. Suppose now $i = n$, i.e., the offer is zero. Then an (i, j)-player earns zero as Proposer, since no Responder accepts the zero offer. Furthermore, a mutant using the strategy $(1 - j, j)$ earns the same as Responder as the (i, j)-player, but strictly more as Proposer (he gets a share $1-(j/n) > 0$ when playing against an (i, j)-player as Proposer). Thus the population is unstable. To prove that stability implies bargaining efficiency, suppose the population is stable, but that some offers are rejected. Then there is in the population a strategy (p, q) and a strategy (s, t) with $1 - p < t$. Suppose first $p < n$. Then a player using strategy $(s, 1 - p)$ earns a strictly higher expected payoff than a (s, t)-player, since the former earns a strictly higher payoff in the Responder role than the latter. This implies that the population is unstable, a contradiction. If $p = n$, then (n, q) is a tough strategy, which, from the previous result, again contradicts stability.

Part (b): Consider a population with support $\{o, (n-1, n-1)\}$ and let s denote the share of $(n-1, n-1)$-players. The expected payoffs to strategy o and strategy $(n-1, n-1)$ are $(1-s)(1/2) + s(1/n)$ and $(1-s)[(n-1)/n]$, respectively. These expected payoffs are equal at $s = (n-2)/n$. As before, we call this population s^*. We note that this population is exactly the same as the ESS for the two-stage Nash Demand Game, so the average payoff is also the same. To verify that s^* is a Nash equilibrium, consider any pure strategy p other than $(n-1, n-1)$ and o. We note that if p is an alternative, or superior, best reply to s^*, then p must be tougher than $(n-1, n-1)$. Thus we need only check the strategy $(n, n-1)$. However, this strategy earns zero in the Proposer role, since no Responder accepts the zero offer. Thus s^* is a Nash equilibrium. The proof that s^* is the unique ESS is exactly as the proof of Proposition 1. ■

6 References

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