New-Keynesian Models and Monetary Policy: A Reexamination of the Stylized Facts

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Abstract

Using an empirical New-Keynesian model with optimal discretionary monetary policy, we calibrate key parameters—the central bank's preference parameters; the degree of forward-looking behavior in the determination of inflation and output; and the variances of inflation and output shocks—to match some broad characteristics of U.S. data. Our preferred parameterizations all imply a small concern for output stability but a large preference for interest rate smoothing, and a small degree of forward-looking behavior in price-setting but a large degree of forward-looking in the determination of output. We provide some intuition for these results and discuss their consequences for practical monetary policy analysis.

Keywords: Interest rate smoothing, central bank objectives, forward-looking behavior.

JEL Classification: E52, E58.

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1 Introduction

Many analyses of monetary policy presented in recent years have been based on the hypothesis that the private sector's behavior can be approximated by aggregate supply and demand relations derived from so-called New-Keynesian models.[1] This is true both for analyses primarily aimed at describing how monetary policy has been conducted, and for analyses intended to provide policy recommendations. The popularity of the New-Keynesian framework in both theoretical and applied work is easy to grasp: while being similar in structure to traditional models used for policy analysis (such as the IS/LM model), it can (under certain assumptions) be derived from microeconomic theory with optimizing agents. It therefore facilitates communication between policymakers and more theoretically oriented researchers.

Since analytical solutions are often not available for this kind of models, and the empirical literature has not yet reached a consensus about key parameters in the model, researchers tend to rely on calibrated models. It is not entirely clear, however, whether these calibrated models are suitable for policy analysis, that is, whether they fit the data. The purpose of this paper, therefore, is to examine whether a New-Keynesian model can be calibrated to match the broad characteristics of U.S. data.

We use a framework (based on Rudebusch, 2002a) where the private sector's behavior is assumed to be described by a model with New-Keynesian features and where the central bank is assumed to solve a well-defined optimization problem. We examine how certain changes in the central bank's preferences or the private sector's behavior affect the time-series properties of inflation, the nominal interest rate and the output gap, and we calibrate the model to fit some broad stylized facts about fluctuations in the U.S. economy. Although the model parameters are not determined by estimation, we show that certain parameter combinations are more consistent with the data than others.

Our analysis sheds some light on several issues that have recently been discussed in the literature. For instance, empirical estimates of Taylor rules typically suggest that central banks have very strong preferences for smoothing nominal interest rates

[1] Early references on how to derive such models from optimizing behavior include McCallum and Nelson (1999) and Rotemberg and Woodford (1997). Clarida et al. (1999) review this literature. To our knowledge there is no generally agreed set of restrictions that characterize models that are presented under the New-Keynesian label. Theoretical models are of course more sparsely parameterized than empirical models. We use the New-Keynesian label because the model that we apply is similar to many earlier models of aggregate supply and demand that have been presented as New-Keynesian.

(e.g., Clarida et al., 2000). Although arguments for why such a policy may be optimal have been presented (see, e.g., Cukierman, 1991; Goodfriend, 1991; Woodford, 1999), it has also been argued that there is little deliberate interest rate smoothing in practice and that the empirical Taylor rules are misspecified descriptions of monetary policy (see Rudebusch, 2002b). In our model with optimal monetary policy, the Taylor rule is indeed misspecified. Nevertheless, our results indicate that a good approximation of central bank behavior can be obtained assuming a large preference for interest rate smoothing but virtually no preference for output stabilization.

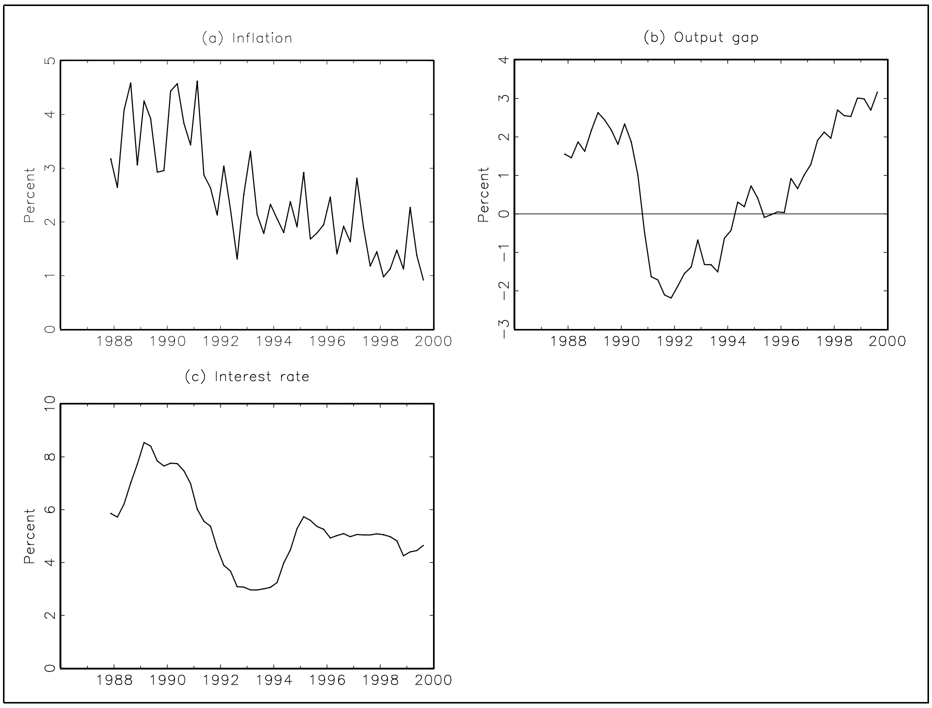
A different issue concerns the importance of forward- versus backward-looking behavior in private agents' decision rules. Many studies of monetary policy are based on the presumption that the private sector is entirely backward-looking (e.g., Rudebusch and Svensson, 1999; Dennis, 2001b; Favero and Rovelli, 2001), raising concerns about the Lucas critique. At the same time, purely forward-looking models have been shown to have difficulties explaining the persistence in the data (see e.g., Fuhrer and Moore, 1995; Estrella and Fuhrer, 2002). Our analysis stresses the fact that the time-series properties of inflation (and other variables in the economy) are affected by both private behavior and monetary policy; therefore answers to questions like those above depend crucially on the assumptions made about the behavior of both central banks and the private sector. For example, we show that the degree of forward-looking behavior in price-setting has important consequences not only for the persistence of inflation, but also for the volatility of output and the interest rate. Likewise, the ability to explain the persistence of inflation depends not only on the degree to which price setters are forward-looking, but also on the objectives of the central bank.

The rest of the paper is organized as follows. We begin by presenting some stylized facts for the U.S. economy in Section 2. Section 3 presents the model and the main results from our calibration exercise, while Section 4 compares our preferred calibration with alternative parameterizations. Section 5 goes through some robustness exercises. Section 6 summarizes our results and discusses the consequences for practical monetary policy analysis.

2 Stylized facts for the U.S. economy

This section presents some stylized facts for the U.S. economy, in terms of standard deviations and autocorrelations for inflation, the output gap, and the 3-month interest rate. These facts will serve as a benchmark with which we want to compare

Figure 1: Data series, 1987Q4–1999Q4



Sources: Bureau of Economic Analysis (inflation); Federal Reserve Bank of St. Louis (output and interest rate).

the implications of the theoretical model in Section 3. We use quarterly data for the period from 1987Q4 to 1999Q4. This sample excludes the disinflationary period of the early 1980s, and is characterized by a rather stable monetary policy regime. It also matches the period used by Rudebusch (2002b) when analyzing recent U.S. monetary policy.

The inflation rate is the annualized quarterly change in the GDP deflator (seasonally adjusted), obtained from the Bureau of Economic Analysis. The output gap is the percent deviation of real GDP (measured in chained 1996 dollars, seasonally adjusted) from potential GDP, as calculated by the Congressional Budget Office (see Congressional Budget Office, 1995). The interest rate is the (annualized) average of daily rates on a 3-month T-bill. Data on interest rates and potential and actual GDP were obtained from the FRED database of the Federal Reserve Bank of St. Louis. The data series are shown in Figure 1. Table 1 shows the standard deviations and autocorrelations for each series, with standard errors in parentheses. The output gap and the interest rate are both more volatile (in terms of standard deviations) and more persistent than the inflation rate.

It is often observed that the large persistence in the interest rate (the instrument of monetary policy) cannot be explained by persistence in inflation and output alone.

Table 1: Standard deviations and autocorrelations in U.S. data

	Standard deviation	Autocorrelations		
		1 lag	2 lags	3 lags
Inflation	1.04	0.65	0.53	0.54
	(0.10)	(0.09)	(0.12)	(0.10)
Output gap	1.67	0.91	0.83	0.75
	(0.15)	(0.05)	(0.09)	(0.13)
Interest rate	1.51	0.94	0.86	0.74
	(0.18)	(0.05)	(0.10)	(0.14)

Note: Quarterly U.S. data, 1987Q4–1999Q4. Standard errors in parentheses are based on GMM and the delta method, using a Newey and West (1987) estimator with one lag.

Table 2: Estimated Taylor rule

Coefficient				Statistics	
γ_0	γ_π	γ_y	ρ_i	R^2	σ_ζ
0.793	1.615	0.959	0.711	0.965	0.350
(0.779)	(0.286)	(0.129)	(0.076)		

Note: Quarterly U.S. data, 1987Q4–1999Q4. Standard errors in parentheses are based on the delta method, using a Newey and West (1987) estimator with two lags.

This is confirmed for our sample by Table 2, which shows the results from estimating a Taylor-type rule of the form

$$i_t = (1 - \rho_i)\left[\gamma_0 + \gamma_\pi \bar{\pi}_t + \gamma_y y_t\right] + \rho_i i_{t-1} + \zeta_t, \tag{1}$$

where i_t is the 3-month interest rate, $\bar{\pi}_t = 1/4 \sum_{j=0}^{3} \pi_{t-j}$ is the four-quarter inflation rate and y_t is the output gap. The estimates imply long-run response coefficients of inflation and the output gap of 1.62 and 0.96, respectively, but the short-run effects are dominated by the lagged interest rate, which has a coefficient of 0.71.[2]

3 Model and calibration

Our main purpose is to examine whether the stylized facts presented in Table 1 can be explained by a New-Keynesian model framework. The standard New-Keynesian model (surveyed by Clarida et al., 1999) is attractive in that it is derived from

[2]While these results are fairly standard in the literature (cf. Clarida et al., 2000), they are very sensitive to the choice of sample period. Excluding the first two years of data from the sample, the estimated coefficient of inflation falls to 0.69. The only coefficient that seems robust to the choice of sample period is that of the lagged interest rate, which is consistently estimated to be around 0.70.

microfoundations and is sparsely parameterized. However, it is well-known that this simple model has problems when confronted with the data, and additional inertia is often introduced in practical applications (see, e.g., Clarida et al., 1999; Estrella and Fuhrer, 2002). For our purposes we need a model that has the potential of matching the stylized facts for the U.S. economy. Therefore we choose the specification of Rudebusch (2002), which is an empirical model in the New-Keynesian tradition, and we extend this model to allow for varying degrees of forward-looking behavior in the determination of both inflation and output.[3] We close the model by assuming that the central bank chooses a path for the short-term interest rate to minimize (under discretion) a standard objective function. Thus, the model is given by the following three equations:[4]

$$\pi_t = \mu_\pi \mathrm{E}_{t-1}\bar{\pi}_{t+3} + (1-\mu_\pi)\sum_{j=1}^{4}\alpha_{\pi j}\pi_{t-j} + \alpha_y y_{t-1} + \varepsilon_t, \tag{2}$$

$$y_t = \mu_y \mathrm{E}_{t-1} y_{t+1} + (1-\mu_y)\sum_{j=1}^{2}\beta_{yj} y_{t-j} - \beta_r\left[i_{t-1} - \mathrm{E}_{t-1}\bar{\pi}_{t+3}\right] + \eta_t, \tag{3}$$

$$\min_{\{i_t\}} \ \mathrm{Var}\left[\bar{\pi}_t\right] + \lambda \mathrm{Var}\left[y_t\right] + \nu \mathrm{Var}\left[\Delta i_t\right]. \tag{4}$$

Equation (2) is an empirical version of a New-Keynesian Phillips curve (or aggregate supply equation), where inflation depends on expected and lagged inflation, the output gap of the previous period, and the "cost-push shock" ε_t. Equation (3) is an aggregate demand equation (or consumption Euler equation) that determines the output gap as a function of the expected and lagged output gap, the real short-term interest rate of the previous period, and the demand shock η_t. Equation (4) is the objective function for monetary policy; the central bank acts to minimize the weighted unconditional variances of inflation, the output gap, and the change in the interest rate. The target level for inflation is normalized to zero, while that of output is given by the potential level, so the target for the output gap is also zero.

[3] The main specification of Rudebusch (2002a) allows for forward-looking behavior in the determination of inflation, but not output. Rotemberg and Woodford (1997), Ireland (2001), Christiano et al. (2001), and Smets and Wouters (2002), among others, estimate New-Keynesian models with better microfoundations for private behavior. In comparison to our model, they do not model central bank behavior as optimizing, but specify a Taylor-type rule for monetary policy. Also, Rotemberg and Woodford (1997) and Ireland (2001) introduce persistence only through serially correlated shocks.

[4] Rudebusch (2002b) uses a similar model, with forward-looking behavior in both inflation, output, and the real interest rate. We choose a slightly different specification, since his model includes expected future interest rates, and therefore cannot be easily mapped into the standard linear RE framework (see, e.g., Söderlind, 1999). Dennis (2001a) suggests an alternative RE framework that is able to handle such model specifications.

Table 3: Parameter values

Inflation		Output gap	
$\alpha_{\pi 1}$	0.67	β_{y1}	1.15
$\alpha_{\pi 2}$	-0.14	β_{y2}	-0.27
$\alpha_{\pi 3}$	0.40	β_r	0.09
$\alpha_{\pi 4}$	0.07		
α_y	0.13		

Note: Parameters estimated by Rudebusch (2002a) on quarterly U.S. data, 1968Q3–1996Q4.

Therefore, although we assume that the central bank acts under discretion, there is no inflation bias, but inflation is on average equal to the target.[5]

Because we want to focus on specific aspects of New-Keynesian models and certain facts (i.e., the volatility and persistence of inflation, output and the interest rate), we have chosen to take certain parameter estimates for granted and to calibrate the remaining parameters to replicate the moments we are interested in as closely as possible. Thus, we choose to estimate those parameters that are crucial for the moments of interest: the importance of forward-looking behavior in the determination of inflation and output (μ_π and μ_y), the relative weights in the central bank's objective function (λ and ν), and the standard deviation of supply and demand shocks (σ_π, σ_y). For the other parameters we simply use the estimates of Rudebusch (2002a), shown in Table 3.[6]

The ideal way to match theory and facts remains a controversial issue. It is well known that point estimates using maximum likelihood methods can be fairly sensitive to small changes in the sample or estimation method (see, e.g., Ireland, 2001; Smets and Wouters, 2002; or Söderlind, 1999). This reflects the fact that theoretical models contain many parameters that are redundant when it comes to

[5]There remains a difference between the discretionary outcome and that under commitment, however: because discretionary policy cannot exploit private agents' expectations, it is less efficient in stabilizing the economy than the optimal policy under commitment. This inefficiency of discretionary policy has been termed "stabilization bias" (see Svensson, 1997, or Woodford, 1999), and is analyzed in detail by Dennis and Söderström (2002).

[6]These parameter values were estimated by Rudebusch (2002a) using OLS on quarterly U.S. data for the period 1968Q3–1996Q4 (with survey data for inflation expectations). Stability tests typically cannot reject the hypothesis of no structural breaks in such estimated equations (Rudebusch and Svensson, 1999; Rudebusch, 2002a; Dennis, 2001b); thus the estimates are likely to be approximately valid also for our shorter sample period. Rudebusch also estimates the value of μ_π to 0.29, but restricts μ_y to zero. Similar estimates for the α and β parameters are obtained restricting also μ_π to zero (Rudebusch and Svensson, 1999; Rudebusch, 2001), or using FIML or SUR techniques (Dennis, 2001b).

fitting the model to data, and that estimation techniques are sensitive to outliers. Rather than trying to find a single "best" estimate for the parameters of interest, we therefore choose to locate a range of parameter values which are broadly consistent with the data. Our approach to calibration is however relatively ambitious and is similar to GMM, except that our objective is to pick out a range of parameter combinations with almost equal fit. A GMM approach to matching the moments of interest gives virtually identical results.[7] In fact, we regard it as a useful alternative to methods that focus on one single "best" parameter combination.

We thus calibrate the parameter vector $(\lambda, \nu, \mu_\pi, \mu_y, \sigma_\pi, \sigma_y)$ to match the stylized facts of the U.S. economy presented in Table 1. This calibration is performed using a grid search over a broad range of parameter values. The grid search goes through all combinations of $\lambda, \nu \in \{0, 0.1, 0.25, 0.5, 1, 2, 5\}$; $\mu_\pi, \mu_y \in \{0.001, 0.1, 0.25, 0.5, 0.75, 0.9, 1\}$; and $\sigma_\pi, \sigma_y \in \{0.1, 0.15, 0.25, 0.5, 0.75, 1, 1.25\}$, resulting in 117,649 configurations. For each configuration we calculate the optimal discretionary policy rule and the resulting unconditional moments (standard deviations and autocorrelations) of inflation, output and the interest rate.[8] We then compare these model standard deviations and autocorrelations with those in actual U.S. data, and pick out the configurations that match the data most closely.

In the end, we choose to identify those configurations where the standard deviations and autocorrelations of inflation, output, and the interest rate lie within ± 1.25 standard errors from the values in the actual U.S. data. This procedure results in eight parameter configurations, shown in Table 4. For comparison, the table also includes a parameterization that is more common in the literature,[9] where $(\lambda, \nu, \mu_\pi, \mu_y) = (1.0, 0.5, 0.25, 0.25)$, but the shock variances are left at their calibrated values $(\sigma_\pi, \sigma_y) = (0.75, 0.5)$. Our results indicate that in order to match the time-series behavior of U.S. data, our model needs to be characterized by

- a fairly small preference for output stabilization: $\lambda \leq 0.10$;
- a large preference for interest rate smoothing: $0.5 \leq \nu \leq 2$;

[7] In the GMM estimation we minimize the function $(\xi_j - \hat{\xi})'(\xi_j - \hat{\xi})$ over our grid of parameter configurations, where ξ_j is the vector of moments for configuration j and $\hat{\xi}$ is the vector of moments in the data. All configurations in Table 4 are among the 25 best configurations according to this criterion, and the other configurations are qualitatively very similar.

[8] Appendix A shows how to use standard methods to calculate the optimal policy rule, the reduced form of the model, and the variance-covariance matrices of the state variables.

[9] The values for λ and ν in the "typical" parameterization are used as a benchmark by Rudebusch and Svensson (1999) and Rudebusch (2001), while the values for μ_π and μ_y are similar to those used by Rudebusch (2002a).

Table 4: Calibrated parameter configurations

Config. No.	λ	ν	μ_π	μ_y	σ_π	σ_y
1	0.00	1.00	0.10	0.50	0.75	0.50
2	0.00	2.00	0.001	0.75	0.75	0.15
3	0.00	2.00	0.001	0.75	0.75	0.25
4	0.00	2.00	0.001	0.75	0.75	0.50
5	0.00	2.00	0.001	0.90	0.75	0.50
6	0.10	0.50	0.10	0.50	0.75	0.50
7	0.10	1.00	0.001	0.75	0.75	0.50
8	0.10	1.00	0.001	0.90	0.75	0.50
Typical	1.00	0.50	0.25	0.25	0.75	0.50

Parameter values calibrated to match moments in U.S. data. The parameterizations all imply unconditional moments within ± 1.25 standard errors from moments in the data.

- a small degree of forward-looking behavior in price-setting: $\mu_\pi \leq 0.1$; and
- a large degree of forward-looking behavior in consumption/aggregate demand: $\mu_y \geq 0.5$.

Some of these calibrated parameter values may seem extreme by conventional standards. In theoretical analyses, authors often assume a larger preference for output stability than for interest rate smoothing, as in the typical configuration in Table 4. Instead, our calibration indicates that central bank behavior (at least that of the Federal Reserve) is dominated by a preference for interest rate smoothing rather than output stability.

However, this result finds some support in the empirical literature. Dennis (2001b) estimates the preference parameters of the Federal Reserve using full information maximum likelihood (FIML) for the period 1979–2000, and obtains estimates of $(\lambda, \nu) = (0.23, 12.3)$.[10] Favero and Rovelli (2001) use GMM for the period 1980–98 and obtain $(\lambda, \nu) = (0.00125, 0.0085)$. The differences between these results seem to be mainly due to Favero and Rovelli (2001) using a finite policy horizon (of four quarters), while Dennis (2001b) uses an infinite horizon (as in our model). Nevertheless, both studies find a more important role for interest rate smoothing than for output stabilization, as in our calibration.

[10] Matching the volatility of inflation, output, and the change in the interest rate, Dennis obtains $(\lambda, \nu) = (0.46, 0.74)$, but this parameterization implies a variance of the interest rate which is almost twice as large as in the data (Dennis, 2001b, Appendix 2).

The calibrated degrees of forward-looking behavior in inflation and output determination are perhaps less controversial. It is often argued that the purely forward-looking specification of the New-Keynesian model (with $\mu_\pi = \mu_y = 1$) is at odds with the data (Estrella and Fuhrer, 2002), and there is a large literature estimating versions of the New-Keynesian Phillips curve in equation (2). Galí and Gertler (1999) argue that the backward-looking term is not quantitatively important, but many other analyses tend to favor primarily backward-looking specifications, and estimate μ_π to be between 0.1 and 0.4, depending on sample period and estimation technique.[11] Lindé (2002) estimates versions of equations (2) and (3) on quarterly U.S. data for 1960–97, and obtains $\mu_\pi = 0.28$ and $\mu_y = 0.43$. Using a model similar to ours with optimal discretionary policy, Lansing and Trehan (2001) find that a large μ_y is a sufficient condition for the coefficients in the optimal policy rule to match the standard Taylor rule coefficients.

For our calibrated parameter configurations, Table 5 shows the standard deviations and autocorrelations, along with the moments of the data. Inflation and the interest rate are slightly more volatile than in the data, while the output gap is slightly more stable. In terms of autocorrelations, the calibrated parameterizations are very close to the actual data. In the more typical parameterization, inflation is too volatile and too persistent compared with the data, the output gap is too stable and the interest rate is much too volatile. Thus, our favored parameterizations are considerably more successful than more commonly used parameterizations in describing the behavior of all three variables.

To illustrate the behavior of the calibrated model, Figure 2 shows how the economy responds to unit shocks to the three variables at $t = 0$, using a "baseline" parameter configuration given by configuration No. 6 in Table 4. Although we have chosen not to match the cross-correlations in the data, these impulse responses look quite reasonable. After an interest rate disturbance[12] (in the first row), the interest rate is slowly moved back to neutral, and returns after four quarters. By construction (see equations (2)–(3)), there is no immediate response of inflation or output to the policy shock, but from $t = 1$ onwards, the output gap responds more quickly than inflation. The maximum effect on output (approximately -0.35%) comes after

[11] See, e.g., Fuhrer (1997), Roberts (2001), Rudd and Whelan (2001), Lindé (2002), or Jondeau and Le Bihan (2001).

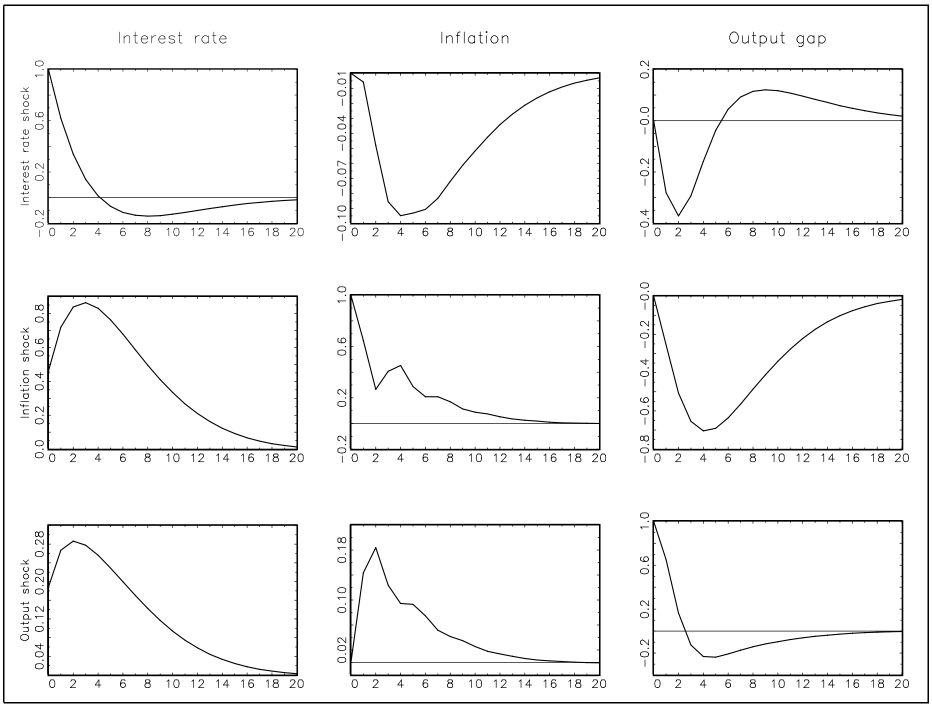
[12] The interest rate disturbance is not part of the model, since the interest rate is set optimally. Nevertheless, an artificial interest rate shock can be constructed by assuming that the interest rate is unexpectedly raised by one percentage point for one period, and that the system follows the reduced-form afterwards (as in Svensson, 2000). See Appendix B for details.

Table 5: Standard deviations and autocorrelations in actual data and model

	Config. No.	Standard deviation	Autocorrelations		
			1 lag	2 lags	3 lags
		(a) Inflation: π_t			
Data		*1.04*	*0.65*	*0.53*	*0.54*
Calibrated	1	1.10	0.69	0.49	0.51
	2	1.13	0.70	0.51	0.55
	3	1.14	0.71	0.51	0.55
	4	1.14	0.71	0.51	0.55
	5	1.13	0.70	0.50	0.55
	6	1.10	0.69	0.49	0.51
	7	1.14	0.71	0.51	0.55
	8	1.13	0.70	0.50	0.55
Typical		1.28	0.77	0.64	0.66
		(b) *Output gap*: y_t			
Data		*1.67*	*0.91*	*0.83*	*0.75*
Calibrated	1	1.57	0.91	0.77	0.65
	2	1.48	0.96	0.90	0.83
	3	1.50	0.95	0.88	0.81
	4	1.57	0.89	0.80	0.74
	5	1.54	0.86	0.79	0.73
	6	1.49	0.89	0.75	0.62
	7	1.53	0.88	0.78	0.72
	8	1.51	0.85	0.78	0.72
Typical		1.06	0.84	0.62	0.43
		(c) *Interest rate*: i_t			
Data		*1.51*	*0.94*	*0.86*	*0.74*
Calibrated	1	1.67	0.97	0.90	0.81
	2	1.57	0.99	0.95	0.90
	3	1.57	0.99	0.95	0.90
	4	1.58	0.99	0.95	0.90
	5	1.56	0.99	0.96	0.91
	6	1.71	0.96	0.89	0.79
	7	1.64	0.98	0.94	0.89
	8	1.63	0.99	0.95	0.90
Typical		2.04	0.93	0.80	0.66

Unconditional moments with calibrated parameters. The typical configuration is given by $(\lambda, \nu, \mu_\pi, \mu_y, \sigma_\pi, \sigma_y) = (1.0, 0.5, 0.25, 0.25, 0.75, 0.5)$.

Figure 2: Impulse responses in baseline model



Unit shocks. Baseline parameter values: $(\lambda, \nu, \mu_\pi, \mu_y, \sigma_\pi, \sigma_y) = (0.1, 0.5, 0.1, 0.5, 0.75, 0.5)$.

two quarters, while that on inflation (approximately -0.10%) comes later, after four to six quarters. This pattern is similar to that obtained from typical VAR models of the U.S. economy (see, e.g., Christiano et al., 1999).

After shocks to inflation and output, monetary policy responds only gradually, since the central bank dislikes large swings in the interest rate (ν is large).[13] After an inflation disturbance, the monetary policy response opens up a negative output gap, which is then closed very slowly (since the weight on output stabilization λ is small). After an output disturbance, the central bank must change the positive output gap into a negative gap in order to fight the inflationary impulse, and this is done fairly quickly (again because of the small λ). In both cases the quarterly inflation rate displays a rather volatile pattern, partly due to the fact that the central bank aims at stabilizing annual inflation.

This analysis indicates that our calibrated model provides a reasonable description of the U.S. economy. In contrast, a parameterization that is more common in the literature is clearly at odds with the data when describing the behavior of

[13]Note that the central bank aims to stabilize annual inflation rather than quarterly inflation (which is shown in the figure), and annual inflation of course responds more slowly to the disturbance than does quarterly inflation. The central bank therefore responds more slowly than would have been the case with a target for quarterly inflation (cf. Nessén, 2002).

output and the interest rate. The next section provides intuition for these results by more carefully scrutinizing the calibrated parameterizations.

4 Inspecting the mechanism: The key parameters

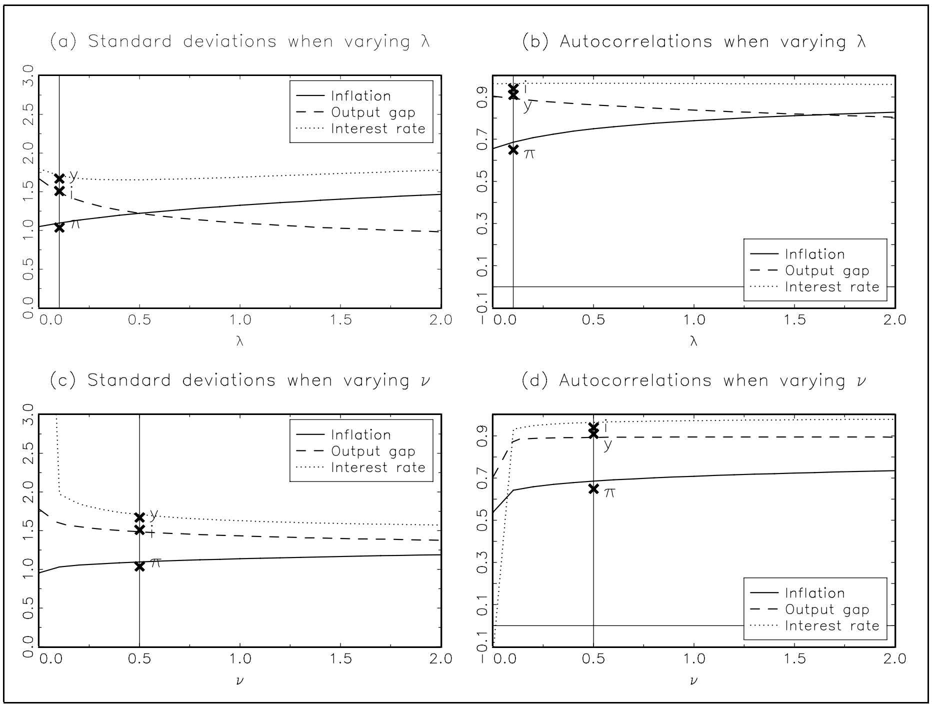
To some extent, the empirical literature gives support for our calibrated parameter configurations. However, this literature typically does not provide much intuition for the final choice of parameters. In contrast, since our approach aims at matching moments from different parameter configurations with those in actual data, it is straightforward to examine the consequences of alternative parameterizations. Because we jointly calibrate several parameters, we will also demonstrate how variations in a parameter in one equation affects the moments of another variable.

In this section we depart from the baseline configuration (No. 6 in Tables 4–5) in one parameter dimension at a time and calculate the resulting standard deviation and first-order autocorrelation of inflation, output and the interest rate.[14] This exercise is intended to explain the calibration in detail, and also to reveal the extent to which the different parameters contribute to the overall fit of the model. The results are reported in Figures 3–5. In each figure, vertical lines represent the baseline parameter values, and the three stars represent the moments of the actual data.

For the central bank's preference parameters, we would a priori expect that increasing the weight of one variable in the objective function would make that variable more stable and less persistent, since the central bank will act more strongly to offset the effects of shocks on that particular variable. For the other variables, one would expect the opposite pattern. Figure 3*a*–*b* shows that this intuition holds when varying the weight on output stabilization, λ. As λ increases (keeping the other parameters fixed), output becomes more stable and less persistent, while inflation and (to some extent) the interest rate become more volatile and more persistent. For inflation and output, increasing λ makes the standard deviation and autocorrelations move away from the values in the data, leading to a worse fit of the model. Decreasing λ towards zero has no important effect on the behavior of any variable; since ν is large, a zero weight on output stabilization does not lead to much volatility in the interest rate and output, as would have been the case with $\nu = 0$. Thus, Figure 3 indicates that *larger values of λ than in our calibrated configurations imply too high volatility and persistence of inflation and too low volatility and persistence of output*

[14]The higher-order autocorrelations give the same qualitative picture as first-order autocorrelations.

Figure 3: Varying preference parameters from the baseline configuration



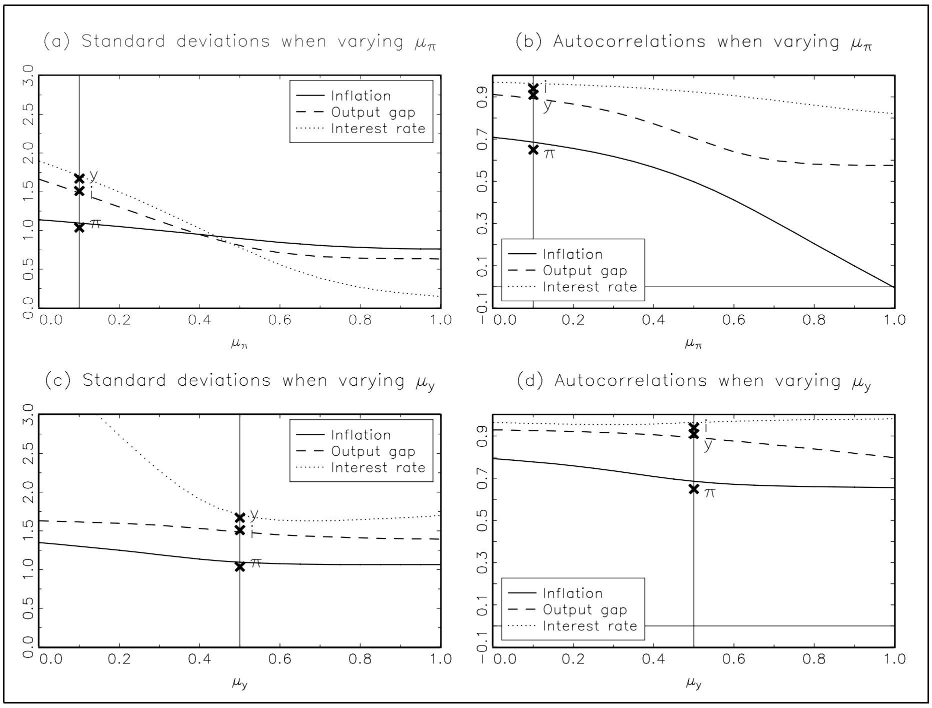
Unconditional standard deviations and first-order autocorrelations as the weight on output stabilization λ (upper panels) and interest rate smoothing ν (lower panels) vary from the baseline configuration (No. 6 in Table 4). Vertical lines represent baseline values, stars represent moments in actual data.

compared with U.S. data.

When it comes to the weight on interest rate smoothing, ν, Figure 3c–d reveal that our intuition needs to be somewhat refined. As ν is decreased from 0.5 towards zero, inflation becomes more stable and the interest rate more volatile, in accordance with the intuition. However, the volatility in output increases slightly: the direct effect of the increased interest rate volatility on output seems to dominate the larger weight on output stability (relative to interest rate smoothing) in the objective function. As ν falls, the standard deviation of inflation and output tend to approach the data and the standard deviation of the interest rate moves away from the data. The autocorrelation in all variables is not much affected by the decrease in ν, as long as $\nu > 0$. As ν approaches zero, the interest rate becomes extremely volatile (for $\nu = 0$, its standard deviation is close to 8.5%), and the autocorrelation of the interest rate approaches zero. Thus, *smaller values of ν lead to excessive volatility in the interest rate* relative to the data.

As for the importance of forward-looking behavior in the determination of inflation and output, we would a priori expect that more forward-looking in the determination of one variable would make that variable less persistent, while it is difficult

Figure 4: Varying the degree of forward-looking behavior from the baseline configuration

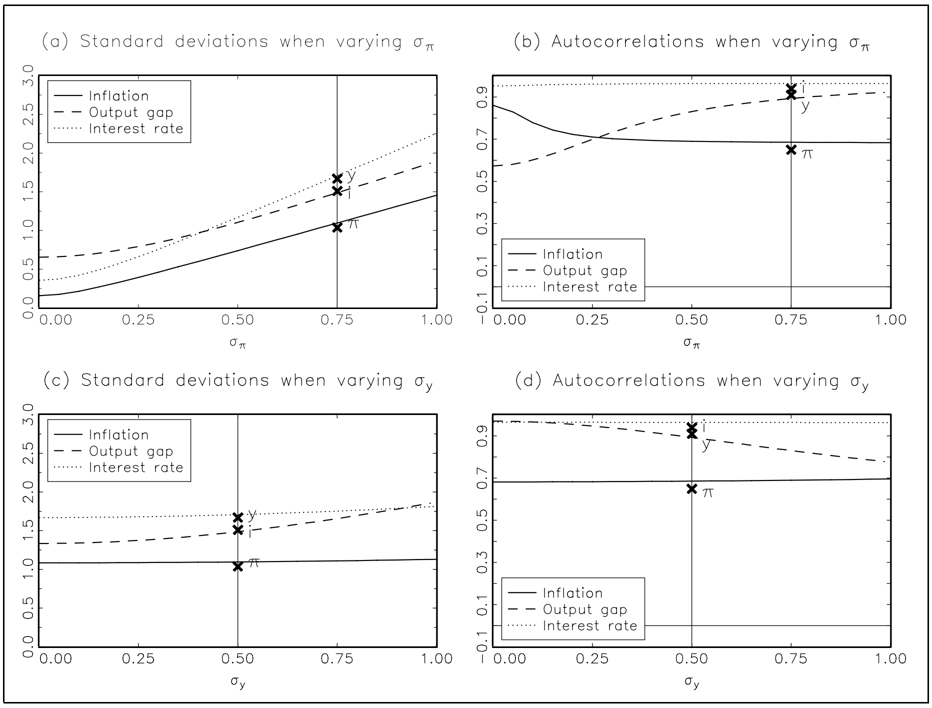


Unconditional standard deviations and first-order autocorrelations as the degree of forward-looking behavior in inflation μ_π (upper panels) and in output μ_y (lower panels) vary from the baseline configuration (No. 6 in Table 4). Vertical lines represent baseline values, stars represent moments in actual data.

to predict the effects on volatility. Figure 4*a*–*b* shows that increasing μ_π quickly reduces the persistence in inflation, but also output and the interest rate become less persistent. At the same time the volatility in all variables falls, with a particularly large effect on the interest rate. For all variables this decrease in volatility and persistence tends to move the model moments away from those found in the data. As μ_π approaches unity (a common case in the theoretical literature), the interest rate becomes very stable, and the autocorrelation of inflation approaches zero. While the previous literature has focused on the need for backward-looking elements in inflation to match the persistence of inflation, our results show that a small degree of forward-looking is needed also to match the behavior of output and the interest rate. Thus, *larger values of μ_π than in our calibration imply too low volatility of output and the interest rate and too low persistence of inflation and output.*

Figure 4*c*–*d* show that a larger degree of forward-looking in output makes the output gap less volatile and persistent, but here the effects are very small. As μ_y falls also inflation and the interest rate become more volatile, and the standard deviation of inflation and the interest rate move away from the values in the data. This effect is

Figure 5: Varying the standard deviation of shocks from the baseline configuration



Unconditional standard deviations and first-order autocorrelations as the standard deviation of inflation shocks σ_π (upper panels) and output shocks σ_y (lower panels) vary from the baseline configuration (No. 6 in Table 4). Vertical lines represent baseline values, stars represent moments in actual data.

particularly strong for the volatility of the interest rate, which increases considerably as μ_y falls. Indeed, letting μ_y approach zero has no important effect on inflation or output, while it leads to a very volatile interest rate (with a standard deviation of 3.7% when $\mu_y = 0$). As output (and consumption) becomes more inertial and less forward-looking, larger movements in the interest rate are needed to persuade consumers to adjust.[15] We conclude that *smaller values of μ_y lead to excessive volatility in the interest rate and inflation* relative to the data. Surprisingly, the behavior of output is not much affected by changes in μ_y.

Finally, Figure 5 shows the effects of varying the standard deviation of inflation and output shocks. While increasing σ_π leads to more volatility in all variables, increasing σ_y only affects the volatility in output. This reflects the fact that supply shocks pose a more serious trade-off to policymakers, who must contract output to decrease inflation. Demand shocks, on the other hand, are more easily mitigated. It is thus clear from Figure 5 that *the calibrated value of σ_π serves to match the*

[15]This is consistent with Lansing and Trehan (2001), who argue that a small degree of forward-looking in output leads to very aggressive policy behavior, i.e., large coefficients in an optimized Taylor-type rule.

volatility in all variables whereas σ_y is chosen to match the volatility in output.

While the main purpose of this section was to explain the parameters resulting from our calibration exercise, the discussion has also highlighted some important aspects of this class of models. When the economy is given by a system of simultaneous equations, the largest effects of changing a given parameter in one equation may well be on the behavior of some other variable in the system. Our results make clear that to explain the behavior of the interest rate (and thus monetary policy) we need not only a large weight on interest rate smoothing, but also a fairly large degree of forward-looking behavior in the determination of output and a small degree of forward-looking in price-setting. The time-series properties of inflation can be explained only if our model includes a small degree of forward-looking behavior in price-setting (as noted elsewhere), but also a small weight on output stabilization and a large degree of forward-looking behavior in output. Finally, to match the behavior of the output gap we need a small weight on output stabilization and a small degree of forward-looking in price-setting, while the degree of forward-looking in output is less important. These results strengthen the argument that results from single-equation analyses may not be sufficient to pin down the value of any one parameter; a systems approach is often more appropriate.

5 Robustness issues

Our calibration in the previous sections is conditioned on a number of choices regarding data, parameter values, model specification and matching criteria. This section discusses the extent to which these choices affect our calibration results.

First, our calibrated parameter configurations are chosen so that the moments of the model lie within ± 1.25 standard errors from those in the data. Figure 6 shows how the calibrated parameter values vary as the range varies from ± 1 to ± 3.5 standard errors. This gives an idea of the robustness of the different parameter values. It seems that some parameter values are more robust than others: when we increase the accepted range, the set of some parameter values grows faster than for other values. Up to ± 1.5 standard errors, the results are very much the same as in our preferred calibrations, while above ± 2 standard errors, ν and μ_y cover almost the entire permitted interval (except $\nu = 0$). However, $\nu = 0$ is only picked out for ± 3.25 standard errors; loosely speaking the hypothesis of no interest rate smoothing is "rejected" using any reasonable level of significance.

Second, while calibrating the model to match data from 1987 to 1999, we use

Figure 6: Parameter ranges for varying criteria

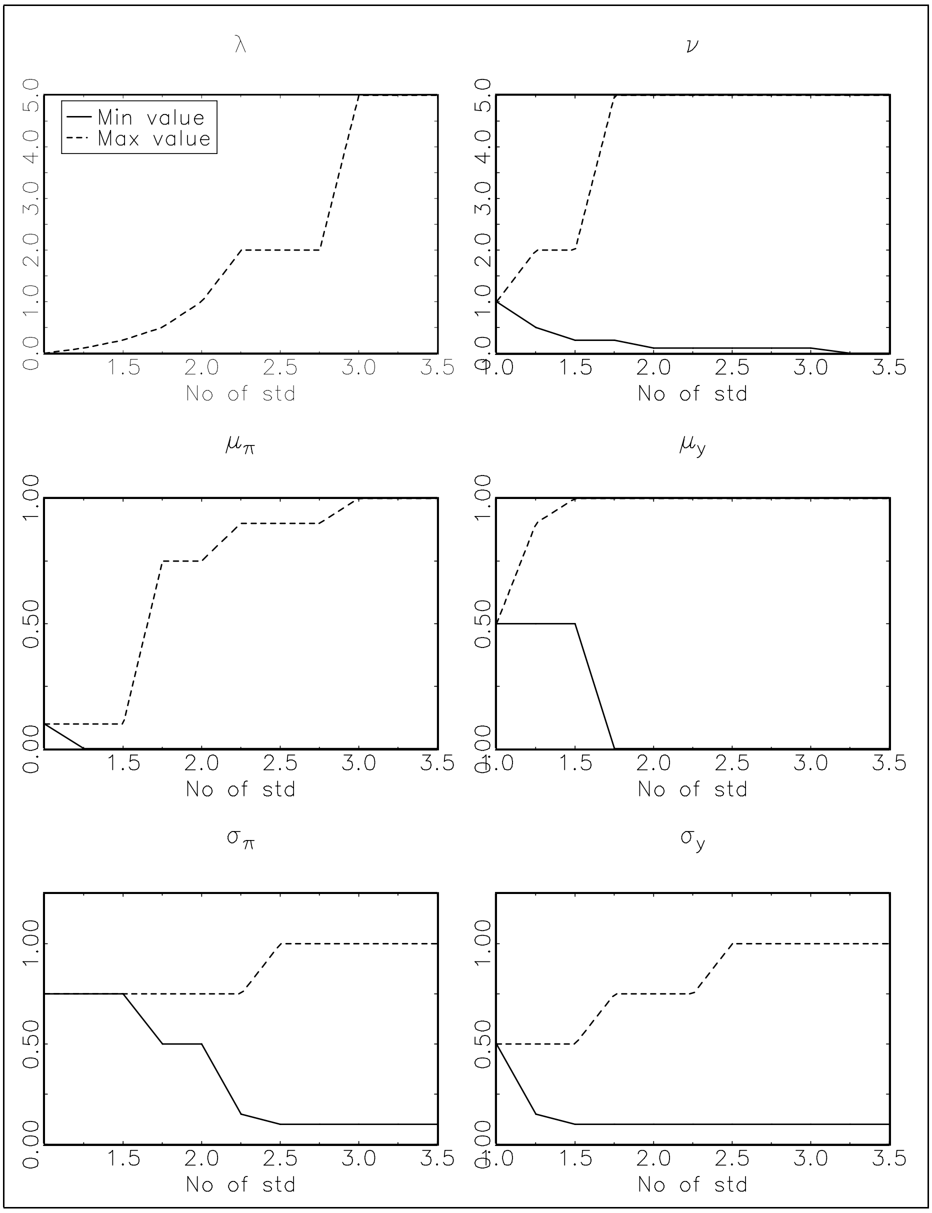


Table 6: Parameter values, 1987–2001

Inflation		Output gap	
$\alpha_{\pi 1}$	0.282	β_{y1}	1.229
$\alpha_{\pi 2}$	−0.025	β_{y2}	−0.244
$\alpha_{\pi 3}$	0.292	β_r	0.073
$\alpha_{\pi 4}$	0.385		
α_y	0.141		

Note: Parameters estimated by Castelnuovo (2002) on quarterly U.S. data, 1987Q3–2001Q1. The parameters μ_π and μ_y are restricted to zero.

some parameter estimates from the period 1968–1996. The latter period includes the period of high and variable inflation during the 1970s, so if the parameters estimated by Rudebusch (2002a) are not truly structural, they may not be representative for the more recent sample period. As an alternative, Table 6 shows parameter estimates obtained by Castelnuovo (2002) when estimating the purely backward-looking version of the model ($\mu_\pi = \mu_y = 0$) for the sample 1987Q3–2001Q1. The main difference from the values in Table 3 relates to the autoregressive parameters in the Phillips curve: in the more recent sample the weights on lagged inflation are shifted somewhat towards the longer lags than when including also the earlier period. Using these parameter values, we obtain similar results for λ, σ_π and σ_y, even larger values for ν (above 2), and smaller values for μ_y (around 0.25). Most interestingly, this calibration results in inflation being either purely backward-looking ($\mu_\pi = 0$) or, more often, purely forward-looking ($\mu_\pi = 1$). This suggests that the parameters in the inflation and output equations are important when calibrating the degree of forward-looking behavior, but less so when determining the preference parameters of the central bank.

A third issue concerns the measurement of the output gap. It is rather standard in the literature to define the output gap as the deviation of real GDP from potential, calculated by the Congressional Budget Office. However, the CBO's methodology to calculate potential output leads to a rather smooth series, and so may exaggerate the volatility of the output gap. The results in Section 4 suggest that calibrating the model to match a less volatile output gap would imply a slightly larger λ and μ_π and a smaller σ_π. Still, since the standard deviation of the output gap is not the only moment that ties down the calibration (the small value of λ is also related to the volatility of inflation and the persistence in the output gap), our qualitative results are unlikely to be very sensitive to the choice of output gap measure.

Fourth, estimated Taylor rules are often used to discuss the issue of interest rate smoothing. However, Section 3 focuses on matching only the volatility of key economic variables, not the coefficients in the Taylor rule. If instead we try to match the estimated rule in Table 2 our calibration yields virtually any value for λ, while for ν, μ_π and μ_y the basic results from the previous calibration remain unaltered. In particular, we still get a large value for ν: to match the degree of persistence in the Taylor rule, we must allow for a very large weight on interest rate smoothing (even larger than in Section 3). Furthermore, the preference for interest rate smoothing is always at least as large as the preference for output stabilization.

Finally, in addition to introducing more lags to the theoretical version of the New-Keynesian model, the Rudebusch (2002a) model also uses a slightly different dating of expectations. While this seemingly innocent specification is rather common in empirical modeling (see, e.g., Rotemberg and Woodford, 1997, or Christiano et al., 2001), it has important consequences for the dynamics of the model (Dennis and Söderström, 2002). We therefore examine also the more standard dating of expectations using the specification

$$\pi_t = \mu_\pi \mathrm{E}_t \bar{\pi}_{t+3} + (1-\mu_\pi) \sum_{j=1}^{4} \alpha_{\pi j} \pi_{t-j} + \alpha_y y_{t-1} + \varepsilon_t, \tag{5}$$

$$y_t = \mu_y \mathrm{E}_t y_{t+1} + (1-\mu_y) \sum_{j=1}^{2} \beta_{yj} y_{t-j} - \beta_r \left[i_{t-1} - \mathrm{E}_{t-1} \bar{\pi}_{t+3} \right] + \eta_t. \tag{6}$$

Calibrating this model gives virtually identical results to those in Section 3, the main difference being that this calibration always yields $\lambda = 0$ and $\nu > 1$.

These robustness exercises suggest that the results for the degree of forward-looking in inflation and output may not be entirely robust: some alternative calibrations favor parameterizations with a large degree of forward-looking in inflation and a fairly small degree of forward-looking in output. The parameters governing central bank behavior, on the other hand, seem very robust. Although some calibrations yield a larger preference for output stabilization than in Section 3, the preference for interest rate smoothing is always at least as large as (and in most parameterizations many times larger than) the weight on output stabilization.

6 Concluding remarks

The main purpose of this paper has been to examine whether a suitably calibrated New-Keynesian model of optimal discretionary monetary policy can match some broad characteristics of the U.S. economy. This is an important issue, since calibrated models of this kind are frequently used to analyze monetary policy and as a basis for policy recommendations. Our analysis shows that it is indeed possible to match some important stylized facts using a model with New-Keynesian features, but empirically relevant calibrations entail some controversial parameter values.

First, frequently used calibrated models often assume that the central bank's objective function is characterized by a low preference for interest rate smoothing. Our results show—like some earlier papers—that this makes it hard to match the low volatility of the interest rate in actual data.

Second, a standard assumption is that it is appropriate to assume that the central bank has a relatively strong preference for output stabilization. We find that unless there is a small (virtually zero) preference for output stabilization, our New-Keynesian model can hardly match the low volatility and persistence in inflation, or the high volatility and persistence in the output gap.

A third result from our exercises is that a large degree of backward-looking behavior in the Phillips curve is needed to match the high persistence in inflation. This is well known from earlier studies. But since these studies have often focused on the time series properties of inflation, it has not been noted that backward-looking behavior in inflation is important also for explaining the volatility and persistence of output and the interest rate.

Finally, we find that an empirically relevant New-Keynesian model needs a fairly large degree of forward-looking behavior in the aggregate demand equation. Specifically, this is needed to match the low volatility in the interest rate: with a more persistent output gap, larger interest rate movements are needed to affect aggregate demand.

Although our preferred parameter values differ from those typically used in the literature, they are by no means counter-intuitive. The tradition to work with models with a relatively strong preference for output stability and a weak preference for interest rate smoothing (a high λ in relation to ν) does not seem to be based on economic theory. Rather, it seems to be based on the simple observation that central banks do not pursue "strict" inflation targeting (so λ and ν are not both zero): inflation is too persistent and interest rates are too stable to be consistent with "strict" inflation targeting. Our results suggest that it is better to describe "flexible" monetary policy as reflecting a preference for interest rate smoothing rather than output stability.

There are good reasons to believe that central banks have an interest rate smoothing objective in addition to their preferences for price stability. A preference for interest rate smoothing can be motivated by central bank concerns about stability on financial markets and the payment system in particular.[16] Listening to central bank rhetoric also gives the impression that financial stability is a more important objective than output stability. During the disinflation that many developed countries have experienced since the early 1980s, central banks have been rather unwilling to admit that they care about output stability. Meanwhile, there have been many actions taken with the explicit intent to promote financial stability (see Estrella, 2001, for an overview).

It should be stressed that much work remains before central banks' responsibility for financial stability and payment system stability can be analyzed using formal models with both good micro-foundations and empirical support. A relatively large weight on interest rate smoothing in relation to output stability in a quadratic loss function is of course a very crude way to model central banks' behavior. Furthermore, central banks do not seem to behave in the linear way assumed in our model and most other analyses of monetary policy. Rather, they seem to change interest rates in a step-wise fashion, and the most common policy decision is to leave the instrument rate unchanged. The rationale for such a policy remains to be discovered, but it does imply a high degree of interest rate smoothing. What we suggest is that, within the linear-quadratic framework commonly applied, policy should be described in terms of a high ν in relation to λ, not the other way around. We think that earlier exercises with calibrated models may have missed this point because they have paid too little attention to the time-series properties of nominal interest rates (in relation to, e.g., inflation).

We do not think that the relatively high degree of forward-looking behavior in the aggregate demand relation is unreasonable either. The microfoundations of the aggregate demand equation stem from the representative individual's desire to smooth consumption over time (an Euler equation). With the deregulations and innovations in financial markets that have taken place since the 1980s, it has become easier for households to smooth their consumption over time and to implement the forward-looking behavior that would be optimal in the absence of credit market restrictions. The growing interest of the general public in the development of the stock market also suggests that consumption plans nowadays are largely forward-looking. The low degree of forward-looking in the aggregate supply equation suggested by our

[16] See, e.g., Cukierman (1991) and Goodfriend (1991). Lorenzoni (2001) presents a theoretical analysis of the dual objectives of price stability and payment system stability, and their connection to interest rate smoothing.

results is more difficult to understand. It has been recognized earlier that this may be needed to make the New-Keynesian model consistent with the data, but there is no convincing theoretical argument for a very low μ_π.

Our results suggest that policy recommendations and other conclusions based on New-Keynesian models with parameter values that have now become standard should be taken with a grain of salt. Before such exercises are discussed seriously, the consistency between the calibrated models and reality needs more careful examination. Steps in this direction, in the form of econometrically estimated New-Keynesian models of aggregate supply, aggregate demand and monetary policy, have recently been presented by Dennis (2001b), Favero and Rovelli (2001), and Lindé (2002). Their results are not entirely consistent, however, and more work along these lines is clearly needed.

A Model appendix

To solve the model we first write it on state-space form. Lead (2) and (3) one period:

$$\begin{aligned}\pi_{t+1} &= \frac{\mu_\pi}{4}\mathrm{E}_t\left[\pi_{t+1}+\pi_{t+2}+\pi_{t+3}+\pi_{t+4}\right] \\ &\quad +(1-\mu_\pi)\left[\alpha_{\pi 1}\pi_t+\alpha_{\pi 2}\pi_{t-1}+\alpha_{\pi 3}\pi_{t-2}+\alpha_{\pi 4}\pi_{t-3}\right]+\alpha_y y_t+\varepsilon_{t+1}, \qquad \text{(A1)} \\ y_{t+1} &= \mu_y \mathrm{E}_t y_{t+2}+(1-\mu_y)\left[\beta_{y1}y_t+\beta_{y2}y_{t-1}\right] \\ &\quad -\beta_r\left[i_t-\frac{1}{4}\mathrm{E}_t\left(\pi_{t+1}+\pi_{t+2}+\pi_{t+3}+\pi_{t+4}\right)\right]+\eta_{t+1}. \qquad \text{(A2)}\end{aligned}$$

Then solve for the forward-looking variables $\mathrm{E}_t\pi_{t+4}$ and $\mathrm{E}_t y_{t+2}$ and take expectations as of period t:

$$\begin{aligned}\frac{\mu_\pi}{4}\mathrm{E}_t\pi_{t+4} &= \left(1-\frac{\mu_\pi}{4}\right)\mathrm{E}_t\pi_{t+1}-\frac{\mu_\pi}{4}\mathrm{E}_t\pi_{t+2}-\frac{\mu_\pi}{4}\mathrm{E}_t\pi_{t+3} \\ &\quad -(1-\mu_\pi)\left[\alpha_{\pi 1}\pi_t+\alpha_{\pi 2}\pi_{t-1}+\alpha_{\pi 3}\pi_{t-2}+\alpha_{\pi 4}\pi_{t-3}\right]-\alpha_y y_t \qquad \text{(A3)} \\ \mu_y\mathrm{E}_t y_{t+2} &+ \frac{\beta_r}{4}\mathrm{E}_t\pi_{t+4} = \mathrm{E}_t y_{t+1}-(1-\mu_y)\left[\beta_{y1}y_t+\beta_{y2}y_{t-1}\right] \\ &\quad +\beta_r\left[i_t-\frac{1}{4}\mathrm{E}_t\left(\pi_{t+1}+\pi_{t+2}+\pi_{t+3}\right)\right], \qquad \text{(A4)}\end{aligned}$$

and reintroduce the disturbances via the (predetermined) variables

$$\pi_{t+1} = \mathrm{E}_t\pi_{t+1}+\varepsilon_{t+1}, \qquad \text{(A5)}$$

$$y_{t+1} = \mathrm{E}_t y_{t+1}+\eta_{t+1}. \qquad \text{(A6)}$$

Define an $(n_1 \times 1)$ vector $(n_1 = 11)$ of predetermined state variables as[17]

$$x_{1t} = \left\{\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}, y_{t-2}, y_{t-3}, i_{t-1}, i_{t-2}, i_{t-3}\right\}', \qquad \text{(A7)}$$

an $(n_2 \times 1)$ vector $(n_2 = 4)$ of forward-looking jump variables as

$$x_{2t} = \left\{\mathrm{E}_t\pi_{t+1}, \mathrm{E}_t\pi_{t+2}, \mathrm{E}_t\pi_{t+3}, \mathrm{E}_t y_{t+1}\right\}', \qquad \text{(A8)}$$

and an $(n_1 \times 1)$ vector of shocks to the predetermined variables as

$$v_{1t} = \left\{\varepsilon_t, \mathbf{0}'_{3\times 1}, \eta_t, \mathbf{0}'_{6\times 1}\right\}'. \qquad \text{(A9)}$$

We can then write the model in compact form as

$$A_0\begin{bmatrix} x_{1t+1} \\ \mathrm{E}_t x_{2t+1}\end{bmatrix} = A_1\begin{bmatrix} x_{1t} \\ x_{2t}\end{bmatrix}+B_1 i_t+v_{t+1}, \qquad \text{(A10)}$$

[17] The additional lags of the output gap and the interest rate are not state variables, but are needed to calculate the unconditional autocorrelations below.

where

$$v_{t+1} = \begin{bmatrix} v_{1t+1} \\ \mathbf{0}_{n2\times 1} \end{bmatrix}, \tag{A11}$$

and where the matrices A_0, A_1 and B_1 contain the parameters of the model. The shock vector v_{1t} has covariance matrix Σ_{v1}, which is a diagonal matrix with diagonal $\left\{\sigma_\varepsilon^2, \mathbf{0}'_{3\times 1}, \sigma_\eta^2, \mathbf{0}'_{6\times 1}\right\}$ and zeros elsewhere.

To obtain the usual state-space form, premultiply (A10) by A_0^{-1} to get[18]

$$\begin{bmatrix} x_{1t+1} \\ \mathrm{E}_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bi_t + v_{t+1}, \tag{A12}$$

where $A = A_0^{-1}A_1$ and $B = A_0^{-1}B_1$.[19]

To write the central bank's objective function (4), it is convenient to define a vector of target variables as

$$z_t = \{\bar{\pi}_t, y_t, \Delta i_t\}', \tag{A13}$$

which can be calculated by

$$z_t = C_x x_t + C_i i_t. \tag{A14}$$

The central bank's period loss function in (4) can then be written as

$$\begin{aligned} L_t &= \bar{\pi}_t^2 + \lambda y_t^2 + \nu (\Delta i_t)^2 \\ &= z_t' K z_t, \end{aligned} \tag{A15}$$

where K is a matrix of preference parameters. Using (A14), the loss function can be expressed as

$$\begin{aligned} L_t &= z_t' K z_t \\ &= \begin{bmatrix} x_t' & i_t' \end{bmatrix} \begin{bmatrix} C_x' \\ C_i' \end{bmatrix} K \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix} \\ &= x_t' C_x' K C_x x_t + x_t' C_x' K C_i i_t + i_t' C_i' K C_x x_t + i_t' C_i' K C_i i_t \\ &= x_t' Q x_t + x_t' U i_t + i_t' U' x_t + i_t' R i_t, \end{aligned} \tag{A16}$$

[18]This means that A_0 must be non-singular, i.e., $\mu_\pi, \mu_y \neq 0$.

[19]Note that $A_0^{-1} v_{t+1} = v_{t+1}$ since A_0 is block diagonal with an identity matrix as its upper left block and the lower block of v_{t+1} is zero.

where

$$x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}, \tag{A17}$$

and where

$$\begin{aligned} Q &= C_x' K C_x, & \text{(A18)} \\ U &= C_x' K C_i, & \text{(A19)} \\ R &= C_i' K C_i. & \text{(A20)} \end{aligned}$$

Thus the central bank's control problem is given by the conventional Bellman equation

$$J(x_t) = \min_{i_t} \left\{ x_t' Q x_t + x_t' U i_t + i_t' U' x_t + i_t' R i_t + \delta \mathrm{E}_t J(x_{t+1}) \right\}, \tag{A21}$$

subject to the transition equation (A12), and the optimal policy rule can be calculated using standard methods (see Söderlind, 1999, for an overview).

The optimal policy under discretion is a rule for the interest rate as a linear function of the predetermined variables:

$$i_t = F x_{1t}, \tag{A22}$$

resulting in the reduced form

$$\begin{aligned} x_{1t+1} &= M x_{1t} + v_{1t+1}, & \text{(A23)} \\ x_{2t} &= N x_{1t}. & \text{(A24)} \end{aligned}$$

See Söderlind (1999) for details. The target variables in z_t then follow

$$z_t = C x_{1t}, \tag{A25}$$

where

$$C = C_{x1} + C_{x2} N + C_i F. \tag{A26}$$

The reduced form (A23) implies that the unconditional variance-covariance matrix of x_{1t} satisfies

$$\Sigma_{x1} = M \Sigma_{x1} M' + \Sigma_{v1}, \tag{A27}$$

and using the vec operator and solving for $\mathrm{vec}\,(\Sigma_{x1})$, we get

$$\mathrm{vec}\,(\Sigma_{x1}) = (I - M \otimes M)^{-1}\, \mathrm{vec}\,(\Sigma_{v1})\,. \tag{A28}$$

The covariance matrix of x_{2t} is then given by

$$\Sigma_{x2} = N\Sigma_{x1}N', \tag{A29}$$

and that of z_t is

$$\Sigma_z = C\Sigma_{x1}C'. \tag{A30}$$

B Responses to an interest rate shock

In order to model a monetary policy shock, i.e., a one-time shock to the interest rate, suppose the central bank changes the interest rate at time $t = 0$ by di_t, and from then on follows its optimal policy rule $i_t = Fx_{1t}$ for all $t > 0$. How does the economy respond to such a shock?

Note first that the predetermined variables in x_{1t} do not respond to a change in i_t, so $dx_{1t} = 0$. The forward-looking variables in x_{2t}, on the other hand, respond immediately. But the response of x_{2t} depends on the response of $\mathrm{E}_t x_{2t+1}$. Partition A and B conformably with x_{1t} and x_{2t}. Then the response of $\mathrm{E}_t x_{2t+1}$ is, using (A24) and (A12),

$$\begin{aligned} d\mathrm{E}_t x_{2t+1} &= Nd\mathrm{E}_t x_{1t+1} \\ &= N\left[A_{11}dx_{1t} + A_{12}dx_{2t} + B_1 di_t\right]. \end{aligned} \tag{B1}$$

From (A12) we also get

$$d\mathrm{E}_t x_{2t+1} = A_{21}dx_{1t} + A_{22}dx_{2t} + B_2 di_t. \tag{B2}$$

Combining these expressions and using $dx_{1t} = 0$ we get

$$dx_{2t} = \left[A_{22} - NA_{12}\right]^{-1}\left[NB_1 - B_2\right] di_t. \tag{B3}$$

The variables in x_{1t+1} then respond by

$$\begin{aligned} dx_{1t+1} &= A_{12}dx_{2t} + B_1 di_t \\ &= \left\{A_{12}\left[A_{22} - NA_{12}\right]^{-1}\left[NB_1 - B_2\right] + B_1\right\} di_t, \end{aligned} \tag{B4}$$

and from then on the system follows (A23) and (A24).

References

Castelnuovo, Efrem, "Squeezing the interest rate smoothing weight with a hybrid expectations model," Working Paper No. 2002-6, IEP, Università Bocconi, November 2002.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Monetary policy shocks: What have we learned and to what end?" in Woodford, Michael and John B. Taylor (eds.), *Handbook of Macroeconomics*, Elsevier Science North-Holland, Amsterdam, 1999.

———, "Nominal rigidities and the dynamic effects of a shock to monetary policy," Working Paper No. 8403, National Bureau of Economic Research, July 2001.

Clarida, Richard, Jordi Galí, and Mark Gertler, "The science of monetary policy: A New Keynesian perspective," *Journal of Economic Literature* 37 (4), 1661–1707, December 1999.

———, "Monetary policy rules and macroeconomic stability: Evidence and some theory," *Quarterly Journal of Economics* 115 (1), 147–180, February 2000.

Congressional Budget Office, "CBO's method for estimating potential output," memorandum, Congressional Budget Office, October 1995.

Cukierman, Alex, "Why does the Fed smooth interest rates?" in Belongia, Michael T. (ed.), *Monetary policy on the 75th anniversary of the Federal Reserve System*, Kluwer Academic Publishers, Boston, 1991.

Dennis, Richard, "Optimal policy in rational-expectations models: New solution algorithms," Working Paper No. 2001-09, Federal Reserve Bank of San Francisco, July 2001a.

———, "The policy preferences of the U.S. Federal Reserve," Working Paper No. 2001-08, Federal Reserve Bank of San Francisco, July 2001b.

Dennis, Richard and Ulf Söderström, "How important is precommitment for monetary policy?" Working Paper No. 139, Sveriges Riksbank, September 2002.

Estrella, Arturo, "Dealing with financial instability: The central bank's tool kit," *Sveriges Riksbank Economic Review* 2, 34–49, 2001.

Estrella, Arturo and Jeffrey C. Fuhrer, "Dynamic inconsistencies: Counterfactual implications of a class of rational-expectations models," *American Economic Review* 92 (4), 1013–1028, September 2002.

Favero, Carlo A. and Riccardo Rovelli, "Macroeconomic stability and the preferences of the Fed. A formal analysis, 1961–98," Manuscript, IGIER, Università Bocconi, July 2001. Forthcoming, *Journal of Money, Credit, and Banking.*

Fuhrer, Jeffrey C., "The (un)importance of forward-looking behavior in price specifications," *Journal of Money, Credit, and Banking* 29 (3), 338–350, August 1997.

Fuhrer, Jeffrey C. and George Moore, "Inflation persistence," *Quarterly Journal of Economics* 110 (1), 127–159, February 1995.

Galí, Jordi and Mark Gertler, "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics* 44 (2), 195–222, October 1999.

Goodfriend, Marvin, "Interest rates and the conduct of monetary policy," *Carnegie-Rochester Conference Series on Public Policy* 34, 7–30, Spring 1991.

Ireland, Peter N., "Sticky-price models of the business cycle: Specification and stability," *Journal of Monetary Economics* 41 (1), 3–18, February 2001.

Jondeau, Eric and Hervé Le Bihan, "Testing for a forward-looking Phillips curve: Additional evidence from European and U.S. data," Manuscript, Banque de France, August 2001.

Lansing, Kevin J. and Bharat Trehan, "Forward-looking behavior and the optimality of the Taylor rule," Working Paper No. 2001-03, Federal Reserve Bank of San Francisco, February 2001.

Lindé, Jesper, "Estimating New-Keynesian Phillips curves: A full information maximum likelihood approach," Working Paper No. 129, Sveriges Riksbank, December 2002. (Revised version).

Lorenzoni, Guido, "Interest rate stabilization and monetary control: A reconciliation," Manuscript, Princeton University, October 2001.

McCallum, Bennett T. and Edward Nelson, "An optimizing IS-LM specification for monetary policy and business cycle analysis," *Journal of Money, Credit, and Banking* 31 (3), 296–316, August 1999.

Nessén, Marianne, "Targeting inflation over the short, medium and long term," *Journal of Macroeconomics* 24 (3), 313–329, September 2002.

Newey, Whitney K. and Kenneth D. West, "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica* 55 (3), 703–708, May 1987.

Roberts, John M., "How well does the New Keynesian sticky-price model fit the data?" Finance and Economics Discussion Paper No. 2001-13, Board of Governors of the Federal Reserve System, February 2001.

Rotemberg, Julio J. and Michael Woodford, "An optimization-based econometric framework for the evaluation of monetary policy," in Bernanke, Ben S. and Julio J. Rotemberg (eds.), *NBER Macroeconomics Annual*, The MIT Press, Cambridge, Mass., 1997.

Rudd, Jeremy and Karl Whelan, "New tests of the New-Keynesian Phillips curve," Finance and Economics Discussion Paper No. 2001-30, Board of Governors of the Federal Reserve System, June 2001.

Rudebusch, Glenn D., "Is the Fed too timid? Monetary policy in an uncertain world," *Review of Economics and Statistics* 83 (2), 203–217, May 2001.

———, "Assessing nominal income rules for monetary policy with model and data uncertainty," *Economic Journal* 112 (479), 1–31, April 2002a.

———, "Term structure evidence on interest rate smoothing and monetary policy inertia," *Journal of Monetary Economics* 49 (6), 1161–1187, September 2002b.

Rudebusch, Glenn D. and Lars E. O. Svensson, "Policy rules for inflation targeting," in Taylor, John B. (ed.), *Monetary Policy Rules*, University of Chicago Press, 1999.

Smets, Frank and Raf Wouters, "An estimated stochastic dynamic general equilibrium model of the euro area," Working Paper No. 171, European Central Bank, August 2002. Forthcoming, *Journal of the European Economic Association.*

Söderlind, Paul, "Solution and estimation of RE macromodels with optimal policy," *European Economic Review Papers and Proceedings* 43 (4–6), 813–823, April 1999.

Svensson, Lars E. O., "Optimal inflation targets, 'conservative' central banks, and linear inflation contracts," *American Economic Review* 87 (1), 98–114, March 1997.

———, "Open-economy inflation targeting," *Journal of International Economics* 50 (1), 155–184, February 2000.

Woodford, Michael, "Optimal monetary policy inertia," Working Paper No. 7261, National Bureau of Economic Research, July 1999.