

# Decoherence induced CPT violation and entangled neutral mesons

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We discuss two classes of semi-microscopic theoretical models of stochastic space-time foam in quantum gravity and the associated effects on entangled states of neutral mesons, signalling an intrinsic breakdown of CPT invariance. One class of models deals with a specific model of foam, initially constructed in the context of non-critical (Liouville) string theory, but viewed here in the more general context of effective quantum-gravity models. The relevant Hamiltonian perturbation, describing the interaction of the meson with the foam medium, consists of off-diagonal stochastic metric fluctuations, connecting distinct mass eigenstates (or the appropriate generalisation thereof in the case of K-mesons), and it is proportional to the relevant momentum transfer (along the direction of motion of the meson pair). There are two kinds of CPT-violating effects in this case, which can be experimentally disentangled: one (termed “ $\omega$ -effect”) is associated with the failure of the indistinguishability between the neutral meson and its antiparticle, and affects certain symmetry properties of the initial state of the two-meson system; the second effect is generated by the time evolution of the system in the medium of the space-time foam, and can result in time-dependent contributions of the  $\omega$ -effect type in the time profile of the two meson state. Estimates of both effects are given, which show that, at least in certain models, such effects are not far from the sensitivity of experimental facilities available currently or in the near future. The other class of quantum gravity models involves a medium of gravitational fluctuations which behaves like a “thermal bath”. In this model both of the above-mentioned intrinsic CPT violation effects are not valid.

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## I. INTRODUCTION AND MOTIVATION

There have been two strands of research in the last few years which have only recently made contact. One is the subject of bipartite entanglement and the other is the role of space-time foam for decoherence of elementary particles. The latter was first championed by Wheeler within the context of microscopic horizons of radius of the order of the Planck length which may induce in space-time a fuzzy structure. This has been further developed in [1] where it was suggested that topological fluctuations in the space-time background and microscopic black holes can lead to non-unitary evolution and a breakdown of the S-matrix description in field theory. If this is correct then the usual formulation of quantum mechanics has to be modified. Arguments have been put forward for this modification of the Liouville equation to take the form [2]

$$\partial_t \rho = \frac{i}{\hbar} [\rho, H] + \delta H \rho. \quad (1.1)$$

Equations of this form are frequently required to describe the time evolution of an open quantum mechanical system where  $\delta H \rho$  has a Lindblad form [3]. In such systems observable degrees of freedom are coupled to unobservable components which are effectively integrated over. Initial pure states evolve into mixed ones and so the S-matrix  $\mathcal{S}$  relating initial and final density matrices does not factorise, i.e.

$$\mathcal{S} \neq S S^\dagger \quad (1.2)$$

where  $S = e^{iHt}$ . In these circumstances Wald [4] has shown that CPT is violated, at least in its strong form, i.e. there is *no* unitary invertible operator  $\Theta$  such that

$$\Theta \bar{\rho}_{\text{in}} = \rho_{\text{out}}. \quad (1.3)$$

This result is due to the entanglement of the gravitational fluctuations with the matter system. Such entanglement is not generally a perturbative effect.

It was pointed out in [5], that if the CPT operator is not well defined this has implications for the symmetry structure of the initial entangled state of two neutral mesons in meson factories. Indeed, if CPT can be defined as a quantum mechanical operator, then the decay of a (generic) meson with quantum numbers  $J^{PC} = 1^{--}$  [6], leads to

a pair state of neutral mesons  $|i\rangle$  having the form of the entangled state

$$|i\rangle = \frac{1}{\sqrt{2}} \left( |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle - |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle \right). \quad (1.4)$$

This state has the Bose symmetry associated with particle-antiparticle indistinguishability  $C\mathcal{P} = +$ , where  $C$  is the charge conjugation and  $\mathcal{P}$  is the permutation operation. If, however, CPT is not a good symmetry (i.e. ill-defined due to space-time foam), then  $M_0$  and  $\overline{M}_0$  may not be identified and the requirement of  $C\mathcal{P} = +$  is relaxed [5]. Consequently, in a perturbative framework, the state of the meson pair can be parametrised to have the following form:

$$|i\rangle = \frac{1}{\sqrt{2}} \left( |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle - |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle \right) + \frac{\omega}{\sqrt{2}} \left( |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle + |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle \right)$$

where  $\omega = |\omega| e^{i\Omega}$  is a complex CPT violating (CPTV) parameter. For definiteness in what follows we shall term this quantum-gravity effect in the initial state as the “ $\omega$ -effect” [5].

Given the possible breakdown of conventional unitary quantum mechanics and non-invariance of CPT as a consequence of space-time foam, it is interesting to see what, if any, type of foam can generate this type of effect from a fundamental and non-phenomenological stance. This is a somewhat delicate matter since evolution using the popular Lindblad approaches [3],[7],[8] generate effects, which, however, have essential differences in both form and interpretation from the “ $\omega$ -effect”, and thus can be experimentally disentangled from it [8]. However the Lindblad-type approach to quantum gravity is primarily based on mathematical considerations of quantum dynamical semigroups (required for irreversibility) and Markov processes and does not claim any other physical motivation. It is certainly not a microscopic theory of quantum gravity. There is need for a more detailed microscopic and model dependent approach in order to arrive at reasonable estimates for the  $\omega$ -(initial state) and its evolution. In this work we shall study and estimate such effects in the context of specific models of space-time foam which are motivated by underlying theoretical considerations.

Space-time foam is a generic term that covers quite distinct points of view. One of the most plausible reasons for considering quantum decoherence models of quantum gravity, comes from recent astrophysical evidence for the acceleration of our Universe during the current-era. Observations of distant supernovae [9], as well as WMAP data [10] on thermal fluctuations in the cosmic microwave background (CMB), indicate that our Universe is at present accelerating, and that 74% of its energy-density budget consists of an unknown entity, termed *Dark Energy*. Best-fit models of such data include Einstein-Friedman-Robertson-Walker Universes with a non-zero cosmological *constant*. However, the data are currently compatible also with (cosmic) time-dependent vacuum-energy-density components, relaxing asymptotically to zero [11]. Such relaxation mechanisms may be due to extra scalar *quintessence* fields [12], which in the case of some models inspired by non-critical string theory might be the dilaton itself [13]. Identification of the dark energy component of the Universe with the central charge surplus of the supercritical  $\sigma$ -models describing the (recoil) string excitations of colliding brane-worlds leads to a non-equilibrium energy density of the (observable) brane world [14, 15] and a relaxation scenario for the dark energy. The associated dilaton field during the present era of the Universe may even be constant, in which case the relaxation of the dark-energy density component is a purely stringy feature of the logarithmic conformal field theory [16] describing the D-brane recoil [17] in a (perturbative)  $\sigma$ -model framework. This is compatible with a de Sitter space, with scale factor  $a(t) = \exp(\sqrt{\Omega_0/3}t)$ , which implies an asymptotic Hubble horizon

$$\delta_H \sim \int_{t_0}^{\infty} d(ct') a^{-1}(t') < \infty \quad (1.5)$$

One suggestion for the quantisation of such systems is through analogies with open systems in quantum mechanics. For some simple cases, such as conformally coupled scalar fields [18] in de Sitter spaces it has been shown explicitly that the system modes decohere for wavelengths longer than a critical value, which is of the order of the Hubble horizon. From the theorem by Wald [4], the CPT operator is ill-defined in such decoherent field theories. Non-critical (Liouville) string [19] provides a rather unified formalism for dealing not only with cosmological constant Universes in string theory, but also in general with decoherent quantum space-time foam backgrounds, that include microscopic quantum-fluctuating black holes [20]. Within this framework a particularly simple and tractable background is given by D-particles. Low energy matter is represented as open or closed strings and moves in a  $D + 1$  dimensional target space. The string states collide with massive D-particle defects embedded in target space. The recoil fluctuations of the D-particle induce a space-time distortion given by the metric tensor

$$g_{ij} = \delta_{ij}, g_{00} = -1, g_{0i} = \varepsilon(\varepsilon y_i + u_i t) \Theta_\varepsilon(t), i = 1, \dots, D \quad (1.6)$$

where the suffix 0 denotes temporal (Liouville) components and

$$\Theta_\varepsilon(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{dq}{q - i\varepsilon} e^{iqt}, \quad (1.7)$$

$$u_i = (k_1 - k_2)_i,$$

with  $k_1$  ( $k_2$ ) the momentum of the propagating closed-string state before (after) the recoil;  $y_i$  are the spatial collective coordinates of the D particle and  $\varepsilon^{-2}$  is identified with the target Minkowski time  $t$  for  $t \gg 0$  after the collision [17]. These relations have been calculated for non-relativistic branes where  $u_i$  is small. Now for large  $t$ , to leading order,

$$g_{0i} \simeq \bar{u}_i \equiv \frac{u_i}{\varepsilon} \propto \frac{\Delta p_i}{M_P} \quad (1.8)$$

where  $\Delta p_i$  is the momentum transfer during a collision and  $M_P$  is the Planck mass (actually, to be more precise  $M_P = M_s/g_s$ , where  $g_s < 1$  is the (weak) string coupling, and  $M_s$  is a string mass scale); so  $g_{0i}$  is constant in space-time but depends on the energy content of the low energy particle [21]. Such a feature does not arise in conventional approaches to space-time foam and will be important in our formulation of one of the microscopic models that we will consider.

The above model of space-time foam refers to a specific string-inspired construction. However the form of the induced back reaction (1.8) onto the space-time has some generic features, and can be understood more generally in the context of effective theories of such models, which allows one to go beyond a specific non-critical (Liouville) model. Indeed, the D-particle defect can be viewed as an idealisation of some (virtual, quantum) black hole defect of the ground state of quantum gravity, viewed as a membrane wrapped around some small extra dimensions of the (stringy) space time, and thus appearing to a four-dimensional observer as an “effectively” point like defect. The back reaction on space-time due to the interaction of a pair of neutral mesons, such as those produced in a meson factory, with such defects can be studied generically as follows: consider the non-relativistic recoil motion of the heavy defect, whose coordinates in space-time, in the laboratory frame, are  $y^i = y_0^i + u^i t$ , with  $u^i$  the (small) recoil velocity. One can then perform a (infinitesimal) general coordinate transformation  $y^\mu \rightarrow x^\mu + \xi^\mu$  so as to go to the rest (or co-moving) frame of the defect after the scattering. From a passive point of view, for one of the mesons, this corresponds to an induced change in metric of space-time of the form (in the usual notation, where the parenthesis in indices denote symmetrisation)  $\delta g_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}$ , which in the specific case of non-relativistic defect motion yields the off-diagonal metric elements (1.8). Such transformations cannot be performed simultaneously for both mesons, and moreover in a full theory of quantum gravity the recoil velocities fluctuate randomly, as we shall discuss later on. This means that the effects of the recoil of the space-time defect are observable. The mesons will feel such effects in the form of induced fluctuating metrics (1.8). It is crucial to note that the interaction of the matter particle (meson) with the foam defect may also result in a “flavour” change of the particle (e.g. the change of a neutral meson to its antiparticle). This feature can be understood in a D-particle Liouville model by noting that the scattering of the matter probe off the defect involves first a splitting of a closed string representing matter into two open ones, but with their ends attached to the D-particle, and then a joining of the string ends in order to re-emit a closed string matter state. The re-emitted (scattered) state may in general be characterised by phase, flavour and other quantum charges which may not be required to be conserved during black hole evaporation and disparate space-time-foam processes. In our application we shall restrict ourselves only to effects that lead to flavour changes. The modified form of the metric fluctuations (1.8), which characterise our specific model of Liouville decoherence is given in the next section.

The second model to be considered by us shares a concern with the effect of horizons and the consequent absence of unitarity but the formulation is not supported by a formal theory like string theory. A different effective theory of space-time foam has been proposed by Garay [22]. The fuzziness of space-time at the Planck scale is described by a non-fluctuating background which is supplemented by non-local interactions. The latter reflects the fact that at Planck scales space-time points lose their meaning and so these fluctuations present themselves in the non-fluctuating coarse grained background as non-local interactions. These non-local interactions are then rephrased as a quantum thermal bath with a Planckian temperature. The quantum entanglement of the gravitational bath and the two meson (entangled) state is explicit in this model. Consequently issues of back reaction can be readily examined. Since the evolution resulting from the standard Lindblad formulation does not lead to the  $\omega$  effect, this manifestation of CPTV is not the result of some arbitrary non-unitary evolution. Hence it is interesting to study the above two quite distinct models (one motivated by string theory and the other by field theory) for clues concerning the appearance of (and an estimate for the order of magnitude) of  $\omega$ .

The structure of this article is as follows: In the next section we will construct the model incorporating ideas from Liouville string theory that were mentioned earlier. It will be referred to as the Liouville stochastic metric (LSM) model. It will be demonstrated that this model leads to the appearance of  $\omega$ -type effects both in the initial state and during evolution in the foam medium. In fact, there are some subtleties concerning strangeness conservation

(on choosing the  $M$  meson to be the  $K$  meson for definiteness); consequently, the precise form of the pertinent CPT Violating terms in the decay products of a meson factory, constitutes a sensitive probe of any non-conservation. In models of space-time foam such as LSM strangeness is not always conserved, as there is no corresponding no-hair theorem for the associated (singular) space-time fluctuations. This is a feature that is model dependent, and, in this work, we pay particular attention to determining the conditions under which this happens. Qualitative estimates of the effects are given and the rôle of time dependence in disentangling the effects is discussed. The neutral meson system and the thermal bath, representing space-time foam, together form one large hamiltonian system whose evolution can be calculated exactly. This implies in principle an exact knowledge of the dynamics of the neutral mesons. Consequently in the third section, entanglement that is induced in the state of the neutral meson pair by the thermal bath, is calculated. However, as discussed there, no  $\omega$  type effect is generated by the evolution in such a type of foam. Moreover owing to the details of the entanglement of bath and neutral mesons the stationary states of the system cannot be interpreted in terms of an initial  $\omega$  effect. Conclusions are presented in the final section. Technical aspects of our work are given in two appendices.

## II. LIOUVILLE INSPIRED DECOHERENCE

### A. Liouville-Stochastic-Metric (LSM) fluctuations and Meson systems: formalism

Polchinski's realisation [23] that solitonic string backgrounds (D-branes) can be described in a conformally invariant way in terms of world sheets with boundaries has significantly changed the understanding of target space structure. Collective target space coordinates of the soliton have Dirichlet boundary conditions on these boundaries. A model of space-time foam [15] can be based on a number (determined by target space supersymmetry) of parallel brane worlds with three large spatial dimensions which move in a bulk space-time containing a "gas" of D-particles. One of these branes is the observable Universe. For an observer on the brane the crossing D-particles will appear as twinkling space-time defects, i.e. microscopic space-time fluctuations. This will give the four-dimensional brane world a "D-foamy" structure. Following some recent work on gravitational decoherence [24],[25] the target space metric state, which is close to being flat, can be represented schematically as a density matrix

$$\rho_{\text{grav}} = \int d^5r f(r_\mu) |g(r_\mu)\rangle \langle g(r_\mu)|. \quad (2.1)$$

The parameters  $r_\mu$  ( $\mu = 0, \dots, 5$ ) are stochastic with a gaussian distribution  $f(r_\mu)$  characterised by the averages

$$\langle r_\mu \rangle = 0, \quad \langle r_\mu r_\nu \rangle = \Delta_\mu \delta_{\mu\nu}.$$

The fluctuations experienced by the two entangled neutral mesons will be assumed to be independent and  $\Delta_\mu \sim O\left(\frac{E^2}{M_P^2}\right)$  i.e. very small. As matter moves through the space-time foam in a typical ergodic picture the effect of time averaging is assumed to be equivalent to an ensemble average. As far as our present discussion is concerned we will consider a semi-classical picture for the metric and so  $|g(r_\mu)\rangle$  in 2.1 will be a coherent state. In the future we will also address non-classical fluctuations where the  $|g(r_\mu)\rangle$  could represent squeezed states of gravitons.

In order to address oscillation and MSW-like phenomena [24] [26],[27] the fluctuations of each component of the metric tensor  $g^{\alpha\beta}$  will not be simply given by the simple recoil distortion (1.8), but instead will be taken to have a  $2 \times 2$  ("flavour") structure:

$$\begin{aligned} g^{00} &= (-1 + r_4) \mathbf{1} \\ g^{01} &= g^{10} = r_0 \mathbf{1} + r_1 \sigma_1 + r_2 \sigma_2 + r_3 \sigma_3 \\ g^{11} &= (1 + r_5) \mathbf{1} \end{aligned} \quad (2.2)$$

where  $\mathbf{1}$ , is the identity and  $\sigma_i$  are the Pauli matrices. The above parametrisation has been taken for simplicity and we will also consider motion to be in the  $x$ - direction which is natural since the meson pair moves collinearly in the Center-of-Mass (C.M.) frame. A metric with this type of structure is compatible with the view that the D-particle defect is a "point-like" approximation for a compactified higher-dimensional brany black hole, whose no hair theorems permit non-conservation of flavour. In the case of neutral mesons the concept of "flavour" refers to either particle/antiparticle species or the two mass eigenstates, by changing appropriately the relevant coefficients.

The Klein-Gordon equation for a spinless neutral meson field  $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$  with mass matrix  $m = \frac{1}{2}(m_1 + m_2)1 + \frac{1}{2}(m_1 - m_2)\sigma_3$  in a gravitational background is

$$(g^{\alpha\beta}D_\alpha D_\beta - m^2)\Phi = 0 \quad (2.3)$$

where  $D_\alpha$  is the covariant derivative. Since the Christoffel symbols vanish for  $a_i$  independent of space time the  $D_\alpha$  coincide with  $\partial_\alpha$ . Hence

$$(g^{00}\partial_0^2 + 2g^{01}\partial_0\partial_1 + g^{11}\partial_1^2)\Phi - m^2\Phi = 0. \quad (2.4)$$

It is useful at this stage to rewrite the state  $|i\rangle$  in terms of the mass eigenstates. To be specific, from now on we shall restrict ourselves to the neutral Kaon system  $K_0 - \bar{K}_0$ , which is produced by a  $\phi$ -meson at rest, i.e.  $K_0 - \bar{K}_0$  in their C.M. frame. The CP eigenstates (on choosing a suitable phase convention for the states  $|K_0\rangle$  and  $|\bar{K}_0\rangle$ ) are, in standard notation,  $|K_\pm\rangle$  with

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}(|K_0\rangle \pm |\bar{K}_0\rangle). \quad (2.5)$$

The mass eigensates  $|K_S\rangle$  and  $|K_L\rangle$  are written in terms of  $|K_\pm\rangle$  as

$$|K_L\rangle = \frac{1}{\sqrt{1 + |\varepsilon_2|^2}}[|K_-\rangle + \varepsilon_2|K_+\rangle] \quad (2.6)$$

and

$$|K_S\rangle = \frac{1}{\sqrt{1 + |\varepsilon_1|^2}}[|K_+\rangle + \varepsilon_1|K_-\rangle]. \quad (2.7)$$

In terms of the mass eigenstates

$$|i\rangle = \mathcal{C} \left\{ \begin{aligned} & (|K_L(\vec{k})\rangle|K_S(-\vec{k})\rangle - |K_S(\vec{k})\rangle|K_L(-\vec{k})\rangle) + \\ & \omega (|K_S(\vec{k})\rangle|K_S(-\vec{k})\rangle - |K_L(\vec{k})\rangle|K_L(-\vec{k})\rangle) \end{aligned} \right\} \quad (2.8)$$

where  $\mathcal{C} = \frac{\sqrt{(1+|\varepsilon_1|^2)(1+|\varepsilon_2|^2)}}{\sqrt{2}(1-\varepsilon_1\varepsilon_2)}$  [5]. In the notation of two level systems (on suppressing the  $\vec{k}$  label) we write

$$\begin{aligned} |K_L\rangle &= |\uparrow\rangle \\ |K_S\rangle &= |\downarrow\rangle. \end{aligned} \quad (2.9)$$

The unnormalised state  $|i\rangle$  will then be an example of an initial state

$$|\psi\rangle = |k, \uparrow\rangle^{(1)}|-\!k, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)}|-\!k, \uparrow\rangle^{(2)} + \xi|k, \uparrow\rangle^{(1)}|-\!k, \uparrow\rangle^{(2)} + \xi'|k, \downarrow\rangle^{(1)}|-\!k, \downarrow\rangle^{(2)} \quad (2.10)$$

where  $|M_L(\vec{k})\rangle = |k, \uparrow\rangle$  and we have taken  $\vec{k}$  to have only a non-zero component  $k$  in the  $x$ -direction; superscripts label the two separated detectors of the collinear meson pair,  $\xi$  and  $\xi'$  are complex constants and we have left the state  $|\psi\rangle$  unnormalised. The evolution of this state is governed by a hamiltonian  $\hat{H}$

$$\hat{H} = g^{01}(g^{00})^{-1}\hat{k} - (g^{00})^{-1}\sqrt{(g^{01})^2k^2 - g^{00}(g^{11}k^2 + m^2)} \quad (2.11)$$

which is the natural generalisation of the standard Klein Gordon hamiltonian in a one particle situation. Moreover  $\hat{k}|\pm k, \uparrow\rangle = \pm k|k, \uparrow\rangle$  together with the corresponding relation for  $\downarrow$ .

**B. Gravitationally-dressed initial entangled state: stationary perturbation theory and order of magnitude estimates of the  $\omega$ -effect.**

The effect of space-time foam on the initial entangled state of two neutral mesons is conceptually difficult to isolate, given that the meson state is itself entangled with the bath. Nevertheless, in the context of our specific model, which is written as a stochastic hamiltonian, one can estimate the order of the associated  $\omega$ -effect of [5] by applying non-degenerate perturbation theory to the states  $|k, \uparrow\rangle^{(i)}$ ,  $|k, \downarrow\rangle^{(i)}$ ,  $i = 1, 2$ . Although it would be more rigorous to consider the corresponding density matrices, traced over the unobserved gravitational degrees of freedom, in order to obtain estimates it will suffice formally to work with single-meson state vectors).

Owing to the form of the hamiltonian (2.11) the gravitationally perturbed states will still be momentum eigenstates. The dominant features of a possible  $\omega$ -effect can be seen from a term  $\widehat{H}_I$  in the single-particle interaction hamiltonian

$$\widehat{H}_I = - (r_1\sigma_1 + r_2\sigma_2) \widehat{k} \quad (2.12)$$

which is the leading order contribution in the small parameters  $r_i$  (c.f. (2.2),(2.11)) in  $H$  (i.e.  $\sqrt{\Delta_i}$  are small). This implies a modification of the mass eigenstates by the medium of quantum gravity, in analogy with the celebrated MSW effect of neutrino propagating in matter media [27], but with the important difference here that the effects of the medium are directional.

In first order in perturbation theory the gravitational dressing of  $|k, \downarrow\rangle^{(i)}$  leads to a state:

$$|k^{(i)}, \downarrow\rangle_{QG}^{(i)} = |k^{(i)}, \downarrow\rangle^{(i)} + |k^{(i)}, \uparrow\rangle^{(i)} \alpha^{(i)} \quad (2.13)$$

where

$$\alpha^{(i)} = \frac{{}^{(i)}\langle \uparrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1} \quad (2.14)$$

and correspondingly for  $|k^{(i)}, \uparrow\rangle^{(i)}$  the dressed state is obtained from 2.14 by  $|\downarrow\rangle \leftrightarrow |\uparrow\rangle$  and  $\alpha \rightarrow \beta$  where

$$\beta^{(i)} = \frac{{}^{(i)}\langle \downarrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2} \quad (2.15)$$

Here the quantities  $E_i = (m_i^2 + k^2)^{1/2}$  denote the energy eigenvalues, and  $i = 1$  is associated with the up state and  $i = 2$  with the down state. With this in mind the totally antisymmetric ‘‘gravitationally-dressed’’ state can be expressed in terms of the unperturbed single-particle states as:

$$\begin{aligned} & |k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow\rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow\rangle_{QG}^{(2)} = \\ & |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} \\ & + |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) + |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\ & + \beta^{(1)}\alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} - \alpha^{(1)}\beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} \end{aligned}$$

It should be noted that for  $r_i \propto \delta_{i1}$  the generated  $\omega$ -like effect corresponds to the case  $\xi = \xi'$  in (2.10) since  $\alpha^{(i)} = -\beta^{(i)}$ , while the  $\omega$ -effect of [5] (2.8) corresponds to  $r_i \propto \delta_{i2}$  (and the generation of  $\xi = -\xi'$ ) since  $\alpha^{(i)} = \beta^{(i)}$ . In the density matrix these cases can be distinguished by the off-diagonal terms.

These two cases are physically very different. In the case of  $\phi$ -factories, the former corresponds to non-definite strangeness in the initial state of the neutral Kaons (seen explicitly when written in terms of  $K_0 - \overline{K}_0$ ), and hence strangeness nonconservation in the initial decay of the  $\phi$ -meson, while the latter conserves this quantum number. We remind the reader that in a stochastic quantum-gravity situation, strangeness, or, in that matter, the appropriate quantum number in the case of other neutral mesons, is not necessarily conserved, and this is reflected in the above-described general parametrisation of the interaction Hamiltonian (2.12) in ‘‘flavour’’ space.

As we shall discuss in the next subsection, the (decoherent) time evolution of these two cases causes the appearance of terms with the opposite effects, as far as the quantum numbers in question are concerned. Namely, the strangeness-conserving initial state leads to the appearance of CPT violating terms with a strangeness violating form, while an

initially strangeness-violating combination generates, under evolution in the foam, a strangeness-conserving  $\omega$ -effect of the form proposed in [5].

We next remark that on averaging the density matrix over the random variables  $r_i$ , we observe that only terms of order  $|\omega|^2$  will survive, with the order of  $|\omega|^2$  being

$$|\omega|^2 = \mathcal{O}\left(\frac{1}{(E_1 - E_2)} \langle (\downarrow, k | H_I | k, \uparrow) \rangle^2 \right) = \mathcal{O}\left(\frac{\Delta_2 k^2}{(E_1 - E_2)^2}\right) \sim \frac{\Delta_2 k^2}{(m_1 - m_2)^2} \quad (2.16)$$

for the physically interesting case in which the momenta are of order of the rest energies.

Recalling (c.f. (1.8)) that the variance  $\Delta_2$  (and also  $\Delta_1$ ) is of the order of the square of the momentum transfer (in units of the Planck mass scale  $M_P$ ) during the scattering of the single particle state off a space-time-foam defect, i.e.  $\Delta_2 = \zeta^2 k^2 / M_P^2$ , where  $\zeta$  is at present a phenomenological parameter. It cannot be further determined due to the lack of a complete theory of quantum gravity, which would in principle determine the order of the momentum transfer. We arrive at the following estimate of the order of  $\omega$  in this model of foam:

$$|\omega|^2 \sim \frac{\zeta^2 k^4}{M_P^2 (m_1 - m_2)^2} \quad (2.17)$$

Consequently for neutral kaons, with momenta of the order of the rest energies  $|\omega| \sim 10^{-4} |\zeta|$ , whilst for  $B$ -mesons we have  $|\omega| \sim 10^{-6} |\zeta|$ . For  $1 > \zeta \geq 10^{-2}$  these values for  $\omega$  are not far below the sensitivity of current facilities, such DAΦNE, and  $\zeta$  may be constrained by future data owing to upgrades of the DAΦNE facility or a Super B factory. If the universality of quantum gravity is assumed then  $\zeta$  can also be restricted by data from other sensitive probes, such as terrestrial and extraterrestrial neutrinos [24].

### C. Time-evolution generated $\omega$ -like effects

We next discuss a similar CPT violating (CPTV) effect generated by the time evolution of the system. The evolution of the two particle state  $|\psi\rangle$  is given by

$$|\psi(t)\rangle = \exp\left[-i\left(\widehat{H}^{(1)} + \widehat{H}^{(2)}\right)\frac{t}{\hbar}\right] |\psi\rangle \quad (2.18)$$

where the superscripts on the  $\widehat{H}$  indicate that part of the two particle state that is being acted on by  $\widehat{H}$ . On using (4.20) and (4.21) of Appendix A we find

$$|\psi(t)\rangle = e^{-i(\lambda_0^{(1)} + \lambda_0^{(2)})t} \left\{ \begin{aligned} &\hat{a}_{\uparrow\downarrow}(t) |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} + \hat{a}_{\uparrow\uparrow}(t) |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} \\ &+ \hat{a}_{\downarrow\downarrow}(t) |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} + \hat{a}_{\downarrow\uparrow}(t) |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} \end{aligned} \right\} \quad (2.19)$$

where

$$\hat{a}_{\uparrow\downarrow}(t) = \hat{f}^{(1)}(t) \hat{f}^{(2)}(t)^* + \hat{g}^{(1)}(t)^* \hat{g}^{(2)}(t) - i\xi_1 \hat{f}^{(1)}(t) \hat{g}^{(2)}(t) - i\xi_2 \hat{g}^{(1)}(t)^* \hat{f}^{(2)}(t)^*, \quad (2.20)$$

$$\hat{a}_{\uparrow\uparrow}(t) = -i\hat{f}^{(1)}(t) \hat{g}^{(2)}(t)^* + i\hat{g}^{(1)}(t)^* \hat{f}^{(2)}(t) + \xi_1 \hat{f}^{(1)}(t) \hat{f}^{(2)}(t) - \xi_2 \hat{g}^{(1)}(t)^* \hat{g}^{(2)}(t)^*, \quad (2.21)$$

$$\hat{a}_{\downarrow\downarrow}(t) = -i\hat{g}^{(1)}(t) \hat{f}^{(2)}(t)^* + i\hat{f}^{(1)}(t)^* \hat{g}^{(2)}(t) - \xi_1 \hat{g}^{(1)}(t) \hat{g}^{(2)}(t) + \xi_2 \hat{f}^{(1)}(t)^* \hat{f}^{(2)}(t)^*, \quad (2.22)$$

and

$$\hat{a}_{\downarrow\uparrow}(t) = -\hat{g}^{(1)}(t) \hat{g}^{(2)}(t)^* - \hat{f}^{(1)}(t)^* \hat{f}^{(2)}(t) - i\xi_1 \hat{g}^{(1)}(t) \hat{f}^{(2)}(t) - i\xi_2 \hat{f}^{(1)}(t)^* \hat{g}^{(2)}(t)^*. \quad (2.23)$$

The operators  $\widehat{f}^{(i)}, \widehat{f}^{(i)*}, \widehat{g}^{(i)}$  and  $\widehat{g}^{(i)*}$  here act on eigenstates and so they produce c-number eigenvalues  $f^{(i)}, f^{(i)*}, g^{(i)}$  and  $g^{(i)*}$  respectively. A similar convention will be used for other related operators. In the evolution given by (2.19) we shall examine terms involving  $|k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)}$  and  $|k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)}$  but independent of  $\xi_1$  and  $\xi_2$ . Consequently these may be regarded as a generation of the  $\omega$  contribution as a consequence of entanglement

with the gravitational space-time foam rather than as a modification of an existing term. Since the hamiltonian generating the evolution is stochastic, the state of the system at any time is given by  $(|\psi(t)\rangle\langle\psi(t)|)_{avg}$  where the averaging is over the stochastic parameters in the metric. The resulting state is a mixed density matrix.

Just as in the estimates of the order of the  $\omega$ -effect for the stationary states, it is adequate to work in terms of the wavefunctions. The magnitude of the  $\omega$  generated in (2.19) will now be estimated. We define  $\varpi$  (2.21) and  $\varpi'$  (2.22) as

$$\varpi(t) = i \left( f^{(2)}(t) g^{(1)*}(t) - f^{(1)}(t) g^{(2)*}(t) \right) \quad (2.24)$$

and

$$\varpi'(t) = i \left( f^{(1)*}(t) g^{(2)}(t) - g^{(1)}(t) f^{(2)*}(t) \right). \quad (2.25)$$

From (4.22) and (4.23) of Appendix A we deduce that

$$\varpi(t) = i \left[ \left( \cos(|\lambda^{(2)}|t) - i \sin(|\lambda^{(2)}|t) \frac{\lambda_3^{(2)}}{|\lambda^{(2)}|} \right) \sin(|\lambda^{(1)}|t) \frac{\lambda_1^{(1)} - i\lambda_2^{(1)}}{|\lambda^{(1)}|} - (1 \leftrightarrow 2) \right] \quad (2.26)$$

and

$$\varpi'(t) = -i \left[ \left( \cos(|\lambda^{(2)}|t) + i \sin(|\lambda^{(2)}|t) \frac{\lambda_3^{(2)}}{|\lambda^{(2)}|} \right) \sin(|\lambda^{(1)}|t) \frac{\lambda_1^{(1)} + i\lambda_2^{(1)}}{|\lambda^{(1)}|} - (1 \leftrightarrow 2) \right]. \quad (2.27)$$

Clearly we have an exact relation

$$\varpi'(t) = \varpi(t)^*. \quad (2.28)$$

Moreover

$$\begin{aligned} \text{Re}(\varpi(t)) &= \sin(|\lambda^{(2)}|t) \sin(|\lambda^{(1)}|t) \frac{\lambda_1^{(1)}\lambda_3^{(2)} - \lambda_1^{(2)}\lambda_3^{(1)}}{|\lambda^{(1)}||\lambda^{(2)}|} \\ &\quad + \cos(|\lambda^{(2)}|t) \sin(|\lambda^{(1)}|t) \frac{\lambda_2^{(1)}}{|\lambda^{(1)}|} - \cos(|\lambda^{(1)}|t) \sin(|\lambda^{(2)}|t) \frac{\lambda_2^{(2)}}{|\lambda^{(2)}|} \end{aligned} \quad (2.29)$$

and

$$\begin{aligned} \text{Im}(\varpi(t)) &= \sin(|\lambda^{(2)}|t) \sin(|\lambda^{(1)}|t) \frac{\lambda_2^{(2)}\lambda_3^{(1)} - \lambda_2^{(1)}\lambda_3^{(2)}}{|\lambda^{(1)}||\lambda^{(2)}|} \\ &\quad + \cos(|\lambda^{(2)}|t) \sin(|\lambda^{(1)}|t) \frac{\lambda_1^{(1)}}{|\lambda^{(1)}|} - \cos(|\lambda^{(1)}|t) \sin(|\lambda^{(2)}|t) \frac{\lambda_1^{(2)}}{|\lambda^{(2)}|}. \end{aligned} \quad (2.30)$$

To lowest order ( in the strength of the metric fluctuations)

$$\lambda_3^{(2)}\lambda_1^{(1)} - \lambda_3^{(1)}\lambda_1^{(2)} \approx -2r_1 k \chi_3 \quad (2.31)$$

and  $\chi_3 \approx \frac{1}{2} \left( \sqrt{k^2 + m_1^2} - \sqrt{k^2 + m_2^2} \right)$  (which is small for  $(m_1 - m_2) \ll k$ ) and

$$\begin{aligned} &\cos(|\lambda^{(2)}|t) \sin(|\lambda^{(1)}|t) \frac{\lambda_2^{(1)}}{|\lambda^{(1)}|} - \cos(|\lambda^{(1)}|t) \sin(|\lambda^{(2)}|t) \frac{\lambda_2^{(2)}}{|\lambda^{(2)}|} \\ &\approx -\sin \left( 2 \left( |\lambda^{(1)}| + r_3 k \right) t \right) \frac{r_2 k}{\chi_3}. \end{aligned} \quad (2.32)$$

Hence for  $r_2 \neq 0$   $\text{Re}(\varpi(t)) \gg \text{Im}(\varpi(t))$  and  $\varpi(t) \approx \varpi'(t)$ . By contrast when  $r_2 = 0$  and  $r_1 \neq 0$   $\text{Im}(\varpi(t)) \gg \text{Re}(\varpi(t))$  and so  $\varpi(t)^* \sim -\varpi(t)$  which is the permutation symmetry necessary for the  $\omega$ -effect [5]. This leads to the simplification

$$\varpi(t) \sim i \cos(|\lambda^{(2)}|t) \sin(|\lambda^{(1)}|t) \left[ \frac{\lambda_1^{(1)}}{|\lambda^{(1)}|} - \frac{\lambda_1^{(2)}}{|\lambda^{(2)}|} \right] \quad (2.33)$$



and so the leading contribution to  $|\psi(t)\rangle$  of the CPT violating type is

$$|\psi(t)\rangle \sim e^{-i(\lambda_0^{(1)} + \lambda_0^{(2)})t} \varpi(t) \left\{ |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} \right\} \quad (2.34)$$

From (2.33)  $O(\varpi) \simeq \frac{\lambda^{(1)} - \lambda^{(2)}}{|\lambda^{(1)}|} \sim 2 \left(1 + \Delta_1^{\frac{1}{2}}\right) \frac{\Delta_1^{\frac{1}{2}} k}{|\lambda^{(1)}|}$  and  $|\lambda^{(1)}| \sim \left(1 + \Delta_1^{\frac{1}{2}}\right) \sqrt{\chi_1^2 + \chi_3^2}$ . From (4.3), (4.4) and (4.5) of Appendix A we observe that, to leading order,  $\chi_3 \sim (k^2 + m_1^2)^{\frac{1}{2}} - (k^2 + m_2^2)^{\frac{1}{2}}$  and so

$$O(\varpi) \simeq \frac{2\Delta_1^{\frac{1}{2}} k}{(k^2 + m_1^2)^{\frac{1}{2}} - (k^2 + m_2^2)^{\frac{1}{2}}} \cos\left(|\lambda^{(1)}|t\right) \sin\left(|\lambda^{(1)}|t\right) = \varpi_0 \sin\left(2|\lambda^{(1)}|t\right). \quad (2.35)$$

$$\text{with } \varpi_0 \equiv \frac{\Delta_1^{\frac{1}{2}} k}{(k^2 + m_1^2)^{\frac{1}{2}} - (k^2 + m_2^2)^{\frac{1}{2}}}.$$

Although  $\Delta_1$  is a parameter, our model has its origins in models of D-particle foam; as previously discussed the estimate for  $\Delta_1$  that arises in such models is given in terms of the momentum transfer during the scattering of the matter state with the space-time defect

$$\Delta_1^{1/2} \sim |\zeta| \frac{|k|}{M_P}.$$

This yields an  $|\varpi_0|$  of the same order as  $|\omega|$  in (2.17).

The time dependence of this effect allows its experimental disentanglement from the  $\omega$ -effect appearing in the initial state of the two neutral mesons. The situation should be compared with the analogous one within the context of a Lindblad approach to the foam, considered in [8], where again the evolution effects can be disentangled from the initial-state symmetry CPTV  $\omega$ -effects.

### III. THE THERMAL MASTER EQUATION

Master equations with a thermal bath have been argued to be relevant to decoherence with space time foam [22]. A thermal field represents a bath about which there is minimal information since only the mean energy of the bath is known, a situation valid also for space time foam. In applications of quantum information it has been shown that a system of two qubits (or two-level systems) initially in a separable state (and interacting with a thermal bath) can actually be entangled by such a single mode bath [28]. As the system evolves the degree of entanglement is sensitive to the initial state. The close analogy between two-level systems and neutral meson systems, together with the modelling by a phenomenological thermal bath of space time foam, makes the study of thermal master equations a rather intriguing one for the generation of  $\omega$ . The hamiltonian  $\mathcal{H}$  representing the interaction of two such two-level systems with a single mode thermal field is

$$\mathcal{H} = \hbar\nu a^\dagger a + \frac{1}{2}\hbar\Omega\sigma_3^{(1)} + \frac{1}{2}\hbar\Omega\sigma_3^{(2)} + \hbar\gamma \sum_{i=1}^2 \left( a\sigma_+^{(i)} + a^\dagger\sigma_-^{(i)} \right) \quad (3.1)$$

where  $a$  is the annihilation operator for the mode of the thermal field and the  $\sigma$ 's are the standard Pauli matrices for the 2 level systems. The operators  $a$  and  $a^\dagger$  satisfy

$$[a, a^\dagger] = 1, [a^\dagger, a^\dagger] = [a, a] = 0. \quad (3.2)$$

The superscripts label the particle. This hamiltonian is the Jaynes-Cummings hamiltonian [29] and explicitly incorporates the back reaction or entanglement between system and bath. This is in contrast with the Lindblad model and the Liouville stochastic metric model. In common with the Lindblad model it is non-geometric. In the former the entanglement with the bath has been in principle integrated over while in the latter it is represented by a stochastic

effect. It is useful to go to the interaction picture in which operators are labelled by a subscript I; in terms of the Schrödinger picture the interaction picture operators are given by

$$\begin{aligned} a_I(t) &= \exp(-i\nu t) a \\ \sigma_{3I}(t) &= \sigma_3 \\ \sigma_{+I}(t) &= \exp(i\Omega t) \sigma_+. \end{aligned} \quad (3.3)$$

Moreover the interaction part of the hamiltonian  $V = \hbar\gamma \sum_{i=1}^2 \left( a\sigma_+^{(i)} + a^\dagger\sigma_-^{(i)} \right)$  transforms to

$$V_I = \hbar\gamma \sum_{i=1}^2 \left( \exp(-i\delta t) a\sigma_+^{(i)} + \exp(i\delta t) a^\dagger\sigma_-^{(i)} \right) \quad (3.4)$$

where  $\delta = \nu - \Omega$ . The initial density matrix  $\rho(0)$  is taken as

$$\rho(0) = \rho_M \otimes \rho_F. \quad (3.5)$$

Here

$$\rho_M = |\mathfrak{A}\rangle \langle \mathfrak{A}|, \quad (3.6)$$

$$|\mathfrak{A}\rangle = |\uparrow\rangle^{(1)} |\downarrow\rangle^{(2)} - |\downarrow\rangle^{(1)} |\uparrow\rangle^{(2)} + \xi_1 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} + \xi_2 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)}, \quad (3.7)$$

and

$$\rho_F = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} |n\rangle \langle n|. \quad (3.8)$$

Hence

$$\rho(0) = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} |\mathfrak{A}\rangle |n\rangle \langle n| \langle \mathfrak{A}|. \quad (3.9)$$

From Bose-Einstein statistics

$$\bar{n} = \frac{1}{-1 + e^{\frac{\hbar\nu}{k_B T}}} \quad (3.10)$$

where  $T$  is the temperature of the heat bath. The stationary states of the total matter-bath system are non-separable as noted in Appendix B. Consequently the effect of the bath cannot be found by means of an effective hamiltonian (for the matter system on its own) perturbed from the hamiltonian in the absence of the bath. The dynamics of the matter system on its own is described by a non-Markovian master equation. However we will describe the evolution of the matter directly by considering the hamiltonian evolution of the combined matter-bath system and then by tracing over the bath degrees of freedom. As described in Appendix B the combined evolution takes place in four dimensional subspaces  $\mathcal{S}_n$ . Hence even though the full Hilbert space is infinite dimensional the dynamical evolution can be obtained as the direct sum of the evolved vectors in each  $\mathcal{S}_n$ .  $|\mathfrak{A}\rangle |n\rangle$  evolves to  $|\Phi_n(t)\rangle$  where

$$\begin{aligned} |\Phi_n(t)\rangle &= X_1^{(n-2,4)}(t) \xi_2 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} |n-2\rangle \\ &+ \left\{ \begin{aligned} &\left( X_1^{(n-1,2)}(t) - X_1^{(n-1,3)}(t) \right) |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} + \xi_2 X_2^{(n-2,4)}(t) |\uparrow\rangle^{(1)} |\downarrow\rangle^{(2)} \\ &+ \xi_2 X_3^{(n-2,4)}(t) |\downarrow\rangle^{(1)} |\uparrow\rangle^{(2)} \end{aligned} \right\} |n-1\rangle \\ &+ \left\{ \begin{aligned} &\xi_1 X_1^{(n,1)}(t) |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} + \left( X_2^{(n-1,2)}(t) - X_2^{(n-1,3)}(t) \right) |\uparrow\rangle^{(1)} |\downarrow\rangle^{(2)} \\ &+ \left( X_3^{(n-1,2)}(t) - X_3^{(n-1,3)}(t) \right) |\downarrow\rangle^{(1)} |\uparrow\rangle^{(2)} + \xi_2 X_4^{(n-2,4)}(t) |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \end{aligned} \right\} |n\rangle \\ &+ \left\{ \begin{aligned} &\xi_1 X_2^{(n,1)}(t) |\uparrow\rangle^{(1)} |\downarrow\rangle^{(2)} + \xi_1 X_3^{(n,1)}(t) |\downarrow\rangle^{(1)} |\uparrow\rangle^{(2)} \\ &+ \left( X_4^{(n-1,2)}(t) - X_4^{(n-1,3)}(t) \right) |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \end{aligned} \right\} |n+1\rangle \\ &+ \xi_1 X_4^{(n,1)}(t) |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} |n+2\rangle. \end{aligned} \quad (3.11)$$

Consequently at time  $t$  the density matrix for the matter by itself is

$$\rho_M(t) = \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \langle l | \Phi_n(t) \rangle \langle \Phi_n(t) | l \rangle. \quad (3.12)$$

We are interested primarily in the terms in  $\rho_M(t)$  which involve  $|\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)}$  and  $|\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)}$ . If we denote these terms by  $\rho_{ii}$  then it is straightforward to show that

$$\begin{aligned} \rho_{ii}(t) = & \sum_{n=1}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left| X_1^{(n-1,2)}(t) - X_1^{(n-1,3)}(t) \right|^2 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} \langle \uparrow|^{(1)} \langle \uparrow| \\ & + \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left| X_4^{(n-1,2)}(t) - X_4^{(n-1,3)}(t) \right|^2 |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \langle \downarrow|^{(1)} \langle \downarrow| \\ & + |\xi_2|^2 \sum_{n=2}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left| X_1^{(n-2,4)}(t) \right|^2 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} \langle \uparrow|^{(1)} \langle \uparrow| \\ & + |\xi_1|^2 \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left| X_4^{(n,1)}(t) \right|^2 |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \langle \downarrow|^{(1)} \langle \downarrow| \\ & + \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left( \begin{aligned} & |\xi_1|^2 \left| X_1^{(n,1)}(t) \right|^2 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} \langle \uparrow|^{(1)} \langle \uparrow| + |\xi_2|^2 \left| X_4^{(n-2,4)}(t) \right|^2 |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \langle \downarrow|^{(1)} \langle \downarrow| \\ & + \xi_1 \xi_2^* \left[ X_4^{(n-2,4)}(t) \right]^* X_1^{(n,1)}(t) |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} \langle \downarrow|^{(1)} \langle \downarrow| \\ & + \xi_1^* \xi_2 \left[ X_1^{(n,1)}(t) \right]^* X_4^{(n-2,4)}(t) |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \langle \uparrow|^{(1)} \langle \uparrow| \end{aligned} \right). \end{aligned}$$

In  $\rho_{ii}(t)$  only the terms  $\rho'_{ii}(t)$

$$\begin{aligned} \rho'_{ii}(t) = & \sum_{n=1}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left| X_1^{(n-1,2)}(t) - X_1^{(n-1,3)}(t) \right|^2 |\uparrow\rangle^{(1)} |\uparrow\rangle^{(2)} \langle \uparrow|^{(1)} \langle \uparrow| \\ & + \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \left| X_4^{(n-1,2)}(t) - X_4^{(n-1,3)}(t) \right|^2 |\downarrow\rangle^{(1)} |\downarrow\rangle^{(2)} \langle \downarrow|^{(1)} \langle \downarrow| \end{aligned} \quad (3.13)$$

are generated during the evolution from the CPT symmetric part of  $|\mathfrak{A}\rangle$ . In Appendix B we noted that  $a_i^{(n-1,2)} = a_i^{(n-1,3)}$ ,  $i = 1, \dots, 3$  and so  $X_1^{(n-1,2)}(t) = X_1^{(n-1,3)}(t)$ ; by the same reasoning  $X_4^{(n-1,2)}(t) = X_4^{(n-1,3)}(t)$ . Consequently  $\rho'_{ii}(t) = 0$  and so the thermal bath model does not generate the entanglement implied by CPTV.

#### IV. CONCLUSIONS

In this work we have discussed two classes of space-time foam models, which, conceivably, may characterise realistic situations of the (still elusive) theory of quantum gravity. In one of the models (LSM), inspired by non-critical string theory models of foam, but placed here in the more general context of semi-microscopic effective theories of quantum gravity, there is an appropriate metric distortion caused by the recoil of the space-time defect (microscopic black hole) during the scattering with the matter probe. The distortion is such that it connects the different mass eigenstates of neutral mesons, and is proportional to the momentum transfer of the matter probe during its scattering with the space-time defect. The latter is assumed to fluctuate randomly, with a dispersion which at present is viewed as a phenomenological parameter to be constrained by data.

This causes a CPTV  $\omega$ -like effect in the initial entangled state of two neutral mesons in a meson factory, of the type conjectured in [5]. Using stationary (non-degenerate) perturbation theory it was possible to give an order of magnitude estimate of the effect: the latter is momentum dependent, and of an order which may not be far from the sensitivity of experimental facilities in the near future, such as a possible upgrade of the DAΦNE facility or a Super B factory. A similar effect, but with a sinusoidal time dependence, and hence experimentally distinguishable from the initial-state effect, is also generated in this model of foam by the evolution of the system. The situation needs to be

compared with Lindblad-type phenomenological models of quantum gravity foam, where again evolution-generated CPT Violating effects can be disentangled from the  $\omega$ -effect characterising the initial state.

As we have discussed, there are two physically very different cases for the initial state in a quantum gravity situation of an LSM-like model. One involves an initial state of two mesons with definite ‘‘strangeness’’ (i.e. the appropriate quantum number for the meson factory in question), which the space-time foam evolves into an indefinite ‘‘strangeness’’ combination (i.e. strangeness violation). The other case corresponds to an indefinite strangeness initial state, whose (decoherent) time evolution yields time-dependent  $\omega$ -effect terms with definite strangeness of the type considered in [5]. These two cases are produced by the different terms in the initial perturbation Hamiltonian (2.12).

A second model of space-time foam, that of a thermal bath of gravitational degrees of freedom, is also considered in our work, which, however, does not lead to the generation of an  $\omega$ -like effect.

It is interesting to continue the search for more realistic models of quantum gravity, either in the context of string theory or in other approaches, such as the canonical approach or the loop quantum gravity, in order to search for intrinsic CPT violating effects in sensitive matter probes. Detailed analyses of global data, including very sensitive probes such as high energy neutrinos, is the only way forward in order to obtain some clues on the elusive theory of quantum gravity. Decoherence, induced by quantum gravity, far from being ruled out at present, may indeed provide the link between theory and experiment in this intriguing area of physics.

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### Appendix A: details of LSM evolution

In order to simplify  $\hat{H}$  (2.11) we note the decomposition

$$\sqrt{(\hat{A}\mathbf{1} + \hat{B}\sigma_3 + \hat{C}\sigma_1 + \hat{D}\sigma_2)} = \hat{\chi}_0\mathbf{1} + \sum_{j=1}^3 \hat{\chi}_j\sigma_j \quad (4.1)$$

where

$$\hat{\chi}_0 = \frac{1}{2} \left[ \left( \hat{A} + \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} + \left( \hat{A} - \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} \right], \quad (4.2)$$

$$\hat{\chi}_1 = \frac{\hat{C}}{2\sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2}} \left[ \left( \hat{A} + \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} - \left( \hat{A} - \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} \right], \quad (4.3)$$

$$\hat{\chi}_2 = \frac{\hat{D}}{2\sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2}} \left[ \left( \hat{A} + \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} - \left( \hat{A} - \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} \right], \quad (4.4)$$

and

$$\hat{\chi}_3 = \frac{\hat{B}}{2\sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2}} \left[ \left( \hat{A} + \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} - \left( \hat{A} - \sqrt{\hat{B}^2 + \hat{C}^2 + \hat{D}^2} \right)^{1/2} \right]. \quad (4.5)$$

Using (4.1) we can write the single particle hamiltonian as

$$\hat{H} = (1 + r_4) \left\{ - (r_0\mathbf{1} + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3) \hat{k} + \left( \hat{\chi}_0\mathbf{1} + \sum_{j=1}^3 \hat{\chi}_j\sigma_j \right) \right\} \quad (4.6)$$

for

$$\widehat{A} = \widehat{k}^2 [(r_0^2 + r_1^2 + r_2^2 + r_3^2) + (1 - r_4)(1 + r_2)] + \frac{1}{2}(1 - r_4)(m_1^2 + m_2^2) \quad (4.7)$$

$$\widehat{B} = 2\widehat{k}^2 r_0 r_3 + \frac{1}{2}(m_1^2 - m_2^2)(1 - r_4) \quad (4.8)$$

$$\widehat{C} = 2\widehat{k}^2 r_0 r_1. \quad (4.9)$$

and

$$\widehat{D} = 2\widehat{k}^2 r_0 r_2. \quad (4.10)$$

The operators  $\widehat{A}, \widehat{B}, \widehat{C}$  and  $\widehat{D}$  will always act on eigenstates  $|p\rangle$  (where  $\widehat{k}|p\rangle = p|p\rangle$ ). The evolution operator  $e^{-i\widehat{H}t}$  in turn has the decomposition

$$e^{-i\widehat{H}t} = e^{-i\widehat{\lambda}_0 t} e^{-i\sum_{j=1}^3 \widehat{\lambda}_j \sigma_j t} \quad (4.11)$$

where

$$\widehat{\lambda}_\mu = (1 + r_4) (\widehat{\chi}_\mu - r_\mu \widehat{k}), \quad \mu = 0, \dots, 3. \quad (4.12)$$

Now

$$e^{-i\sum_{j=1}^3 \widehat{\lambda}_j \sigma_j t} = \cos\left(|\widehat{\lambda}|t\right) - i \sum_{j=1}^3 \widehat{\lambda}_j \sigma_j \left(|\widehat{\lambda}|t\right)^{-1} \sin\left(|\widehat{\lambda}|t\right) \quad (4.13)$$

where  $|\widehat{\lambda}| = \sqrt{\sum_{i=1}^3 \widehat{\lambda}_i^2}$  and again it is assumed that the operator  $e^{-i\sum_{j=1}^3 \widehat{\lambda}_j \sigma_j t}$  acts on an eigenstate  $|p\rangle$  of  $\widehat{k}$ . Consequently

$$|\widehat{\lambda}| |p\rangle = \sqrt{(\lambda_1^2(p) + \lambda_2^2(p) + \lambda_3^2(p))} |p\rangle \quad (4.14)$$

with

$$\widehat{\lambda}_j |p\rangle = \lambda_j(p) |p\rangle \quad (4.15)$$

for  $j = 1, \dots, 3$ . Now

$$\sum_{j=1}^3 \widehat{\lambda}_j \sigma_j |k, \uparrow\rangle = (\widehat{\lambda}_1 + i\widehat{\lambda}_2) |k, \downarrow\rangle + \widehat{\lambda}_3 |k, \uparrow\rangle, \quad (4.16)$$

$$\sum_{j=1}^3 \widehat{\lambda}_j \sigma_j |k, \downarrow\rangle = (\widehat{\lambda}_1 - i\widehat{\lambda}_2) |k, \uparrow\rangle - \widehat{\lambda}_3 |k, \downarrow\rangle, \quad (4.17)$$

$$\widehat{\lambda}_j^{(1)} |k, \alpha\rangle^{(1)} = (1 + r_4) (\widehat{\chi}_j - r_j k) |k, \alpha\rangle^{(1)}, \quad (4.18)$$

and

$$\widehat{\lambda}_j^{(2)} |-k, \alpha\rangle^{(2)} = (1 + r_4) (\widehat{\chi}_j + r_j k) |-k, \alpha\rangle^{(2)} \quad (4.19)$$

where  $\alpha = \uparrow, \downarrow$ .

Consequently

$$e^{-i\widehat{H}^{(1)}t} |k, \uparrow\rangle^{(1)} = e^{-i\widehat{\lambda}_0^{(1)}t} \left( \widehat{f}^{(1)}(t) |k, \uparrow\rangle^{(1)} - i\widehat{g}^{(1)}(t) |k, \downarrow\rangle^{(1)} \right) \quad (4.20)$$

and

$$e^{-i\widehat{H}^{(1)}t} |k, \downarrow\rangle^{(1)} = e^{-i\widehat{\lambda}_0^{(1)}t} \left( \widehat{f}^{(1)*}(t) |k, \uparrow\rangle^{(1)} - i\widehat{g}^{(1)}(t) |k, \downarrow\rangle^{(1)} \right) \quad (4.21)$$

where for  $j = 1, 2$

$$\widehat{f}^{(j)}(t) = \cos\left(\left|\widehat{\lambda}^{(j)}\right|t\right) - i \sin\left(\left|\widehat{\lambda}^{(j)}\right|t\right) \left|\widehat{\lambda}^{(j)}\right|^{-1} \widehat{\lambda}_3^{(j)} \quad (4.22)$$

and

$$\widehat{g}^{(j)}(t) = \left(\widehat{\lambda}_1^{(j)} + i\widehat{\lambda}_2^{(j)}\right) \left|\widehat{\lambda}^{(j)}\right|^{-1} \sin\left(\left|\widehat{\lambda}^{(j)}\right|t\right). \quad (4.23)$$

$\widehat{f}^{(j)*}(t)$ ,  $\widehat{g}^{(j)*}(t)$  and  $\widehat{f}^{(j)}(t)$ ,  $\widehat{g}^{(j)}(t)$  share the same eigenvectors but with complex conjugate eigenvalues. The formulae (4.20) and (4.21) hold for states also with superfix 2 when  $k \rightarrow -k$ .

### Appendix B: details of thermal bath evolution

In the analysis of the eigenstates of the Jaynes-Cummings hamiltonian  $\mathcal{H}$  it is clear that certain subspaces  $\mathcal{S}_n$  of the Hilbert space  $H$  for the states of the system and bath are invariant under the action of  $\mathcal{H}$ . Moreover  $\cup_n \mathcal{S}_n = H$ .  $\mathcal{S}_n$  is characterised by states of the form

$$|\Psi(t)\rangle = X_1^n(t) |\uparrow, \uparrow, n\rangle + X_2^n(t) |\uparrow, \downarrow, n+1\rangle + X_3^n(t) |\downarrow, \uparrow, n+1\rangle + X_4^n(t) |\downarrow, \downarrow, n+2\rangle. \quad (4.24)$$

We shall work in the interaction picture but eschew the cumbersome subscript I. The evolution of the  $X_i^n$  is governed by

$$\begin{aligned} i \dot{X}_1^n(t) &= \gamma \sqrt{n+1} \exp(-i\delta t) (X_2^n(t) + X_3^n(t)), \\ i \dot{X}_2^n(t) &= \gamma (\exp(-i\delta t) \sqrt{n+2} X_4^n(t) + \exp(i\delta t) \sqrt{n+1} X_1^n(t)), \\ i \dot{X}_3^n(t) &= \gamma (\exp(-i\delta t) \sqrt{n+2} X_4^n(t) + \exp(i\delta t) \sqrt{n+1} X_1^n(t)), \\ i \dot{X}_4^n(t) &= \gamma \exp(i\delta t) \sqrt{n+2} (X_2^n(t) + X_3^n(t)). \end{aligned} \quad (4.25)$$

On writing

$$\begin{aligned} y_5^n(t) &= X_2^n(t) + X_3^n(t) \\ y_1^n(t) &= \sqrt{n+1} X_1^n(t) \\ y_4^n(t) &= \sqrt{n+2} X_4^n(t) \end{aligned} \quad (4.26)$$

we have from (4.25)

$$\begin{aligned} i \dot{y}_1^n(t) &= \gamma (n+1) \exp(-i\delta t) y_5^n(t), \\ i \dot{y}_4^n(t) &= \gamma (n+2) \exp(i\delta t) y_5^n(t), \\ i \dot{y}_5^n(t) &= 2\gamma (\exp(-i\delta t) y_4^n(t) + \exp(i\delta t) y_1^n(t)). \end{aligned} \quad (4.27)$$

Any solution of these equations satisfies

$$i \ddot{y}_5^n(t) + i(\delta^2 + (4n+6)\gamma^2) \dot{y}_5^n(t) + 2\delta\gamma^2 y_5^n(t) = 0. \quad (4.28)$$

Hence  $y_5^n \propto e^{i\lambda^{(n)}t}$  where  $\lambda = \lambda^{(n)}$  is a solution of

$$\lambda^3 - (\delta^2 + (4n+6)\gamma^2) \lambda + 2\delta\gamma^2 = 0. \quad (4.29)$$

If we denote the three solutions by  $\lambda_i^{(n)}$  ( $i = 1, \dots, 3$ ) then the system of equations of (4.25) has a solution

$$\begin{aligned} X_1^n(t) &= -\gamma \sqrt{n+1} \left( \frac{\mathbf{a}_1^{(n)} e^{i(\lambda_1^{(n)} - \delta)t}}{\lambda_1^{(n)} - \delta} + \frac{\mathbf{a}_2^{(n)} e^{i(\lambda_2^{(n)} - \delta)t}}{\lambda_2^{(n)} - \delta} + \frac{\mathbf{a}_3^{(n)} e^{i(\lambda_3^{(n)} - \delta)t}}{\lambda_3^{(n)} - \delta} \right) \\ X_2^n(t) &= \frac{1}{2} \mathbf{b}_2 + \frac{1}{2} \left( \mathbf{a}_1^{(n)} e^{i\lambda_1^{(n)}t} + \mathbf{a}_2^{(n)} e^{i\lambda_2^{(n)}t} + \mathbf{a}_3^{(n)} e^{i\lambda_3^{(n)}t} \right) \\ X_3^n(t) &= -\frac{1}{2} \mathbf{b}_2 + \frac{1}{2} \left( \mathbf{a}_1^{(n)} e^{i\lambda_1^{(n)}t} + \mathbf{a}_2^{(n)} e^{i\lambda_2^{(n)}t} + \mathbf{a}_3^{(n)} e^{i\lambda_3^{(n)}t} \right) \\ X_4^n(t) &= -\gamma \sqrt{n+2} \left( \frac{\mathbf{a}_1^{(n)}}{\lambda_1^{(n)} + \delta} e^{i(\lambda_1^{(n)} + \delta)t} + \frac{\mathbf{a}_2^{(n)}}{\lambda_2^{(n)} + \delta} e^{i(\lambda_2^{(n)} + \delta)t} + \frac{\mathbf{a}_3^{(n)}}{\lambda_3^{(n)} + \delta} e^{i(\lambda_3^{(n)} + \delta)t} \right) \end{aligned} \quad (4.30)$$

The constants  $\mathbf{a}_i^{(n)}$  and  $\mathbf{b}_2$  are determined by initial conditions. A set of independent initial conditions is given by  $X_j^n(0) = \delta_{jk}$  ( $k = 1, \dots, 4$ ); the set of associated solutions is denoted by  $a_i^{(n,k)}$ ,  $b_2^{(k)}$  and  $X_i^{(n,k)}(t)$ . Also for consistency we define for  $k = 1, \dots, 4$

$$X_1^{(-1,k)} = X_1^{(-2,k)} = X_2^{(-2,k)} = X_3^{(-2,k)} = 0. \quad (4.31)$$

The  $\lambda_i^{(n)}$  are related to the energy eigenvalues of stationary states of the combined system and bath. Owing to the entanglement between the bath and the matter it is not useful to consider the bath as leading to a small perturbation of the matter state.

From (4.25) and (4.30) we have for  $k = 1$

$$\begin{aligned} 1 &= -\gamma\sqrt{n+1} \left( \frac{a_1^{(n,1)}}{\lambda_1^{(n)} - \delta} + \frac{a_2^{(n,1)}}{\lambda_2^{(n)} - \delta} + \frac{a_3^{(n,1)}}{\lambda_3^{(n)} - \delta} \right), \\ 0 &= a_1^{(n,1)} + a_2^{(n,1)} + a_3^{(n,1)}, \\ 0 &= \frac{a_1^{(n,1)}}{\lambda_1^{(n)} + \delta} + \frac{a_2^{(n,1)}}{\lambda_2^{(n)} + \delta} + \frac{a_3^{(n,1)}}{\lambda_3^{(n)} + \delta}, \\ b_2^{(1)} &= 0. \end{aligned}$$

Similarly for  $k = 2$

$$\begin{aligned} 0 &= \frac{a_1^{(n,2)}}{\lambda_1^{(n)} - \delta} + \frac{a_2^{(n,2)}}{\lambda_2^{(n)} - \delta} + \frac{a_3^{(n,2)}}{\lambda_3^{(n)} - \delta}, \\ 1 &= a_1^{(n,2)} + a_2^{(n,2)} + a_3^{(n,2)}, \\ 0 &= \frac{a_1^{(n,2)}}{\lambda_1^{(n)} + \delta} + \frac{a_2^{(n,2)}}{\lambda_2^{(n)} + \delta} + \frac{a_3^{(n,2)}}{\lambda_3^{(n)} + \delta}, \\ b_2^{(2)} &= 0. \end{aligned}$$

For  $k = 3$

$$\begin{aligned} 0 &= \frac{a_1^{(n,3)}}{\lambda_1^{(n)} - \delta} + \frac{a_2^{(n,3)}}{\lambda_2^{(n)} - \delta} + \frac{a_3^{(n,3)}}{\lambda_3^{(n)} - \delta}, \\ 1 &= a_1^{(n,3)} + a_2^{(n,3)} + a_3^{(n,3)}, \\ 0 &= \frac{a_1^{(n,3)}}{\lambda_1^{(n)} + \delta} + \frac{a_2^{(n,3)}}{\lambda_2^{(n)} + \delta} + \frac{a_3^{(n,3)}}{\lambda_3^{(n)} + \delta}, \\ b_2^{(3)} &= -1. \end{aligned}$$

For  $k = 4$

$$\begin{aligned} 0 &= \frac{a_1^{(n,4)}}{\lambda_1^{(n)} - \delta} + \frac{a_2^{(n,4)}}{\lambda_2^{(n)} - \delta} + \frac{a_3^{(n,4)}}{\lambda_3^{(n)} - \delta}, \\ 0 &= a_1^{(n,4)} + a_2^{(n,4)} + a_3^{(n,4)}, \\ 1 &= -\gamma\sqrt{n+2} \left( \frac{a_1^{(n,4)}}{\lambda_1^{(n)} + \delta} + \frac{a_2^{(n,4)}}{\lambda_2^{(n)} + \delta} + \frac{a_3^{(n,4)}}{\lambda_3^{(n)} + \delta} \right), \\ b_2^{(4)} &= 0. \end{aligned}$$

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