An Industrial Organization Analysis
for the Colombian Banking System

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Abstract

This paper presents two versions of a spatial competition model for the banking sector. The first version, describes a framework that follows closely Salop's spatial competition model. This version is modified in the second part by introducing the loan market and default risk probabilities for credit. Both theoretical approaches are analyzed empirically for the Colombian data,covering the period 1996-2005. Our results allow us to construct a deviation of the observed number of branches from an optimal number of branches for the banking system throughout the period of study. The deviation indicates that in the last years the number of branches is below the optimum which suggest that political measures should focus in increasing the number of branches in the country. Additionally, we found empirical evidence of market separability between the loan and deposit markets, and finally, we were able to determine the signs of the relations between credit collateral, payment probability and interest rates.

1 Introduction

In the last years, international literature has outlined the importance of spatial variables in the analysis of banks' strategic behavior[1]. Three main facts explain their introduction in competitive and efficiency models. First, they recognize the service variable within the competitive variables of banking institutions. Second, they introduce the access costs that customers assume when they demand banking services. Even though technology has presented a huge progress in the last years, the negative relation between distance from a bank and utility for the customer is still obvious[2]. Finally, they evaluate banks' efficiency per branch. In this way, they allow the authorities to identify if the market is over dimensioned or if there is a deficit of banking services in a geographical space.

This paper intends to make use of these *spatial variables* in a scenario consistent with the New Empirical Industrial Organization (NEIO) approach, which analyzes markets with differentiated products. More specifically, this work focuses in studying competition under a scenario with spatial differentiation. Some examples of this type of literature are Hotelling (1929), Salop (1979) and Economides (1989). Of these models, the most widely used in spatial literature is Salop's approach of the circular city. For instance, Fuentelsaz and Salas (1992), apply it to the Spanish case to study the effects of spatial competition in banks' efficiency; Chiappori et al. (1995), use it to analyze the effects of regulation over the credit market in Europe; Freixas and Rochet (1997) employ it to find the optimal number of branches; and Zanna (2003) uses the model to identify the consequences of introducing financial innovations.

Traditionally in Colombia, the empirical work that analyzes banks' behavior and efficiency has focus on price and quantity variables. Therefore, spatial variables have received little attention. Some examples include, Levy and Micco (2003), Mora (2004), Estrada (2005) and Salamanca (2005). This

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[1]For instance, Chiappori et al. (1993) propose a framework in which banks compete simultaneously with interest rates and branches to analyze the effect of regulation, Barros (1997) exposes a spatial competition model to explain price differences across banks in the deposit market and Kim and Vale (2001) set up an oligopolistic model to test for the role of the branch network as a non-price strategic variable in the Norway banking sector.

[2]This argument does not imply that banks can not use technology to mitigate this relation.

work seeks to emphasize their importance by analyzing the dimension of the banking system branching network in the context of efficiency and competition. *In particular, the purpose of this document is to prove the hypothesis that suggests that Colombian banks obey as well to a spatial strategic behavior when they make their decisions. Meaning from this perspective, that the relations between interest rates, default probabilities, deposits and credits with number of branches can be better understand from this point of view.* Additionally, the proposed framework allows to calculate the optimal number of branches for the banking system.

The paper is structured in five additional sections. The first section, presents a deposit model that follows the framework first proposed by Salop (1969) of spatial competition around the unit circle. In this section, the model is generalized to any geographical space, following Fuentelsaz and Salaz (1992). The next section, modifies the previous model to introduce the loan market and default risk. Section 4 deals with functional forms, samples and data. Section 5, exposes the empirical estimation of both models, and finally, section 6 concludes.

2   Deposits Market Model

We begin describing a location-based differentiation scheme as the one model by Fuentelsaz and Salas (1992)[3]. In this framework, we have a continuum of depositors distributed uniformly along a unit circle, and n banks (indexed by $i = 1, ..., n$) which only have one branch. This assumption does not generate a theoretical problem, because branches compete for depositors even within the same bank. The difference is the strength of the competition. A branch director prefers to loose a client with other branch of the same bank than with other bank. Additionally, he has global policies that limit the competition with branches of the same bank. For practical reasons, and given that in the model a branch represents a bank we will use this terms arbitrarily. In this scenario, the opening of an additional branch generates a vicious externality over the market shares of the remaining banks[4].

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[3]The authors propose a model inspired in Salop's work. Salop (1979) was the first one to introduce the idea of a circle market in which firms compete for deposits.

[4]See Chiappori et al. (1995) and Tirole (1988).

Banks collect deposits and invest them into riskless projects that return a constant rate. Depositors do not have direct access to the riskless investments which in turn justifies the financial intermediation of banks. The location of banks is symmetrical, so the distance between two adjacent banks would be $\frac{1}{n}$, as shown in Figure 1.

Figure 1: Location of Agents in the Circle
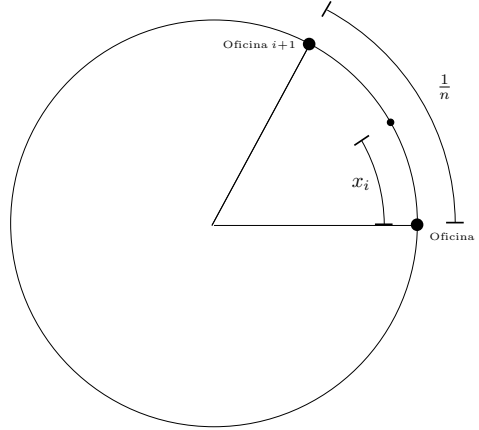
Furthermore, customers assume a positive transportation cost (t) by unit distance (x) when they make their deposits in a bank. However, they receive an interest rate for their deposits (r_{di}). From this assumptions we can express depositors utility as:

$$U_i = \max[(r_{di} - tx_i), 0]$$

Given that utility could be zero for those costumers that walk a distance larger than $\frac{1}{n}$, and given that the customers will only put their money in a bank if the interest rate offered allow them to make some positive utility, the relevant market of a bank would be only this extension. Additionally as in Fuentelaz and Salas (1992) a depositor, may or may not have information of the banks that surround him with certain probability[5]. In this way, if the customer does not have full information he would deposit his money in the bank that he knows, otherwise he would deposit his money in the bank that offers him the highest rate and is nearer. The probability that a customer

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[5]This could be due to the places he visits every day or the information he may have in certain moments.

has information of the existence of bank i would be denoted by Φ_i with $\Phi_i \in (0,1)$. This probability allows us to define two different scenarios of the model: one in which customers have perfect information and other with imperfect information.

2.1 Model with Perfect Information

Under a scenario with perfect information, an individual located between two banks would deposit its money into bank i by analyzing two variables. In first place, he would consider which of the banks around him offers a higher rate for his deposits, and in second place, he would choose the nearest bank. This way, he would choose bank i only if:

$$r_{di} - tx_i \geq r_d - t \left[\frac{1}{n} - x_i \right]$$

Where r_{di} represents the interest rate that the i bank pays to the deposits and r_d is the interest rate of the competitor bank. Thus, the customer would be indifferent between i and its adjacent bank if:

$$r_{di} - tx_i = r_d - t \left[\frac{1}{n} - x_i \right] \tag{1}$$

From equation (1) we can derive an expression for the deposit supply of bank i that lies between its location and the location of the bank $i+1$. This supply function would be given by the distance x_i:

$$x_i = \frac{r_{di} - r_d + \frac{t}{n}}{2t} \tag{2}$$

Equation (2) does not represent the total deposit supply available for the bank i, because it excludes the segment that lies down to the other side of the bank in the symmetric circle. Hence, the total supply of bank i would be given by equation (2) multiplied by two:

$$d_i(r_{di}, r_i) = \frac{1}{n} + \frac{r_{di} - r_d}{t} \tag{3}$$

2.2 Model with Imperfect Information

Under this scenario, banks i and $i + 1$ compete only for customers that have information of both banks. Thus, if Φ_i is the probability of knowing

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the existence of bank i, the probability that a depositor knows both banks around him would be given by $\Phi_i\Phi$, where Φ denotes the probability of having information of the bank that is different from i. Therefore, the probability that a depositor only knows the bank i, would be given by $\Phi_i(1-\Phi)$. Keeping in mind the symmetry of this problem, the deposit supply under imperfect information could be written as follows:

$$d_i(r_{di}, r_d) = 2[\Phi_i(1-\Phi)]\frac{1}{n} + \Phi_i\Phi\left[\frac{r_{di} - r_d}{t} + \frac{1}{n}\right] \tag{4}$$

Then, bank i has to solve the following problem under monopolistic competition:

$$\max_{r_{di} \geq 0} \pi = [s - r_{di}]d_i(r_{di}, r_d) - f \tag{5}$$

where s accounts for the riskless interest rate and f represents the fixed costs associated with the installment of a new branch [6]. The first order condition of the profit function is given by:

$$[s - r_{di}]\frac{dd_i}{dr_{di}} - d_i = 0$$

which could be written as:

$$[s - r_{di}]\frac{\Phi_i\Phi}{t} - \left[2\Phi_i(1-\Phi_i)\frac{1}{n} + \left(\Phi_i\Phi\left(\frac{r_{di} - r_d}{t} + \frac{1}{n}\right)\right)\right] = 0 \tag{6}$$

Equation (6) represents the reaction function of bank i. Additionally, in a symmetrical Nash equilibrium every bank offers the same interest rate ($r_{di} = r_d$), and every individual has the same information ($\Phi_i = \Phi$). Therefore, the optimal financial margin, represented by the difference between the riskless rate of investments and the deposit interest rate, can be written as follows:

$$[s - r_{di}]^* = \frac{t}{n}\left(\frac{2 - \Phi}{\Phi}\right) \tag{7}$$

The expression reveals that the financial margin is positively related with transportation costs, which is intuitive given that banks have to compensate for the costs that individuals assume when they approach to a bank. The equation also unveils a negative relation between the financial margin and the

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[6]In this scenario, costs are only associated with deposits and riskless investments.

knowledge probability $\left(\frac{\delta[s-r_{di}]^*}{\delta\Phi} \leq 0\right)$, which can be explained by competition forces. If agents have more information then banks are forced to compete harder by reducing their margins. Finally, we observe that the total number of branches for the geographical area (n) is inversely related with the banks' financial margin. This relation is due to the fact that a higher number of branches is associated with a more competitive behaviour.

By replacing equation (7) into the profit function, and making the function equal to zero, for a competitive solution, the optimal number of branches for the banking system is:

$$n^* = (2 - \Phi)\sqrt{\frac{t}{f}} \tag{8}$$

Equation (8) shows that the total number of branches of the system decreases with fixed costs (f) and with the knowledge probability (Φ), while it increases with costumers' transportation costs (t). The intuition of the inverse relation between the fixed costs and the number of branches is obvious. When the cost per branch increases, the number of branches decreases, given the banks' budget constraint. However, the relation between the number of branches and the knowledge probability is not straight forward.

If individuals have more information (which would be represented in a higher Φ), then the total number of branches may decline because customers can take greater advantage from the available branches. Moreover, when transportation costs are higher, new branches are opened: some customers may choose not to deposit their money if the transportation costs exceed the utility that the interest rate generates to them. Under this circumstances, a new bank may acknowledge the situation and decide to enter the market, keeping these customers.

2.3 De-normalizing the Model

Until now we have assumed that banks and customers are located in a unit circle. In this subsection we generalize the market dimension to a geographical space of size Km, such that $Km > 0$, which represents a more consistent scenario and allows us to rewrite equation (5) that represents the profit function as:

$$\pi_i(n) = [s - r_d]\left(\frac{D_i}{Km}\right) - F$$

where from equation (7) we can express:

$$[s - r_{di}] = \frac{t}{n}\left(\frac{2 - \Phi}{\Phi}\right)$$

and F represents banks' costs per opened branch. For simplicity a parameter τ is defined, the parameter comprises the transportation costs and the probability of knowledge Φ, such that:

$$\tau = t\left(\frac{2 - \Phi}{\Phi}\right)$$

Following Fuentelaz and Salas (1992), it is assumed that the costs per branch have a positive relation with its relative size. Intuitively, it can be justified by the higher number of employees and other resources a branch with more deposits should use. Thus, the cost function per branch can be written as:

$$F(D_i) = f\left(\frac{D_i}{Km}\right)^{\beta}$$

Where β accounts for scale economies. If $\beta = 0$, it would indicate that banks have constant returns in costs; if $\beta < 1$ it would represent increasing returns to scale; and, if $\beta > 1$, it would point decreasing returns to scale.

By using τ and $F(D_i)$ the profit function could be reexpressed as:

$$\pi(n) = \tau\left(\frac{1}{n}\right)\left(\frac{D_i}{Km}\right) - f\left(\frac{D_i}{Km}\right)^{\beta} \tag{9}$$

By making the profits of the representative bank equal to zero for the competitive solution, the optimal number of branches for the banking system is:

$$n^* = \left(\frac{\tau}{f}\right)\left(\frac{D_i}{KM}\right)^{1-\beta} \tag{10}$$

The equation confirms the relations that were found under the normalized model. First, the total number of branches of the banking system increases with the total supply of deposits per unit area in the market. Second, it

decreases with the costs per branch, and finally, it grows with the total costs assumed by the customers when they move towards a bank. The previous subsection explained these relations.

Replacing (10) in the first order condition for the financial margin, we obtain the optimal financial margin for the generalized model:

$$[s - r_{di}]^* = f\left(\frac{Km}{D_i}\right)^{1-\beta} \tag{11}$$

This equation represents the optimal financial margin of bank i. It shows that the relation between the financial margin and the stock of deposits per unity of geographical area for the bank i depends of the signs of the parameters β and f. Equations (10) and (11) will be used for the empirical analysis in the next sections.

3   Loan-Deposit Model

In order to propose a more complete model that recognizes the importance of loans in banks strategic behavior, we follow Chiappori et al. (1995). In this paper, the credit market is added to Salop's circle city where most of the assumptions made in the deposit model introduced earlier are maintained. Again, n banks are located around a unit circle uniformly and each customer assumes a transportation cost per unit distance when he is moving towards a bank to deposit money or to take a loan. However, the transportation cost is not necessarily the same for each activity; the transportation cost to make a deposit is denoted by t and the one incurred to take a loan would be t_l. When a customer receives a loan he will pay the bank a interest rate h, and similarly, he will receive an interest rate of r for each deposit.

Taking into account the balance sheet restriction that banks face, the total volume of loans that each bank may offer, denoted by V, is assumed to be less than the total amount of deposits each bank holds.

This paper modifies Chiappori's model by introducing default risk in the loan market. We assume that for each loan, the representative bank assumes a default probability of non-fulfilment. Therefore, the bank may receive the money it lends with a probability of p and, on the other hand, with a

probability of $1-p$ it would just receive the collateral (η) of the credit. In this context, the collateral is the minimum payment that the bank will receive. We assume as well, that banks are constantly monitoring each of the credits to prevent moral hazard in agents[7]. This last assumption implies that banks create incentives for individuals and firms in such a way that they have good will to pay back credits. Naturally, this monitoring represents an additional cost for banks which is transferred directly to the customers when they pay the credit. This is a crucial element of the model because it allows us to leave banks risk management aside.

The characterization of the deposit supply follows the same analysis of the model that only includes the deposit market mentioned in the previous section. Hence, the deposit supply of bank i can be written as:

$$D_i(r_i, r_{di}) = 2x = \frac{1}{n} + \frac{r_i - r_{di}}{t} \tag{12}$$

where x stands for the distance between the individual and the bank and r_{di} represents the interest rate offered by i's competitor.

To obtain the credit demand, the behavior of the indifferent customer must be characterized. This customer faces a condition that satisfies the following equation:

$$t_l Y + p h_i L - (1-p)(L - \eta) = t_l \left(\frac{1}{n} - Y \right) + p(h_{di} L) - (1-p)(L-\eta) \tag{13}$$

In this expression Y represents the distance of the customer to the bank; η stands for the collateral, which has a lower face value than the loan $(\eta < L)$; $1 - p$ is the default probability; and h_{di} represents the loan interest rate of the neighbor competitor of bank i. The equation implies that the cost of taking a loan for the indifferent customer is equivalent to the transportation cost plus the interests payed on the loan. Nevertheless, in the expression the interest rate is discounted from the net benefit that the customer may gain when he has a loss and only pays the collateral η.

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[7]Almazan (1999) proposes a theoretical model of competition that takes into account the level of capitalization of banks and their ability to monitor different types of projects. Using this two concepts, the author solves the problem of risk management that each bank faces when it lends money for a specific project. However, this type of analysis was not incorporated to our model in order to maintain empirical applicability.

From equation (13) we obtain the total demand for loans,which could be written as follows:

$$L_i(h_i, h_{di}) = 2VY = V\left(\frac{1}{n} + \left[\frac{h_i - h_{di}}{t_l}\right]Lp\right) \tag{14}$$

Letting s be the interest rate of the riskless investments and C the cost function of the bank, the following profit function can be obtained:

$$\Pi_i(h_i, r_i) = s(D_i - L_i) - r_i(D_i) + ph_i(L_i) - (1-p)(L_i - \eta) - C \tag{15}$$

The expression is only different from the previous profit functions because it includes default probabilities. As it is shown, if the customer pays the credit the bank gets the interest on the loan as expected, nonetheless, if the customer defaults, the bank looses the amount that is not covered by the collateral of the loan.

By taking the first order conditions of the profit function with respect to the interest rates of deposits (r_i) and of loans (h_i) and replacing $r_i = r_{di}$ and $h_i = h_{di}$ for the Nash solution we obtain the following expressions:

$$r_i = s - \frac{t}{n} \tag{16}$$

$$h_i = \frac{1}{p}\left(\frac{t_l}{Ln} + s + (1-p)\right) \tag{17}$$

Surprisingly, equation (16) is the same expression that Chiappori et al.(1995) obtained for the model that did not includes default risk. As expected, the interest rate on deposits depends positively on the number of competitors and on the interest rate of the riskless investments and on the number of competitors. This last is due to the competition practices that banks perform through deposits interest rates.

As equation (17)shows, the interest rate on loans decreases with the probability of payment and the number of competitors. The first relation could be explained by the known trade-off that exists between risk and loan interest rates: if the probability of default decreases the bank is facing less risk which in turn allows it to reduce the interest rate. On the other hand, the inverse relation between number of branches and the interest rates, responds to the

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competitive behavior of banks. Additionally, the equation reveals that the interest rate on loans increases with the riskless rate and the transportation costs. The effect of the riskless rate is accounting for the opportunity cost that the bank incurs when it makes a loan. Finally, if a customer has to assume higher transportation costs he would demand lower interest rates for the same credit.

By substituting equations (16) and (17) into the profit function and assuming the free entry condition ($\Pi = 0$), the optimal number of banks in the market (n^*) for the short run is:

$$n^* = \sqrt{\left(t + \frac{V t_l}{L}\right)\left(\frac{1}{C - (1 - p)\eta}\right)} \tag{18}$$

The equation shows that the number of banks in the market increases with the transportation costs associated with loans or deposits and with the total number volume of loans available in the market (V). Yet, the number of branches is not directly affected by the stock of deposits in the market in the short run. This last indicates, that the relevant market for taking decisions concerning the opening of new branches is the loan market. The above represents an important result that must be taken into account by regulators, given that when they introduce policies that intend to affect the number of branches in the territory, they can focus them on the loan market to achieve more instant results.

The expression also unveils that the number of banks in the market is a decreasing function of costs, of the collateral of loans and of the default probability. The first of these relations is obvious, nonetheless, the inverse effect of the collateral of loans and of the default probability in the number of branches deserves an explanation. On the one hand, when the collateral increases, individuals that are risk averse are going to be more cautious when they take a credit, reducing loan demand and therefore, the number of branches. On the other hand, when the probability of default is higher, banks are the ones that would be more cautious lending, reason for which they will not open new branches even if there is high demand for credits.

Replacing equation (18) into the first order conditions of the profit function, given by (16) and (17), gives the long run equilibrium conditions for the interest rates of the market, which can be written as follows:

$$r_i^* = s - \frac{t}{\sqrt{(t + \frac{Vt_l}{L})(\frac{1}{C-(1-p)\eta})}} \tag{19}$$

$$h_i^* = \frac{1}{p}\left(\frac{t_l}{L\sqrt{(t + \frac{Vt_l}{L})(\frac{1}{C-(1-p)\eta})}} + s + (1-p)\right) \tag{20}$$

The cost function is specified following Fuentelaz y Salas (1992). The authors assume that costs per branch have a positive relation with its relative size, which is justified by the increase in the number of employees and resources each branch must incur when the deposits or credit stock grows. Therefore, the cost function is given by:

$$C = f(\frac{D_i}{km})^\gamma (\frac{L_i}{km})^\phi \tag{21}$$

where γ, ϕ and f stand for parameters of the equation.

By replacing C in equations (19) and (20) we derive the empirical analysis of the model in the next sections.

4 Functional Forms and Data

4.1 Functional Forms

The empirical estimation of the model is divided in two parts that correspond to both of the theoretical models exposed in the previous sections.

Deposits Market Model

For the empirical estimation of the model that only includes the deposit market (exposed in section 2) we estimate the linearized versions of equations (10) and (11)[8]. The linearization of equation (10), that shows the optimal

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[8]The equations are given by the following expressions:

$$n^* = \left(\frac{\tau}{f}\right)\left(\frac{D_i}{KM}\right)^{1-\beta}$$

$$[s - r_{di}]^* = f\left(\frac{Km}{D_i}\right)^{1-\beta}$$

number of branches for the banking system per square kilometer for bank i in the t period, would be given by:

$$\ln(n_t) = a_0 + a_1 \ln\left(\frac{D_{it}}{Km}\right) + \varepsilon_{it}, \tag{22}$$

where $a_0 = ln\left(\frac{\tau}{f}\right)$, and $a_1 = (1 - \beta)$. In this equation, n_t stands for the optimal number of branches of the banking system per square kilometer; D_{it}, accounts for the deposits stock of the bank i in period t, and Km represents the geographical area in square kilometers.

In the same way, the linear functional form for equation (11), that explains the financial margin of bank i in period t is:

$$\ln(s - r_{it}) = b_0 + b_1 \ln\left(\frac{Km}{D_{it}}\right) + \epsilon_{it} \tag{23}$$

where $b_0 = ln(f)$, and $b_1 = (1 - \beta)$. Here, $(s - r^{it})$ represents the optimal financial margin. As can be seen in (22) and (23), the linearization of the theoretical model suggest that a_1 must be equal to b_1 .

Loan-Deposit Model

The theoretical model exposed in section 3, gives two functional forms for the empirical estimation. These are equations (19) and (20)[9] after replacing C with the functional form proposed in expression (21). Given that in this model t, γ, Φ and t_l are unknown parameters, this two equations can be expressed as:

$$r_i^* = r_i^*\left(s, V, L, D, p, \eta\right) \tag{24}$$

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[9]The equations were given by the following expressions:

$$r_i^* = s - \frac{t}{\sqrt{(t + \frac{Vt_l}{L})(\frac{1}{C-(1-p)\eta})}}$$

$$h_i^* = \frac{1}{p}\left(\frac{t_l}{L\sqrt{(t + \frac{Vt_l}{L})(\frac{1}{C-(1-p)\eta})}} + s + (1-p)\right)$$

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$$h_i^* = h_i^*\bigg(s, V, L, D, p, \eta\bigg) \tag{25}$$

We take the linear expressions of these functions in order to develop a more rigorous econometric method and make full use of all the available data. This does not imply a loss of generality for our purposes, because we would be able to validate the model if the symbols of the coefficients correspond with the theoretical analysis.This is a common practice in related literature. For instance, Angbazo (1997) applies this method to linearize the reported net interest margins. However, we will not be able to analyze the magnitude of the parameters obtained. As Melvyn et al.(1978) mention: *The choice of a functional form should be based on an integrated consideration of the economic problem.*

4.2   Sample and Data Description

In order to fully exploit the available data, different samples were used for the estimation of each of the models exposed above. In particular, the number of branches is only available quarterly from January of 1994 to September of 2005. Since it is a variable required for the empirical estimation of the model of section 2, we use this sample to estimate that model. However, the number of branches is not necessary for the model that was introduced in section 3, for this reason we used another sample with higher periodicity and larger span for the estimation of the second model. Each of the samples is described next.

Deposits Market Model - Sample

The data used for the estimation of equations (22) and (23) covers the period between January 1994 and September 2005. It has quarterly frequency and it was obtained from the information published by the Colombian Financial Superintendence [10].

The deposit interest rate (r) was constructed as the ratio between the interest expenditures and the total amount of deposits taken from the bank's balance

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[10]The information is available in the Financial Superintendence web page: http://www.superfinanciera.gov.co/

sheets and the loss and profit accounts[11]. Similarly, the loan interest rate was calculated as the ratio between the interest rents and the total stock of credits. The financial margin $(s - r_{id})$ was computed as the difference between both of these rates. These estimations correspond to the implicit interest rates, since there is no available information for each bank's marginal rate.

Finally, information concerning the square kilometers of the country was taken from the Colombian Atlas of the Geographical Institution Agustin Codazzi for 2005.

Four variables were constructed from the sample to offer some preliminary information of the size and efficiency of Colombian banks during the period of analysis. As can be seen in Figure 2, the four variables are gross domestic product per branch, population per branch, deposits per branch and branches per square kilometer. The values in the graphics correspond to the averages of all the observations for each period.

As can be seen in the figures, the variables of gross domestic product per branch and deposits per branch exhibit a similar behavior. In 2002, this variables reached a peak, however, this increase was not caused by a fall in the number of branches because the number of branches grew that year. Thus, we may conclude that 2002 was a year in which efficiency per office increased, since the increase in the number of branches was not reflected in a reduction in the stock of deposits per branch. The series reached a higher peak in 2004, nevertheless, this time the increase was explained by a reduction in the number of branches as can be observed in the graph of branches per square kilometer. It is possible that from this date on, banks were motivated with the positive results increasing the number of branches in the market and causing a reduction of deposits per branch for 2005.

Additionally, it most be noted that the number of branches per square kilometer presented a positive trend till 1998, from this year on it has been near to a constant with mild fluctuations.

Loan-Deposits Model - Sample

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[11]This type of estimation for the interest rates has been applied widely in empirical literature, see Barajas et al.(2000), Reyes (2004), Uchida (2005) or Salamanca (2005).

Figure 2: Comparative analysis of size and efficiency
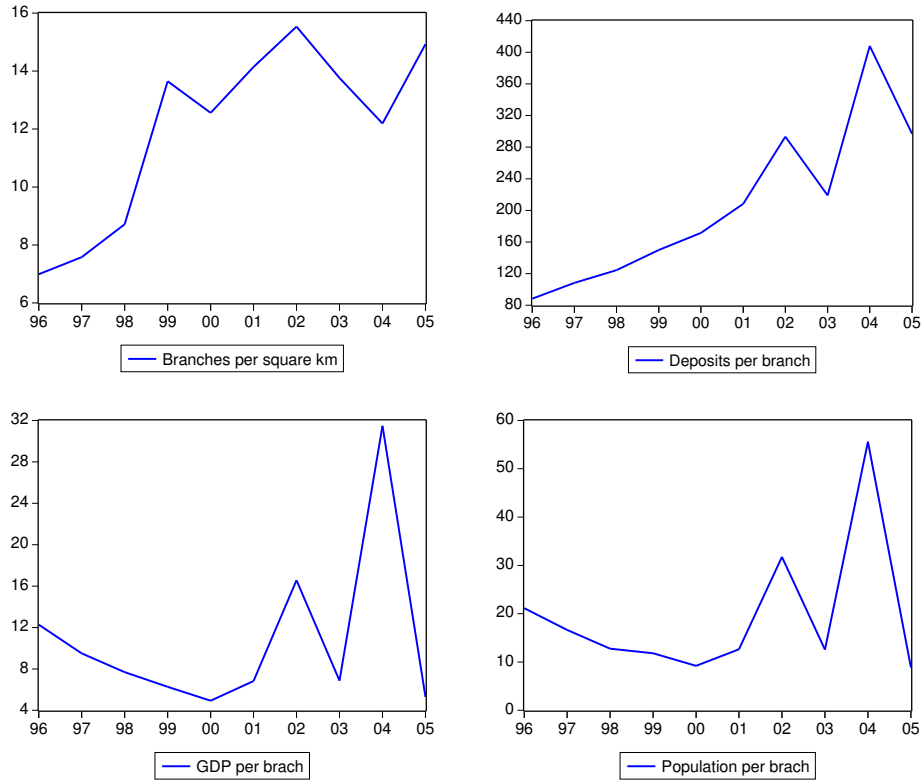
On the first three figures the series are divided by a 100,000, whereas, the series of population per branch was multiplied by this value. Therefore, GDP per branch is in thousand of millions of pesos; population per branch is in 100,000 habitants per branch; deposits per branch are in thousands of millions and branches per square kilometer indicate the number of branches per 100,000 square kilometers.

As was mentioned previously, a different sample was used to estimate the model that includes the loan market, because there was more data available for the required variables. The sample employed has monthly frequency and it covers the period between March 1995 and July 2006. The series were obtained from the information published by the Colombian Financial Superintendence.

Similarly, loan interest rates were calculated as the ratio between the interest rents to the total amount of credits, and the deposits rates were computed as the ratio of interest expenditures to the total amount of deposits. Furthermore, The riskless rate for banks investment (s) was obtained from a weighted average rate for government bonds constructed by the *Bolsa de Valores de Colombia* called $IPTES$.[12].

Given that in Colombia only mortgage and commercial loans have collateral (η), this last one, was constructed as a ratio between these type of loans and the total stock of credits for each bank. Likewise, to construct the default probability for each bank ($1 - p$) in each of the periods we divide non-performing loans by total loans.

Finally, to estimate the V, the total volume of loans that a bank may offer, we calculate the total amount of deposits that a bank does not require to maintain in reserves or as obligatory investments.

5   Empirical Results

Given the structure of the data, the estimation of both models is based on the methodology of Biørn (1999). The author, considers the estimation of regression systems with *random individual effects* in the intercept from unbalanced panel data, i.e. data where individual time series have unequal length[13]. This method, is described in Appendix 1, with some detail.

As in Judge et al. (1985), in order to obtain unbiased and consistent estimators under Biørn's methodology, a fundamental assumptions of no correlation between the explanatory variables and the latent random individual effect (or

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[12]The statistic was constructed since March of 2005 and it is published in the diary bulletin of the institution.

[13]Single equation models with unbalanced panel data are studied in Biørn (1981).

unobserved effect) has to be made. If this assumption does not holds, the individual effects estimator (dummy variable or within estimator) should be used.

Given the multi-equational (SUR) nature of the model under unbalanced panel data structure, Vásquez (2007) proposes an extended version of the *Hausman* specification test for the correlation between the independent variables and the latent effect. As usual, the correspondent test statistic is based in the comparison between the covariance matrices of the random and fixed individual effects estimators. In order to compute this test statistic, the fixed effects estimator for regression systems from unbalanced panel must be estimated reason for which it is proposed as well in Vásquez (2007), including a brief extension of restricted seemingly unrelated regression estimators and the correspondent test statistic to the case of unbalanced panel data a brief description on restricted seemingly unrelated regression estimators and the correspondent test statistic is found in Judge et al.(1985, 1988). This analysis was extended by Vásquez(2007) to the case of unbalanced panel data. This methodology is described in Appendixes 2 and 3.

5.1   Estimation of Model I

For the model of section 1, we estimate jointly equations (22) and (23), by using the above methodology. As can be seen, these equations constitute, under certain conditions, a system of seemingly unrelated equations (SUR) with equality linear restrictions. Such restrictions are imposed by the theoretical model that requires that a_1 is equal to b_1.

Results

By comparing the value obtained by the Hausman specification test statistic (81.00636) against its correspondent critical value for the χ^2 distribution with two degrees of freedom (DF) (9.21034) at 1 percent of significance, the null hypothesis of no correlation was rejected. Therefore, the fixed effects estimator was used. The estimates are outlined in Table 1. As can be seen, the explanatory variables are statistically significant. Additionally, the value obtained for the χ^2 statistic test (0.02494), used for checking the validity of the linear equality restriction, compared with its respective critical value (6.63490) indicates that the mentioned restriction has empirical support.

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Table 1: Estimation results for the deposit model

	Dependent variable	Variable	Coefficient	St. Error	T-stat
Equation 1	$ln(n_t)$	Mean intercept	-7.38		
		$ln(\frac{Q}{Km})$	0.19	0.014	13.86
Equation 2	$ln(s - r_{id})$	Mean intercept	-1.7		
		$ln(\frac{Km}{Q})$	0.19	0.014	13.86

The two estimated equations were:

$$\ln(n_t) = a_0 + a_1 * \ln\left(\frac{D_{it}}{Km}\right) + \varepsilon_{it},$$

where $a_0 = ln\left(\frac{\tau}{f}\right)$ and $a_1 = (1 - \beta)$, and:

$$\ln(s - r_{id}) = b_0 + b_1 * \ln\left(\frac{Km}{D_{it}}\right) + \epsilon_{it}$$

where $b_0 = ln(f)$ and $b_1 = (1-\beta)$. The sample has 3302 observations that cover the period between January 2006 and September 2005. It includes 46 banks that were in the market in some or in all of the years of analysis.

This last result implies that the restriction is statistically valid, at 1 percent of significance level. Since the coefficients fulfill the linear equality, they have the same value in the table.

Based on the estimation, the value of the parameter β was computed, following the theoretical framework, as follows:

$$a_1 = (1 - \beta)$$

$$b_1 = (1 - \beta)$$

By replacing a_1 and b_1 by 0.19 in the last equations, the obtained value for the parameter β is 0.81. Recalling the functional form for the cost function,

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this value indicates that banks have increasing returns to scale[14]. This result could be interpreted as a sign of *growing efficiency in the banking system throughout the period in study.* This result is consistent with Estrada (2005).

Moreover, from the estimation we were able to construct the number of branches per square kilometer that every bank view as optimal in each period for the whole banking system. This was done by replacing the values of the parameters in equation (22) for each observation and then taking an average of the constructed dependent variables[15]. The constructed average for all the observations in the sample was of **2573**. Table 2, shows the difference between the optimal number of branches and the observed number of branches for each of the years analyzed in the estimation. As can be seen, the number of branches increased in the first years until 2001, date from which it started to decrease. Although, the number of branches has decline since 2001, it is still higher than the optimal number computed in the estimation. This conclusion needs to be carefully analyzed given that the estimation only takes into account the effective demand and supply for credits and deposits. Thus, several areas in the country were banks are not present are not included.

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[14]The cost function was given by:

$$C(q) = f \left(\frac{Q}{L} \right)^{\beta}$$

[15]For a_0 the value of the mean intercept was used. Additionally equation (22) is:

$$\ln(n_t) = a_0 + a_1 \ln \left(\frac{\mathrm{D}_{it}}{\mathrm{Km}} \right) + \varepsilon_{it},$$

Table 2: Difference between the optimal and the observed number of branches

Period	Branches	Dist.**
1996 09	1783	-790
1997 09	1913	-660
1998 09	2512	-61
1999 09	2838	265
2000 09	2850	277
2001 09	3121	548
2002 09	3029	456
2003 09	2631	200
2004 09	2801	228
2005 09	2723	150

* Distance from the estimated average. The optimal average is of 2573.

5.2   Estimation of Model II

For the model that includes the loan market presented in section 2 the linear versions of equations (26) and (27) were estimated[16]. Again, we jointly estimate the equations by using the methodology proposed by Vásquez (2007). However, this estimation does not include the extension for linear restrictions.

Results

As in the previous model, the fixed effects estimator was employed given that the value obtained with the *Hausman* specification test statistic (200.65859) compared with the critical value for the χ^2 distribution with zero degrees of freedom (DF) (23.20925) at 1 percent of significance, allows to reject the null hypothesis of no correlation between the explanatory variables and the latent effect.

---

[16]The equations were given by the linear versions of the following expressions:

$$r_i^* = r_i^*\left(s, V, L, D, p, \eta\right) \tag{26}$$

$$h_i^* = h_i^*\left(s, V, L, D, p, \eta\right) \tag{27}$$

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	Dependent Var.	Variable	Coefficient	St. Error	T-Stat
Equation 1	h	intercept (mean)	0.2463		
		s	0.1668	0.0173	9.6081
		L	-0.0289	0.0059	-4.8748
		D	0.0049	0.0045	1.0828
		p	0.5788	0.0269	21.491
		η	-0.1146	0.0162	-7.0403
Equation 2	r	intercept (mean)	0.1182		
		s	0.1800	0.0149	12.079
		L	-0.0001	0.0051	-0.0219
		D	0.0001	0.0038	3.7334
		p	0.5415	0.0231	23.421
		η	-0.0218	0.0139	-1.5596

The two estimated equations were:

$$t_i^* = t_i^*\left(s, L, D, p, \eta\right)$$

$$r_i^* = r_i^*\left(s, L, D, p, \eta\right)$$

The sample has 3302 observations that cover the period between January of 2006 and September of 2005. It includes 46 banks that were in the market in some or in all of the years in analysis.

The estimation results are outlined in Table 3. The variable V (that represents the total volume of loans that a bank may offer) which caused severe multicolinearity problems was not included in the estimation. This is not a crucial consequence from the theoretical point of view because each bank determines the total volume of loans it may lend as a percentage of the total stock of deposits, thus, the relevant variable, the stocks of deposits an loans, contains all the relevant information for determining the interest rates, meaning that from this two variables V could be obtained.

In both equations the only variables that are not statistical significant are the total stock of deposits (D) in equation 1 and the stock of loans (L) in the

second equation. This results can be interpreted as evidence that deposits do not affect the loan interest rate directly, and viceversa, that loans do not affect the deposit interest rate directly. This is an important conclusion for understanding banks strategic behavior when they fix each of these rates, this reinforces the idea that market separability in some theoretical models is not absolutely mistaken.

The results also unveil a positive relation between the riskless rate of investment (s) and the loan interest rate (h), which can be explained by the opportunity cost of credits. If safe investments pay a higher rate then a rational bank would demand a higher remuneration for its money. Likewise, the estimation shows, as was expected, that the total stock of loans affects inversely the interest rate. This relation can be explained probably in the other direction: when a bank offers lower interest rates on credits it increases its net demand for loans[17]. With respect to the collateral (η), the estimation indicates that when there is an increase in the required collateral the interest rate declines, which is intuitive given the lower risk that a bank faces when it increases the collateral for credits.

On the other hand, the second equation shows that the deposits interest rate (r), depends positively on the riskless rate of investment (s) and on the total stock of deposits (D) while it is negatively affected by the collateral (η). The first relation can be explained by competitive behavior: if banks have higher remuneration for their investments they are able to compete with higher deposit rates. The positive effect of deposits on the interest rate can be seen more clearly in the other direction as in equation 1 with loans, when the interest rate on deposits is higher the deposits supply increases. Last, the inverse relation between the collateral and the deposits interest rate can be interpreted as a rent effect: when banks have a higher collateral they are going to recover the invested money with a higher probability, thus, they are able to compete in the deposit market with higher rates.

Finally, both equations show a positive effect of the payment probability (p) on the interest rates. This outcome is easily explained for the deposits rate given that when the probability of payment is higher the banks have safer investments and they are able to compete with higher rates in the

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[17]It is important to clarify that econometric estimations should never be viewed as causality relations but as pure relations between variables.

deposit market. However, the positive relation between this probability and the interest rate on loans is not that clear. At first, the economic intuition suggest an inverse relation of these variables given the reduction on risk, but a second explanation may be suggested, if a bank is aware of the rationality of its clients it knows that a reduction on loan rates leads to higher probability of payment and viceversa. That is, if a client has high rates on credits and he faces any difficulty he may default the credit with a higher certainty than the one he would have if he faces lower rates.

In conclusion, the data validates both of the theoretical models exposed in sections 2 and 3. Therefore, the hypothesis that suggested that Colombian banks develop a strategic spatial behavior from which relations of banking variables can be better understood could be proven.

6   Conclusions

This paper intends to determine if Colombian banks present a strategic spatial behavior. Furthermore, this work seeks to analyze if the spatial perspective allows to improve our understanding of banking variable relations.

In order to achieve this objective this paper proposes two versions of Salop´s model of spatial competition. The first one, follows closely the methodology of Fuentelsaz and Salas (1992) in which banks compete for customers in a geographical space. The second version, incorporates the loan market into the spatial framework by following the methodology developed in Chiappori et al. (1995). However, this model was modified to introduce default risk probabilities into the loan market.

The estimation of both models was achieved by using regression systems for seemingly unrelated equations for unbalanced panel data. The results, lead to two important conclusions. First, banks present growing efficiency in the banking system throughout the period in study, and second, the financial system presents evidence of over dimension with respect to the number of branches since 1999. Additionally, the empirical estimations clarified the following relations:

- There is a positive relation between the riskless rate of investment and he loan interest rate, which can be explained by the opportunity cost of credits.

- There is an inverse relation between the collateral and the interest rate on loans, which is due to the lower risk that a bank faces when it increases the collateral for credits.

- An increase in the required collateral has an inverse relation on deposits interest rate which can be interpreted as a rent effect: when banks have higher collateral they recover the invested money with higher probability, thus, they are able to compete in the deposit market with higher rates.

- There is a positive effect of the payment probability on the interest rates. In the case of deposits rates it can be explained given that when the probability of payment is higher, the banks have safer investments

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and they are able to compete with higher rates in the deposit market. In the case of the loans rate, it is explained by banks awareness of clients rationality.

- Empirical support was found to the idea of market separability between the credit and deposit markets.

A   Appendix 1: Estimating Regression Systems from Unbalanced Unbalanced Panel Data (Random Effects)

The objective of this appendix is to briefly describe the key steps of the estimation with random individual effects proposed by Biørn (1999).

As the author points out, let a system of G equations (indexed by $g = 1, ..., G$) with observations from unbalanced panel data with N individuals (indexed by $i = 1, ..., N$). Each individual is observed in at least one and at most p periods. Let N_p denote the number of individuals observed p periods (not necessary the same and not necessarily consecutive, $p = 1, ..., P$), and let N be the total number of observations, i.e., $N = \sum_{p=1}^{P} N_p$ and $n = \sum_{p=1}^{P} N_p p$.

Ordering the individuals in P groups such that N_1 are observed once, and N_2 twice and so on, M_p denotes the cumulated number of individuals observed up to p times:

$$M_p = N_1 + N_2 + ... + N_p, \qquad p = 1, 2, ..., P \qquad (28)$$

In this case $M_1 = N_1$ and $M_p = N$. Consider as well that I_p stands for the index set of the individuals observed p times, such that:

$$I_1 = [1, ..., M_1]$$
$$I_2 = [M_1 + 1, ..., M_2]$$
$$\vdots \qquad\qquad (29)$$
$$I_p = [M_{p-1} + 1, ..., M_p]$$

Where, I_1 could be considered as a cross section and I_2, I_3,..., I_P as balanced sub-panels with $2, 3, ..., P$ observations for each individual respectively. Hence, the $g'th$ equation for the i individual in the t period, given H_g regressors is be given by:

$$y_{git} = x_{git}\beta_g + \alpha_{gi} + u_{git} \qquad (30)$$
$$g = 1, ..., G; \qquad i\epsilon I_p; \qquad t = 1, ..., p; \qquad p = 1, ..., P$$

where y_{git} represents the dependent variable; x_{git} is the $(1 \times H_g)$ vector that contains the independent variables; β_g stands the coefficient vector of dimensions $(H_g \times 1)$ of the $g'th$ equation including the intercept term; α_{gi} represents the latent effect specific to each individual i in the $g'th$ equation; and μ_{git} is the disturbance of i in the $g'th$ equation for the t period. The G equations system for each individual in period t can be written as:

$$
\begin{pmatrix} y_{1it} \\ . \\ . \\ . \\ y_{Git} \end{pmatrix} = \begin{pmatrix} x_{1it} & 0 & . & . & 0 \\ 0 & x_{2it} & 0 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & . & x_{Git} \end{pmatrix} \begin{pmatrix} \beta_1 \\ . \\ . \\ . \\ \beta_G \end{pmatrix} + \begin{pmatrix} \alpha_{1i} \\ . \\ . \\ . \\ \alpha_{Gi} \end{pmatrix} + \begin{pmatrix} u_{1it} \\ . \\ . \\ . \\ u_{Git} \end{pmatrix} \tag{31}
$$

or in compact form:

$$
y_{it} = X_{it}\beta + \epsilon_{it}, \qquad \epsilon_{it} = \alpha_i + u_{it} = \begin{pmatrix} \epsilon_{1it} \\ . \\ . \\ . \\ \epsilon_{Git} \end{pmatrix} \tag{32}
$$

In the above expression, the dimensions of x_{it} and of β are $(G \times H)$ and $(H \times 1)$, respectively. It is assumed that:

$$
E(\alpha_i) = 0_{G,1}
$$
$$
E(\mu_{it}) = 0_{G,1}
$$
$$
E(\alpha_i \alpha_j') = \delta_{ij}\Sigma_\alpha \tag{33}
$$
$$
E(\mu_{it}\mu_{js}') = \delta_{ij}\delta_{ts}\Sigma_\mu
$$

where the $\delta's$ are kronecker deltas and:

$$
\Sigma_\alpha = \begin{pmatrix} \sigma_{11}^\alpha & \cdots & \sigma_{1G}^\alpha \\ \vdots & \cdots & \vdots \\ \sigma_{G1}^\alpha & \cdots & \sigma_{GG}^\alpha \end{pmatrix}
$$

$$
\Sigma_\mu = \begin{pmatrix} \sigma_{11}^\mu & \cdots & \sigma_{1G}^\mu \\ \vdots & \cdots & \vdots \\ \sigma_{G1}^\mu & \cdots & \sigma_{GG}^\mu \end{pmatrix}
$$

From the assumptions made the composite disturbance vectors defined in (32) satisfy:

$$E(\epsilon_{it}) = 0_{G,1} \qquad E(\epsilon_{it}\epsilon_{js}) = \delta_{ij}(\Sigma_\alpha + \delta_{ts}\Sigma_u)$$

In this way the individual mean for ϵ's could be defined by:

$$\bar{\epsilon}_{i.} = \begin{cases} \epsilon_{i1} & \text{for } i \in I_1, \\ (1/2) \sum_{t=1}^{2} \epsilon_{it} & \text{for } i \in I_2 \\ \vdots & \\ (1/p) \sum_{t=1}^{p} \epsilon_{it} & \text{for } i \in I_p \end{cases} \tag{34}$$

the global mean is:

$$\bar{\epsilon} = \frac{1}{n} \sum_{p=1}^{P} \sum_{i \in I_p} \sum_{t=1}^{p} \epsilon_{it} = \frac{1}{n} \sum_{p=1}^{P} \sum_{i \in I_p} p\bar{\epsilon}_{i.} \tag{35}$$

By defining the within individual $(W_{\epsilon\epsilon})$, and between individual variation$(B_{\epsilon\epsilon})$ as:

$$W_{\epsilon\epsilon} = \sum_{p=1}^{P} \sum_{i \in I_p} \sum_{t=1}^{p} (\epsilon_{it} - \bar{\epsilon}_{i.})(\epsilon_{it} - \bar{\epsilon}_{i.})'$$

$$B_{\epsilon\epsilon} = \sum_{p=1}^{P} \sum_{i \in I_p} p(\bar{\epsilon}_{i.} - \bar{\epsilon})(\bar{\epsilon}_{i.} - \bar{\epsilon})'$$

the author shows that the unbiased estimators for $\widehat{\Sigma_u}$ and $\widehat{\Sigma_\alpha}$ are given by:

$$\widehat{\Sigma_u} = \frac{W_{\epsilon\epsilon}}{n - N} \tag{36}$$

$$\widehat{\Sigma_\alpha} = \frac{B_{\epsilon\epsilon} - \frac{n-1}{n-N}W_{\epsilon\epsilon}}{n - \frac{\sum_{p=1}^{P} N_p p^2}{n}} \tag{37}$$

To get the G.L.S estimates by groups, which is the fundamental base of the G.L.S estimation for the set of total information, the next matrices are defined:

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$$y_{i(p)} = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{ip} \end{bmatrix} \qquad X_{i(p)} = \begin{bmatrix} X_{i1} \\ \vdots \\ X_{ip} \end{bmatrix} \qquad \epsilon_{i(p)} = \begin{bmatrix} \epsilon_{i1} \\ \vdots \\ \epsilon_{ip} \end{bmatrix} \tag{38}$$

For $i \in I_p$, $p = 1, 2, \ldots, P$. Where y_{ip}, $X_{i(p)}$ and $\epsilon_{i(p)}$ are the stacked $(G_p \times 1)$ vector, $(G_p \times H)$ matrix and $(G_p \times 1)$ vector of $y's$, $X's$ and $\epsilon's$, respectively of the p observations of the i individual $i \in I_p$. In compact matrix notation for the set of observations, the model for each individual could be written as:

$$Y_{i(p)} = X_{i(p)}\beta + (e_p \otimes \alpha_i) = X_{i(p)}\beta + \epsilon_{i(p)} \tag{39}$$

Where e_p is a vector $(p \times 1)$ with ones. For the last equation it is supposed that:

$$E[\epsilon_{i(p)}] = 0, \qquad E[\epsilon_{i(p)}\epsilon'_{i(p)}] = I_p \otimes \Sigma_u + E_p \otimes \Sigma_\alpha = \Omega_{\epsilon(p)} \tag{40}$$

Where I_p is the p dimensional identity matrix and $E_p = e_p e'_p$ is a matrix of ones of dimensions (p x p). Accordingly, the generalized least squares (GLS) problem for estimating the coefficient vector β_{GLS} is obtained by minimizing the quadratic form, given by:

$$Q = \sum_{p=1}^{P} \sum_{i \epsilon I_p} \epsilon'_{ip} \Omega_{\epsilon(p)}^{-1} \epsilon_{ip} \tag{41}$$

where:

$$\epsilon_{ip} = y_{ip} - x_{ip}\beta$$

$$\Omega_{\epsilon(p)}^{-1} = B_p \otimes \Sigma_u^{-1} + A_p \otimes (\Sigma_u + p\Sigma_\alpha)^{-1}$$

with $B_p = I_p - (\frac{1}{p})E_p$ and $A_p = (\frac{1}{p})E_p$. Thus, the generalized least square estimator β_{GLS} is:

$$\beta_{GLS} = \left(\sum_{p=1}^{P} \sum_{i \epsilon I_p} X'_{i(p)} \Omega_{\epsilon(p)}^{-1} X_{i(p)} \right)^{-1} \left(\sum_{p=1}^{P} \sum_{i \epsilon I_p} X'_{i(p)} \Omega_{\epsilon(p)}^{-1} Y_{i(p)} \right) \tag{42}$$

The covariance matrix is written as:

$$V(\beta_{GLS}) = \left(\sum_{p=1}^{P} \sum_{i \epsilon I_p} X'_{i(p)} \Omega_{\epsilon(p)}^{-1} X_{i(p)} \right)^{-1} \tag{43}$$

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Finally, the algorithm suggested to obtain the corresponding GLS estimator (β_{GLS}) for all the observations could be described as:

1. Obtain consistent estimators for the first residuals. $\widehat{\epsilon}_{it} = y_{it} - X_{it}\widehat{\beta}_{OLS}$, with $\widehat{\beta}_{OLS}$ taken from a regression with all the observations y_{it} y X_{it}.

2. Substitute ϵ_{it} by $\widehat{\epsilon}_{it}$ and get the estimators. $\widehat{W}_{\widehat{\epsilon}\widehat{\epsilon}}$ y $\widehat{\beta}_{\widehat{\epsilon}\widehat{\epsilon}}$.

3. Replace $W_{\epsilon\epsilon}$ and $\beta_{\epsilon\epsilon}$ by $\widehat{W}_{\widehat{\epsilon}\widehat{\epsilon}}$ and $\widehat{\beta}_{\widehat{\epsilon}\widehat{\epsilon}}$ and get the estimators $\widehat{\Sigma_u^{-1}}$, $\widehat{\Sigma_\alpha^{-1}}$ and $\widehat{\Sigma_{(p)}^{-1}}$ of the matrices Σ_u^{-1}, Σ_α^{-1} and $\Sigma_{(p)}^{-1}$.

4. Substitute Σ_u^{-1}, Σ_α^{-1} and $\Sigma_{(p)}^{-1}$ by $\widehat{\Sigma_u^{-1}}$, $\widehat{\Sigma_\alpha^{-1}}$ and $\widehat{\Sigma_{(p)}^{-1}}$ and obtain $\widehat{\beta}_{GLS}$ and $V(\widehat{\beta}_{GLS})$.

In order to obtain $\widehat{\beta}_{GLS}$ (random effects estimator) under linear equality restrictions [18]($\widehat{\beta}^*_{GLS}$) , the extension about testing equality linear restrictions on the coefficient vector for the SUR case described in Judge et al. (1985) is followed. A set of linear coefficients is defined in the following expression:

$$R\beta = r \tag{44}$$

where R and r are known matrices of dimensions $(j \times k)$ and $(j \times 1)$, respectively, j denotes the number of restrictions and k the number of explanatory variables (which includes the intercept term). Under the assumption that the vector ϵ is normally distributed and the null hypothesis $R\beta = r$ is true it can be shown that:

$$g = (R\widehat{\beta}_{GLS} - r)'(R\widehat{C}R')^{-1}(R\widehat{\beta}_{GLS} - r) \backsim \chi^2_{(j)} \tag{45}$$

where $\widehat{C}$ represents the covariance matrix of the estimator of random effects described in (43). Therefore, the correspondent test statistic is given by g. In this way, the linear restricted estimator would be given by:

$$\widehat{\beta}^*_{GLS} = \widehat{\beta}_{GLS} + \overline{C}R'(R\widehat{C}R')^{-1}(r - R\widehat{\beta}_{GLS}) \tag{46}$$

---

[18]It is important to note two important issues: i) This kind of test will only have a large sample justification given that the relevant test statistic depends on an estimator of the variance-covariance residual matrix. ii) In the context of multiple equations it is now possible to test restrictions that relate the coefficients in one equation with the coefficients in other equations.

B   Appendix 2: Estimating Regression Systems from Unbalanced Panel Data (Fixed Effects)

This appendix is a summary of the methodology proposed by Vásquez (2007) concerning with fixed effects estimation in regression systems from unbalanced panel data. Additionally, this appendix considers his extension of the *Hausman* specification test for this kind of models. Given that this work is in the same fashion of Biørn's methodology, described in Appendix 1, this appendix uses, as much as possible, his notation, organization of the data and model specification.

As in appendix 1, consider the same G equations system and the same indexes for periods and individuals. Equations (28), (29) remain the same, nevertheless the specification of the model under the equation (30) has some modifications, so it could be rewritten in the following way:

$$y_{git} = \overline{\beta}_{1g} + x_{git}^s \beta_g^s + \alpha_{gi} + u_{git}, \qquad g = 1, ..., G; \qquad i\epsilon I_p; \qquad t = 1, ..., p; \ (47)$$

$$t = 1, ..., p; \qquad p = 1, ..., P$$

where y_{git} stands for the dependent variable, x_{git}^s is a $(1 \times H_g)$ regressor vector (note that the supra-index s allow to differentiate this vector from the vector that contains the 1 associated with the intercept), β_g^s is the $(H_g \times 1)$ vector of unknown coefficients which does not includes the intercept term and is common to all individuals, u_{git} is the genuine stochastic disturbance term of individual i in the $g'th$ equation in the observation t, finally, $\beta_{1g} = \overline{\beta}_{1g} + \alpha_{gi}$ is the intercept for the gth equation for the individual i in moment t. Thus, $\overline{\beta}_{1g}$ represents the average intercept for the gth equation and α_{gi} represents the difference of this average for the ith individual in the gth equation. In this model it is assumed that α_{gi} is fixed (not stochastic).

As in Appendix 1, the stacked system for expressing equation (29) could be reexpressed as follows:

$$
\begin{pmatrix} y_{1it} \\ \vdots \\ y_{Git} \end{pmatrix} = \begin{pmatrix} \overline{\beta}_{11} \\ \vdots \\ \overline{\beta}_{1G} \end{pmatrix} + \begin{pmatrix} \alpha_{1i} \\ \vdots \\ \alpha_{Gi} \end{pmatrix} + \begin{pmatrix} X^s_{1it} & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & X^s_{Git} \end{pmatrix} \begin{pmatrix} \beta^s_1 \\ \vdots \\ \beta^s_G \end{pmatrix} + \begin{pmatrix} u_{1it} \\ \vdots \\ u_{Git} \end{pmatrix} \qquad (48)
$$

or in compact notation:

$$
y_{it} = \overline{\beta}_1 + \alpha_i + X^s_{it}\beta^s + u_{it} \qquad (49)
$$

In the above expression, the dimensions of X_{its} and of β are $(G \times H)$ and $(H \times 1)$, respectively. It is assumed that:

$$
E(u_{it}) = 0_{G,1} \qquad (50)
$$

$$
E(u_{it}u^{'}_{is}) = \delta_{ij}\delta_{ts}\Sigma_u
$$

where X_{it} and u_{it} are not correlated and the $\delta's$ are kronecker deltas and:

$$
\Sigma_u = \begin{pmatrix} \sigma^u_{11} & \cdots & \sigma^u_{1G} \\ \vdots & \vdots & \vdots \\ \sigma^u_{G1} & \cdots & \sigma^u_{GG} \end{pmatrix} \qquad (51)
$$

By substituting ϵ by u in equations (34) and (35) the individual specific mean $(\overline{u}_{i.})$ and the global of the error term and $(\overline{u})$ are respectively defined. The corresponding $(G \times G)$ matrix of within individual covariation could be expressed as:

$$
W_{uu} = \sum_{p=1}^{P} \sum_{i\in I_p} \sum_{t=1}^{p} (u_{it} - \overline{u}_{i.})(u_{it} - \overline{u}_{i.})' \qquad (52)
$$

Following Biørn (1999)it is possible to show that:

$$
E(W_{uu}) = (n - N)\Sigma_u
$$

such that:

$$
\widehat{\Sigma}_u = \frac{W_{uu}}{n - N} \qquad (53)
$$

Specifying the $(G_p \times 1)$ the vectors $y_{i(p)}$, $u_{i(p)}$ and the $(G_p \times H)$ matrix $X_{i(p)}$ in the same way that in equation (38), the corresponding model for the p

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observations of the individual $i \in I_p$ could be expressed in compact form as follows:

$$y_{i(p)} = \{[e_p \otimes (\overline{\beta}_1 + \alpha_i)] + X^s_{i(p)}\beta^s + u_{i(p)}\}$$

Where e_p is the $(p \times 1)$ vector of ones, $\overline{\beta}_1$ is a $(G \times 1)$ vector which represents the mean intercept common to all individuals that differs for each equation, β^s is an stack of vectors with dimensions $(\sum_{g=1}^{H} H_g \times 1)$ which contains the unknown coefficients associated to the explanatory variables in all the equations of the system except the intercept. $X^s_{i(p)}$ is a $(Gp \times \sum_{g=1}^{H} H_g)$ matrix which the observations of the explanatory variables for all the equations of the system except for the vector of ones associated to the intercept of each equation. $u_{i(p)}$ is a $(Gp \times 1)$ vector of the stochastic disturbance term. Finally, it is possible to define $\overline{\beta}_{1i} = (\overline{\beta}_1 + \alpha_i)$ the $(G \times 1)$ vector of intercepts for the *ith* individual belonging to the *pth* group.

From equation (50) it follows that:

$$E(u_{i(p)}u'_{i(p)}) = I_p \otimes \Sigma_u = \Omega_{u(p)}$$

where I_p is an identity matrix of order p and $\Omega_{u(p)}$ is the $(Gp \times Gp)$ covariance matrix of the disturbances.

The fixed effects estimator (β^s_{FE}) for all the equations using all observation when the Σ_u matrix is known, could be obtained by minimizing the quadratic form:

$$Q = \sum_{p=1}^{P} \sum_{i \in I_p} u'_{i(p)} V^{-1}_{u(p)} u_{i(p)}, \qquad u_{i(p)} = y_{i(p)} - [e_p \otimes (\overline{\beta}_1 + \alpha_i) - X^s_{i(p)}]$$

where:

$$V^{-1}_{u(p)} = D_{ps}\Omega^{-1}_{u(p)}, \qquad \Omega^{-1}_{u(p)} = I_p \otimes \Sigma^{-1}_u, \qquad D_{ps} = D_p \otimes I_g, \qquad D_p = I_p - (\frac{1}{p})e_p e'_p$$

where I_p is the p dimensional identity matrix and $e_p e'_p$ is a $(p \times p)$ matrix of ones. Therefore, the generalized least square estimator β^s_{FE} is:

$$\widehat{\beta}_{FE}^{s} = \left(\sum_{p=1}^{P}\sum_{i\in I_p} X_{i(p)}^{s'} V_{u(p)}^{-1} X_{i(p)}^{s}\right)^{-1} \left(\sum_{p=1}^{P}\sum_{i\epsilon I_p} X_{i(p)}^{s'} V_{u(p)}^{-1} y_{i(p)}'\right) \quad (54)$$

Finally, the covariance matrix is written as:

$$V(\widehat{\beta}_{(FE)}^{s}) = \left(\sum_{p=1}^{P}\sum_{i\epsilon I_p} X_{i(p)}^{s'} V_{u(p)}^{-1} X_{i(p)}^{s}\right)^{-1} \quad (55)$$

In order to estimate β_{FE}^{s} and its corresponding covariance matrix $V(\beta_{FE}^{s})$, the following algorithm is recommended:

1. Run OLS separately for all of the G equations using the observations in y_{it} and X_{it}^{s}. By stacking the estimators, obtain the joint estimator $\widehat{\beta}_{OLS}^{s}$. Compute the corresponding consistent residual vectors $\widehat{u}_{it} = y_{it} - X_{it}\widehat{\beta}_{OLS}^{s}$ for all i and t.

2. Estimate the within matrix of residuals by inserting $u_{it} = \widehat{u}_{it}$ in (52).The estimator is denoted by $W_{\widehat{u}\widehat{u}}$.

3. Estimate Σ_u by replacing $W_{\widehat{u}\widehat{u}}$ in (53) for $p = 1, ..., P$. The estimator is denoted by $\widehat{\Sigma}_u$.

4. Obtain the feasible GLS estimator of β_s by replacing $\widehat{\Sigma}_u$ into (54).

In order to obtain $\widehat{\beta}_{FE}^{s}$(fixed effects estimator) under linear equality restrictions $(\widehat{\beta}_{FE}^{*})$, the extension about testing equality linear restrictions on the coefficient vector as in the case of the previous appendix is applied. In this way, equation (44) remains the same and (45) could be rewriten as:

$$g = (R\widehat{\beta}_{FE}^{s} - r)'(R\widehat{C}R')^{-1}(R\widehat{\beta}_{FE}^{s} - r) \curvearrowright \chi_{(j)}^{2} \quad (56)$$

where $\widehat{C}$ represents the covariance matrix of the estimator of fixed effects described in (55). In this way, the linear restricted estimator would be given by:

$$\widehat{\beta}_{FE}^{*} = \widehat{\beta}_{FE}^{s} + \overline{C}R'(R\widehat{C}R')^{-1}(r - R\widehat{\beta}_{FE}^{s}) \quad (57)$$

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C   Appendix 3: Hausman Specification Test

In order to choose between the fixed or random effects estimator for the unknown parameters of the models given by the systems of equations (22)-(23) and (24)-(25) an extension of the Hausman (1978) specification test, proposed by Vásquez (2007) is used given that the data constitutes a framework of unbalanced panel data. This test is used to determine if the latent variable has correlation with the independent variables or with the error term, i.e. to test the null hypothesis (H_0) of no correlation of the explanatory variables and the individual effects. If H_0 is rejected the fixed effects estimator should be used.

As Baltagi (1995), Wooldridge (2002) note and Judge et al. (1985), remark: "It would appear that a reasonable prescription is to use the error components model if $\alpha_i \sim i.i.d(0, \sigma_\alpha^2)$ assumption is a reasonable one and N is sufficiently large for reliable estimation of σ_u; otherwise, particularly when α_i, and X_i^s are correlated or N is small they recommend the within estimator".

Furthermore, Hausman (1978) states that under the random effects specification $\widehat{\beta}_{GLS}$ is the asymptotically efficient estimator while the fixed effects estimator $\widehat{\beta}_{FE}^s$ is unbiased and consistent but not efficient[19]. If $E\left(\alpha_i | X_i^s\right) \neq 0$ the random effects estimator is biased and inconsistent while the fixed effects estimator is not affected by this failure of orthogonality. For this reason, it is crucial to use the mentioned extension of this test in order to choose between $\widehat{\beta}_{FE}^s$ and $\widehat{\beta}_{GLS}$. The statistic could be computed as follows:

$$H = [\beta_{FE}^s - \beta_{GLS}^s]'[V(\beta_{FE}^s) - V(\beta_{GLS})]^{-1}[\beta_{FE}^s - \beta_{GLS}^s]$$

The H statistic has an asymptotic $\chi_{(k)}^2$ distribution, where k denotes the total number of explanatory variables without the intercept term. Additionally, in this expression β_{FE}^s denotes the fixed effects estimator given by equation (54), β_{GLS}^s represents the random effects estimator expressed in equation (42) and $V(\beta_{FE}^s)$ and $V(\beta_{GLS})$ stand for the covariance matrices of those estimators, described in equations (55) and (43), respectively.

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[19]Additionally, a problem for the fixed effects estimator is its sensitivity to errors in variables. In this case, when the variables are obtained in deviations from their individual means, the amount of inconsistency increases.

Following Judge et al. (1985), it is important to note that the most important problem using the *Within* estimator is : "when α_i and X_i^s are correlated, the *Within* estimator may be less efficient than the alternative consistent estimators that exploit information on the relationship between α_i and X_i^s and that do not ignore sample variation across individuals". However, literature on such estimators in the context of linear regression system equations (SUR) under unbalanced panel data seems to be not available until now.

References

Almazan, A.(1999). A Model of Competition in Banking: Bank Capital vs Expertise. *Journal of Financial Intermediation..* Vol. 11, Pg. 87-121.

Angbanzo, L.(1997). Commercial bank interest rate margins, default risk, interest rate risk, and off balance sheet banking. *Journal of Banking and finance..* Vol. 21, Pg. 55-87.

Baltagi,B., H. (1999). Econometric Analysis of Panel Data. *John Wiley and Sons.*

Barajas A., Salazar, N. and Steiner, R. (1999). Interest spreads in banking in Colombia: 1974-1996. *Staff Papers, International Monetary Fund..* Vol. 46.

Barros, P. (1997). Multimarket competition in banking, with an example from the Portuguese markat *International Journal of Industrial Organization.* Vol.17, pp. 335-352.

Berg S. A., and Kim M., (1994). Oligopolistic interdependence and the structure of production in banking: An empirical evaluation. *Journal of Money, Credit and Banking.* Vol.26 pp. 309-322.

Berg S. A., and Kim M., (1996). Banks as multioutput oligopolies: An empirical evaluation of the retail and corporate banking markets. *Federal reserve bank of Chicago.*

Biorn E. Estimating Regression Systems from Unbalanced Panel Data:A Stepwise Maximun Likelihood Procedure. *Department of Economics. University of Oslo*

Chiappori, P. A., D. Perez-Castrillo, and Verdier D., (1993). Spatial competition in the banking system: Localization, cross subsidies and the regulation of deposit rates. *European Economic Review.* Vol. 39, pp. 889-918.

Economides, N.(1989). Desirability of Compatibility in the Absence of Network Externalities. *Banco de la República de Colombia.*

Estrada, D.(2005). Efectos de las fusiones sobre el sector financiero colombiano. *American Economic Review.* Vol. 79, pp.1165-81.

Freixas, X., and Rochet, J., (1997). Microeconomics of banking.*Cambridge: MIT Press.*

Fuentelesaz, L., and Salas, V., (1992). Competencia Espacial en la banca al por menor.*Findación BBVA.*

Gilbert A.(1984). Bank market structure and competition.*Journal of Money, Credit and Banking.* Vol. 16, pp. 205-218

Hannan T., And Liang, (1993). Inferring market power from time series data.*International Journal of Industrial Organization.* Vol. 11,No.4.

Hausman., A.(1978). Specification Tests in Econometrics.*Econometrica.* Vol. 46, No.6., pp.1251-1271.

Hotelling H.(1929). Stability in Competition.*The Econommic Journal.* Vol. 39, No.153., pp.41-57.

Iwata, G.(1974). Measurement of conjectural variations in olipoly.*Econometrica.* Vol. 42, No. 5, pp.947-966.

Judge, G., Griffiths, E., Hill, k., Lüktepohl, H., and Lee, T.(1985). The theory and Practice of Econometrics. Second Edition.*Wiley Series in Probability and Mathematical Statistics.*

Kim, M., and Vale, M.(2001). Non-price strategic behavior: the case of bank branches.*International Journal of Industrial Organization.* Vol. 19, pp. 1583-1602.

Kim, M., Vivas, A. L., and Morales A.(2003). Multistrategic spatial competition with application to banking. *University of Malaga.*

Lau, L. (1982). On identifyng the degree of competitiveness from industry price and industrial data.*Economic Letters.* Vol.10, pp. 93-99.

Levy, E., and Micco, A.(2003). Concentration and foregin penetration in Latin American banking sectors: Impact on competition and risk. *Escuela de negocios. Universidad Torcuatto Di Tella.*

Melvyn, F., McFadden D. and Mundlack Y. (1978). Production Economics: a Dual Approach to Theory and Aplicattions. North Holand.

Mora, H. (2004). Eficiencia de los sistemas bancarios de los países miembros del FLAR. *FLAR.*

Neven, D., and Roller, L.(1999). An aggegate structural model of competition in the European banking industry.*International Journal of Industrial Organization.* Vol.17, pp. 1059-1074.

Panzar, J., C., and Rosee, J., N.(1987). Testing for monopoly equilibrium.*Journal of Industrial Economics.* Vol.35, pp. 443-456.

Reyes, C.(2004). El poder de mercado de la banca privada colombiana. *Tesis de Maestría, Universidad de los Andes.*

Salop, S.(1979). Monopolistic competition with outside goods.*The Bell Journal of Economics.* Vol. 10, No. 1, pp. 141-156.

Schmalensee, R. (1989). Good Regulatory Regimes. *Rand Journal of Economics.* Vol. 20, No. 3, pp. 417-436.

Salamanca, D.,(2005). Competencia en los mercados de credito y depositos en Colombia: aplicacion de un modelo de oligopolio fijador de precios. *Universidad de los Andes.*

Shaffer, S.(1989). Competition in the US banking industry *Economic Letters.* Vol. 29, pp.321.

Shaffer, S.(1993). A test of competition in Canadian banking. *Journal of Money, Credit and Banking.* Vol. 25, pp.49-61.

Swank, J.(1995). Oligopoly in loan and deposit markets: An econometric application to the Netherlans. *De Economist.* Vol. 143, pp.353-366.

Suonemin, N.(1994). Measuring competition in banking: A two product model.*Scandinavian Journal of Economics.* Vol. 96, pp.95-110.

Toolsema L.,(2002). Competition in the Dutch consumer credit market.*Journal of Banking and Finance.* Vol. 26.

Uchida, H., y Tsutsui, Y., (2005). Has competition in the Japanese banking sector improved?. *Journal of Banking and Finance.* Vol. 29.

Vásquez, D., (2007). A extension of the methos for estimating regression systems from unbalanced panel data. *Unpublished.* Banco de la República .

Vesala J., (1995). Testing competition in banking: behavioral evidence from Finland. *Bank of Finland Studies.*

Wooldridge J., M., (2002). Econometric Analysis of Cross Section and Panel Data. *MIT Press.*

L.F. (2003)Interest Rates, Promotional Prizes and Competition in the Banking Industry *Documentos CEDE.* Facultad de Economía, Universidad de los Andes. 2003-08.