Determinants of Interest Margins in Colombia

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Abstract

This paper analyzes the determinants of interest margins in the Colombian Financial System. Based on the model by Ho and Saunders (1981), interest margins are modelled as a function of the *pure* spread and bank-specific institutional imperfections using quarterly data for the period 1994:IV-2005:III. Additionally, the *pure* spread is estimated as a function of market power and interest rate volatility. Results indicate that interest margins are mainly affected by credit institutions' inefficiency and to a lesser extent by credit risk exposure and market power. This implies that public policies should be oriented towards creating the necessary market conditions for banks to enhance their efficiency.

Key words: Interest Margins, Competition, Credit Risk, Interest Rate Risk.

JEL Classification: L11; L41; L89, G21, G28.

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1 Introduction

The present paper is focused on modelling the interest margin of credit institutions, considering the effect of variables related to risk, market competition and operational and intermediation costs.

Based on the model developed by Ho and Saunders (1981), banks are modelled as dealers in the credit market acting as intermediaries between suppliers and demanders of financial funds. The interest margin that results of the bank's maximization problem is a function of the degree of market competition and interest rate volatility.

However, this paper differs from the aforementioned work, both in the estimation techniques used in the empirical tests as well as in the variables used to determine interest margins. Specifically, Lerner (1981) comments how the original paper by Ho and Saunders (1981) does not include any type of intermediation costs by the bank different from traditional financial costs¹. Wong (1997) explicitly introduces exploitation costs in the explanation of the interest margin as does the model in this paper when estimating the latter.

More importantly, the theoretical model here used includes an asset and an interbank market, as well as a market structure which resembles a model of monopolistic competition with product differentiation, given that agents cannot willingly substitute one bank for another. In this sense, the model here presented can be viewed as an extension of the original model, such as has been done by Allen (1988), McShane and Sharpe (1985) and Angbazo (1997), among others.

Identifying the determinants of interest margins in Colombia is a relevant public policy issue, since monetary authorities are concerned about the efficiency and competitiveness of the payment system and the degree of financial depth. Therefore, policy recommendations oriented towards achieving a lower interest margin can be derived once the main factors affecting the spread are determined.

This paper will focus on the microeconomic determinants of interest mar-

¹Other intermediation costs include expenses related with enhancing existing costumerbank relationships and attracting new clients (e.g. offering different goods for opening new accounts or getting credit cards, among others). Such expenses could lead banks to increase their margin in order to compensate for the additional cost.

gins, in order to capture the heterogeneity between credit institutions. No macroeconomic variables are included in the determination of interest margins, since changes in the former should be captured in changes in bank-specific variables. For instance, the financial liberalization process that the Colombian Financial System has undergone in the past years must reflect changes in banks efficiency.

Using a two step estimation approach, in which the interest margin is first estimated, using an unbalanced fixed effects panel data with a time-varying intercept, as a function of the *pure* spread (the intercept in the regression) and four microeconomic variables (credit risk, operational costs, opportunity costs and net commissions). In the second step, the estimated *pure* spread is regressed against interest rate volatility and market power following the theoretical model using Recursive OLS estimates.

The first step results indicate that the interest margin is positively and highly affected by inefficiency (i.e. operational costs) both for the Financial System as a whole and for Commercial and Mortgage banks. It is also found that the other intermediation costs are substitutes of interest rates for banks and complementary for other credit institutions, since the latter operate in more specialized markets. Finally, the estimated *pure* spread is also found to be a determinant of interest margins and closely follows its evolution.

In the second step regression market power is found to directly affect the *pure* spread. This indicates that market power is also a determinant of interest margins through its effect on the *pure* spread. However, this methodology does not allow to quantify this effect.

The paper is organized as follows: The theoretical model is developed in section two. The third section presents a review of the evolution of the main interest margin determinants. The two step empirical estimation and its results are presented in section four. The last section concludes.

2 The Model

Based on the model developed by Ho and Saunders (1981), banks are viewed as risk adverse intermediaries between demanders and suppliers of funds². When undertaking this role, the bank faces an important uncertainty. Mainly, that deposit supply inflows arrive at different moments in time from loan demand outflows. This generates a cost for the intermediary, given that it will have to hold either a long or short position in the money market, being therefore exposed to changes in this market's rate (i.e. interest rate risk)³.

Loans and deposits arrive at different moments in time, although both are assumed to have the same maturity date (i.e. after the end of the decision period). Additionally, they are assumed to be long-term operations, contrary to the short-term money market where investment and borrowing mature at the end of the decision period. This difference in maturity in fund sources for the bank, creates exposure to interest rate risk. Such risk will be faced whenever the intermediary has an unmatched portfolio of deposits and loans at the end of the decision period and the money market rate changes⁴. Suppose a new deposit (loan) is contracted at a long-term interest rate R_D (R_L). If this deposit (loan) arrives earlier than a new loan (deposit), the bank will have to invest (borrow) the funds at the short-term money market rate r. In doing so, the bank faces a reinvestment (refinancing) risk at the end of the decision period should the short-term rate, r, fall (rise).

Therefore, Financial Intermediaries will transfer these financial costs,

²Financial Intermediaries' risk aversion can be explained by different factors: First of all, profit (and thus wealth) volatility, can cause greater costs associated with its administration. Second, guarantees required by Financial Authorities, such as deposit insurance, may lead banks to limit their willingness to engage in risky activities. Finally, banks operating in non competitive systems, with a certain degree of market power, may give up greater benefits associated with a higher exposure to risk, therefore concentrating their lending portfolio. This type of behavior is widely analyzed in what is known as agency problems.

³Note that this exposure exists due to the fact that banks do not need deposits to grant loans, since funds can be borrowed from the money market.

⁴Such interest rate risk is explained in the literature by what is denominated as *mismatching* in financial intermediation operations. This phenomena is explained only by the difference in timing in which loans and deposits arrive for each bank during the period of analysis, since the model does not take into account the possibility of different maturities between instruments.

which arise from the uncertainty in the provision of deposit and loan operations (i.e. fees for administering financial resources), to economic agents. Consequently, each bank participates in the market by setting a loan and deposit interest rate, R_L and R_D , that depends on these financial costs:

$$R_{Di} = r - a_i \quad ; \quad R_{Li} = r + b_i \tag{1}$$

where r is the expected interest rate in the money market, while a_i and b_i are the financial costs associated with the provision of loans and deposits for bank i, respectively.

After the bank sets its interest rates it takes a passive role in the market. In other words, the bank waits for loans and deposits to arrive randomly at different times. The former are modelled as Poisson processes with probabilities λ_{L_i} and λ_{D_i} , respectively. Linear functions are considered for the determination of these variables. The main difference of this model with the one proposed by Ho and Saunders (1981) is that a different specification for these parameters is considered. In this case, bank i competes with other banks by setting its financial costs over loans and deposits, b_i and a_i , taking into account those charged by its competitors. Hence, the probabilities of granting a credit or receiving a deposit are given by⁵:

$$\lambda_{D_i} = \alpha - \beta (a_i - \frac{1}{N} \sum_{j=1}^N a_j)$$

$$\lambda_{L_i} = \alpha - \beta (b_i - \frac{1}{N} \sum_{j=1}^N b_j)$$
(2)

This approach can be considered as a version of the original model in which product differentiation by banks is taken into account ⁶. Additionally, banks have a certain degree of market power in the loan and deposit markets, given that agents cannot freely choose their bank nor can they perfectly

⁵Ho and Saunders (1981) assume a symmetric linear specification in which $\lambda_D = \alpha - \beta a$ and $\lambda_L = \alpha - \beta b$. In this case there exists one monopolist bank which fixes a and b. Given these costs, economic agents will randomly offer and demand funds.

⁶See Shubik (1980).

substitute between them⁷.

Probabilities λ_{L_i} and λ_{D_i} are decreasing functions of the financial costs charged by each bank and a positive function of those charged by competing institutions. The $\alpha > 0$ parameter is the probability of arrival of deposits and loans in a market were all banks have the same cost structure (i.e. charge the same a and b). The $\beta > 0$ parameter represents the degree of substitution between financial costs charged by banks. A higher β implies a higher degree of substitution between the values of a and b set by different institutions. Consequently, the value of β reflects the market power of bank i, since it can charge a higher financial cost than its competitors and still have a probability different from zero of receiving (granting) funds.

For simplicity, new loans and deposits all have the same positive size Q and the same duration, thus avoiding transformation problems that may arise between assets and liabilities. Each bank is interested in the difference between loans and deposits $(L_i - D_i) = I_i$, which is interpreted as the net credit inventory. A one-period decision model is assumed, in which banks maximize their expected utility as a function of terminal wealth.

In what follows, the determination of interest margins in the case where N banks, identical with respect to their degree of risk aversion, but different in their cost structure, will be analyzed⁸. Furthermore, the effect of mergers over interest margins will be discussed.

2.1 Intermediation Model with Identical Banks

In this section, banks are assumed to have an identical and constant risk aversion coefficient. Thus, the problem is solved for bank i taking into account that wealth is represented by three components: the net credit inventory $(L_i - D_i) = I_i$, the short-run position in the money market, M_i , and a diversi-

⁷In practice, various reasons can explain this limited substitution: the long-lasting relationship between banks and their clients, the degree of bank specialization in certain operations, and the strategies followed by banks in different geographic zones.

⁸In this model, only the costs arising from administering funds under uncertainty are considered. A more complete model could also include operational costs (other costs directly related to the intermediation activity different from the financial costs defined here). See Fernandez (2002) for an analysis in which such intermediation costs are also included.

fied portfolio of liquid investments γ_i . M_i is defined as the difference between supply and demand of liquidity in the interbank market: $M_i = M_i^S - M_i^D$, both of which mature at the end of the decision period. In this way, a bank can have a short position in the money market if this difference is negative (need to fund loans), and a long position if the difference is positive (need to invest excess liquidity). Nonetheless, the bank approaches this market only when there are imbalances in his net credit inventory, since funds lent or deposited in the credit market are more profitable. γ_i , on the other hand, is an exogenous portfolio of liquid investments that constitutes part of the banks base or initial wealth at the beginning of the decision period.

As mentioned above, in this model loans and deposits have the same maturity in one period of time, implying that the net credit position will be subject to interest rate risk since rates on loans and deposits are not fixed simultaneously. Hence, bank i will participate in the market by setting a loan and deposit interest rate, R_L and R_D , that depends on the fees arising from administering loan and deposit operations under this uncertainty. From equation (1), the interest margin can be defined as:

$$R_{Li} - R_{Di} = a_i + b_i$$

Bank i fixes the values of a_i and b_i that maximize the expected utility as a function of expected wealth at the end of the period.

As bank i is risk averse, its utility function is assumed to be of the mean-variance type 9 :

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This is equivalent to maximizing a constant absolute risk aversion (CARA) or exponential utility function when the returns are normally distributed. For example: Assume a negative exponential utility function of the form

$$U(W) = -e^{-\rho W}$$

Terminal wealth W has a normal distribution with mean μ and standard deviation σ . Using the generalized method of moments (GMM) for a normal distribution, a utility function that depends on the mean and the variance of wealth is obtained:

$$EU(W) = E[-e^{-\rho W}] \quad = -e^{[-\rho(\mu - \frac{1}{2}\rho\sigma^2)]} \quad = U[E(W) - \frac{\rho}{2}Var(W)]$$

where ρ is the risk aversion coefficient.

$$U_i = E\left(\widetilde{W}_i\right) - \frac{1}{2}\rho_i var\left(\widetilde{W}_i\right) \tag{3}$$

Total wealth (\widetilde{W}_i) at the end of the period is defined as:

$$\widetilde{W}_{i} = \gamma_{i} \left(1 + \widetilde{r}_{\gamma} \right) + M_{i} \left(1 + r \right) + I_{i} \left(1 + \widetilde{r}_{I} \right) \tag{4}$$

Where, \widetilde{r}_{γ} , r, \widetilde{r}_{I} are the interest rates associated with the returns on investments, the money market and net credit operations¹⁰, and where ρ_{i} is the risk aversion coefficient for bank i.

Using (3) and (4), U_i is obtained as a function of I_i and M_i :

$$U_{i} = U(I_{i}, M_{i}) = \gamma_{i} (1 + r_{\gamma}) + M_{i} (1 + r) + I_{i} (1 + r_{I})$$
$$-\frac{1}{2} \rho_{i} \left[\sigma_{\gamma}^{2} \gamma_{i}^{2} + \sigma_{I}^{2} I_{i}^{2} + 2 \sigma_{\gamma I} \gamma_{i} I_{i} \right]$$
(5)

Where:

 $r_{\gamma} = E(\widetilde{r}_{\gamma}); r_{I} = E(\widetilde{r}_{I}); \sigma_{\gamma}^{2} = var(\widetilde{r}_{\gamma}); \sigma_{I}^{2} = var(\widetilde{r}_{I}); \sigma_{\gamma I} = cov(\widetilde{r}_{\gamma}, \widetilde{r}_{I}).$ For N > 1, the *pure* interest margin¹¹ $(s_{i} = a_{i} + b_{i})$ is given by ¹²:

$$s_i = \frac{N}{(N-1)} \frac{2\alpha}{\beta} + \rho_i \frac{Q\sigma_I^2}{(1+r)} \tag{6}$$

Equation (6) includes each of the determinants of the *pure* interest margin: The first term, captures the effect of the level of competition in the market over the *pure* spread. That is, as in the original model, α/β measures the effect of the elasticities of supply and demand. A higher value of

 $^{^{10}}$ McShane and Sharpe (1985) consider a model in which banks' wealth is only represented by the position in the money market M and by the net credit inventory I. The results do not differ much from those found in these paper.

¹¹Following Ho and Saunders (1981), in this paper the *pure* spread represents the fraction of interest margins that is not affected by certain institutional imperfections. In fact, this margin exists solely due to the fact that there is transactions uncertainty.

 $^{^{12}}$ See the theoretical appendix for a detailed analysis on the derivation of the pure spread.

this ratio implies that the bank faces more inelastic supply and demand functions. This version differs from the Ho and Saunders (1981) model in that market power is also affected by the number of institutions in the banking sector N/(N-1). If banks where risk neutral, *pure* interest margins would only be affected by the level of competition. In this approximation, mergers have a positive effect over the *pure* spread. When mergers occur, the number of institutions falls causing a rise in the pure interest margin.

The second term of the equation shows that the *pure* spread is positively affected by the institutions attitude towards risk.

Thus, the *pure* interest margin depends positively on the risk aversion coefficient (ρ_i) and interest rate volatility (σ_i^2) and negatively on the number of institutions and the degree of elasticity of supply and demand curves. It is worth noting that in the second term of the equation, the *pure* spread seems to be negatively affected by the money market interest rate r. However, it must be taken into account that since the money market interest rate affects both loan and deposit rates, it also affects σ_i^2 , hence the net effect is ambiguous¹³.

3 Evolution of Interest Margin Determinants

In this section, the evolution of the main variables used to estimate interest margins is described. Mean levels of all variables refer to weighted averages¹⁴ and standard deviations are included to allow for a measure of dispersion between groups.

The different variables show similar trends both for the Total Financial System and for Commercial and Mortgage Banks. As shown in Table 1, the mean interest margin level is currently below the pre-crisis period (before 1998), although its standard deviation has increased in the last years. As can be seen in Figure 1, the interest margin has not been steadily falling since the early 90's. In fact, it started to increase since 2002, a trend that

 $^{^{13}}$ For a detailed analysis on the effect of the money market rate on the pure spread see Wong (1997). Specifically, the author states that the effect of an increase in r is either positive or ambiguous depending on whether the bank is a net borrower or lender in the interbank market, respectively, as well as on the size of the income and substitution effects.

¹⁴All variables are weighted by size, as explained in the following section.

was reversed during 2004 until finally stabilizing around 9% during 2005.

Credit risk shows the most interesting behavior of all variables, presenting a sharp increase in the crisis period followed by falling levels which are now similar to those present before 1998. Banks mean credit risk level is not below the pre-crisis period due to the inclusion of Mortgage Banks in the calculations, whose portfolio was the most sensitive during the crisis. However, their current portfolio diversification and the increase in the issue of mortgage backed securities have decreased their exposure towards this risk. Additionally, improvements in credit risk in the last year are also explained by the good domestic macroeconomic situation and healthier debtors' balance sheets.

The efficiency measure (inefficiency) shows a similar trend to that of credit risk, increasing sharply during the crisis period and decreasing afterwards. The fact that mean levels of operational costs over assets are at their historical lows, implies that financial institutions have experienced positive efficiency innovations in their cost structure.

The interesting fact about the net commissions over assets ratio¹⁵ is that banks seems to charge, on average, higher exploitation costs as shown in the weighted average of this variable in Table 1. This could be in line with the assumption that in more competitive markets, commissions might act as a substitute of interest rates.

The reserves over assets ratios has been decreasing since the 1990's and continues with this trend during the first half of this decade. This behavior is explained by the decline in the reserve requirement coefficient that the Central Bank imposes on financial institutions.

Finally, real total assets and equity decreased sharply during the crisis, as shown in Table 2. However, average assets and equity did not plummet in this period since the fall in the total value of the variables was overcompensated by the decrease in the number of institutions. In 2005, the level of real total assets is greater than the pre-crisis period, while real equity is still below the levels of the mid nineties.

 $^{^{15}\}mathrm{Net}$ commission expenses are defined as the difference between commission income and expenses.

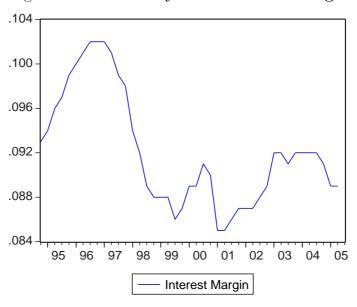
Table 1: Descriptive Statistics: 1996-2005

Financial System								
		Dic-96	Dic-99	Dic-02	Sep-05			
Interest Margin	Mean	0.102	0.086	0.088	0.089			
	Std. Dev	0.055	0.067	0.061	0.058			
Credit Risk	Mean	0.03	0.09	0.02	0.02			
	Std. Dev	0.052	0.176	0.052	0.052			
Reserves/Assets	Mean	0.05	0.03	0.04	0.03			
·	Std. Dev	0.031	0.022	0.020	0.018			
Operational Costs/Assets	Mean	0.06	0.07	0.07	0.06			
	Std. Dev	0.034	0.037	0.034	0.032			
Net Comissions/Assets	Mean	0.01	0.01	0.01	0.01			
,	Std. Dev	0.023	0.022	0.021	0.022			
Commercial and Mortgage Banks								
		Dic-96	Dic-99	Dic-02	Sep-05			
Interest Margin	Mean	0.111	0.094	0.095	0.093			
	Std. Dev	0.025	0.063	0.047	0.032			
Credit Risk	Mean	0.02	0.11	0.03	0.03			
	Std. Dev	0.021	0.292	0.018	0.025			
Reserves/Assets	Mean	0.07	0.05	0.05	0.04			
	Std. Dev	0.029	0.022	0.016	0.018			
Operational Costs/Assets	Mean	0.07	0.08	0.07	0.06			
	Std. Dev	0.024	0.033	0.027	0.020			
Net Comissions/Assets	Mean	0.01	0.01	0.01	0.01			
	Std. Dev	0.020	0.013	0.012	0.013			

Table 2: Assets and Equity:(millions of pesos of September 2005)

Total System							
	Dic-96	Dic-99	Dic-02	Sep-05			
Total Assets	116,577	103,384	101,358	106,485			
Average Assets	1,295	1,752	$2,\!252$	3,132			
Equity	17,066	12,153	10,979	13,412			
Average Equity	190	206	244	394			
Commercial and Mortgage Banks							
Commercial al	na Mortg	gage Ban	KS				
Commercial at	Dic-96	Dic-99	$\frac{\mathrm{ks}}{\mathrm{Dic}\text{-}02}$	Sep-05			
Total Assets				Sep-05 98,323			
	Dic-96	Dic-99	Dic-02	-			
Total Assets	Dic-96 94,639	Dic-99 86,926	Dic-02 89,611	98,323			

 $\label{eq:Figure 1: Financial System: Interest Margin} Figure 1: \textbf{Financial System: Interest Margin}$



4 Empirical Estimation

4.1 The Data

In this section the interest margin, the *pure* spread and market power are estimated in two different steps¹⁶. This two-step approach to panel data on bank spreads has also been used by McShane and Sharpe (1985) Allen (1988), Angbazo (1997), Wong (1997) and Saunders and Schumacher (2000).

Aside from this study, only the paper by Brock and Rojas (2000) has tackled the issue of interest margins using a two-step approach for the case of Colombia¹⁷. Specifically, their analysis shows that both high levels of operating costs and non-performing loans raise interest spreads. In addition, reserve requirements act as a tax on banks that gets translated into a higher spread.

Other studies of bank interest margins in the Colombian Financial System present a different approach. Salazar (2005) estimates a linear equation using a time series analysis for the aggregate of the banking sector to identify the determinants of interest margins (no market power variable is used in this approach). Barajas et al. (1999), on the other hand, use both a Full Information Maximum Likelihood procedure, to jointly estimate a spread equation and a demand function for loans, and a Random Coefficients Model using Panel Data for 22 banks, in which the spread equation alone is estimated. The former, finds that financial repression is a main determinant of the high levels of interest margins. The latter, establishes that interest margins have become more responsive to loan quality (i.e. credit risk). However, both studies coincide in that interest spreads are highly sensitive to bank efficiency.

This paper, contrary to the papers mentioned above for Colombia, will not only focus on the determinants of interest margins, but on those of the

¹⁶In this paper this approach is used in order to analyze the evolution of *pure* spreads. Other studies by Fernandez (2002) and Maudos and Fernandez (2002), estimate interest margins in a one step regression using proxies of determinants of this variable, both from the theoretical model as well as other variables which are assumed to affect margins, which does not allow this type of analysis.

¹⁷Although their paper does not only focus in the case of Colombia, but other Latin American Countries as well

pure spread as well. Given that the latter is a determinant of interest margins, and that market power's evolution in explaining the pure spread is also estimated, the dynamic relationship between interest margins and market power is accounted for.

The estimation in this paper is as follows. In the first step, an unbalanced panel of quarterly data for 85 financial institutions in the period 1994:IV-2005:III¹⁸ is estimated to obtain the determinants of interest margins and a series for the *pure* spread. In the second step, this series is used as a dependent variable in a Recursive OLS estimation using macro data in order to obtain the time-varying coefficient of market power. Four different types of credit institutions are included in the estimation: Commercial Banks, Mortgage Banks, Commercial Finance Corporations and Investment Banks. Since all these institutions vary importantly both between and within groups, the data was weighted by size¹⁹ in order to control the bias that many small institutions may induce on the estimation. Additionally, the exercise was replicated using only Commercial and Mortgage Banks, since these sectors are most relevant in the Colombian Financial System.

The bank interest margin is measured as:

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\frac{Interest\ Income}{Performing\ Loans} - \frac{Interest\ Expense}{Cost\ Bearing\ Liabilities}
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However, this interest margin is not strictly the same *pure* spread defined in the theoretical model, since institutional imperfections generate additional costs which may lead for implicit rates to include other interest expenses (or mark-ups). To calculate the *pure* spread, such institutional imperfections must be accounted for in the estimation of the interest margin. Following Ho and Saunders (1981), the model estimated controls for credit risk, opportunity cost and efficiency. Also, this model controls for additional costs related to the intermediation activity of financial institutions, which have become increasingly important in the local market.

The variables are defined as:

$$Credit\ Risk = \frac{Provision\ Expenses}{Total\ Loans}$$

Opportunity
$$Cost = \frac{Reserves\ at\ the\ Central\ Bank}{Total\ Assets}$$

¹⁸The data employed in the estimation comes from balance sheet data reported by financial institutions to the Financial Superintendency.

 $^{^{19}}$ Size is measured as the ratio between bank i's assets over total assets for a given year.

$$Efficiency = \frac{Operational\ and\ Labor\ Costs}{Total\ Assets}$$

Other Intermediation $Costs = \frac{Net\ Commission\ Expenses}{Total\ Assets}$

Credit Risk is expected to have a positive impact on interest margins since a greater exposure by financial intermediaries to this risk will be reflected on higher loan rates. The volume of reserves at the Central Bank also has a positive impact over interest margins given that it poses an opportunity cost to the financial intermediaries by forcing them to deviate resources from potentially more profitable activities. The efficiency measure is expected to have a positive impact on interest margins, given that higher administration costs will be partially transferred to agents. Net commission expenses can have either a positive or negative effect on interest margins, depending on whether these expenses/profits are complementary or substitute, respectively, to the interest rate²⁰.

4.2 Estimating the Interest Margin

As mentioned above, the interest spread calculated from balance sheet data, differs from the *pure* spread derived in the theoretical model due to certain institutional imperfections. Thus, prior to estimating the determinants of the *pure* spread, this variable must be obtained from an estimation of interest margins.

The implicit assumption is therefore, that at any moment in time, bank interest margins are a function of the *pure* spread plus additional mark-ups due to institutional imperfections (Credit Risk, Opportunity Cost, Efficiency and Other Intermediation Costs). Following Ho and Saunders (1981), it is assumed that if banks share a similar market structure, attitudes towards risk and interest rate volatility, their *pure* spreads will be practically the

²⁰In a highly competitive market, where banks cannot perfectly fix their interest rates, commission expenses are expected to be a substitute of interest rates. On the other hand, markets in which institutions have a certain degree of market power, due for example to product differentiation, will be characterized by complementary commission expenses and interest rates.

same. Given this, it is possible to approximate the interest margin by the following equation:

$$IM_{it} = \theta_{0t} + \theta_1 CR_{it} + \theta_2 OC_{it} + \theta_3 Eff_{it} + \theta_4 Com_{it} + \varepsilon_{it}$$
 (7)

Where θ_{0t} is the *pure* spread which varies over time but not across individuals, CR_{it} is credit risk, OC_{it} is opportunity cost Eff_{it} is the measure of efficiency and Com_{it} are net commission expenses for bank i at time t. The model is estimated using an unbalanced panel fixed effects regression with a variable intercept over time (but constant across individuals)²¹. It is worth noting that this estimation technic differs significantly from that of Ho and Saunders (1981) where the *pure* spread series was constructed from different cross section regressions for each period of time.

Results are presented for both the total of the Financial System as well as only for Commercial and Mortgage Banks in Table 3.

All variables are significant at the 1% level except the opportunity cost variable in the Total System (TS) regression. The credit risk and efficiency coefficients present the expected sign, implying that a higher exposure to credit risk and greater inefficiency effectively raises the interest margin. The net commissions coefficient presents a negative sign, implying that these expenses are a substitute to interest rates in the Colombian Financial System.

In the Only Banks (OB) regression results are very similar, although in the case of the opportunity cost variable, the coefficient is significant and has the expected sign. The net commissions coefficient is also a substitute of interest rates in this sub-sample.

It is worth noting that when a model for Commercial Finance Corporations and Finance Banks is estimated, the net commissions coefficient is positive and significant at the 1% level, indicating complementarity among this variable and interest rates. This result is not surprising since many of these institutions are concentrated in very specific markets, which allows them to exercise a higher degree of market power compared to banks.

Concerning the intercept of the regression, it is worth mentioning that the output reports an *overall* value, which is not the average of the fixed effects "estimates". The estimated intercepts (i.e. the *pure* spread at each

²¹See Econometric Appendix for estimation details.

Table 3: Estimation Results

Total Financial System							
Interest Margin	Coef.	Std. Error	P-Value				
Constant	0.0002	0.000	0.000				
Credit Risk	0.1600	0.010	0.000				
Reserves/Assets	-0.0111	0.017	0.508				
Operational Costs/Assets	1.2024	0.035	0.000				
Net Commissions/Assets	-0.4411	0.125	0.000				
R^2 within		0.541					
R^2 between		0.915					
R^2 overall		0.862					
No. Of Obs.		2,755					
No. Of Groups		103					
Commercial and Mortgage Banks							
Interest Margin	Coef.	Std. Error	P-Value				
Constant	0.0233	0.005	0.000				
Credit Risk	0.1571	0.016	0.000				
Reserves/Assets	0.3893	0.030	0.000				
Operational Costs/Assets	0.6483	0.066	0.000				
Net Commissions/Assets	-0.4551	0.141	0.001				
·							
R^2 within		0.251					
R^2 between		0.210					
R^2 overall		0.245					
No. Of Obs.		1,392					
No. Of Groups		46					

moment in time) are presented in the next subsection.

As a final remark with respect to these estimations, it is important to note that the efficiency variable presented the highest estimated coefficients in both regressions, indicating that financial institutions transfer a large part of their administrative costs to interest margins. These result is consistent with what is found by Salazar (2005), Brock and Rojas (2000) and Barajas et al. (1999), where efficiency was also a key determinant of interest margins.

Concerning public policy, this result implies that authorities should direct their efforts to creating the basic market conditions needed for banks to enhance their efficiency.

4.3 Calculating the Pure Spread

The *pure* spread is the estimated constant coefficient of the above regressions. Given that this coefficient varies over time, it is possible to construct a series of the *pure* spread as follows:

$$\theta_{0t} = \bar{Y}_t - \theta' \bar{X}_t$$
Where $\bar{Y}_t = \frac{1}{N} \sum_{i=1}^N Y_{it}$ and $\bar{X}_t = \frac{1}{N} \sum_{i=1}^N X_{it}$

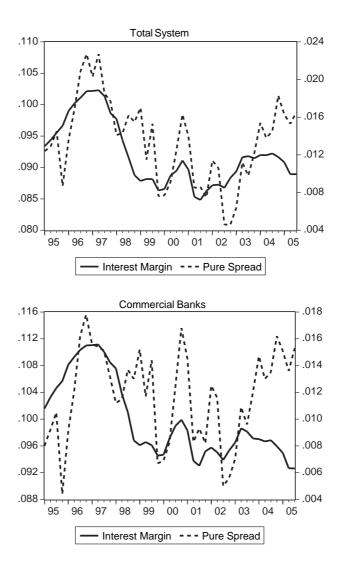
The evolution of the *pure* spread of TS and OB over time is presented in Figure 2.

As can be seen, the level of the *pure* spread is always inferior to that of the interest margin, which implies that institutional imperfections faced by credit intermediaries effectively represent a positive mark-up. Additionally, the evolution of the *pure* spread follows closely that of the interest margin, indicating that this variable is fundamental in explaining the latter. However, in the last year the interest margin has slightly decreased while the pure spread shows an increase, thus the behavior of the former is mainly explained in this period by the lower levels of credit risk and the recent improvements in efficiency.

4.4 Calculating Market Power

As stated in the theoretical model, the *pure* interest margin is a function of market power and interest rate volatility. Following Ho and Saunders (1981),

Figure 2: Interest Margin and the Pure Spread



a model to calculate market power can be derived from an estimation of the $pure \text{ spread}^{22}$ of the form:

$$\theta_{0t} = \delta_0 + \delta_1 \sigma_t^2 + \nu_t \tag{8}$$

From equation (6)

$$\delta_0 = \frac{N}{(N-1)} \frac{2\alpha}{\beta}$$

$$\delta_1 = \rho_i \frac{Q\sigma_I^2}{(1+r)}$$

 σ_t^2 is defined in the theoretical model as the volatility of the net credit inventory I interest rate. In order to capture this, the model was estimated using the volatility of the monthly exante interest margin²³. Since the last volatility measure may have certain problems (i.e. different maturities of loans and deposits, whereas the theoretical model assumes the same maturity for both) a second estimation was performed using the volatility of the weekly Interbank Interest Rate (TIB) as a proxy of σ_t^2 . It is reasonable to use the volatility of the TIB as a proxy of the volatility of the Net Credit Inventory interest rate, given that the interbank, loan and deposit markets are closely linked. Hence, variations in the TIB directly translate into variations of the other interest rates. Results do not vary significantly between both specifications.

The model is estimated using Recursive OLS in order to obtain varying coefficients of market power, allowing an analysis of its evolution over time.

²²Since the *pure* spread is estimated from a prior regression model, the dependent variable of the second-step estimation will have a measurement error. However, when such a problem arises in the dependent variable consequences are not serious. The standard errors and tests will remain valid. However, the standard errors will tend to be larger than they would have been if there had been no measurement error, resulting only on efficiency problems. But the larger error variance does not violate any of the assumptions needed for OLS estimation. In fact, the OLS estimators are consistent (and possibly unbiased as well). For further discussion see Wooldridge (2001).

²³The *exante* interest margin is the difference between the marginal loan and deposit rates, which are the rates at which banks grant new loans and receive new deposits, respectively.

As can be seen in Figure 3, both results (TS and OB) indicate that market power seems to be closely linked to the *pure* spread.

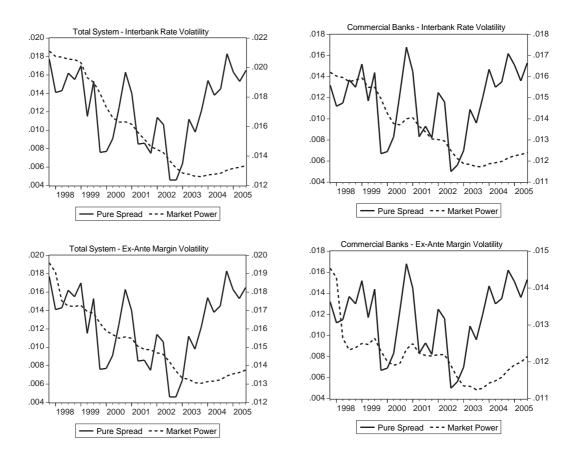
Results are presented from the last quarter of 1997, given that findings for these estimations for the first periods of time may be misleading, since the coefficients are calculated with few observations.

The increase in market power during the first years was accompanied by an increase in the *pure* spread of 1.4 percentage points (pp). Market power started to decrease since the beginning of 1997 coinciding with the entrance of two major foreign banks (BBVA and Banco Santander), which could indicate an increase in competition. However, the fall in market power continued for several years including the 1998-2000 crisis.

Even when deriving concrete conclusions from this period may be erratic, the model may shed some light as to this decrease in market power. Although the number of institutions fell dramatically during the crisis period (i.e. N/(N-1) rises), this change is not induced by mergers, but by an adverse economic situation. Since the theoretical model does not consider the possibility of bankruptcy (i.e. a fall in N is only derived from a merger), this component of market power is not readily fit to explain this specific situation. On the other hand, the model does capture changes in economic conditions, which are reflected in the elasticity of supply and demand (α/β) . During a recession, when loan demand and deposit supply are expected to be relatively more elastic (i.e. lower α/β), market power falls and so does the pure spread, ceteris paribus. Again the fall in market power was in line with a decrease in the pure interest margin, which fell from 2.3% in June 1997 to 0.5% in December 2002 (see Figure 3).

The decreasing trend of market power stopped at the beginning of 2003, staying constant for almost a year. Starting 2004, market power began increasing, while the *pure* spread started to rise since the beginning of 2003 (rising 1.1 pp till September 2005). The former may be due to important mergers that have occurred during the last years, which is consistent with what the theoretical model of the *pure* interest margin predicts. Nonetheless, stating that the increase in market power is only due to mergers would be a bold statement, taking into account that the model here used has a coefficient of market power that may be capturing effects that are not easily discriminated. Mainly, that the evolution of market power is closely related to the economic cycle. Thus, in moments of economic growth, when loan

Figure 3: Market Power and the Pure Spread



demand and deposit supply are relatively inelastic (i.e. a high α/β), market power also increases.

Finally, it is worth mentioning that the above result carries an important implication in identifying the determinants of interest margins. In the previous subsection, the importance of the *pure* spread in explaining interest margins was discussed. Thus, if market power is one of the driving forces of the *pure* spread, it is reasonable to asses that it is also a determinant of banks interest margins. However, quantifying the direct effect market power has on the latter is beyond the scope of this analysis²⁴.

5 Concluding Remarks

This paper extends the Ho and Saunders (1981) model by including an asset and an interbank market, as well as a market structure which resembles a model of monopolistic competition with product differentiation. Additionally, the estimation of the interest margin includes a new variable which captures other costs associated with banks' commercial activities.

Results indicate that the interest margin is determined by operational and other financial costs, which act as substitutes of interest rates for banks and as a complementary for other credit institutions, as well as by credit risk.

In all regressions, the most important of the microeconomic factors is credit institutions' inefficiency. This result is consistent with similar findings by Salazar (2005), Brock and Rojas (2000) and Barajas et al. (1999), where efficiency was also a key determinant of interest spreads. This result is relevant for policy makers, since it implies that in order to achieve lower interest margins, public policy should be oriented towards creating the necessary market conditions for banks to enhance their efficiency.

It is also found that interest margins are positively affected by the *pure* spread, which is common for all institutions in each period of time.

Empirical exercises suggest that market power directly affects the pure

²⁴The reason why this quantification cannot be carried out is simply that both coefficients are estimated using two separate regression models, each with a different econometric technique.

spread, as suggested by the theoretical model developed in section two. This result indicates that market power is also a determinant of interest margins through its effect on the *pure* spread.

The evolution of market power in the last years raises important issues concerning the effect that mergers have in the Colombian Financial System, as well as on the impact that changes in the business cycle, which affect elasticity of deposit supply and loan demand, have on interest margins.

Concerning the effect mergers have on interest margins, it is important to note that even when mergers could imply a higher degree of market power, they could also be a source of higher efficiency²⁵. Results here found suggest that efficiency is the main determinant of interest margins, and therefore, mergers which increase market power and improve efficiency simultaneously, could have a negative net impact on margins (i.e. may end up reducing it). Therefore, future research should focus on the policies needed to effectively generate the market conditions that may lead to a more efficient payment system.

 $^{^{25}}$ Estrada (2005) shows that for the Colombian case, mergers between big banks effectively enhance efficiency.

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A Theoretical Appendix

Assume an objective function for a risk averse bank of the form:

$$U_i = E\left(\widetilde{W}_i\right) - \frac{1}{2}\rho_i var\left(\widetilde{W}_i\right) \tag{9}$$

At the end of the period, terminal wealth is represented by the returns on held assets by bank i; financial, liquid and net credit inventory $(I_i = L_i - D_i)$, where L are loans and D deposits:

$$\widetilde{W}_{i} = \gamma_{i} \left(1 + \widetilde{r}_{\gamma} \right) + M_{i} \left(1 + r \right) + I_{i} \left(1 + \widetilde{r}_{I} \right) \tag{10}$$

Using the last two expressions, and assuming that there exists at least one loan and deposit transaction, utility for each bank can be calculated as a function of I_i and M_i :

$$U_{i} = U(I_{i}, M_{i}) = \gamma_{i} (1 + r_{\gamma}) + M_{i} (1 + r) + I_{i} (1 + r_{I})$$
$$-\frac{1}{2} \rho_{i} \left[\sigma_{\gamma}^{2} \gamma_{i}^{2} + \sigma_{I}^{2} I_{i}^{2} + 2 \sigma_{\gamma I} \gamma_{i} I_{i} \right]$$
(11)

When a bank attracts a new deposit, its utility is changed due to a modification in its net credit position: $I_{i_0} - Q$, where Q is the size of the new deposit transaction. The new liquidity position for bank i will be given by $M_{i_0} + Q + Qa_i$, which represents the initial money market position plus the new deposit flow plus the cost charged by the bank by the size of the deposit. Substituting in 11 one has:

$$(\Delta U_i/Deposit) = U_i(I_i - Q, M_i + Q(1 + a_i)) - U(I_i, M_i)$$

$$= Q[(1 + a_i)(1 + r) - (1 + r_I)]$$

$$- \frac{1}{2}\rho_i[\sigma_I^2(Q^2 - 2QI_i) - 2\sigma_{\gamma I}\gamma_i Q]$$
(12)

Similarly, when a new loan is granted, the bank has a new net credit inventory position $I_{i_0} + Q$, where Q stands for the size of the new credit

transaction; the new money market position will then be given by M_{i_0} – $Q + Qb_i$, which just as in the prior case, also incorporates the size of the credit transaction and the charges associated to it. Substituting in the utility equation yields:

$$(\Delta U_i/Loan) = U_i(I_i + Q, M_i - Q(1 - b_i)) - U(I_i, M_i)$$

$$= Q[(1 + r_I) - (1 - b_i)(1 + r)]$$

$$- \frac{1}{2}\rho_i[\sigma_I^2(Q^2 + 2QI_i) + 2\sigma_{\gamma I}\gamma_i Q]$$
(13)

If depositors and borrowers arrive randomly at bank i, following a Poisson process, then λ_{D_i} and λ_{L_i} will be decreasing functions of the provision fees a_i and b_i :

$$\lambda_{D_i} = \alpha - \beta \left(a_i - \frac{1}{N} \sum_{j=1}^{N} a_j \right) \tag{14}$$

$$\lambda_{L_i} = \alpha - \beta (b_i - \frac{1}{N} \sum_{j=1}^{N} b_j)$$

$$\tag{15}$$

The optimum margin will be obtained from the problem associated to the election of the optimal charges a_i and b_i which maximize the increase in the expected utility of banks:

$$(\triangle U_i \mid a_i, b_i) = \lambda_{D_i}(\triangle U_i / Deposit) + \lambda_{L_i}(\triangle U_i / Loan)$$
 (16)

First order conditions with respect to a_i and b_i are:

$$\frac{d\lambda_{D_i}}{da_i}(\Delta U_i/Deposit) + \lambda_{D_i}Q(1+r) = 0$$
(17)

$$\frac{d\lambda_{L_i}}{db_i}(\Delta U_i/Loan) + \lambda_{L_i}Q(1+r) = 0$$
(18)

Adding the last two equations yields:

$$-\beta \left(\frac{N-1}{N}\right) (\Delta U_i/Dep. + \Delta U_i/Loan) + (\lambda_{D_i} + \lambda_{L_i})Q(1+r) = 0$$
 (19)

Since:

$$(\Delta U_i/Deposit + \Delta U_i/Loan) = Q\{(1+r)(a_i+b_i)\} - \rho_i \sigma_I^2 Q^2$$
 (20)

and 26 :

$$\lambda_{D_i} + \lambda_{L_i} = 2\alpha \tag{21}$$

So that:

$$-\beta \left(\frac{N-1}{N}\right) \left[Q(1+r)s_i - \rho_i \sigma_I^2 Q^2 \right] + 2\alpha Q(1+r) = 0$$
 (22)

Dividing the last expression by Q(1+r) and rearranging terms, yields an expression for the *pure* spread $(s_i = a_i + b_i)$.

$$s_i = \frac{N}{(N-1)} \frac{2\alpha}{\beta} + \rho_i \frac{Q\sigma_I^2}{(1+r)}$$
 (23)

For simplicity, it is assumed that $\sum_{j=1}^{N} a_j = Na_i$ and $\sum_{j=1}^{N} b_j = Nb_i$. These are simple specifications and results are identical to the case when elastic demand functions are used.

B Econometric Appendix

The econometric estimation used in this paper was performed in two steps. In the second step, the *pure* margin was estimated as a function of market power and interest rate volatility using Recursive OLS estimates, in order to capture the evolution in time of market power. In the first step an unbalanced panel was estimated using fixed effects with a time-varying intercept. The objective of this estimation was to obtain a series for the *pure* spread (the intercept in the regression) that would be used afterwards as the dependent variable of the second step model.

The model used in the first step was of the form:

$$y_{it} = \alpha_t^* + \beta' x_{it} + \mu_{it}$$

$$i = 1, ..., N; t = 1, ..., T$$
(24)

where β' is a 1xK vector of constants and α_t^* is a 1x1 scalar constant representing the effects of those variables peculiar to the t-th period, but common across individuals. The error term measures the effects of the omitted variables that are peculiar to both individual units and time periods.

To calculate α_t^* , we have:

$$\alpha_t^* = \bar{y}_t - \beta' \bar{x}_t \tag{25}$$

$$i = 1, \dots, N$$

where

$$\bar{y}_t = \frac{1}{N} \sum_{i=1}^{N} y_{it} \qquad \bar{x}_t = \frac{1}{N} \sum_{i=1}^{N} x_{it}$$
 (26)

and where

$$\hat{\beta} = \left[\sum_{t=1}^{T} \sum_{i=1}^{N} (x_{it} - \bar{x}_t)(x_{it} - \bar{x}_t)' \right]^{-1} \left[\sum_{t=1}^{T} \sum_{i=1}^{N} (x_{it} - \bar{x}_t)(y_{it} - \bar{y}_t) \right]$$
(27)

Only the means of cross-sectional observations need to be found for each time unit. Then, the variables are transformed by subtracting out the appropriate cross-section mean and then the Least Squares Method is applied to the transformed data. Finally, the covariance matrix is calculated as:

$$Var(\hat{\beta}) = \sigma_u^2 \left[\sum_{t=1}^T \sum_{i=1}^N (x_{it} - \bar{x}_t)(x_{it} - \bar{x}_t)' \right]^{-1}$$
 (28)