Voting over tax schedules in the presence of tax avoidance

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Abstract

This paper reconsiders the classical problem of majority voting over tax schedules, adding the possibility to avoid taxes. In this setting preferences over tax schedules are not determined by earned income, but rather by *taxable* income, which depends on the joint decisions of labor supply and tax avoidance investments. The ordering of earned- and taxable income are shown to be the same if the tax avoidance function is log concave.

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1 Introduction

A standard result in the literature on collective choice over redistributive policies is that political conflict is a mirror image of earned income. Individuals with a high income favor low taxes, while those with a low income favor high taxes, and the median income earner is decisive in a majority rule election. However, if money can be used to avoid taxation, this is no longer self-evident since *earned* income is no longer necessarily the same as *taxable* income.

This paper reconsiders the classical problem of majority voting over linear income tax schedules studied in Roberts (1977). Here, heterogenous individuals face a joint decision on how much to work and how much of the earned, pre-tax income to spend on avoiding taxation. Tax avoidance activities have a general form, and can be considered as all costly actions taken by an individual with the sole purpose of reducing his tax burden.[1]

As in Roberts' article, the main concern is the existence of a majority voting equilibrium. Using the single-crossing condition developed in Gans and Smart (1996), it will be shown that a majority voting equilibrium exists if *taxable income,* rather than labor income, is order restricted. The relation between the ordering of labor income and that of taxable income turns out to depend on the tax avoidance opportunities. As will be shown, a technical condition on the avoidance possibilities can ensure that they are the same, but there are also situations where these orderings are never the same, and those with the highest labor income pay no taxes in equilibrium.[2]

The paper connects two strands of the literature. On the one hand, following Romer (1975), Roberts (1977) and Meltzer and Richard (1981), much of the literature on voting over income tax schedules has focused on the interaction between labor supply and majority rule. On the other hand, many have pointed out that tax avoidance responses are at least as important as changes in labor supply, leading

[1] Tax *avoidance* is usually defined as all *legal* measures taken to reduce taxes without altering real variables (see e.g. Slemrod and Yitzhaki (2001)). Here the focus is not on the legal aspect of the activity, but rather on all aspects which can be modeled as choices under certainty, (for example, costly, illegal *tax evasion* with a zero probability of detection).

[2] That such a situation can indeed be a political equilibrium is shown in Roine (2002).

to studies of the interaction between labor supply and tax avoidance (e.g. Mayshar (1991) and Agell and Persson (2000)).[3] In this paper, labor supply, tax avoidance, and the majority-rule determination of the tax/transfer scheme are present simultaneously.

2 The model

Consider a situation with n individuals, who have preferences over two goods, consumption c and leisure l. Every individual has a time endowment of one unit, which can be divided between work y (that pays a wage normalized to 1) and leisure $l = 1 - y$.

It is assumed that preferences can be represented by a differentiable utility function $u(c, l; \alpha)$, with $u_c > 0$ and $u_l < 0$, where α is an index representing an individual characteristic (e.g., ability or preference for work).

Individuals are to collectively decide on a tax schedule, which consists of a lump-sum transfer, $T \in \Re$, received by everyone, and a proportional tax rate $t \in [0, 1]$ on labor income y.[4] However, individuals can avoid paying a share of their tax through investing in tax avoidance. More precisely, an investment, $A \in \Re$, reduces the tax-payment by a factor $\delta(A) \in [0, 1]$, since individuals investing in avoidance only pay $\delta(A)ty$, instead of ty, in taxes.

Tax avoidance possibilities, described by the function $\delta(A)$, are assumed to be such that larger avoidance investments lead to smaller actual tax payments, but with diminishing returns, i.e., $\delta'(A) < 0$ and $\delta''(A) > 0$. Furthermore, $\delta(0) = 1$ and $\lim_{A\to\infty} \delta(A) \geq 0$. That is, making no investment in tax avoidance gives no reduction, and complete avoidance is not possible.

An individual's consumption is given by

$$c = (1 - \delta(A)t)y + T - A \ , \tag{1}$$

[3] Examples of studies ranking different responses to taxation are Slemrod (1992) and Auerbach and Slemrod (1997).

[4] Note that it is not necessary to assume a balanced budget. T can be any real in the space of possible tax/transfer schemes, $[0, 1] \times \Re$.

and an individual's problem, given any tax/transfer scheme (t, T), is to

$$\max_{A \geq 0, y > 0} u((1 - \delta(A)t)y + T - A, 1 - y; \alpha) \ . \tag{2}$$

Taking the derivative with respect to A gives the first-order condition

$$\frac{du}{dA} = u_c(-\delta'(A)ty - 1) \leq 0 \ .$$

Noting that the second-order condition holds, since $\delta''(A) > 0$, the optimal investment in tax avoidance A^* is given by

$$A^* = \max\{0, A\} \ ,$$

where A is implicitly given as the unique solution to the equation

$$-\delta'(A) = \frac{1}{ty} \ . \tag{3}$$

The corner solution $A = 0$ is relevant whenever y and t are such that $\frac{1}{ty} > -\delta'(0)$, that is, when an individual's tax payment is so small that it does not pay to invest in tax avoidance at all. The interpretation of (3) is that the optimal tax avoidance investment A^* is increasing in labor income y and the tax rate t (lower values of $-\delta'(A^*)$ are associated with higher values of A^*). Furthermore, it shows that there is an income level

$$\tilde{y} = -1/\delta'(0)t \ , \tag{4}$$

such that those with a lower income ($y < \tilde{y}$) do not invest in tax avoidance, while those with a higher income ($y > \tilde{y}$) do.

For those avoiding taxes, at any (t, T), the derivative with respect to y gives the first-order condition

$$\frac{du}{dy} = u_c[(1 - \delta(A)t) - \delta'(A)\frac{dA}{dt}ty - \frac{dA}{dt}] - u_l \leq 0 \ ,$$

with equality if y is positive. Using that $-\delta'(A^*) = \frac{1}{ty}$ for optimal tax avoidance decisions gives that optimal labor supply y^* is implicitly given by

$$u_c(1 - \delta(A^*)t) - u_l = 0 \ . \tag{5}$$

Compared to the standard expression, $u_c(1-t)-u_l=0$ (which gives the labor supply for those who do not avoid taxes, since $\delta(0)=1$) the tax wedge is now altered by a factor $\delta(A^*)$, due to the optimal tax avoidance investment.[5]

Individual preferences over tax schedules (pairs of (t,T)) are given by the indirect utility function

$$v(t,T;\alpha)=u((1-\delta(A^*)t)y^*+T-A^*,1-y^*;\alpha)\ , \tag{6}$$

where A^* and y^* satisfy (3) and (5) for those avoiding taxes, and where $\delta(A^*)=1$ if $A^*=0$.[6] As shown in Gans and Smart (1996), a sufficient condition for a majority voting equilibrium to exist, is that the indirect utility function $v(t,T,\alpha)$ satisfies the Spence-Mirrlees condition, that is, voters' marginal rates of substitution between t and T are globally increasing in α. The marginal rate of substitution is given by

$$-\frac{\partial v/\partial t}{\partial v/\partial T}=-\frac{\partial u((1-\delta(A^*)t)y^*+T-A^*,1-y^*;\alpha)/\partial t}{\partial u((1-\delta(A^*)t)y^*+T-A^*,1-y^*;\alpha)/\partial T}\ ,$$

which, given (3) and (5), simplifies to

$$-\frac{\partial v/\partial t}{\partial v/\partial T}=\delta(A^*)y^*\ . \tag{7}$$

Hence, the slope of an individual's indifference curve in the space of all possible tax schedules is given by $\delta(A^*)y^*$, that is, an individual's taxable income. This implies that the Spence-Mirrlees condition holds if and only if *taxable income*, $\delta(A^*)y^*$, is increasing in α.

Proposition 1 *Suppose that majority rule is used to choose a tax schedule, consisting of a lump-sum transfer and a proportional tax rate on labor income, and that*

[5] Since y is not explicit in equation (5), it is not possible to solve for y^*, nor explicitly express $\partial y/\partial\alpha$. However, using (3), (5) and the total differential of (1), one can show that $\partial y/\partial\alpha$, $\partial A/\partial\alpha$, and $\partial c/\partial\alpha$ must always have the same sign. This means that if $u(c,l;\alpha)$ is such that $\partial c/\partial\alpha$ is always positive, so is $\partial y/\partial\alpha$.

[6] It should be noted that the envelope function $v(\cdot)$ is differentiable even if the optimal choice of $\delta(A^*)$ is non-differentiable at the point $\tilde{y}=-1/\delta'(0)t$. The function is well defined for those with income $y^*<\tilde{y}$ where $-\frac{\partial v/\partial t}{\partial v/\partial T}=y^*$, while it is $-\frac{\partial v/\partial t}{\partial v/\partial T}=\delta(A^*)y^*$ for those with income $y^*>\tilde{y}$. Since $\delta(A^*)\to 1$ as $y\to\tilde{y}$ from above, the left differentiable at $\tilde{y}$, $(\lim_{y\to\tilde{y}-}(y^*))$ equals the right differentiable at $\tilde{y}$ $(\lim_{y\to\tilde{y}+}(\delta(A^*)y^*)$, which means that the value function is also differentiable at $\tilde{y}$. See, for example, Milgrom and Segal (2000).

individuals have the possibility to invest in avoiding taxation. Then, a sufficient condition for a majority-voting equilibrium to exist is that taxable income be increasing in some invariant order of the voters for all possible tax schedules.

This result can be seen as a modified version of Proposition 1 in Gans and Smart (1996). The difference is that in the presence of tax avoidance, it is not the order of *optimal labor supply,* y^* that is of importance but rather, the order of optimally chosen *taxable income,* $\delta(A^*)y^*$, which depends on the joint decisions of labor supply and tax avoidance. In the absence of avoidance possibilities the two are, of course, always equal.

That the ordering of labor income (labor supply) is monotonically increasing irrespective of the tax schedule is normally guaranteed under relatively mild conditions. But this is not equally obvious in a setting with tax avoidance. In the standard case, if the index α, ordering individuals by type, stands for higher abilities or wage rates, then higher values of α are associated with higher *labor* income under standard assumptions. However, in the presence of tax avoidance opportunities, higher labor income is also associated with larger investments in tax avoidance, i.e., a higher value of y^* means a lower value of $\delta(A^*)$. For the ordering of *taxable* income $\delta(A^*)y^*$ to remain equal to the ordering of labor income y^*, increases in y must always dominate the decreasing effect of $\delta(A)$, due to an increased avoidance at higher incomes. This raises the obvious question of when these orderings are the same, i.e. given some ordering of earned income y, when is the ordering of taxable income $\delta(A^*)y$ the same? It turns out that a technical condition on the avoidance function guarantees that the ordering remains unchanged.

Proposition 2 *Assume that there is a fixed ordering of labor income y over all tax schedules and that a fraction $\delta(A)$ of taxes can be avoided at a cost A, where $\delta'(A) < 0$ and $\delta''(A) > 0$. Then, taxable income $\delta(A^*)y$ has the same ordering over all tax schedules, if the tax avoidance function $\delta(A)$ is such that* $\log \delta(A)$ *is concave.*

Proof. What is to be shown is under what conditions $\delta(A^*)y$ is increasing in y, or equivalently, when

$$\frac{\partial}{\partial y}[\delta(A^*)y] = \delta(A^*) + \delta'(A^*)\frac{dA^*}{dy}y \geq 0 \ .$$

Using the condition for the optimal avoidance decision (given by equation (3) above)

$$-\delta'(A^*) = \frac{1}{ty}$$

and the total derivative of this,

$$\delta''(A^*)dA^* = \frac{1}{ty^2}dy \Rightarrow \frac{dA^*}{dy} = \frac{1}{\delta''(A^*)ty^2} \ ,$$

the condition can be written as

$$\delta(A^*) + \frac{\delta'(A^*)}{\delta''(A^*)}\frac{1}{ty} \geq 0 \ .$$

Again, using that $-\delta'(A^*) = \frac{1}{ty}$ and rearranging gives

$$\delta(A)\delta''(A) - \delta'(A)\delta'(A) \leq 0.$$

This condition is equivalent to

$$\frac{d}{dA}(\frac{\delta'(A)}{\delta(A)}) \leq 0 \ .$$

which holds if

$$\frac{d^2}{dA^2}\log(\delta(A)) \leq 0.$$

Hence, $\delta(A^*)y$ is increasing in y, if the logarithm of $\delta(A)$ is concave. ∎

The interpretation of the log concavity requirement is that for a very convex tax avoidance function, the optimal avoidance expenditure increases very fast with income for some individuals at least for sufficiently high tax rates. This implies that the reported income of those individuals may fall with productivity, violating the Spence-Mirrlees condition.[7]

[7] A number of tax avoidance functions fulfill the log concavity condition. For example, all functions of the form $\delta(A) = e^{-nA}$ (for positive A and $n > 0$) and functions of the form $\delta(A) = n^{-A}$ (for positive A and $n > 1$). One convex function which does not fulfill the condition is $\delta(A) = 1/(1+A)^n$, (for positive A and n).

What is most important to note is perhaps not the precise condition that guarantees order-restriction but rather, the fact that the tax avoidance possibilities may alter the preference ordering, even if there is a fixed order of labor income. A situation where the potential consequences become particularly stark is when the returns to tax avoidance are increasing rather than decreasing as assumed before, i.e. when $\delta(A)$ is such that $\delta'(A) < 0$ and $\delta''(A) < 0$ instead of $\delta''(A) > 0$. Optimal tax avoidance is still given by

$$\max_A u((1-\delta(A)t)y + T - A, 1-y; \alpha)$$

and the first-order condition is

$$\frac{du}{dA} = u_c(-\delta'(A)ty - 1) \leq 0,$$

as before. However, when $\delta''(A) < 0$, this means that the choice of A, which maximizes utility, is given by either of the corner solutions $\delta = 1$ or $\delta = 0$, with the corresponding optimal tax avoidance investments, $A = 0$ or $A = A_{\max}$. Figure 1 illustrates this fact. The left hand graph shows a convex avoidance function with the interior solution to the optimization problem for an individual with income y at tax rate t. Increases in t as well as in y cause the slope of the "indifference line" to fall, implying that increases in the tax rate induce more avoidance investments and that the richer an individual is the more he invests in avoidance. The right-hand graph shows a situation where an individual with a lower income (y_1) chooses to invest nothing while the high income individual (y_2) optimally invests $A_{\max}$ and, consequently, pays no taxes.

This binary choice of either investing so as to completely avoid taxes, or not avoid taxes at all, leads to a situation where preferences over tax schedules are never order restricted over all possible tax schedules. A simple way of seeing why this is the case is to consider a tax schedule (t, T) where t is sufficiently close to zero for no one to invest in tax avoidance. At this point, the taxable income of individuals is ordered according to their labor income. But if, as t increases, at least the richest individual starts investing in tax avoidance, this alters the ordering of taxable income. The richest individual, who has the highest taxable income before the investment, now has a taxable income of zero.[8] In contrast to Proposition 2, the following can now be concluded:

Proposition 3 *Assume that there is a fixed ordering of labor income y over all tax schedules. Then, taxable income $\delta(A^*)y$ never has the same ordering as y if the tax avoidance function $\delta(A)$ is concave and, at least, one individual invests in tax avoidance.*

If a majority voting equilibrium exists in this situation, it will be one where the rich (tax avoiders) and the poor favor an increase of the tax rate. The reason for this unusual coalition is, of course, that the rich pay no taxes due to their investment in tax avoidance.

3 Conclusion

This paper has considered the problem of majority voting over income-tax schedules in the presence of tax avoidance. The first key result is that when money can be used to invest in activities that decrease the tax base, the preference ordering over tax/transfer schemes is no longer given by earned, pre-tax income, but by *taxable income*. A sufficient condition for a majority-voting equilibrium to exist in such a setting is, hence, that *taxable income* be increasing in some invariant order of the voters, for all possible tax schedules. The second key result concerns when the ordering of earned income remains unchanged by the introduction of tax avoidance possibilities. As has been shown the orderings are unchanged if the tax avoidance function - as defined in this paper - is log concave. This means that adding the

[8] Another way of showing the same thing would be to consider the indifference curves of individuals in the space of all possible tax schedules $[0, 1] \times \Re$. In a range where neither of two individuals invests in tax avoidance, the indifference curves of the richer individual are steeper and hence, cross those of the individual with lower incomes from below. Given that at some point, the high income individual chooses to completely avoid taxes, while the low income individual does not, the indifference curves for the high income individual become flat and will cross those of the individual with low income again, this time from above. Hence, the indifference curves fail to satisfy the single crossing property.

possibility of avoiding taxes will not change the result of the median-voter theorem[9] as long as the avoidance function is log concave.

Equally important, however, are the cases where this condition is not fulfilled, since this can affect the preference ordering of voters in sometimes surprising ways. For example, tax avoidance opportunities that exhibit increasing returns to scale induce behavior where the rich pay no taxes, only the avoidance cost, which in turn means that they support any further increases in the tax rate. More generally, any non-convexities in the tax avoidance function, such as fixed costs, lead to a change in the orderings of earned and taxable income.[10] This implies that, just as tax avoidance can alter the standard analysis of labor supply, it may also change the view of redistributive politics. In particular, the political conflict over redistributive taxes may no longer be between the rich and the poor.

[9] As formulated by Roberts (1977).

[10] Roine (2002) studies such a situation and shows how to solve for the political equilibrium, even though the median-voter theorems do not apply in general.

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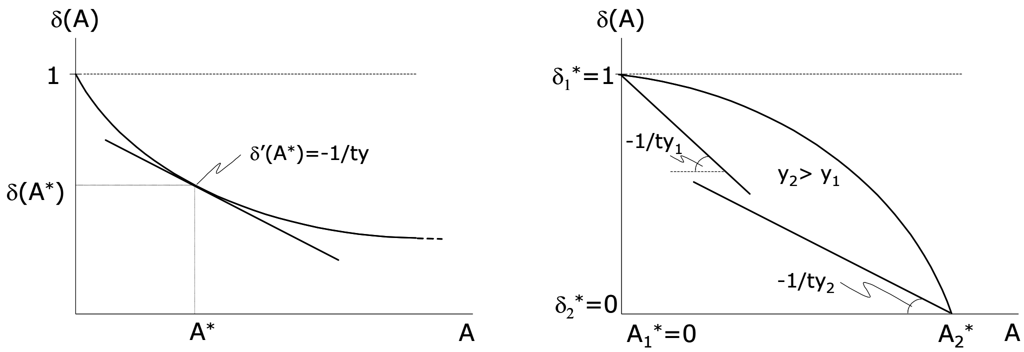

Figure 1: Optimal tax avoidance for different functional forms. To the left, a convex function ($\delta'' > 0$), with an interior optimum and to the right a concave function ($\delta'' < 0$), where only corner solutions can be optimal.