On the Definition and Age-Dependency of the Value of a Statistical Life. A Review and Extension

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Abstract

The value of preventing a fatality or (saving) a statistical life is an important question in health economics as well as environmental economics. This paper reviews and adds new insights to several of the issues discussed in the literature. For example, how do we define the value of a (statistical) life? Are there really strong theoretical reasons for believing that the value of a life declines with age? The paper derives definitions of the value of a statistical life in both single-period models and life-cycle models. Models with and without actuarially fair annuities are examined, as well as the age-profile of the value of a statistical life.

Key words: Value of a statistical life, value of preventing a fatality, age-specific values, willingness to pay.

JEL classification: I10, D61, C61.

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1. Introduction

In many cases, such as environmental pollution and new medical treatments we are interested in estimating the benefits and costs of measures reducing the risk of death. A quite natural way of formulating the problem is in terms of the benefits and costs of a measure expected to save one life. If the value of saving one (statistical) life exceeds the costs incurred, undertaking the measure would seem worthwhile. It should also be mentioned that nowadays many authors seem to prefer to speak of the value of preventing a fatality rather than the value of a statistical life. In this paper, I will stick to the old fashioned terminology, however.

There seems to be no universally agreed estimate of the value of a statistical life. According to Viscusi's (1992) extensive survey, most reliable estimates are clustered in the \$3 to \$7 million interval. A recent study by the EU's DG Environment recommends the use of a value in the interval €0.9 to €3.5 million. (In June 2001, 1€≈0.85\$.) The best estimate according to this study is a figure of around €1.4 million. DG Environment also concludes that there "are strong theoretical and empirical grounds for believing that the value for preventing a fatality declines with age" (p. 2).

However, even if we set aside all the problems faced in arriving at a reasonable empirical estimate of the value of preventing a fatality, many questions still remain. For example, how do we define the value of a (statistical) life? Are there really strong theoretical reasons for believing that the value of a life is declining with age? The purpose of this paper is to derive definitions of the value of a statistical life in single-period as well as in life cycle models. Some of the results derived are new. In particular, the paper replaces earlier approximations of the effect of a drop in the hazard rate by an exact definition. Moreover, in contrast to previous contributions, it provides a detailed analysis of the age-dependency of the value of a statistical life[1].

There are good reasons for exploring several different models, for example with respect to the availability of actuarially fair annuities. First, we do not know what model people have in mind when making decisions. Therefore, the mechanical use of one definition or another of the value of a statistical life in a cost-benefit analysis of a measure preventing a fatality, might cause a seriously biased estimate of benefits. Second, the institutional set-up varies between countries and a definition appropriate for one country might be less relevant for another.

The paper is structured as follows. Section 2 derives definitions of the value of a (statistical) life within two different single-period models. One definition refers to the case with no atemporal equivalent of an actuarially fair annuity (and no bequest motive). The second definition refers to the case where the wealth of a deceased is transferred to the survivors. Thus, there is a kind of (inverse) life insurance. Section 3 is devoted to a discussion of the definition of the value of a statistical life in a life-cycle model without actuarially fair annuities, while Section 4 considers the case where such annuities are available. An analysis of the age-dependency of the value of a statistical life is found in Section 5. A few concluding remarks can be found in Section 6.

2. The value of a statistical life: The single-period case

Throughout this paper, I will consider individuals that derive utility from consuming a single commodity if alive. The probability of survival is denoted μ, i.e. a fraction $1-\mu$ will die. Individuals are assumed to act as if they maximise their expected utility. In the single-period case, their expected utility is defined as follows:

$$U^E = \mu f(c) + (1-\mu)u^d \qquad (1)$$

where $f(c)$ is the utility enjoyed if alive, c denotes consumption if alive, and u^d denotes a fixed and finite level of "utility" assigned to the state dead; see, for example, Jones-Lee (1976) or Rosen (1988) for details. Thus, bequests are ignored here, but a variation with intentional bequests is considered in the Appendix. Deducting u^d from the utility derived in each state of the world in equation (1) yields $V^E=\mu u(c)$, where $u(c)=f(c)-u^d$.

Each individual is endowed with wealth k. Two different assumptions are employed with respect to ownership of k if the individual dies. According to the first variation wealth is passed on to the individual's heirs (who are not further considered here). Therefore, the budget constraint of a survivor is $k=c$, where the price of the single good is normalised to unity. Expected utility as a function of wealth is defined as $V^E=\mu u(k)$.

According to the second variation, borrowed from Rosen (1988), the wealth of a deceased person is transferred to those surviving. Since a fraction $1-\mu$ dies, each survivor will receive $k(1-\mu)/\mu$. Conditional on survival, the budget constraint is $k/\mu=c$. In this case, the expected utility as a function of wealth is defined as $V^E=\mu u(k/\mu)$.

Let us first examine the case with no atemporal equivalent of an actuarially fair annuity. Expected utility is defined as follows:

$$V^E = \mu u\left[k - CV(\mu)\right] \tag{2}$$

where $CV(\mu)$ is a payment, and $CV(\mu)=0$ initially.

Consider a small increase in the survival probability μ. Using equation (2), the WTP for such a risk reduction is defined as follows:

$$u(k)d\mu - \mu u_k(k)dCV = 0 \tag{3}$$

where $\mu u_k(.)=dV^E/dk$ is the expected marginal utility of wealth/income, and dCV is a payment such that the individual remains at the initial level of expected utility following a small increase $d\mu$ in the probability of survival.

Thus I have defined the WTP for a risk reduction. The value of a (statistical) life remains to be defined, however. Rosen (1988, p. 287) defines the *value of a life* as the marginal rate of substitution between wealth and risk, i.e.:

$$MRS_{k,\mu} = \frac{\partial V^E / \partial \mu}{\partial V^E / \partial k} = \frac{dCV}{d\mu} \tag{4}$$

Jones-Lee (1991, 1994) defines the value of *statistical* life as the population mean of $MRS_{k,\mu}$. Since I consider a cohort of ex ante identical individuals (facing identical risk reductions), I will here interpret equation (4) as providing a definition of the value of a statistical life[2] (VSL). Thus, we have the following definition of the VSL:

$$\frac{u(k)}{\mu u_k(k)} = \frac{1}{d\mu} dCV \tag{5}$$

where $\mu u_k(.)=dV^E/dk$ is the expected marginal utility of wealth. The left-hand side expression in equation (5) yields the gain in expected utility due to a small risk reduction converted from units of utility to monetary units by division by the expected marginal utility of wealth. The right-hand side expression in equation (5) yields the WTP for a risk reduction saving $d\mu$ lives multiplied by $1/d\mu$. Thus, the right-hand side expression yields the WTP for a measure expected to save one life.

Next, let us turn to the case where a survivor gets a tontine share. Drawing on Rosen (1988), expected utility in this case is equal to:

$$V^E = \mu u\left[\frac{k - CV(\mu)}{\mu}\right] \tag{6}$$

where $CV(\mu)$ initially is equal to zero.

The WTP for a small risk reduction is given by the following equation:

$$u(k/\mu)d\mu + \mu u_k(.)\left[-\frac{\mu dCV}{\mu^2} - \frac{kd\mu}{\mu^2}\right] = 0 \qquad (7)$$

where $u_k(k/\mu)=dV^E/dk$ denotes the expected marginal utility of wealth. Thus, the value of a statistical life is defined as follows:

$$\frac{u(.)}{u_k(.)} - c = \frac{1}{d\mu}dCV \qquad (8)$$

where $c=k/\mu$. In this case, the value of consumption is deducted from the monetary value of the direct gain in expected utility if a life is saved; the initial survivors will get fewer transfers from deceased individuals when the probability of death declines.

Equation (8) captures the value of unintended bequests. Therefore, this variation might seem more useful than equation (5) if the ultimate goal is to undertake a social cost-benefit analysis. In fact, the rule stated in equation (8) comes quite close to the rule generated by a simple single-period model with intentional bequests. This is further demonstrated in the Appendix at the end of the paper. However, if people express altruism, for example toward other household members the outcome is changed. In a social cost-benefit analysis the WTP for altruistic motives would have to be added. Therefore, it is not entirely self-evident that the rule in equation (8) is more useful than the rule in (5) if the purpose is to undertake a cost-benefit analysis. For further discussion of the concept of altruism, see Jones-Lee (1991).

Next, I turn to life cycle models where actuarially fair life-assured annuities are available and not available, respectively. This seems to be a legitimate approach since empirical estimates of dCV might refer to either of the two models. In particular, if survey methods such as contingent valuation are used to collect information on the WTP for risk reductions, we do not necessarily know what model a respondent might have in mind.

3. A life-cycle model without life insurance

In this section, a life-cycle model where individuals face age-specific death rates replaces the single-period model. However, individuals are still assumed to derive utility from the consumption of a single commodity. Therefore, instantaneous utility at age t is equal to $u[c(t)]$. For simplicity, the utility discount rate $\theta \geq 0$ is assumed to be age-independent. The hazard rate $\delta(t)$, which yields the conditional probability of death in a short time interval $(t, t+dt)$, is assumed to be non-decreasing in age.

The remaining expected present value utility, given the survival of an individual until age τ, is defined as follows[3]:

$$U_{\tau}^{E} = \int_{\tau}^{\infty} u[c(t)] e^{-\theta(t-\tau)} \frac{e^{-\int_0^t \delta(s)ds}}{e^{-\int_0^{\tau} \delta(s)ds}} dt = \int_{\tau}^{\infty} u[c(t)] e^{-\theta(t-\tau)} \mu(t;\tau) dt \qquad (9)$$

where $\mu(t;\tau)$ denotes the probability of becoming at least t years old, conditional on surviving until the age of τ years. The consumption path is chosen so as to maximise (9), subject to the dynamic budget constraint stated in equations (A.6) in the Appendix. The individual has a capital income, i.e. interest on his wealth, and a wage/pension income. If less (more) than current income is spent on the single consumption good, then the individual will have a positive (negative) net accumulation of wealth. Necessary conditions for a solution to the above optimisation problem are stated in equations (A.8).

The question is how to define the value of a statistical life within this framework. For the moment, let us simply assume that this value is defined as follows:

$$\frac{\int_{\tau}^{\infty} u[c^*(t)]e^{-\theta(t-\tau)}\mu(t;\tau)dt}{\lambda^*(\tau;\tau)} = \frac{V(\tau)}{\lambda^*(\tau;\tau)} \tag{10}$$

where an asterisk denotes a value along the optimal path, $\lambda^*(\tau;\tau)$ is a costate variable (dynamic Lagrange multiplier) yielding the marginal utility of consumption at age τ, as is further explained below equation (A.8) in the Appendix[4], and $V(\tau)$ denotes the value function, i.e. the value function yields the expected remaining present value utility of a utility-maximising individual aged τ years (and can be interpreted as the intertemporal counterpart to the single-period indirect utility function). Equation (10) measures the expected remaining present value utility converted to monetary units by division by the marginal utility of consumption at age τ.

Equation (10) yields a definition of the VSL corresponding to the one (in the case without actuarially fair life-assured annuities) suggested by, for example, Shepard and Zeckhauser[5] (1984). If a measure, say medical or environmental, "saves" one life, the gain in expected present value utility is given by the value function $V(\tau)$. Dividing through by the marginal utility of consumption at age τ will convert the expression from units of utility to monetary units. Rosen (1988) defines the VSL as the marginal rate of substitution between risk and wealth. Such a definition results in equation (10) if the attention is restricted to drops in the hazard rate lasting over very short periods of time. This result will be demonstrated below.

Let us now address the question of how to find a way of measuring the VSL as defined in equation (10). As a first step, let us consider an infinitesimally small change in the hazard rate lasting over a certain interval of time (beginning at age τ). This change will affect the survivor function also beyond this time interval, since the survival probability at any particular point in time depends on the integral (sum) of all previous hazard rates. The maximal once-and-for-all WTP at age τ, here denoted $dCV(\tau)$, in exchange for an increase in remaining expected present value utility is given by the following equation[6]:

$$dV(\tau) = \int_{\tau}^{\infty} u[c^*(t)]e^{-\theta(t-\tau)}d\mu(t;\tau)dt - \lambda^*(\tau;\tau)dCV(\tau) = 0 \tag{11}$$

where $d\mu(t;\tau)$ is the change in the probability of survival at age t conditional on being alive at age τ.

One would like to transform this equation so that it reflects the monetary value of the value function, i.e. $V(\tau)/\lambda^*(\tau;\tau)$. Johansson (2001) shows that equation (11) cannot be used to arrive at an unbiased measure of $V(\tau)/\lambda^*(\tau;\tau)$, unless consumption is constant across the entire life cycle. The problem is that instantaneous utility cannot be factored out from the integral in equation (11) if optimal consumption is age-dependent. This prevents any attempts to manipulate equation (11) so as to yield a variation of equation (10). The reader is referred to equations (A.9)-(A.10) in the Appendix for details.

There is an interesting case, however, where we can arrive at an unbiased estimate of VSL even if optimal consumption follows a non-constant pattern across the life cycle. This is the case where the drop in the hazard rate lasts over a very short time interval. (Blomqvist (2001) has recently considered this kind of a "blip" case, but similar cases have also been considered by, for example, Shepard and Zeckhauser (1982, 1984) and Rosen (1988).)

A possibility is to model such a change in the survivor function as suggested in Johannesson et al. (1997). In this case, illustrated in Figure 1, there is a drop equal to $d\kappa$ in the hazard rate lasting over the interval $[\tau, \tau+\varepsilon]$ for an individual who has survived until age τ. At age $\tau+\varepsilon$, the hazard rate returns to its initial path.

FIGURE 1 ABOUT HERE

I claim that this approach, which is further explained in equation (A.11) in the Appendix, is exact (in contrast to, for example, the approximations used by Shepard and Zeckhauser (1982, 1984), and Blomqvist (2001)).

Using this way of modelling a change in the hazard rate, equation (11) above would read:

$$d\kappa \int_{\tau}^{\tau+\varepsilon} u[c^*(t)]e^{-\theta(t-\tau)}(t-\tau)\mu(t;\tau)dt + \varepsilon d\kappa \int_{\tau+\varepsilon}^{\infty} u[c^*(t)]e^{-\theta(t-\tau)}\mu(t;\tau)dt - \lambda^*(\tau;\tau)dCV(\tau) = 0 \tag{12}$$

Next, multiply through this equation by $1/\varepsilon d\kappa$. As $\varepsilon \to 0$, the first term in this reformulated version of equation (12) tends to zero. This result follows from L'Hôpital's rule; see equations (A.12) in the Appendix. The second term in (12) multiplied by $1/\varepsilon d\kappa$ yields the expected remaining present value utility of a τ-year old person (since $\tau+\varepsilon \to \tau$ as $\varepsilon \to 0$), i.e. the value function $V(\tau)$.

Thus, in the case of a true "blip", i.e. where $\varepsilon \to 0$, equation (12) reduces to:

$$\frac{V(\tau)}{\lambda^*(\tau;\tau)} = \frac{dCV(\tau)}{\varepsilon d\kappa} \tag{13}$$

This expression yields a seemingly conventional single-period definition of the VSL, equal to the WTP for a risk reduction multiplied by one over the risk reduction (adjusted for the duration ε of the drop in the hazard rate). This result also holds if consumption is age-dependent.

Equation (13) reflects the MRS between initial wealth and initial risk. This result can be obtained by noting that $dCV(\tau)/\varepsilon d\kappa = dCV(\tau)/d\mu^{\kappa}(\tau+\varepsilon;\tau)$ as $\varepsilon \to 0$, where $d\mu^{\kappa}(\tau+\varepsilon;\tau)$ yields the change in the survival probability at age $\tau+\varepsilon$ caused by a small change in κ and conditional on being alive at age τ[7]. The expression $dCV(\tau)/d\mu^{\kappa}(\tau;\tau)$ is analogous to the one used to define the MRS between wealth and risk in equation (4). This result confirms Rosen's (1988) interpretation of the value of a (statistical) life.

If the individual is asked to pay for a change $d\kappa$ in the hazard rate lasting over a longer period of time than "ε goes to zero", the approach suggested above will obviously provide a biased estimate of the value of a statistical life. This is due to the fact that the assumption of the first term in the left-hand side expression of equation (12) being equal to zero cannot be defended if the drop $d\kappa$ lasts over a "longer" ("$\varepsilon>0$") period of time. However, the approach provides an *upper bound* for the "true" value of a statistical life, since the following result can be shown to hold (using equations (A.13)-(A.15) in the Appendix):

$$\frac{V(\tau+\varepsilon)}{\lambda^*(\tau+\varepsilon;\tau+\varepsilon)} \leq \frac{dCV(\tau)}{e^{-r\varepsilon}\varepsilon d\kappa} \tag{14}$$

where, for simplicity, r is the constant market rate of interest. Thus, $e^{r\varepsilon}dCV(\tau)/\varepsilon d\kappa$ provides an upper bound for the monetary value of the expected remaining present value utility of a person surviving until the age of $\tau+\varepsilon$, i.e. $V(\tau+\varepsilon)/\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$. This result holds regardless of if consumption is constant or non-constant over the life cycle.

The only problem I see in using (13) or (14) in an empirical study is the following. We do not know the magnitude of the bias we introduce in the measurement of the VSL. Applying the result stated in (13) might result in a considerable overestimation or underestimation of $V(\tau)/\lambda^*(\tau;\tau)$. The problem is that "ignoring" the first term in the left-hand expression of equation (12), as is done for obtaining (13), will cause a bias (unless ε is extremely small). This bias is increasing in ε, i.e. the WTP for a risk reduction is non-linear in ε. Similarly, applying (14) might result in a considerable overestimation of $V(\tau+\varepsilon)/\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$. We simply do not know.

In sum, using empirical data to arrive at an unbiased estimate of the VSL seems possible if the WTP-measure refers to a true blip in the hazard rate. However, if the drop in the hazard rate lasts for a longer period of time, say a year, the measure might be biased, unless optimal consumption is constant across the life cycle. Unfortunately, there seems to be no obvious way of stating whether the bias is "small" or "large".

4. A life-cycle model with actuarially fair life-assured annuities

This section assumes that there are insurance companies offering actuarially fair insurance, i.e. the intertemporal equivalent of a tontine. There is a large number of identical individuals of age t. Following Yaari (1965) and Blanchard (1985), it is assumed that these individuals will contract to have all of their wealth return to the insurer, contingent on their death. While alive, they will in exchange receive a return per (short) time period equal to the hazard rate times their wealth. In order to provide a simple illustration, let us assume that the hazard rate is age-independent, i.e. $\delta(t)=\delta$ for all t. Then the remaining life expectancy is equal to $1/\delta$ (independently of current age). If optimal wealth is also age-independent and equal to k, then the individual will receive an amount δk at each point in time (or rather over a short interval of time t, $t+dt$). Since his remaining life expectancy is equal to $1/\delta$, in total the individual can expect to receive $(1/\delta)\delta k=k$ from his insurance company. Therefore, if the contract specifies that the insurance company will receive his wealth, i.e. k, when he dies, then the contract is actuarially fair.

The utility maximisation problem for the case with actuarially fair life-assured annuities corresponds to maximising equation (9) subject to equations (A.17) in the Appendix. The first-order conditions for an optimum are stated in equations (A.20) in the Appendix.

In a model with this kind of "inverted" life insurance (and where the hazard rate is non-declining in age), it can, using equation (A.22) in the Appendix, be shown that optimal consumption increases (decreases) if the market rate of interest r exceeds (falls short of) the utility discount rate θ. Thus, consumption is independent of risk; risk is insured away. This is in sharp contrast to the case without insurance, where the rate of change of consumption is driven by the sign of $r-\theta-\delta(t)$; a result which can be established using equations (A.14) in the Appendix.

Let us now consider a change in the hazard rate modelled as an infinitesimally small drop $d\kappa$ during an interval ε (beginning at the current age τ). Since the individual receives a capital income related to his hazard rate, a change in the hazard rate will have a direct impact on his budget constraint. This is the main difference between this case and the one considered in the previous section; it can be seen by comparing the Hamiltonians in equations (A.19) and (A.7), respectively. Proceeding in the same way as in equation (12), the WTP for the considered change in the hazard rate $d\kappa$ is defined as follows:

$$\begin{aligned} & d\kappa \int_{\tau}^{\tau+\varepsilon} u[c^{**}(t)] e^{-\theta(t-\tau)} (t-\tau) \mu(t;\tau) dt + \varepsilon d\kappa \int_{\tau+\varepsilon}^{\infty} u[c^{**}(t)] e^{-\theta(t-\tau)} \mu(t;\tau) dt - \\ & d\kappa \int_{\tau}^{\tau+\varepsilon} \eta^{**}(t;\tau) k^{**}(t) dt - \eta^{**}(\tau;\tau) dCV^{a}(\tau) = 0 \end{aligned} \tag{15}$$

where a double-asterisk denotes an optimal value, $k^{**}(t)$ denotes (optimal) wealth at age t, $\eta^{**}(t;\tau)$ is a costate variable (with $\eta^{**}(\tau;\tau)$ equal to the marginal utility of consumption at age τ, conditional on being alive at age τ, as seen from equation (A.22) in the Appendix), and $dCV^{a}(\tau)$ is the WTP for the considered risk reduction.

According to the third term in equation (15) there is now an effect through the budget constraint since the "annual" amount received from the insurance company changes when the hazard rate drops over the time interval ε. Multiplying this budget-related term by $1/\varepsilon d\kappa$ and assuming that $\varepsilon \to 0$, one obtains:

$$\begin{aligned} & \lim_{\varepsilon \to 0} \frac{\int_{\tau}^{\tau+\varepsilon} \eta^{**}(t;\tau) k^{**}(t) dt}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{\eta^{**}(\tau+\varepsilon;\tau) k^{**}(\tau+\varepsilon)}{1} = \\ & -\eta^{**}(\tau;\tau) \int_{\tau}^{\infty} [w(t) - c^{**}(t)] e^{-\theta(t-\tau)} \mu(t;\tau) dt \end{aligned} \tag{16}$$

where the first equality follows from L'Hôpital's rule, $w(t)$ denotes any wage/pension income at age t, and the final equality has been obtained by exploiting the life-time budget constraint, see equation (A.18) in the Appendix[8].

The first term in equation (15) multiplied by $1/\varepsilon d\kappa$ tends to zero as $\varepsilon \to 0$, see equation (A.12a) in the Appendix, while the second term multiplied by $1/\varepsilon d\kappa$ tends to the value function of a τ-year old person as $\varepsilon \to 0$. Thus in the special case of a true “blip", equation (15) reduces to:

$$\frac{V^{a}(\tau)}{\eta^{**}(\tau;\tau)} + \int_{\tau}^{\infty} \left[w(t) - c^{**}(t)\right] e^{-\theta(t-\tau)} \mu(t;\tau) dt = \frac{dCV^{a}(\tau)}{\varepsilon d\kappa} \tag{17}$$

where $V^{a}(\tau)$ denotes the value function, i.e. the expected remaining present value utility conditional on survival until age τ. The VSL-result stated in equation (17) parallels the result in Rosen’s (1988) equation (32). It parallels equation (34) in Shepard and Zeckhauser (1984) if $r=\theta$; they only consider the case where optimal consumption is constant for all t.

The VSL-measure in (17) differs from the one stated in (13), since the expected present value of the difference between current income and current consumption enters (17) but not (13). The term in question reflects the fact that reducing the risk of death will also mean fewer transfers from deceased to survivors, at least if assets $k(\tau)$ at age τ are strictly positive, as can be seen from (16). Whether assuming such a (“inverted”) life insurance is realistic is another question. It might also be noted that the VSL as defined by (17) might be larger or smaller than the VSL obtained through (13); see Shepard and Zeckhauser (1984, p. 430) for details. This result is hardly surprising. After all, the two approaches draw on different assumptions with respect to the institutional set-up of the economy. Hence, one would expect them to generate different optimal paths for consumption, different utility levels, and different valuations of marginal additions to wealth.

Oftentimes, a survey method, such as the contingent valuation method, is used to collect information on the WTP for a risk reduction. Then, there is the question whether an individual calculates his WTP-measure according to equation (15) or equation (12) (or uses some other model). To illustrate a possible problem, let us assume that the investigator wants to base a social cost-benefit analysis on (17). Moreover, the investigator incorrectly assumes that the respondent has calculated his WTP using equation (12); in fact, the respondent uses (15). Then, the investigator will deduct an amount reflecting the value of the integral[9] in (17) to arrive at a rough estimate of the VSL. As a consequence, a ceteris paribus underestimation of the VSL will result.

Finally, the reader should recall the assumptions used in obtaining the VSL-measure in (17). The drop in the hazard rate must last over a very short period of time for the first integral in equation (15) to equal zero and for (16) to work. It is not self-evident that this assumption is reasonable in empirical applications, where risk reductions might last over considerable time intervals.

5. On the age-dependency of the value of a statistical life

An important issue for decision-makers is whether the value of a statistical life increases or decreases in age. Assuming that the value declines with age might seem quite reasonable. For example, the DG Environment of the EU claims that there "are strong theoretical and empirical grounds for believing that the value for preventing a fatality declines with age" (p.2), an issue addressed in this section.

Let us first consider the case without life insurance. According to equation (13), the monetary value of the expected remaining present value utility of a person having survived until age τ is defined as:

$$VSL(\tau) = \frac{V(\tau)}{\lambda^*(\tau;\tau)} \quad (18)$$

Differentiating this expression with respect to τ indicates whether the *VSL* is increasing or decreasing in age. One obtains:

$$\frac{dVSL(\tau)}{d\tau} = \frac{V_\tau(\tau)}{\lambda^*(\tau;\tau)} - \frac{V(\tau)}{\lambda^*(\tau;\tau)} \frac{\lambda^*_\tau(\tau;\tau)}{\lambda^*(\tau;\tau)} =$$

$$[\theta + \delta(\tau)] \frac{V(\tau)}{\lambda^*(\tau;\tau)} - \frac{u[c^*(\tau)]}{\lambda^*(\tau;\tau)} - \frac{V(\tau)}{\lambda^*(\tau;\tau)} \frac{\lambda^*_\tau(\tau;\tau)}{\lambda^*(\tau;\tau)} \tag{19}$$

where a subscript τ refers to a partial derivative with respect to current age τ, and $\lambda^*_\tau(\tau;\tau)$ = $-[r-\theta-\delta(\tau)]\lambda^*(\tau;\tau)$; see equation (A.16) in the Appendix. Whether the value of a statistical life is increasing or decreasing in age depends on several factors. The first term on the right-hand side of equation (19) yields the gain in expected present value utility (converted to monetary units) as the future comes closer when the age of the individual increases marginally. This is a pure "discounting" effect, since both θ and $\delta(\tau)$ actually work like discount factors. The second term on the right-hand side of equation (19) captures the loss in instantaneous utility (converted to monetary units) when the individual becomes marginally older. The third term captures the fact that the marginal utility of income is, in general, age-dependent. Recall that the age-pattern of $\lambda^*(\tau;\tau)$ is determined by the difference between the sum of the utility discount rate plus the hazard rate and the market rate of interest; the reader is referred to equation (A.16) in the Appendix for details. In turn, an age-dependency of $\lambda^*(\tau;\tau)$ will affect the monetary value of the remaining expected present value utility when age is marginally increased[10].

In order to shed further light on the sign of equation (19), assume that the probability of death is age-independent, so that δ is a constant. Moreover, assume that $r=\delta+\theta$. Then, optimal consumption remains constant across the entire life cycle, i.e. $c^*(t)=c^*$ for all t. In this case, instantaneous utility is constant for all ages, i.e. $u[c^*(t)]=u[c^*]$ for all t. Therefore, $V(\tau)=u[c^*]/[\delta+\theta]$ and the first two terms on the right-hand side of (19) net out. Moreover, $\lambda^*(\tau;\tau)$ is age-independent when $r=\delta+\theta$. Thus, *VSL* is independent of age in the case considered where optimal consumption is constant across the entire life cycle. Next, let us consider the case where optimal consumption decreases with age, i.e. $r<\delta(t)+\theta$. Then[11], $[\delta(\tau)+\theta]V(\tau)<u[c^*(\tau)]$ and

$\lambda_{\tau}^{*}(\tau;\tau) > 0$. Therefore, in the case of decreasing optimal consumption across the entire life cycle, *VSL* declines with age.

More generally, equation (19) indicates that *VSL* might be increasing, constant, or decreasing with age, depending on the age-pattern of optimal consumption. The value might also be increasing (decreasing) with age over a certain age interval and then decreasing (increasing). Therefore, claims that there are strong theoretical reasons for assuming that the *VSL* declines with age seem somewhat premature.

However, it remains to check whether the introduction of actuarially fair life-assured annuities will affect the age-dependency of the *VSL*. In this latter case, the *VSL* is defined as follows:

$$VSL^{a}(\tau) = \frac{V^{a}(\tau)}{\eta^{**}(\tau;\tau)} + \int_{\tau}^{\infty}\left[w(t) - c^{**}(t)\right]e^{-\theta(t-\tau)}\mu(t;\tau)dt \qquad (20)$$

Using equation (A.23) in the Appendix, the following expression for the age-dependency of VSL^{a} is obtained:

$$\frac{dVSL^{a}(\tau)}{d\tau} = \left[\delta(\tau) + \theta\right]VSL^{a}(\tau) - \frac{u\left[c^{**}(\tau)\right]}{\eta^{**}(\tau;\tau)} - \left[w(\tau) - c^{**}(\tau)\right] - \frac{V^{a}(\tau)}{\eta^{**}(\tau;\tau)}\frac{\eta_{\tau}^{**}(\tau;\tau)}{\eta^{**}(\tau;\tau)} \qquad (21)$$

where $\eta_{\tau}^{**}(\tau;\tau) = (\theta - r)\eta^{**}(\tau;\tau)$ as is shown below equation (A.22) in the Appendix. The sign of $dVSL^{a}(\tau)/d\tau$ seems to be ambiguous. In other words, equation (21) provides no obvious age-pattern for the value of a statistical life. Once again, one must reasonably conclude that the value of a statistical life might be increasing, constant, or decreasing (or show a more complicated pattern) in age.

Let us therefore consider a special case. First of all, assume that $r=\theta$. Then, consumption as well as the variable $\eta^{**}(\tau;\tau)$ are age-independent; see equation (A.22) in the Appendix. Moreover, let us assume that the wage/pension income $w(t)$ is constant across the life cycle.

Finally, assume that the hazard rate is non-decreasing in age, i.e. that $d\delta(t)/dt \geq 0$. Then, in terms of equation (21), it holds that:

$$\frac{[\delta(\tau)+\theta]V(\tau)}{\eta^{**}(\tau;\tau)} - \frac{u[c^{**}(\tau)]}{\eta^{**}(\tau;\tau)} \leq 0 \tag{22a}$$

and:

$$[\delta(\tau)+\theta]\int_{\tau}^{\infty}[w(t)-c^{**}(t)]e^{-\theta(t-\tau)}\mu(t;\tau)dt - [w(\tau)-c^{**}(\tau)] \leq 0 \tag{22b}$$

where equation (20) has been used to "decompose" $VSL^{a}(\tau)$. If optimal consumption, labour/pension income and the hazard rate are age-independent, i.e. if $c^{**}(t)=c^{**}$, $w(t)=w$ and $\delta(t)=\delta$ for all $t \geq \tau$, then the weak inequalities in (22) reduce to equalities, and VSL^{a} is age-independent[12]. If the hazard rate increases with age, while $c^{**}(t)=c^{**}$ and $w(t)=w$, (22a) and (22b) reduce to strict inequalities[13] and VSL^{a} declines with age.

Thus, VSL^{a} is age-independent if $r=\theta$, the hazard rate is age-independent, and non-capital income is constant across the life cycle. On the other hand, VSL^{a} declines with age, for example if the hazard rate increases with age (while $r=\theta$, and $w(t)=w$, $\forall t$). Thus, one can find cases where the value of a statistical life declines with age. However, according to the examination undertaken in this section, it seems far from self-evident that there are strong theoretical reasons for believing that *VSL* and/or VSL^{a} declines with age. Further theoretical (as well as empirical) investigations seem warranted. For example, it might be fruitful to look at cases where some age groups, not the least the very old, face borrowing constraints; see, for example, Leung (1994) for an analysis of the implications of borrowing constraints.

6. Concluding comments

This paper has been devoted to an examination of different definitions of the value of a statistical life. Both single-period and intertemporal models have been examined in some detail. It is possible to generalise the simple single-period definition of the VSL to a life-cycle perspective.

That is, even in an intertemporal model of utility maximisation one can in a sense speak of a programme saving, say, 1 life out of 10,000 and use the WTP for such a risk reduction to define the VSL.

However, the practical problem seems to be that these theoretical definitions must be based on "blips", i.e. drops in the hazard rate lasting over very short time intervals. Therefore, and according to this paper, one would expect real world computations of the VSL to be biased, in general, at least if such applications are based on the contingent valuation method. Typically, people are asked to value reductions in the hazard rate lasting over "long" (say a year?) periods of time. Unless optimal consumption remains constant over the entire life cycle, the resulting measures will not reflect the monetary value of expected remaining present value utility.

More generally, many changes in death risks are more or less permanent, for example due to long-run changes in the degree of pollution of air or water. Similarly, a new medical drug might have a long run impact on a patient's hazard rate. If we use the WTP for such long-term drops in the hazard rate in order to calculate the VSL, one would expect the resulting VSL-estimates to be seriously biased. Therefore, it is important to further examine if and how "parametric" changes in death risks can be used to estimate the VSL.

Moreover, the paper has explored the implications for the definition of the VSL of different assumptions with respect to the availability of actuarially fair life-assured annuities. In particular, whether the expected present value of remaining income less the expected present value of remaining consumption should be added to the value of saving a life depends critically on the availability of actuarially fair life-assured annuities. In empirical studies, it is of importance to check whether an estimate of the VSL assumes the presence of actuarially fair life-assured annuities. Otherwise, there is a risk of double counting, if the estimates are used for a cost-benefit analysis.

The model with actuarially fair life-assured annuities considered in Section 4 seems to generate a VSL-definition that comes quite close to the cost-benefit rule derived by Arthur (1981). He used an intergenerational general equilibrium model with population growth to derive the value of a statistical life. Therefore, it seems as if the approximation provided in (17) is more useful than (13) for cost-benefit analysis. However, it should be stressed that the presence of bequest motives and (one-sided or double-sided) altruism might change this conclusion. Therefore, deriving cost-benefit rules for dynamic economies with population growth seems to be an important question.

The age-dependency of the VSL has also been analysed in this paper. It is sometimes claimed that there are strong theoretical and empirical reasons for believing that the value of a statistical life declines with age. According to the analysis of this paper, the VSL might increase, be constant, or decline with age or even show a more complicated age-pattern. For example, the VSL might increase with age to peak at a certain age, and then start to decline. Shepard and Zeckhauser (1984) and Rosen (1984) have also noted the possibility of such more complex patterns across the life cycle. Therefore, the claim that there are strong *theoretical* grounds for the view that the VSL declines with age, put forward by, for example, the DG Environment of the EU (2001, p.2), seem premature.

Finally, it remains to investigate how to define and estimate the value of a statistical life in more complex models. For example, intentional bequest motives have been ignored. Chang (1991) introduces such motives in a model with a perfect annuity market, as in Section 4 above. It turns out that the welfare effect of longevity on bequests is ambiguous. Therefore, in the presence of bequest motives of the kind assumed by Chang (1991), the conclusion seems to emerge that the WTP for longevity is also ambiguous. It seems to be an important task for future research to examine the consequences of intentional bequests on the definition of the value of a statistical life. Similarly, it seems important to broaden the analysis so as to cover more general instantaneous utility functions, where, for example, the quality of life is valued.

Appendix

Drawing on Jones-Lee (1976), let us consider an individual who cares for his heirs within a single-period model. His expected utility is defined as follows:

$$V^E = \mu f(k) + (1-\mu)g(k) \tag{A.1}$$

where $g(.)$ is a bequest function.

Next, consider the WTP for an increase in the survival probability:

$$dV^E = [f(k) - g(k)]d\mu - [\mu f_k(.) + (1-\mu)g_k(.)]dCV = 0 \tag{A.2}$$

Thus:

$$\frac{f(k) - g(k)}{\mu f_k(k) + (1-\mu)g_k(k)} = \frac{dCV}{d\mu} \tag{A.3}$$

where $\mu f_k(k)+(1-\mu)g_k(k)$ is the expected marginal utility of wealth. If:

$$\frac{g(k)}{\mu f_k(k) + (1-\mu)g_k(k)} \approx k \tag{A.4}$$

then:

$$\frac{f(k)}{\mu f_k(k) + (1-\mu)g_k(k)} - k = \frac{dCV}{d\mu} \tag{A.5}$$

This equation is quite similar to equation (9).

The dynamic budget constraint for the model considered in Section 3 is as follows:

$$\dot{k}(t) = rk(t) + w(t) - c(t) - CV(t) \tag{A.6a}$$

$$k(\tau) = k_\tau \tag{A.6b}$$

$$\lim_{t\to\infty} k(t)e^{-r(t-\tau)} = 0 \tag{A.6c}$$

where $k(t)$ is assets at age t, r is the for simplicity constant rate of interest, $w(t)$ denotes labour (and/or pension) income at age t, $CV(t)$ is a payment made at age t, initially $CV(t)=0$, $\forall t$, and k_τ

denotes initial assets. The condition (A.6c), usually referred to as a No Ponzi Game condition, is imposed to prevent unlimited borrowing.

The present value Hamiltonian corresponding to this maximisation problem is:

$$H(t) = u\left[c(t)\right]e^{-\theta(t-\tau)}\mu(t;\tau) + \lambda(t;\tau)\left[rk(t) + w(t) - c(t) - CV(t)\right] \tag{A.7}$$

where $t \geq \tau$, and $\lambda(t;\tau)$ denotes a costate variable.

Necessary conditions for a solution to the problem of maximising equation (9) subject to (A.6) include (compare, for example, Seierstad and Sydsaeter (1987)):

$$\partial H(t) / \partial c(t) = u_c\left[c(t)\right]e^{-\theta(t-\tau)}\mu(t;\tau) = \lambda(t;\tau)$$

$$\dot{\lambda}(t;\tau) = -\partial H(t) / \partial k(t) = -r\lambda(t;\tau) \tag{A.8}$$

$$\lim_{t\to\infty} \lambda(t;\tau) = 0$$

Using these necessary conditions for an interior solution to the maximisation problem, it can be shown that $u_c[c^*(\tau)]=\lambda^*(\tau;\tau)$, where a subscript c refers to a partial derivative with respect to consumption, and an asterisk refers to a value along the optimal path. Thus, in optimum, the costate variable at age τ, i.e. $\lambda^*(\tau;\tau)$, is equal to the marginal utility of consumption at age τ, conditional on being alive at this age.

If consumption is constant across the entire life cycle, equation (11) in the main text can be rearranged so as to read:

$$\frac{u\left[c^*\right]}{\lambda^*(\tau;\tau)} = \frac{dCV(\tau)}{\int_\tau^\infty e^{-\theta(t-\tau)}d\mu(t;\tau)} \tag{A.9}$$

where c^* denotes the constant optimal level of consumption. Next, multiply through equation (A.9) by the number of remaining discounted life years. This operation yields:

$$\frac{\int_{\tau}^{\infty} u[c^*]e^{-\theta(t-\tau)}\mu(t;\tau)dt}{\lambda^*(\tau;\tau)} = \frac{V(\tau)}{\lambda^*(\tau;\tau)} = \frac{\int_{\tau}^{\infty} e^{-\theta(t-\tau)}\mu(t;\tau)dt}{\int_{\tau}^{\infty} e^{-\theta(t-\tau)}d\mu(t;\tau)} dCV(\tau) \tag{A.10}$$

Thus, if consumption is constant, the monetary value of the expected remaining present value utility could be estimated empirically. The WTP for the considered risk reduction should be multiplied by the number of remaining discounted life years and divided by the number of discounted life years gained due to the measure considered.

To illustrate the approach in Figure 1, define the survivor function, conditional on having survived until age τ, as follows:

$$\begin{aligned} \mu^{\kappa}(t;\tau) &= \mu(t;\tau)e^{\kappa(t-\tau)} && ;t \in [\tau, \tau+\varepsilon] \\ \mu^{\kappa}(t;\tau) &= \mu(t;\tau)e^{\kappa\varepsilon} && ;t > \tau+\varepsilon \end{aligned} \tag{A.11}$$

where the parameter κ is used to model a change in the hazard rate lasting over the interval $[\tau, \tau+\varepsilon]$. The hazard rate is equal to $\delta(t)$-κ for $t\in[\tau, \tau+\varepsilon)$ and to $\delta(t)$ for $t\geq\tau+\varepsilon$; see Johannesson et al. (1997) for details.

Let us consider a small change $d\kappa$ in the hazard rate (evaluated at $\kappa=0$). Using (A.11), one finds that $d\mu(t;\tau)^{\kappa}=\mu(t;\tau)(t-\tau)d\kappa$ for $t\in[\tau, \tau+\varepsilon)$, and $d\mu(t;\tau)^{\kappa}=\mu(t;\tau)\varepsilon d\kappa$ for $t\in[\tau+\varepsilon, \infty)$. This way of modelling a drop in the hazard rate is used in obtaining equation (12).

In order to illustrate the variation of *L'Hôpital's rule* used in this paper, let us consider two continuously differentiable functions $f(\varepsilon)$ and $g(\varepsilon)$, such that $f(0)=g(0)=0$. Thus, at $\varepsilon=0$ the ratio $f(\varepsilon)/g(\varepsilon)$ takes on the indeterminate form $0/0$. However, according to L'Hôpital's rule, it holds that $lim_{\varepsilon\to0} f(\varepsilon)/g(\varepsilon)=lim_{\varepsilon\to0} f'(\varepsilon)/g'(\varepsilon)$, where a prime refers to a derivative with respect to ε, and it is assumed that the limit on the right-hand side exists (e.g., $g'(0)\neq0$). This result can be obtained by using the generalised mean value theorem; see, for example, Courant and

John (1965, pp. 464-465). Multiplying the first term in equation (12) by $1/\varepsilon d\kappa$ and applying L'Hôpital's rule, one obtains:

$$\lim_{\varepsilon \to 0} \frac{\int_{\tau}^{\tau+\varepsilon} u[c^*(t)] e^{-\theta(t-\tau)} (t-\tau)\mu(t;\tau)dt}{\varepsilon} =$$

$$\lim_{\varepsilon \to 0} \frac{u[c^*(\tau+\varepsilon)] e^{-\theta\varepsilon} (\varepsilon)\mu(\tau+\varepsilon;\tau)}{1} = 0 \tag{A.12a}$$

Recall that the derivative of the integral in (A.12a) with respect to ε is equal to the derivative of the integral with respect to the upper limit of integration, i.e. reduces to the integrand evaluated at $t=\tau+\varepsilon$.

In order to check the behaviour of the ratio $dCV(\tau)/\varepsilon d\kappa$ as $\varepsilon \to 0$, differentiate equation (12) in the main text with respect to ε and assume that the WTP, i.e. $dCV(\tau)/d\kappa$, is adjusted so as to preserve the equality. One obtains:

$$\int_{\tau+\varepsilon}^{\infty} u[c^*(t)] e^{-\theta(t-\tau)} \mu(t;\tau)dt = \lambda^*(\tau;\tau) \frac{d}{d\varepsilon} \frac{dCV(\tau)}{d\kappa} \tag{A.12b}$$

As $\varepsilon \to 0$, the left-hand side expression in (A.12b) reduces to the value function at age τ, i.e. $V(\tau)$. Using L'Hôpital's rule, one obtains the following result:

$$\lim_{\varepsilon \to 0} \frac{dCV(\tau)}{d\kappa} \frac{1}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \frac{dCV(\tau)}{d\kappa} \frac{1}{1} = \frac{V(\tau)}{\lambda^*(\tau;\tau)} \tag{A.12c}$$

This equation confirms that $dCV(\varepsilon)/\varepsilon d\kappa$ is the appropriate willingness-to-pay measure as $\varepsilon \to 0$.

To obtain equation (14), let us rearrange equation (12):

$$\frac{\int_{\tau+\varepsilon}^{\infty} u[c^*(t)] e^{-\theta(t-\tau)} \mu(t;\tau)dt}{\lambda^*(\tau;\tau)} =$$

$$\frac{dCV(\tau)}{\varepsilon d\kappa} - \frac{\int_{\tau}^{\tau+\varepsilon} u[c^*(t)] e^{-\theta(t-\tau)} (t-\tau)\mu(t;\tau)dt}{\lambda^*(\tau;\tau)\varepsilon} \tag{A.13}$$

We want to rewrite the left-hand side expression in (A.13) so that it reflects $V(\tau+\varepsilon)/\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$. To obtain an expression for $\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$, let us introduce the following definition of the costate variable $\lambda^*(\tau+\varepsilon;\tau)$, i.e. the present value costate variable at age $\tau+\varepsilon$, conditional on being alive at age τ:

$$\lambda^*(\tau+\varepsilon;\tau)=\lambda^*(\tau;\tau)e^{-r(\tau+\varepsilon-\tau)}=u_c\left[c^*(\tau+\varepsilon)\right]e^{-\theta\varepsilon}\mu(\tau+\varepsilon;\tau) \quad \text{(A.14a)}$$

where the two first lines in equations (A.8) have been used to establish the equalities. Multiplying through by $e^{\theta\varepsilon}/\mu(\tau+\varepsilon;\tau)$ will shift the costate variable forward to age $\tau+\varepsilon$. Thus:

$$\lambda^*(\tau+\varepsilon;\tau+\varepsilon)=\lambda^*(\tau;\tau)e^{-r(\tau+\varepsilon-\tau)}e^{\theta\varepsilon}/\mu(\tau+\varepsilon;\tau)=u_c\left[c^*(\tau+\varepsilon)\right] \quad \text{(A.14b)}$$

i.e. $\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$ expresses the marginal utility of consumption at age $\tau+\varepsilon$, conditional on being alive at that age. Multiplying through the nominator of equation (A.13) by $e^{\theta\varepsilon}/\mu(\tau+\varepsilon)$ and the denominator of the equation by $e^{\theta\varepsilon}/(\mu(\tau+\varepsilon)e^{-r\varepsilon})$ yields:

$$\frac{\int_{\tau+\varepsilon}^{\infty}u\left[c^*(t)\right]e^{-\theta(t-\tau-\varepsilon)}\mu(t;\tau+\varepsilon)dt}{\lambda^*(\tau+\varepsilon;\tau+\varepsilon)}=$$

$$\frac{dCV(\tau)}{\varepsilon d\kappa e^{-r\varepsilon}}-\frac{\int_{\tau}^{\tau+\varepsilon}u\left[c^*(t)\right]e^{-\theta(t-\tau)}(t-\tau)\mu(t;\tau)dt}{\lambda^*(\tau+\varepsilon;\tau+\varepsilon)\varepsilon e^{-r\varepsilon}} \quad \text{(A.15)}$$

Using (A.15), one arrives at the weak inequality stated in (14) in the main text.

Note that equation (A.14b) implies that the age-dependency of $\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$ is as follows:

$$\frac{d\lambda^*(\tau+\varepsilon;\tau+\varepsilon)}{d(\tau+\varepsilon)}=\lambda^*_{\tau+\varepsilon}(\tau+\varepsilon;\tau+\varepsilon)=-\left[r-\theta-\delta(\tau+\varepsilon)\right]\lambda^*(\tau+\varepsilon;\tau+\varepsilon) \quad \text{(A.16)}$$

Therefore, $\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$ increases (decreases) with age if the market interest rate falls short of (exceeds) the sum of the utility discount rate plus the hazard rate. Equation (A.16) evaluated at $\varepsilon=0$ is used in Section 5.

In Section 4, the individual is assumed to maximise the remaining expected present value utility, as expressed in equation (9), subject to:

$$\dot{k}(t) = [r + \delta(t) - D\kappa]k(t) + w(t) - c(t) - CV(t) \tag{A.17a}$$

$$k(\tau) = k_\tau \tag{A.17b}$$

$$\lim_{t\to\infty} k(t)e^{-\int_\tau^t (r+\delta(s)(s-\tau))ds+\kappa\varepsilon} = 0 \tag{A.17c}$$

where $D=1$ for $t\in[\tau, \tau+\varepsilon)$ and zero for $\underline{t>}\,\tau+\varepsilon$. The wealth accumulation equation (A.17a) reflect the fact that the individual receives both interest income and an income from the insurance company. In addition, he receives labour (or possibly pension) income $w(t)$. As before, it is assumed that $CV(t)=0,\ \forall t$. Integrating equation (A.17a), and using the initial condition in equation (A.17b) and the No Ponzi Game condition in equation (A.17c) yields the remaining life time budget constraint:

$$k(\tau) = \int_\tau^{\tau+\varepsilon} [c(t) - w(t) + CV(t)]e^{-r(t-\tau)}\mu(t;\tau)e^{\kappa(t-\tau)}dt + \\ \int_{\tau+\varepsilon}^{\infty} [c(t) - w(t) + CV(t)]e^{-r(t-\tau-\varepsilon)}\mu(t;\tau)e^{\kappa\varepsilon}dt \tag{A.18}$$

The present value Hamiltonian at age t corresponding to the maximisation problem in equation (9), subject to (A.17), is defined as follows:

$$H(t) = u[c(t)]e^{-\theta(t-\tau)}\mu^\kappa(t;\tau) + \eta(t;\tau)[(r + \delta(t) - D\kappa)k(t) + w(t) - c(t) - CV(t)] \tag{A.19}$$

where $\eta(t;\tau)$ is a costate variable. In what follows, it is assumed that $\kappa=0$.

Necessary conditions for a solution to the maximisation problem stated in equation (9) subject to equations (A.17) with $\kappa=0$ include:

$$\partial H(t)/\partial c(t) = u_c[c(t)]e^{-\theta(t-\tau)}\mu(t;\tau) = \eta(t;\tau)$$

$$\dot{\eta}(t;\tau) = -\partial H(t)/\partial k(t) = -(r + \delta(t))\eta(t;\tau) \tag{A.20}$$

$$\lim_{t\to\infty} \eta(t;\tau) = 0$$

Using these equations, it can be shown that:

$$\eta^{**}(t;\tau) = \eta^{**}(\tau;\tau)e^{-\int_\tau^t (r+\delta(s))(s-\tau)ds} \tag{A.21}$$

Thus, the present value costate variable at age t conditional on survival until age τ is "driven" by the integral from age τ to age t of the interest rate plus the hazard rate. Moreover, using (A.20) and (A.21), and setting $t=\tau+\varepsilon$, it can be shown that:

$$u_c\left[c^{**}(\tau+\varepsilon)\right] = \eta^{**}(\tau;\tau)e^{(\theta-r)\varepsilon} = \eta^{**}(\tau+\varepsilon;\tau+\varepsilon) \tag{A.22}$$

where $u_c[c^{**}(\tau+\varepsilon)]$ denotes the marginal utility of consumption at age $\tau+\varepsilon$, and a double-asterisk refers to an optimal value.

Note that equation (A.22) implies that $d\eta^{**}(\tau+\varepsilon;\tau+\varepsilon)/d(\tau+\varepsilon) = \eta^{**}_{\tau+\varepsilon}(\tau+\varepsilon;\tau+\varepsilon) = (\theta-r)\eta^{**}(\tau+\varepsilon;\tau+\varepsilon)$. This age-dependency result evaluated at $\varepsilon=0$ is used in equation (21).

To obtain equation (21) in the main text, differentiate equation (20) with respect to age to obtain:

$$\frac{dVSL^a(\tau)}{d\tau} = \frac{\left[\delta(\tau)+\theta\right]V^a(\tau)}{\eta^{**}(\tau;\tau)} - \frac{u\left[c^{**}(\tau)\right]}{\eta^{**}(\tau;\tau)} +$$

$$\left[\delta(\tau)+\theta\right]\int_\tau^\infty \left[w(t)-c^{**}(t)\right]e^{-\theta(t-\tau)}\mu(t;\tau)dt - \left[w(\tau)-c^{**}(\tau)\right] - \frac{V^a(\tau)\eta^{**}_\tau(\tau;\tau)}{(\eta^{**}(\tau;\tau))^2} \tag{A.23}$$

Using the definition of $VSL^a(\tau)$, equation (A.23) can be rearranged so as to obtain equation (21) in the main text.

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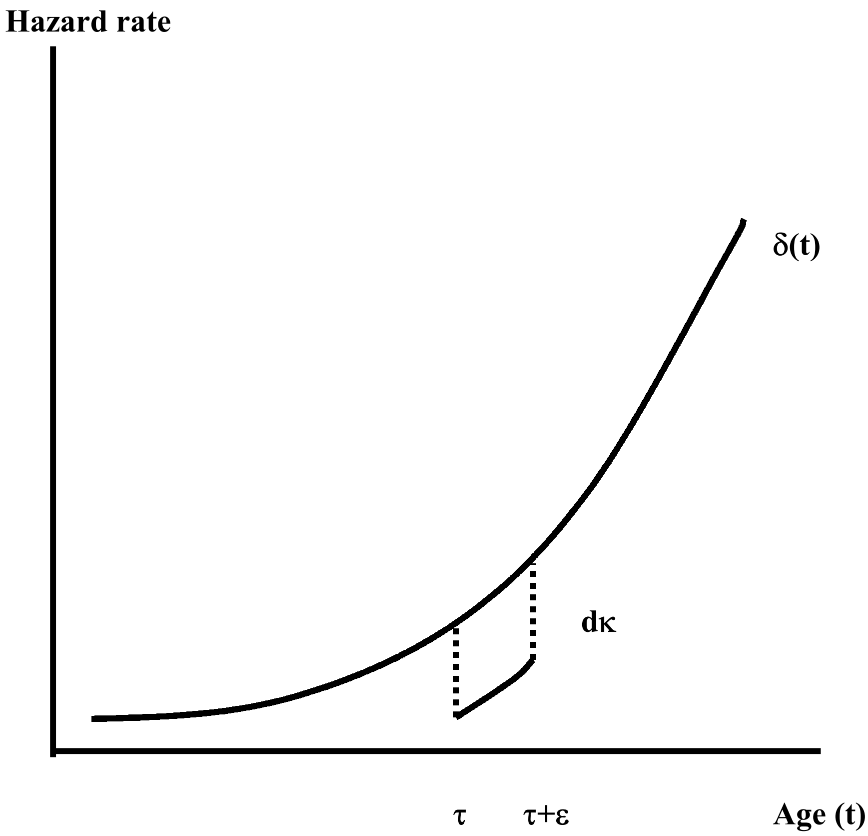

Figure 1. A drop $d\kappa$ in the hazard rate occurring at age τ and lasting until age $\tau+\varepsilon$.

Endnotes

[1] An early and brilliant survey and extension of the definition of the value of a statistical life can be found in Jones-Lee (1976). Later work that has inspired this paper includes Arthur (1981), Shepard and Zeckhauser (1982, 1984), Rosen (1988), Johansson (2001), and Blomqvist (2001).
[2] Obviously, however, in an intertemporal world with people of different ages, the question is whether the VSL refers to a population mean or an age-specific mean. The definitions used in Sections 2-3 below refer to the value of life of an ex ante homogenous group rather than to (possibly age-specific) population means.
[3] There might be a time inconsistency issue when $\delta(t)$ is age-dependent; see, for example, Blanchard (1985) for details. This issue is ignored in this paper.
[4] Léonard and Long (1992, pp. 152-154) show that $\lambda^*(\tau;\tau)$ is the worth, or imputed value, of one unit of initial stock of assets, i.e. k_τ; see equation (A.17b) in the Appendix.
[5] Shepard and Zeckhauser (1984) introduce a borrowing constraint. According to Leung (1994, p. 1236) they mistreat this constraint when formulating the individual's decision problem. Leung (1994) shows that a borrowing constraint means that savings must be exhausted at some time before the maximum lifetime. Thereafter, consumption $c(t)$ is equal to $w(t)\geq 0$ at each point in time.
[6] The total effect on the value function of a small change in a parameter is obtained by taking the partial derivative of the present value Hamiltonian (or more generally the Lagrangean), see equation (A.7) in the Appendix, with respect to the parameter, and integrating the result along the optimal path over the planning horizon. For details on this dynamic envelope theorem, the reader is referred to, for example, Caputo (1990) and LaFrance and Barney (1991).
[7] Using equation (A.11) in the Appendix one obtains: $d\mu^{\kappa}(\tau+\varepsilon;\tau)/\varepsilon=\mu(\tau+\varepsilon;\tau)\varepsilon d\kappa/\varepsilon=\mu(\tau+\varepsilon;\tau)d\kappa$, where the differentiation is with respect to κ and the expressions are evaluated at $\kappa=0$.
Thus $lim_{\varepsilon\to 0}\, d\mu^{\kappa}(\tau+\varepsilon;\tau)/\varepsilon=d\kappa$ since $\mu(\tau;\tau)=1$.
[8] $CV(\tau)$ is suppressed from the final integral in (16) since $CV(\tau)=0$, by assumption. Moreover, in terms of equation (A.18) in the Appendix, the integral assumes that $\kappa=0$.
[9] An amount is deducted (added) if the sign of the integral in (17) is negative (positive). The sign of the integral is negative (positive) if $k(\tau)>0$ ($k(\tau)<0$).
[10] Using equation (A.16) in the Appendix, equation (19) can be further simplified so as to equal $dVSL(\tau)/d\tau=[rV(\tau)-u[c^*(\tau)]]/\lambda^*(\tau;\tau)$. However, this variation does not seem to add much to the question of whether or not $VSL(\tau)$ declines with age.
[11] $V(\tau)<u[c^*(\tau)]/[\delta(\tau)+\theta]$ if $\delta(t)>\delta(\tau)$ and $u[c^*(t)]<u[c^*(\tau)]$ for $t>\tau$ (while $V(\tau)=u[c^*]/[\delta+\theta]$ if $\delta(t)=\delta$ and $u[c^*(t)]=u[c^*]$ for all t).
[12] The final term of equation (21) equals zero since $\eta^{**}(\tau;\tau)$ is a constant whenever $r=\theta$.
[13] For example, $V(\tau)<u[c^{**}]/[\delta(\tau)+\theta]$ if $\delta(t)>\delta(\tau)$ for $t>\tau$ (while $V(\tau)=u[c^{**}]/[\delta+\theta]$ if $\delta(t)=\delta$ for all t).