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Is there Evidence of Pessimism and Doubt in Subjective Distributions? A Comment on Abel*

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Abstract

Abel (2002) shows that pessimism and doubt in the subjective distribution of the growth rate of consumption reduce the riskfree rate puzzle and the equity premium puzzle. We quantify the amount of pessimism and doubt in survey data on US consumption and income. Individual forecasters are in fact pessimistic, but show marked overconfidence rather than doubt. Whether this implies that overconfidence should be built into Abel's model depends on how the empirically heterogeneous subjective distributions are mapped into the distribution of a fictitious representative agent. We work out the form of this mapping in an Arrow-Debreu economy and show that the equity premium increases with the dispersion of beliefs. We then estimate this aggregate distribution and find little evidence of either overconfidence or doubt.

Keywords: equity premium, riskfree rate, aggregation of beliefs, Survey of Professional Forecasters, Livingston Survey.

JEL: C42, G12, E44.

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1 Introduction

A number of recent papers on the riskfree rate and equity premium puzzles explore departures from the neoclassical paradigm in which the puzzle was originally formulated. For example, Barberis, Huang, and Santos (2001) adopt a non-standard utility function, motivated by prospect theory; Anderson, Hansen, and Sargent (2000) and Tornell (2000) relax the rational expectation hypothesis, postulating ambiguity-averse agents; and Benartzi and Thaler (1995) consider myopic loss aversion. While several theoretical explanations have been proposed, empirical attempts at discriminating the more successful models are lagging behind. This paper is one such attempt.

We concentrate on the work of Abel (2002), who studies two deviations from rational expectations in an otherwise standard neoclassical framework. Starting from the Lucas (1978) fruit-tree asset pricing model, he shows that uniform pessimism and doubt enhance the empirical performance of the model, in particular by reducing the equity premium and riskfree rate puzzles. Uniform pessimism is defined as (the subjective distribution being) a leftward translation of the objective distribution, doubt as a mean-preserving spread of the objective distribution.

Given the crucial role played by the behavioral assumptions, an evaluation of their empirical plausibility is desirable. In Abel's words "...this demonstration leads naturally to the next question: How much pessimism and doubt might characterize subjective distributions?" (page 1088). In this paper we take on the question. We commence our investigation with no strong prior on the presence of pessimism in actual expectations. As for doubt, surveys and experimental studies typically conclude that people are prone to overconfidence, that is, its opposite.[1] Using methods discussed in Giordani and Söderlind (2003), we study data from the Livingston Survey and, in particular, from the Survey of Professional Forecasters, looking for evidence for and against pessimism and doubt in the subjective distributions of US consumption and real output growth.

The plan of the paper is as follows: Section 2 summarizes the model in Abel (2002); Section 3 describes the survey data; Section 4 looks at pessimism; Section 5 is concerned with doubt in individual distributions; Section 6 develops a simple model to argue that

[1] Rabin (1998) and Hirshleifer (2001) discuss overconfidence in their surveys of behavioral economics and finance. In this literature, the term "overconfidence" usually describes overly narrow confidence bands (the opposite of Abel's doubt), but it can also stand for inadequate adjustment of one's forecast when given knowledge of other agents' forecasts.

when the focus is on asset pricing it may be more appropriate to look for doubt in the average (across forecasters) distribution; and Section 7 summarizes our findings.

2 A Short Recap of Abel's Model

This section presents a simplified version of the model in Abel (2002). It shows the risk premium on a consumption claim when the representative investor has pessimism and doubt.

Under the assumption of lognormality of both the objective and subjective distributions, Abel shows that pessimism and doubt enter linearly in the determination of the riskfree rate and of the equity premium. The same assumption also implies that uniform pessimism and doubt reduce to a comparison between the means and variances of the two distributions, which is also the approach taken here.

With a constant relative risk aversion γ, the price of a one-period asset that gives the payoff D_{t+1} in the next period is $P_{Dt} = \beta \, \mathrm{E}_t^*(C_{t+1}/C_t)^{-\gamma} D_{t+1}$, where β is the discount factor, and E_t^* is the expectation according to the representative investor's subjective beliefs. This result comes directly from the first order condition for optimal investment in any standard model (for instance, the Lucas (1978) model).

Assume that the subjective beliefs of the investor are that $\ln(C_{t+1}/C_t)$ is normally distributed with mean μ^* and variance σ^{*2}. It is then straightforward to calculate the prices of a one-period consumption claim ($D_{t+1} = C_{t+1}$) and a one-period real bond ($D_{t+1} = 1$). See Appendix A for details.

In a large sample, the average return coincides with the (true) expected value of D_{t+1}/P_{Dt}, that is, $\mathrm{E}(D_{t+1}/P_{Dt})$, where the price is determined by the subjective beliefs (as described above). Assume that the true distribution of consumption growth is normal, but with variance $\sigma^2 = \sigma^{*2} - \theta$ and mean $\mu = \mu^* + \Delta + \theta/2$. If θ and Δ are positive, then the true distribution has a smaller variance (investors have doubt) and a higher mean (investors are pessimistic). Define the excess return as the return on the consumption claim (R_{ct}) divided by the return on the real bond (R_{rt}). Straightforward calculations (see Appendix A) give the log expected excess return as

$$\ln \mathrm{E}(R_{ct}/R_{rt}) = \gamma\sigma^2 + \Delta + \gamma\theta, \tag{1}$$

where γ is the coefficient of relative risk aversion. The first term on the right hand side is the risk premium under rational expectations, the second term shows that pessimism ($\Delta > 0$) generates higher average excess returns, and the third term shows that doubt ($\theta > 0$) does the same. The doubt term is potentially more promising since it is multiplied by the risk aversion coefficient, which is often believed to be substantially higher than unity. The rest of this paper is an attempt to measure the degree of doubt and pessimism directly.

3 Data

This section describes the survey data we use and how it relates to Abel's model.

In the Lucas (1978) fruit-tree model, consumption, income, and dividends are the same. As Abel notices, this poses a problem at the stage of calibration and testing. Consumption and income exhibit similar behaviors in the US. Dividends stand apart, however, having decreased in real terms in a century of data (see page 1088 in Abel), and having exhibited more volatile growth rates than consumption and output.

We follow the standard approach in analyzing the consumption-based asset pricing model by considering consumption and income. They are equivalent in fruit-tree model of Lucas, but it is the consumption path that is most relevant for asset prices in a more general framework (see Section 2). However, survey data on complete subjective distributions (as opposed to point forecasts) is only available for income. We therefore study doubt on real GDP forecasts. The high correlation in both actual and expected growth rates of consumption and income (which we document in Section 4) gives us some confidence that statements concerning doubt made for income may apply to consumption as well.

We base most of our analysis on survey data from the *Survey of Professional Forecasters* (SPF), which provides forecasts of both real consumption (real consumption expenditures) and the growth rate of real GDP[2], both available from 1981Q3. The SPF is a quarterly survey of key economic variables. The participants are professional forecasters from the business and financial community, and are screened prior to inclusion in the survey. Forecasters can be identified by a number, but are otherwise anonymous, which lessens the suspect that forecasters have incentives to misreport.[3]

[2]The SPF data refers to real GNP from 1981Q3 to 1991Q4, and to real GDP since 1992Q1. For our purposes the difference can be disregarded.

[3]Studies of stock analysts' forecasts find a large dose of optimism in forecasting earnings (Chan, Karceski, and Lakonishok (2003)). However, these analysts have an incentive to release optimistic fore-

The growth rates of real GDP and consumption are defined (in the SPF) as the value in year t divided by the value in year $t-1$, minus one. We use forecasts of this growth rate at four forecasting horizons: one to four quarters ahead. In practice, the four quarters ahead forecast is made in Q1 of year t, the three quarters ahead forecast is made in Q2 and so forth.

A unique feature of the SPF is that, for a handful of variables, which include real GDP growth (but not consumption), forecasters provide a histogram of their subjective probability distribution. The histogram bins are set by the survey manager, and forecasters decide on the amount of probability mass assigned to each bin. Before 1992, the bins boundaries were -2, 0, 2, 4, 6. Since 1992, they have been -2, -1, 0, 1, 2, 3, 4, 5, 6.

The *Livingston survey* summarizes the forecasts of economists from industry, government, banking, and academia. It is the oldest continuous survey of economists' forecasts.[4] It provides point forecasts for real GDP, but not for consumption. Since the Livingston survey only supplies point forecasts, it is not helpful to discuss doubt. We use it in addition to the SPF to evaluate evidence of pessimism. In particular, we wish to check whether the two surveys give quantitatively similar answers with regard to pessimism during the same period. Moreover, because the Livingston series on real GDP is longer (starting in 1972), we can ask whether the degree of pessimism is sensitive to the sample.

4 Pessimism (too low expected growth rate)

This section studies whether US forecasters have been pessimists historically, in the sense that they have consistently underestimated the growth rates of real GDP and consumption. We compare the forecast errors using the median (across forecasters) point forecast.[5]

The results, summarized in *Table 1*, show evidence of pessimism. The SPF forecast of GDP growth is lower than the mean of the actual series at all four horizons, with the bias increasing with the horizon. At the four-quarter horizon, output growth was on average 0.64% higher than forecasted (the average is 2.31%). Over the same period, the

casts, as testified by recent events.

[4]Both the SPF and the Livingston survey are now administered by the Federal Reserve Bank of Philadelphia. The Livingston forecasts refer to the level at a one year horizon, and are released semiannually.

[5]This is common practice in studies of survey forecasts, since the median is less sensitive to outliers. For consistency with the rest of the paper, we use the means of the fitted normal distribution as point forecasts (see Section 5). Forecasters in the SPF also provide a point forecast (besides the histogram). Results using these original point forecasts are nearly identical.

	4 quarters	3 quarters	2 quarters	1 quarter
SPF, GDP, 82-02	0.64 (0.26)	0.38 (0.21)	0.31 (0.18)	0.26 (0.17)
SPF, consumption, 82-02	0.72 (0.11)	0.35 (0.09)	0.08 (0.07)	0.04 (0.04)
Livingston, GDP, 82-02	0.67 (0.35)			
Livingston, GDP, 72-02	0.20 (0.37)			

Table 1: Average forecast error of the growth rates of real GDP and real consumption. The forecast error is defined as outcome minus median forecast. Standard deviations (in parenthesis) assume MA(1) errors, and a Newey-West estimator has been used. SPF forecasts made in 1985Q1 and 1986Q1 are excluded.

Livingston median forecast underestimates growth by 0.67%. One would suspect that the strong growth in the 1990s contributes strongly to the amount of ex-post pessimism. In fact, using the full sample from the Livingston survey (1972-2002), the average underestimation falls to 0.20%.

The forecasts of consumption growth tell a similar story: the average underestimation (sample 1982-2002) is 0.72% for the four-quarter horizon. The forecast errors of consumption and GDP have a correlation of 0.71.

This evidence suggests that US forecasters have been pessimistic. For the 1982–2002 sample, this pessimism would have contributed around 0.7% to the average excess return in (1), that is, $\Delta = 0.007$. While this is not a trivial amount, it does not cover much ground in improving the empirical performance of the model: Abel's Table 1 shows that, in the absence of doubt, Δ needs to be at least five times higher to account for the equity premium—unless the coefficient of relative risk aversion takes (what are commonly thought to be) unreasonable values.

5 Doubt (too high variance of the subjective distribution)

This section studies whether US forecasters have had doubt historically, that is, if they have overestimated the uncertainty of consumption growth rates. We check if subjective confidence bands actually cover the correct number of actual outcomes.

In order to derive confidence intervals from the histograms for GDP growth in the SPF, we estimate the density functions in two different ways: *(i)* using the histogram data directly—combined with the assumption that the distribution is uniform within each bin; *(ii)* fitting a normal distribution to the histogram data.

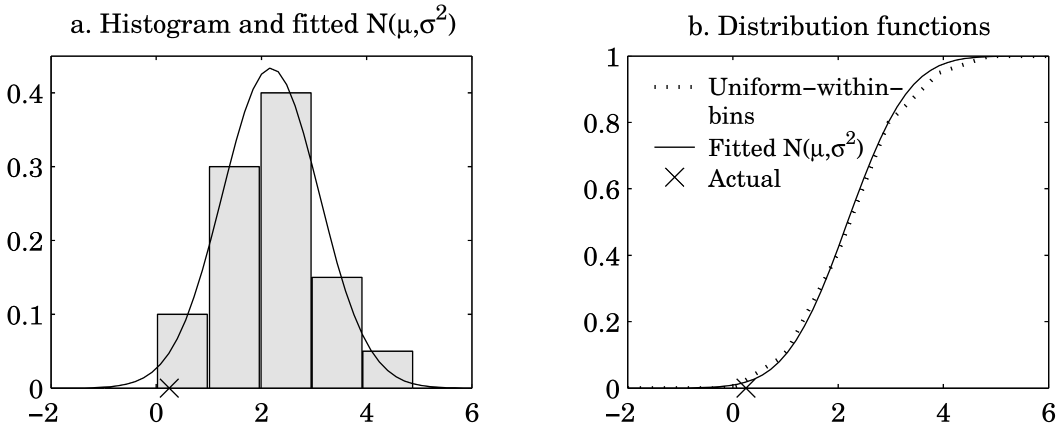

Figure 1: Example of an individual histogram. Subfigure a shows the histogram of GDP growth 2001, as reported by a typical forecaster in 2001Q1. The fitted normal distribution is also shown. Subfigure b shows the distribution function calculated assuming a uniform distribution within each bin, and the fitted normal distribution. The actual GDP growth is also marked.

Figure 1 illustrates the estimation methods. Subfigure a shows the histogram (for GDP growth 2001) reported by a typical forecaster in 2001Q1—typical because he/she reported non-zero probabilities for five intervals, and because he/she was much too optimistic that particular year. To calculate confidence bands from the histograms we assume that the distribution is uniform within each interval. While this is a convenient assumption, it squares poorly with the overall shape of most histograms in the data set: they often look bell-shaped. As a consequence, the assumption may make the distributions look wider than they really are. To capture that feature, we also provide results from normal distributions fitted to the histograms—which is also illustrated in the figure.

The moments of the fitted normal distribution are estimated by minimizing the sum of the squared differences between the survey probabilities and the probabilities implied by a normal distribution.[6] For comparison with Abel's model, we transform all data to natural logarithms.

Subfigure b shows the corresponding cumulative distribution functions. They look fairly similar for this (and most) observation. The actual outcome (0.25% GDP growth) is far out in the left tail (the 1.8 percentile according to the fitted normal and the 2.5

[6] A more detailed description of the data and of the nonlinear least squared estimation procedure can be found in Giordani and Söderlind (2003).

percentile according to the histogram). However, had the outcome been 3.5% instead, then difference would have been larger (92 and 88 percentiles respectively).

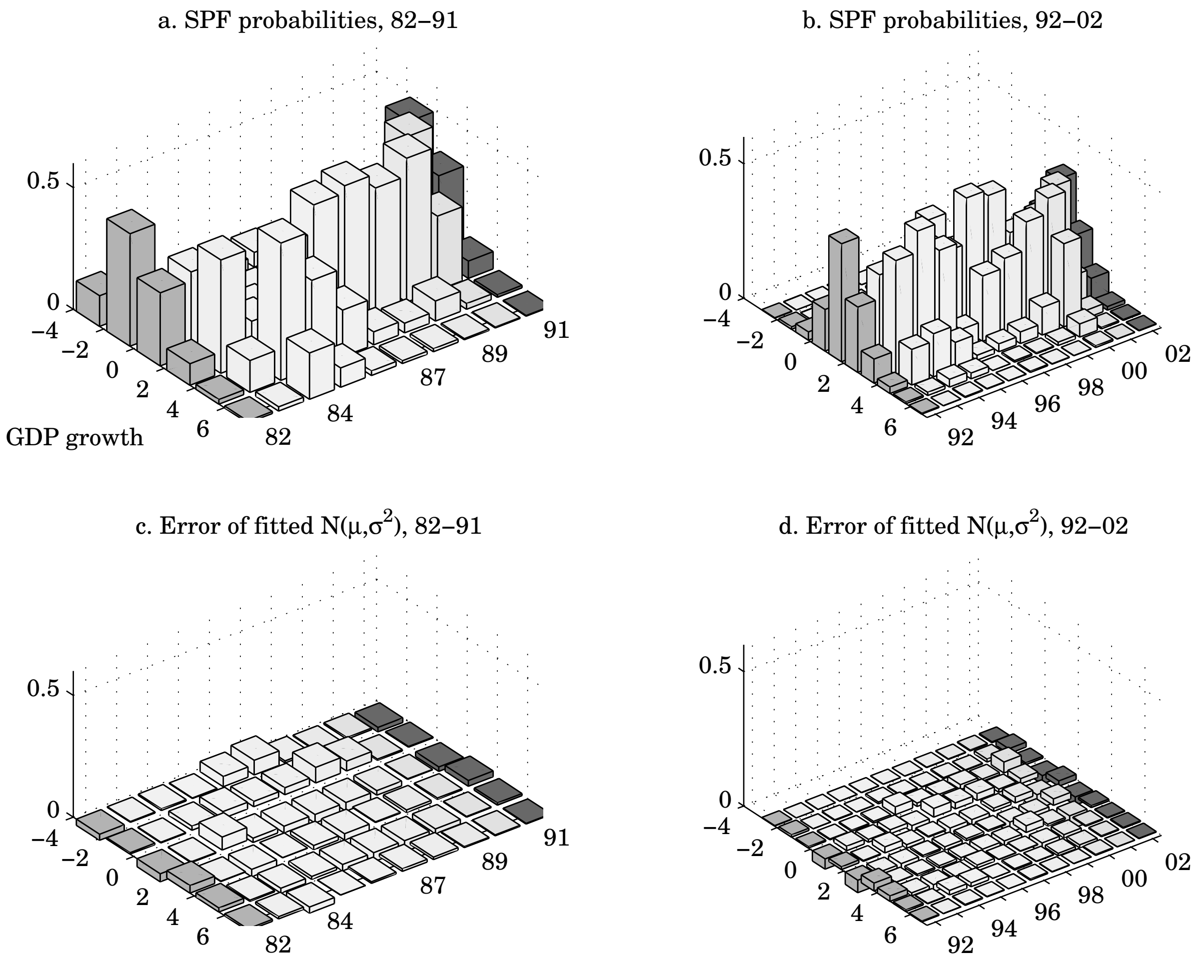

Figure 2: Average probabilities in survey and fitted normal distributions. Subfigures a and b show the average (across forecasters) probabilities of GDP growth according to SPF in quarter 1. Subfigures c and d show the difference between the actual probability mass contained in each bin and the corresponding probability implied by the fitted normal distribution.

The main drawback with the fitted normal distribution is that it does not work well on the few histograms that look very different from a bell shape. Overall, this is not a big problem—as can be gauged from the average (across forecasters) histograms in *Figure 2*. Subfigures a–b show the average (across forecasters) histogram for the first quarter of each year, and subfigures c–d show the difference between the actual probability mass contained in each bin and the corresponding probability implied by the fitted normal distribution. The differences are small, which suggests that the distributions are approximately normal. However, we report all results for both assumptions (normal and uniform-within-bin).

Having estimated the mean and variance for each forecaster (and each quarter), we can keep track of how often the realized outcome (the actual GDP growth) falls within a given confidence band (90%, 80%, or 66%). We will then say that there is doubt (overconfidence) if the actual GDP growth is inside the 90% (say) confidence band more (less) than 90% of the time. Since the number of forecasters has changed from quarter to quarter, we normalize the number to one, so each period receives the same weight in the time average.

The results are summarized in *Table 2*. We discard data for 1985Q1 and 1986Q1 since the survey probably had serious errors in those quarters (see Federal Reserve Bank of Philadelphia (2000)). There is a strong consistency in coverage ratios at all forecast horizons, so we show results for all horizons jointly.

	Fitted normal distributions			Histograms, assuming flat pdf within bin		
Confidence level:	90%	80%	66%	90%	80%	66%
No bias adjustment	0.58	0.48	0.38	0.71	0.60	0.49
Common bias adjustment	0.63	0.55	0.44	0.77	0.68	0.59
Individual bias adjustment	0.68	0.59	0.49	0.81	0.73	0.63

Table 2: Comparison of confidence bands and actual GDP growth, individual distributions. This table shows the fraction of forecasts when actual GDP growth is inside the respective $x\%$ confidence band. Confidence bands are from the individual distributions. The number of forecasters is normalized to one in each quarter. In the common adjustment, the bands of all forecasters are adjusted by the same amount (different values for Q1, Q2, Q3, and Q4). In the individual adjustment, the band of a forecaster is adjusted by his/her own average bias (different values for Q1, Q2, Q3, and Q4). The sample is 1982Q1–2002Q4, excluding 1985Q1 and 1986Q1.

The main result is that forecasters underestimate uncertainty. This remains true also after adjusting the point forecasts (the mid points of the confidence bands) for the bias in the mean growth rate. Two different adjustments are used. First, in the "common" adjustment the confidence bands of all forecasters are shifted up by the same amount, which in practice means 0.64% for all Q1 forecasts, 0.38% for all Q2 forecasts and so forth (see Table 1). Second, in the "individual" adjustment, each forecaster is adjusted by his own bias.

It is clear from Table 2 that there is more overconfidence according to the fitted normal distributions than according to the uniform-within-bin. The difference is explained by how the two methods deal with the distribution *within* an interval: whereas our application of the histogram data assumes that the density function is flat within each interval, the fitted normal distributions put relatively more probability mass close to the center. This means that the fitted normal distributions effectively have less fat tails. For instance, with the individual bias adjustment, the 90% confidence bands cover 68% of the outcomes assuming a normal distribution, and 81% assuming a uniform-within-bin distribution. Although we tend to believe that the truth is somewhere in between, both approaches indicate overconfidence rather than doubt.

To test these findings statistically we apply the Christoffersen (1998) test. It tests for correct coverage by checking if an $x\%$ confidence band indeed covers $x\%$ of the outcomes. Under the null hypothesis and the additional assumption that the forecast errors are serially uncorrelated, the test statistic has a chi-square distribution with one degree of freedom. To avoid serial correlation problems, we focus on the results for Q1. Even with individual bias adjustment, the result from the uniform-within-bin confidence bands is that the null hypothesis of correct coverage can be rejected (using a 5% critical value) for 22% of the forecasters. If we focus on forecasters with 5 or more replies (to increase the power of the test), the null hypothesis can be rejected for 34% of the forecasters. The corresponding numbers for the fitted normal distributions are 28% and 38% respectively.

These results are in line with those found by Giordani and Söderlind (2003) for inflation forecasts from the SPF (in that case the sample was 1969-2001). In fact, the finding of undersized confidence bands is a typical result in the behavioral literature (see, for example, Hirshleifer (2001), Rabin (1998), and Thaler (2000)).

To see the importance for asset pricing, consider the results for the fitted normal distributions at the 90% confidence level. To get an actual coverage of 68% as in Table 2, the investors must have believed in a variance which is approximately a third of the actual (ex post) variance.[7] In terms of the asset pricing equation (1) this gives overconfidence instead of doubt: θ is negative and equal to $-(2/3)\sigma^2$. Combining this number with the finding in Table 1 of a 0.7% pessimism (at most), our "empirical version" of (1) is

[7] For instance, the probability of $x \leq -0.95$ is 5% according to a $N(0, 1/3)$ distribution and 17% according to a $N(0, 1)$ distribution.

therefore

$$\ln \mathrm{E}(R_c/R_r) = \gamma\sigma^2/3 + 0.007. \tag{2}$$

Although the pessimism contributes somewhat to explaining the equity premium puzzle, the overconfidence works strongly in the opposite direction. For instance, suppose the equity premium is 5.7%. Without pessimism and doubt (set $\Delta = \theta = 0$ in (1)), $\gamma\sigma^2$ must then be 5.7%—which is a challenge since it requires very high risk aversion and/or very high consumption uncertainty. With the pessimism and overconfidence documented here, $\gamma\sigma^2$ must instead be 15%—which is an even bigger challenge.

Of course, this discussion should not be taken literally since the stock market index is not a claim on the consumption process—but the main point is that the joint evidence of pessimism and overconfidence seems to worsen the equity premium puzzle. That conclusion may be premature, however. One factor complicating the analysis is disagreement among forecasters, which we discuss in the next section.

6 Disagreement and Aggregation of Beliefs

This section incorporates heterogeneous beliefs in the study of pessimism and doubt. The statistics reported in Table 2 add to the existing stock of evidence of overconfidence in individual distributions. Does this imply that we must rule out doubt in Abel's model? Not necessarily. Abel's fictitious agents have identical subjective distributions by assumption. Our real forecasters do not, and mapping heterogeneous beliefs into the distribution of a representative agent is not a straightforward exercise. This section suggests a simple approach to take the heterogeneity into account and provides further empirical evidence.

Varian (1985) studies the pricing of Arrow-Debreu assets when opinions differ. The optimization problem for individual i is

$$\begin{aligned} &\max \frac{C_{i1}^{1-\gamma}}{1-\gamma} + \beta\,\mathrm{E}_i\,\frac{C_{i2}^{1-\gamma}}{1-\gamma}, \text{ subject to} \\ &Y_{i1} = C_{i1} + \int_s p(s)B_i(s)ds, \text{ and} \\ &C_{i2}(s) = Y_{i2}(s) + B_i(s) \text{ for all } s. \end{aligned} \tag{3}$$

In these equations, C_{i1} and C_{i2} (Y_{i1} and Y_{i2}) are consumption (income) of individual i in periods 1 and 2. The values in period 2 are uncertain, since it is not known in which "state of the world" (indexed by s) we will end up. The assets (one for each state) are indexed by s, $p(s)$ denoting the price of an asset which delivers one unit in state s, and $B_i(s)$ the amount of asset s purchased by individual i. The expectations operator, E_i, carries a subscript to indicate that the investors have different beliefs (otherwise all investors are identical). The second equation says that period-one income (of individual i) equals consumption and the net purchase of assets. The third equation says that period-two consumption in state s (of individual i) equals income (in that state) plus the number of asset s held.

Given the amount of public attention/debate on macro forecasting, we think that the individual histograms in the SPF should be thought of as representing beliefs "at and after trading." The forecasters probably arrived at those beliefs by incorporating prior beliefs/information and then updating them with public information (as revealed by media, other forecasts, and the trading process)—but the beliefs did not converge: it is probably fair to say that most macro forecasters have very similar information sets, but still arrive at different forecasts.

We wish to derive the price of a real bond which delivers one unit in all states in period 2, and the price of a claim on consumption which delivers the average (per capita) consumption (equal to average income) in period two. We assume that individual distributions of the (gross) growth rate of aggregate consumption are lognormal (following Abel), and that the investors differ only with respect to the means (point forecasts) of their distributions. In particular, we assume that the cross-sectional (across investors) distribution of the individual point forecasts is normal. In reality, the forecasters in SPF disagree also on the volatility (and other moments of data), but to a smaller extent.

To derive the asset prices, we have to find the equilibrium allocation. The simplest case is when the utility function is logarithmic ($\gamma = 1$). Then, all investors will choose the same consumption level in period 1, so aggregation is straightforward (see Rubinstein (1974) and also Detemple and Murthy (1994) for a more recent application). Appendix B shows that the price of a consumption claim (P_c) relative to the price of a real bond (P_r) then is

$$\ln(P_c/P_r) = \mu - (\sigma^2 + \delta^2)/2, \tag{4}$$

where μ is the mean (across individuals) growth rate of average consumption, σ^2 is the variance of individual distributions of consumption growth, and δ^2 is the variance (dispersion/disagreement) of individual point forecasts. The relative asset price in this heterogeneous-agent economy is the same of a representative-agent economy in which the distribution of the representative agent's beliefs is described by a normal distribution with mean μ, the average mean across agents, and variance $\sigma^2+\delta^2$, the average individual uncertainty plus a measure of disagreement. This is the distribution we get by averaging individual probabilities.

This suggests that it makes sense to assess pessimism by comparing the average (or median) point forecasts with the outcome—as we did in Section 4. It also suggests that we should take disagreement into account when constructing the confidence bands.

When the utility functions are not logarithmic, we arrive at approximate results only—since the investors now differ in their choice of period 1 consumption. However, Appendix B shows that the kind of disagreement found in the SPF means that a very good approximation is given by

$$\ln(P_c/P_r) \approx \mu + (1-2\gamma)(\sigma^2 + \delta^2/\gamma)/2. \tag{5}$$

The difference between the log utility case (4) is that disagreement (δ^2) becomes relatively less important compared to the individual uncertainty (σ^2) when γ is large. Although disagreement does contribute to the risk premium, its role is much smaller than that of the individual uncertainty. The intuition is that the risk individual investors perceive (and which will be priced) is their own uncertainty. The important empirical implication is that the log utility case provides an upper bound (if we rule out $\gamma < 1$) of the relative error we make by forgetting disagreement.

We therefore construct confidence bands by averaging the probabilities of individual forecasters—as an upper bound on the importance of disagreement. Because the amount of disagreement is non-negligible (on average, δ^2 makes up a third of $\sigma^2 + \delta^2$ for the 4-quarter horizon), the coverage ratios from the average probabilities, reported in *Table 3*, are higher than those found for individual forecasters (see Table 2)—they are (with bias adjustment) fairly close to their nominal values. We also find that the average probabilities (with bias adjustment) have no problem passing the Christoffersen's test of correct coverage (the p-values are much higher than 5%).

However, there is still no evidence of doubt—and (as argued above) this should be considered as an upper bound on the importance of disagreement.

	Fitted normal distributions			Histograms, assuming flat pdf within bin		
Confidence level:	90%	80%	66%	90%	80%	66%
No bias adjustment	0.76	0.62	0.48	0.89	0.74	0.55
Bias adjustment	0.87	0.73	0.59	0.91	0.83	0.61

Table 3: Comparison of confidence bands and actual GDP growth, average probabilities. This table shows the fraction of forecasts when actual GDP growth is inside the respective $x\%$ confidence band. Confidence bands are from the average (across forecasters) probabilities of each interval. The bias adjusted confidence bands are formed by adjusting the mean by the average bias (different values for Q1, Q2, Q3, and Q4). The sample is 1982Q1–2002Q4, excluding 1985Q1 and 1986Q1.

7 Summary

Pessimism and doubt in the subjective probability distributions of consumption and income growth can improve the empirical performance of standard asset pricing models. Using expectations from the Survey of Professional Forecasters and from the Livingston Survey, we are able to study pessimism and doubt in subjective distributions of real forecasters. There is some evidence of pessimism in our data, but individual forecasters clearly exhibit overconfidence rather than doubt. However, building on a simple model we argue that doubt may be more appropriately measured with reference to the aggregate (averaged across individuals) probability distribution. The average distribution shows no statistically significant sign of either overconfidence or doubt.

Our conclusion is therefore that doubt is not a promising explanation of the equity premium puzzle, and that the amount of pessimism we document provides only a rather small improvement in the empirical performance of the model. This conclusion must be considered tentative, however, for at least two reasons. The first is that results may differ if we could use data on consumption or dividend growth. The second is that there is no guarantee that the beliefs of professional forecasters (as opposed to say, those of the general public or of professional traders) are the most relevant for asset pricing.

A Derivation of Equation (1)

The consumption claim has the price $P_{ct} = C_t \beta \exp[(1-\gamma)\mu^* + (1-\gamma)^2 \sigma^{*2}/2]$, and the real bond has the price $P_{rt} = \beta \exp(-\gamma\mu^* + \gamma^2\sigma^{*2}/2)$.

The true expectation of the return on the consumption claim is therefore $\mathrm{E}\, R_c = \mathrm{E}\, C_{t+1}/P_{Ct} = \exp(\mu + \sigma^2/2)/\{\beta \exp[(1-\gamma)\mu^* + (1-\gamma)^2\sigma^{*2}/2]\}$, where μ and σ^2 are the true moments. For the one-period real bond we have $R_r = 1/P_r = \exp(\gamma\mu^* - \gamma^2\sigma^{*2}/2)/\beta$.

The average realized excess return, $\mathrm{E}(R_c/R_r)$, follows directly as $\mathrm{E}(R_c/R_r) = \exp(\mu - \mu^* + \sigma^2/2 - (1-2\gamma)\sigma^{*2}/2)$. This simplifies to equation (1).

B Derivation of Equation (5)

B.1 First Order Conditions and Results for Log Utility

We assume that all investors are identical, except that they have different beliefs. The optimization problem is as in (3). Investor m's first order condition for AD asset s can be written

$$p\,(s)^{1/\gamma}\, C_{m2}\,(s) = \beta^{1/\gamma} f_m(s)^{1/\gamma} C_{m1},$$

where $f_m(s)$ is investor m's subjective pdf evaluated at s (if the number of states is finite, $f_m(s)$ is the probability assigned to state s) . To simplify the notation slightly, we henceforth assume that $\beta = 1$ and that the aggregate output in state s equals s, that is, $Y_2(s) = s$.

Let g_m be the distribution (pdf) of different investors. Integrate over investors

$$p\,(s)^{1/\gamma} \int_m C_{m2}\,(s)\, g_m dm = \int_m f_m(s)^{1/\gamma} C_{m1} g_m dm. \tag{6}$$

Use the market clearing condition that in state s output equals aggregate consumption (output), $s = \int_m C_{m2}\,(s)\, g_m dm$, and solve the equation for the price of asset s as

$$p\,(s) = \left(\int_m f_m(s)^{1/\gamma} C_{m1} g_m dm \right)^{\gamma} / s^{\gamma}. \tag{7}$$

Most of the terms in parenthesis are parameters (exogenous): the aggregate output in period 1, the states, the subjective distributions and the distribution of the investors. However, the consumption choice in period 1 of agent m, C_{m1}, must be determined in equilibrium. For the case of $\gamma = 1$ (log utility), it is established (see Rubinstein (1974)) that C_{m1} is the same across investors so

$$p\,(s) = \frac{Y_1}{s} \int_m f_m(s) g_m dm, \text{ if } \gamma = 1.$$

In this case, it is the average (across investors) probabilities that matter for asset pricing.

B.2 Numerical Results on Period 1 Consumption for $\gamma \neq 1$

To get results for $\gamma \neq 1$, we assume that $f_m(s)$ is lognormal so investor m's beliefs about $\ln Y_2$ is $N(m, \sigma^2)$. We also assume that investors only differ with respect to the mean of their subjective distributions, and that those means are normally distributed: g_m is the pdf of $N(\mu, \delta^2)$. These assumptions have the advantages of being consistent with the lognormal model used in the first half of the paper and to give relatively simple results.

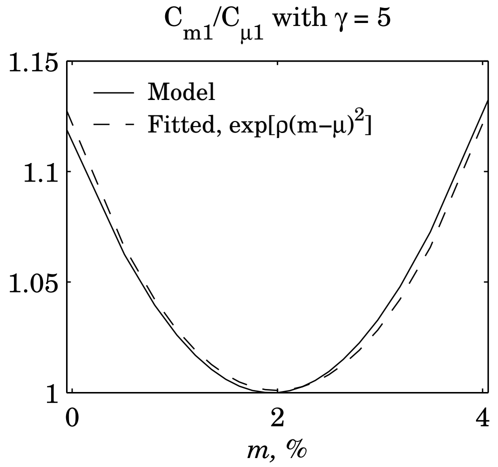

Figure 3: Period 1 consumption relative to the median investor, $C_{m1}/C_{\mu 1}$**,** as a function of the mean of the individual distribution (average mean 2%).

To find the equilibrium values of C_{m1}, we use a numerical routine. The lognormal distributions of investors are approximated by discrete distributions with 35 points, and the normal distribution across investors is approximated by a discrete distribution with 21 points. We calculate results for the following values which square well with the SPF data: median expected growth rate (μ) of 2%, cross-sectional standard deviation of means (δ) of 1%, and individual standard deviations (σ) of 1%.

The main results are illustrated in *Figure 3*. For $\gamma > 1$, investors with non-typical means consume more in period 1—and the pattern is such that $\ln(C_{m1}/C_{\mu 1})$ is very well approximated by $\rho(m - \mu)^2$, where ρ is positive. The numerical calculations verify that $C_{m1}/C_{\mu 1} = 1$ for $\gamma = 1$, and also show that $\gamma < 1$ gives the reverse pattern compared to $\gamma > 1$.

The main point of the simulations, however, is that period 1 consumption does not differ very much between investors—so we could approximate (7) by setting C_{m1} to a constant. A more formal argument is as follows. Consider $C_{m1} g_m$ in (7), where g_m is a normal pdf $\phi(m; \mu, \delta^2)$. Multiplying this pdf by $\exp[\rho(m - \mu)^2]$ gives $\phi(m; \mu, \delta_\rho^2)\delta_p/\delta$ where $\delta_\rho = \delta/(1 - 2\rho\delta^2)^{1/2}$. This shows that $C_{m1} g_m$ is also a normal pdf (times the factor δ_p/δ), with a higher standard deviation than g_m. However, the difference is small. Keeping the other parameters constant, we have $\delta = 0.01$ and the numerical results in Figure 3 (for $\gamma = 5$) give $\rho = 250$, so $\delta_p \approx 1.025 \times \delta$.

To make C_{m1} really differ between investors, we would need to make the cross-sectional variation (δ) several times larger than the individual uncertainty (σ). This is certainly not the case in SPF, so we believe that setting $C_{m1} = Y_1$ provides a good approximation.

B.3 Approximate Asset Pricing Results for $\gamma \neq 1$

We now assume that $C_{m1} = Y_1$ for all investors to arrive at simple approximate results. The following remarks turn out to be useful for calculating (7). (Proving the remarks involves straightforward calculations).

Remark 1 *Let $\varphi(s; m, \sigma^2)$ be a lognormal pdf where* $\mathrm{E} \ln s = m$ *and* $Var(\ln s) = \sigma^2$. *Raising the lognormal pdf to the power of α gives*

$$\varphi(s; m, \sigma^2)^\alpha = (2\pi\sigma^2)^{(1-\alpha)/2}\alpha^{-1/2}s^{1-\alpha}\varphi(s; m, \sigma^2/\alpha).$$

Remark 2 *Let $\phi(y; \mu, \delta^2)$ be a normal pdf. Integrating the lognormal pdf over normally distributed means gives*

$$\int_m \varphi(s; m, \sigma^2)\phi(m; \mu, \delta^2)dm = \varphi(s; \mu, \sigma^2 + \delta^2).$$

Remark 3 *By combining the two previous remarks, we have*

$$\int_m \varphi(s; m, \sigma^2)^\alpha \phi(m; \mu, \delta^2)dm = (2\pi\sigma^2)^{(1-\alpha)/2}\alpha^{-1/2}s^{1-\alpha}\varphi(s; \mu, \sigma^2/\alpha + \delta^2),$$

which we raise to the power of $1/\alpha$ (and simplify by using the first remark) to get

$$\left[\int_m \varphi(s; m, \sigma^2)^\alpha \phi(m; \mu, \delta^2)dm\right]^{1/\alpha} = \sigma^{(1/\alpha-1)}(\sigma^2/\alpha + \delta^2)^{(1-1/\alpha)/2}(\sqrt{\alpha})^{1-1/\alpha}\varphi(s; m, \sigma^2 + \alpha\delta^2).$$

To find $p(s)$ in (7), note that the term in parenthesis can be calculated by applying Remark 3 with $\alpha = 1/\gamma$

$$p(s) = \frac{Y_1^\gamma}{s^\gamma}(\sqrt{\gamma}\sigma)^{\gamma-1}(\gamma\sigma^2 + \delta^2)^{(1-\gamma)/2}\varphi(s; \mu, \sigma^2 + \delta^2/\gamma). \tag{8}$$

The price of the consumption claim is found by multiplying (8) by s and then integrating over s,

$$p_c = Y_1^\gamma(\sqrt{\gamma}\sigma)^{\gamma-1}(\gamma\sigma^2 + \delta^2)^{(1-\gamma)/2}e^{(1-\gamma)\mu+(1-\gamma)^2(\sigma^2+\delta^2/\gamma)/2}, \tag{9}$$

which uses the standard formula $\mathrm{E}\, y^k = \exp(k\mu + k^2\sigma^2/2)$.

The price of a real bond is found by integrating (8) over s

$$p_r = Y_1^\gamma (\sqrt{\gamma}\sigma)^{\gamma-1}(\gamma\sigma^2 + \delta^2)^{(1-\gamma)/2} e^{-\gamma\mu + \gamma^2(\sigma^2+\delta^2/\gamma)/2}. \tag{10}$$

We finally relate the prices of the two assets to get (5).

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