On the possibility of political change - outcomes in between local and global equilibria

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Abstract

We study voting over education subsidies where poor individuals may be excluded and the rich may chose private alternatives. With plausible changes of the standard game we show that this problem typically has multiple equilibria; one with low taxes, many excluded, and many in private schooling; another with high taxes, everyone in schooling, and few choosing the private alternative. Shifts between these equilibria can only happen through jumps in policy, not through gradual change. The method we develop identifies the global, as well as all local majority rule equilibria, and it characterizes "stability regions" around each local equilibrium. Introducing costs into the political system can make the local equilibria the globally stable outcome which, for example, implies that identical countries with different starting points could end up with completely different redistributive systems. Outcomes change in intuitive ways with the parameters and several insights with respect to the possibilities of political change seem general for problems of redistribution with excludability.

Keywords: political economy, political equilibrium, voting, redistribution, education subsidies, local equilibrium, non-median voter equilibrium

JEL classification: D7, H2

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1 Introduction

In standard majority rule problems of redistribution, individuals with income below the median gain from redistribution while those above lose. This splits the population into two opposing halves, making the voter with median-income decisive. There are, however, a number of well-known extensions of such problems where this conflict between the rich and the poor is not necessarily sustained. If, for example, a poor individual must reach some minimum level of income to benefit from redistribution, he may dislike small tax increases at low levels, but favour them given that the level is sufficiently high. This, in turn, affects individual preferences in such a way that they are not necessarily single-peaked, nor single-crossing, which in turn may affect the possibility to evoke any of the median-voter theorems.[1]

A classic situation of this kind is voting over school subsidies. For low levels of taxation, subsidies may not be sufficient to enable the poorest to attend school and, hence, they do not gain anything from the system. This means that they may oppose a higher tax rate in coalition with the rich. However, if taxes increase and transfers become larger, education becomes accessible for the poor and they now change their attitude to further tax increases. In a situation where everyone attends school and hence gets the subsidy, the political conflict returns to the standard rich-against-the-poor outcome.

In this paper, we revisit the problem of voting over education subsidies using a new approach. We develop a method which allows us to study the problem regardless of the applicability of the median-voter theorems. This is, of course, desirable in itself but it also turns out that the method enables us to gain a number of new insights. Specifically, we find that a typical situation has *multiple local equilibria*; one with zero taxes and no subsidies; one with low taxes, a large number of individuals still outside the schooling system and many in private schooling; and yet another one with high taxes, everyone in schooling, and few choosing the private alternative.

Clearly, only one of these outcomes can be the Condorcet winner, and usually (though not always) this turns out to be the high tax equilibrium. However, when introducing small changes to the standard

[1] The distinction referred to is, of course, that between the two versions of the median voter theorem; one restricting preferences to being single-peaked (based on Black, 1948 and Downs, 1957), the other restricting the preference ordering (variously stated in Roberts,1977, Grandmont, 1978, Rothstein, 1990 and 1991, and unified in Gans and Smart, 1996). See Austen-Smith and Banks (1999) and Persson and Tabellini (2000) for overviews. In problems of the type studied here both versions can fail to hold and even though one can extend the number of situations for which we can guarantee equilibria by using both versions of the theorem, it does typically not cover all relevant situations.

political game, such as costs associated with altering the incumbent policy (borne either by the candidates or the electorate), each local equilibrium may actually become the stable outcome. As we will show, the local equilibria usually do not only defeat neighbouring policies, but often a broad range of policies around itself. We refer to such a range of policies as a *stability region.* If an economy starts out in a local equilibrium, the relationship between costs of changing policy and the size of the stability region, determines whether change will occur or not. Consequently, two otherwise identical economies could exhibit very different transfer systems as a result of different initial policies. Furthermore, the only way to shift from a local equilibrium to another is by a "jump" in policy, while gradual change typically is not possible. Our method allows for the evaluation of exactly how small (or large) perturbations of the existing situation that are needed for political change to take place.

Under realistic assumptions, we find the low tax equilibrium to have a very broad stability region - defeating all tax rates between zero and 68 percent. Hence, relatively small costs are sufficient to make it the stable outcome in a majority vote, even though the high tax-transfer state is the Condorcet-winner policy. The stability regions, as well as political coalitions finally, change in economically intuitive ways with the parameters.

The awareness of potential problems with majority rule equilibrium in standard redistributive situations go back to classics such as Bowen (1944) and Musgrave (1959, Chapter 6).[2] One partial solution to the potential problem of non-existence of voting equilibria in these cases, is to note that when the problem is one dimensional there will, under very general assumptions, at least be local equilibria.[3] This was discussed in the context of schooling by Stiglitz (1974) and used by Klevorick and Kramer (1973) in a study of social choice on pollution management. However, in their classic textbook *Public Economics,* Atkinson and Stiglitz (1980) deemed the local equilibrium concept unsatisfactory. As they put it: "whether it provides a persuasive resolution to the "majority-voting paradox" depends on the extent to which choices are limited to small perturbations of the existing situation" (p. 307).

[2] Bowen (1944) is particularly interesting as this is an early recognition of the fact that education is an example of a social good which is not equally beneficial to everyone. He also points out the difference between voting over preferred levels of public goods and "voting on increments to existing outputs". Again he points to schooling as an example where individuals typically vote "not on how much of the good they prefer, but rather on whether or not they wish a given increment of decrement to the quantity already provided" (p.40). Finally Bowen also mentions the potential problems with strategic voting (though he does not use the term).

[3] E.g. Theorem 2 in Klevorick and Kramer (1973). See also Kats and Nitzan (1977) on the relations between global and local equilibria.

Ever since, the typical way of dealing with this kind of problem has been to focus on cases where some version of the median voter theorem applies. Examples are Glomm and Ravikumar (1992), Perotti (1993), Epple and Romano (1996a) and (1996b), Gouveia (1997), Glomm and Ravikumar (1998) and others.[4] An exception is Fernadez and Rogerson (1995), who solve the problem by introducing restrictions on the underlying distribution of the voters.[5]

All of the articles above, only consider policies which defeat *all* other alternatives (i.e. the Condorcet winner), while the possibility of local equilibria - or something in between local and global equilibria - have not been considered.[6] In this paper, we argue that minor (and plausible) changes of the standard game, in many cases can be sufficient to make the previously discarded local equilibrium concept relevant as a stable equilibrium outcome, even though agents are not myopic. Finally, the method we present in this paper is general is applicable to many one-dimensional majority voting problem where the median voter theorems do not apply.

The remainder of the paper is structured as follows: In Section 2 we apply the method to a problem of majority rule decisions over school subsidies where credit constraints for investing in education can be binding, and where public and private schooling coexist. In Section 3 we define relevant political equilibria (in particular global and local equilibria) and the versions of majority rule competition discussed above. In Section 4 we solve the problem and show how the outcome varies with the parameters of the problem. In particular we show how the stability regions around local equilibria change with the costs of schooling and the initial distribution of income. In Section 5 we discuss some more general insights that can be drawn from our example and, finally, Section 6 concludes the paper.

[4] Typically, these articles give conditions for when the median voter theorem applies and, in some of them, the equilibrium (if it exists) is characterized for the cases when the theorem does not apply. However, this is not to say that they have only looked at cases where preferences are single-peaked. There are cases where the median voter theorem holds even though preferences are not single-peaked (as in Glomm and Ravikumar 1998) and there are cases where there is majority voting equilibrium but the median (income earner) is not decisive (as in Epple and Romano 1996a). Both single-peakedness and order restriction are sufficient, not necessary, conditions for a majority voting equilibrium.

[5] The list in Fernandez and Rogerson (1995) has 25 different cases and, as they point out, this list would become longer if they had more than three income groups (footnote 17, p. 257).

[6] Local equilibria have been considered in articles on other related topics. Examples of such articles are Alós-Ferrer and Ania (2001) and Crémer and Palfrey (2002).

2 A simple model of educational choice

Consider an economy with a continuum of individuals who differ in initial income y_i. The distribution of income is given by a cummulative distribution function, $F(y)$, with a corresponding probability density function denoted $f(y)$. The function $f(y)$ is assumed to be continuous and positive over it's support $[0, \infty)$. The number of individuals is normalized to one and hence, aggregate income $Y = \int_0^\infty y f(y)\, dy$ is equal to average income.

Individuals are assumed to live for two periods. For simplicity utility is linear in income and there is no discounting between periods.[7] In the first period each individual has to decide whether to invest in public, private or no schooling. Investing in public schooling has the fixed cost E, whereas investing in private schooling has the fixed cost P, with $P > E > 0$.[8] The return to schooling is realized in the second period, and given by $g(y_i)$, and $h(y_i)$ for public and private schooling respectively. Those who do not attend school get the same income as in the first period, y_i. Furthermore, it is assumed that $h(y_i) - P > g(y_i) - E > y_i$ for all i, which implies that all individuals prefer private education to public education, and that all individuals prefer public education to no education.[9] However, in the absence of perfect credit markets (and without special government intervention) individuals would sort into three groups depending on whether they can afford private schooling, public schooling, or no schooling at all.[10]

To enable schooling for a larger share of the population a uniform tax, $\tau \in [0, 1)$, chosen in a majority rule election, is raised to subsidize public education. Specifically, we assume that agents who choose public schooling get a subsidy, $s(\tau)$, while those who choose no schooling and private schooling respectively get no subsidy.[11]

Given initial income, individual utility (over both periods) is given by

[7] These simplifications are not in anyway necessary for the application of our method. It is straight forward to use it with any well-behaved utility function. We only choose the simplest possible setting for expositional purposes.

[8] These costs can be thought of as timecosts, foregone income or acctual schooling fees or some combination of these. We assume that the costs can be treated as one fixed, lump-sum cost.

[9] Discounting can be thought of as included in the payoff functions.

[10] Note that what we call "absence of government intervention" does not imply that there is no government, and in particular it does not imply the absence of public schooling. It should only be taken to mean that at this point there is no specific subsidy to those attending the public school.

[11] We could generalize this to a situation where individuals who choose private school get at least a fraction $\delta s(t), \delta \in [0, 1]$.To simplify the exposition of our method and avoid carrying a number of possible cases throughout the analysis we simplify the setting to one where $\delta = 0$.

$$u_i = \begin{cases} (1-\tau)\, y_i + y_i & \text{if no schooling} \\ (1-\tau)\, y_i + s\,(\tau) - E + g(y_i) & \text{if public schooling} \\ (1-\tau)\, y_i - P + h(y_i) & \text{if private schooling} \end{cases} \tag{1}$$

Assuming that the ordinal relationship between the different choices is not affected by the subsidy, i.e. that, $h(y_i) - P > g(y_i) - E + s(\tau) > y_i$ for all i and all τ, the population will split into three groups defined by two thresholds.[12] If an individual has an income equal to or above the critical income

$$y^*(\tau) = \begin{cases} 0 & \text{if } E \leq s \\ \frac{E-s(\tau)}{1-\tau} & \text{if } E > s \end{cases}, \tag{2}$$

he will chose public schooling, unless he has an income higher than

$$y^{**} = \frac{P}{1-\tau}, \tag{3}$$

in which case he choses private schooling.

Denoting the share of the population in private school by

$$N_{priv} = \int_{y^{**}}^{\infty} f\,(y)\,dy = 1 - F\,(y^{**}\,(t))\,,$$

and the share investing in public schooling

$$N_{pub} = \int_{y^{*}}^{y^{**}} f\,(y)\,dy = F(y^{**}(t)) - F(y^{*}(t)),$$

the share unable to afford schooling is given by $1 - N_{pub} - N_{priv}$. The size of the subsidy $s\,(\tau)$, which goes only to those in public school, depends on the tax rate τ, but also on the number of individuals optimally choosing public schooling. Assuming a balanced budget

[12] The case which we assume away is the possibility that there are those who could pay for the private education with the higher return, but who would abstain from this so as to get the subsidy in period one. This can happen in general but, as we will confirm, does not happen for any relevant cases in our example. It would also be possible to solve the model with this additional possibility but it would add a number of cases but no additional insights.

$$s(\tau) = \frac{\tau Y}{N_{pub}} = \frac{\tau Y}{F(\frac{P}{1-\tau}) - F(\frac{E-s(\tau)}{1-\tau})} \tag{4}$$

(or zero if no one chooses public schooling). This implicit function can be shown to have a fix-point (the proof has been placed below in connection to where we solve the model and find the equilibria).

Given the critical income levels above, we can define the utility function in terms of these threshold and as a function of the tax rate. Differentiating (1) with respect to the tax rate gives

$$\frac{\partial u_i}{\partial \tau} = \begin{cases} -y_i & \text{if } y_i < y^* \\ -y_i + s'(\tau) & \text{if } y^* \leq y_i \leq y^{**} \\ -y_i & \text{if } y_i \geq y^{**} \end{cases} \tag{5}$$

The interpretation of these expressions are straightforward: Individuals who cannot afford the investment despite the subsidy, would oppose a marginal increases in the tax rate, since this will only lead to an increase of their tax payment ($-y_i$). The same is true for those who chose the private alternative since they also pay taxes and get nothing in return. Those who chose public schooling, on the other hand, also experience an increase in their tax burden, but at the same time they get an increased subsidy (as long as $s'(\tau) > 0$).[13] Within this group, the preference toward a marginal increase therefore clearly depends on the relative size of the increase of the subsidy compared to the increased individual tax burden. Given the obvious monotonicity of this relation, the share of the population in favour of a marginal increase at τ would be those with income $y_i \in (y^*, \widehat{y})$, where $\widehat{y}$ is the income of an indifferent agent given by $\widehat{y} = s'(\tau)$, given that $\widehat{y} \in (y^*, y^{**})$.[14]

The above does not say anything about individual preferences over (all) tax rates. It is simply a description of how the population would be split into different groups in favor of, or opposed to, a marginal change of the tax rate (at any tax rate). Nevertheless, this way of describing the marginal (or local) preferences of the population is, as we will show below, useful when studying how the population is divided in their views of different tax rates and in finding local as well as global political equilibria.

[13] Obviously, if $s'(t) < 0$, no one is in favor of a marginal increase of the tax rate.

[14] We can disregard the weight of those who shift between groups (i.e. shift from not being able to afford schooling, to investing in public school, or shift between public and private school) as their weight goes to zero for an infinitesimally small change of the tax rate. We will discuss this in more detail below.

2.1 Majority rule over educational subsidies

Given the individual alternatives described above, what level of taxation would be chosen in a majority rule election? What would be the political equilibrium? The answer to this depends on the exact political game. We will consider four variations of standard Downsian competition below. There are however some things we can note without specifying the precise rules of political competition. First, the shifts in how an individual evaluates a marginal change of the tax rate (depending on her choice of education) clearly illustrates why there may be problems with applying either version of the median voter theorems.[15] The failure to satisfy single peaked preferences over tax rates is easily illustrated. As taxes increase an individual who is too poor to make the investment will only pay increasing taxes with decreasing utility as a consequence. At some point, however, the size of the subsidy can be large enough to enable the poor individual to make the investment leading utility to (possibly) increase. Figure 1a shows an individual with an initial income (endowment) $y < y^*$ for all $\tau < \tau'$. At τ' the subsidy becomes just large enough for $(1-\tau')y + s(\tau') = E$ which allows the individual with initial income y to invest and (under a certain subsidy function) experience increasing utility for higher tax rates. Similarily we can illustrate how individual indifference curves (in the tax-subsidy space) can be non-single crossing. Figure 1b illustrates the fact that an individual who at tax rate τ requires at least $s(\tau)$ to make the investment has an indifference curve which crosses the indifference curve of a richer person, who invests regardless of the subsidy, twice. The fact that preferences are not necessarily single peaked nor single crossing means that none of the median voter theorems can be applied in general.[16]

Even though the requirements for applying the MVTs do not hold, there is, as noted above, no ambiguity in terms of how individuals evaluate *marginal changes* of the tax rate. This means that for any tax rate τ, we can aggregate the shares of the population into the total support for and against any marginal change in the tax rate $\tau \pm \epsilon$. Furthermore, we can define a tax rate as a *local equilibrium* if it is

[15] As mentioned in the introduction, one may distinguish between the theorem relying on preferences being single peaked and the theorem which requires single crossing (shown to be equivalent to order-restriction in Gans and Smart, 1996).

[16] The exact form of the utility function, as well as the indifference curves, of course depend on how subsidies evolve over tax rates. For violation of single-peaked preferences to occur in this type of setting, what is needed is that there be some segment of the policy where the poor person has decreasing utility from increasing the subsidy because they do not (can not) participate, and that in some other segment where they do participate, they gain from redistribution. For single-crossing to be violated what is needed is that poor individuals, who normally have indifference curves with a smaller slope than richer once (as they require smaller subsidies to be indifferent to a certain tax increase), for low enough tax rates can not participate in the redistributive system and, hence have vertical indifference curves at this point.

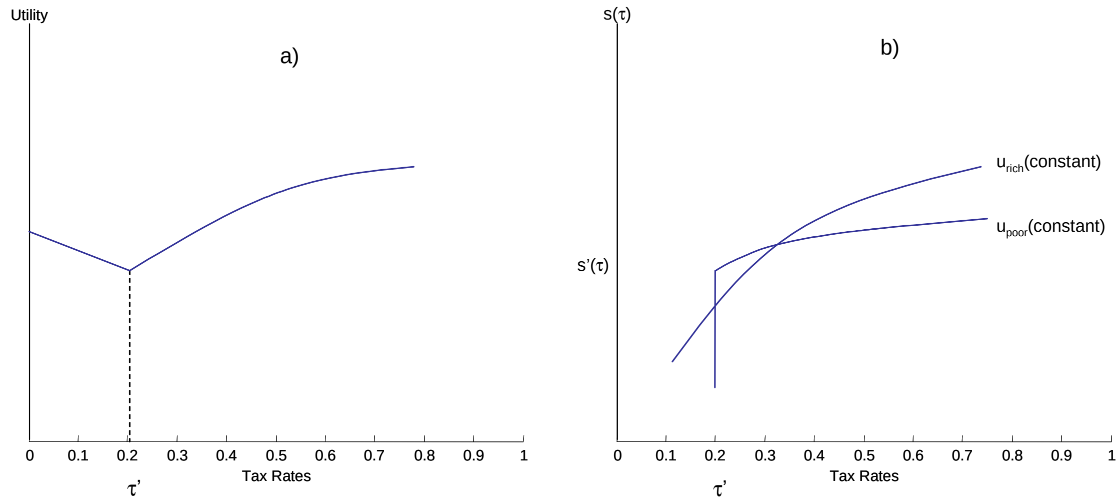

Figure 1: Illustration of failure to satisfy conditions for the Median Voter Theorems.

majority preferred to its' neighboring tax rates $\tau \pm \epsilon$ (in the case of the possible corner solutions only one neighboring point is relevant, with an obvious corresponding definition).[17] We now make the following observations regarding local equilibrium outcomes and their relationship to global equilibrium outcomes (i.e. the Condorcet winner, if it exists):

Lemma 1 *For a tax rate to be a global equilibrium it must also be a local equilibrium (while the reverse is obviously not true).*

Lemma 2 *If a local equilibrium tax rate also defeats all other tax rates, then this local equilibrium is also a global equilibrium.*

Lemma 3 *If no local equilibrium tax rate is majority preferred to all other tax rates, then there exists no global equilibrium.*

As we can construct a function for the *aggregate support for a marginal increase* over the whole policy space (the relevant space here being the one dimensional set of alternatives $\tau \in [0; 1)$) we can use this information to determine in which direction a majority would like to push the tax rate at any point (given

[17] See Kramer and Klevorick (1973) and (1974), Kats and Nitzan (1976), or more recently Alós-Ferrer and Ania (2001) or Crémer and Palfrey (2001) for more rigorous definitions of local equilibrium in majority rule games. As shown by Kramer and Klevorick (1973) a local equilibrium will exist in one dimensional settings under very general circumstances.

that we only consider marginal changes).[18] This is of course not sufficient to solve the problem but it does provide a very useful starting point due to the following simple fact: If there is a majority in favour of a marginal change of the tax rate, such a policy is not even a local equilibrium and consequently it can not be a global equilibrium. Put differently, it suffices to consider local equilibria when searching for a global one.[19] In the problem above, the fraction of the population that favours an increase of the tax rate at any τ is simply:

$$H(\tau) = \begin{cases} 0 & \text{if } \widehat{y} < y^* \\ \int_{y^*}^{\widehat{y}} f(y)dy & \text{if } \widehat{y} \in (y^*, y^{**}) \end{cases}$$

For this characterization to be correct we must show that the critical incomes are continuos in τ, that is

$$\lim_{\epsilon \longrightarrow 0} y^* (\tau) = y^* (\tau + \epsilon) \tag{6}$$

$$\lim_{\epsilon \longrightarrow 0} \widehat{y} (\tau) = \widehat{y} (\tau + \epsilon)$$

$$\lim_{\epsilon \longrightarrow 0} y^{**} (\tau) = y^{**} (\tau + \epsilon) \tag{7}$$

which in turn depends on the properties of $s(\tau)$. The limit for investing in private schooling is given by $y^{**} = \frac{P}{1-\tau}$ and is obviously contious in τ. The critical income for investing in public schooling, $y^*(\tau)$ is given by (2) above and the continuity of this depends on the continuity of $s(\tau)$, and as $\widehat{y}(\tau) = s'(\tau)$ we must also show that the function $s(\tau)$ is continously differentiable for all critical incomes to be continous in the tax rate. Using the implicit function theorem we can show that $s(\tau)$ exists and is a continously differentiable function with a unique solution for every τ.

Proof. We want to prove that for the implicit function $s(\tau)\left[1 - F\left(\frac{E-s(\tau)}{1-\tau}\right)\right] - \tau Y = 0$ there exists a unique continously differentiable function s which has a solution for every τ. Consider the continously

[18] Clearly we could just as well focus on the group in favor of a marginal decrease, which is just the complement of those in favor of a marginal increase. (Also, considering tax rates smaller than unity is just for convenience so as to avoid carrying the additional notation for cases when $\tau = 1$ would create conditions where terms are undefined due to division by zero).

[19] Note that this is not saying that it is enough to compare local equilibria (if there are more than one) as it may well be the case that a local equilibrium tax rate is defeted by some other tax rate which is not itself even a local equilibrium. It does however say that for a tax rate to be a global equilibrium it must also be a localequilibrium (as noted in Lemma 3).

differentiable function $\Phi(x^o, \tau^o) = \tau Y$ where

$$\Phi(x^o, \tau^o) = x\left[1 - F\left(\frac{E - x}{1 - \tau}\right)\right].$$

Taking the derivative w.r.t x (at x^o with τ^o fixed) gives

$$\frac{\partial \Phi}{\partial x} = \left[1 - F\left(\frac{E - x}{1 - \tau}\right)\right] + \frac{x}{1 - \tau} \times f\left(\frac{E - x}{1 - \tau}\right)$$

which is always positive (for the relevant domains of x, τ and E and given the functions F and f) and hence is a bijection. By the implicit function theorem we then know that there exists a neighborhood of τ and a unique continously differentiable function s such that $s(\tau^o) = x^o$ and $\Phi(s(\tau), \tau) = 0 \ \forall \tau$. ■

3 Solving the Model

To explicitly solve the model we need to choose values for the parameters (such as the schooling costs) and the initial income distribution. We do not set out to calibrate the model to any specific country or situation, but instead chose reasonable values to illustrate a possible outcome, and then study how the equilibria move with the parameters. Below we analyse which policy that will be announced by the office-seeking politicians in four different political games.

3.1 Parameterization

First, a distribution function for pre-tax income must be chosen. The model can be solved for any continuous and well-behaved initial income distribution. The specific shape of the distribution function will however, be an important determinant of the political support for redistribution. Second, the costs of investing in public and private schooling respectively (i.e., E and P) must be chosen. Third, we must specify the functions for the return to public and private schooling, i.e., $g(y_i)$ and $h(y_i)$.

To approximate this, we assume pre-tax income to be Weibull distributed with parameters ($b = 100, c = 1.4$), which generates a Gini Coefficient of 0.39.[20]

[20] The average market (pre-tax) income Gini is 0.39 according to data from the Luxembourg Income Study (based on

The fixed cost of investing in public and private schooling may respectively be expressed as shares of the average income:

$$E = \mu Y, \qquad P = \varphi E \qquad \mu, \varphi \in \mathbb{R}^{+}, \quad \varphi > 1. \tag{8}$$

In the benchmark calibration, we set $\mu = 0.5$ and $\varphi = 2.8$, but again, we solve for equilibria over a wide range of values.[21] The return to public and private shooling finally, are set to $h(y_i) = E + \alpha y_i$ and $g(y_i) = P + \beta y_i$, with $\alpha = 2$ and $\beta = 3$. Note that the exact functional forms of these functions are not important for the political outcomes locally, since neither y^*, y^{**} or $\widehat{y}$ depend on these functions. However, they do enter the utility functions and will therefore play a role in the comparisons between alternatives where individuals choose different (or no) schooling between the respective policies being compared. Hence, they affect the global equilibrium as well as the stability regions.

3.2 The Aggregate Support for Changing the Policy

Given the chosen parameters we are now ready to construct the function $H(\tau)$, which shows how the aggregate support for a local increase of the tax rate varies with the tax rate. This is illustrated in Figure 2. Recall that that at any point where $H(\tau)$ is above the 0.5-line, there is a majority (more than 50 percent of the population) in favour of a marginal increase of the tax rate, and vice versa for points below the 0.5-line. Starting from the very left in Figure 2, i.e., at $\tau = 0$, we see that only around 43 percent prefers a marginaly higher tax rate to the zero tax rate. As we move to the right, the aggregate support for a local increase of the tax rate first increases to 57 percent at the tax rate 11 percent, and then decreases and reaches a minimum around 37 percent at the tax rate 40 percent and finally increases again to a maximum at the tax rate 1.

For interior tax rates, only points where $H(\tau)$ cuts the 0.5 line from above (reading left to right) are local equilibria. Since there is only one such point - at the tax rate $\tau = 0.1429$ - this is the only interior local equilibrium. At this point, exactly half the population is in favour of an increase, while the other half would like a decrease. In addition, the tax rate $\tau = 0.1429$ is majority preferred to marginal changes

34 observations taken from the OECD Economic Studies, 1997). The *two-parameter* Weibull *pdf* is given by $p(y) = \left(\frac{y}{b}\right)^c \exp\left[-\left(\frac{y}{b}\right)^c\right]$, where $y > 0, b > 0$ and $c > 0$. The parameter b is just a scale parameter, and c is a shape parameter determining the degree of inequality.

[21] A discussion of the benchmark calibration can be found in the Appendix.

($\pm\epsilon$) in the tax rate. There are also two tax rates for which $H(\tau)$ cuts the 0.5 line from below, i.e., at $\tau = 0.05$ and $\tau = 0.49$. It is important to note that these tax rates never constitute local equilibria. In fact, they are both defeated by both their neighbours, implying that they are locally unstable in both directions. Note finally that both corners also are local equilibria, since a majority of the population is against a local increase at $\tau = 0$, while a majority of the population is in favour of an increase at $\tau = 1$. Hence, to sum up, there are three local equilibria, at $\tau = 0, \tau = 0.1429$ and at $\tau = 1$, and from Lemma 1 it immedeately follows these are also the only candidates for being the global equilibrium, since no other tax rate is even a local equilibrium.[22]

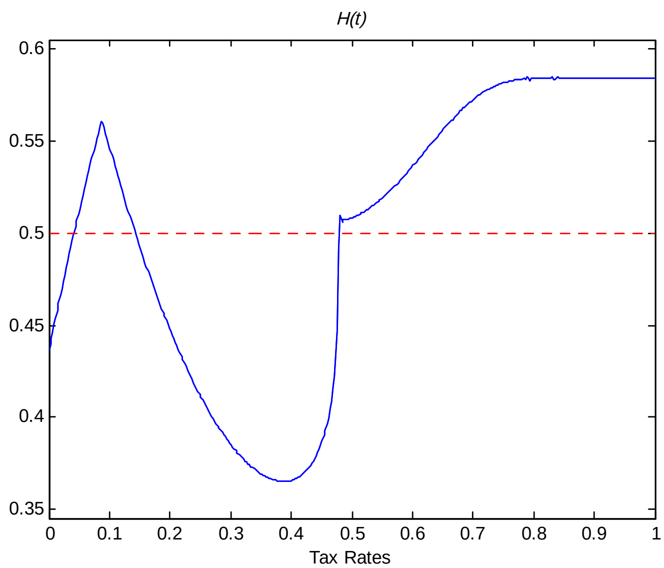

Figure 2: The Aggregate Support for a Marginal Increase in the Tax Rate

Looking at the division of the population at each local equilibrium, as shown in Figure 3, is illustrative of what drives the support for change in different regions of the tax rate. The top panel shows the intuitively clear division at $\tau = 0$. First, there is a group at the lower end of the distribution (those with $y \in [0, y^*]$) characterized by individuals so poor that they can not afford education at this point. They have nothing to gain from a marginal change in the subsidy which they do not get and consequently they

[22]Recall that there are no disincentive effects from taxation in the model. Hence, in a purely redistributive setting as long as the median is poorer than the mean there will always be a majority in favor of more redistribution (given that everyone participates and gets the subsidy). Introducing effects which bound the maximum away from one does not change the qualitative results.

oppose marginal increases of the tax rate (again, note that the wight of those who shift between groups as a consequence of a marginal change is zero). The second group (those with $y \in [y^*, \widehat{y}])$ consists of individuals who at this point gains from increased taxes as they get a subsidy which is larger than their tax payment, while the third group are those at the top of the distribution who, at this point, pay more than they get and consequently oppose a tax increase. (This third group consists of those who attend private school and therefore get no subsidy as well as those who attend public school and receive a subsidy but pay more than they get). The middle panel shows how the composition has gradually changed as we have moved to a higher tax rate. It is still a coalision between the poorest and the rich who at this local equilibrium oppose a marginal increase of the tax rate, but the share at the lower end is now smaller than before as more individuals at this tax level can afford education thanks to the subsidy. Finally, the bottom panel shows the division at $\tau = 1$ where everyone receives the subsidy and the situation is one where everyone with an endowment below the mean wants to maximize redistribution.

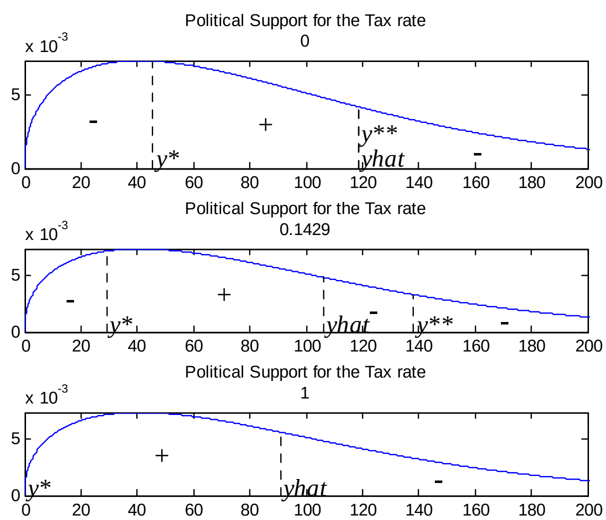

Figure 3: Political Compositions that Favor a Marginal Increase in the Tax Rate

Fig. 2 and 3 are both silent about which (if any) of the three equilibria that is the global equilibrium. To determine this, we compare the three candidates to (a dense grid of) all other tax rates. Doing this, we find the global equilibrium to be $\tau = 1$. In this search process, we also get information about exactly how each local equilibrium compares to all other tax rates. This allows us to create stability regions around each local equilibrium. Fig.4 shows the results from such a comparison for $\tau = 0$ and $\tau = 0.1429$ with stars indicating policies that defeat the respective local equilibrium in a majority vote. We refer to the the interval of tax rates without stars as a local equilibriums *stability region*, since all these tax rates are defeated by the local equilibrium in a majority vote. The top panel shows that $\tau = 0$ is not very stable since it is defeated by any tax rate higher than 7 percent. However, this is not the case for the local equilibrium $\tau = 0.1429$, which is majority prefered to all tax rates between zero and $\tau = 0.68$. As we will see below, these stability regions might be interesting in a number of situations.

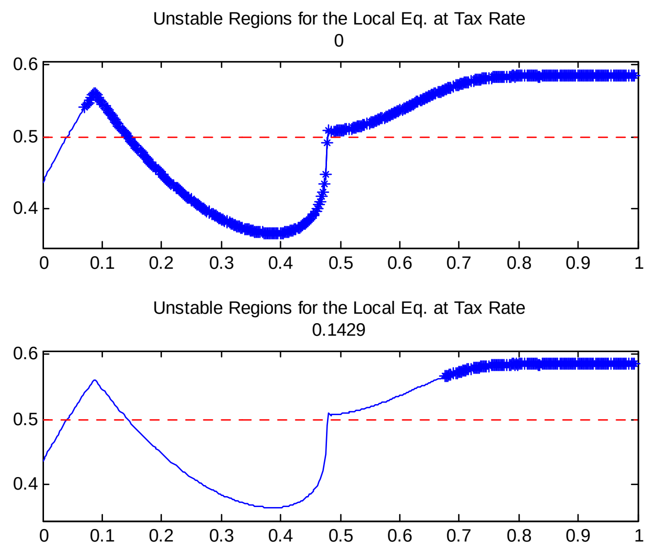

Figure 4: Stability Properties for the Local Equilibria

In a standard setting of Downsian competition the global equilibrium is the obvious outcome. There are, however, plausible modifications of the standard game which lead to situations where this is not necessarily the case. We therefore introduce the following concept:

Definition 1 *The* ***stable outcome*** *(or Nash equilibrium) of a majority rule election game is a tax rate announced by the politicians from which none of them has an incentive do deviate.*

In the following section we consider four different specifications of the political game that illustrate how the global equilibrium does not always coincide with the stable outcome, and in particular, why local equilibria - depending on the stability regions - are likely to be the stable outcome.

4 Political Games

4.1 Standard Downsian competition

As a first benchmark we consider the setting where two parties chose a tax rate as their platform and the party which gets the majority of the votes wins the election. In this case the relevant political equilibrium is the *majority voting equilibrium* and the policy outcome will be the "Condorcet winner". In terms of the method above the $H(\tau)$-function provides information on which tax rates that are local equilibria (and hence candidates for being the global equilibrium). Since we assume that there are no cost associated either with campaigning or changing the policy, both parties will, in the example above, announce the tax rate $\tau = 1$ as their platform. Hence, the stable outcome is the global equilibrium. The local equilibria are not really interesting in this game, but merley a step on the way to finding the global equilibrium (if it exists).[23]

4.2 Myopic political competition

Consider now instead the polar extreme where the candidates for some reason only can alter the policy in small steps. Under this assumption the $H(\tau)$-function also reveals the transition path to the equilibrium. Which of the local equilibra that actually becomes the stable outcome will depend on the initial tax rate. Fig. 2 shows that if the economy starts out with a tax rate in the interval $\tau = (0.05, 0.49)$, it will eventually end up at the local equilirium $\tau = 0.1429$, whereas if the initial tax rate is below 0.05, it will converge to the tax rate $\tau = 0$. If the initial tax rate is above 0.49 finally, the policy will instead be pushed toward 1.

It is important to note that in this type of game the restriction on the candidates alternatives is very strict. Even if one thinks that it is reasonable to limit the politicians possibility to suggest very large moves from the starting point (at least without incuring any costs) this is not ideally captured. The marginal conditions behind the $H(\tau)$-function only evaluate each tax rate against $\varepsilon-$changes and can not in itself say anything about the aggregate preference for a slightly larger (but still small) move. As was

[23] If there is no global equilibrium the situation is one usually thought of as a case of "policy cycling". Roine (2006) however presents some arguments for local equilibria in Downsian competition in the absence of a Condorcet winner.

noted in the introduction, this is likely to be the reason for why local equilibria in majority voting games have been dismissed in the literature.

4.3 Downsian competition with costs of altering the policy

Now consider instead a standard Downsian competition game between two parties starting at some status quo position, but where we add the reasonable assumption that there are costs involved in shifting policy (or altering policy platform). The outcome of the game will then typically come to depend on the *stability regions* around the local equilibria. The set up depends on what we assume about the costs of moving and in particular on whether these costs are borne by the candidates or by the voters. We analyse both cases in turn.

4.3.1 Costs Borne by the Candidates

Consider the following reduced form of the problem facing each candidate at the status quo: If the candidate sticks to the status quo policy and the other candidate does the same they both have a 50/50 chance of winning. If one candidate moves to a policy which is majority preferred to the status quo and the other one does not, the candidate that moves wins with certainty but bears a cost, c, of moving. Finally, if both move (to the same policy) they are back to a situation where both have a 50 percent chance of winning, but now both of them will have to bear the cost of moving.[24] Normalizing the payoff from winning to one, this game has the following normal form:

$$\begin{array}{ccc} & \text{Move} & \text{Stay} \\ \text{Move} & (0.5-c;\, 0.5-c) & (1-c;\, 0) \\ \text{Stay} & (0;\, 1-c) & (0.5;\, 0.5) \end{array} \cdot \tag{9}$$

[24] To illustrate our point we assume a one-shot game where we only consider two possible actions; "Move" (implicitly to the nearest point which defeats the status quo) or "Stay" (at the status quo). In a full version of the game there are of course an infinite number of possible moves. Some, such as moving to policies which are defeated by the status quo, are not interesting but others may be. However, as our aim here is to give conditions for when both candidates would chose to stick to the status quo this simplified version is sufficient.

Assume now that the economy starts out at a local equilibrium, and that the parties move simultanuously in a one-shoot game.[25] Obviously, if the initial tax rate is $\tau = 1$, the economy stays there, but if we start at any of the other local equilibria the stable outcome clearly depends on the cost. More precisly, if $c < 0.5$, moving is a strictly dominant strategy but if instead $c > 0.5$ staying with the status quo is optimal (with obvious indifference at $c = 0.5$). If we, which we think is realistic, assume that the cost of moving is increasing in the distance moved, this implies a negative relation between the size of the stability region and the "cost per unit moved" needed to make a local equilibrium the stable outcome of the game.[26] In other words, the broader is the stability region, the further the parties must move from the status quo to get a majority of the votes. Since this is increasingly costly, the more likely it is that the local equilibrium becomes stable. To see this explicitly, assume that the candidates face the following simple cost function when moving

$$c(\Delta\tau) = \xi\Delta\tau, \tag{10}$$

where $\Delta\tau$ is the Euclidian distance between the status quo policy and the policy the candidate is moving to. If we now compare the two local equilibria $\tau = 0$ and $\tau = 0.1429$, the marginal cost needed to make each of them the stable outcome is more than 8 times higher for the zero tax equilibrium than the equilibrium with $\tau = 0.1429$.[27] This is expected since the former eqilibrium has a very short stability region, whereas the latter already without costs defeats more than 2/3 of all tax rates. Note that the tax rate $\tau = 1$ is still the global equilibrium, but that the stable outcome of the game can differ from that, depending on the starting point and the cost of altering the platform. This simple example illustrates that the relevance of a local equilibrium policy as a plausible outcome depends crucially on precisly *how* locally stable it is.

[25] For simplicity, we assume that the starting point is a local equilibrium. This is, however, not very restrictive since a policy which is not a local equilibrium can be defeted by an infinitesimally small move (which, if costs are related to distance moved, has a very low c). In fact, if we only require the cost of moving to be an increasing continous function c of the distance moved $(\Delta\tau)$ such that $c(0) = 0$, $\{stay; stay\}$ can never be the equilibrium.

[26] An example of a reason for why large moves should be more costly is that it is likely to be more costly for parties to communicate major changes in their programs compared to small ones.

[27] The total cost is not really interesting since payoffs have been normalized.

4.3.2 Transition costs borne by the voters

Finally, we consider the case where transition costs are incurred by the citizens.[28] Specifically, assume that the transition cost function is still given by (10), but that the cost of moving just a marginal unit is zero (i.e., $c(\epsilon) = 0$) and that the transition is financed with a tax on the income in period 2. These assumptions are sufficient to make sure that the $H(\tau)$-function in Fig. 3 remains unchanged.[29] Voters face the exact same problem *locally*, but the comparison of the equilibria to all other tax rates is different, due to the transition costs.

As in the previous section, if the economy starts out with a tax rate $\tau = 1$, it stays there and the stable outcome is $\tau = 1$. If, however, the economy starts out with any other tax rate, the stable outcome could again differ from $\tau = 1$. It is of course possible to consider a number of different games but what we are interested in here is to illustrate the relation between the stability region and the transition cost. In particular, we ask the following question: given that the economy starts out in a local equilibrium (that is not also the global equilibrium, how large must the transition cost be for this to be the stable outcome. Furthermore, we want to get a sense of whether the order of magnitude of the cost is such that this is a realistic case. With knowledge of the stability regions we can calculate the minimum cost required to make the respective local equilibria stable. We can then express these as fractions of total income to get a sense of their size. The results for such an exercise in the above benchmark problem are presented in Table 1.

Table 1: Transition Cost Needed to Make the Local Equilibrium Stable

LOCAL EQUILIBRIUM	$\tau = 0$	$\tau = 0.1429$
Min. Transition Cost for Stability (% of Tot. Inc.)	17.64%	2.2%

Consequently, if the economy starts out at the local equilibrium of $\tau = 0.1429$ and the total cost of moving to a majority prefered policy is larger than 2.2 percent of total income, the status quo remains

[28] These might be administration costs, or simply costs associated with expanding the system.

[29] This assumption can easily be relaxed. If there was a positive cost for a marginal move this would simply shift the $H(\tau)$-function as there would be an additional cost (which would be a fixed number for the $\varepsilon-$sized move) to every individuals utility function. It is also straightforward to show that the tax rate needed to finance the transition is $\tau^t = \xi \Delta\tau / Y$.

the stable outcome.[30] However, if the initial tax rate is $\tau = 0$, the total transition cost needed to make it the stable outcome is more than 17.64 percent of GDP. These results are similar to the previous case in the sense that transition costs must be seven times higher to make the zero-tax equilibrium stable compared to the interior equilibrium ($\tau = 0.1429$). Also the general point is again that relatively small transition costs may be sufficient to make a local equilibrium, with a broad stability region around it, the stable outcome.

5 Changing the parameters

Having established the potential importence of, not just the global equilibrium tax rate, but also the local equilibria and their respective stability regions we now move on to study how the possible outcomes depend on the parameters. More precisely, we will illustrate how the outcomes change over ranges of different starting values of one parameter at a time, keeping the others at their benchmark values from above.

5.1 The cost of public schooling

Fig. 5 shows the different equilibria and their stability regions as we change the cost of public education (expressed as a fraction of total income and keeping the cost of private schooling fixed at the benchmark level).

In the considered range of public schooling costs, the high tax rate equilibrium is the global equilibrium. However, as the cost of public schooling goes up, the share of the population at the bottom of the distribution who cannot afford it grows. At some point they are able to form a locally stable coalition with the rich for a low interior tax rate. Increasing the tax rate further also inroduces zero as a local equilibrium.[31] The stability regions around these respective local equilibria increases as the cost of public

[30] It is of course possible to be more general and compute the transition cost needed to make $\tau = 0.1429$ the stable outcome given that the economy starts out without any public subsidies to schooling.

[31] At first glance the graph may look contradictory as the interior tax rate is stable inside the zero tax rate's region of stability. However, this is perfectly possible as the starting point for the comparision is each local equilibrium respectively. In the region where the interior equilibrium is inside the zero tax rate's stability region, there are points which are defeated by both the zero and the interior local equilibria depending on where we start. However, by definition, the zero tax rate would defeat the interior local equilibrium in this region. This can also be seen by the fact that when the local equilibrium interior tax rate crosses the border of the zero tax rate's stability region, the interior is no longer stable agains zero (it's stability region starts moving up from zero).

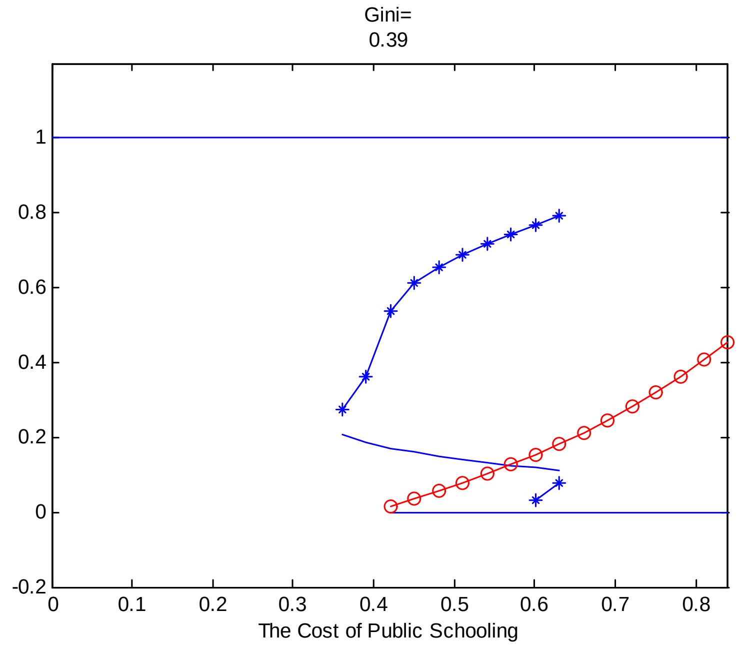

Figure 5: Local Equilibria and Stability Regions as Functions of the Cost of Schooling

schooling goes up. Since the minimum subsidy needed for the poor to benefit goes up as public schooling becomes more expensive, only a large enough subsidy will enable the population at the bottom of the distribution to participate and benefit. The implication of this in terms of political change is that the policy must jump from zero to a higher level of taxation/redistribution for there to be a majority in favor of such a policy. Attempts to suggest moderate increases would be defeated by the zero tax rate. We can also note that the stability region for the zero tax equilibrium seems to increase in the cost, and at some point (outsider the graph) it becomes the global equilibrium. This will indeed happen at the point where the cost of public schooling exceeds the return to public schooling.

These findings are interesting in light of the discussion in Fernandez and Rogerson (1995), p. 260, where it is conjectured that rich individuals would support increasing the "height" of the barrier to education. Figure 5 illustrates precisely how the stability region of the zero tax equilibrium increases as the barrier to education goes up. A related interpretation of the above lies in the comparison between rich and poor countries (rich and poor in the sense that the cost of public schooling is a smaller or larger share of the average income). Our results suggest that even in a setting where this kind of redistributive system is majority preferred (globally), it is less likely that the country would end up with this system the poorer is the country. Furthermore, if change is to take place this must be drastic rather than gradual. These conclusions could not be reached in a setting where the focus was on the global equilibrium only. The only thing that could happen in such an environment is that the global equilibrium jumps from one to zero at the point where a majority of the population no longer finds it affordable to educate themselves at any level of taxation.

5.2 Changing the initial distribution

Another obvious question is how the results change with initial income inequality. To illustrate the most important effects, the two panels in Fig. 6 shows a replication of Fig. 5 above, with the difference that the initial distribution to the left has a Gini coefficient of 0.3 (i.e. a more equal distribution) and the panel to the right has a Gini coefficient of 0.5 (that is, a more unequal distribution compared to the figure above). Starting first with comparing the possibility of a zero tax outcome, it seems that the lower the Gini is, the less likely is a situation where a majoriy favors no system at all. The reason for this is that increasing inequality puts more weight on both ends of the distribution thereby increasing the likelyhood of a ends-against-the-middle equilibrium. This effect goes in the opposite direction compared to the standard effect of increasing inequality leading to higher demand for redistribution because in the setting considered here being poor potentially means not having access to the redistributive system at all.

There also seems to be an effect making the low interior equilibrium more likely when the initial distribution is more even. This effect is due to a larger share of the population being part of the middle-class in favour of some redistribution toward public schooling. Together these effects illustrate the general insight that an uneven initial distribution increases the likelyhood of an extreme outcome. Either in the form of a collapse toward complete redistribution as soon as schooling becomes affordable for the poor, or a collapse toward no redistribution at all when schooling is too expensive unless the redistributive system is sufficiently large. When the initial distribution is more even, this gradual move between the extremes involves a range of schooling costs where the middle-class - which in this case is a larger fraction of the

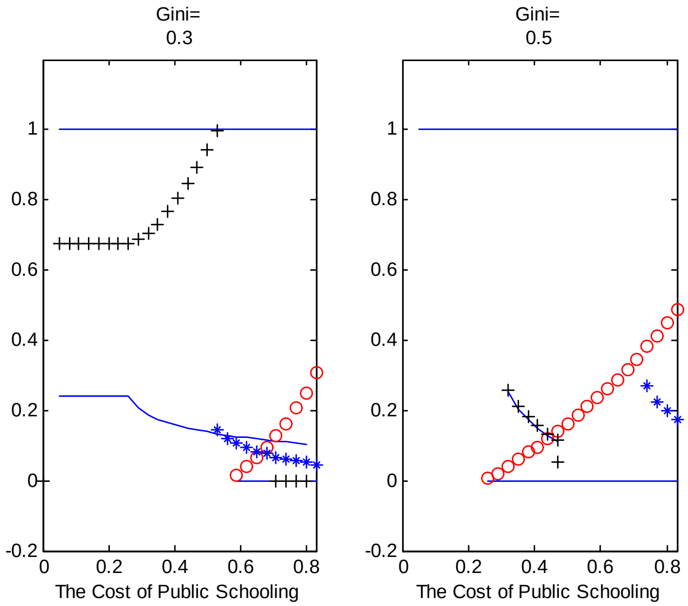

Figure 6: Local Equilibria and Stability Regions for Different Initial Distributions

population - gains the most from limited redistribution, leading to a majority favoring an interior tax rate.

Our results confirm the finding in Fernandez and Rogerson (1995) that an ends-against-the-middle equilibrium, where the poor are excluded, is more likely the higher is the initial income inequality even if we consider a continuous distribution. Furthermore, our way of studying the problem allows us to see how this changes the stability regions of the local equilibria. Such aspects are of course not considered in a setting such as theirs where only the global equilibrium is considered, but nevertheless these results are qualitatively in line with the mechanisms suggested in their paper.

6 Concluding discussion

The model of voting over education subsidies is an example of a reoccuring type of problem in public economics. The common feature is that individuals shift between participating and not participating in some redistributive scheme as the policy changes, which in turn typically lead to difficulties when trying to determine the political support for redistribution. Often - as in the above - the problem involves an investment of some kind which is partially subsidiesed but paid for by a universal tax, and where poor individuals can not (or choose not to) invest if the subsidy is too small. However, similar problems may also arise when studying consumption problems where a good is not consumed at all by poor individuals unless it is sufficiently subsidiezed ("culture goods" are the standard example).[32] Furthermore, even though it is most common that the exclusion happens for the poor and on the side of receiving the subsidy, it is also possible to envision situations where individuals "exit" the system on the side of paying the tax (while still recieving subsidies).[33]

If one insits that there is only one possible majority rule equilibrium in this type of problem (if an equilibrium exists) the method exemplified above can solve the problem in the sense that it finds the global equilibrium tax rate (the Condorcet-winner).[34] However, if one is prepared to consider settings where, for example, there are costs involved in shifting the policy, the stability regions around the local equilibria introduces the possibility to consider outcomes in between local and global equilibria.[35]

Considering first what we have learnt about the specific problem of majority decisions over education subsidies it is illustarative to, once again, compare our results with the insights from Fernandez and Rogerson (1995), who study essentially the same problem as the one above. They reach three main conclusions: First, they show that there can be equilibria where transfers go from lower income groups to higher income groups (that is the case when the poor are effectively excluded from the benefits). Second, they find that this kind of situation becomes more likely the higher is the initial inequality of income, and third, they show that wealthier individuals may gain from higher costs of education as this may enable them to exclude poorer individuals from the redistributive system. Our analysis of the problem come to the same conclusions. However, our way of studying the problem introduces additional insights, which also have implications for this type of problem more generally.

First, we show that when introducing costs of altering the status quo policy in the political game, the possibility of multiple equilibria arises. Depending on the starting point (or on small initial differences) the stable outcome can be very different even if the situation is such that there is only one global equilibrium.

[32] Subsidized "culture consumption" is indeed the lead example in Austen-Smith (2003) which studies majority preferences for subsidies rather than direct income distribution.

[33] Roine (2006a) is an example where fix-cost investments in tax avoidance can cause violations of conditions required for the median voter theorems to hold. In this paper the rich and the poor may in equilibrium favor increased redistribution.

[34] This is in it self important as the alternative has previously been to simplyfy the problem in various ways.

[35] The other obvious way we see to introduce differences between moves close to the status quo and moves far from it is informational aspects. These may of course be linked to costs as overcoming information problems can be seen as possible but costly.

In terms of the example above, two otherwise equal countries (or regions) could based on, for example, differences in the cost of changing the policy end up choosing either a low tax equilibrium with limited subsidies and a large share of the population excluded from education (an outcome which is not a global equilibrium but which may never the less be stable), or a high tax equilibrium with full subsidies and everyone attending school. Similar situations can be envisioned for countries with different initial income distribution, starting at different status quo policies, or with different entry barriers for participating. In all these cases it is worth noting that the way in which the stability regions around local equilibria change correspond to the basic insights above. As inequality increases, or the entry barriers become higher, the stability region around a low tax equilibrium increases making such an outcome more likely. However, we can also see finer aspects of the problem such as the incresed possibility of an interior low tax equilibrium for a more equal initial distributions, and increased likelyhood of extreame outcomes when the initial inequality is greater. These changes happen in a range where a tax rate of one is the global equilibrium and hence the only considered outcome in previous work.

Second, our way of studying the problem also illustrates that policy can not always be shifted gradually. If a redistributive system where exclusion may happen is to be introduced it is possible that it must immediately have some minimum size. If it does not, there can be a majority who preferres the zero tax rate since too many of the poor would oppose the introduction. Similarily shifting from a situation where the system is small in scope to a higer tax-transfer state may also only be possible through a very large shift as all intermediate states would be defeated by the smaller system in a pair-wise competition. It is worth noting that in our example above the stability region of the low tax rate had a range of up to over 50 percentage points which would represent a major shift.

Third, the method gives a "continuous picture" of political support for different policies ranging from local to global equilibrium. The aggregation of the support for marginal (local) change over the policy space gives a map of the direction in which the policy would move starting at any point. The stability regions around each local equilibrium indicates precisely how far policy would have to move for change to be majority preferred, and finally, the last step in the procedure finds the global equilibrium (if it exists). We belive that this, besides solving for the global equilibrium outcome, gives a richer understanding of the possibilities of political change.

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A Appendix

B Proof of continuity of $y^*(\tau), y^{**}(\tau)$ and $\widehat{y}(\tau)$.

As the tax rate changes so does the support for additional changes. Given the assumption that $\delta = 0$, the function which traces the aggregate support for a marginal increase of the tax rate is bounded downwards by y^*, that is, the threshold at which individuals choose public schooling instead of no schooling, and uppwards by y^{**}, the threshold at which individuals instead choose private school.

$$H(\tau) = \begin{cases} 0 & \text{if } \widehat{y} < y^* \\ \int_{y^*}^{\widehat{y}} f(y)dy & \text{if } \widehat{y} \in [y^*, y^{**}] \\ \int_{y^*}^{y^{**}} f(y)dy & \text{if } \widehat{y} \geqslant y^* \end{cases}$$

The applicability of the function which traces the aggregate support for a marginal increase of the tax rate for finding tax rates depends crucially on the continuity of the function. Our claim is that the above integral constitute the fraction of the population which would vote in favor of a suggested marginal increase. As the limits are themselves functions of the tax rate, this is only true if

$$\begin{aligned} \lim_{\epsilon \to 0} y^*(\tau) &= y^*(\tau + \epsilon) \\ \lim_{\epsilon \to 0} \widehat{y}(\tau) &= \widehat{y}(\tau + \epsilon) \\ \lim_{\epsilon \to 0} y^{**}(\tau) &= y^{**}(\tau + \epsilon) \end{aligned}$$

that is if they are continous in τ.Formally these critical incomes are given by

$$\begin{aligned} y^*(\tau) &= \begin{cases} 0 & \text{if } E \leq s \\ \frac{E-s(\tau)}{1-\tau} & \text{if } E > s \end{cases}, \\ \widehat{y}(\tau) &= s'(\tau), \text{ and} \\ y^{**}(\tau) &= \frac{P}{1-\tau}. \end{aligned}$$

First note that y^{**} is obviously continous in τ, $\widehat{y}(\tau)$ is continous if $s(\tau)$ is continously differentiable and since $\lim_{s\to E} \frac{E-s(\tau)}{1-\tau} = 0$, y^* is continuous in τ if s is continuous in τ. The proof hence concernes the properties of s.

Proof. The subsidy is given by[36]

$$s = \begin{cases} \frac{\tau Y}{F\left(\frac{P}{1-\tau}\right)} & \text{if } E \leq s \\ \frac{\tau Y}{F\left(\frac{P}{1-\tau}\right)-F\left(\frac{E-s}{1-\tau}\right)} & \text{if } E > s \end{cases} \tag{11}$$

Note first that s is continuous in τ whenever $E \leq s$ since both τY and $F\left(\frac{P}{1-\tau}\right)$ are continuous in τ. When $s(\tau) = \frac{\tau Y}{F\left(\frac{P}{1-\tau}\right)-F\left(\frac{E-s}{1-\tau}\right)}$ we can use the following implicit function theorem in the following way:

Consider the implicit function

$$\begin{aligned} \Phi(x,\tau) - \tau Y &= 0, \\ \text{where } \Phi(x,\tau) &= x\left[F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-x}{1-\tau}\right)\right] \end{aligned}$$

By the Implicit Function Theorem, if the derivative $\frac{\partial\Phi}{\partial x}$ (with τ fixed) is a bijection there exists a unique continosly differentiable function s, such that $s(\tau) = x$. This solution is also locally unique.

We know that $\frac{\partial\Phi}{\partial x}$ is a bijection if $\frac{\partial\Phi}{\partial x} \neq 0$ and since

$$\frac{\partial\Phi}{\partial x} = F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-x}{1-\tau}\right) + \frac{x}{1-\tau} f\left(\frac{E-x}{1-\tau}\right) > 0$$

[36] Note that if $E \leq s$, everybody is investing in schooling so $y^* = \Phi(y^*) = 0$.

it is indeed a bijection. This means there consists a continously differentiable function $s(\tau)$ which for every τ has a unique solution $s(\tau) = x$ which solves $\Phi(x, \tau) - \tau Y = 0$. In our setting, hence, for every tax rate τ there exists a continously differentiable function $s(\tau)$ which uniquely determines a subsidy that solves the implicit function $s(\tau) = \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s(\tau)}{1-\tau}\right)\right]}$.

....

Finally, note that $\lim\limits_{s \to E} \frac{\tau Y}{F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)} = \frac{\tau Y}{F\left(\frac{P}{1-\tau}\right)}$ so there are no discontinuities at the point $E = s$. Hence, s is continuous in τ, $\forall \tau \in [0, 1)$. It then immediately follows that y^* is continuous in τ, implying that $\lim\limits_{\epsilon \to 0} y^*(\tau) = y^*(\tau + \epsilon)$, which completes the first part of the proof. Consider now $\widehat{y} = \frac{\partial s}{\partial \tau}$. Differentiate s w.r.t. τ to get

$$\frac{\partial s}{\partial \tau} = \begin{cases} \frac{Y}{F\left(\frac{P}{1-\tau}\right)} - \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right)\right]^2} \frac{\partial F\left(\frac{P}{1-\tau}\right)}{\partial \tau} & \text{if } E \leq s \\ \frac{\frac{Y}{F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)} - \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)\right]^2} \left[\frac{\partial F\left(\frac{P}{1-\tau}\right)}{\partial \tau} - \frac{\partial F\left(\frac{E-s}{1-\tau}\right)}{\partial \tau}\right]}{\left[1 - \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)\right]^2} \frac{\partial F\left(\frac{E-s}{1-\tau}\right)}{\partial s}\right]} & \text{if } E > s \end{cases}$$

Both these expressions are continuous in τ. In addition, note that because $\lim\limits_{s \to E} \frac{\partial F\left(\frac{E-s}{1-\tau}\right)}{\partial s} = 0$, and $\lim\limits_{s \to E} \frac{\partial F\left(\frac{E-s}{1-\tau}\right)}{\partial \tau} = 0$, we have that

$$\begin{aligned} & \lim_{s \to E} \frac{\frac{Y}{F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)} - \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)\right]^2} \left[\frac{\partial F\left(\frac{P}{1-\tau}\right)}{\partial \tau} - \frac{\partial F\left(\frac{E-s}{1-\tau}\right)}{\partial \tau}\right]}{\left[1 - \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E-s}{1-\tau}\right)\right]^2} \frac{\partial F\left(\frac{E-s}{1-\tau}\right)}{\partial s}\right]} \\ = & \frac{Y}{F\left(\frac{P}{1-\tau}\right)} - \frac{\tau Y}{\left[F\left(\frac{P}{1-\tau}\right)\right]^2} \frac{\partial F\left(\frac{P}{1-\tau}\right)}{\partial \tau} \end{aligned}$$

again implying that there are no discontinuities at $E = s$, that $\widehat{y}$ is continuous in τ and consequently that $\lim\limits_{\epsilon \to 0} \widehat{y}(\tau) = \widehat{y}(\tau + \epsilon)$. ■

B.1 Proof of Proposition XX

Proof. The proof concerns the case $E > s$, since existence is obvious whenever $E \leq s$. Since $P > E$, and $\tau < 1$, s belongs to the interval $s \in \left[0, \frac{\tau Y}{F\left(\frac{P}{1-\tau}\right) - F\left(\frac{E}{1-\tau}\right)}\right]$ and is thus bounded. From continuity and the intermediate value theorem, s then has a fixed point in this interval. ■

C The Benchmark Calibration

To calibrate μ in (10),' we note that the fixed cost of investing in public schooling E may be decomposed into direct and indirect costs associated with schooling. Direct costs includes tuition and non-tuition spending, such as other school fees, textbooks, supplementary study guides, uniforms, writing supplies, transportation etc. Indirect costs on the other hand, include the value of lost labor income, as well as the economic value of all the unpaid work related to schooling, that parents and community members may carry out.[37]

The direct costs for schooling may be very large. Bray (1999) report for instance that household expenditure on primary (public) education per child in Cambodia is up to 20 percent of the household income. In addition to these direct costs, there are other large costs associated with schooling that are met by community financing and government subsidies. Bray argues that the situation is similar for a number of other developing countries. Taking this as a benchmark of what the direct cost of schooling would be to the household in the absence of donations and subsidies, we set the direct cost of schooling to be 25 percent of average income.

To this direct cost, the indirect cost of schooling, i.e., the economic value of foregone opportunities of schooling must be added. There do not really seem to exist any available studies quantifying the total indirect cost of schooling. Bray (1999) report the value of lost labor income of attaining primary education in Cambodia to be of almost the same magnitude as the direct cost. In a broad sense, we are considering both primary and secondary education and the value of labor generally gets larger as agents get older. Moreover, to the value of lost labor income, the value of all unpaid work related to schooling carried out by parents and community members should be added. Taking this into account and lacking other estimates, we set the indirect cost to be as large as the direct cost, generating a total fixed cost of investing in public schooling of 50 percent of average income, i.e., $\mu = 0.5$.

Now consider the parameter φ. Unfortunately, there are not many available studies on relative costs in private and public schools. One exception is Tsang (2002), who compares the costs of public and private

[37] For example the time parents spend on helping their kids with homework and transportation to school related activities. Parents and community members may also be asked to provide labor and/or materials for construction and maintenance of the school.

schools in developing countries. He report that the direct private cost, i.e., the cost that households have to pay up front to be allowed to enrol the school, is between 1.83 and 8.02, times higher for private than for public schools. Lacking other estimates, we set $\varphi = 2.6$.

Finally, we need to specify the returns to schooling. We have assumed that the return to private schooling is higher than the return to public schooling, i.e., that $h(y_i) > g(y_i)$, $\forall y_i$. In our calibration, we set the returns to $E + \alpha y_i$ and $P + \beta y_i$ for public and private schooling respectively, with $\beta > \alpha > 1$. Note that the exact functional forms of these functions are not important for the political outcomes locally, since neither y^*, y^{**} or $\widehat{y}$ depend upon these functions. However, they do enter the utility functions and will therefore play a role in the comparisons between alternatives where individuals choose different (or no) schooling between the respective policies being compared.

The empirical evidence supporting the assumption that agents from private schools would perform better than agents from public schools is somewhat weak, but the assumption at least seem to have some empirical support (Rouse, 1998 and Long, 2004).[38]

D Numerical Computation of the Equilibrium

1. Set up a grid for policy $[\tau_1, ..., \tau_J]$.
2. At each grid point solve for y_j^*, the subsidy $s_j = \tau_j Y/N_j$ and the share of the population that participates $N_j = \int_{y_j^*}^{\infty} f(y)\,dy$; $j = 1, ..., J$
3. Approximate the functions $y^*(\tau), s(\tau)$ and $N(\tau)$. We use cubic splines
4. At each grid point, compute $\widehat{y}_j = ds(\tau_j)\,d\tau$, and $H_j = \int_{y_j^*}^{\widehat{y}_j} f(y)\,dy$; $j = 1, ..., J$
5. Approximate the functions $\widehat{y}(\tau)$ and $H(\tau)$. Again, we use cubic splines
6. Find all equilibria, i.e., find $H(\tau) = 0.5$ and cuts the 0.5 line from above. Also check corners.
7. Compare these candidates to a dense grid of all other tax rates and report the result.

[38] Generally there is a substantial selection problem involved when trying to estimate the relative return to private and public schooling.